

"EFFECTIVE DEMAND AND UNEMPLOYMENT"

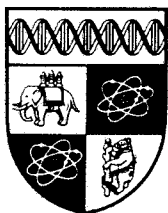
BY

TORBEN M ANDERSEN

UNIVERSITY OF AARHUS

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TORBEN M ANDERSEN

UNIVERSITY OF AARHUS

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ABSTRACT

The coordination of production and consumption decisions is analysed in a static model where the interdependence between production, income and demand is explicitly modelled. With imperfect competition and non-convex production technologies it is shown that there exists a zero pure profit equilibrium with a positive level of employment and possible involuntary employment. Finally, it is shown how to arrange a form of external intervention so as to ensure full employment.

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

## 1. Introduction

A crucial feature of decentralized market economies is the fact that firms are specialists in production and households are generalists in consumption, i.e. firms produce specific products and households consume a variety of products. Consequently the income generated in a particular firm has only a negligible effect on the demand for the product produced by the firm and firms do not perceive that they can affect aggregate income and hence demand (i.e. no Ford Effect). However, aggregate income and effective demand depend critically on the aggregate activity in the economy. Hence, we find at the aggregate level that production depends on effective demand, which in turn depends on the income households earn by selling productive factors to fulfill the factor requirements of the production plans of firms. This creates an interdependence between production, income and demand which is crucial to the level of activity. A potential reason for business cycle fluctuations arises since production and consumption plans are not coordinated in a decentralized market economy.

We shall consider a static model where goods are produced by one homogeneous type of input, viz. labour. There are no non-produced goods or outside money in the model. The neglect of intertemporal considerations is not made to deny their importance for understanding business cycle phenomena. A proper modelling of these things would, however, be beyond the scope of this paper which focusses on the coordination problem in the most simple environment which could be thought of.

In a static model of the sort analysed here there are no demand leakages since all income goes to current demand. Consequently, Say's law holds such that aggregate supply always creates its own demand. It follows that if structural imbalances are disregarded the goods market will clear at any level of activity. To determine the level of activity further assumptions need to be introduced.

A simple way by which to close the model is to assume firms and households to be price-takers. In a competitive environment the auctioneer will adjust the real-wage so as to ensure full employment, and the economy settles at the classical outcome with no unemployment.

Suppose instead that price decisions are delegated to well-defined agents as the firms. If we consider a setting of imperfect competition<sup>1</sup> where prices are flexible but determined by the profit maximizing behaviour of firms we may end up in a different situation.<sup>2</sup> The potential coordination problem is therefore linked to the market power of firms in the present analysis.<sup>3</sup> The price decisions will in a static goods-labour model effectively determine the real-wage rate, as this is the only relative price. In itself this is not sufficient to pin down the level of activity. The reason being that given Say's law the real-wage rate is unimportant to the level of aggregate demand. Consequently any combination of employment and real-wages consistent with profit-maximizing behaviour on the part of firms and the available labour supply is a possible equilibrium. That is, there exists a continuum of equilibria (see Cooper and John (1985)).

Given that market power and hence the adjustment burden is given to the firms it is, however, natural to go one step further and look for equilibria with zero pure profits. This has two motivations. First, an equilibrium with non-zero profits is not viable if entry or exit can take place. Secondly, in a zero-pure profit equilibrium Say's law is effectively broken since demand is determined solely by labour income. Hence, a zero pure profit equilibrium implies a non-trivial problem of coordinating consumption and production decisions. Our major interest is to analyse whether a zero-pure profit equilibrium exists and to explore whether it sustains a situation with involuntary unemployment.<sup>4</sup>

The present analysis is also relevant to the discussion of whether Keynes' (1936) analysis of aggregate demand and supply in a static model is consistent. Previously a foundation of this

analysis has been produced (see e.g. Casarosa (1981)) by combining a static supply model with a demand side with leakages (savings). Implicitly this construction relies on an intertemporal formulation of the demand side and as such it is not internally consistent. The present analysis shows which assumptions are needed in a static setting to construct a rigorous model of aggregate supply and demand so as to explain certain "Keynesian" phenomena.

A simple Cournot-Nash model for a static goods-labour production economy is set-up in Section 2. The determination of aggregate employment is considered in section 3, and the existence of a zero-pure profit equilibrium is analysed in section 4. The role for stabilization policy is discussed in section 5 and concluding remarks are to be found in section 6.

## 2. The Economy

Let there be  $H$  consumers ( $h=1,\dots,H$ ),  $N$  types of commodities ( $i=1,\dots,N$ ) and  $M$  firms ( $j=1,\dots,M$ ). The set of firms is denoted by  $F=\{1,\dots,M\}$ , and  $F(i)$  denotes the subset of firms producing commodity type  $i$ , i.e.  $\cup_{i \in F} F(i) = F$ . Each firm produces only one type of commodity, i.e.  $F(i) \cap F(k) = \emptyset \forall i \neq k \in F$ . Without loss of generality we can simplify and assume that all markets are of equal size in terms of the number of firms, i.e. the number of firms in any product market  $i$  is  $m = M/N \geq 1$ .

Each consumer supplies inelastically one homogenous unit of labour, i.e. total labour supply equals  $H$ . The income is spent on the consumption of the  $N$  commodities. Specifically, the consumer  $h$  is assumed to allocate its income among the  $N$  commodities according to the solution of the following constrained optimization problem.

$$\text{Max}_{\{c_{ih}\}} \sum_{i=1}^N \alpha_i \log c_{ih} \quad , \quad \sum_{i=1}^N \alpha_i = 1 \quad (1)$$

subject to

$$\sum_{i=1}^N P_i c_{ih} = I_h \quad (2)$$

Where  $c_{ih}$  is consumer  $h$ 's consumption of commodity  $i$ ,  $P_i$  is the price of commodity  $i$ , and  $I_h$  is the income of household  $h$ .

Maximizing the utility function (1)<sup>5</sup> subject to the budget restriction (2) yields the following demand functions

$$c_{ih} = \alpha_i \frac{I_h}{P_i} \quad \forall i, h \quad (3)$$

If  $I$  denotes the total income in the economy the aggregate demand for product  $i$  can be written

$$C_i = \alpha_i \frac{I}{P_i} \quad (4)$$

where  $I = \sum_{h=1}^H I_h$

Since we shall not be interested in analysing structural unemployment we impose the symmetry condition that  $\alpha_i = \alpha = 1/N$  for all  $i$ , that is, the markets are of equal size.

All firms produce subject to the same production technology by use of one homogenous type of input, viz. labour. If  $Y_j$  denotes the output of firm  $j$ , the production technology is given as

$$Y_j = f(L_j), \quad f' \geq 0 \quad (5)$$

where  $L_j$  is the use of labour in firm  $j$ . We refrain from imposing further restrictions on the production technology as its specific properties are crucial to the results of the analysis (see section 4).

It is assumed that each firm acts as a Cournot-Nash oligopolist in the product market, that is, each firm chooses its output to

maximize profits taking the output level of the other firms in the same product market for given. An implication of the economy being divided into many submarkets is that each firm takes its effect on the total income in the economy to be negligible, i.e. total income determining demand ( $I$ ) is taken to be exogenous by the firm.

The problem for a firm  $j \in F(i)$  is to maximize

$$\pi_j = P_i f(L_j) - WL_j \quad (6)$$

Firms know the objective demand functions for their products and hence that

$$P_i = [\alpha_i I] / [\sum_{k \in F(i)} f(L_k)] \quad (7)$$

The first-order condition to this problem reads

$$f'(L_j) \left[ P_i - \frac{\alpha_i I f(L_j)}{[\sum_{k \in F(i)} f(L_k)]^2} \right] = W \quad \forall j \in F(i) \quad (8)$$

Since all firms are identical we shall look for a symmetric equilibrium, i.e.  $L_j = L \forall j \in F(i)$ . Hence, (8) can be written<sup>6</sup>

$$f'(L) P_i \varrho = W \quad \forall i \in \{1, \dots, N\} \quad (9)$$

where  $\varrho = \frac{m-1}{m} \leq 1$

Equation (9) shows how the real-wage in product market  $i$  is related to the marginal product of labour. The higher  $\varrho$  the closer we are to the traditional case of perfect competition where the real-wage equals the marginal product of labour. As expected  $\varrho$  is increasing in the number of firms in a given output market, and in the limit with an infinite number of firms we approach the case of perfect competition.

Notice, that there are no non-produced commodities (e.g. money) in the model. Consequently we are only able to solve for relative

prices (here the real-wage rate). As market power by assumption is concentrated in the product markets it follows that firms by their price decisions conditional on a given nominal wage rate fix a real-wage rate. The labour market has for this reason a passive role in the model, firms offer jobs and are effectively determining the real-wage rate (cf. below).

Equation (9) says that firms by their price decision (for the given wage rate) effectively determine the real-wage rate to prevail at any employment level. The real-wage may, however, vary with the state of the economy as we have not yet determined the employment level. An implication of the model is that output is demand determined and we have that the market clearing condition for product market  $i$  reads

$$m f(L) = \alpha_i \frac{I}{P_i} \quad (10)$$

### 3. Determination of Aggregate Employment

The equilibrium conditions for each product market are given by equations (9) and (10). The interdependence between markets is seen clearly from the fact that employment in any product market depends on aggregate income ( $I$ ) and hence the level of activity in other markets. To solve for the equilibrium level of activity in the economy we sum over (10) to find

$$\sum_{i=1}^N m f(L) = E \sum_{i=1}^N \alpha_i \frac{I}{P_i} \quad (11)$$

To proceed we have to specify how aggregate income is determined.

#### 3.1. Effective Demand Determined by Total Factor Income

Given the static set-up all factor income goes to demand, i.e. there are no demand leakages.<sup>7</sup> Aggregate income is therefore made



up of total wage and profit income in the economy, i.e.

$$\begin{aligned}
 I &= \sum_{j=1}^N \pi_j + WE \\
 &= \sum_{i=1}^N P_i^m f(L)
 \end{aligned} \tag{12}$$

where  $E$  is total employment in the economy ( $E=ML$ ).

Equation (12) can by use of expression (9) be reduced to an identity stating that

$$f(L) \equiv f(L) \tag{13}$$

Expression (13) is a statement of Say's law saying that aggregate supply creates its own demand.

It follows that we have a continuum of equilibria since any employment level in each firm  $L \in [0, HM^{-1}]$  and a corresponding real-wage determined from (9) as  $W/P = pf'(L)$  constitutes an equilibrium to the model<sup>8</sup> (compare to Cooper and John (1985)). Notice, that the set of equilibria includes the full employment case where each firm employs its fraction of the labour force ( $= M^{-1}H$ ).

However, not all of the potential equilibria outlined above are viable equilibria to the model as they may imply negative or positive profits as is seen by noting that<sup>9</sup>

$$\begin{aligned}
 \pi &= p[f(L) - \frac{w}{p} L] \\
 &= p[f(L) - pf'(L)L] \stackrel{>}{<} 0 \text{ for } L \in [0, M^{-1}E]
 \end{aligned} \tag{14}$$

### 3.2. Effective Demand Determined by Total Wage Income

With zero pure profit aggregate income and hence demand will

equal total wage income,  $I=WE$ . This implies that Say's law is effectively broken since aggregate supply not necessarily generates its own demand, viz. wage income. Rather the equality of supply and demand becomes a binding constraint on the system which may help us pin down a unique level of activity. To find a zero pure profit equilibrium to our model we insert  $I=WE$  into (9) and (11) to find that such an equilibrium is characterized by the condition

$$f(L) = \rho f'(L)L \quad (15)$$

Equation (15) determines the zero pure profit equilibrium employment level in the economy in terms of the employment level in each firm  $L$ , and hence  $E=ML$ . We find that the equilibrium employment level is determined by the condition that total output ( $f(L)$ ) should equal employment ( $L$ ) times the real remuneration of labour, given as the marginal product of labour times the inverse mark-up parameter  $\rho$  ( $\rho f'(L)$ ). This condition has a simple intuitive interpretation. The LHS of (15) gives aggregate supply (per firm) and the RHS gives aggregate demand (per firm). Equilibrium is thus characterized by a balancing of the supply and demand effects of employment.

The zero pure profit equilibrium can be interpreted in either of two ways. It can be seen as a Kaldor-type of equilibrium under the assumption that the marginal propensity to consume out of wage income is one and zero out of profit income. The zero pure profit equilibrium sustains the widows cruise argument that capitalists get what they consume, that is, when capitalists do not consume they end up with zero profit income. Given this interpretation we are looking for involuntary unemployment equilibria being consistent with Kaldors theory of the functional distribution of income.

Alternatively, the zero profit equilibrium can be said to be the only plausible equilibrium to the model. With non-zero profits entry or exit will induce an adjustment which should affect the

level of activity. Given this argument the burden of adjustment is placed on the number of firms and we shall discuss the dynamics implied by this mechanism as we proceed.

As can be seen from (15) the condition for a zero pure profit equilibrium is crucially dependent on the production technology. To proceed we have to introduce a more specific assumption on the production technology of firms.

#### 4. Existence of Zero-Pure Profit Equilibrium under Different Production Technologies

We shall consider the implications of convex and non-convex production technologies for the existence of a zero pure profit equilibrium. A primary aim is to investigate whether such an equilibrium entails involuntary unemployment.

##### 4.1. Convex Production Technologies

Assume that  $f''(L) \leq 0 \forall L \geq 0$  such that

$$f(L) \geq f'(L)L \quad \forall L \geq 0 \quad (15)$$

According to (15) a zero pure profit equilibrium is characterized by  $f(L) < f'(L)L$  for  $\rho < 1$ . Hence, no such equilibrium exists to the model given a convex production technology.

At all possible levels of employment profits will be positive with a convex production technology. Hence, if we allow for entry the number of firms should increase and eventually  $\rho$  should approach one. We find that in the limiting case of perfect competition ( $\rho=1$ ) we have a zero profit equilibrium only in the case of constant returns to scale ( $f''=0 \forall L \geq 0$ ). In fact we have a continuum of equilibria in this case since (15) will be fulfilled for any employment level  $L \in [0, HM^{-1}]$ . But this is only to repeat the

result found in section 3.1. since with constant returns to scale and perfect competition no further constraints are put on the potential equilibria by looking for a zero pure equilibrium since they all imply zero profit.

#### 4.2. Non-Convex Production Technology

We shall consider an example of non-convex production technologies (see Weitzman (1982)).

Let the production function be given as

$$Y_i = a L_i - b \quad ; \quad a, b > 0 \quad (16)$$

defined for  $L_i \geq b/a$ .

Inserting in the zero profit equilibrium condition (15) we find

$$L = \frac{b}{a} m \geq \frac{b}{a} \quad (17)$$

Hence, total employment equals

$$E = \begin{cases} H & \text{if } \frac{b}{a} m M \geq H \\ \frac{b}{a} m M & \text{if } \frac{b}{a} m M < H \end{cases}$$

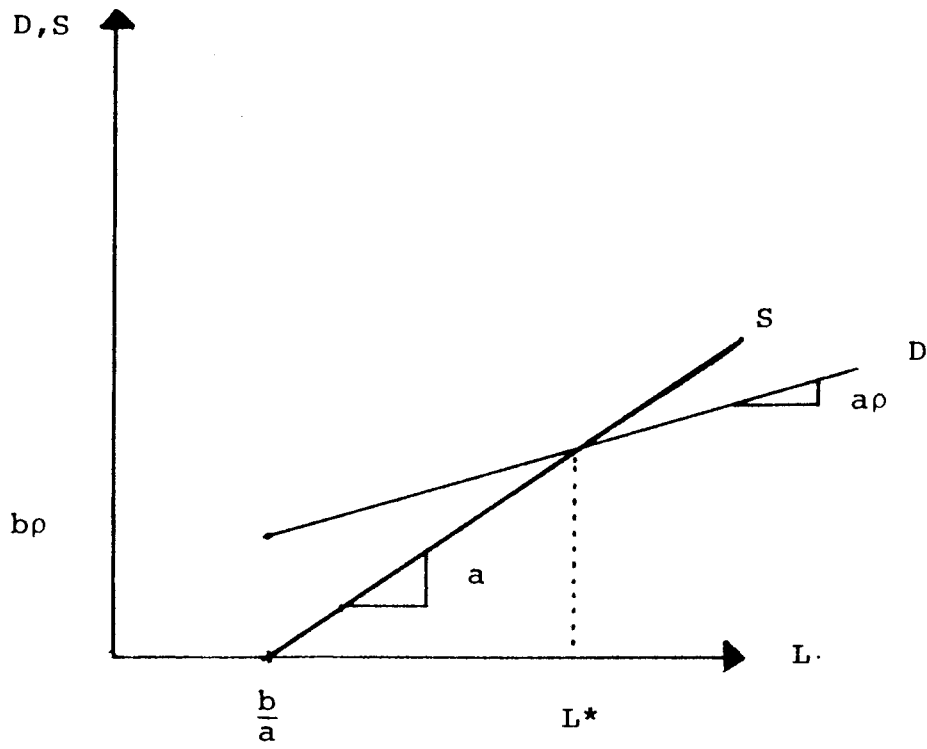
We find that the zero profit equilibrium implies unemployment if

$$\frac{b}{a} \frac{M^2}{N} < H$$

It follows that unemployment is decreasing in  $b$  and  $M$  but increasing in  $a$  and  $N$ .

The intuition behind this result can be explained with the aid of figure 1 which can be seen as a variant of the well-known 45°-diagram. In the figure aggregate supply (the LHS of (12)) and aggregate demand (the RHS of (12)) are shown as depending on the employment level  $L$ .

Figure 1. Aggregate Demand and Supply under a Linear Production Technology with Set-Up Costs



The aggregate supply schedule  $S$  starts out at  $b/a$  and increases with a slope parameter  $a$ . The aggregate demand schedule starts out in  $b_0$  and has a slope parameter  $a_0$ . Hence, we have in general an equilibrium employment level  $L^* > b/a$ .

The figure reveals two important properties which ensure a well defined equilibrium. First, aggregate supply should fall short of aggregate demand at low levels of demand. If that is not the case there does not exist an employment level which will create sufficient demand to absorb the corresponding output. In the present example the set-up costs ( $b$ ) implied by the production technology ensures that such a situation does not arise.

Secondly, aggregate demand should increase by less than aggregate supply to an increase in employment ( $\rho < 1$ ). That is, with the present set-up some degree of imperfect competition is needed to ensure a well-defined equilibrium. If  $\rho = 1$ , i.e. perfect competition, we find that aggregate demand exceeds aggregate supply at any employment level.

It is seen right away from the figure that the higher the real-wage rate relative to the marginal product of labour, i.e. the higher  $\rho$ , the higher becomes the employment level. Hence, employment is lower the lower is the real-wage rate. Consequently, the present unemployment equilibrium does not have any classical properties but rather a Keynesian flavour being due to insufficient effective demand caused by too low real-wages.

It is important to notice that it is the zero pure profit equilibrium condition which breaks Say's law, whereas the assumptions of imperfect competition and a non-convex production technology ensure that we for a given number of firms have a unique zero-pure profit equilibrium (compare to Weitzman (1982)).

If entry and exit is allowed it becomes an issue which zero pure profit equilibrium the economy settles at. A proper dynamic analysis is not, however, possible for two reasons. First, the inde-

terminacy of a non-zero profit equilibrium (see section 3.1.) makes it arbitrary from where to start the dynamic analysis. Secondly, it is seen that there exists multiple zero profit equilibria to the model since the equilibrium condition (15) is equivalent to the zero profit condition. Hence, for any  $M (\leq \sqrt{\frac{a}{b}} HN)$  there exists a zero profit equilibrium with employment determined by (15). The full employment equilibrium belongs to this set of equilibria.

Notice, that in any of these zero profit equilibria there is no incentive for the number of firms to change and hence they are sustainable. It is important to point out that deviations from a zero profit equilibrium would not necessarily put the economy on an adjustment path leading to full employment. Consider small perturbations to an economy in a zero-pure profit Cournot-Nash equilibrium with  $M$  firms and each employing  $L^* = \frac{b}{a} \frac{M}{N}$  workers. If all firms happened to choose an employment level  $L > L^*$  profits would be positive, the number of firms will tend to increase, and eventually aggregate employment would increase. On the other hand if all firms choose to operate at an activity level  $L < L^*$ , profits would be negative, the number of firms would decrease, and eventually employment in the corresponding zero profit equilibrium would be lower. It can thus be concluded that there is no way by which an endogenous determination of the number of firms can be said to induce a dynamic adjustment mechanism which would necessarily lead to full employment.

## 5. Stabilization Policy

Given that uncoordinated production and consumption decisions can cause unemployment it becomes of interest to analyse whether any form of stabilization policy can overcome this coordination problem. Consider external intervention in the form of a subsidy ( $S$ ) on each person employed in a firm financed by a lump-sum tax ( $T$ ) payable by each firm.

Under this scheme profits to a firm producing in output market  $i$ ,

$j \in F(i)$ , becomes

$$\Pi_j = P_i f(L_j) - (W-S)L_j - T$$

Proceeding as in section 3 we find in a symmetric equilibrium that

$$\frac{W-S}{P_i} = \rho f'(L) \quad (15)$$

It is easily shown that it is possible to subsidize the use of labour in such a way as to ensure full employment in a zero-pure profit equilibrium. For aggregate demand to balance aggregate supply at full employment ( $E=H$ ) the real-wage must be such that

$$f(L^H) = L^H \left(\frac{W}{P}\right)^* \quad (16)$$

where  $L^H = M^{-1}H$

To ensure that firms are willing to hire their share of the labour force  $L^H$  the subsidy must be such that (cf. (9))

$$\begin{aligned} \left(\frac{S}{P}\right)^* &= \left(\frac{W}{P}\right)^* - \rho f'(L^H) \\ &= \frac{f(L^H)}{L^H} - \rho f'(L^H) \end{aligned} \quad (17)$$

and the lump-sum tax payable by each firm must be such that  $MT=SE$ .

The employment subsidy works because it makes it possible to enhance the purchasing power of workers without curtailing the profitability of employing labour for the firms. Hence, it has been shown how to arrange a form of external intervention so as to overcome the coordination problem arising from uncoordinated production and consumption plans.

It is moreover seen that the employment subsidy can be arranged so as to overcome the non-existence problem under a convex



production technology. That is, the employment subsidy can be arranged so as to entail a zero pure profit full employment equilibrium even when no zero-pure profit equilibrium exists to the model in the absence of intervention.

## 6. Conclusion

This paper has analysed the assumptions necessary to generate involuntary unemployment in a static economy due to lack of effective demand. It was shown that a non-trivial coordination problem arises in a zero-pure profit equilibrium where Say's law is effectively broken. A well-defined zero-pure profit equilibrium with involuntary unemployment arises if there is imperfect competition in product markets and non-convex production technologies. Unemployment is of a Keynesian nature since it is due to insufficient demand caused by too low real-wages.

## Notes

- 1) The importance of imperfect competition for macroeconomics has been stressed in e.g. d'Aspremont et al. (1984), Benassy (1987), Dixon (1986) and Hart (1982).
- 2) Heller (1985) also follows this approach but maintains the auctioneer construction that the real-wage should adjust so as to clear the labour market in the sense of being on the labour supply curve. However, a possibility of multiple equilibria exists due to the fact that different levels of real wages and employment may be consistent with profit maximizing behaviour.
- 3) Coordination failures related to differential information in a competitive model is analysed in Andersen (1987a,b).
- 4) Although the focus is different the present model is closely

related to Weitzman (1982) in that imperfect competition and non-convex production technologies are crucial.

- 5) The specific utility function facilitates the solution of the model. Without much difficulty the same arguments could be made for a homothetic demand system  $c_{ih} = f_i(P_1, \dots, P_N) W \delta_h$ .
- 6) Nothing will be changed if we allow for unemployment benefits financed by taxation of the wage income of employed workers. Total effective demand would still be the same.
- 7) Notice, that in this static model with the specified demand system the functional distribution of income has no effect on aggregate demand.
- 8) Assuming the second-order conditions to be fulfilled.
- 9) It is seen that a condition for real-profit to be increasing in the employment level is  $(1-\rho)f'(L) - \rho Lf''(L) > 0$ . If this condition is fulfilled we find that the equilibria can be pareto-ranked with higher levels of employment being preferred by both workers and firms.

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