INTERNAL LABOUR MARKETS AND DEMOCRATIC

LABOUR-MANAGED FIRMS

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

SUMMARY

A two-stage decision-making process is modelled where members of the firm vote for a feasible set of wage rates and then choose which work process to join. It is shown that this system is characterised by allocative inefficiency, non-continuous supply functions and wage discrimination. These could be limited by outside opportunities for members or by members having sympathy for others.

1. INTRODUCTION

The analysis of labour-managed (LM-) firms has generally followed the Illyrian tradition originating in the seminal paper of Ward (1958)[1/]. In this tradition, the objective of the LM-firm is taken as the maximisation of the income or utility of a 'typical' member. In many instances membership is considered homogeneous so that the typical member can be any member. In others membership differences still permit an obvious maximand. For example if individual labour supply varies across members, the maximisation of income per unit of total effort of the membership is still a relatively uncontroversial objective. Problems of identifying such a simple objective arise when political or power-distributing factors impinge on a multi-level economic decision-making process. This paper seeks to examine a democratic firm with an internal labour market, and to demonstrate how the democratic process can lead to an inefficient allocation of resources in the internal labour market, together with discriminatory and unstable behaviour. A democratic firm is taken as egalitarian in that all members have the same opportunities. To the extent that different tasks are performed within the firm, each member must be able to exert his right to choose which task he performs. An internal labour market equates the firms' needs for various kinds of labour with individual members' preferences for labour supply by varying the conditions of work (such as the required productivity, work environment and wage rates) in one task compared to another. The "needs" of the firm and the implied equilibrium conditions of work are deemed to be decided by majority vote.

It is of course an undeniable failure of democracy that the majority can treat a minority unfairly. Unhappy situations have occurred at sometime in many countries where a coalition of voters forming a majority have oppressed an easily identified minority. The extent of such oppression has been limited only by feelings of social responsibility among the majority, built-in constitutional safeguards,and emigration of the minority. The same kind of abuse of majority power and limitations on its application is possible in a democratic LM-firm. Major decisions in such a firm may be taken by a straight vote of the membership and a one-member-one-vote rule would be the norm, since an egalitarian principle would be firmly embedded in the LM-firm's constitution. However, without further safeguards democratic decision making may well yield very inegalitarian outcomes. An obvious but extreme example would be if a majority (say 51%) of the membership voted for a motion expelling the other 49% from the firm. Constitutional safeguards may prevent such an occurrence, although Ward (1967) certainly finds it far from implausible. Of course such a motion requires the identification of those to be expelled (perhaps a last-in first-out principle), otherwise an individual may be voting to set a probability on his <u>own</u> redundancy, and this has been found to be unattractive (see Steinherr and Thisse, 1979).

The particular feature that this paper will focus on is one which is unlikely to be protected by the LM-firm's constitution and where the identification of an individual as belonging to the majority or the minority is unlikely to be an issue. We will consider that all members of the firm have tenure and that membership is fixed for the coming period. The firm can produce using either or both of two production processes. These may relate to producing the same product in different ways or to producing different products, or even to producing within the firm or hiring factors to other firms. The internal labour market is based on free choice. Any member can choose to work in either of the available processes. (The construction of the model will imply that a member will not wish to work in both processes). A particular member's choice will reflect his relative work skills and preferences as well as relative conditions of work, and will be made purely on such individualistic criteria. The conditions of work which we will incorporate here will be largely limited to an explicit treatment of relative wage rates. The wage rates available for working in either process will be set by majority voting and will be feasible in that, given the resulting individualistic work choice, the wage bill can be afforded by the LM-firm out of its revenue net of any fixed cost.

The allocation of labour within the LM-firm is thus the result of two stages of decision making. First wage rates are set and then individual labour allocations are made. Obviously there is a sense in which the members as a whole act as a Stackelberg leader in setting wage rates so that members as individuals can respond by allocating their labour to a particular process. Given that individuals adopt the (Nash) conjecture that a change in their behaviour will not affect the behaviour of others, a perfect equilibrium in this two-stage process is a strategy pair for each individual member : a 'yes' or 'no' vote to a motion on wage rates followed by a labour allocation choice resulting from the motion being passed. In this equilibrium, no individual can credibly threaten to make a labour choice other than the one he actually takes.

There are three principal questions which arise. First, will the firm allocate labour in a Pareto efficient way? If not will it tend to over or under specialise?[2/] Secondly, will the equilibrium react continuously to external and exogenous changes? Thirdly, the largely normative issue arises as to whether some members (a minority) are discriminated against by the majority.

If adverse answers to these questions are found then they constitute a further explanation[3] for the small size of most cooperatives in western economies, particularly in the UK.[4] There are three strands to this argument. First, to the extent that growth may mean branching into new work processes, it may be seen as unattractive just because it may lead to the kinds of difficulties outlined above. Second, growth may affect the balance of workers within the firm since it would require the recruitment of new members and thus may be seen as undesirable by the current majority (see also Furubotn, 1976). Finally the kinds of selfish preferences assumed for much of our analysis may be less evident in a small firm where members all know each other. In such a firm members may be more "sympathetic" (see Sen, 1966) and have concern for other members' interests. Firms may decide to stay small in order to retain this social concern which might be lost in a larger, more anonymous, membership.

Our analysis is initially based on a very simple model. This is described in Section 2 and is such that an explicit description of equilibrium can be made. The questions concerning efficiency, behaviour and discrimination are resolved in Section 3. In Section 4 a number of generalisations and extensions are considered sequentially, and conclusions are summarised in a final section.

2. THE MODEL

As the analysis of a two-stage decision-making process tends to become extremely complex, a better focus on the principal issues that have been identified will be obtained by initially abstracting from undue complications; a number of generalisations will be considered in Section 4. Thus we will assume that members as individuals only choose whether to work in process 1 or process 2 (or neither). The amount of work supplied (e.g. the length of the working day) is either fixed or is independent of the parameters of the model. A member will receive a wage w_i if he works for one period in process i. His utility from that work is assumed to be simply

$$U_i = w_i - x_i \qquad i = 1, 2 \tag{1}$$

and his maximum utility is thus

$$U = \max \{0, w_1 - x_1, w_2 - x_2\} \tag{2}$$

The values of x_1 and x_2 vary across individuals. x_i could be interpreted as the (income equivalent) disutility from supplying work effort, or alternatively the training or equipment cost, of working in process i. Assuming for the moment that $w_i - x_i$ is greater than zero for some i the individual will choose process 1 if

$$w_1 - x_1 > w_2 - x_2$$

i.e. $$w_1 - w_2 > x \equiv x_1 - x_2$$

and in process 2 if

$$w_1 - w_2 < x$$

We will assume that individual members vary according to their x characteristics, and that the distribution of x is continuous with a uniform density function, $g(x)$. The effects of a more general distribution of x are considered as an extension to the analysis in Section 4. Thus

$$g(x) = g \qquad \underline{x} < x < \bar{x}$$
$$= 0 \qquad \text{otherwise}$$

It may be helpful to consider $\underline{x} < 0 < \bar{x}$, although this is not necessary. Since the distribution of x is continuous, there is no positive mass of members indifferent between the two work processes. The number of members choosing the first work process is

$$L_1 = (w_1 - w_2 - \underline{x})g \qquad \text{if} \quad w_1 - w_2 \geqslant \underline{x} \tag{3}$$
$$= 0 \qquad \text{otherwise}$$

and the number choosing the second work process is

$$L_2 = (\bar{x} - w_1 + w_2)g \qquad \text{if} \quad w_1 - w_2 \leqslant \bar{x} \tag{4}$$
$$= 0 \qquad \text{otherwise}$$

The simplest view of the two work processes is that they each earn a constant revenue for the firm per unit labour supplied. Let these constant per unit revenues be p_1 and p_2 respectively. The firm uses this revenue to pay its wage bill and a fixed cost of F.[5/] Thus w_1 and w_2 have to satisfy

$$(p_1 - w_1) L_1 + (p_2 - w_2) L_2 - F = 0 \qquad (5)$$

Obviously a continuum of different combinations of w_1 and w_2 may be feasible. Let the set of such combinations be $W = \{W_1, W_2\}$. In the first stage of the decision making process the membership votes on a proposal concerning a pair of feasible wage rates. Since any member works in only one process, each will vote for an extreme among the feasible set. The only possible winning proposals are

(1) $(w_1^*, w_2^*) \in W$ such that $w_1^* \geqslant w_1$ for all $w_1 \in W_1$.

(2) $(w_1^{**}, w_2^{**}) \in W$ such that $w_2^{**} \geqslant w_2$ for all $w_2 \in W_2$.

There may be side conditions relating to the threat by individuals to leave the firm and obtain the reservation utility of 0. Leaving these aside for the moment, the two possible proposals are shown in Figure 1. That $w_2^* > 0$ and $w_1^{**} > 0$ is due to the heterogeneity of members. It is a w_1-maximising policy to set w_2^* greater than zero, but less that p_2. The surplus generated by those members, who still choose process (2) work due to their high x characteristics, is distributed to other members in terms of a higher w_1^*. A similar justification for an interior w_1^{**} can be made. This argument is fairly general but in order to examine the full solution and to determine which proposal will succeed, it will be useful to solve for (w_1^*, w_2^*) and (w_1^{**}, w_2^{**}) from our specific model.

The w_1-maximising strategy involves maximising the firm's net revenue (5), using (3) and (4), with respect to w_2 and then solving for w_1 from (5). Similarly the w_2-maximising strategy involves maximising the firm's net revenue (5) with respect to w_1, and then solving for w_2. In the first case, the vertical slope of the w schedule in Figure 1 is identified, in the second, the horizontal slope is identified.[6/]

Assuming that $\underline{x} < p_1 - p_2 < \bar{x}$ to allow an interior solution,[7/] we have for the w_1-maximising proposal (1),

$$w_1^* = p_1 + \{(L_2^*/g)^2 - f\}/(\bar{x} - \underline{x}) \quad (6i)$$

$$w_2^* = w_1^* - (\bar{x} - p_2 + p_1)/2 \quad (7i)$$

$$L_1^* = g(\bar{x} - \underline{x}) - L_2^* \quad (8i)$$

$$L_2^* = g(\bar{x} + p_2 - p_1)/2 \quad (9i)$$

where $f = F/g$

while for the w_2-maximising proposal (2)

$$w_1^{**} = w_2^{**} + (\underline{x} - p_2 + p_1)/2 \quad (6ii)$$

$$w_2^{**} = p_2 + \{(L_1^{**}/g)^2 - f\}/(\bar{x} - \underline{x}) \quad (7ii)$$

$$L_1^{**} = g(p_1 - p_2 - \underline{x})/2 \quad (8ii)$$

$$L_2^{**} = g(\bar{x} - \underline{x}) - L_1^{**} \quad (9ii)$$

For the derivation of (6i) - (9ii), and a discussion of the shape of the wage frontier, including a numerical example, see the Appendix.

To decide which of the above proposals to support with his vote, each member compares the outcome in terms of his own utility.

Thus, for a given characteristic x, he will vote for or against proposal (1) as

$$w_1^* - w_2^{**} \gtrless x$$

Note that it would be irrational to vote for proposal (1) if the member intended to work in process 2 if the proposal was accepted. It would be similarly irrational to vote for proposal (2) if the member would choose to work in process 1 if proposal (2) was accepted. In casting his vote the member simply compares the outcome for him given that his vote determines the outcome. There is no role for strategic voting and voting is incentive-compatible. Of course it is not generally true that any one member would determine the voting outcome. However since it is true that if $w_1^* - w_2^{**} > x'$ for some $x' \epsilon (\underline{x}, \bar{x})$, then it is also true that $w_1^* - w_2^{**} > x$ for all $x \epsilon (\underline{x}, x')$, the monotonic structure of differences across individual members implies that the median member (i.e. the member with the median x characteristic) will determine the voting outcome.[8/] Thus if x^m is the median x then

Proposal (1) will be implemented if

$$w_1^* - w_2^{**} > x^m \qquad (10i)$$

Proposal (2) will be implemented if

$$w_1^* - w_2^{**} < x^m \qquad (10ii)$$

The median member simply compares the two outcomes in the knowledge that he has the casting vote. All members on one side of him in the x-distribution will vote the same way as he does. In the absence of undemocratic bribery and corruption he has no incentive to do other than vote for the best outcome for himself. Thus Proposal (1) will be adopted if, using (6i) and (7ii) in (10) and noting that the uniform distribution of x implies that $x^m = (\bar{x} + \underline{x})/2$, the following condition holds:

$$p_1 - p_2 > x^m \tag{11}$$

while Proposal (2) will be adopted otherwise. In the next section, the efficiency and other properties of the resulting equilibrium will be considered.

3. IMPLICATIONS

a) Allocative Inefficiency

That a member may vote for Proposal (1), that is for high w_1 does not mean that he will still choose to work in process 1 if Proposal (1) is defeated in favour of Proposal (2). We would expect some Proposal (1) supporters (those with relatively high x characteristics) to choose to work in process 2 if Proposal (2) were adopted and some Proposal (2) supporters (those with relatively low x characteristics) to choose to work in process 1 if Proposal (1) were adopted. This points to the wage setting leading to allocative inefficiency and the extent of such inefficiency is discussed here.

For the LM-firm as a whole, the socially-optimal allocation of labour would lead to the maximisation of the sum of all members' utilities. This sum amounts to the total earnings of the firm net of fixed costs and minus the work supply disutilities. These latter can be represented as the disutility from all members working in process 2 plus the extra disutility (positive or negative) of those members selecting process 1. If L_1 members are assigned to process 1 then this is:

$$Z = p_1L_1 + p_2(L - L_1) - F - \int_{\underline{x}}^{\bar{x}} x_2h(x_2)dx_2 - \int_{\underline{x}}^{\underline{x}+L_1/g} gxdx \qquad (12)$$

where $L = g(\bar{x} - \underline{x})$ is the total membership and $h(x_2)$ is the density function for x_2. Thus Z is equal to the firm's revenue minus fixed cost minus the disutility involved if all members worked in process 2 minus the added disutility (negative or positive) from the lowest x members working in process 1 rather than process 2. Maximising Z with respect to L_1 yields the socially-optimal allocation of labour:

$$L_1^o = g(p_1 - p_2 - \underline{x}) \qquad (13)$$

$$L_2^o = g(\bar{x} - p_1 + p_2) \qquad (14)$$

Comparing (13) and (14) with (9i) and (8ii) we see that

$$L_1^o = 2L_1^{**} \qquad (15)$$

$$L_2^o = 2L_2^{*} \qquad (16)$$

Now as Proposal (1) is chosen when $p_1 - p_2 > x^m$, it is also chosen when $L_1^o > L/2$. Then the smaller employer, process 2, employs only half the number of members that it should do in a social optimum (since $L_2^* = L_2^o/2$). Similarly if Proposal (2) is chosen, only half the socially optimal number of members are employed in process 1. The favourable wage rate for the majority process leads to too large an employment level compared to the social optimum. The welfare loss of the two-stage equilibrium compared with the social optimum can be easily derived (see Appendix) as

$$\text{Loss} = \frac{1}{8g} \min \{\min (L_1^o, L_2^o)\}^2$$

$$\text{Loss} = \frac{g}{8} \min \{(p_1-p_2-\underline{x})^2, (x-p_1+p2)^2\} \qquad (17)$$

so that the welfare loss is small when one process is relatively dominated (L_1^o or L_2^o very small) and the largest where the two processes employ a similar number of members in the social optimum.

b) Discontinuous Behaviour

If prices change sufficiently for the median member to change his vote then the equilibrium will shift discontinuously as the change in wage regime leads to a possibly large number of members shifting from one work process to the other. Thus for a given p_2, the output from process 1 as a function of p_1 follows the supply schedule S_1 as drawn in Figure 2. Note that the socially-optimal supply given by (13) is continuous and that the largest welfare loss is where $p_1 - p_2 \cong x^m$; then the distortion from the majority power of the first stage of the decision-making process is at its greatest.

A result specific to this particular model is that the value of L_1^{**} where p_1 is (just) below $p_2 + x^m$ is from (8ii) $g(x^m - \underline{x})/2 = g(\bar{x} - \underline{x})/4$, while the value of L_1^* when p_1 is (just) above $p_2 + x^m$ is from (8i) $g(\bar{x} - \underline{x}) + g(\bar{x} - x^m)/2 = 3g(\bar{x} - \underline{x})/4$. Thus the larger sector of the firm will always employ at least 3/4 of the membership.

Or course a shift in regimes could be prompted by changes in parameters other than prices. Note however that neither the level of fixed costs nor the density of members plays any role, although a change in the range of the distribution of characteristics may change the 'identity' of the median member and thus tip the balance. Also productivity parameters, subsumed here in the prices, would play an equivalent role to that of prices.

c) Discrimination

The literature on product price discrimination is considerable. Phlips (1983) defines such price discrimination as occurring where price minus marginal cost charged to consumers is higher for some than for others. Such a definition is objective and does not depend on the relative utility effects caused by the discrimination. In a similar approach we may measure wage discrimination within the firm as the extent of differences in the product price to wage margins. Thus define

$$D = (p_1 - w_1) - (p_2 - w_2)$$

$$D = p_1 - p_2 - (w_1 - w_2) \qquad (18)$$

and $|D| = |p_1 - p_2 - (w_1 - w_2)|$ (19)

The inference is that $|D|$ measures the extent of discrimination. If D is positive then this is discrimination against process 1 workers since they are contributing more per member than process 2 workers towards the fixed cost. Using (7i) and (6ii) respectively yields

$$D^{\star} = (p_1 - p_2 - \bar{x})/2 \qquad \text{if } p_1 - p_2 > x^m \tag{20}$$

$$D^{\star\star} = (p_1 - p_2 - \underline{x})/2 \qquad \text{if } p_1 - p_2 < x^m \tag{21}$$

Graphing discrimination D against the price difference $p_1 - p_2$ in Figure 3 shows that discrimination is greatest when the price difference is near to x^m. The explanation for this is that the price difference is then furthest from the tails of the distribution $(\underline{x}, \bar{x})$ and thus individuals with extreme values of x are more committed to choosing a particular work process even though wage dicrimination is being practised against them.

4. GENERALISING THE MODEL

Our model has been the simplest possible to generate the results described in Section 3. In this section we will show the effects of generalising the model in a number of ways. Each generalisation will be considered in turn, using our initial model as a benchmark. The extent of allocative inefficiency and discrimination will be assessed by the sign of the discontinuity in labour allocation when the median member switches his allegiance from one regime to another. As Figures 2 and 3 clearly show, when this is large there must be significant labour misallocation (since optimal allocation is continuous) and discrimination (since many members are avoiding being in the minority sector despite their comparative advantage in labour supply).

a) Alternative Designs for Decision-Making

It is interesting to note that our results are not sensitive to some variations on the 2-stage decision making process we have adopted, while other variations actually increase the inefficiency and discrimination. Suppose for instance that members commit themselves to particular work processes prior to wage rates being determined - that is the two stages are reversed. Then given any labour allocation there is more scope for wage discrimination since members cannot switch from the minority to majority group. Since this greater wage discrimination would be foreseen by members, many fewer would opt for the minority group and the labour misallocation would be more serious. Note that the majority may try to make commitments ex ante to encourage more members to select the minority work process, but that they would have an incentive to renége ex post so that these commitments may not be credible.

Finally note that simultaneous voting between two wage rate regimes and work process selection choices by individual members can only yield an equilibrium if the firm's budget constraint is satisfied at either regime. Since the voting outcome can be foreseen, either the outcome of Section 2 or the outcome from the reverse of the two stages (with a wider difference in wage rate regimes) as described above could constitute an equilibrium outcome.

b) A General Technology

Let revenue generation be defined by the concave function $R(L_1, L_2)$ so that the firm's budget constraint (5) is replaced by

$$R(L_1, L_2) - w_1L_1 - w_2L_2 - F = 0 \quad (5b)$$

and let R_i denote the marginal revenue product of type i labour. A w_1-maximising wage régime is achieved when the derivative of (5b) with respect to w_2 is zero. Thus using (3) and (4) in (5b), differentiating with respect to w_2 and setting to zero yields

$$L_2^* = g(R_2^* - R_1^* - (w_2^* - w_1^*)) \quad (22)$$

where R_2^* denotes R_2 evaluated at (L_1^*, L_2^*). Using (4) again leads to

$$L_2^* = g(R_2^* - R_1^* + \bar{x})/2 \quad (23)$$

which is an equation in L_2^* alone (since $L_1^* + L_2^*$ is the fixed membership). The right-hand-side of (23) is decreasing in L_2^* since it has a slope proportional to $Q^* = R_{22}^* - 2R_{12}^* + R_{11}^* \leqslant 0$[9]. Thus if a solution to (23) exists it is unique. A sufficient condition for a solution is that $R_i^* \to \infty$ as $L_i^* \to 0$, i = 1, 2. In an exactly analogous way, L_1^{**}, the labour allocation to process 2 under the w_2-maximising wage régime can be derived as

$$L_1^{**} = g(R_1^{**} - R_2^{**} - \underline{x})/2 \quad (24)$$

so that, deducting (24) from total membership yields

$$L_2^{**} = g(R_2^{**} - R_1^{**} + 2\bar{x} - \underline{x})/2 \tag{25}$$

and a unique solution L_2^{**} exists to (25) under the same conditions as ensure a unique L_2^{*}. To generate the results of Section 2 from this general technology we note that explicit solutions from (23) and (25) are not possible and rather write L_2^{**} in terms of L_2^{*} by using a first-order Taylor's expansion:

$$L_2^{**} \simeq L_2^{*} + g(\bar{x} - \underline{x})/2 + gQ^{*}(L_2^{**} - L_2^{*})/2 \tag{26}$$

so that

$$L_2^{**} - L_2^{*} \simeq g(\bar{x} - \underline{x})/(2 - gQ^{*}) \tag{27}$$

In the special case of Section 2, $Q \equiv 0$ so that (27) indicates a discontinuity in labour supply, when the wage régime switches, of half the total membership (see Figure 2). As $Q^{*} \leqslant 0$ any technology other than that of perfect substitutability will yield a smaller discontinuity. However unless Q^{*} is infinite (zero substitutability), some discontinuity will persist. Since this discontinuity is prompted by a switch in wage rate régimes, resource misallocation and wage discrimination continue to exist. For example, if R were a symmetric function of L_1 and L_2 and if $x^m = 0$ then allocative efficiency would require that L_1 and L_2 were equal and thus equal to $g(\bar{x} - \underline{x})/2$.

c) A General Density Function

Suppose that the density function of members' x characteristic $g(x)$ is no longer assumed to be uniform but instead is any continuous density function with cumulative distribution function $G(x)$. Then equations (3) and (4) are replaced by

$$L_1 = G(w_1 - w_2) \tag{3c}$$

$$L_2 = N - G(w_1 - w_2) \tag{4c}$$

where N is the total membership size. The w_1-maximising equilibrium is found by maximising (5) with respect to w_2, using (3c) and (4c). The first order condition is

$$-(p_1 - p_2 - \Delta)\ g(\Delta) - (N - G(\Delta)) = 0 \tag{28}$$

where $\Delta \equiv w_1 - w_2$. Note that equation (28) is an equation in Δ alone. Assume a unique solution Δ^* to (28). Then substituting this in (5), (3c) and (4c) yields the w_1-maximising equilibrium. An explicit solution for Δ^* is not possible.

In an exactly analogous way, the w_2-maximising equilibrium can be found as the solution to (5), (3c), (4c) and (29), which is obtained by differentiating (5) with respect to w_1, using (3c) and (4c) and setting to zero, as

$$-\ (p_2 - p_1 + \Delta)\ g(\Delta) - G(\Delta) = 0 \tag{29}$$

Assume a unique solution to (29) and denote this Δ^{**}.

Now, using g^* as $g(\Delta^*)$, etc., from (28) and (29):

$$\Delta^* = p_1 - p_2 + N/g^* - G^*/g^* \tag{30}$$

$$\Delta^{**} = p_1 - p_2 - G^{**}/g^{**} \tag{31}$$

Expanding Δ^{**} around Δ^* yields

$$\Delta^{**} - \Delta^* \simeq - N/g^* - (1 - g^{*\prime} . G^*/g^{*2})(\Delta^{**} - \Delta^*)$$

$$\simeq - \frac{N/g^*}{2 - g^{*\prime} G^*/g^{*2}}$$

where $g^{*\prime}$ is the slope of the density function at Δ^*. Since from (4c).

$$L_2^{**} - L_2^* \simeq -g^*(\Delta^{**} - \Delta^*) \tag{33}$$

we have combining (32) and (33)

$$L_2^{**} - L_2^* \simeq N/(2 - g^{*\prime}G^*/g^{*2}) \tag{34}$$

Again the size of the discontinuity reverts to half the membership if $g^{*\prime} = 0$, for which a sufficient but not necessary condition is that the density function is uniform. With other density functions the discontinuity could be greater or smaller. In Figure 4a a symmetric distribution implies that, since for (w_1^*, w_2^*) to be accepted $L_2^* < N/2$, $g^{*\prime} < 0$ and the discontinuity is reduced. This would also be the case for positively-skewed distributions. Note however that providing $g^{*'}$ is finite, the discontinuity will still exist. In Figure 4b, a negatively-skewed distribution has x^m typically less that the modal x and $g^{*'} > 0$, can occur as shown.

d) More General Conditions of Work

Differences in relative wage rates may be limited by the firm's constitution or social norms. Other conditions of work may still discriminate sufficiently to create inefficient labour allocation. We consider here just one slight generalisation from our basic model. Suppose $w_1 = w_2 = w$ by rule, and that the disutility of working in process i is

$$x_i' = x_i - s_i \tag{35}$$

where x_i varies across individuals as before and s_i is a constant for all individuals and relates to savings in disutility from additional equipment, pleasanter working environment, etc. Then an individual with characteristic x will choose to work in process 1 if

$$x < s_1 - s_2$$

and labour allocation to process 1 is

$$L_1 = g(s_1 - s_2 - \underline{x}) \tag{36}$$

The firm's budget constraint can be considered as

$$p_1L_1 + p_2L_2 - w(L_1 + L_2) - cs_1L_1 - cs_2L_2 - F = 0 \qquad (37)$$

where c is the cost of saving a unit disutility for one worker in process i. Define

$$w_i = w + cs_i \qquad i = 1,2 \qquad (38)$$

and let $k = g/c$, $y = cx$. Then labour supply equations like (36) and the firm's budget constraint (37) have exactly the same form as in the initial problem and the same analysis follows, except that the first stage relates to choosing conditions of work rather than wage rates.

If working conditions affect productivity as well as labour allocation then further analysis is required. However if higher productivity results from better equipment in the majority process, then this will ease the budget constraint and permit larger differences in treatment, increasing allocative inefficiency.

e) Bounds on Wage Discrimination

So far our analysis has omitted the possibility that in a firm where process 2 is dominant, w_1 could not be set at w_1^{**} without members working in process 1 leaving the firm to obtain work elsewhere. If such alternative work exists but members would not leave the firm for any $w_1 > \underline{w}_1 < p_1$, then an equilibrium will occur with less wage discrimination and $w_1 = \underline{w}_1$. An analogous possibility exists for w_2^* to be similarly unfeasible. Such bounds reduce rather than remove the properties of the firm's equilibrium that we have been considering. However if such bounds do exist then the supply function from a process has a downward-sloping segment at low prices for that process's output. Such a case is demonstrated in Figure 5. At prices $p_1 < p_1^b$, and given p_2, $w_1 = \underline{w}_1$. If p_1 decreases further then all the cost of absorbing the lower revenue falls on process 2 workers since $\underline{w}_1$ is a lower bound. This leads to a shift in workers from process 2 to process 1, since w_2 is falling while w_1 is stationary, and thus yields the perverse supply response.

f) Lay-offs

A number of papers[10/] have sought to portray the LM-firm as a pool of labour; some members work within the firm while the rest are 'laid off', receiving unemployment benefit or alternative income while retaining membership in the firm. It is argued that if lay-off periods are distributed among the membership in an egalitarian way then such a firm will allocate the membership between internal and non-internal employment so as to maximise the total income of the membership pool. This would imply equating the value marginal product of a worker within the firm with that earned by those workers laid off. In our model, process 1 could be interpreted as within-firm production while process 2 could be interpreted as income generated by laid-off workers. Then in the absence of differential x-characteristics (i.e. $\underline{x} = \bar{x} = \hat{x}$), all members would be allocated to one process or all to the other due to the assumed constancy of value marginal products. Thus if $p_1 - p_2 > \hat{x}$, there would be no lay-offs while if $p_1 - p_2 < \hat{x}$ there would be no within-firm production.

If individuals differ as to their x-characteristic, then an immediate problem arises. Suppose we have some members working in the firm while others volunteer to be laid off and receive the net income w_2 (composed of p_2 'outside' earnings on unemployment benefit plus $w_2 - p_2$ compensation from within the firm). Then the laid-off members will tend to be the same members period after period since high x individuals require less compensation. If laid-off members form a minority then they will be discriminated against in terms of wage levels. Also if $w_2 - p_2 > 0$ such members constitute a charge on the rest and in that case the 'rest' may use their majority voting power to attempt to turn those voluntarily laid-off into compulsorily-redundant workers with insufficient compensation. Such a problem does not arise if $p_2 > w_2$ since then process 2 workers are contributing net income to the firm. Also if members perceive the threat of redundancy, they may not regularly volunteer to be laid off. In this situation the internal labour market mechanism would break down.

g) Sympathy

In voting for wage regimes our analysis assumed that members have selfish preferences. However Sen (1966) has argued that sympathy may mean that members include other members' utilities in their own utility function. With "perfect sympathy", each member would wish to maximise the sum of all members' utilities, i.e. Z as defined in (12). Then wage rates will be chosen to lead to a Pareto efficient choice of labour allocation. With less than perfect sympathy we would expect an outcome between that of no sympathy and perfect sympathy.

To show this we can calculate

$$\phi = \Delta^{*} - \Delta^{**}$$

$$\phi = (w_1^{*} - w_1^{**}) + (w_2^{**} - w_2^{*}) \tag{39}$$

for various degrees of sympathy represented by the parameter s in the individual's revised maximand

$$U(x) = (1-s) \max \{w_1 - x,\ w_2\} + sg\{\int_{\underline{x}}^{w_1-w_2} w_1 - x\,dx + \int_{w_1-w_2}^{\bar{x}} w_2\,dx\} \tag{40}$$

If s is zero for all individuals then these have no sympathy; if $s = 1$ then perfect sympathy. The expression ϕ represents the sum of two non-negative quantities since $w_1^{*} \geqslant w_1^{**}$ and $w_1^{**} \geqslant w_2^{*}$ by definition. As ϕ becomes smaller so the differences between the two wage regimes decreases. When s is zero, w_i^{*}, w_i^{**} $(i = 1, 2)$ are as given in Figure 1. When s is equal to one then the first part of $U(x)$ is eliminated and $w_i^{*} = w_i^{**}$ $(i = 1, 2)$, so that $\phi = 0$. In general ϕ can be calculated (see Appendix) as

$$\phi = (1 - s)(\bar{x} - \underline{x})/\{sg(\bar{x} - \underline{x}) + 2(1 - s)\}$$

$$= \frac{(\bar{x} - \underline{x})}{\frac{s}{1-s}\, g(\bar{x} - \underline{x}) + 2} \tag{41}$$

So that, as $d\phi/ds < 0$, so more sympathy reduces the differences in the wage regimes. The extent of the discontinuity in labour allocation is given by (4) as

$$L_2^{**} - L_2^{*} = g\phi$$

and, as in subsection (b) and (c), the model returns to that initially analysed if the parameter relating to the generalisation (s) is set to zero.

5. CONCLUSIONS

Decision making by an assembly of worker-members may be democratic but may not be always desirable. The possibility exists that a majority of members can evolve to further their own interests even at the expense of the rest of the membership. Even if an egalitarian constitution enables all members to choose among the available process options and constrains wage rates to be the same for all workers in the same process, wage discrimination across processes can still arise. The results are allocative inefficiency in the form of over-specialisation and the possibility of discontinuous supply curves.

The extent of wage differentials between processes may be limited by feelings of 'sympathy' and by outside opportunities. Alternatively, the problems of conflict and ill-feeling which may arise from wage discrimination might be avoided altogether by removing wage-setting from the agenda of the workers' assembly. Instead the firm's constitution could for example enforce equal wages for all members of the firm, or even job rotation for all members if discrimination could also involve non-pecuniary advantages. Such rigid constitutional features would certainly lead to problems of allocative inefficiency, not just in terms of the quantity allocation but also in terms of lost benefits from utilising members' comparative advantages.

Even if a more general technology or distribution of members is considered, the problems of majority voting are likely to persist. Avoidance is best achieved by remaining a small firm with a stable membership, leading to high sympathy. Thus we have demonstrated an explanation for the small size of labour managed firms in western economies.

Finally, it should be stressed that our analysis should not be taken to infer that the efficient labour allocation identified in (13) and (14) would result from any other particular form of firm organisation. Firms with management-union bargaining have an element of labour management (see Law, 1977) and in any case often involve inefficient contracts (see papers in the tradition of McDonald and Solow, 1981). However, our analysis does serve as a reminder that democratic decision-making does not necessarily lead to economic efficiency.

FOOTNOTES

* Previous version of this paper were presented to seminars at the University of Warwick, University of Newcastle and to the Annual Conference of the European Association for Research in Industrial Economics, Madrid, September 1987. My thanks to participants for their comments, and to Peter Law and anonymous referees of this Journal for a number of valuable suggestions.

1/ See Ireland and Law (1982) for a discussion of a number of aspects of the Illyrian model.

2/ We will use the term over-specialisation to mean a too asymmetric or unequal activity vector, while under-specialisation is analogously a too symmetric or equal activity vector.

3/ Other explanations include capital rationing, lack of property rights in capital and the Ward-Vanek effect. See Ireland and Law, 1982.

4/ In the last decade the number of cooperatives in the UK has risen from about 100 to over 1500. However 90% have less than 20 members and 70% have less than 10. Some discussion of individual cases is given in Cockerton and Whyatt (1984).

5/ The fixed cost provides a rationale for members to work in a firm rather than independently. We assume that the number of workers in the firm is constrained by the nature of the capital stock, so that the firm cannot profitably recruit more members despite the constancy of average revenue.

6/ It is an important constraint on the conduct of the firm that a common wage be paid for work on a particular process. Otherwise, once a majority (say those in process 1) are in control they could behave like a perfectly discriminating monopsonist and pay all the process 2 workers specific individual wages so that they each earn only their reservation utilities.

7/ If $p_1 - p_2 > \bar{x}$ then all members will work in process 1; if $p_1 - p_2 < \underline{x}$ then all will work in process 2.

8/ The importance of the median member is also stressed by Montias (1986), Askildsen, Ireland and Law (1987), and Ireland and Law (1988).

9/ Q is a qradratic form, $Q = \sum_i \sum_j R_{ij} x_i x_j$ where $x_i = 1$, $x_j = -1$. Since R is concave, and Q is a quadratic form composed of R's hessian matrix, Q is negative semi-definite.

10/ See for example Miyazaki and Neary (1983).

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APPENDIX

(a) Derivation of (6i) - (9ii).

Using (3) and (4) in (5) for $\underline{x} \leqslant w_1 - w_2 \leqslant x$ yields

$$-gw_1^2 + 2gw_1w_2 - gw_2^2 + g(p_1 - p_2 + \underline{x})\, w_1 + g(-\bar{x} - p_1 + p_2)\, w_2 + (g\bar{x}p_2 - g\underline{x}p_1 - F) = 0 \qquad \text{(A1)}$$

(A1) is a general equation of the second degree and has the form of a parabola on a twisted axis, as drawn in Figure 1. Differentiating (A1) with respect to w_2 and setting equal to zero gives the condition for the parabola to be vertical:

$$-L_2 + (p_2 - w_2 - p_1 + w_1)g = 0$$

or, solving for w_2,

$$w_2 = w_1 - (\bar{x} - p_2 + p_1)/2 \qquad \text{(A2)}$$

Substituting (A2) back into (A1) yields

$$w_1^* = p_1 + \frac{\{((\bar{x}+p_2-p_1)/2\}^2 - F/g}{\bar{x}-\underline{x}}$$

Then combining (A2) and (A3):

$$w_2^* = w_1^* - (\bar{x} - p_2 + p_1)/2 \qquad \text{(A4)}$$

Also using (A2) in (3) and (4) defines the associated labour allocations:

$$L_1^* = \{(\bar{x} - p_2 + p_1)/2 - \underline{x}\}g \qquad \text{(A5)}$$

$$L_2^* = g(\bar{x} + p_2 - p_1)/2 \qquad \text{(A6)}$$

The outcome from Proposal (1) then yields (6i) → (9i) by using (A6) in (A3). Equations (6ii) - (9ii) are obtained in an analogous way. Using (3) and (4) in (A1) and differentiating with respect to w_1 yields

$$-L_1 - (p_2 - w_2 - p_1 + w_1)g = 0 \qquad \text{(A7)}$$

and

$$w_1 = w_2 + (\underline{x} - p_2 + p_1)/2 \qquad \text{(A8)}$$

Substituting (A8) into (A1) and solving for w_2 yields

$$w_2^{**} = p_2 + \{((p_1 - p_2 - \underline{x})/2)^2 - f\}/(\bar{x} - \underline{x}) \qquad \text{(A9)}$$

and using (A9) in (A8) yields (7ii)

$$w_1^{**} = w_2^{**} + (\underline{x} - p_2 + p_1)/2$$

Finally using (A8) in (3) and (4) yields (8ii) and (9ii) and hence (6ii).

A numerical example may assist. Suppose that $F = 0$, $g = 1/2$, $p_1 = p_2 = 1$ and $\underline{x} = -1$, $\bar{x} = 1$. Then (A1) is that graphed in Figure 1, and has the explicit form

$$w_1 = w_2 - \frac{1}{2} \pm \sqrt{9/4 - 2w_2}$$

Also $w_1^* = 9/8$, $w_2^* = 5/8$ while $w_1^{**} = 5/8$, $w_2^{**} = 9/8$. Associated labour allocations are $L_1^* = 3/4$, $L_2^* = 1/4$ and $L_1^{**} = 1/4$, $L_2^{**} = 3/4$.

In this example $L_1, L_2 > 0$ for all w_1, w_2 satisfying (A1), as can be checked from (3) and (4). In general, this would not necessarily be the case. Also the option of leaving the firm may constrain the ranges of wage levels for which (A1) is valid.

(b) Derivation of the Welfare Loss

Let ΔL_1 be the excess of equilibrium sector 1 employment over the welfare optimal L_1^o. Then, using (12),

$$\Delta Z = Z(L_1^o + \Delta L_1) - Z(L_1^o)$$

$$= (p_1 - p_2)\,\Delta L_1 - \int_{\underline{x} + L_1^o/g}^{\underline{x} + L_1^o/g + \Delta L/g} gx\,dx \qquad \text{(A10)}$$

Integrating (A10) and using (13) yields

$$\Delta Z = -(\Delta L_1)^2/2g \qquad \text{(A11)}$$

Now if $p_1 - p_2 > x^m$ then

$$\begin{aligned} \Delta L_1 &= L_1^* - L_1^o \\ &= L - L_2^* - L_1^o \\ &= L - 1/2\,(L - L_1^o) - L_1^o = 1/2(L - L_1^o) \end{aligned}$$

while if $p_1 - p_2 < x^m$ then

$$\begin{aligned} \Delta L_1 &= L_1^{**} - L_1^o \\ &= -1/2\,L_1^o \end{aligned}$$

Thus $\Delta L_1 = 1/2(L - L_1^o)$ if $p_1 - p_2 > x^m$
$\qquad -1/2\,L_1^o$ if $p_1 - p_2 < x^m$

or alternatively, using (13) and (14)

$$\Delta L_1 = \begin{cases} 1/2(L - L_1^o) & \text{if } L_1^o > L/2 \\ -1/2\,L_1^o & \text{if } L_1^o < L/2 \end{cases}$$

so that

$$\Delta Z = -\{\min L_1^o, L_2^o\}^2/8g \qquad (A12)$$

from which (17) is obtained by using (13) and (14), and noting that $-\Delta Z$ is the welfare loss.

(c) Derivation of (41)

First set up the Lagrangean function

$$\Lambda = (1-s)\ w_2 + sg\{\int_{\underline{x}}^{w_1-w_2} w_1-x^2\ dx + \int_{w_1-w_2}^{\bar{x}} w_2 dx - \int_{\underline{x}}^{\bar{x}} x_2 h(x_2) dx_2\}$$

$$- \lambda g\{(p_1-w_1)(w_1-w_2-\underline{x}) + (p_1-w_2)(\bar{x}-w_1+w_2) - F/g\} \qquad (A13)$$

First order condition for maximising (40) subject to (5) when the individual is to work in process 2 then include

$$\partial\Lambda/\partial w_1 = sg(w_1-w_2-\underline{x}) - \lambda g\{-(w_1-w_2-\underline{x}) + p_1-w_1-p_2+w_2\} = 0 \quad (A14)$$

$$\partial\Lambda/\partial w_2 = 1-s + sg(\bar{x}-w_1+w_2) - \lambda g\{-(\bar{x}-w_1+w_2)$$

$$- p_1+w_1+p_2-w_2\} = 0 \qquad (A15)$$

adding (A14) and (A15) and solving λ yields

$$\lambda = -\{1-s + sg(\bar{x}-\underline{x})\}/g(\bar{x}-\underline{x}) \qquad (A16)$$

and then using (A16) in (A14) yields

$$w_1^{**} - w_2^{**} = \frac{sg(\bar{x}-\underline{x})(p_1-p_2) + (1-s)(\underline{x}+p_1-p_2)}{(sg(\bar{x}-\underline{x}) + 2(1-s)} \qquad (A17)$$

The analogous Lagrangean for the case where the individual chooses to work in process 1 is

$$\Lambda' = \Lambda - (1-s)w_2 + (1-s)(w_1-x) \qquad (A18)$$

An equivalent procedure to that above yields that

$$w_1^{*} - w_2^{*} = \frac{sg(\bar{x}-\underline{x})(p_1-p_2) + (1-s)(\bar{x}+p_1-p_2}{sg(\bar{x}-\underline{x}) + 2(1-s)} \qquad (A19)$$

and subtracting (A17) from (A19) yields (41).

Figure 1

The Wage Frontier

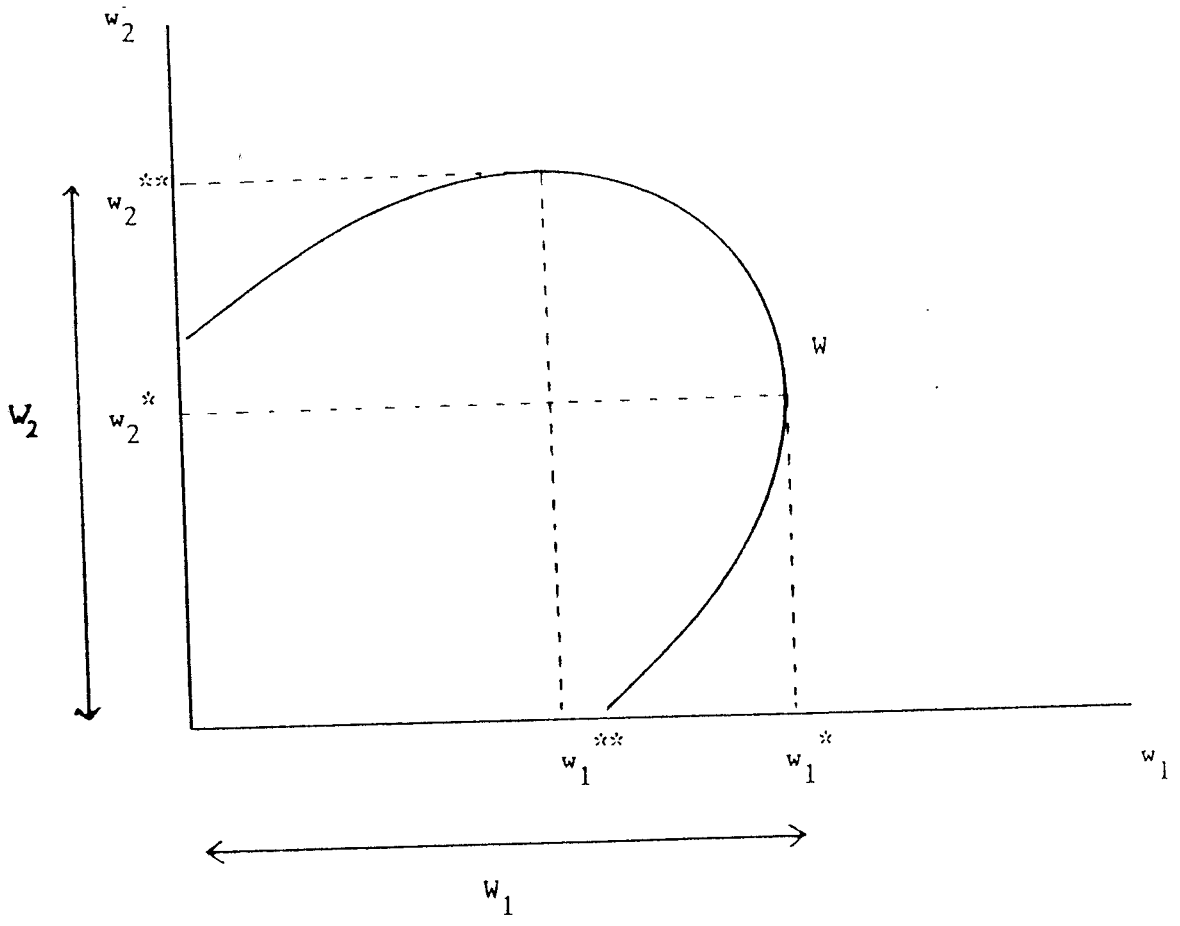

Figure 2

Employment in Process 1

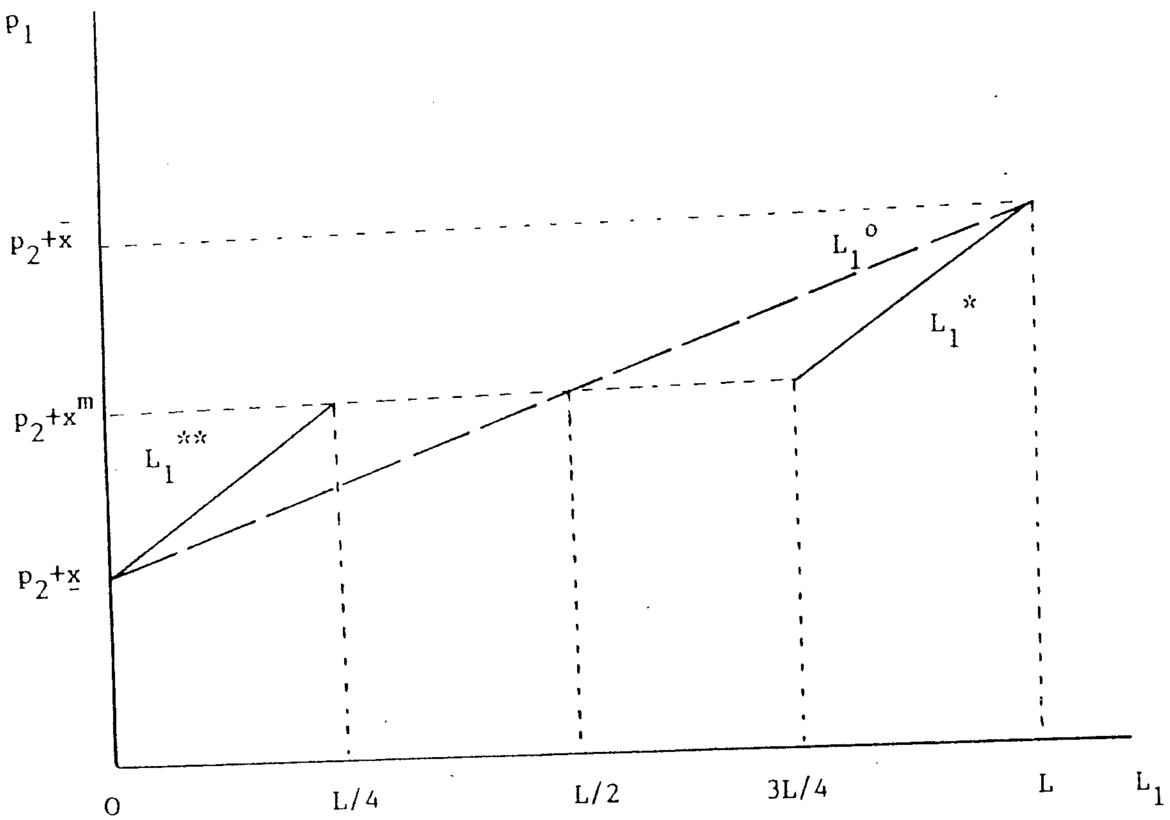

Figure 3

Wage Discrimination

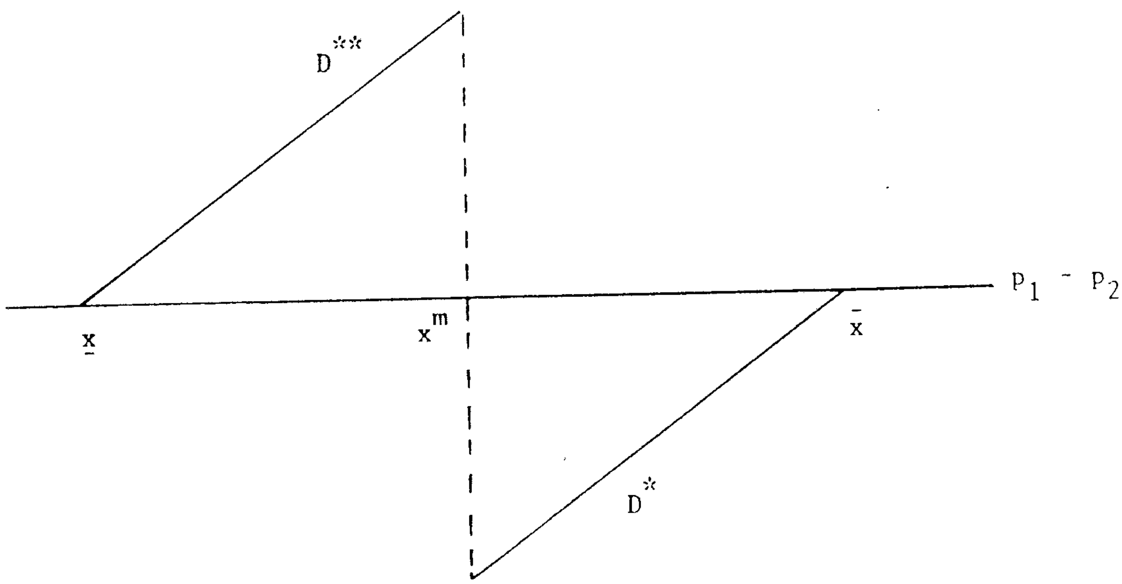

Figure 4

Labour Allocation and Density g(x)

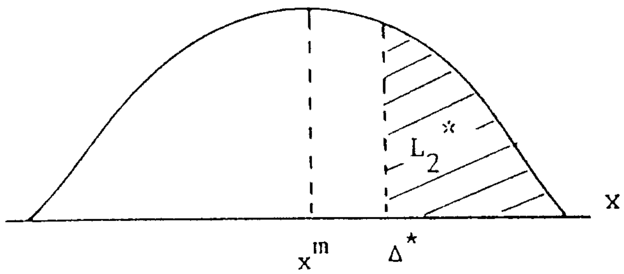

Figure 4a : Symmetric Distribution

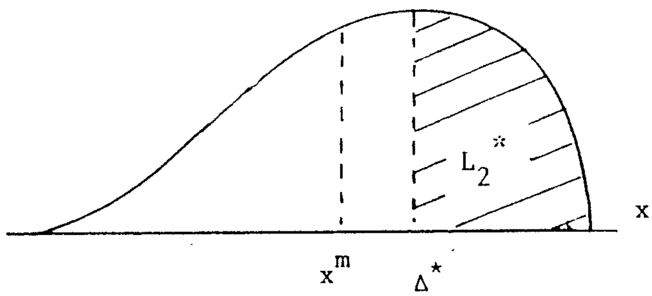

Figure 4b : Negative-skewed Distributi

Figure 5

Downward Sloping Supply at Low Prices