HUMAN CAPITAL AND SELF-ENFORCING CONTRACTS*

by

Costas Azariadis**
University of Pennsylvania

Number 281

WARWICK ECONOMIC RESEARCH PAPERS
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May 1987

* Presented at the Warwick Summer Research Workshop on Microeconomic Explanation of Macroeconomic Phenomena.

** Preliminary Draft. Research support from the National Science Foundation is acknowledged with thanks, and so are comments from the Penn Macro Lunch Group and Asher Wolinsky. The usual caveats apply.

This paper is circulated for discussion purposes only and its contents should be considered preliminary.


ABSTRACT

This essay analyzes labor contracts as a device for rearranging factor incomes over time when the lack of verifiable public information about future compensation prevents finitely-lived workers from borrowing against their earnings. Specific human capital is used as an incentive to implement intertemporal self-enforcing contracts between workers and firms. I propose a necessary and sufficient condition for the existence of such contracts, explore the resulting equilibrium earnings profiles, and investigate how imperfections in the credit market influence the way workers allocate time between current production and training.

J. Econ. Literature classification numbers: 022, 821.

Address all correspondence to the author at:

The Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297
USA
1. INTRODUCTION

This essay explores implicit labor contracts as devices for redistributing factor rewards over time, especially when workers face some form of credit rationing because financial intermediaries cannot verify individual compensation. Motivated partly by earlier work of Becker (1962, 1964), Mincer (1962), and Ben-Porath (1967) on human capital, Williamson (1979) on transactions costs, as well as more recent research by Holmstrom (1983), Ioannides and Pissarides (1983), and Topel and Welch (1984) on intertemporal contracts, this exploration is the dynamical analog of the more familiar atemporal approach which views contracts as mechanisms for rearranging factor incomes over states of nature. In either case contracting is a response to some form of asymmetric information: the acquisition of reliable information about one or more aspects of the exchange relation between an employer and an employee (say, final product demand, labor effort or earnings, employment status of workers) turns out to be too expensive for third parties; consequently, the capital market cannot be relied on to smooth the consumption stream of workers. Hence, the role of financial intermediary—insurance broker or creditor—devolves naturally upon firms which possess a good deal of private information, not ordinarily available to third parties, about persons in their employ.

Cutting off all employees from any direct access to unsecured credit is, of course, a crude assumption meant as a first approximation to imperfections in actual capital markets. It accords with the difficulties one will experience in obtaining lines of credit large enough to help in lifecycle

1. For excellent surveys of that literature see Rosen (1985), Cooper (1986) and Hart and Holmstrom (1986). Among the primary references, an unpublished paper by Timmis and Bernhardt (1985) comes close to the approach followed here.
REFERENCES


(1964), Human Capital. New York: Columbia U. P.


consumption smoothing or, even, in tiding an unemployed person over a severe recession. Credit rationing, in fact, is consistent with recent evidence in empirical macroeconomics [Flavin (1981), Bernanke (1985), Hayashi (1985), Zeldes (1985)] that both aggregate and individual consumption shows "excessive" sensitivity to current income relative to what a stochastic lifecycle theory like Hall's (1978) would predict for a smoothly functioning credit market.

Contracts over dated labor services are analytically similar to atemporal or time-invariant ones over state-contingent labor services: compensation is not tied to the marginal-revenue product of labor and may include implicit net insurance indemnities or net loans disbursed by an employer to an employee. Intertemporal aspects of exchange, however, become paramount when contractants make capital-like decisions that will involve them in sequential trades. Investment in human capital is a leading example of such a sequence of trades, for it requires an agreement as to how a worker and a firm will share over time the cost of and returns from acquiring skills.

Designing intertemporal contracts has been an open issue since Gary Becker's seminal early work on the accumulation of human capital. To advance this issue one needs to describe with some rigor the exchange and the implementation of contracts. How do equilibrium contracts emerge in a dynamic economy? What mechanism enforces them on contractants?

With the exception of the papers mentioned at the very beginning of this essay, these questions remain unanswered, which suggests that the questions themselves are not particularly tractable. By their very nature, intertemporal contracts must specify—not always in an explicit or verifiable manner—the entire process of human capital formation: technology, inputs and outputs. That includes how much training a worker is to receive, at whose
cost, and how the resulting factor rewards are to be distributed in the future.

Among the by-products of such a complete description would be age-earnings profiles, the rate of return on training, the allocation of time between direct production and the acquisition of skills, and other theoretical results of interest to labor economists. At a more fundamental level, however, intertemporal contracts describe the interaction between the workers' lifecycle saving plans and their labor supply decisions, and thus provide a natural setting for studying potential spillovers from the credit market to the labor market when individual factor incomes and net trades are private information. For instance, how does investment in training respond to changes in the rate of interest and, more generally, in the worker's ability to borrow against future earnings? For it is quite possible that some credit-rationed workers will make their intertemporal plans in response to the shadow rate of interest which they face, not on the basis of the (generally lower) market interest rate. The end result may be too little investment in training, or a mismatch of a worker with a job for which she is overqualified; these misallocations will vanish only if employers are able to intermediate freely in the credit market, borrowing on behalf of their employees at the going rate

2. Earnings rise with age, and more training is associated with more steeply rising age-earnings profiles; see Becker (1964).

3. Both Becker (1964) and Mincer (1962) find that the rate of return on human capital exceeds considerably the yield on physical or financial capital. For example, white males graduating from high school in 1939 and 1949 realized a nominal rate of return to four or more years of college education in the neighborhood of 12% p.a. after tax. This rate was a cohort average corrected for ability differences between college and high-school graduates. Rates of return to on-the-job-training seem to have been 9-11% during the same period while the yield on completed high-school education varied between 15 and 20%. Note for contrast that the nominal after-tax return on U.S. manufacturing capital was about 7% in the forties.
Whatever the division of human capital, the earnings profiles associated with dynamical self-enforcing contracts are less steep than the corresponding productivity profiles because of the intermediary activity of employers. Simple as it sounds, this property deserves some comment for it runs counter to a fairly large literature on the earnings profiles of individuals who do not accumulate skills, as well as against many economists' perception of what the facts are.

We know from Holmstrom (1983) and Ioannides and Pissarides (1983) how to construct equilibrium contracts in which earnings rise faster than productivity: wages at first lie below the marginal product of labor and later grow above it in order to deter workers from breach of contract. This profile ceases to be an equilibrium if firms, too, can breach; Holmstrom, Ioannides and Pissarides assume that employers are honest because they are more likely than employees to suffer from the adverse effect of a poor reputation.

In Harris and Holmstrom (1982), wages rise faster over time because employers become surer (form more precise estimates) of the true ability of an individual worker; true ability does not improve over time but its certainty-equivalent estimate does. Lazear (1981), on the other hand, attributes faster growth of wages to firms attempting to discourage workers from shirking on the job when effort per unit time varies and cannot be observed in public. Shirking is detected randomly, results in immediate dismissal, and denies workers the right to an increasing income stream.

17. I assume that labor productivity rises over time or, formally, that \( e_2 + k(\tau) > e_1 - \tau \).

18. One exception is Oswald (1981) in whose model workers accumulate only specific human capital. Here earnings exceed marginal product during training because firms need to attract trainees away from jobs involving less training and, therefore, higher marginal product.
Lazar's idea suggests a natural extension of this essay. When effort or labor supply are privately observed variables, the self-enforcing contracts developed here must be redesigned to coax out of contractants a credible commitment not only about compensation and training, but also on hours worked or intensity of effort.
exactly as much as the employees themselves desire.

From the interaction of the credit and labor markets, it is only a step, albeit a somewhat long one, to study the general equilibrium aspects of idiosyncratic exchange in labor markets. The determination of interest rates, the dynamic efficiency of competitive equilibrium (including the possibility of overaccumulating physical capital and underaccumulating human capital), and the effects of various monetary and fiscal policies are among the main concerns in this area.

This paper emphasizes the microeconomic structure of intertemporal labor contracts, leaving aside their possible implications for macroeconomics and growth theory. There is no government in what follows, interest rates are an exogenous sequence, and the state of information (who knows what about whom) is also an unchanging datum. The key component of the structures which I study here is specific human capital and its role in supporting self-enforcing contracts of finite duration between employers and employees.

In technical language, a contract is self-enforcing if it is individually rational for all contractants both ex ante (before it is agreed to) and ex post (at all dates after the agreement). Contractants trade with each other according to prespecified rules not because they are legally bound to do so (as they might under an explicit contract) but, rather, because following these rules is always in each contractant's own self-interest. Breach of contract, which we define here as a unilateral termination of trading with a fellow-contractant, results in the deadweight loss of all returns to specific human capital, a type of transaction cost that contractants must absorb when they walk away from each other.

4. A start in this direction has been made by Grossman, Hart and Maskin (1983), Farmer (1984), Canzoneri and Siebert (1986) and other writers whose primary focus is unemployment.
Injuring one's reputation is another deterrent against breach of contract which has been studied extensively, both in the general context of repeated games [Radner (1981), Kreps and Wilson (1982)] and in the specific case of labor services [Carmichael (1984), Bull (1987)]. Reputational considerations are an important safeguard against a type of breach that we do not analyze here, namely a unilateral offer to continue a contractual relationship at terms different from those which the contractants originally agreed to. However, reputational enforcement mechanisms require continued exchange to be valued by all contracting parties, and work best for either infinite planning horizons or for finite horizons with learning.

When the planning horizon is finite, it appears more natural (or, at least, simpler) to focus on deadweight losses like specific human capital and related transaction costs. This approach has met some success in industrial organization, following the work of Williamson (1979) who defines a contractual relation by the specific assets that are sunk into it; and in studies of loan contracts like Gale and Hellwig (1985) who rely heavily on deadweight losses from bankruptcy. Oddly enough, specific human capital has not attracted similar attention in the theory of labor contracts. This essay is partly meant as an attempt to correct that oversight.

5. Footnote 11 and section 6 broach the possibility of post-contract bargaining.

6. Otherwise contracts tend to unravel, as one can see from the following simple example. Consider a relationship between a finitely-lived borrower and a lender in which the only penalty for bankruptcy is exclusion from all subsequent trading in the credit market. The privilege of participating in the credit market has no value in the last period of the borrower's life (which is assumed here to be public knowledge). Therefore the borrower will choose to default in the penultimate period. Potential lenders know this and will refuse to extend that borrower any credit two periods before her death, which reduces to zero the value of participating in the credit market, and so on to the beginning of the planning horizon.
net trades or other actions correlated with compensation is inaccessible to financial intermediaries, employees will be unable to borrow from lenders because their loan collateral consists of unverifiable job skills.

Employers, whose net trade positions and profits we assume to be in the public domain, do know both the human capital and the compensation of long-time employees. That knowledge makes them natural candidates for intermediating between their labor force and the credit market. Financial intermediation in this indirect sense works by allowing employee compensation to differ from contribution to output: workers are typically paid more than their net marginal-revenue product while they train, and less than marginal product afterward. One may interpret these wedges between compensation and marginal product as a loan from the employer followed by repayment of principal with interest.

The financial and labor components of an employer-employee relation are entwined in a labor contract, whose execution is observed only by the contractants themselves and cannot be verified by outsiders. Equilibrium labor contracts must therefore be self-enforcing: both contractants must prefer continuation to breach at each stage of their relationship.

Self-enforcement puts certain constraints on the compensation profiles of individuals and, implicitly, on how much firms can lend their employees. A worker who is paid far above her product during training, and considerably below thereafter, has a strong incentive to leave at the end of her training period, especially if a large fraction of the skills she accumulated were general, i.e., transferable from one job to another.

Preferences over intertemporal consumption bundles as well as the composition of human capital become potentially important considerations in contract design under private information. Workers who would find it
profitable to invest a relatively large fraction of their youthful time and resources in training, might find such plans frustrated under private information by a shortage of unsecured direct credit. Heavy investments in training will typically mean large indirect loans from employers, especially for workers with a high rate of time preference in consumption; repaying such loans will not be in the interest of any worker who can breach her contract with impunity, unless she is deterred by the loss of specific skills.

As a result, workers with a high rate of time preference, small first-period resources, high ability and small specific component of total human capital are the most likely to suffer the consequences of private information. These workers will be both credit-rationed and undertrained at a private information equilibrium. Each of them faces a personal (gross) "shadow" rate of interest that equals her marginal rate of substitution between current and future consumption; equals the yield on her human capital; and clearly exceeds the market interest rate.

At the other extreme, workers whose traits include time patience, large first-period resources, low ability and a large component of firm-specific skills are more likely to be immune to rationing: given the rate of interest, their consumption plans and training will be exactly the same as under public information. All yields and marginal rates of substitution will equal the (gross) market rate of interest.

In either case, private information has a visible shrinking effect on the consumption possibilities of the individual worker. The shrinkage is minimal when all human capital is specific, largest and most likely to hurt when all skills are general. Figure 5 demonstrates that loss to equal the triangle area (YXY') in the specific-capital case, and (YXY') plus (YFE) in the general-capital case.
We begin the formal development of intertemporal contracts in section 2 with the description of a dynamical economy whose equilibria we study first in section 3 under the assumption that all relevant information is public and verifiable. Here no individual is credit-rationed, and contracts are legally binding both in the credit and the labor market. Section 4 introduces a specific form of private information in the labor market which restricts to the contractants themselves knowledge of the actual quantities exchanged under an intertemporal contract. Any investments in training, productivity and earnings are observed by the relevant worker-employer pair and may not be verified at finite cost by any third party. Binding agreements are no longer viable, and the execution of contracts is left to the self-interest of the contractants. I develop a necessary and sufficient condition for self-enforcing contracts to exist, and explore how existence depends on the rate of interest and the "ratio" of specific to total human skills.

Section 5 studies equilibrium in the labor market when all employers have free entry in the business of training workers, and identifies the circumstances under which employees will be credit-rationed. Section 6 looks at extensions of the basic results to heterogeneous employers or jobs, to more comprehensive definitions of breach and a number of other issues. Section 7 sums up the main results and compares them with the existing literature.

2. HUMAN CAPITAL IN A DYNAMIC ECONOMY

In what follows we study an economy of overlapping generations whose population remains constant. Tastes, endowments and technology are completely stationary so that all generations, with the exception of the very first, are exact replicas of each other. Time periods are indexed $t = 1, 2, \ldots$, ad inf.; one generation is born at the beginning of each period and is indexed by its
The economic lifespan of all households extends to two full periods, except for those of generation "zero" who are born at \( t = 1 \) and live one period only. There is one perishable consumption good which can be produced under constant returns to scale from a single input, "efficiency labor services." As a first approximation we neglect physical capital in order to direct our full attention to human capital.

Each generation \( t = 1, 2, \ldots \), contains \( I \) households indexed \( i = 1, \ldots, I \). These come in two "classes": "employees" indexed \( i = 1, \ldots, H \), and "employers" indexed \( i = H+1, \ldots, I \). The two classes are mutually exclusive, the distinction between them being permanent in our economy.\(^7\)

Both \( I \) and \( H \) are large numbers, and \( H \) is large relative to \( I - H \).

Let \( \mathbf{c}_i^t = (c_{1i}^t, c_{2i}^t) \geq 0 \) be the consumption vector of household \( i \) in generation \( t \), \( \mathbf{j}_i = (j_{1i}, j_{2i}) \geq 0 \) be the corresponding endowment vector of the consumption good, and \( u_i^f : \mathbb{R}_+^2 \to \mathbb{R} \) be the (direct) utility function over first-period and second-period consumption. We assume that all \( u_i^f \) satisfy standard assumptions on monotonicity, differentiability and concavity, and that \( \omega_i^f = 0 \) for all workers. Note that \( u_i^f \) and \( \omega_i^f \) bear no time label because endowments and preferences are stationary. We may also define for each household an indirect utility function

\[
(1) \quad v_i^f(y_1, y_2, R) = \max_{-y_2/R \leq s \leq y_1} u_i^f(y_1 - s, y_2 + Rs)
\]

where \( s \) is saving, \( R \) is one plus the interest rate on loans and \((y_1, y_2) \geq 0 \) is a vector of total (i.e., endowment plus factor) income. One shows readily that the function \( v_i^f \) is strictly increasing in \((y_1, y_2)\) for

\(^7\) See Kihlstrom and Laffont (1983) for a model that allows individuals to choose their role in the production process.
required to add to the public information equilibrium of section (3) is a
definition of saving behavior for workers and entrepreneurs, together with a
history of government deficits and how they are financed. In a simple economy
of zero government deficits and no public indebtedness (interest-bearing or
otherwise) to households, general equilibrium consists of a cleared labor
market plus zero aggregate saving by young workers and entrepreneurs alike.
If there is some public debt, then equilibrium requires that the stock of debt
equal aggregate saving by households.

The resulting dynamical equilibria are completely analogous to those
described by Diamond (1965); one may investigate stability, dynamic efficiency
and related issues in much the same way as he did. High interest rates, for
instance, will be associated with an efficient allocation of time between
training and production even though they imply a small stock of human
skills. 16

What seems a more original endeavor is to study the accumulation of human
skills under conditions of private information, like those of sections 4 and
5, and find out how the accumulation process unfolds when contract breach and
credit rationing are possible. High rates of interest will no longer be an
automatic signal of dynamic efficiency unless it is also the case that the
rate of interest equals the yield on human capital for every worker; then no
worker will be credit-rationed or undertrained. What is an appropriate
definition of constrained dynamic efficiency in these circumstances? Are
there policies that help decentralize (or achieve) such constrained optima as
competitive equilibria? These extensions of the essay at hand are far beyond

16. The reason is that building up a higher stock of skills requires a higher
investment in training, less productive work by every generation of young
workers and, hence, less consumption by a transitional generation of
older workers.
its introductory intent.

7. CONCLUSIONS

When the endowments and net trades of both employers and employees are verifiable public information, the accumulation of human skills proceeds to the point where the returns on human and non-human capital are equalized. How much is invested in on-the-job training depends on the technology of training, i.e., on the ability of the worker herself and the nature of the job. Investment depends neither on the division of skills into specific and general components nor on preferences over intertemporal consumption bundles.

This independence holds because abundant public information enables individuals to write binding loan and labor contracts. The credit market operates perfectly in this setting, and training programs are designed to maximize present value independently of tastes. Each worker's intertemporal consumption possibilities are captured in an ordinary budget set, and are as large as her ability permits.

In one sense at least, this essay is the human-capital counterpart of much earlier work on the joint determination of consumption and labor supply. Topel and Welch (1984), for instance, investigated how decisions on labor supply, compensation and consumption interact under technological uncertainty; even earlier, Seater (1977) and Ioannides (1981) studied the impact of financial intermediation on job choice, especially on job search activity.

The process of acquiring and exploiting job skills, however, requires employers and employees to form long-term attachments in the course of which they come to possess information about each other that outsiders cannot collect easily. If, in particular, information about individual compensation,
every non-satiated consumer.

Entrepreneurs are endowed with a constant-returns production technology \( y_t = \frac{L_t}{L_t} \) which permits them to extract up to one unit of the consumption good from each unit of efficiency labor services. Each worker comes with a vector \( e^i = (e_{11}, e_{21}) > 0 \) of leisure time which she supplies inelastically to the labor market. Any worker may augment the factor input value of her time on the job by training, i.e., by devoting some of her first-period labor endowment to the acquisition of skills that enhance her second-period productivity. As a result, the worker's supply of efficiency labor units falls short of \( e_{11} \) in the first period, exceeds \( e_{21} \) in the second.

The training possibilities available to worker \( i \) and any employer are captured by a pair of continuous, decreasing, concave functions \( f^G_i : [0, e_{11}] \rightarrow \mathbb{R}_+ \) and \( f^S_i : [0, e_{11}] \rightarrow \mathbb{R}_+ \) which describe, respectively, the general training and specific training choices open to the worker-firm pair. In particular, suppose that the worker supplies \( L_1 \in [0, e_{11}] \) units of efficiency labor to the production process during her first period of life, and invests \( e_{11} - L_1 \) units in the acquisition of skills; then her endowment of efficiency labor units in the second period is \( f^G_i(L_1) + f^S_i(L_1) > e_{21} \) if she remains with the same firm, and \( f^G_i(L_1) > e_{21} \) if she switches employers. The sum \( f_i = f^G_i + f^S_i \) defines the input possibility frontier (IPF) for worker \( i \); we draw it in Figure 1.

An input possibility frontier bounds the input possibilities set (IPS), i.e., the set of all lifetime labor input combinations, and therefore of net contributions to output, which are technically feasible for worker \( i \). IPF's relate the returns and costs of human investments; among the costs that Figure 1 illustrates, one may include foregone earnings (leisure) as well as some direct outlays, but not other inputs of the type considered in Ben-Porath
Figure 1: The Input Possibility Frontier
other than \( j \) can afford to offer worker \( i \) a profitable contract that she prefers to \( \delta_{ij} \).

Potential job mismatching is one undesirable consequence of private information here: some employees will not train for the jobs for which they are best suited (in the sense of maximizing \( x^*_{ij} \) over \( j \)) because no self-enforcing contracts may exist for these jobs. One example is a highly productive potential match that involves little specific training; another, perhaps more interesting, example is a job requiring a large investment in training and, hence, an initial wage payment so far in excess of net productivity as to court subsequent breach of contract by the worker.

Breach of contract, a key consideration in this essay, describes a number of possible events. The one on which we have focused so far is the abandonment of a long-term trading relation in favor of a shorter-term one. What about breach not so much as a desire to change trading partners but merely to alter in one's favor the terms of trade specified in advance? This situation is an apt description of a long-term contract between a worker and a firm who have sunk certain resources into training and, consequently, possess some monopoly power over each other in the second period of the contract. The two contractants may feel quite free to reject whatever terms they had originally agreed to and bargain anew over how to divide the "surplus" from the relationship, i.e., the returns to specific human capital.

A similar bargaining problem has been analyzed by Rubinstein (1982) who studies the division of a given surplus by two individuals under a variety of assumptions about rates of time preference, the sequencing of actions and bargaining costs. If we allow bargaining costs and the time between offers to go to zero and if, in addition, the two sides are equally patient, then the unique subgame-perfect equilibrium in Rubinstein's game converges to the
solution originally suggested by Nash: the surplus is divided equally between the two sides. In our context, this means that the double inequality (7) which guarantees breachproofness must be replaced by the equality

\[ y_2 = e_2 + \frac{1}{2} (k(\tau) + g(\tau)) \]

given the value \( \tau \) chosen in the first period.\(^{15}\) The earnings profile associated with any breachproof zero profit contract is then uniquely described by \((e_1 - \tau + (k(\tau) - g(\tau))/2R, e_2 + [k(\tau) + g(\tau)]/2)\); one can easily amend the results we derived in sections 4 and 5 to accommodate post-investment bargaining.

To explore human capital accumulation in a general equilibrium setting one must first complement the apparatus developed in this essay with a description of how the rate of interest itself is set in a competitive credit market. Having thus "closed" the model, we may proceed to the standard issues of existence, stability and efficiency of dynamical equilibria, much as one does in ordinary capital theory. This task has been attempted by Uzawa (1965), Razin (1972), Drazen (1978) and other authors who have extended the neoclassical one-sector, one-household growth model to study the joint dynamics of physical and human capital.

It is not a difficult undertaking to replicate this effort for an overlapping generations model of public information, many households, and possibly many goods as well, in an economy whose inhabitants must decide how to allocate their fixed supply of time between training and work. All one is

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15. Another solution to the post-investment bargaining game divides total human capital, \( k \), between the contractants, with general capital being an "outside option" for the worker. In this case the worker's second-period income would not follow equation (21) but rather would equal \( e_2 + \max(g, k/2) \). See Binmore (1986) or Sutton (1986) on this point.
(1967), e.g., physical capital or one's own commodity holdings. The concavity of the IPF reflects the effect of considerations not explicitly articulated in this essay, e.g., the increasing marginal disutility of effort by trainees, cooperating inputs, etc.

The functions \( f_i^g, f_i^s \) are not identical for all workers because of differences in ability and other individual characteristics; they are, however, assumed to be independent of both time and of the firm at (or job for) which training takes place. These are not innocuous assumptions, and we shall discuss them again in section 6.

The time-independence of input possibility frontiers, for instance, means that the skills accumulated by one generation depreciate completely and do not pass on to the next one. Ruling out such external effects suppresses increasing returns to human capital, which were studied by Romer (1986) in the context of a neoclassical growth model.

An employer-independent IPF, on the other hand, implies that job matching is irrelevant and that employees have a decided advantage in wage bargaining. For example, the physical productivity of a trained worker depends neither on the nature of her job (e.g., location or industry) nor on the individual characteristics of the employer, just on worker ability and the length of the labor contract. Before training commences, therefore, firms are perfect substitutes for one another while potential trainees, who are indexed by ability, face different training "costs." That, in turn, means that employees will extract from employers all net returns to training.

To describe costs and returns from training, we denote by \( T_{ii} = e_{ii} - L_{1i} \) worker i's investment in training, by \( k_i^1 = L_{2i} - e_{2i} \) the accumulated stock of human capital, and by \( k_i^g = L_{2i}^g - e_{2i} \) the stock of general human capital. In the sequel, we frequently use accumulation functions for worker
(2) \[ k_i(\tau) = f_i(e_{1i} - \tau) - e_{2i}, \quad g_i(\tau) = f_i^g(e_{1i} - \tau) - e_{2i} \]

Here the function \( g_i \) refers to general capital and \( f_i \) to all human capital. Each of these functions is continuous, increasing, concave, and its graph contains the origin; both are drawn in Figure 1(b).

For future use we also define

(3) \[ \tau^*_i(R) = \arg \max_{0 < \tau \leq e_{1i}} [k_i(\tau) - R\tau] \]
6. EXTENSIONS

This section outlines how the basic results derived in previous sections stand up as we consider more general technologies of training, permit richer definitions of breach of contract, examine standard equilibrium issues like existence and efficiency of dynamical equilibrium, or grapple with the effects of various fiscal policies on the accumulation of skills.

There are at least two directions along which one may fruitfully generalize the input possibility frontiers, described in this essay. A direct, and relatively straightforward, extension will permit investment in training to exceed, in principle, the worker's first period time endowment; the IPF may well extend outside the non-negative orthant, reflecting direct expenditure of material resources by the employer and a strictly negative net contribution to output by a trainee. With public information and a perfect
credit market, negative net output by a young worker poses no serious problems; the employer simply extends to the trainee a large enough loan, which ensures that workers will be able to afford the best consumption bundle to which their ability and wealth entitles them. However, when factor incomes are private information, large loans to employees become an inducement to breach; individuals who desire such loans are likely to be victims of credit-rationing and of undertraining as well.

It is slightly more complicated to index the training technology on both worker and employer. Rosen (1972) has already done so in a theoretical study of how workers accumulate general skills under constant returns to training and a perfect credit market. The aim here is to permit the return to training to reflect the ability of both trainer and trainee, and perhaps the nature of the job itself. Suppose, for instance, that \( k_{ij}(t) \) denotes total skills accumulated by worker \( i \) in training at firm \( j \) (or job \( j \)), where the investment expenditure \( t \) is in some well-specified input possibility frontier \( T_{ij} \). The profit from training, \( \pi_{ij} = k_{ij}(t) - Rt \), attains a maximal value

\[
\pi^*_{ij}(R) = \max_{t \in T_{ij}} \left[ k_{ij}(t) - Rt \right]
\]

When information is public, equilibrium matches between employees and employers are easy to define: given the interest rate, worker \( i \) will work for firm \( j \) if \( \pi^*_{ij}(R) \geq \pi^*_{ik}(R) \) for all employers \( k \neq j \). The equilibrium contract \( \delta_{ij} = (y_{ij,1}, y_{ij,2}, t_{ij}) \) between \( i \) and \( j \) will set \( t \in T_{ij} \) to maximize \( \pi^*_{ij} \), but will no longer yield zero profit for the firm. In fact, the worker's earnings profile will be arranged to yield zero profit for that firm \( k \) whose \( \pi^*_{ik}(R) \) is second-largest among all employers; then no firm
to be that amount of training which maximizes the joint surplus of the employer-employee pair from the process of accumulating skills. If the IPF is differentiable and satisfies the boundary conditions

(4) \[ k'_1(e_1) < R < k'_1(0) \]

on the derivatives of \( k_1 \), then at \( \tau^*_1 \) we have equality between the (gross) rate of interest, \( R \), and the (gross) marginal yield on human capital, \( k_1(\tau) \).

3. TRAINING WITH PERFECT INFORMATION

We work out first the implications of the supposition that the commodity endowment, factor income and net trades of any person are verifiable public information. This rules out credit rationing, for it means that all loan and labor contracts are legally enforceable.

An intertemporal labor contract \( \delta = (y_1, y_2, \tau) \) between worker 1 and an entrepreneur specifies an investment in training, \( \tau \), a real earnings profile \( (y_1, y_2) \) over the worker's lifetime, and entitles the firm to the proceeds from the sale of that worker's contribution to output. Because no household is credit rationed, contracts are evaluated by the present value of the factor incomes paid out. Given the gross interest rate \( R > 0 \), for instance, contract \( \delta = (y_1, y_2, \tau) \) has present value \( p(\delta) \) for household 1 and \( \pi^1(\delta) \) for any firm, where

(5a) \[ p(\delta) = y_1 + y_2/R \]

(5b) \[ \pi^1(\delta) + p(\delta) = e_{11} - \tau + [e_{21} + k_1(\tau)]/R \]
Recall that the technology of production we are using here equates output with efficiency labor input. We are thus led directly to the following definition of labor market equilibrium.

**Definition 1:** Given the interest rate, a contract \( \delta^i \) is equilibrium for worker \( i \) and some firm if there is no other contract \( \delta \) such that \( p(\delta) \geq p(\delta^i) \) and \( \pi^i(\delta) \geq 0 \), with at least one inequality strict.

The contract \( \delta^i \) is an equilibrium one, then, if every contract that worker \( i \) prefers to \( \delta^i \) earns a negative profit for each firm. Because firms have free entry into, and face equal costs at, the business of training any one worker, their profit from the combined process of training, producing and selling will be zero in equilibrium. Equilibrium contracts are in effect chosen to maximize the present value of labor income subject to a non-negative profit constraint. As a result, the equilibrium investment in training given by equation (3) maximizes the combined present value \( p + \pi^i \) on the LHS of equation (5b). We have thus proved

**Theorem 1:** Given the gross interest rate \( R \), any equilibrium contract for worker \( i \) satisfies \( \tau^*_1 = \tau^*_i(R) \) and \( \pi^i(\delta) = 0 \).

Equation (5b) says that there are infinitely many (strictly speaking, a continuum of) equilibrium contracts for worker \( i \), all of them featuring the same investment in training, \( \tau^*_i \), and the same present value of factor income, \( y_{1i} + y_{2i}/R = e_{1i} - \tau^*_i + [e_{2i} + k_i(\tau^*_i)]/R \). Equilibrium contracts differ only in the timing of the earnings profile, a detail of no possible
preferred bundle on the CPF will be a point somewhat between F and A in Figure 5.

Inequality (19a) will also hold for a person whose maximal element over the CPF lies somewhere between points F and F'. Think of someone who wishes to borrow a relatively modest amount and whose training produces a relatively large amount of job-specific skills. Because of the large potential deadweight loss from breach, employers can lend substantial sums to a worker of this type.

As a formal check on this intuitive exposition, we find easily that the LHS of inequality (19a) is a decreasing function of the interest rate, provided that consumption in each period is a normal good. A given worker is therefore more likely to face credit rationing at a low rate of interest, for that is when she is more likely to desire to borrow large sums.

When all human capital is specific, the earnings possibility frontier coincides with point Y in Figure 5, and the corresponding CPF is the segment (AY) of the budget line. As predicted by the corollary to Theorem 2, an equilibrium self-enforcing contract exists here whenever \( \tau_i^*(R) > 0 \). Whether or not worker \( i \) is rationed depends again on the validity of inequality (19a). Here the endowment profile \( x_i(\tau_i) = (e_{1i} - \tau_i^* + k_i(\tau_i^*)/R, e_{2i}) \) represents point F on the budget line.

Suppose, to the contrary, that all human capital is general, which means that the EPF is the IPF from point E to F, and coincides with the budget line beyond F. Then, roughly speaking, the equilibrium contract corresponds to a tangency of the worker's indifference curve with either the budget line along the segment (FA), or with the input possibility frontier along the segment (EF). Formally, we have
Theorem 4: Suppose that all human capital is general and let 
\[ T_i = \text{arg max } u(e_{1i} - T_i e_{1i} + k_i(T)) \quad 0 < T_i < e_{1i} \]
define the utility maximizing element on the input possibility frontier of worker \( i \). Then the equilibrium contract under private information will invest \( \min\{T^*(R), T_i\} \) in the training of this worker.

Even though it refers to a limiting special case, this result is rather instructive, for general skills have overwhelming importance in the process of accumulating human capital, especially if one includes schooling. The unconstrained equilibrium investment, \( T_i^* \), differs fundamentally from the constrained equilibrium value, \( T_i \): both depend on the technology of training, i.e., the ability of the worker, but there the similarity ends. The value \( T_i^* \) responds to the rate of interest but is quite independent of the worker's tastes; \( T_i \), on the other hand, is an upper bound on training which directly reflects the utility function and not the rate of interest. Figure 6 illustrates the equilibrium response of investment in training to interest-rate variations for a worker accumulating only general skills.

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14. Paul Taubman has pointed out to me that Bartel (1980) presents indirect evidence on this issue: in her sample earnings growth depends on individual job history. Mobile workers gain smaller absolute raises on any given job than do more permanently attached workers but make up for this by faster gains in earnings from changing jobs.
relevance under a perfect credit market. Equally irrelevant here is the division of human capital into specific and general components.

Like investment projects financed in a perfect capital market, equilibrium contracts in this economy possess a separation property which makes them responsive to the underlying cost and return structure but not to the preferences of the individuals involved. From equation (3) we see that investment in training reflects the worker's ability—i.e., the steepness of the input possibility frontier—but not her indifference map or the division of human capital into specific and general components. This situation will change under private information, for then the possibility of credit rationing makes the timing of income potentially important for workers while human capital becomes the only deterrent to breach of contract.

4. SELF-ENFORCING CONTRACTS

To analyze what credit rationing does to the exchange of labor services, we assume in the sequel that the commodity endowments, factor incomes and net trades of all entrepreneurs are verifiable public information while the income and net trades of each worker, observed costlessly by himself and her employer, cannot be monitored by a third party at any finite cost. If the deadweight costs of bankruptcy are zero, then this admittedly drastic assumption means that workers cannot borrow directly from ultimate lenders.

8. To understand the allocative irrelevance of this indeterminacy, recall that output of the consumption good (and total income) depends only on the stock of human capital, and consumption decisions reflect present value alone. Hence, the timing of factor incomes does not influence aggregate saving and should have no bearing on equilibrium rates of interest.

9. Among a firm's net trades one must include total employment and profit, which may reveal some information about the wage income and employment status of individual workers. These signals are not valuable if the number of workers per firm is large; we ignore them in what follows.
i.e., other workers or third-party entrepreneurs.

Financial intermediation is conducted by employers, who may borrow up to the present value of their own verifiable future income on behalf of their workers. Like any bonafide commercial bank, employers in this paper are agents partly specialized in the business of gathering information about borrowers and disseminating information about themselves to potential lenders. An employer will lend an employee by paying first period compensation above first-period productivity \((y_1 > e_1 - T)\), and recoup principal plus interest from the excess of second-period productivity over labor earnings \((y_2 < e_2 + k(T))\).

Credit rationing may arise here because a worker with a vector \((y_1, y_2)\) of total income, earned plus unearned, cannot consume more than \(y_1\) in the first stage of her life. Therefore, the value of a contract \(\delta = (y_1, y_2, t)\) to any worker is not fully captured by the expression \(y_1 + y_2 / R\), but may depend on the timing of the income stream as well. By analogy with equation (1), we define an indirect utility function for a credit-rationed worker

\[
v^i(y_1, y_2, R) = \max_{0 \leq s \leq y_1} u^i(y_1 - s, y_2 + Rs)
\]

Again we recall that the indirect utility function \(v^i\) is increasing in \((y_1, y_2)\) for every worker. Entrepreneurs continue to evaluate contracts by present value alone, i.e., by the expression \(w^i(\delta)\) given in equation (5b).

Since third parties cannot verify net trades between contractants, workers and firms may in principle abrogate any agreements between them. A contract \(\delta = (y_1, y_2, t)\) is breachproof if neither contractant finds it rational to terminate the relationship at the beginning of the second period. If worker \(i\) were to breach \(\delta\), her second period income would
We underline the word "an" to stress that the number of equilibrium earnings profiles is infinite whenever the worker is not credit rationed. The case of no-credit rationing is very similar to the one in section 3: workers judge an earnings profile by its present value, not by the timing of income. And any contract that is a private-information equilibrium for an unrationed worker is also a public-information equilibrium.

The situation changes drastically if the maximal element on the CPF lies somewhere between the tangency point $F'$ and the initial endowment point $E.$ Here the worker is credit-rationed, the equilibrium contract $(x^1_1(T^*), x^1_2(T^*), x^2(T^*))$ is unique and corresponds to a tangency between the EPF and an indifference curve. At the tangency point, the absolute value of the slopes involved exceed $R$, investment in training falls short of the public information equilibrium, and the rate of return on human capital is above the rate of interest.

To describe the equilibrium contract, therefore, we need to know how the interest factor $R$ compares with the worker's marginal rate of substitution at the consumption vector $x^1(T^*) = (x^1(T^*), x^2(T^*))$, i.e., whether or not the inequality

$$(19a) \quad u^1_1(x^1(T^*)) / u^2_2(x^1(T^*)) < R$$

12. If the initial endowment point $E$ is maximal on the CPF, then Theorem 2 asserts that a self-enforcing contract does not exist.
holds true. We summarize the preceding two paragraphs in

**Theorem 3:** Suppose $T^*_1(R) > 0$ and let $s_1(\tau)$ be the specific human capital accumulated by worker 1 when she invests $\tau$ in her training. If inequality (19a) holds, then $\delta^* = (x^*_1(t_1), x^*_2(t_1), t_1)$ is an equilibrium contract, worker 1 is not credit-rationed, and the yield on human capital equals the interest rate. There is also a continuum of other equilibrium contracts, all with $\tau = t_1^*$ and with the same present value of earnings as $\delta^*$.

If, on the other hand, inequality (19a) fails, then the equilibrium contract, $\hat{\delta} = (x^*_1(\hat{\tau}), x^*_2(\hat{\tau}), \hat{\tau})$, is unique, the worker is credit-rationed, and the yield on human capital exceeds the interest rate. Investment in training is uniquely determined from the equation

\[(19b) \quad u^*_1(x^*_1(\hat{\tau}))/u^*_2(x^*_2(\hat{\tau})) = s^*_2(\hat{\tau})/(1 - s^*_1(\hat{\tau})/R)\]

For a fixed rate of interest, any worker who satisfies inequality (19a) is not injured by private information about earnings. Both her investment in training and her consumption bundle are identical to their corresponding equilibrium values under public information. Employers are successful here as middlemen between workers and ultimate lenders for two reasons. The more obvious one is that a worker with earnings profile $(e_1 - \tau^*, e_2 + k(\tau^*))$ may desire to lend, not borrow, at the going rate of interest; that worker's

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13. The uniqueness of $\delta^*$ is a consequence of the unique tangency point between a concave EPF and a convex indifference curve.
change from $y_2$ to the product, $e_{21} + g_1(t)$, of her general human capital. Breach of the same contract by any firm lowers its wage bill by $y_2$ and its sales revenue by the product, $e_{21} + k_1(t)$, of the terminated worker's entire human capital. The contract is safe against breach (i.e., individually rational ex post) if

$$e_{21} + g_1(t) \leq y_2 \leq e_{21} + k_1(t)$$

For the sake of comparison, we define a "spot" contract $(e_{11}, e_{21}, 0)$ for worker $i$ which describes the earnings of that worker if she chooses to acquire no training whatever and hires her services instead to a different firm each period; note that a spot contract does not permit employers to lend employees. A non-trivial contract with positive training, then, is individually rational ex ante (i.e., dominates the "spot" contract) if

$$(8a) \quad v^i(y_1, y_2, R) \geq v^i(e_{11}, e_{21}, R)$$

and

$$(8b) \quad x_i(t) \geq 0$$

The first inequality expresses individual rationality for a possibly credit-rationed worker; the second one states that a training contract yields no less to the firm than the zero profit of a "spot" contract which pays workers their total product in each period.

A contract is called self-enforcing if it is individually rational for worker $i$ and some firm, both ex ante and ex post. Formally we have
Definition 2: A contract \( \delta = (y_1, y_2, \tau) \) is self-enforcing for some worker-firm pair if \( \tau > 0 \) and inequalities (7), (8a) and (8b) hold simultaneously.

The earnings profile \((y_1, y_2)\) contained in a self-enforcing contract (SEC) is designed to achieve simultaneously two somewhat contradictory objectives: one, to set first-period earnings high enough so that workers are relieved from credit-rationing; two, to fix second-period income at a value that is high enough to discourage quits. Any firm's willingness to intermediate between its credit-rationed employees and ultimate creditors is limited by the fact that "subsidized" and unsecured loans to junior employees \((y_1 \text{ in excess of } e_{11} - \tau)\) must reduce their subsequent labor earnings \((y_2 \text{ less than } e_{21} + k(\tau))\) and thereby raise their incentives to quit.

Contracts must therefore strike a balance between financial intermediation and incentives against breach. Such a balance may be possible at some, but not all, levels of investment in training, or it may be altogether impossible to achieve. If that is the case for some worker, we shall say that a SEC does not exist for her; she will not receive any training even if training has a large rate of return.

As an example, we shall try to find out if any of the contracts \((y_1, y_2, \tau^*)\) which are equilibria for worker \(i\) under public information are self-enforcing and, hence, implementable under private information as well. Here \(\tau^*_i\) is defined in equation (3), and the profile \((y_1, y_2)\) leaves every firm with zero profit, i.e.,

\[
y_1 + y_2/R = e_{11} - \tau^*_1 + [e_{21} + k_1(\tau^*_1)]/R.
\]
Definition 4: Given the interest rate, a contract \( \delta^i = (y_1^i, y_2^i, T_i^i) \) is equilibrium for worker 1 and some firm if there is no other contract \( \delta = (y_1, y_2, T) \) such that \( v^i(y_1^i, y_2^i, R) > v^i(y_1, y_2, R) \) and \( \pi^i(\delta) > 0 \) with at least one strict inequality.

To maximize the indirect utility of workers subject to a zero profit constraint, equilibrium contracts minimize the chance of credit rationing for workers by paying them in the first period as large a wage income as the breachproofness constraint (7) will allow. As a result, the earnings profile for worker 1, \( (e_1 - \tau_k + [k(T) - g_1(T)]/R, e_2 + g_1(T)) \), will lie on the EPF of Definition 3. We denote by \( x_1(T) \) and \( x_2(T) \), respectively, the first and second components of that profile.

For each value of \( \tau \in [0, e_1] \) we have a point on the EPF, and every one of these points corresponds to a budget set of the form

\[
\{(c_1, c_2) > 0 | c_1 + c_2 / R \leq x_1 + x_2 / R, c_1 \leq x_1\} \text{ for worker 1.}
\]

The envelope of all possible budget sets is called the consumption possibility frontier (CPF) for worker 1. Formally, the CPF is the boundary of the relevant consumption possibility set (CPS) defined below

\[
(17) \quad B_1^i = \{(c_1, c_2) > 0 | c_1 + c_2 / R \leq x_1^i(T) + x_2^i(T), c_1 \leq x_1^i(T), 0 \leq \tau \leq \tau^*_1(R)\}
\]

and drawn in Figure 5.

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11. To justify paying workers in the second period as little as required to avoid breach of contract, we have ruled out post-contract renegotiation. The choice we gave contractants in section 1 was either to carry out the agreement at the original terms or to terminate. Section 6 looks briefly at second-period renegotiation as a bargaining game between contractants who possess some monopoly power over each other after training costs are sunk.
The CPF coincides with the EPF for all values $\tau \in [0, \tau^*_1]$ until the point $F'$ in Figure 5 where the EPF becomes tangent to the budget line that is itself tangent to the IPF. Beyond point $E$, the CPF no longer follows the EPF since it is not rational for any worker-firm pair to invest in training more than the sum $\tau^*_1$ which represents the public information equilibrium. Indeed, any point on the EPF corresponding to $\tau > \tau^*_1$ is decidedly inferior to point $F'$ from the worker's standpoint for it has lower present value of earnings and worse timing (smaller first-period income) to boot.

The worker's choice over vectors on the consumption possibility frontier directly dictates the equilibrium contract. For example, if the worker's most preferred consumption vector lies to the northwest of point $F'$ on Figure 5, then an equilibrium contract for worker $i$ under private information is $(x^i_1(\tau^*_1), x^i_2(\tau^*_1), \tau^*_1)$. Here the earnings functions $x_1$ and $x_2$ are defined...
The issue is whether a contract of this type satisfies inequalities (7) and (8a) for \( \tau = \tau^*_1 \) at some given \( R > 0 \).

If all human capital is specific, i.e., \( g_1(t) = 0 \) for all \( t \), then such a contract clearly exists because incentives against breach are very strong. The set of all breachproof zero-profit contracts corresponds to the line segment (FY) in Figure 2(a); among them we may choose the one that is associated with point Y by setting \( y_1 = e_{11} - \tau^*_1 + k_1(\tau^*_1)/R \) and \( y_2 = e_2 \). As an income profile, this point clearly satisfies inequality (8a) and dominates the profile \((e_{11}, e_{21})\) associated with point E; Y has both greater present value and earlier timing than E.\(^{10}\)

![Figure 2: The Implementability of Public Information Equilibria](image)

On the other hand, if all human capital is general, then a SEC may very well fail to exist, particularly for low rates of interest. The theory of human capital in this case says that firms cannot intermediate on behalf of

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\(^{10}\) Even in this extremely favorable case, asymmetric information results in a shrinkage of the worker's consumption opportunities relative to what is available under public information. In particular, the shaded triangle \((YXY')\) is no longer available with private information because contracts for which \( y_2 < e_2 \) are not safe from breach by workers.
their workers; employees will bear the full cost of on-the-job-training, and reap all the rewards from it. To see this more clearly, note that

\[ k_i(T) = g_i(T) \text{ for all } T \]

simplifies equations (7) and (9) to

\[ y_1 = e_{11} - \tau_i^* \quad y_2 = e_{21} + k_i(T_i^*) \]

Now, if either \( R \) or \( e_{11} \) are very small, then \( y_1 \) is also very small, and virtually all earnings will accrue in the second period of the worker's life. The worker's budget set reduces to a narrow strip, like the one shaded in Figure 2(b), with a maximal point at \( F \) that is inferior to what the worker can achieve with the spot endowment \( E \). No SEC exists at \( \tau = \tau_i^* \).

Even if we could not implement under private information any of the contracts that represent public information equilibria, we might still be able to find a self-enforcing contract for worker \( i \) if we were to invest in her training an amount different from \( \tau_i^* \), in particular, less than \( \tau_i^* \). As we reduce human investment, the yield on human capital rises above the rate of interest, and a wedge opens between the returns to financial and human assets.

The reduced acquisition of job skills is not entirely without merit for it succeeds in providing at least partial relief from credit rationing. Less training, in fact, means that employers contribute more than they otherwise might have to net output early in their careers, and less later on. The resulting productivity profiles are "less steep" and employees become less inclined to borrow at any rate of interest.

To grasp what earnings profiles are possible at different levels of human investment, we define for each worker an earnings possibility set (or EPS) bounded by the two axes and the earnings possibility frontier (or EPF). This frontier is the set of all earnings profiles consistent with zero profits for
panel 4(a), then it is always possible to find a contract that has positive investment in training and an earnings profile which the worker prefers to the endowment point \( E \). In fact, any earnings profile that lies on the upward-sloped segment of the earnings possibility frontier dominates \( E \): it gives the worker more income in every stage of her lifecycle.

On the other hand, if \( T_i - T_i > 0 \), then it is possible that no self-enforcing contract will exist for worker \( i \) if her indifference curve through the initial endowment point \( E \) is sufficiently steep, as shown in panel 4(b). This situation corresponds to spot-market workers who are severely rationed, i.e., face an abnormally high "shadow" rate of interest. If the shadow rate exceeds the market rate, an untrained worker will prefer consuming the vector \((e_{i1}, e_{i2})\) to any other point in her budget set. If, in addition, the shadow interest factor is higher than the absolute value of the right-hand side in equation (15), then there is no feasible earnings profile on the entire EPF which worker \( i \) will prefer to \((e_{i1}, e_{i2})\).

Employees who are this severely credit-rationed will prefer to remain untrained, even if the relevant rate of return may exceed the market yield on loans. We have thus demonstrated

**Theorem 2:** For a given interest factor \( R > 0 \), denote by \( T_i^* \) the equilibrium investment in the training of employee \( i \) under full information and by \( R_i > 0 \) the absolute slope of her indifference curve at the initial endowment point \((e_{i1}, e_{i2})\). If \( T_i^* > 0 \), then a self-enforcing contract fails to exist for worker \( i \) if, and only if, both of the following two inequalities hold:

\[
R > k_i'(0) - g_i'(0) \quad \text{and} \quad R_i > Rg_i'(0)/(g_i'(0) + R - k_i'(0))
\]
The first of these inequalities simply implies $\bar{T}_i = 0$; the second one insures that the indifference curve is steeper than the EPF at $(e_{11}, e_{21})$. Whether or not these relations hold will depend on the rate of interest as well as the tastes and training technology of the individual worker in question. *Ceteris paribus,* a self-enforcing contract is less likely to exist for relatively high rates of interest and for negligible amounts of specific human capital. This is because high interest rates mean low present values for all training programs; and a relatively unimportant specific human capital component reduces the deadweight loss from breach.

Theorem 2 yields sharper predictions in the familiar polar cases. If all human capital is specific, a self-enforcing contract exists for worker $i$ whenever $\tau_i^* > 0$ or, equivalently, $R < k'_i(0)$; this rules out the first inequality in (16). In the opposite case of no specific capital, inequalities (16) become $R_i > k'_i(0)$. This proves

**Corollary:** If all human capital is specific, a self-enforcing contract exists for worker $i$ whenever $\tau_i^*(R) > 0$. If all human capital is general, a self-enforcing contract exists for worker $i$ whenever $\tau_i^*(R) > 0$ and $R_i < k'_i(0)$.

5. EQUILIBRIUM WITH PRIVATE INFORMATION

Among all self-enforcing contracts, a labor market equilibrium will choose ones that maximize worker utility and make at least zero profit for firms. Extending the definition of equilibrium from the public information case, we have
firms and with the breachproofness constraint (7). More formally, we have

**Definition 3:** Given the interest factor $R$, the earnings possibility set for worker $i$ is

$$
\psi_i = \{(y_1, y_2) > 0 | y_1 \leq e_{i1} - \tau + \frac{k_i(\tau) - g_i(\tau)}{R}, y_2 \leq e_{i2} + g_i(\tau), 0 \leq \tau \leq e_{i1}\}
$$

Recall from equation (8) that $\tau^*_i$ is that level of investment in skills which maximizes the return $k_i(\tau) - R\tau$ from all human capital; similarly we define

$$
(11) \quad \overline{\tau}_i(R) = \arg \max_{0 \leq \tau \leq e_{i1}} [k_i(\tau) - g_i(\tau) - R\tau]
$$

(11) to be the maximizer of the net return from specific human capital alone. Because $k_i$ and $g_i$ are both concave functions, it follows that, for each $R > 0$ and $i$, we have

$$
(12) \quad 0 \leq \overline{\tau}_i(R) \leq \tau^*_i(R)
$$

The EPS lists all earnings profiles that are feasible for each worker and, therefore, describes indirectly that worker's consumption possibilities. Before we study qualitatively the properties of the EPS, we define formally two related concepts for worker $i$: her input possibility set (IPS)

$$
(13) \quad I_i = (L_1, L_2) | 0 \leq L_1 \leq e_{i1}, L_2 \leq e_{i2} + k_i(e_i - L_1)
$$
and her public information budget set

\[ B_0^1 = \left\{ (c_1, c_2) \geq 0 | c_1 + c_2 / R \leq e_{11} - \tau^* + (e_{21} + k_1(\tau^*)) / R \right\} \]

The IPS we know already: it is bounded by the axes and the input possibility frontier, lies in the interior of \( B_0^1 \) and represents the consumption possibilities of a hypothetical autarkic worker. Such a person by definition cannot trade in the credit market, either directly or indirectly through her employer, and consumes all earnings. Therefore, the difference between the IPS and the consumption possibility set \( B_0^1 \) measures the value of a perfectly functioning credit market to any worker, independently of the specific tastes involved. Figure 3 illustrates.

By construction, the present value of any point \((y_1, y_2)\) on the earnings possibility frontier is less than or equal to

\[ e_{11} - \tau^* + (e_{21} + k_1(\tau^*)) / R \]

hence the EPS is contained in the budget set \( B_0^1 \). The remaining qualitative properties of the EPS are easily established once we understand how to construct it. Figure 3 shows the procedure: for any \( \tau = \hat{\tau} \) we draw first the line \( L_1 = e_{11} - \hat{\tau} \), which defines a point \( \alpha \) on the IPF. Next we draw the firm's zero profit locus for \( \tau = \hat{\tau} \), which is a straight line through \( \alpha \) with slope \(-R\). On that zero profit line we locate the point \( \alpha' \) which represents the earnings profile \((e_{11} - \hat{\tau} + (k_1(\hat{\tau}) - g_1(\hat{\tau}))/R, e_{21} + g_1(\hat{\tau}))\).

Among all earnings profiles of a given present value which are safe from breach, this one pays the worker most in period one and, hence, provides the best possible protection against credit rationing.

As we repeat this procedure for all possible values \( \tau \in [0, e_{11}] \) we trace the entire EPF. For instance, if all human capital were general, then the earnings possibility set is the area (DEFBD) in Figure 3, i.e., coincides with
FIGURE 3: CONSUMPTION POSSIBILITIES UNDER PUBLIC INFORMATION

FIGURE 4: EARNINGS POSSIBILITIES UNDER PRIVATE INFORMATION
the input possibility set above the line \( y_2 = e_2 \). Supposing, on the other hand, that all human capital is specific and non-transferable means that the EPS shrinks to the line segment (DY) in Figure 3, and the EPF consists of just point \( Y! \)

When human capital possesses both specific and general components, the EPS is bounded by the earnings possibility frontier, the vertical axis, and the line \( y_2 = e_2 \). The EPF is drawn in Figure 4, where panel (a) represents the case \( \overline{t}_i > 0 \) and panel (b) corresponds to \( \overline{t}_i = 0 \). In either case, the EPF begins at the worker's initial endowment point \( E \) where \( \tau = 0 \); is tangent to the consumption possibility frontier at the point \( F' \) where \( \tau = \tau^*_i \); eventually crosses the IPF at some \( \tau > \tau^*_i \) and remains inside for all \( \tau \in [\tau^*_i, e_{i1}] \).

Panel 4(a) corresponds to the case \( \overline{t}_i > 0 \) (\( \overline{t}_i \) is defined in equation (12)). As \( \tau \) increases in the interval \([0, \overline{t}_i]\), both specific and general capital accumulate faster than the interest factor \( R \). The EPF initially slopes upward at an increasing rate until it reaches point \( \theta \) where \( \tau = \overline{t}_i \) and the slope is infinite; thereafter the frontier is negatively sloped.

The case \( \overline{t}_i = 0 \) appears in panel 4(b) from which is missing the upward-sloped, convex initial segment of the EPF. Assuming differentiability, the EPF has slope

\[
(15) \quad - \frac{dy_2}{dy_1} \bigg|_E = -R \frac{g'(0)}{g'(0) + R - k'(0)}
\]

at the no-training point \( E \). If \( 0 = \overline{t}_i < \tau^*_i \), then the absolute value of that slope exceeds the interest factor \( R \).

Whether or not self-enforcing contracts will exist depends to a certain degree on the values of \( \overline{t}_i \) and \( \tau^*_i \). If \( \overline{t}_i > 0 \) for worker 1, as in