Product Variety and Imperfectly Competitive Free-Entry Industries: Policy Design, Conjectural Equilibria and Consumption Externalities

G D Myles

University of Warwick

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by

G D Myles*

University of Warwick

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*Postal Address: Department of Economics, University of Warwick, Coventry, CV4 7AL

This paper is circulated for discussion purposes only and its contents should be considered preliminary.

ABSTRACT

This paper analyses the properties of a model of imperfect competition in conjunction with a preference for product variety. When the consumer treats product variety as parametric, a tax scheme is described that generates a socially optimal equilibrium from market behaviour. Welfare-improving and optimal commodity taxes are also discussed.

An alternative, conjectural, definition of equilibrium is introduced; for a single consumer model this is argued to result in greater variety and utility. If further consumers are introduced, variety causes externalities in consumption; their effects are analysed and policies to overcome them discussed.

When production involves a fixed cost, and consumers possess a preference for variety, there will always be a conflict between the exploitation of economies of scale and the provision of numerous product variants. A body of literature has developed that seeks to determine the efficiency of the free-market in resolving this conflict; the focus being placed on whether too 'much' or too 'little' variety is produced, see Spence (1976), Dixit and Stiglitz (1977) and Hart (1985). However, the implications of these results for policy design have received little attention. Perry (1984) provides a characterisation of when policy should aim to expand or contract the industry in question, but without describing how this should be achieved. Two modelling issues also arise in this context: with the exception of Hart (1985) the models represent the demand side by a single, aggregate, consumer and this consumer is assumed to act in an entirely passive manner, treating available product variety as parametric. This paper aims to make progress on each of these issues.

The central theme is the policy treatment of industries of the form described above. Optimal taxation is discussed in the context of a standard, single aggregate consumer, model of free-entry Cournot oligopoly . A distinction is drawn between policy design when a lump-sum subsidy to fixed costs is an available policy instrument and policy choice when it is not. With the lump-sum subsidy available I demonstrate that with the correct choice of policy, the market equilibrium achieves socially optimal levels of product variety. Without the subsidy, policy design requires the extension of recent results on optimal commodity taxation with imperfect competition (Myles (1987)a, b) to encompass the importance of variety. Following this, an alternative definition of equilibrium is discussed; this involves the consumer forming a conjecture regarding the functional relationship between the level of his demand and the resulting number of active firms. It is argued that if these conjectures are rational, or 'correct', the conjectural equilibrium will have greater product variety than the standard equilibrium.

Finally, when further consumers are introduced into the model, variety has the nature of a public good with externalities linking consumers. As each consumer will ignore the externality effect of their demand choice upon their peers, it is proved that the market equilibrium has less variety, and utility, than the welfare-maximising equilibrium with optimally chosen demands. A number of policy schemes are described that generate this social optimum from market behaviour.

The structure of the model is discussed in Section I. Section II presents the standard model; existence of equilibrium is demonstrated for a restricted case and optimal taxation is described. The conjectural equilibrium is introduced in Section III, again for a single consumer. Further consumers are introduced in Section IV and the externality effect of variety is analysed. Conclusions are contained in Section V.

I. DISCUSSION OF MODEL

In constructing the model I have attempted to product the simplest structure that is able to capture the features of interest: imperfectly competitive production and preference for variety. Accordingly, I have made two strong assumptions; this section discusses these and places them into context.

The production side of the model consists of an imperfectly competitive industry, which permits free-entry and employs a production process that involves a fixed set-up cost, and a competitive industry, producting with constant returns to scale, whose output is taken to be numeraire. I choose to assume that the imperfectly competitive industry can be modelled as a quantity-setting Cournot oligopoly that faces as single aggregate demand function; in effect, each firm will charge the same price in equilibrium. As the example presented below will illustrate this formulation is not inconsistent with the notion of product differentiation.

Demand originates from a single utility maximising consumer, with a fixed endowment of income, in Section II and III. In Section IV further consumers are introduced. The second substantive assumption is to represent the preference for variety by introducing n, the number of active firms in the oligopoly, directly into the utility function; in detail, I assume

$$U = U(X, n, Y) \qquad (1)$$

so that utility is a function of consumption, X, of the oligopoly's output, the number of firms, n, and total consumption, Y, of the numeraire. Before the specification of utility is discussed, note that (1) produces a demand function as described above; maxmisation of (1) will face the oligopoly with a single aggregate demand. In turn, this supports an equilibrium with a single price.

(1) assumes that the consumer is concerned with aggregate consumption and the number of firms in the market, as opposed to the standard specification which assumes utility to be a function of the consumption of each of the n goods. The inclusion of n to represent the preference for variety is the most direct approach that can be taken; Ireland (1985) presents a variant of this approach. As a modelling aid it removes problems concerning 'neighbouring' good effects, this is achieved by enforcing an extreme form of symmetry between goods. The major advantage however, is the clarity that it gives the value of extra variety. It also enables straightforward specification of social optima.

To illustrate these points consider an example: let X represent a quantity of restaurant meals, and n the number of alternative restaurants. The assumptions I have placed upon industrial conduct imply that, although firms may supply either French, Italian or Chinese cuisine, each regards itself to be active simply in the market for restaurant meals, and treats other restaurants equally as competitors. Now consider the consumer. At any point in time he may be indifferent as to precisely which restaurant he patronises on any given evening, but in the passage of time, it is reasonable to expect that he would choose to dine in a number of different restaurants and that his utility would be increased by doing so. This is the scenario that is envisaged by the assumptions of the model.

Finally in support of (1), if the outputs of the n firms are labelled $x_1, \ldots, x_n$, note that any utility function that satisfies

$$U = U(x_1, \ldots, x_n, Y) = U(\underset{\sim}{P} \cdot \underset{\sim}{x}, Y) \qquad (2)$$

for all permutation matrices P, can be written

$$U = U(x, Y) = U(\frac{X}{n}, Y) = U(X , n, Y) \qquad (3)$$

when $x_i = x$, all $i = 1, \ldots, n$. This condition will be satisfied in the model due to the assumption of Cournot behaviour and the single equilibrium price. Symmetric functions have been commonly employed in previous analyses of this form of model.

II. TAXATION IN THE ONE-CONSUMER MODEL

This section will follow the practice of previous analyses of free-entry oligopoly and assume that the single consumer treats the number of active firms as parametric. This assumption will be relaxed in the next section. The aim here is to introduce the method of analysis adopted, stressing the relationship between derivatives of demand and utility, to provide an existence proof for a simplified case and to consider the optimal tax treatment of free-entry industries.

The model consists of a single consumer who allocates a fixed income M for expenditure on goods X and Y to maximise a strictly concave utility function $U = U(X, n, Y)$. X is an homogeneous good produced by a Cournot oligopoly of n firms into which entry is unrestricted. Each firm i in this industry has cost function $C(x^i)$, x^i being the firm's output, with $C(0) > 0$, and $C' > 0$. In contrast, Y is produced by a perfectly competitive industry operating with constant returns to scale. Y acts as a numeraire throughout and its pre-tax price is normalised at unity.

II.1 Social Optimum (SO) and Market Equilibrium (ME)

The market equilibrium is described by three conditions: profit maximisation of individual firms, the free-entry rule and the consumer's budget constraint. In order to describe these it is first necessary to derive the demand function facing the industry.

Treating n as parametric, the consumer faces the problem:

$$\max_{X,Y} U(X, n, Y) \quad \text{s.t } M = p_X X + Y$$

Optimal choices are described by the two conditions

$$p_X = U_X/U_Y, \quad M = p_X X + Y \tag{4}$$

and hence

$$p_X = \frac{U_X[X, n, M - p_X X]}{U_Y[X, n, M - p_X X]}$$

which is an implicit representation of the direct, and inverse, demand functions. The first derivative of inverse demand is given by:

$$\frac{dp_X}{dX} = \frac{U_{XX} - U_{XY}p_X - p_X U_{XY} + p_X^2 U_{YY}}{U_Y + XU_{XY} - p_X XU_{YY}} < 0 \qquad (5)$$

Recalling that I am treating the industry as a Cournot oligopoly with homogenous output, each individual firm's profit maximisation is described by:

$$p_X - C' + x^i \frac{dp_X}{dX} = 0, \; i = 1, \ldots, n \qquad (6)$$

Entry is constrained by the rule

$$x^i p_X - C(x^i) = 0, \; i = 1, \ldots, n \qquad (7)$$

and the consumer's budget constraint governs total demand:

$$M = \sum_{i=1}^{n} x^i . \, p_X + Y \qquad (8)$$

Since all the active firms produce with the same cost function it is reasonable to assume that the equilibrium described by

(6), (7) and (8) will be symmetric, hence $x^i = x$ all $i = 1, \ldots, n$ and $X = nx$. Note that the equality in (7) implies that n is being treated as a continuous variable, if it is sufficiently large this should be a valid approximation.

Using (4) and (5) the three conditions for market equilibrium may finally be written:

Market Equilibrium (ME)

(a) $$\frac{U_X}{U_Y} - C' + x \frac{[U_{XX} - 2(\frac{U_X}{U_Y}) . U_{XY} + (\frac{U_X}{U_Y})^2 U_{YY}]}{[U_Y + nxU_{XY} - (\frac{U_X}{U_Y}) nxU_{YY}]} = 0$$

(b) $$C - x(\frac{U_X}{U_Y}) = 0$$

(c) $$M = nx(\frac{U_X}{U_Y}) + Y$$

a, b and c constitute a three-equation system in the three dependent variables x, n and Y. Existence of such a solution will be discussed in the next sub-section.

The social optimum involves choosing the number, n, of operational firms, allocating to each a level of output, x, to be produced and using any residual income for production of numeraire.

This optimum is the solution to

$$\max_{x,n,Y} L = u(xn,\ n,\ Y) + \lambda(M - nC(x) - y)$$

Eliminating λ from the necessary conditions:

Social Optimum (SO)

(α) $U_X - U_Y C' = 0$

(β) $xU_X + U_n - U_Y C = 0$

(γ) $M = nC + Y$

Before discussing existence of market equilibrium I will first present a result concerning the possible positioning of the ME and SO in $x - n - Y$ space. Although the derivation is obvious this result is of importance for the existence of tax schemes that replicate the SO from market behaviour.

Result 1

The ME and SO occur on the same $x - n - Y$ transformation surface.

Proof

Substituting (b) in (c) for ME gives (γ) for SO.

This result reveals that a policy designed to move the market equilibrium closer to the socially desirable one will only involve moving the equilibrium of the economy around the transformation frontier rather than first having to change from one transformation to another or from the interior to the boundary.

II.2 Existence of Market Equilibrium

As the system represented by (a), (b) and (c) is rather impenetrable in its most general form I will instead consider the existence problem for a slightly restricted variant. Despite this restriction the analysis will still indicate the critical dependence of equilibrium on the shape of the consumers' indifference curves.

I will restrict the utility function to be additive in Y, so

$$U = U(nx, n) + Y$$

where I am retaining the assumption that the equilibrium involves equal output from all active firms. With square brackets indicating functional dependence, the description of market equilibrium is reduced to the two equation system:

(i) $$C[x] - x\, U_X[nx, n] = 0$$

(ii) $$U_X[nx, n] - C'[x] + xU_{XX}[nx, n] = 0$$

To prove existence of a solution to (i) and (ii) six further assumptions are required. These are:

(1) $\gamma U_X[n\gamma, n] \to \infty$ as $\gamma \to 0$, for $0 \leqslant n \leqslant \infty$

(2) $\delta U_X[n\delta, n] \to 0$ as $\delta \to \infty$, for $0 \leqslant n \leqslant \infty$

(3) $x^2 U_{XX} + x U_{Xn} < 0$ all $0 < x < \infty$, $0 < n < \infty$

(4) $xC'(x) \to \infty$ as $x \to \infty$, $xC'' + x > 0$

(5) $\exists \xi < \infty$ s.t. $|U_{XX}[\alpha_1, \alpha_2]| < \xi$ all $0 \leqslant \alpha_1 \leqslant \infty$, $0 \leqslant \alpha_2 \leqslant \infty$

(6) $x^m U_X[nx^m, n] - C(x^m) > 0$ when x^m satisfies

$$U_X[nx^m, n] - C'[x^m] + x^m U_{XX}[nx^m, n] = 0$$

$$2U_{XX}[nx^m, n] - C''[x^m] + x^m U_{XXX}[nx^m,] < 0$$

and $n = 1$.

Assumptions 1 and 2 constrain the behaviour of marginal utility, in a sense this must change faster than the quantity of the good. (3) states that marginal utility must decrease as more firms are introduced whilst holding the output of each firm constant and (5) places a bound on the rate of decrease of marginal utility. Assumption (4) limits returns to scale and (6) states that if the industry were monopolised, the monopoly would make a strictly positive profit.

These assumptions are now used to prove the following theorem:

Theorem 1

Under assumptions A1 - A6 there exists a solution x^*, n^* to (i) and (ii).

Proof

Throught the proof I will consider only $n > 1$. Fix n at 1. By A1, as $x \to 0$, $xU_X[x, 1] \to \infty$ and A2 => as $x \to \infty$ $xU_X[x, 1] \to 0$, also $xU_X > 0$ all x. As xU_X is continuous it must intersect $C(x)$, at a value of x, say x^1. Now consider $n \to \infty$, $C(x) > 0$ by assumption. Assume $x \to \delta > 0$ as $n \to \infty$. Hence $xU_X[nx, n] \to 0$ as $x \to \delta$, and $n \to \infty$, but this implies $C - xU_X > 0$ sox $< \delta$. But δ was arbitrary, thus $x \to 0$. For each n there can be associated a value of x, x^n. By A3 and $0 > C' > \infty$ these are so $x < \delta$. But δ was arbitrary, thus $x \to 0$. For each n there can be associated a value of x, x^n. By A3 and $0 > C' > \infty$ these are one-to-one with n and, using the implicit function theorem, $\exists$ a function $\theta(n)$ s.t. $x = \theta(n)$ with $x^1 = \theta(1)$ etc. and $(\theta(1) > 0$, $\lim_{n \to \infty} \theta(n) = 0$.

Now write (ii) as $xU_X - xC' + x^2U_{XX} = 0$. Again take $n = 1$. By A1 as $x \to 0$, $xU_X \to \infty$ and, by A5, $x^2U_{XX} \to 0$. Therefore $\lim_{x \to 0} xU_X + x^2U_{XX} = \infty$, similarly $\lim_{x \to \infty} xU_{XX} + x^2U_{XX} = -\infty$. By A4 $xC'(x)$ is continuous and strictly monotonic in x, therefore (ii) possesses a solution $x^1 > 0$. Consider $xU_X + x^2U_{XX}$ as $n \to \infty$. For any $x = \varepsilon > 0$, $\varepsilon U_X[n\varepsilon, n] \to 0$ and $\varepsilon^2U_{XX}[n\varepsilon, n] \to \nu < 0$ hence for $x > 0$, $\lim_{n \to \infty} xU_X + x^2U_{XX} < 0$. But $\varepsilon C'(\varepsilon) > 0$ all $\varepsilon > 0$, therefore

$x < \epsilon$ if (ii) is to be satisfied. Again ϵ was arbitrary, therefore $x \to 0$, as $n \to \infty$ employing the implicit function theorem again $\exists \phi(n)$ s.t. $x = \phi(n)$ with $\phi(1) > 0$, $\lim_{n\to\infty}\phi(n) = 0$.

Again, take $n = 1$. By definition $x = \phi(1) = x^m$ and $C(\theta(1)) - \theta(1)U_X[\theta(1), 1] = 0$. Using A6 $\theta(1) > \phi(1)$. Now consider some sequence $n^1, \ldots, n^k, \ldots$ with $n^k > n^{k-1} > \ldots > n^1$ and $n^k \to \infty$ as $k \to \infty$ and assume there corresponds to this a sequence $x^1, \ldots, x^k, \ldots,$ with $x^k < x^{k-1} \ldots < x^1$, $x^k \to 0$ as $k \to \infty$ such that x^k, n^k satisfy both (i) and (ii). Now let $\lim_{k\to\infty} x^k U_X[n^k x^k, n^k] = \delta$ say which, if (i) is to hold must equal $\lim_{k\to\infty} C(x^k) > 0$. But $\lim_{k\to\infty} x^k(C'(x^k) = 0$ and $\lim_{k\to\infty}(x^k)^2 U_{XX}(n^k x^k, n^k) = 0$. Therefore x^k, n^k cannot satisfy both (i) and (ii), in fact if two sequences are considered $x^{\theta k}$ and $x^{\phi k}$ $\exists k^*$ such that $x^{\theta k^*} < x^{\phi k^*}$. Hence $\theta(n^{k^*}) < \phi(n^{k^*})$.

As θ and ϕ are both continuous, combining $\theta(1) > \phi(1)$ and $\theta(n^{k^*}) < \phi(n^{k^*})$ implies $\exists n^*$ such that $\theta(n^*) = \phi(n^*)$ and $\theta(N^*) = \phi(N^*)$ and $x^* = \theta(n^*) = \phi(n^*)$ which is the required solution.

To extend this method of proof to the model described by (a), (b) and (c), first note that

$$nx\left(\frac{UX}{UY}\right) + Y$$

is strictly increasing in Y for given x and n. Hence, for all x, n (c) has a unique solution for Y, writing this as

$$Y = g(x, n, M)$$

$g(\cdot)$ can be substituted in (a) and (b) to eliminate Y. A proof can then be constructed as above. Although conceptually straightforward, the details of this proof would be rather awkward and it is not attempted here.

II.3 Optimal Taxation : Lump-Sum Taxes Available

The presence of fixed costs are critical to the existence of a variety problem, without them it would be possible for the market to supply an unlimited number of product variants. In a similar manner, the possibility for a tax scheme to subsidise, or even add to, the burden of fixed costs is a critical determinant of the effectiveness of policy. Consequently, the discussion of optimal taxation presented here is divided into two parts. This sub-section will analyse tax schemes where the lump-sum tax/subsidy is a valid policy option, in the next it will be unavailable.

I shall now analyse two candidates for tax schemes that generate the social optimum from the market equilibrium. The first proposition to note is:

Proposition 1

An output tax on X coupled with a subsidy to fixed costs, when restricted to meet a balanced budget, cannot generate the social optimum from the market equilibrium.

Proof

Writing the subsidy paid to each firm as S and the output tax as t_X, the market equilibrium becomes the solution to:

(a') $$\frac{U_X}{U_Y} - C' - t_X + x \frac{[U_{XX} - 2(\frac{U_X}{U_Y}) U_{XY} + (\frac{U_X}{U_Y})^2 U_{YY}]}{[U_Y - (\frac{U_X}{U_Y}) nxU_{YY} + nxU_{XY}]} = 0$$

(b') $$t_X x + C - x \frac{U_X}{U_Y} - S = 0$$

(c') $$M = nx(\frac{U_X}{U_Y}) + Y + nS - t_X xn$$

Note that due to the balance budget requirement, $nS = t_X xn$, S and t_X will cancel from (b') and (c'). The scheme described cannot then affect x and n in the manner required.

The second alternative is to subsidise fixed costs and place taxes on both Y and X. Maintaining budget balance between the three policy instruments allows two degrees of independence and, as proposition 2 demonstrates, allows the desired modification of the market equilibrium.

Proposition 2

Employing a tax on Y, an output tax on X and a subsidy to fixed cost it is possible to replicate the social optimum from the market equilibrium.

Proof

If the tax on Y is written t_Y, the consumer chooses X to maximise U(X, n, Y) subject to $M - Sn = q_X X + (1 + t_Y)Y$, where q_X is the post tax price of X. Proceeding in the manner used to derive (5)

$$\frac{dq_X}{dX} = (1 + t_Y) \frac{[U_{XX} - 2(\frac{U_X}{U_Y}) U_{XY} + (\frac{U_X}{U_Y})^2 U_{YY}]}{[U_Y + nxU_{XY} - nx (\frac{U_X}{U_Y}) U_{YY}]}$$

Using this definition the market equilibrium is the solution to:

(a") $$(1 + t_Y) \frac{U_X}{U_Y} - C' + x.(1+t_Y) \frac{[U_{XX} - 2(\frac{U_X}{U_Y}) U_{XY} + (\frac{U_X}{U_Y})^2 U_{YY}]}{[U_Y + nxU_{XY} - nx (\frac{U_X}{U_Y}) U_{YY}]} = 0$$

(b") $$t_X x + C - x(1 + t_Y)(\frac{U_X}{U_Y}) - S = 0$$

(c") $$M = nx(1 + t_Y)(\frac{U_X}{U_Y}) + (1 + t_Y) Y + nS - t_X xn - t_Y Y$$

In (c") the budget balance condition allows the final three terms to be eliminated, using (b") demonstrates that it describes an identical production constraint to that of S.O. Using budget-balance again allows S to be eliminated from (a") and (b"), that is, write

$$S = t_X x + \frac{t_Y Y}{n} .$$

Thus

$$\text{(a")} \qquad (1 + t_Y) \frac{U_X}{U_Y} - C' + x.(1+t_Y) \frac{[U_{XX} - 2(\frac{U_X}{U_Y}) U_{XY} + (\frac{U_X}{U_Y})^2 U_{YY}]}{[U_Y + nxU_{XY} - nx (\frac{U_X}{U_Y}) U_{YY}]} = 0$$

and

$$\text{(b")} \qquad C - x(1 + t_Y)(\frac{U_X}{U_Y}) + \frac{t_Y Y}{n} = 0.$$

Setting the latter equation equal to S.O.(β) gives

$$t_Y = \frac{[U_Y x - U_X x - U_n]}{[\frac{U_Y Y}{n} - U_Y x]}$$

where all terms are evaluated at the socially optimal values. Substituting this definition of t_Y into (a") gives the desired value of t_X. From budget balance S can be determined and the equations for M.E. then describe the social optimum.

In summary, it has been demonstrated that if a lump-sum subsidy is available as a policy option then it is possible to design a tax scheme that generates the social optimum from market behaviour.

II.4 Commodity Taxation

The exact replication described in proposition 2 is only possible when the lump-sum subsidy is available as a policy instrument. If it is not available such replication is ruled out and the important task is to determine those factors that influence the direction of taxation. To proceed I will first analyse the direction of welfare-improving tax changes from an initial zero-tax position and then discuss optimal commodity taxes, in both cases the assumption that there is a single consumer implies that the arguments that follow are concerned solely with efficiency.

The direction of welfare-improving tax changes are the solution to:

$$\text{Find } dt_X,\ dt_Y \quad \text{s.t.} \quad dV > 0,\ dR = 0$$

where

$$V = V(q_X, q_Y, n, M)$$

is the consumer's indirect utility function and

$$R = t_X X + t_Y Y$$

Note that via (a) and (b) q_X is dependent upon both t_X and q_Y. Calling the derivatives $\partial q_X/\partial q_Y$ and $\partial q_X/\partial t_X$ h_1 and h_2

respectively,

$$dV = \left[\frac{\partial V}{\partial q_Y} + \frac{\partial V}{\partial q_X} h_1 + \frac{\partial V}{\partial n}\frac{\partial n}{\partial t_Y}\right] dt_Y + \left[\frac{\partial V}{\partial q_X} h_2 + \frac{\partial V}{\partial n}\frac{\partial n}{\partial t_X}\right] dt_X \quad (9)$$

and, from the budget constraint,

$$0 = Xdt_X + Ydt_Y.$$

Hence, $dt_X < 0$ if

$$Y\left[\frac{\partial V}{\partial q_X} h_2 + \frac{\partial V}{\partial n}\frac{\partial n}{\partial t_X}\right] - X\left[\frac{\partial V}{\partial q_Y} + \frac{\partial V}{\partial q_X} h_1 + \frac{\partial V}{\partial n}\frac{\partial n}{\partial t_Y}\right] < 0 \quad (10)$$

Now using Roy's identity, this can be written as:

$$Xh_1 + Y(1-h_2) + \frac{\partial V}{\partial n}\left(\frac{\partial V}{\partial M}\right)^{-1}\left[\frac{Y}{X}\frac{\partial n}{\partial t_x} - \frac{\partial n}{\partial q_Y}\right] < 0 . \quad (11)$$

Evidently, if variety was of no concern, i.e. $\frac{\partial V}{\partial n} = 0$, the corresponding condition would be

$$dt_X < 0 \quad \text{if} \quad Xh_1 + Y(1 - h_2) < 0 \quad (12)$$

[cf. eq(23) Myles (1987a)].

The interesting question here is whether the preference for variety increases the likelihood of subsidising X production. Contrasting (11) and (12) this will indeed be the case if

$$\frac{Y}{X}\frac{\partial n}{\partial t_X} - \frac{\partial n}{\partial q_Y} < 0. \quad (13)$$

As

$$\frac{\partial n}{\partial q_Y} = \frac{1}{x}\frac{\partial X}{\partial q_Y} - \frac{\dfrac{\partial^2 X}{\partial q_Y \partial q_X}}{2\dfrac{\partial X}{\partial q_X} + (q_X - t_X - C'\,\dfrac{\partial^2 X}{\partial q_X^2} - C''\,(\dfrac{\partial X}{\partial q_X})^2} \qquad (14)$$

and

$$\frac{\partial n}{\partial t_X} = \frac{1}{x}\frac{\partial X}{\partial q_X} - \frac{(n-1)\,\dfrac{\partial^2 X}{\partial q_X^2}}{2\dfrac{\partial X}{\partial q_X} + (q_X - t_X - C')\,\dfrac{\partial 2X}{\partial q_X^2} - C''(\dfrac{\partial X}{\partial q_X})^2} \qquad (15)$$

noting that the denominator of the second term in both (14) and (15) is $(\frac{\partial X}{\partial q_X})^2$ times the second-order condition for profit maximisation of each firm, sufficient conditions for inequality (13) to be satisfied are:

$$\frac{\partial^2 X}{\partial q_Y \partial q_X} > 0, \quad \frac{\partial^2 X}{\partial q_X^2} < 0.$$

The conditions are fairly mild, for instance a linear demand function will satisfy them and, as (14) and (15) make clear, they can be weakened still further.

The conclusion to be drawn from this brief analysis is that the preference for variety does indeed increase the possibility that taxation should move towards the subsidisation of X production, provided that (13) is satisfied.

Optimal commodity taxes are found by maximising

$$L = V(q_X, q_Y, n, M) + \lambda(t_X X + t_Y Y)$$

with respect to t_X and t_Y. The necessary conditions for this maximisation are:

$$\frac{\partial V}{\partial q_X} h_1 + \frac{\partial V}{\partial q_Y} + \frac{\partial V}{\partial n}\frac{\partial n}{\partial q_Y} + \lambda[t_X \frac{\partial X}{\partial q_X} h_1 + t_X \frac{\partial X}{\partial q_Y} + t_Y \frac{\partial Y}{\partial q_X} h_1 + t_Y \frac{\partial Y}{\partial q_Y}$$

$$+ Y + t_X \frac{\partial X}{\partial n}\frac{\partial n}{\partial t_Y} + t_Y \frac{\partial Y}{\partial n}\frac{\partial n}{\partial t_Y}] = 0$$

$$\frac{\partial V}{\partial q_X} h_2 + \frac{\partial V}{\partial n}\frac{\partial n}{\partial t_X} + \lambda\ [X + t_X \frac{\partial X}{\partial q_X} h_2 + t_Y \frac{\partial Y}{\partial q_X} h_2 + t_X \frac{\partial X}{\partial n}\frac{\partial n}{\partial t_X}$$

$$+ t_Y \frac{\partial Y}{\partial n}\frac{\partial n}{\partial t_X}\ ] = 0$$

and

$$t_X X + t_Y Y = 0$$

Eliminating λ and t_Y from these equations the solution for t_X may be written

$$t_X = \frac{X[\frac{\partial V}{\partial q_X} h_1 + \frac{\partial V}{\partial q_Y} + \frac{\partial V}{\partial n}\frac{\partial n}{\partial q_Y}] - Y[\frac{\partial V}{\partial q_X} h_2 + \frac{\partial V}{\partial n}\frac{\partial n}{\partial t_X}]}{A} \tag{16}$$

where

$$A = h_1[Y.[\frac{\partial V}{\partial q_X}\frac{\partial X}{\partial n}\frac{\partial n}{\partial t_X} - \frac{\partial V}{\partial n}\frac{\partial n}{\partial t_X}\frac{\partial X}{\partial q_X}] + X.[\frac{\partial V}{\partial n}\frac{\partial n}{\partial t_X}\frac{\partial Y}{\partial q_X}$$

$$- \frac{\partial V}{\partial q_X}\frac{\partial Y}{\partial n}\frac{\partial n}{\partial t_X}]] + h_2[Y.[\frac{\partial V}{\partial q_Y}\frac{\partial X}{\partial q_X} + \frac{\partial V}{\partial n}\frac{\partial n}{\partial q_Y}\frac{\partial X}{\partial q_X} - \frac{\partial V}{\partial q_X}\frac{\partial X}{\partial q_Y}$$

$$- \frac{\partial V}{\partial q_X}\frac{\partial X}{\partial n}\frac{\partial n}{\partial t_Y}] + X.[\frac{\partial V}{\partial q_X}\frac{\partial Y}{\partial q_Y} + \frac{\partial V}{\partial q_X}\frac{\partial Y}{\partial n}\frac{\partial n}{\partial t_Y} - \frac{\partial V}{\partial q_Y}\frac{\partial Y}{\partial q_X}$$

$$- \frac{\partial V}{\partial n}\frac{\partial n}{\partial q_Y}\frac{\partial Y}{\partial q_X}]] + [Y.[\frac{\partial V}{\partial q_Y} + \frac{\partial X}{\partial n}\frac{\partial n}{\partial t_X} + \frac{\partial V}{\partial n}\frac{\partial n}{\partial t_X}\frac{\partial X}{\partial q_Y}]$$

$$- X[\frac{\partial V}{\partial q_Y}\frac{\partial Y}{\partial n}\frac{\partial n}{\partial t_X} + \frac{\partial V}{\partial n}\frac{\partial n}{\partial t_X}\frac{\partial Y}{\partial q_Y}]] \qquad (17)$$

The factors determining the sign of the numerator of (16) have been discussed above. However, if optimal taxes are to possess the sign of welfare-improving taxes A must be negative. Unfortunately, as inspection of (17) makes clear, this is not a fact that can be established at this level of generality. This leaves the awkward conclusion that the direction of welfare-improving taxes and the sign of optimal ones need not be in agreement.

III. AN ALTERNATIVE, CONJECTURE-BASED, EQUILIBRIUM

The form of equilibrium analysed in Section II, and in previous discussions of this model, suffers from the defect that the consumer is assumed to act in too passive a manner. In analysing the maximisation that led to (4), n was treated as parametric and hence a demand function of the form $X = X(p_X, n)$ derived. However, given the structure of the model, it appears more appropriate that the consumer, rather than treating n as parametric, should be aware that the level of his demand for X affects the equilibrium number of firms n via the action of the market.

To analyse this form of awareness, I will assume that it is possible to associate with the consumer a conjecture function, $\psi(p_X, X)$, where, at price p_X and demand X, $n = \psi(p_X, X)$ is the number of active firms the consumer believes will exist. To avoid the problem of falsification of beliefs, I will further assume that the conjectures always prove correct, as the model is developed it will be appreciated that this is not an unreasonable requirement.

Two propositions are stated below, Proposition 3 demonstrates that incorporating the conjecture into the consumer's maximisation results in higher demand for X when $\frac{\partial \psi}{\partial X} > 0$, at given n and p_X, than for the maximisation of Section II. Proposition 4 is concerned with finding conditions for which the equilibrium number of firms increases with demand, the conditions found are in fact very weak. Taking these propositions together, it is argued that the conjectural equilibrium will have greater product variety and utility than the standard equilibrium.

III.1 Consumer's Choice for the Two Equilibria

For the model including conjectures the consumer's choice problem is defined as:

$$\max_{X,Y} \; U(X, \psi(X, p_X), Y) \qquad \text{s.t. } M = p_X X + Y \tag{18}$$

where the substitution $n = \psi(X, p_X)$ has been made. Optimal choices,

X^*, Y^*, are characterised by

$$U_X + U_n\psi_X = U_Y p_X, \ M = p_X X^* + Y^* \tag{19}$$

It is evident that this maximisation will lead to different choices than that of Section III, calling those arising from (4), when $n = \tilde{n}$, $\tilde{X}$ and $\tilde{Y}$, proposition 3 follows:

Proposition 3

At given p_X and $\tilde{n} = \psi(X^*, p_X)$, if $\psi_X > 0$ then

$\tilde{X} < X^*$.

Proof

Substituting for $\tilde{n}$ in (4)

$$U_X[\tilde{X}, \psi(X^*, p_X), Y] = p_X U_Y[\tilde{X}, \psi(X^*, p_X), \tilde{Y}]$$

and, from (19),

$$U_X[X^*, \psi(X^*, p_X), Y^*] + U_n[X^*, \psi(X^*, p_X), Y^*] \ \psi_X[X^*, p_X]$$

$$= p_X U_Y[X^*, \psi(X^*, p_X), Y^*]$$

Combining the two expressions

$$\frac{U_X[\psi(X^*,p_X),\tilde{Y}]}{U_Y[\tilde{X},\psi(X^*,p_X),\tilde{Y}} = \frac{U_X[X^*,\psi(X^*,p_X),Y^*]}{U_Y[X^*,\psi(X^*,p_X),Y^*]} + \frac{U_n[X^*,\psi(X^*,\psi(X^*,p_X),Y^*]}{U_Y[X^*,\psi(X^*,p_X),Y^*]} \tag{22}$$

Assume $\tilde{X} = X^*$, thus $\tilde{Y} = Y^*$. However, to then satisfy (22) implies $U_n = 0$. Therefore $\tilde{X} \neq X^*$. Now take $\tilde{X} > X^*$ and $\tilde{Y} < Y^*$. These imply $\tilde{U}_X < U_X^*$ and $\tilde{U}_Y > U_Y^*$ provided U_{XX}, $U_{YY} < 0$, $U_{XY} > 0$ which I assume hold. With these values

$$\frac{U_X^*}{U_Y^*} > \frac{\tilde{U}_X}{\tilde{U}_Y} \Rightarrow \frac{U_n^*}{U_Y^*} < 0.$$

Hence $X^* > \tilde{X}$.

As Proposition 3 is dependent upon ψ_X being positive, the validity of this assumption must now be demonstrated.

The solution of (19) will be a demand function

$$X = e(p_X)$$

Now consider an expansion of this function to

$$X' = (1 + \varepsilon)\, e(p_X), \quad \varepsilon > 0$$

These two demand functions will lead to distinct market equilibria;

one with n firms, the other with n' firms. Rather than demonstrate directly that $n' > n$, I proceed as follows: Let $\varepsilon \to 0$ then $\psi_X > 0$ is equivalent to $dn/d\varepsilon \,|_{\varepsilon=0} > 0$. Proposition 4 provides a characterisation of the conditions for which $dn/d\varepsilon \,|_{\varepsilon=0} > 0$

Proposition 4

$\frac{dn}{d\varepsilon} > 0$ when $C''n + (n+1)\,\phi' + n\phi'' < 0$ where $\phi = e^{-1}(\frac{X}{1+\varepsilon})$.

Note that this condition is fairly mild. As $\phi' < 0$ it is certainly satisfied by constant marginal cost $(C'' = 0)$ and concave $(\phi'' \leq 0)$ demand.

Proof

The proof involves working through the comparative statics of the free-entry model. First invert $X = (1+\varepsilon)\,e(p_X)$ to give

$$p_X = e^{-1}\left(\frac{X}{1+\varepsilon}\right) = \phi\left(\frac{X}{1+\varepsilon}\right)$$

The expressions equivalent to (6) and (7) are:

$$\phi\left(\frac{nx}{1+\varepsilon}\right) x - C(x) = 0$$

and

$$\phi(\frac{nx}{1+\varepsilon}) - C'(x) + \frac{x}{1+\varepsilon}\phi'(\frac{nx}{1+\varepsilon}) = 0$$

Taking the total differential of these and forming a matrix expression

$$\begin{bmatrix} \phi + \frac{xn}{(1+\varepsilon)}\phi' - C' & \frac{x^2}{1+\varepsilon}\phi' \\ \frac{(n+1)\phi'}{(1+\varepsilon)} - C'' + \frac{xn\phi''}{(1+\varepsilon)}\frac{x}{(1+\varepsilon)}\phi' & + \frac{x^2}{(1+\varepsilon)^2}\phi'' \end{bmatrix} \begin{bmatrix} dx \\ dn \end{bmatrix}$$

$$= \begin{bmatrix} \frac{x^2 n\phi'}{(1+\varepsilon)^2} \\ \frac{nx^2\phi''}{(1+\varepsilon)^3} + \frac{(n+1)}{(1+\varepsilon)^2} x\,\phi \end{bmatrix} d\varepsilon \qquad (23)$$

Evaluating at $\varepsilon = 0$ and solving

$$dn = \frac{1}{|A|}\cdot \; [C'' x^2 n\phi' + (n+1)x\phi' [\phi - C'] + x^2 n\phi'' [\phi - C']]\; d\varepsilon$$

where $|A|$ is the determinant of the matrix in (23); if the model is stable $|A| > 0$. Also, from profit maximisation $\phi - C' = x\phi'$. Hence

$$\frac{dn}{d\varepsilon} = \frac{1}{|A|} x^2\phi' [C'' n + (n+1)\phi' + n\phi'']$$

As $\phi' < 0$ this completes the proof.

The implications of these propositions are discussed in the following sub-section.

III.2 Comparison of Standard and Conjectural Equilibria

It would be satisfactory to provide a definite statement on the relative values of endogenous values for the two equilibria. However, this does not appear possible without placing further restriction upon the model. Instead, I will provide an argument to support the claim that the conjectural equilibrium has a greater number of active firms, there is no claim that this constitutes a rigorous proof.

The difficulty involves in comparing the equilibrium is that (4) generates a demand function $X = X(p_X, n)$, whereas the form of function from (19) is $X = e(p_X)$; the different dimensionality of these prevents direct comparison. Proposition 3 has given one point of comparison and it is this that will be exploited below.

Inverting the two demand functions to give

$$p_X = \pi(X, n)$$

and

$$p_X = \sigma(X)$$

repsectively, the two equilibria are characterised by

$$\left.\begin{array}{l} \pi(X, n)\, X - nC(\tfrac{X}{n}) = 0 \\ n\pi(X, n) - nC'(\tfrac{X}{n}) + X\pi_X(X, n) = 0 \\ M = \pi(X, n)\, X + Y \end{array}\right\} \quad \text{Standard}$$

$$\left.\begin{array}{l} \sigma(X)\, X - nC(\tfrac{X}{n}) = 0 \\ n\sigma(X) - nC'(\tfrac{X}{n}) + X\sigma_X(X) = 0 \\ M = \sigma(X)\, X + Y \end{array}\right\} \quad \text{Conjectural}$$

Calling the equilibrium values for the conjectural model X^*, n^*, proposition 3 implies that $\pi(X^*, n^*) < \sigma(X^*)$ [at price $p_X = e(X^*)$, proposition 3 => $\tilde{X} < X^*$ is chosen for maximisation (4), hence to obtain $\tilde{X} = X^*$, the price, say $\tilde{p}_X$, must be less than $e(X^*)$, but $\tilde{p}_X = \pi(X^*, n^*)$.]

Now consider $\pi(X^*, n^*)\, X^* - n^*C(\frac{X^*}{n^*})$. As $\sigma(X^*)\, X - nC(\frac{X^*}{n}) = 0$

by the definition of equilibrium, and as $\pi(X^*, n^*) < \sigma(X^*)$,

$$\pi(X^*, n^*)\, X^* - n^*C(\frac{X^*}{n^*}) < 0$$

X^*, n^* are hence not an equilibrium of the standard system. Now,

$$\frac{\partial}{\partial n} [\pi(X, n)\, X - nC(\tfrac{X}{n})] = X\,\pi_n - C + \frac{X}{n} C'$$

Assuming $xC'(x) > C(x)$, evaluated at $x = \frac{X^*}{n^*}$, then this derivative is positive. Similarly,

$$\frac{\partial}{\partial X}\left[\pi(X, n)\, X - nC\left(\frac{X}{n}\right)\right] = \pi + X\pi_X - C' < 0.$$

As any standard equilibrium $\tilde{X}$, $\tilde{n}$ must satisfy $\pi(\tilde{X},\tilde{n})\, \tilde{X} - \tilde{n}C\left(\frac{\tilde{X}}{\tilde{n}}\right) = 0$ these derivates suggest that such an equilibrium is reached from X^*, n^* by increasing n and reducing X. The claim that the conjectural equilibrium utility follows from the claim that the consumer cannot be made one off by accounting for the dependence of variety upon demand.

IV SOCIALLY INEFFICIENT CONSUMPTION DECISIONS

When the conjectural model developed above is extended to accommodate more than one consumer an interesting feature becomes apparent: when any consumer changes the level of their demand this affects variety which, in turn, has a [illegible]pon the utility of all consumers. In essence, because of the preference for variety, there is an externality linking consumers. Obviously, in planning their consumption no consumer will take account of this effect.

If variety increases with demand, the externality will be positive and this suggests that X will be under-demanded; Proposition 5 demonstrates that this is indeed the case. Proposition 6 then follows almost trivially from 5: as the market demand for X is too low the social optimum, restricted in a sense made precise below, has both greater variety and utility than the market equilibrium. This result is of some interest for the debate on the efficiency of the free-market in producing the correct level of variety.

This section concludes with a brief review of compensation and taxation schemes that generate the Restricted Social optimum from market behaviour.

IV.1 Inefficient Consumption

The Restricted Social Optimum analysed below is constructed as follows: each consumer h is instructed to present to the market a pair of socially optimal demand functions, $X^h = \theta^h(p_X, M^h)$, $Y^h = \eta^h(P_X, M^h)$, and the market then operates freely to determine the equilibrium allocation n, X^h, X^h, all h. It is evident that this optimum will have greater utility than the market equilibrium, but less than the optimum which results when n, X^h, Y^h, all h, are chosen subject only to the aggregate budget constraint.

To provide the simplest discussion of the consumption inefficiency I will assume that there are H identical consumers each with a utility function of the form

$$U = U(X^h, n, Y^h)$$

and a fixed income M. Social welfare is a utilitarian sum

$$S.W. = \sum_{h \in H} U(X^h, n, Y^h)$$

The effect of changes in consumers' demands upon the number of active firms is again captured by a conjecture function, is again assumed to be the correct conjecture, of the form

$$n = \phi(X, p_X) = \phi(\sum_{h \in H} X^h, p_X)$$

I will also assume that the condition given in proposition 3 is binding; $\phi_X > 0$. With this definition each consumer's utility function may be written

$$U = U(X^h, \phi(\sum_{h \in H} X^h, p_X), Y^h)$$

The maximising choices X^h, Y^h are determined by

$$U_Y p_X = U_X + U_n \phi_X, \quad M = p_X X^h + Y^h \qquad (24)$$

The important result is the following:

Proposition 5

The restricted socially optimal demand function $X^h = \theta^h(p_X, M^h)$ for each consumer $h \in H$ is such that $\theta^h(p_X, M^h) > e^h(p_X, M^h)$, all p_X, where $e^h(p_X, M^h)$ is the demand determined by (22).

Proof

One further assumption is required for the proof; this places a bound upon the convexity of $\phi(X, p_X)$:

<u>Assumption 'C'</u>

For all $X^* < \tilde{X}\frac{M}{p_X}$, $\phi_X(H\tilde{X}, p_X) < H\phi_X(HX^*, p_X)$

If $H = 1$ this is equivalent to requiring that $\phi(\cdot)$ is at most linear, as H increases greater convexity is permitted.

The restricted socially optimal demand functions are the solution to:

$$\max_{X^h,\ Y^y,\ h=1,\ldots,M} \sum_{h \epsilon H} U(X^h, h, Y^h)$$

$$\text{s.t.} \quad M = p_X X^h + Y^h \qquad h = 1, \ldots, H$$

$$n = \phi(\sum_{h \epsilon H} X^h, p_X)$$

Focussing on consumer 1, X^1 and Y^1 solve:

$$\max_{X^1, Y^1} U(X^1, \phi(X^1 + \sum_{h=2}^{H} X^h, p_x), Y^1) + \sum_{h=2}^{H} U(X^h, \phi(X^1 + \sum_{j=2}^{H} X^h, p_X), Y^h)$$

$$\text{s.t.} \quad M = p_X X^1 + Y^1$$

Hence X^1, Y^1 satisfy

$$p_X U_Y = U_X + U_n \psi_X + \sum_{h=2}^{H} U_n \psi_X, \quad M = p_X X^1 + Y^1$$

As the consumers are identical and the utility functions strictly concave the optimal allocation will be such that $X^h = X^*$, $Y^h = Y^*$ all h. The above equation then becomes:

$$p_X U_Y = U_X + HU_n \psi_X, \quad M = p_X X^* + Y^* \tag{25}$$

Now call the solution to (24) $\tilde{X}$, $\tilde{Y}$, all h. I wish to show that $X^* > \tilde{X}$ given p_X.

First, assume $X^* = \tilde{X} \Rightarrow Y^* = \tilde{Y}$ and $\psi^* = \psi(X^*, p_X) = \psi(\tilde{X}, p_X) = \tilde{\psi}$. Hence, $\tilde{U}_X = U_X^*$, $\tilde{U}_Y = U_X^*$, $\tilde{U}_n = U_n^*$ and $\tilde{\psi}_X = \tilde{\psi}_X$ but these cannot satisfy (24) and (25) simultaneously, hence $\tilde{X} \neq X^*$.

Now assume $\tilde{X} > X^* \Rightarrow \tilde{Y} < Y^*$ and $\psi^* < \tilde{\psi}$. Hence $\frac{U_X}{\ddot{U}_Y} < \frac{U_X^*}{U_Y^*}$ which, if (24) and (25) are to be satisfied $\Rightarrow \tilde{\psi}_X > H\psi_X^*$ as $\frac{\tilde{U}_n}{\tilde{U}_X} < \frac{U_n}{U_Y^*}$. But this contradicts assumption C'. Therefore, $\tilde{X} > \ddot{X}$ for all $\alpha > p_X > 0$. Associating a value of X^* to p_X by the function $\theta^h(p_X, M^h)$, for income M^h, and $\tilde{X}$ by $e^h(p_X, M^h)$ the proposition follows.

The next proposition follows trivially:

Proposition 6

The number of active firms n^* determined by the restricted social optimum is greater than the number $\tilde{n}$ resulting from the market equilibrium.

Proof

To derive proposition 5 I assumed n increased with demand (as discussed in proposition 4). As $\sum_{h \epsilon M} \theta^h(p_X, M^h) > \sum_{h \epsilon M} e^h(p_X, M^h)$ all p_X, $n^* > \tilde{n}$.

One final point is worth making: I have assumed to this point that $\psi_X > 0$. However, the same conclusion to proposition 6 would hold if $\psi_X < 0$. In that case assume 'C' would be changed to $\tilde{\psi}_X > H\psi_X^*$, this would imply $\theta^h(p_X, M^h) < e^h(p_X, M^h)$ and then by $\psi_X < 0$, $R^* > \tilde{n}$.

IV.2 Compensation and Taxation Schemes

Proposition 5 and 6 have demonstrated how the free-market consumption decisions of consumers are inefficient; an alternative set of feasible decisions have been shown to exist which result in a higher level of utility for each consumer. This section reviews three policy schemes designed to overcome this inefficiency. Each of the schemes modifies the consumer's decision problem in such a way that the restricted social optimum is generated from market behaviour.

The first scheme involves side-payments between consumers:

each consumer receives a payment K from each of his peers for each unit of X he purchases, in return he pays out K for each unit bought by other consumers. Under these rules his decision problem is:

$$\max_{X^h,\ Y^h} U(X^h, \psi(\sum_{h\epsilon M} X^h, p_X), Y^h)$$

$$\text{s.t. } M + (H - 1)\ KX^h = p_X X^h + \sum_{i\epsilon H, i\neq h} KX^i$$

Retaining the assumption that consumers are identical $X^h = X^i = X$ all $i \epsilon H$ and the solution to the maximisation is determined by

$$U_X = U_n\psi_X = U_Y p_X - U_Y(H-1)K,\ M = p_X X + Y \qquad (26)$$

Now setting $K = \frac{U_n^*}{U_Y^*}\psi_X^* > 0$, U_n^*, U_Y^* and ψ_X^* the values at the solution to (25), generates (25) from (26) and the consumer will then choose the optimal demands.

An identical result is achieved by a lump-sum tax T on each consumer and a subsidy payment L for each unit of X consumed. The budget constraint becomes

$$M - T = (p_X - L)\ X^h + Y^h$$

but, as the government budget is balanced, the solution to the maximisation with this constraint is:

$$U_X + U_n \psi_X = U_Y(p_X - L), \quad M = p_X X + Y$$

The required values are then $L = (H - 1) \dfrac{U_n^* \psi_X^*}{U_Y^*}$ and $T = LX$.

The same result may also be achieved by commodity taxation when forward-shifting by the oligopoly is 100%. Letting the tax on Y be τ and the subsidy on X be ξ, the restricted social optimum is derived from market behaviour if

$$\tau = \frac{(H-1)\, U_n^* \psi_X^* Y^*}{M U_Y^* - (H-1)\, U_n^* \psi_X^* X^*}$$

and

$$\xi = \frac{(H-1)\, U_n^* \psi_X^* X^*}{M U_Y^* - (H-1)\, U_n^* \psi_X^* X^*}$$

V. SUMMARY AND CONCLUSIONS

This paper has analysed the properties of a simplified model of imperfect competition in conjunction with a preference for product variety. The existence of equilibrium has been demonstrated for a restricted version of the general model, under the standard assumption that the consumer treats variety as parametric. When a lump-sum subsidy may be paid towards fixed costs, an optimal tax scheme was described that generates the socially optimal equilibrium from market behaviour. The direction of welfare-improving commodity taxes has been characterised, and it was noted that the preference for variety acts to increase the likelihood of subsiding the oligopoly. Expressions for optimal commodity taxes were also given.

An alternative, conjecture-based definition of equilibrium was introduced; this is most appropriate when the consumer is 'large' relative to the economy. It was argued, although not strictly proven, that the conjectural equilibrium involves greater product variety than the standard equilibrium. When the conjectural equilibrium was applied to a many-consumer economy, variety could be viewed as a form of public good with externalities linking consumers. Due to this externality, the free-market solution was characterised by insufficient aggregate demand for the oligopoly's product and, in consequence, too little product variety, in contrast to the equilibrium socially optimal aggregate demand. Policy schemes designed to counter the externality were noted.

The major conclusion drawn from the paper is that the specification of utility employed has been successful in permitting the analysis of taxation, when variety must be accounted for, and in highlighting the externalities connected with variety. Two points of discontent remain: firstly, that welfare-improving and optimal taxes may be of opposite sign; secondly, that the comparison between the standard and conjectural equilibria has been made on a too informal basis.

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