THE INFLUENCE OF TECHNOLOGY AND DEMAND CONDITIONS ON FUTURES PRICES AND HEDGING*

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Paul Weller and Makoto Yano

University of Warwick
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ABSTRACT

We examine the determination of spot and futures prices in rational expectations equilibrium in a model with three groups of agents, agricultural producers, processing firms and speculators. We find necessary and sufficient conditions for producers to be short, processors to be long, and for the futures price to lie below the expected future spot price (normal backwardation). The conditions impose plausible restrictions on demand elasticities, and on the elasticity of substitution in the processing technology. We use a new technique of analysis which, in contrast to much of the literature does not require restrictive assumptions to be imposed upon the structure of preferences.

INTRODUCTION

The purpose of this paper is to throw further light upon three widely acknowledged "stylised facts" in the economics of futures markets. They are:

(1) Producers and processors of an agricultural commodity hedge against risks in the futures market for that commodity.

(2) Producers are sellers of futures contracts whereas processors are buyers.

(3) Hedgers as a group are net sellers of futures contracts.

The origins of interest in these observations can be traced back to Keynes (1930). He pointed to the existence of speculators who were willing to purchase the net supply of contracts from hedgers, and argued that this willingness could only be explained by the existence of a compensating risk premium, which would take the form of a positive expected gain to holding the contract to maturity; in other words the expected future spot price of the commodity should exceed the current price of a futures contract stipulating future delivery of one unit of the commodity. This is the essence of Keynes' theory of normal backwardation.[1/]

Although Keynes used observations (1)-(3) as a foundation for his theory of the relationship between spot and futures prices, it was left to Hicks (1946) to seek an explanation for these observations. He argued that the nature of production processes was such that producers had a greater degree of flexibility in the purchase of inputs than in the completion of outputs. In the case we wish to focus upon, that of agricultural production and processing, this is clearly so. Decisions on inputs to processing can be made after the "harvest", whereas decisions on inputs to agricultural production must necessarily be made before. This means that processors can provide themselves with partial insurance against risk generated by variable input prices by suitable adjustment of the input level. By purchasing futures contracts a processor can reduce input price variability, but demand for such insurance is inherently weaker, because of the additional source of insurance mentioned above, than that from agricultural producers. Hence the state of affairs in which producers wish to sell forward more than processors wish to purchase.

We construct an equilibrium model in which to examine these arguments. We obtain a strikingly simple necessary and sufficient condition for observations (1)-(3), together with normal backwardation. The condition is that the elasticity of demand for the processed commodity exceed the elasticity of substitution in the processing industry and that the derived demand for the agricultural commodity be strictly inelastic, the degree of inelasticity depending upon technological and demand parameters. We are able also to sharpen the distinction between speculative and hedging activity, and to show how this depends upon the degree of risk aversion and the relative income variability of the groups in the market.

There have been several recent theoretical investigations of the determinants of hedging behaviour. (Anderson and Danthine (1983), Newbery and Stiglitz (1981), Ch.13, O'Hara (1985), Stein (1979) and Stiglitz (1983)). A number of these papers have pointed to the importance of the demand elasticity as a determinant of backwardation, but none distinguish between the elasticity of demand for the processed commodity and the (derived) elasticity of demand for the unprocessed commodity. With the exception of Stiglitz (1983) and O'Hara (1985), all the work cited above is carried out in the framework of mean-variance analysis, with its attendant restrictions. Our technique of analysis is novel and requires no restriction to be imposed upon the form of individual preferences. It has intrinsic interest, and we have already employed it to examine the general equilibrium effects on spot price variability resulting from the opening of a futures market (Weller and Yano (1985)). The paper by O'Hara is the first, so far as we are aware, to present a formal analysis of the role played by the nature of the production technology in the determination of hedging decisions. Her treatment however is rather different from ours and our results are not directly comparable.

I. Model

The model describes the behaviour of three groups of individuals, agricultural producers, or farmers, (X), processors (Y), and pure speculators (S). A commodity, X, is produced by farmers and is subject to random variation in supply. We allow for the possibility that there are two distinct sources of demand for X, final consumption demand and input demand from processors, whose output we denote by Y.[2/] $D^X(p^X)$ represents final consumption demand for the unprocessed commodity X, whose price is p^X. The quantity of X used as an input to processing is denoted F, and is transformed into Y according to the production relation $Y = g(F,N)$. The variable N represents an aggregate of all other factors of production. We assume that $g(F,N)$ displays constant returns to scale. However the timescale over which processing decisions are made is sufficiently short to justify treating N as fixed at $\bar{N}$. In addition, g(F,N) is assumed to be strictly concave in F.[3/] $D^Y(p^Y)$ represents demand for the processed commodity, Y, whose price is p^Y.

Activity in the model takes place within a single period. The futures market for good X opens at the beginning of the period, before the supply of good X is known. Exchange on the spot market occurs at the end of the period, after the supply of good X is known. A futures contract stipulates that one unit of good X is to be delivered when the spot market opens. We shall examine a rational expectations equilibrium in which each group is assumed to know the correct distribution of spot prices when futures trading takes place.

The above setting is intended to formalise the following facts. In general, agricultural production takes a relatively long time and is subject to considerable influence by random factors. In contrast, the processing of agricultural products needs a much shorter time and is subject to a much smaller degree of risk. In order to focus upon these facts we ignore the input decisions of farmers and consider only those of processors explicitly.[4/]

The supply of good X is affected by the state of nature, i, and is therefore written X_i. State i is realised with probability ϕ_i. The preferences of each group are represented by von Neumann-Morgenstern utility functions, $\Sigma_i \phi_i u^j(M_i^j)$, where M_i^j is the income of group j (=X,Y,S) in state i. We assume $u_M^j = \partial u^j(M)/\partial M > 0$ and $u_{MM}^j = \partial^2 u^j(M)/\partial M^2 < 0$. Let q be the price of a futures contract, and let Z^j be group j's demand for futures contracts. Denote also by I_i^j the income of group j before trade in the future market. Then group j's objective in the futures market is to

(1) $$\max_{z^j} \Sigma_i \phi_i u^j(M_i^j)$$

$$\text{s.t. } M_i^j = I_i^j + (p_i^X - q)z^j$$

where

(2) $$I_i^X = p_i^X X_i$$

(3) $$I_i^Y = p_i^Y g(F_i, N) - p_i^X F_i$$

(4) $$I_i^S = I^S$$

Note that this formulation allows processors to make decisions about the level of input F after the state of nature has been realised and so makes their output Y a random variable. Costs associated with N are fixed in the short run and so can be ignored. We assume that pure speculators have exogenously determined non-random income.

The market clearing condition in the futures market is

(5) $$\Sigma_j z^j = 0$$

As is discussed in the Introduction, normal backwardation is said to occur in the futures market for X if the price of a good X futures contract is lower than the expected spot price of X or if, in other words, $q < E(p^X)$. Since speculators have state independent income I^S, they buy futures contracts $(z^S > 0)$ if and only if the expected return from purchasing contracts, $E(p^X)Z^S$, exceeds the cost qZ^S. Moreover, a group's position is said to be long (short) in the futures market for X if it is a buyer (seller) of futures contracts. Thus, we have

<u>Proposition 1</u> (Keynes-Kaldor). Speculators are long in good X futures if and only if normal backwardation occurs.

II. Spot Markets : Preliminary Analysis

We first examine the way in which variations in supply affect spot prices and incomes. Our model takes the following form:

$$(6) \qquad D^X(p_i^X) + F_i = X_i$$

$$(7) \qquad D^Y(p_i^Y) = g(F_i, \bar{N})$$

$$(8) \qquad p_i^X g_F(F_i, \bar{N}) = p_i^X$$

where $g_F(F_i, \bar{N}) = \frac{\partial g}{\partial F_i}(F_i, \bar{N})$. Input F_i is selected so as to maximise I_i^Y, implying (8).

In order to analyse the comparative statics of system (6)-(8), we denote a proportional change in x by $\hat{x} = dx/x$. Then, suppressing subscript i for clarity, we have

$$(9) \qquad -\lambda^X \varepsilon^X \hat{p}^X + \lambda^F \hat{F} = \hat{X}$$

$$(10) \qquad -\varepsilon^Y \hat{p}^Y = \theta \hat{F}$$

$$(11) \quad \varepsilon^F \hat{p}^Y - \varepsilon^F \hat{p}^X = \hat{F}$$

where $\lambda^X = \frac{D^X}{X}$ and $\lambda^F = \frac{F}{X}$ are the shares in total output X of consumption and factor demands, respectively; $\varepsilon^j = -\frac{p^j}{D^j}\frac{dD^j}{dp^j}$ is the elasticity of demand for good j (j = X,Y); $\varepsilon^F = -\frac{g_F}{Fg_{FF}}$ is the elasticity of factor demand, and $\Theta = (p^X F)/(p^Y Y)$ is the cost-revenue ratio.

We find from (10) and (11) that

$$(12) \quad \hat{p}^Y = \frac{\Theta\varepsilon^F}{\varepsilon^Y + \Theta\varepsilon^F}\hat{p}^X$$

and from (9), (10) and (11) that

$$(13) \quad \hat{p}^X = -\frac{1}{\eta^X}\hat{X}$$

where

$$(14) \quad \eta^X = \lambda^X\varepsilon^X + \frac{\lambda^F\varepsilon^Y\varepsilon^F}{\varepsilon^Y + \Theta\varepsilon^F}$$

is the elasticity of (Marshallian) derived demand for X.

Now we define the following terms:

$$(15) \quad \delta^X = - \frac{dI^X}{p^X dX}$$

$$(16) \quad \delta^Y = - \frac{dI^Y}{p^X dX}$$

$$(17) \quad \delta = \delta^X + \delta^Y = - \frac{d(I^X + I^Y)}{p^X \, dX}$$

Each of these terms is a measure of the income change brought about by a change in the supply of good X. Observing that

$$(18) \quad \sigma^Y = (1-\Theta)\varepsilon^F$$

where σ^Y is the elasticity of substitution between F and N[5/], we may write

$$(19) \quad \delta^X = \frac{1}{\eta^X} (1 - \eta^X)$$

$$(20) \quad \delta^Y = - \frac{1}{\eta^X} \frac{\lambda^F}{(\varepsilon^Y + \Theta\varepsilon^F)} (\varepsilon^Y - \sigma^Y)$$

In the case of farmers, the sign of δ^X, a measure of the extent of impoverishment in time of good harvest, depends, as one would expect, upon whether the elasticity of derived demand for X is greater than or less than unity. In the case of processors, we find more interestingly that the sign of δ^Y depends upon whether the elasticity of demand for their output is greater or less than the elasticity of substitution between X and all other factors of production.

III. Futures Market

Now we are in a position to relate the above analysis to behaviour in the futures market for commodity X. The techniques we shall use are similar to those we have employed in Weller and Yano (1985) to examine the general equilibrium effects of opening a futures market. We assume that there are only two states of nature $(i = 1,2)$ and confine our analysis to the case in which the variability in X is small. We may assume without loss of generality that $\phi_1 = \phi_2$. Thus $\bar{X} = (X_1 - X_2)/2$ is the mean of random output (X_1, X_2); if we define $R = X_1 - \bar{X} = \bar{X} - X_2$, the randomness in the output of X can be measured by the absolute value of R. This generates variability in spot incomes I^j which is measured by $\delta^j (j = X,Y)$. By conditions (6), (7) and (8) therefore, the spot market equilibrium can be written as

$$e_{si}(R) = (I_i^X(R),\ I_i^Y(R);\ F_i(R),\ D_i^X(R),\ p_i^X(R);\ Y_i(R),\ D_i^Y(R),\ p_i^Y(R)),\quad i = 1,2.$$

Note that, by the Implicit Function Theorem,

(21) $e_{si}(R)$, $i = 1,2$ is continuously differentiable in R, $-\bar{X} < R < \bar{X}$.

The optimisation problem (1) reveals that an equilibrium in the futures market also depends upon R since the income distribution $I_i^j(R)$ depends upon R. We see from (13) that the spot price of X depends upon the state of nature; i.e. $p_1^X(R) \neq p_2^X(R)$. Thus, for each given futures price the demand for futures of any given group is uniquely determined given $R \neq 0$.

A difficulty arises in the case where $R = 0$, since demand for futures contracts, Z^j, is indeterminate. Since for $R = 0$ the spot price of X must be non-random, the price of a futures contract has to be equal to the spot price; that is, $q(0) = p_1^X(0) = p_2^X(0) = \bar{p}^X$. Thus, neither gains nor losses occur by trading futures contracts; such a contract does not function as a security. The set of demands for futures, denoted formally by $Z^j(0)$, is equal to the set of all real numbers.

We avoid this difficulty by a transformation of the optimisation problem (1) similar to that used in Weller and Yano (1985). Define $B_i^j = M_i^j - I_i^j$. Then, since by assumption $\phi_1 = \phi_2$, the maximisation problem (1) takes the form

$$(22)\quad \max_{B_i^j} \frac{1}{2} u^j(I_1^j + B_1^j) + \frac{1}{2} u^j(I_2^j + B_2^j)$$

$$\text{s.t. } rB_1^j + B_2^j = 0$$

where r is the implicit price of state 1 income in terms of state 2 income; i.e. $r = (q - p_2^X)/(p_1^X - q)$ if $p_1^X \neq p_2^X$. Since $B_2^j = -rB_1^j$ by (22) and $I_i^j = I_i^j(R)$, we obtain

$$(23)\quad \frac{u_M^j(I_1^j(R) + B_1^j)}{u_M^j(I_2^j(R) - rB_1^j)} = r, \quad j = X, Y, S.$$

$$(24)\quad B_1^X + B_1^Y + B_1^S = 0.$$

Equation (23) follows from the first order condition for (22), and (24) from the condition for market clearing. Equations (23) and (24) implicitly determine B_1^j and r as unique functions of R, which we denote by $B_1^j(R)$

and r(R). Crucial to our analysis is the fact that, by the Implicit Function Theorem,

(25) $B_1^j(R)$ and $r(R)$ are continuously differentiable in R, $\bar{X} < R < \bar{X}$.

Since $I_1^j(0) = I_2^j(0)$ for $j = X, Y, S$, we have

(26) $B_1^j(0) = 0$

(27) $r(0) = 1.$

Equation (27) follows from (23) and (26). We now study the comparative statics of (23) and (24) around $R = 0$. A change in R at $R = 0$ induces income changes $dI_1^j = - dI_2^j$. Totally differentiating (23) and evaluating at $R = 0$ using (26) and (27) we obtain

(28) $dB_1^j = - \frac{1}{2\rho^j} \hat{r} - dI_i^j ,$

where the coefficient of absolute risk aversion, $\rho^j \equiv - u_{MM}^j/u_M^j$ is evaluated at $M = I_1^j(0) = I_2^j(0)$. Since, by assumption, $I_1^S = I_2^S = I^S$ is independent of R, (24) and (28) imply

(29) $dB_1^X = - dI_1^X + \gamma^X(dI_1^X + dI_1^Y)$

(30) $dB_1^Y = - dI_1^Y + \gamma^Y(dI_1^X + dI_1^Y)$

(31) $dB_1^S = \gamma^S(dI_1^X + dI_1^Y)$

at $R = 0$, where

(32) $\gamma^j = (\frac{1}{\rho^j}) / (\Sigma_k \frac{1}{\rho^k})$.

We shall now use the relationships derived in this section in the proof of our main results.

IV Speculation, Hedging and Normal Backwardation : Local Results

In this section we first demonstrate that one may think of the demand for futures contracts for a particular group as composed of distinct hedging and speculative elements. We show how the relative magnitude of these elements is related to the change in income variability resulting from trade in futures. We then use these results to characterise the interrelationship between hedging, short and long positions in the market, and normal backwardation. We first prove

Theorem 1 Let $C = (p^X)^2 \eta^X / \bar{X}$. Then, we have

(33) $\lim_{R \to 0} z^X(R) = C\gamma^X\delta - C\delta^X$

(34) $\lim_{R \to 0} z^Y(R) = C\gamma^Y\delta - C\delta^Y$

(35) $\lim_{R \to 0} z^S(R) = C\gamma^S\delta$

Proof. Since for $R \neq 0$, as is seen above, $r = (p_2^X - q)/(q - p_1^X)$ and since for $R = 0$ we have (27), for all R we have $q(R) = \dfrac{r(R)p_1^X(R) + p_2^X(R)}{1 + r(R)}$. Thus, (20) and (25) imply that

(36) $q(R)$ is continuously differentiable in R, $-\bar{X} < R < \bar{X}$.

Since q and p_1^X are differentiable in R, we may apply L'Hôpital's Rule to $r(R) = \dfrac{p_2^X(R) - q(R)}{q(R) - p_1^X(R)}$. Then, we have

(37) $$\lim_{R \to 0} r(R) = r(0) = 1 = \frac{dp_2^X - dq}{dq - dp_1^X},$$

where the derivatives dq, dp_1^X, and dp_2^X are all evaluated at $R = 0$. Note that at $R = 0$, $de_{s1} = -de_{s2}$. Since this implies $dp_1^X = -dp_2^X$ at $R = 0$, and since, by (37), we have $2dq = dp_1^X + dp_2^X$ at $R = 0$, we have

(38) $dq = 0$ at $R = 0$.

It is clear that $p_1^X(R) \neq p_2^X(R)$ for $R \neq 0$, and so we have $z^j(R) = \dfrac{B_1^j(R)}{p_1^X(R) - q(R)}$ for $R \neq 0$. As already observed $q(R)$, $B_1^j(R)$, and $p_1^X(R)$ are differentiable, and we may apply L'Hôpital's Rule to this expression. Then, using (38) we obtain

(39) $$\lim_{R \to 0} z^j(R) = \frac{dB_1^j}{dp_1^X},$$

where the derivatives dB_1^j and dp_1^X are evaluated at $R = 0$. At $R = 0$,

$\hat{X}_1 = dR/X$, and equations (13), (29), (30) and (31) imply the theorem. Q.E.D.

A number of conclusions may be drawn from Theorem 1. First it is natural to call $C\gamma^i\delta$ the speculative component, and $-C\delta^i$ the hedging component./6 The less risk averse a group is, the larger γ^i and the greater the speculative component. The hedging component is always non-zero if income and agricultural output (and so spot price) are correlated. The term C is a measure of variability in p^X induced by fluctuations in supply. The larger is C the smaller is the variation in p^X. Changes in price variability, which must result from changes in the elasticity of derived demand, η^X, affect hedging and speculative components in equal proportion.

This discussion points to an ambiguity in the use of the terms hedger and speculator. Groups X and Y can be thought of as simultaneously engaging in both hedging and speculative activity. To clarify things, we introduce the concept of a pure hedger. A group i is said to consist of pure hedgers if individuals in the group are infinitely risk averse. This implies that $\gamma^i = 0$, and that their demand for futures consists only of a hedging component $-C\delta^i$. We see that group S, consisting of pure speculators will always trade so as to increase income risk, (except in the special case where $\delta = 0$). Pure hedgers, on the other hand, will always trade so as to reduce income risk i.e. to increase income in the low income state and to reduce it in the high income state.

We shall distinguish between a net hedged position in which income variability is reduced by futures trading, and a net speculative position in which income variability is increased./7 For the case we are concerned with in which R is small, group i, $i = X, Y$, has a net hedged position if $-\frac{dB_1^i}{dI_1^i} > 0$. Introducing the definition

$$(40) \quad h^i = 1 - \frac{\delta/\delta^i}{1/\gamma^i}$$

it is immediate from (29) and (30) that

$$(41) \quad h^i > 0 \text{ if and only if } -\frac{dB_1^i}{dI_1^i} > 0$$

For this reason we term h^i group i's <u>net hedging index</u>. It has a natural interpretation if one thinks of $\frac{1}{\gamma^i}$ as a measure of the extent to which group i is more or less risk averse than average, and of $\frac{\delta}{\delta^i}$ as a measure of the extent to which group i faces more or less income variability than average.

It is instructive to rewrite the results of Theorem 1 as

$$(33') \quad \lim_{R \to 0} Z^X(R) = -Ch^X\delta^X$$

$$(34') \quad \lim_{R \to 0} Z^Y(R) = -Ch^Y\delta^Y$$

If we compare (33) and (34) with (33') and (34') we see that if, for example, $\delta < 0$ and $\delta^i > 0$, it is quite possible for the speculative component to exceed the hedging component in absolute value, and for the group to be in a net hedged position. In fact, we see from (40) that if δ and δ^i have opposite signs, group i is always in a net hedged position, because the hedging and speculative components reinforce each other.

We now use (40) together with (19) and (20) to state

Proposition 2. If R is small, farmers are in a net speculative position if and only if

$$(42)\quad \frac{1-\gamma^X}{\gamma^X} < -\frac{\lambda^F}{\varepsilon^Y+\theta\varepsilon^F}\;\frac{\varepsilon^Y-\sigma^Y}{1-\eta^X}$$

Processors are in a net speculative position if and only if

$$(43)\quad \frac{1-\gamma^Y}{\gamma^Y} < -\left(\frac{\lambda^F}{\varepsilon^Y+\theta\varepsilon^F}\right)^{-1}\left(\frac{\varepsilon^Y-\sigma^Y}{1-\eta^X}\right)^{-1}$$

Remark We see from (42) and (43) that it is not possible for both group X and group Y to be simultaneously in a net speculative position. We already know that $\gamma^i = 0$ implies that group i is a pure hedger. For $0 < \gamma^i < 1$, the inequalities imply $(1-\gamma^X)(1-\gamma^Y) < \gamma^X\gamma^Y$, which in turn must imply $\gamma^S < 0$, a contradiction, since $\sum_j \gamma^j = 1$.

A necessary condition for either group to be in a net speculative position is that $\varepsilon^Y - \sigma^Y$ and $1-\eta^X$ have opposite signs. This is equivalent to saying that δ^X and δ^Y have the same sign. Thus a sufficient condition for both groups to be in a net hedged position is that $\varepsilon^Y - \sigma^Y$ and $1-\eta^X$ have the same sign, or that δ^X and δ^Y have opposite signs. This increases the scope for risk reduction via income stabilisation. That these conditions are not necessary is explained by the presence of pure speculators, who will always be prepared to take on some income risk.

The next result of interest follows from Proposition 1 and equation (35) in Theorem 1:

Proposition 3 If R is small,

(44) $q < E(p^X)$ iff $\delta > 0$

or, normal backwardation occurs in equilibrium if and only if the aggregate income of farmers and processors declines in time of good harvest.

The term net short hedging is commonly used to describe the situation in which producers and processors are net sellers of futures contracts, in other words $Z^X + Z^Y < 0$. We argue that this terminology is misleading, implying as it does that it is always appropriate to regard both producers and processors as hedgers. We will use the term net short hedging to describe a situation in which $Z^X + Z^Y < 0$ and $h^X > 0$ and $h^Y > 0$.

Proposition 1 and the condition for market clearing (5) imply

Proposition 4 Normal backwardation occurs if and only if producers and processors are net short.

Combining Proposition 3 with (19) and (10) we find

Proposition 5 If R is small, normal backwardation occurs if and only if

$$(45)\quad 1 - \eta^X - \lambda^F \frac{(\varepsilon^Y - \sigma^Y)}{\varepsilon^Y + \theta\varepsilon^F} = \lambda^X(1-\varepsilon^X) + \lambda^F \frac{\varepsilon^F}{\varepsilon^Y + \theta\varepsilon^F}(1-\varepsilon^Y) > 0$$

This result confirms and generalises the observations of McKinnon (1967), Anderson and Danthine (1983), and Newbery and Stiglitz (1981) that normal backwardation is likely to be associated with inelastic demand.

We move immediately to a statement of the main theorem of this section:

Theorem 2 If R is small, the following statements are equivalent:

(i) in equilibrium farmers are short, processors are long, and net short hedging and normal backwardation occur.

(ii) $\varepsilon^Y > \sigma^Y$ and $\eta^X < 1 - \frac{\lambda^F}{\varepsilon^Y + \theta\varepsilon^F}(\varepsilon^Y - \sigma^Y)$

Proof The statement (i) is equivalent to the following conditions: $z^X < 0$, $z^Y > 0$, $z^X + z^Y < 0$, $h^X > 0$, $h^Y > 0$, and $q < E(p^X)$. The second inequality in (ii) is necessary and sufficient for normal backwardation by Proposition 5, and so for $\delta > 0$ by Proposition 3. From (20) $\varepsilon^Y > \sigma^Y$ iff $\delta^Y < 0$. From (40) $\delta > 0$ and $\delta^Y < 0$ iff $h^Y > 0$. From (34') $\delta^Y < 0$ and $h^Y > 0$ iff $\lim_{R\to 0} z^Y(R) > 0$. Also $\delta > 0$ and $\delta^Y < 0$ iff $\delta^X > 0$, and $\delta > 0$ iff $\lim_{R\to 0} z^S(R) > 0$ by (35). By market clearing $\lim_{R\to 0} z^X(R) < 0$, and since $\delta^X > 0$, $h^X > 0$ by (33'). Q.E.D.

Corollary If $\lambda^X = 0$, statement (i) of Theorem 2 is equivalent to

(ii)' $1 > \varepsilon^Y > \sigma^Y$

Proof Follows directly by substitution from (13) into the second inequality in (ii).

The interest of Theorem 2 stems from the fact that the circumstances described in the statement of (i) coincide exactly with the stylised facts (1)-(3) in the introduction. In addition, we are able to be more precise about the sense in which farmers and processors are hedged. They are in a net hedged position and so have unambiguously reduced their income variability. The equivalent conditions in the statement of (ii) are quite plausible for the case we consider. The elasticity of substitution between the agricultural input and other factors of production can reasonably be expected to be rather small. The condition on η^X, the elasticity of derived demand is consistent with the fact that estimated demand elasticities for agricultural products are very commonly significantly less than one./8 It is also worth noting that a sufficient condition for this inequality to be satisfied is that ε^X and ε^Y are both less than one (from (45)). In the special case referred to in the Corollary, which applies to a number of commodities for which there are active futures markets (e.g. rubber, grains, sugar) the condition in (ii)' is even more transparent.

Observe also that in the equilibrium characterised above, the incomes of farmers and processors are affected in opposite ways. Farmers suffer a decline in income in time of good harvest, processors on the other hand experience an increase in income.

We conclude with a result which can be thought of as complementing Proposition 4, in that it reveals precisely the link between short hedging and normal backwardation.

Theorem 3 If R is small, and farmers (processors) are in a net speculative position, processors (farmers) are short net hedgers if and only if normal backwardation occurs.

Proof The theorem can be restated as: for small R, if $h^X(h^Y) < 0$, then $h^Y(h^X) > 0$, and $z^Y(z^X) < 0$ if and only if $\delta > 0$, using Proposition 3. From the remark after Proposition 2 it follows that $h^X(h^Y) < 0$ implies $h^Y(h^X) > 0$. It remains only to establish equivalence.
(i) Sufficiency: suppose $h^X < 0$, $z^Y < 0$ and $\delta < 0$. From (35) $\delta < 0$ implies $z^S < 0$, and so $z^X > 0$ by market clearing. But $z^X > 0$ and $h^X < 0$ implies $\delta^X > 0$ from (33'). Then $\delta < 0$, $\delta^X > 0$ implies $\delta^Y < 0$, which together with $z^Y < 0$ requires $h^Y < 0$ by (34'), a contradiction.
(ii) Necessity: suppose $h^X < 0$, $z^Y > 0$ and $\delta > 0$. From (35) $\delta > 0$ implies $z^S > 0$, so by market clearing $z^X < 0$, and since $h^X < 0$, $\delta^X < 0$ from (33'). But $z^Y > 0$ and $h^Y > 0$ implies $\delta^Y < 0$ from (34'), and we have a contradiction. Q.E.D.

What Proposition 4 and Theorem 3 reveal is that there is a sense in which short hedging is always equivalent to the occurrence of normal backwardation. Proposition 4 shows that if both farmers and processors are in net hedged positions, normal backwardation is equivalent to net short hedging. Theorem 3 shows that where only one group is in a net hedged position, (and there must always be such a group), that group is short if and only if normal backwardation occurs.

V. Normal Backwardation : A Global Result

Strictly speaking, the results obtained in Section IV hold only for the case where variability in the output of X is small. However the relationship between normal backwardation and good harvest impoverishment may be extended to the case where production risk is large. That is to say, we may prove

Theorem 4 Suppose that equilibrium is unique. Then, for any $R (R \neq 0)$, normal backwardation occurs if and only if farmers and processors in the aggregate suffer from good harvest impoverishment.

Proof First we calculate excess demand $Z^j(q)$ for futures contracts at $q^0 \equiv E(p^X)$, and denote $Z^{0j} = Z^j(q^0)$. Any risk-averse individual will acquire perfect insurance at these (actuarially fair) terms of trade. This allows us to solve for Z^{0j} by setting total income equal in each state. Thus, $I_1^j + (p_1 - q^0)Z^{0j} = I_2^j + (p_2 - q^0)Z^{0j}$, giving

$$(46) \quad Z^{0j} = - \frac{I_1^j - I_2^j}{p_1^X - p_2^X} .$$

Let $R > 0$. This implies $X_1 > \bar{X} > X_2$. Since the spot demand curve is downward sloping we have $p_1^X < p_2^X$. Thus, by (46) $\Sigma_{j=X,Y}(I_1^j - I_2^j) < 0$ is equivalent to $\Sigma_j Z^{0j} < 0$ at q^0. Allowing q to vary independently of R we obtain $\lim_{q \to p_1^X} \Sigma_{j=X,Y} Z^j = \infty$, $\lim_{q \to p_2^X} \Sigma_{j=X,Y} Z^j = -\infty$. By assumption equation $\Sigma_j Z^j(q) = 0$ has a unique solution, say q^*. Therefore $\Sigma_j Z^{0j} < 0$ at q^0 is equivalent to $q^* < q^0 = E(p^X)$ in equilibrium. Q.E.D.

VI. Concluding Remarks

The theory of normal backwardation, proposed by Keynes as an explanation of an observed interdependence between the spot price of a commodity and the price of a futures contract in that commodity, made appeal to an unexplained regularity in the futures market, that hedgers in the futures market were in aggregate net sellers of futures contracts although users tended to be buyers and producers sellers. We have provided a plausible explanation for this phenomenon: for the case of agricultural commodities demanders of the commodity as an input to processing must face an elasticity of demand for their output which is greater than the elasticity of substitution between the input and other factors of production, and there must be inelastic derived demand for the commodity.

We show also that normal backwardation and net short hedging occur if and only if agricultural producers and processors show a decline in aggregate income when output is high (aggregate good harvest impoverishment).

Footnotes

1/ Normal backwardation will be used throughout the paper to describe the situation in which expected future spot price is greater than current futures price.

2/ Coffee is an example of a commodity which can either be consumed in the form of beans, or processed into instant coffee.

3/ As O'Hara (1985) points out, this assumption implies strict convexity of processors' profit functions in input and output prices, if input decisions are taken after the state of nature is realised. Input flexibility implies that a mean-preserving spread in input or output prices increases expected profitability. Thus if processors are to engage in any hedging activity, they must be risk-averse.

4/ Newbery (1985) points out the important fact that farmers may choose riskier production processes if a futures market for their product exists, because they can reduce income risk by trading in the futures market. Here we do not consider such a relationship since our aim is to characterise the determinants of hedging and speculative activity, and of long and short positions.

5/ This follows from the fact that for a linear homogeneous production function $\sigma^Y = \frac{g_F g_N}{Y g_{FN}}$ (see e.g. Silberberg (1978) p.316), where $g_{FN} = \frac{\partial^2 g}{\partial F \partial N}$. By Euler's Theorem $Fg_{FF} + Ng_{FN} = 0$, and substituting for g_{FN} we obtain (18)).

6/ This decomposition is closely paralleled by that obtained by Newbery and Stiglitz (1981) for the case of mean-variance preferences.

7/ This terminology accords with that adopted by Newbery and Stiglitz (1981).

8/ Newbery and Stiglitz (1981), Table 20.6, p.293, quote their own estimates of price elasticities for cocoa, coffee, cotton, jute, rubber and sugar, which all lie between 0.4 and 0.8.

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