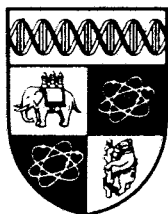Information Revelation in a Market
with Pairwise Meetings

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WARWICK ECONOMIC RESEARCH PAPERS

INFORMATION REVELATION IN A MARKET WITH PAIRWISE MEETINGS

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Abstract

The paper presents a simple pairwise meetings model of trade. The new feature is that agents have asymmetric information about the true state of the world. The focus is on the transmission of the information through the process of trade. The qualitative questions is: to what extent is the information revealed to uninformed agents through the trading process, when the market is in some sense frictionless? In particular: does the decentralized process give rise to full revelation results as derived by the literature on rational expectations for centralized and competitive environments? In the context of the model of this paper, it turns out that the information is not fully revealed to uninformed agents, even when the market is in some sense approximately frictionless.

Information Revelation in a Market with Pairwise Meetings

1. Introduction

In markets in which trade takes place under conditions of asymmetric information the process of trade itself may transmit some of the relevant information, and if agents have the opportunity to extract this information from the process before they trade, the outcomes will reflect this fact.

This observation lies, of course, at the heart of the literature on rational expectations. In the context of a market characterized by centralized trading the price may aggregate and transmit information. The concept of rational expectations equilibrium refers to a centralized market which is also competitive and requires that agents indeed utilize whatever information that can be extracted from the equilibrium price.

However, many of the models in which we are interested are not characterized by centralized trading in the sense that one price is announced for all, and it is therefore of interest to investigate the transmission of information via the trade process in other environments. The present paper investigates the transmission of information through the process of trade in a market in which transactions are conducted in pairwise meetings of agents. The agents have asymmetric information on some underlying parameter which affects the value of the goods throughout the market, and they are aware of the relationship between the value of this parameter and the distribution of agreements reached in the market. The counterpart of extracting information from price in a competitive market takes here the form of sampling alternative trading partners in an attempt to learn from the distribution of their offers about the value of the parameter of interest. Of course, the extraction of the information from the market behavior of others is not as immediate here as is

the extraction of information from price in a competitive market. However, the same issues can be addressed in the context of the present framework: how is the allocation affected by the fact that the trade process serves not only to transfer goods but also to transmit information? and to what extent is the information revealed to the participants?

Regarding the latter question it is obvious that, in as much as the activity of sampling potential partners and finding out about their bargaining positions is time consuming or otherwise costly, one cannot expect that the information will be fully revealed as might be the case in a competitive market. Yet, one can inquire about the extent of information revelation when such market is approximately frictionless in the sense that the cost of time, for example, is negligible.

To address these questions we use a simple model with the following features. There are two populations of agents which will be thought of as sellers, who have one unit of an indivisible good for sale, and buyers who seek to buy one unit. The market operates over time. In each period all agents are matched with agents of the opposite type at random. If two matched agents agree on the terms of the transaction, they exchange the good and leave the market and if they disagree, they stay in the market to be rematched. The asymmetric information is about some parameter which we shall think of as the value of all units of the good traded in this market and which can be either high throughout or low throughout. Some of the agents who enter the market each period know the true value of the good and others do not. The range of possible "bargaining positions" that agents can adopt is also restricted to two: a seller can either insist that the true value is high and demand the high price or be willing to concede to the low price, and similarly a buyer can insist that the true value is low and demand the low price or be willing

to agree to the high price.

We characterize the steady state equilibria of this market. The distribution of the agreements depends on the true value. The uninformed agents are aware of this relationship and their equilibrium behavior incorporates the optimal (given their information) amount of search to learn more about the true value. The force that limits the agents' learning at equilibrium is their impatience which is captured by a constant discount factor δ.

To address the question of whether or not information is fully revealed to the uninformed agents when the market becomes approximately frictionless, we consider a sequence of equilibria that obtains when δ approaches 1. We show that the information is not fully revealed in the sense that a significant fraction of those who are uninformed as they come into the market never learn about the true value and end up transacting at the wrong price. This is because when the market is made frictionless there are two opposing effects. On the one hand, it becomes less costly for an uninformed agent to insist on the more favorable price and collect more observations before he concedes to the less favorable price. On the other hand, when everybody behaves in this manner, the market is filled with uninformed agents who stay on in an attempt to acquire more information. Hence the information to be extracted from each meeting is less and the overall effect need not be full revelation of the information.

The related literature includes three lines of work. The first consists of Laffont and Maskin [1986]. In some sense their paper is the closest in its general motivation to the present paper. While the present paper attempts to explore the extension of the rational expectations ideas to a market characterized by decentralized trading, they explore these ideas in the context of centralized but non-competitive market. The second line consists of the work

that looks into the micro-structure of the rational expectations equilibrium. That is, the manner in which information is aggregated into the price in the trade process in a competitive market. This work includes Hellwig [1982] and Dubey, Geanakaplos and Shubik [1984]. Its relation with the present paper is not immediate and is due to the fact that, under certain interpretation, the equilibrium of a pairwise meetings model of the broad type considered here can be viewed as competitive (see Gale [1987]), and hence the present model can be also viewed as an alternative approach to that work. Finally, in terms of its basic model the paper is related to the growing literature on matching and bargaining models, and out of this it is most closely related to the work of Rosenthal and Landau [1981] and to the work of Samuelson [1987]. These works analyze game theoretic matching models with imperfect information and learning. The information theoretic difference between those papers and the present one is that they are of the independent values variety while ours of the common values variety.

2. The Model

The market is envisioned as an ocean of agents. There are two populations, 1 and 2, with equal numbers. We shall refer to the members of population 1 as sellers who are interested in selling a unit of some indivisible good, and to the members of population 2 as buyers who are interested in buying a unit of the good. We shall think of the population size as a continuum with finite measure. But we shall avoid the complications that arise when one considers a continuum of random variables by assuming that there is no aggregate randomness, so that the random elements that are introduced below are random only from the point of view of the individual.

The interaction in the market takes place in time which is divided into

discrete periods. At each time period each agent is matched with exactly one agent of the other type. In the end of one period a meeting terminates either with an agreement in which case the two agents transact and leave the market, or in disagreement in which case the two agents stay in the market to be rematched. There are constant streams of new arrivals. At each period a measure M of new buyers and a measure M of new sellers join in.

The payoffs to agents who reach an agreement are affected by some parameter which can be thought of as the true value of the good. This parameter is of one of two values H or L, which will be interpreted as the high value and the low value respectively. The true value is the same for all units of the good traded throughout the market and it does not change over time. It is either H for all units throughout or L for all units throughout.

The true state is reflected in the payoffs accruing to agents upon completing a transaction. To understand the role of the true state it is useful to describe the "bargaining" component. It is assumed that a bargaining game between two matched agents takes place within one period and that each of the two agents has only two actions available: to adopt position P_H or to adopt position P_L. The payoffs depend on the positions and on the true state as follows:

<div>

True State H

buyer (player 2)

seller (player 1)		P_H	P_L
	P_H	a_H, b_H	disagree
	P_L	c_H, d_H	e_H, f_H

True State L

buyer (player 2)

seller (player 1)		P_H	P_L
	P_H	f_L, e_L	disagree
	P_L	d_L, c_L	b_L, a_L

</div>

The entry "disagree" corresponds to the disagreement situation upon which the

parties return to the market to be rematched. The other three combinations correspond to agreements. The assumptions are that for $i = H,L$: $f_i \geq d_i \geq b_i > 0$ with at least one inequality strict; $a_i \geq c_i \geq e_i$ with at least one inequality strict; and $e_i < 0$.

The reader need not, of course, memorize this structure. It is best to remember the general relations along the lines of the following interpretation. Position p_H is interpreted as offering or demanding the high price which corresponds to state H, while p_L is interpreted as the low price which corresponds to state L. A seller always prefers the high price to the low price, but he still benefits from trading at p_L when the true state is L. Analogously the buyer always prefers p_L to p_H, but when the true state is H he prefers trade at p_H to no trade at all. The assumptions $e_H < 0$ and $e_L < 0$ mean that selling (buying) at the high value state H (low value state L) for the low (high) price is worse than not trading. The purpose of these assumptions is to rule out a class of pooling equilibria in which there is one price on which all agents agree in both states, since these equilibria are uninteresting for the analysis of the present paper.

To obtain an example that gives rise to such pattern of payoffs, suppose that when the true state is H the seller's cost of providing the good will be y while the buyer's valuation will be $y + 2$. When the true state is L the seller's cost will be $z < y$ and the buyer's valuation is $z + 2$. Think of p_H and p_L as price announcements where $p_H = y + 1$ is the price that halves the surplus when the true state is H and $p_L = z + 1$ halves the surplus when the true state is L. Suppose that the trading mechanism is a double auction with the individuals being limited to announcing p_H and p_L. That is, the price is p_H or p_L if both agents announce that

price and it is $\frac{1}{2}(p_H + p_L)$ if the seller and the buyer announce p_L and p_H respectively.

The payoffs to an agreement as depicted by the above matrices are evaluated at the date at which the agreement is reached. It is assumed that all agents are impatient and discount expected future benefits using the constant discount factor $\delta < 1$.

As mentioned above the true state is the same for all units and all agents. However, not all agents are informed about the true state. It is assumed that a fraction x_1 of the type 1 agents (sellers) and a fraction x_2 of the type 2 agents (buyers) who enter each period know the true state. The rest do not know the true state and upon entry they just have (the same) prior beliefs that the true state is H with probability α_H and L with probability $\alpha_L = 1-\alpha_H$.

A strategy for an agent is a sequence of decision rules specifying the agent's position, p_H or p_L, in each meeting, given his personal history.

A steady state is a lasting situation such that the number of agents of each type and the fraction of agents of each type, who adopt a particular position, are constant over time.

A steady state equilibrium consists of four distributions (for each true state the distribution of each agent type between position p_H and p_L)

	Fraction of sellers adopting:		Fraction of Buyers adopting:	
	p_H	p_L	p_H	p_L
True state H	H_1	$1-H_1$	H_2	$1-H_2$
True state L	$1-L_1$	L_1	$1-L_2$	L_2

and and assignment of strategies to agents such that:

(i) Each agent's strategy is optimal given the above distributions and the agent's assessment of what the true state is.

(ii) When all agents employ the assigned strategies the market is in a steady state: if the true state is H, the steady state distributions will be $(H_1, 1-H_1)$ and $(H_2, 1-H_2)$; if the true state is L, the resulting distributions will be $(L_1, 1-L_1)$ and $(L_2, 1-L_2)$.

3. The Equilibrium

Agents know the distribution of the positions P_H and P_L prevailing among agents of the opposite type, conditional on the true state. Thus, a strategy is optimal for an agent if it maximizes his expected payoff, given these distributions and given what the agent knows about the true state. The derivation of an optimal strategy for an informed agent is a relatively simple problem: given the true distribution (which is known to the agent) and the discount factor δ, there is an optimal position to be adopted in perpetuity. The derivation of the optimal strategy of an uninformed agent is somewhat more complicated, since as the agent proceeds he learns more about the true state and the value of this information has to be accounted for in the choice of strategy. In short, an informed agent faces a problem of search from a known two-point distribution, while an uninformed agent faces a problem of search from an unknown distribution which belongs to a family of two such distributions.

In deriving the optimal strategies the first thing to note is that once a type 1 agent (a seller) adopts P_L or a type 2 agent (a buyer) adopts P_H, they reach agreement immediately. Therefore, the only seller's strategies that have to be considered are of the form: offer P_H n times in a row and

then switch to p_L (n can also be 0 or ∞, where 0 means to adopt p_L from the start, and ∞ means to offer p_H perpetually). Similarly, the relevant buyer's strategies are of the form: offer p_L n times in a row and then switch to p_H. Let us refer to a strategy by the integer n that characterizes it. Let $v_1(Q,n)$ denote the expected value of strategy n to a seller who believes with probability Q that the true state is H.

$$(1) \quad V_1(Q,n) = Q\{H_2 a_H \frac{1-[\delta(1-H_2)]^n}{1-\delta(1-H_2)} + [\delta(1-H_2)]^n[H_2 c_H + (1-H_2)e_H]\}$$

$$+ (1-Q)\{(1-L_2)f_L \frac{1-(\delta L_2)^n}{1-\delta L_2} + (\delta L_2)^n[(1-L_2)d_L + L_2 b_L]\}$$

Define the set $N_1(Q)$ by

$$(2) \qquad N_1(Q) = \underset{n}{\text{ArgMax }} V_1(Q,n)$$

where n can assume the value ∞ as well. Now, an optimal strategy for a seller is characterized by an integer in the appropriate set N_1. For an informed seller this is an integer in $N_1(1)$ if the true state is H, and an integer is $N_1(0)$ if the true state is L. For an uninformed seller this is an integer in $N_1(\alpha_H)$.

Observe from (1) that the set $N_1(Q)$ depends on the parameters $\delta, a_H, c_H, e_H, b_L, d_L, f_L$ and on the endogenous variables H_2 and L_2. Below we shall characterize $N_1(Q)$ for the case $1-H_2 < L_2$, which will turn out to be the only case that prevails in equilibrium. The analysis of the remaining cases is deferred to the proof of proposition 1 in Appendix I. Suppose $1-H_2 < L_2$ and let Q_1 be defined as the solution to

(3) $V_1(Q_1,1) = V_1(Q_1,0)$

By examining (1) for the case $1-H_2 < L_2$, the following facts can be verified: (i) $Q_1 < 1$; (ii) for $Q > Q_1$, $V_1(Q,1) > V_1(Q,0)$; (iii) for $Q < Q_1$, $V_1(Q,1) < V_1(Q,0)$; (iv) if there is some n such that $V_1(Q,n) > V_1(Q,0)$, then $V_1(Q,1) > V_1(Q,0)$. These facts imply that if a seller believes with probability $Q > Q_1$ that the true state is H, it will pay him to adopt p_H for at least one more time. Conversely, if the seller believes in H with probability $Q < Q_1$, he will prefer to switch to p_L immediately. Therefore, the optimal strategy for a seller is to adopt p_H as long as the probability that he assigns to the true state being H is more than Q_1. The optimal strategies for an informed seller are simple: $N_1(1) = \{\infty\}$, since always $Q_1 < 1$; $N_1(0) = \{0\}$ or $\{0,\ldots,\infty\}$ or $\{\infty\}$ according to whether $Q_1 > 0$ or $Q_1 = 0$ or $Q_1 < 0$. The optimal strategy for an uninformed seller is characterized by the minimal integer n such that after sampling n buyers who offered p_L, the updated belief that the true state is H is smaller than Q_1. That is,

(4) $$\frac{\alpha_H(1-H_2)^n}{\alpha_H(1-H_2)^n + (1-\alpha_H)L_2^n} \leq Q_1$$

Now, when $Q_1 > 0$ there is some finite n_1 such that (4) holds (since $1-H_2 < L_2$). Then $N_1(\alpha_H) = \{n_1\}$ or $N_1(\alpha_H) = \{n_1,n_1+1\}$ according to whether n_1 satisfies (4) with strict inequality or with equality. When $Q_1 \leq 0$, the updated probability is never below Q_1 and the optimal strategy corresponds to $n_1 = \infty$.

Since the model treats the two types of agent and the two states symmetrically, the buyer's optimal behavior is completely analogous. Let

strategy n of a buyer be the strategy that prescribes offering p_L n times in a row and then switching to p_H, and let Q denote the probability with which a buyer believes in state L. By exchanging the roles of H and L and the roles of 1 and 2 everywhere in the above discussion surrounding (1)-(4), we get the analogous values $V_2(Q,n)$, $N_2(Q)$ and Q_2. When $1-L_1 < H_1$ the cutoff probability Q_2 can be used to characterize the buyer's optimal strategies in the same manner as Q_1 was used above to characterize the sellers' strategies.

The optimal strategies derived above will now be combined with the steady state conditions to characterize the steady state equilibrium. The steady state conditions require that the distributions as captured by H_1 and L_1 are constant over time and that, in each of the states, the flow of arrivals, M, is equal to the flow of departures. Let K_i, i = H,L, denote the measure of agents of each type who are present in the market at state i = H,L (recall that the number of sellers is equal to the number of buyers). In state H the steady state condition is

$$(5) \qquad M = K_H \, H_1 \, H_2 + K_H(1-H_1).$$

In state L the condition is

$$(6) \qquad M = K_L \, L_1 \, L_2 + K_L(1-L_2).$$

At equilibrium the terms on the RHS of (5) and (6) have to be consistent with all agents pursuing their optimal strategies. To examine the implications of this, let us continue to consider the case in which the equilibrium values of H_1 and L_1 satisfy $1-H_2 < L_2$ and $1-L_1 < H_1$. It

follows from the preceding discussion that, in this case, the optimal strategies can be characterized using the cutoff probabilities Q_1.

If $Q_1 > 0$, then $N_1(\alpha_H) = \{n_1\}$ or $\{n_1, n_1+1\}$ where $0 \leq n_1 < \infty$ and where $\{n_1, n_1+1\}$ corresponds to case in which n_1 satisfies (4) with equality. Thus, the behavior of the uninformed seller is characterized by the interger n_1 and a fraction g_1. This fraction differs from 1 only when $N_1(\alpha_H) = \{n_1, n_{1+1}\}$, in which case g_1 specifies the overall fraction of uninformed sellers who will switch from p_H to p_L after n_1 times, while $1-g_1$ is the fraction of the uninformed sellers who will adopt p_H for one more time. Thus, when the true state is H, the fraction $1-H_1$ of the seller population who at a given period adopt p_L is

$$(7) \qquad 1-H_1 = M(1-x_1) \, [g_1(1-H_2)^{n_1} + (1-g_1)(1-H_2)^{n_1+1}]/k_H$$

This is because the sellers who adopt p_L are exactly those uninformed sellers who entered n_1 (and n_1+1 if $g_1 < 1$) periods ago, did not meet a buyer who would agree to p_H and have just switched to p_L.

When the true state is L, the informed sellers adopt p_L, since $Q_1 > 0$ implies $N_1(0) = \{0\}$. Therefore, the fraction L_1 of the seller population who at a given period adopt p_L is

$$(8) \qquad L_1 = \{x_1 M + (1-x_1)M[g_1 L_2^{n_1} + (1-g_1)L_2^{n_1+1}]\}/k_L$$

where the term $x_1 M$ captures the number of the informed, while the other terms capture as before the number of uninformed who entered n_1 (or n_1+1) periods ago.

If $Q_1 \leq 0$ then $N_1(\alpha_H) = \{\infty\}$ and $N_1(0) = \{\infty\}$ or $\{0, \ldots, \infty\}$

according to whether $Q_1 < 0$ or $Q_1 = 0$. In this case equations (7) and (8) are replaced by

(7B) $H_1 = 1$

and

(8B) $L_1 = r_1 x_1 M/k_L$

where $r_1 \in [0,1]$ is the fraction of the informed sellers who adopt strategy 0, and $1-r_1$ is the fraction who adopt strategy ∞. The fraction $r_1 > 0$ only if $Q_1 = 0$ in which case the informed sellers are indifferent among all strategies.

By complete analogy, when $1-L_1 < H_1$ the optimal buyer strategies can be characterized using Q_2 and the equilibrium distributions are as follows. If $Q_2 > 0$, then

(9) $1-L_2 = (1-x_2) M[g_2(1-L_1)^{n_2} + (1-g_2)(1-L_1)^{n_2+1}]/k_L$

(10) $H_2 = \{x_2 M+(1-x_2) M[g_2 H_1^{n_2} + (1-g_2)H_1^{n_2+1}]\}/k_H$.

If $Q_2 \leq 0$, then these equations are replaced by

(9B) $L_2 = 1$

(10B) $H_2 = r_2 x_2 M/k_H$.

Here too the fractions g_2 and r_2 describe how the relevant population is distributed among alternative strategies when there is indifference.

Consider a particular assignment of values (K_H, K_L, H_i, L_i) $i = 1,2$ such that $1-H_2 < L_2$ and $1-L_1 < H_1$. Substitute H_2 and L_2 into (1), (3) and (4) to obtain Q_2 and n_2. Use Q_i and n_i to obtain the appropriate version of (7) - (10) in the manner described above. Now, if there exist $g_i \in [0,1]$ and $r_i \in [0,1]$ such that (5) - (6) and the appropriate version of (7) - (10) are satisfied and if $g_i < 1$ and $r_i > 0$ only in the cases mentioned above, then the values (K_H, K_L, H_i, L_i), $i = 1,2$, together with the strategies characterized by n_i and Q_i constitute a steady state equilibrium.

Recall that so far the analysis of the optimal strategies and the resulting conditions (7) - (10) has been confined to the case of $1-H_i < L_i$. To complete the equilibrium analysis we have to consider other cases as well. This discussion is deferred to the appendix since it does not contribute directly to the intuitive development of the model. The following proposition (which is proved in Appendix I) summarizes the results of the full equilibrium analysis. In particular, it establishes that the case of $1-H_i < L_i$ developed above is the only possible case.

Proposition 1:

(i) If either $\alpha_H e_H + \alpha_L b_L > 0$ or $\alpha_L e_L + \alpha_H b_H > 0$, then there exists a steady state equilibrium

(ii) The equilibrium values of H_i, L_i, K_H, K_L satisfy (5), (6), $1-H_2 < L_2$, $1-L_1 < H_1$ and in addition one of the systems (7) - (10) or (7B), (8B), (9), (10) or (7), (8), (9B), (10B).

First, observe that since part (ii) of the proposition establishes that at any equilibrium the conditions $1-H_i < L_i$ are satisfied, it immediately implies that the analysis preceding the proposition provides a complete characterization of the equilibrium. Second, consider the role and meaning of the condition stated in part (i) that either $\alpha_H e_H + \alpha_L b_L > 0$ or $\alpha_L e_L + \alpha_H b_H > 0$. The meaning of the converse, $\alpha_H e_H + \alpha_L b_L \leq 0$, is that the expected benefit to an uninformed seller from adopting p_L, evaluated at the prior probabilities is non-positive. The meaning of $\alpha_L e_L + \alpha_H b_H \leq 0$ for the buyer is analogous. If both $\alpha_H e_H + \alpha_L b_L \leq 0$ and $\alpha_L e_L + \alpha_H b_H \leq 0$, then one cannot necessarily rule out a disequilibrium situation in which the uninformed agents never risk adopting the less favorable position (the uninformed sellers and buyers always insist on p_H and p_L respectively) and hence the numbers of uninformed agents present in the market keep growing indefinitely. When, for example, $\alpha_H e_H + \alpha_L b_L > 0$ this situation is prevented, since it guarantees that uninformed sellers would rather risk adopting p_L than not trade at all.

Finally, to conclude the equilibrium analysis recall that an equilibrium has quite a simple structure. At each of the states there is a distribution of sellers who adopt positions p_H and p_L, and a corresponding distribution of buyers. The informed agents know the true state and it is in their interest to demand or offer p_H in state H and p_L in state L. The uninformed do not know the true state, but they know what distribution of offers prevails in each one of the states. They start with the position which is more favorable to them (sellers with p_H; buyers with p_L) and learn about the distribution as they proceed with their search. Once they are confident enough that the opposite state is the true one (after n_i unsuccessful meetings), they switch to the less favorable position and

complete their transactions. The equilibrium conditions guarantee that the endogeneous equilibrium distributions are indeed consistent with the optimum behavior of all agents.

4. Revelation of Information

The only external source of information is the information brought by the informed individuals and it is revealed in the market in the different price distributions that prevail in the different states. In principle, any uninformed individual can learn about the true state to any degree of accuracy by searching the market sufficiently intensively. What limits the extent of the search is the friction captured by the discount factor δ, which induces individuals to compromise on the accuracy of the information and quit searching after drawing a number of samples. Thus, with the frictions in place, it is not surprising that the information is not fully revealed in the sense that a non-negligible percentage of the transactions are carried out at the "wrong" price - a price at which one of the parties to the exchange would refuse to trade if he knew the true state. The main point of this discussion is to inquire to what extent the information is revealed at equilibrium when the frictions are made negligible. Obviously, if everything remains the same and just the discount factor of one uninformed individual is made arbitrarily close to 1, then this individual will probably search for somebody who will be willing to trade at the more favorable terms, but at any rate he will be unlikely to end up transacting at the less favorable terms if they are also wrong. However, when the frictions are made negligible for everybody at once, then the extraction of information from search becomes also more difficult since the market is filled with searchers who would like to acquire more information before they yield to a less favorable price. The questions are:

what percentage of total transactions are made at the "correct" price when the frictions are made negligible? Is the information fully revealed in the sense that almost all transactions are carried out at the "correct" price?

Observe that the fraction of all transactions that are made at the "wrong" price is $(1-x_1)[g_1(1-H_2)^{n_1} + (1-g_1)(1-H_2)^{n_1+1}]$ when the true state is H, and is $(1-x_2)[g_2(1-L_1)^{n_2} + (1-g_2)(1-L_1)^{n_2+1}]$ when the true state is L. This is because in state H, for example, the sellers who sell for the low price are exactly those $M(1-x_1)[g_1(1-H_2)^{n_1} + (1-g_1)(1-H_2)^{n_1+1}]$ uninformed sellers whose attempts to sell for a high price failed for n_1 or n_1+1 periods as might be the case. The question of whether the information is fully revealed at equilibrium reduces to the question of whether the limiting equilibrium values of $(1-H_2)^{n_2}$ and $(1-L_1)^{n_2}$ are positive, when the limit is taken as δ approaches 1. The meaning of positive limits is that, even when the market is approximately frictionless, a non-negligible fraction of all transactions are made at the wrong price. The following proposition asserts that at least one of these limits is indeed positive.

Proposition 2:

Consider a sequence of δ converging to 1 and a corresponding sequence of equilibria such that the limits $H_1, L_1, H_1^{n_2}, L_2^{n_1}, (1-H_2)^{n_1}$ and $(1-L_1)^{n_2}$ exist. Then, at least one of the values $\lim(1-H_2)^{n_1}$ and $\lim(1-L_1)^{n_2}$ is positive.

Proof: Upon substituting (11) and (12) into (8), (8B), (10) and (10B) they can be rewritten as follows. For $n_1 > 0$, (8) and (8B) are rewritten as

$$(13) \quad \frac{1}{1-L_1} = \begin{cases} 1 + \dfrac{g_1 L_2^{n_1} + (1-g_1) L_2^{n_1+1} + \dfrac{x_1}{1-x_1}}{1+L_2 +\ldots+ L_2^{n_1-1} + (1-g_1)L_2^{n_1}} & \text{for } 0 < n_1 < \infty \\[3mm] 1 + \dfrac{r_1 x_1 (1-L_2)}{1-r_1 x_1} & \text{for } n_1 = \infty \end{cases}$$

and for $n_1 = 0$, $L_1 = 1$.

Similarly, (10) and (10B) are rewritten as

$$(14) \quad \frac{1}{1-H_2} = \begin{cases} 1 + \dfrac{g_2 H_1^{n_2} + (1-g_2)H_1^{n_2+1} + \dfrac{x_2}{1-x_2}}{1+H_1 +\ldots+ H_1^{n_2-1} + (1-g_2)H_1^{n_1}} & \text{for } 0 < n_2 < \infty \\[3mm] 1 + \dfrac{r_2 x_2 (1-H_1)}{1-r_2 x_2} & \text{for } n_2 = \infty \end{cases}$$

and for $n_2 = 0$, $H_2 = 1$.

Next, let us utilize the observation that, in each state and each period, the number of buyers who buy at a certain price must be equal to the number of sellers who sell at that same price. In state H the equality between the number of buyers (on the LHS) and sellers (on the RHS) who transact at the low price is given by

$$(15) \quad M(1-x_2)[1-g_2 H_1^{n_2} - (1-g_2)H_1^{n_2+1}] = M(1-x_1)[g_1(1-H_2)^{n_1+1} + (1-g_1)(1-H_2)^{n_1+2}]$$

In state L the equality of numbers of buyers and sellers who transact at the high price is

$$(16) \quad M(1-x_2)[g_2(1-L_1)^{n_2+1} + (1-g_2)(1-L_1)^{n_2+2}] = M(1-x_1)[1-g_1 L_2^{n_1} - (1-g_1)L_2^{n_1+1}]$$

Suppose that both $\lim (1-L_1)^{n_2} = 0$ and $\lim(1-H_2)^{n_1} = 0$. This

supposition has two implications. First, equations (15) and (16) imply that $\lim H_1^{n_2} = 1$ and $\lim L_2^{n_1} = 1$. Second, there cannot be a subsequence over which $L_1 = 1$ since then over this sequence $n_1 = 0$ and $\lim (1-H_2)^{n_1} = 1$, in contradiction to the supposition. Similarly, there cannot be a subsequence over which $H_2 = 1$.

We may now use (13) and (14) to evaluate $\lim(1-L_1)^{n_2}$ and $\lim(1-H_2)^{n_1}$ and it follows that

Claim 3:

(i) If both $\lim L_2^{n_1} = 1$ and $\lim(1-L_1)^{n_2} = 0$, then the sequence $\dfrac{n_2}{n_1}$ approaches ∞.

(ii) If both $\lim H_1^{n_2} = 1$ and $\lim(1-H_2)^{n_1} = 0$, then the sequence $\dfrac{n_1}{n_2}$ approaches ∞.

The details of this proof are deferred to Appendix II. However, the idea is quite simple. When $\lim L_2^{n_1} = 1$ the RHS of (13) is of the order of magnitude of $1 + \dfrac{1}{n_1}$. Therefore, $\lim(1-L_1)^{n_2}$ is of the order of $(1 - \dfrac{1}{n_1})^{n_2}$ and this approaches zero only if $\dfrac{n_2}{n_1} \to \infty$.

Claim 3 contradicts the supposition that both $(1-H_2)^{n_1}$ and $(1-L_1)^{n_2}$ approach zero since it is impossible to have both $\dfrac{n_2}{n_1}$ and $\dfrac{n_1}{n_2}$ approaching infinity.

Q.E.D.

Proposition 2 has established that the revelation of information is incomplete. That is, even when the market is approximately frictionless in the sense that δ is close to 1, at least in one of the states a non-negligible fraction of the transactions are made at the wrong price. The intuition behind the result is evident from the proof. In order for uninformed buyers not to

transact often at the wrong price it has to be that their search is relatively thorough, i.e., n_2 is large, and that L_1 is not too small. But the latter requires that n_1 is not too large so that the uninformed sellers are not too thorough in their search and hence are bound to make mistakes. Finally, notice that it need not be that both $\lim(1-H_2)^{n_1}$ and $\lim(1-L_1)^{n_2}$ are positive. If, for example, α_H is very small it is conceivable that all uninformed agents assume that the true state is L and adopt p_L. In this case $L_1 = 1$ and uninformed buyers never err in buying for p_H when the true state is L.

5. Discussion

The main conclusion from above is that, in a market with decentralized trading in which the uninformed acquire information through searching among potential trading partners, a non-negligible fraction of them may end up transacting without learning the true information, even when the market is approximately frictionless.

Before we proceed to contrast this conclusion with the outcomes of a corresponding centralized trading process, it should be mentioned that the findings of the previous sections are of some independent interest even not in the limit and even without the comparison to centralized trading models. This is because some important markets are characterized by decentralized trading and the trade process is by no means frictionless. To the extent that some of uninformed agents acquire information primarily through search among potential partners, a model of the type presented here might be more appropriate to study such markets.

The relationship with rational expectations equilibrium

The familiar results on information revelation through trading were derived for markets characterized by centralized and competitive trading. Therefore, it is obviously of interest to contrast the result of section 4 with the extent of information revelation in a centralized trading version of the present model. The centralized trading version is such that a single price is announced in each period (either the high or the low price) and sellers and buyers respectively decide whether to sell and buy. The steady-state, rational expectations equilibrium is a price for each state such that: the uninformed know the equilibrium state-price relations and use this knowledge in their decisions; demand is equal to supply in each period; and the flows of departing agents are exactly matched by the flows of entering agents.

It is immediate to verify that the steady state rational expectations equilibrium here is fully revealing: the equilibrium price will be the high price in state H, and the low price in state L. This is so long as the fractions of informed agents x_i are positive and regardless of how small they are. The full revelation of the information in the rational expectations equilibrium is, of course, a well known result and the above paragraph just tells us that it appears in the example pursued here as well. From contrasting the above with the result of proposition 2, we conclude that the equilibrium outcome of a decentralized trading process may not approximate the rational expectations equilibrium of the corresponding centralized trading process, even when the market is approximately frictionless.

Gale [1987] has explained the sense in which the equilibria of decentralized trading process, which take place under conditions of perfect information, approximate the Walrasian outcome when the frictions are small. This relationship is true for the perfect information versions of the present

model as well (i.e., when $x_1 = x_2 = 1$). It can be easily verified that the perfect information version of the decentralized process has a unique equilibrium. In this equilibrium all transactions are made at the high price in state H and at the low price in state L, and this outcome coincides of course, with the steady state competitive equilibrium outcome of the centralized process. Our underlined conclusion suggests that this relationship may not extend to a world of asymmetric information due to the imperfect transmission of information by the decentralized process.

Let us conclude this part with two remarks. First, one should of course be very careful in drawing far reaching conclusions from the crude model of the present paper with its dubious notion of price. The above qualitative conclusion should therefore be viewed as a conjecture based on the intuition developed in this limited model. Second, it should be noted that the sense in which we use the term "frictionless" is very specific. Here this term stands for δ close to 1, which means that individual search activity is relatively not costly. If one thinks instead of a market in which it is possible to acquire pieces of aggregate information about transactions throughout the market, and if "frictionless" means that the acquisition of such information is easy, then it is conceivable that, at the equilibrium of an approximately "frictionless" market, the information will be fully revealed. Thus, the above analysis suggests, at the most, that the fully revealing rational expectations equilibrium may not provide a good approximation for the equilibrium outcome of a decentralized process in which the uninformed have access only to limited samples of personal information. However, the rational expectations equilibrium concept may very well provide a good approximation when agents have access to some pieces of aggregate market information.

The relationship between external information and revelation

Proposition 2 does not capture explicitly, in terms of the parameters of the model, the extent to which the information is not revealed. To get some idea on the magnitude of this phenomenon and on how it depends on other parameters, consider the completely symmetric model in which $x_1 = x_2 = x$, $\alpha_H = \frac{1}{2} = \alpha_L$ and the payoffs are symmetric in the sense that $i_H = i_L$, $i = a,b,\ldots,f$. In this case there exists a symmetric equilibrium in which $H_2 = L_1$, $H_1 = L_2$, $g_1 = g_2 = g$ and $n_1 = n_2 = n$. As $\delta \to 1$, let us look at a sequence of symmetric equilibria and let $k = \lim(1-H_2)^n$ over such a sequence.

Proposition 3:

The limit k exists and satisfies

$$(17) \qquad k = (1-k)^{\frac{1}{k(1-x)} - 1}$$

The proposition is proved in Appendix II. The interest in formula (17) is not in its special features but rather in the qualitative relationship that it implies between k and the percentage of informed agents x. It can be verified that k is a decreasing function of x: the more external information there is, the lower the percentage of transactions carried out at the wrong prices. Further, as x varies between 0 and 1, k attains any value between 1/2 and 0. That is, the magnitude of the phenomenon is non-negligible: when the percentage of informed is close to zero, almost half of all transactions (the half owes to the symmetry assumed here) are made at the wrong price.

Proposition 3 and the accompanying discussion point to another aspect of the comparison between the outcomes of the centralized and decentralized

versions of the present model. In the rational expectations equilibrium of the centralized trading version the information is fully revealed, regardless of how small are the magnitudes of the external information as captured by the x_i's. The fact that the full revelation of information is somehow independent of how widely the information is spread is not special to the centralized trading version of the present model, and similar phenomena appear in fully revealing equilibria in other models. In some sense this feature is an artifact of the too perfect transmission of information through prices. In contrast, as proposition 3 shows, in the decentralized trading process of the present paper the extent of information revelation is closely related to the amount of external information. When x is close to 1, so that most agents are informed, the fraction of uninformed agents who end up transacting at the "wrong" price is close to zero.

Note that the last point can be related to the work of Grossman and Stiglitz [1980]. They consider a market model in which agents either purchase the relevant information or try to extract it from the noisy price, and derive the equilibrium amounts of information acquisition and revelation via the price. One can pursue a similar exercise in the present model by specifying the costs of becoming informed and letting the x_i's be determined in the model by the decisions of the agents. As we saw the extent of revelation here increases with the x_i's. At the equilibrium the x_i's will have to be such that the marginal agent is indifferent between acquiring information or not.

6. Concluding Remarks

The framework of a random pairwise-meetings model already incorporates some special assumptions. On top of these, the foregoing analysis has imposed a number of extra assumptions: the populations of buyers and sellers were

assumed equal; an agent was matched in every period; the menu of bargaining positions was limited to two. Although I did not analyze the model in the absence these assumptions, the intuition that I developed leads me to believe that the first two assumptions simplify the analysis significantly but are probably not essential for the qualitative results. If, for example, the assumption on equal populations and/or the choice of the particular matching technology were relaxed, agents may be able to extract information from the frequency of their meetings. This would imply that an agent's strategy will not be characterized by a single integer, n_i, but rather will also depend on the information that can be learned from the frequency of past meetings as well. Nevertheless, this extra complexity does not seem likely to affect the basic forces that prevent full revelation in the present model. However, I do not know how essential the assumption that limits the range of bargaining positions is. It is not intuitively obvious from our analysis whether or not a richer set of prices will facilitate full revelation. Therefore, this feature of the model presents probably the most pressing need for further investigation.

APPENDIX I

Proposition 1

(i) If either $\alpha_H e_H + \alpha_L b_L > 0$ or $\alpha_L e_L + \alpha_H b_H > 0$, then there exists a steady state equilibrium.

(ii) The equilibrium values of H_1, L_1, K_H, K_L are such that beside the steady state conditions (5), (6) they satisfy: $1-H_2 < L_2$, $1-L_1 < H_1$ and either (7) - (10) or (7B), (8B), (9), (10) or (7), (8), (9B), (10B).

Proof of part (i):

The following claim lists the relevant properties of $N_1(Q)$ which are used in the equilibrium analysis.

Claim 1:

(i) If $1-H_2 < L_2$, $N_1(\alpha_H) = \{n_1\}$ or $\{n_1, n_1+1\}$, where $0 \leq n_1 \leq \infty$

(ii) If $1-H_2 \geq L_2$, $N_1(\alpha_H) = \{0\}$ or $\{\infty\}$ or $\{0, \infty\}$ or $\{0, \ldots, \infty\}$.

(iii) $N_1(1) = \{\infty\}$

(iv) $N_1(0) = \{0\}$ or $\{\infty\}$ or $\{0, \ldots, \infty\}$

(v) Max $N_1(0) \leq$ min $N_1(\alpha_H)$

Proof of Claim 1:

Define

$$Y = H_2 a_H - [1-\delta(1-H_2)] [H_2 c_H + (1-H_2)e_H]$$

$$W = (1-\delta L_2) [(1-L_2)d_L + L_2 b_L] - (1-L_2)f_L$$

Observe from (1) that

$$(A.1) \qquad V_1(Q,n) - V_1(Q,0) = \sum_{i=0}^{n-1} \{[\delta(1-H_2)]^i QY - (\delta L_2)^i (1-Q)W\}$$

By assumptions on the payoffs, Y is always positive, but W can be positive or negative. Obviously,

$$N_1(Q) = \text{Arg Max}_n \ [V_1(Q,n) - V_1(Q,0)].$$

(i) Suppose that $1-H_2 < L_2$ and $0 < Q < 1$. Either $W \leq 0$, in which case it follows from (A.1) that $N_1(Q) = \{\infty\}$. If $W > 0$, then there must be some finite n such that

$$(A.2) \qquad [\delta(1-H_2)]^n QY \leq (\delta L_2)^n (1-Q)W$$

Let n_1 denote the minimal such n, we have $N_1(Q) = \{n_1\}$ if the inequality is strict, and $N_1(Q) = \{n_1, n_1+1\}$ if n_1 satisfies (A.2) with equality.

(ii) Suppose $1-H_2 \geq L_2$ and $0 < Q < 1$. If for some n (A.2) holds, then it must hold for any integer smaller than n and hence $V_1(Q,0) \geq V_1(Q,n)$.

If the reverse of (A.2) holds for some n, then it must hold for any larger integer and hence $V_1(Q,\infty) \geq V_1(Q,n)$. Therefore, either 0 or ∞ or both belong to $N_1(Q)$, depending on whether $V_1(Q,0)$ is greater or smaller than $V_1(Q,\infty)$. If $1-H_2 = L_2$ and $QY = (1-Q)W$, then $N_1(Q) = \{0,\ldots,\infty\}$.

(iii) Substitute $Q = 1$ in (A.1) and observe that $N_1(1) = \infty$

(iv) Substitute $Q = 0$ in (A.1) and observe that $N_1(0) = \{0\}$ or $\{\infty\}$ or $\{0,\ldots,\infty\}$ according to whether $W > 0$ or < 0 or $= 0$.

(v) Recall from (i) that, if $\min N_1(\alpha_H) < \infty$, then $W > 0$. But then $N_1(0) = \{0\}$. Therefore, $\text{Max } N_1(0) \leq \min N_1(\alpha_H)$.   Q.E.D.

Claim 1 presents the different configurations of the sellers' optimal strategies. The implications of these different cases to the relations between H_1 and L_1 were already partly examined in the discussion surrounding (7) - (10) in Section 3 above and will now be fully detailed.

Case A1: $N_1(\alpha_H) = \{n_1\}$ or $\{n_1,n_1+1\}$, where $0 \leq n_1 < \infty$.

From parts (iv) and (v) of claim 1 we have $N_1(0) = 0$. Therefore, as explained in section 3

$$(7) \qquad 1-H_1 = M(1-x_1)[g_1(1-H_2)^{n_1} + (1-g_1)(1-H_2)^{n_1+1}]/K_H .$$

and

(8) $\qquad L_1 = \{x_1 M + (1-x_1) M [g_1 L_2^{n_1} + (1-g_1) L_2^{n_1+1}]\}/K_L$

where $g_1 < 1$ only if $N_1(\alpha_H) = \{n_1, n_1+1\}$.

Case B1: $N_1(\alpha_H) = \{\infty\}$.

Obviously, (7) is replaced by

(7B) $\qquad H_1 = 1$

The counterpart of (8) depends on the nature of $N_1(0)$. If $N_1(0) = \{0, \ldots, \infty\}$, letting r_1 denote the fraction of the informed who choose p_L upon entry where the remaining fraction, $1-r_1$, choose p_H forever, equation (8) is replaced by

(8B) $\qquad L_1 = r_1 x_1 M/K_L$.

If $N_1(0) = \{0\}$, then (8) is replaced by (8B) with $r_1 = 1$. If $N_1(0) = \{\infty\}$, then (8) is replaced by

(8BB) $\qquad L_1 = 0$

Case C1: $N_1(\alpha_H) \equiv \{0, \infty\}$ or $\{0, 1, \ldots, \infty\}$.

From parts (iv) and (v) of claim 1 we have $N_1(0) = \{0\}$. If fraction t_1 of the uninformed choose p_L upon entry (the $n_1 = 0$ strategy) and the remaining fraction $1-t_1$ choose p_H in perpetuity (the $n_1 = \infty$ strategy),

then (7) and (8) are replaced by

(7C) $\quad 1-H_1 = (1-x_1)Mt_1/K_H$

(8C) $\quad L_1 = [x_1M + (1-x_1)Mt_1]/K_L.$

Since the model treats the two types of agent and the two states symmetrically, the different configuration of buyer strategies are derived by complete analogy (just interchange everywhere the roles of 1 and 2, H and L). Cases A2, B2 and C2 below are analogous to A1–C1 above.

Case A2: $N_2(\alpha_L) = \{n_2\}$ or $\{n_2, n_2+1\}$, where $0 \leq n_2 < \infty$.

(9) $\quad 1-L_2 = (1-x_2)M[g_2(1-L_1)^{n_2} + (1-g_2)(1-L_1)^{n_2+1}]/K_L$

(10) $\quad H_2 = \{x_2M + (1-x_2)M[g_2H_1^{n_2} + (1-g_2)H_1^{n_2+1}]\}/K_H$

where g_2 is the fraction of uninformed buyers who adopt strategy n_2 when $N_2(\alpha_L) = \{n_2, n_2+1\}$.

Case B2: $N_2(\alpha_L) = \{\infty\}$.

(9B) $\quad L_2 = 1$

If $N_2(0) = \{0, \ldots, \infty\}$,

(10B) $\quad H_2 = r_2 x_2 M/K_H,$

where r_2 is the fraction of entering informed buyers who choose p_H when the true state is H.

If $N_2(0) = \{0\}$, then (10) is replaced by (10B) with $r_2 = 1$. If $N_2(0) = \{\infty\}$, then (10) is replaced by

(10BB) $H_2 = 0$.

Case C2: $N_2(\alpha_L) = \{0,\infty\}$ or $\{0,\ldots,\infty\}$

(9C) $1-L_2 = (1-x_2)Mt_2/K_L$

(10C) $H_2 = [x_2M + (1-x_2)Mt_2]/K_H$

where t_2 is the fraction of the uninformed buyers who adopt strategy $n_2 = 0$, while the remaining fraction $1-t_2$ adopt strategy $n_2 = \infty$.

Finally, recall from section 3 the two steady state conditions (5) and (6)

(5) $M = K_H H_1 H_2 + K_H(1-H_1)$

(6) $M = K_L L_1 L_2 + K_L(1-L_2)$

An equilibrium is an assignment of values (K_H, K_L, H_i, L_i) $i = 1,2$ such that when all agents employ optimal strategies satisfying (1) - (2), the steady state conditions (5) - (6) and the appropriate version of (7) - (10) are satisfied, where the selection of the appropriate version of (7) - (10) is as described by cases A-C above.

<u>**Claim 2**</u>: If either $\alpha_H e_H + \alpha_L b_L > 0$ or $\alpha_L e_L + \alpha_H b_H > 0$, then there exist (K_H, K_L, H_i, L_i), $i = 1, 2$, and $n_1 \in N_1(\alpha_H)$, $n_2 \in N_2(\alpha_L)$, g_i, r_i and t_i in $[0,1]$ such that the steady state conditions (5) - (6) and the appropriate version of system (7) - (10) are satisfied.

Proof: Consider the following point to set mapping from $[0,1]^4$ into its power set. For a given 4-tuple (H_1, H_2, L_1, L_2) use (1) - (2) to calculate $N_1(\alpha_H)$, $N_2(\alpha_L)$ and $N_i(0)$. Let the set G_1 be $\{1\}$ if $N_1(\alpha_H) = \{n_1\}$ and let $G_1 = [0,1]$ if $N_1(\alpha_H) = \{n_1, n_1+1\}$. Similarly, let $G_2 = \{1\}$ if $N_2(\alpha_L) = \{n_2\}$ and $G_2 = [0,1]$ if $N_2(\alpha_L) = \{n_2, n_2+1\}$. If $(H_1, H_2) \neq (1,0)$, solve (5) to obtain

(11) $K_H = M/[H_1 H_2 + (1-H_1)]$

If $(L_1, L_2) \neq (0,1)$, solve (6) to obtain

(12) $K_L = M/[L_2 L_1 + (1-L_2)]$.

Follow the instructions in cases A-C to obtain the appropriate version of (7) - (10). Substitute (11), (12), H_i, L_i, n_i $i = 1,2$ into the RHS of the chosen version of (7) - (10). Let $\hat{H}_i, \hat{L}_i$ denote the sets of H_i's and L_i's obtained from the LHS of this four equation system when g_i is varied over G_i, r_i and t_i are varied over $[0,1]$.

The above defines a point-to-set mapping that maps (H_1, H_2, L_1, L_2) such that $(H_1, H_2) \neq (1,0)$ and $(L_1, L_2) \neq (0,1)$ to $(\hat{H}_1, \hat{H}_2, \hat{L}_1, \hat{L}_2)$. This correspondence will be extended continuously by mapping $(H_1, H_2) = (1,0)$ to $(\hat{H}_1, \hat{H}_2) = (1, \frac{1}{(1-x_2)(n_2+1-g_2)+1})$ and $(L_1, L_2) = (0,1)$ to

$(\hat{L}_1,\hat{L}_2) = (\frac{1}{(1-x_1)(n_1+1-g_1)+1},1)$. The correspondence is convex valued since $\hat{H}_i$ and $\hat{L}_i$ are either singletons or closed intervals. Over the ranges in which $N_1(\alpha_H)$ and $N_2(\alpha_L)$ are singletons the correspondence is in fact a continuous function. The discontinuity points of this function are where either one of the sets N_i is not a singleton. But these gaps are filled by letting g_i,r_i and t_i range over $[0,1]$. Therefore, the correspondence is upper semi-continuous. Thus, the correspondence satisfies the conditions of Kakutani's fixed point theorem (see, e.g., Todd [1976]) and hence has a fixed point.

For the fixed point to be part of the desired solution it has to be that $(H_1,H_2) \neq (1,0)$ and $(L_1,L_2) \neq (0,1)$. Suppose to the contrary that the fixed point has $(H_1,H_2) = (1,0)$. This implies that $n_1 = \infty$, $n_2 = \infty$ and $(L_1,L_2) = (0,1)$. It then follows from (1) that $V_1(\alpha_H,\infty) = 0$ and similarly $V_2(\alpha_L,\infty) = 0$. Observe that $V_1(\alpha_H,0) = \alpha_H e_H + \alpha_L b_L$ and $V_2(\alpha_L,0) = \alpha_L e_L + \alpha_H b_H$. Hence, it follows from the assumption that either $V_1(\alpha_H,0) > 0 = V_1(\alpha_H,\infty)$ or $V_2(\alpha_L,0) > 0 = V_2(\alpha_L,\infty)$, in contradiction to $N_1(\alpha_H) = N_2(\alpha_L) = \{\infty\}$. Therefore, both $(H_1,H_2) \neq (1,0)$ and $(L_1,L_2) \neq (0,1)$. It follows that a fixed point together with the associated values of K_H,K_L,n_i,g_i,r_i and t_i constitute a solution such that $n_1 \in N_1(\alpha_H)$, $n_2 \in N_2(\alpha_L)$ and the appropriate version of (5) - (10) is satisfied.

Q.E.D.

The claim concludes the proof of part (i) since, by construction, a solution to conditions (5) - (6) and the appropriate version of (7) - (10) already involves the use of optimal strategies and hence is an equilibrium.

Proof of part (ii):

Suppose to the contrary that at equilibrium $H_1 \leq 1-L_1$. If Max $N_1(\alpha_H) < \infty$ and

$L_2 > 1-H_2$, then

$$L_1 > M(1-x_1)[g_1 L_2^{n_1}+(1-g_1)L_2^{n_1+1}]/K_L > M(1-x_1)[g_1(1-H_2)^{n_1}+(1-g_1)(1-H_2)^{n_1+1}]/K_L$$

$$> M(1-x_1)[g_1(1-H_2)^{n_1} + (1-g_1)(1-H_2)^{n_1+1}]/K_H = 1-H_1,$$

where the first inequality follows from (8), the second follows from $L_2 > 1-H_2$, the third follows from $H_1 \leq 1-L_1$, $L_2 > 1-H_2$, (11) and (12), and the last equality follows from (7). But $L_1 > 1-H_1$ contradicts the supposition.

If $N_1(\alpha_H) = \{\infty\}$ then $H_1 = 1$ and hence $H_1 \leq 1-L_1$ implies $L_1 = 0$. Now, it may not be that $N_2(\alpha_L) = \{\infty\}$ since then $L_2 = 1$, but at equilibrium $(L_1,L_2) \neq (0,1)$ for otherwise (5) fails to hold. It also may not be that $N_2(\alpha_L) = \{0,\infty\}$, since $H_1 = 1$ and $L_1 = 0$ together with (5) and (6) imply that $K_H H_2 = K_L(1-L_2)$ in contradiction to (7C) and (8C).

The above eliminations leave only the case $N_1(\alpha_H) = \{0,\infty\}$ or $\{0,\ldots,\infty\}$ and the case Max $N_1(\alpha_H) < \infty$ with $L_2 \leq 1-H_2$. Part (ii) of claim 1 implies that in the first case $L_2 \leq 1-H_2$, and in the second case $N_1(\alpha_H) = \{0\}$. Hence, in the first case the relevant equations are (7C) and (8C), while in the second case the relevant equations are (7) and (8) with $n_1 = 0$ and $g_1 = 1$. Observe that, when $H_1 \leq 1-L_1$, both (7C), (8C) and (7), (8) with $n_1 = 0$ and $g_1 = 1$ imply $K_L > K_H$.

Thus, the supposition $H_1 \leq 1-L_1$ implies $L_2 \leq 1-H_2$ and $K_L > K_H$. But by permuting the above arguments (exchanging the roles of H and L and the roles of 1 and 2), $L_2 \leq 1-H_2$ implies $H_1 \leq 1-L_1$ and $K_H > K_L$, contradiction. Therefore, at equilibrium $1-H_2 < L_2$ and $1-L_1 < H_1$.

Now, $1-H_2 < L_2$ and $1-L_1 < H_1$ together with part (ii) of claim 1 rule out any one of the equations (7BB) - (10BB) and (7C) - (10C). Next note that (7B) - (10B) may not prevail together. This is because (7B) and (9B) imply $H_1 = L_2 = 1$ which together with (5) and (6) imply $K_H H_2 = K_L L_1 = M$, in contradiction to (8B) and (10B).

Q.E.D.

Appendix II

Proofs of Claim 3 (in Proposition 2) and Proposition 3:

Both proofs use the results collected in the following lemma.

Lemma:

Let $\{z_n\}$ be a sequence such that for all n, $0 \le z_n \le 1$. Suppose that $\lim z_n^n = z > 0$, and let R be a constant. Then,

(i) $$\lim_{n \to \infty} (1 + R(1-z_n))^n = z^{-R}$$

(ii) $$\lim (1 + \frac{z_n^n + R}{1+z_n +...+ z_n^{n-1}})^n = \begin{cases} z^{-(z+R)/(1-z)} & \text{if } z < 1 \\ \\ e^{1+R} & \text{if } z = 1 \end{cases}$$

Proof of the lemma:

(i) Consider the sequence $\{n(1-z_n)\}$. It must be bounded from above. Otherwise, for any F there is an n such that $n(1-z_n) > F$ and hence there is a subsequence such that $z_n < 1 - \frac{F}{n}$ implying that $z = \lim z_n^n \le \lim(1 - \frac{F}{n})^n = e^{-F}$ for any arbitrarily large F, in contradiction to $z > 0$. Let v be a cluster point of the sequence $n(1-z_n)$, and consider

a subsequence converging to   v.   For any ε there is $N(\varepsilon)$ such that for

$n > N(\varepsilon)$

$$1 - \frac{v+\varepsilon}{n} \leq z_n \leq \frac{v-\varepsilon}{n}$$

Therefore, for any cluster point   v,

$$z = \lim z_n^n = \lim (1 - \frac{v}{n})^n = e^{-v}$$

which means that $v = - \ell nz$ and that $\lim n(1-z_n)$ exists and is equal to

v.   Therefore,

$$\lim (1+R(1-z_n))^n = \lim[1+Rn(1-z_n)/n]^n = e^{-R\ell nz} = z^{-R}$$

(ii)   If $z = 1$ then for any $\varepsilon > 0$ and sufficiently large   n,

$$1 + \frac{z_n^n + R}{n} \leq 1 + \frac{z_n^n + R}{1+z_n +\ldots+ z_n^{n-1}} \leq 1 + \frac{z_n^n + R}{n(1-\varepsilon)}$$

By raising to the power of   n   and taking limits we get that the desired limit

is sandwiched between e^{1+R} and $e^{(1+R)/(1-\varepsilon)}$ and hence it is e^{1+R}.

If $z < 1$,   rewrite

$$\lim(1 + \frac{z_n^n + R}{1+z_n +\ldots+ z_n^{n-1}}) = \lim [1 + \frac{z_n^n + R}{1 - z_n^n} \cdot \frac{n(1-z_n)}{n}]^n .$$

Now, it follows from (i) that the last expression is equal to $z^{-(z+R)/(1-z)}$.

<div align="right">Q.E.D.</div>

Proof of Claim 3:

Raise both sides of (13) to the power of n_2.   Since $\lim(1-L_1)^{n_2} = 0$

the RHS of (13) raised to the power of n_2 approaches ∞. This implies that n_2 approaches ∞. If the sequence of n_1 is bounded from above, then clearly $\dfrac{n_2}{n_1}$ approaches ∞. Suppose that there is a subsequence of n_1 that approaches ∞ and a number F such that over this subsequence $\dfrac{n_2}{n_1} < F$. From parts (i) and (ii) of the lemma, we have that the RHS of (13) when raised to the power of n_1 approaches some finite number 1 or $e^{1/1-x_1}$. Therefore, when it is raised to the power of $n_2 \leq n_1 F$ and the limit is taken over this subsequence, it will not exceed 1 or $e^{F/(1-x_1)}$ contradiction. Therefore, $\dfrac{n_2}{n_1}$ is unbounded over any subsequence.

The second part of the claim is completely analogous. It just uses (14) instead.

$$Q.E.D.$$

Proof of Proposition 3:

In the symmetric equilibrium $H_1 = L_2$, $H_2 = L_1$, $n_1 = n_2 = n$ and $g_1 = g_2 = g$. In this case the appropriate version of (14) is

$$(A.3) \qquad \frac{1}{1-H_2} = 1 + \frac{gH_1^n + (1-g)H_1^{n+1} + \frac{x}{1-x}}{1+H_1 +\ldots+ H_1^{n-1} + (1-g)H_1^n}$$

Consider a subsequence such that $\lim(1-H_2)^n = k$. Proposition 2 implies that $k > 0$ and (15) implies that $\lim H_1^n = 1 - \lim(1-H_2)^n = 1-k$. Raising both sides of (A.3) to the power of n and applying part (ii) of the lemma we have

$$k = (1-k)^{\frac{1}{k(1-x)} - 1}$$

Since this equation has a unique solution, it must be that $\lim(1-H_2)^n$ exists and equal to this solution.

$$Q.E.D.$$

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