GENERAL EQUILIBRIUM AND IMPERFECT COMPETITION: PROFIT FEEDBACK EFFECTS AND PRICE NORMALISATIONS

M.W. CRIPPS AND G.D. MYLES

UNIVERSITY OF WARWICK

No. 295

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
COVENTRY

GENERAL EQUILIBRIUM AND IMPERFECT COMPETITION: PROFIT FEEDBACK EFFECTS AND PRICE NORMALISATIONS

M.W. CRIPPS AND G.D. MYLES

UNIVERSITY OF WARWICK

No. 295

April 1988

This paper is circulated for discussion purposes only and its contents should be con sidered preliminary.

GENERAL EQUILIBRIUM AND IMPERFECT COMPETITION: PROFIT FEEDBACK EFFECTS AND PRICE NORMALISATIONS.

M.W. Cripps and G.D. Myles*

University of Warwick

April 1988

Abstract: There are two general issues which bedevil general equilibrium models with imperfect competition: price normalisation and the feedback from prices to incomes. We present a class of normalisations which do not affect the behaviour of oligopolists, this is argued to be the only class of normalisations with this general property. We also provide a set of necessary conditions for the existence of equilibrium with monopoly and feedback effects.

*Postal Address: Department of Economics, University of Warwick, Coventry, CV4 7AL

1. INTRODUCTION

Despite the abundant body of literature on the properties of perfectly competitive general equilibrium models, analysis of equilibrium with imperfect competition has not yet progressed far. One possible explanation for this is the plethora of technical and existence problems which beset the modeller. This paper addresses the problem of finding an appropriate normalisation in such a general equilibrium model. Arbitrary normalisations can generate an indeterminacy in the equilibrium because they affect oligopolists relative profits. Here we argue for a particular class of normalisations which, by satisfying specified conditions, do not cause such indeterminacy.

An important aspect of this paper, and one that has been emphasised by Nikaido (1975), is the employment of objective demands in formulating the oligopolists' maximisation. In essence, the demand function is chosen by the consumer based on the prices he faces and the value of his profit and wage income; the oligopolist is assumed to know this chosen demand function and to allow for the dependence upon profits when choosing his optimal price. There is, of course, a circularity here in that profit depends upon demand which, in turn, depends upon profit; this is usually termed a feedback effect. Section 2 of the paper contains two theorems which describe when such maximisations have finite solutions in the case of monopoly. It must be noted that in a closed system it would be inconsistent and irrational for a firm with the power to set its own price not to recognise the effect of its distributed profits upon demand. It is one of the purposes of this paper to show that such recognition does not lead to intractable analysis, or to difficulties of interpretation.

Models similar to those in Section 2 have been studied by Nikaido (1975) and Cornwall (1977). In contrast to Nikaido, we have emphasised throughout the structure of the decision problem facing monopolists and, unlike Cornwall, we have chosen to retain profit maximisation as their objective. The retention of profit maximisation implies a dichotomy between the behaviour of consumers as the managers of firms and as recipients of profits trading in the marketplace. This is not a problem in a more general model where the consumers are small and diverse. In this case a rational consumer will have no influence over the firm and will treat profits as a parameter of its decision problem. An alternative rationalisation of this assumption is given in Gabszewicz and Vial (1972).

In the discussion of general equilibrium with imperfect competition Cornwall (1977), Gabszewicz and Vial (1972), and Dierker and Grodal (1986) have argued that the equilibrium reached is dependent upon the price normalisation rule. In their models real variables are dependent upon the absolute level of prices; a conclusion that goes against intuition. In contrast, the example we analyse satisfies standard homogeneity conditions so that it is relative prices that determine equilibrium. Consequently prices can be normalised, providing the normalisation procedure, which we discuss below, is correctly chosen. The features of the example are then generalised in Section 3. We address the issue of normalisation in a full general equilibrium model and characterise a class of normalisation rules that do not influence the imperfectly competitive general equilibrium.

Section 2 of the paper analyses a simple example of a monopolist who recognises the dependence of demand upon profits and considers the existence of a general equilibrium after the introduction of a monopolised industry. Section 3 then introduces a full general equilibrium model and proves the existence of a set of normalisations which do not determine the equilibrium outcome. This class is argued to contain all the normalisations with this property. Conclusions are given in Section 4.

2. AN EXAMPLE: A SINGLE MONOPOLIST.

In this section we establish that a monopolist's profit maximisation problem has a well defined solution when there are the feedback effects from its distributed profits either in a partial equilibrium, or in a general equilibrium. We then discuss the existence of a general equilibrium of this model and the properties of a class of price normalisation rules with respect to this equilibrium.

2.1. PROFIT MAXIMISATION WITH PROFIT FEEDBACK EFFECTS: PARTIAL EQUILIBRIUM.

Assuming a partial equilibrium framework, the monopolist faces demand $X = X(q, \pi)$, where q denotes his chosen price and π the level of the profits. This implies that the monopolist is sufficiently large with respect to the total economy that his profit level has a non-negligible effect upon the demand he faces. Production takes place subject to the cost function $C = C(X)$. Combining these, the profit maximisation problem is

$$\max_q \quad \pi = q.X(q, \pi) - C(X(q, \pi)) \equiv F(q, \pi) \qquad (1)$$

To analyse the maximisation of (1) consider providing consumers with a fixed income of value I and assume that the monopolist's profits remain undistributed. The existence of a maximum level of monopoly profits for the original problem can then be proved by constructing a mapping from $I \rightarrow \pi$ and showing that this mapping has a unique fixed point. Such a fixed point is equivalent to the monopolist distributing profits to some value I* and earning profits to the same value.

The following assumption is required for the proof:

A.1: $\pi^I(I) \equiv \max_q F(q,I)$ is a continuous strictly increasing differentiable function of I, and $\pi^I(0) > 0$.

Theorem 1. If A.1 holds, and $d\pi^I/dI < r < 1$ for all $I \geq 0$, then: $\pi = F(q,\pi)$, has a unique maximum π^*.

Proof: We wish to show that $\pi^I(I)$ is a contraction mapping and hence has a unique fixed-point by the contraction mapping theorem. The function $\pi^I(I)$ is continuous by A.1 and $\pi^I(0)>0$, so all that remains to be shown is that the function $\pi^I(I)$ has a unique intersection with the 45° line. It is clear that provided the gradient of $\pi^I(I)$ is strictly positive and uniformly bounded above by $r<1$, then a unique intersection must exist. (An extension of this method of proof to oligopoly is given in the appendix.) ∎

The important condition of theorem 1 is $d\pi^I(I)/dI < r$ and we now provide an economic interpretation of this. For some fixed I, the optimal q solves

$$X + q\delta X/\delta q - \delta C/\delta X . \delta X/\delta q = 0.$$

Write this choice as $q = q(I)$. Profits as a function of I can then be defined as: $\pi^I(I)=F(q(I),I)$, hence

$$d\pi^I(I)/dI = (X + q\delta X/\delta q - \delta C/\delta X.\delta X/\delta q)dq/dI + (q - \delta C/\delta X)\delta X/\delta I.$$

But, given optimality of q(I) we know $d\pi^I(I)/dI = (q - \delta C/\delta X)\delta X/\delta I$. The theorem requires $d\pi^I(I)/dI < r$, from above this is satisfied if

$$\frac{qX}{X}\frac{\delta X}{\delta I} < -r.\frac{q}{X}\frac{\delta X}{\delta q}. \qquad (2)$$

Assuming qX is approximately equal to I, this becomes $\epsilon^d_I<r|\epsilon^d_q|$. From (2), a maximum profit level exists if the income elasticity of demand is strictly less than the absolute value of the price

elasticity. In effect, the positive feedback must be constrained so that the system does not explode.

2.2 EXISTENCE OF GENERAL EQUILIBRIUM.

We now consider an economy with two consumption goods: one produced by a monopolist who chooses profit levels in the manner described above, the other produced by a perfectly competitive sector under conditions of constant returns to scale. Both goods are produced using labour alone. We establish that there is a solution to the monopolist's optimisation problem in this economy under much weaker conditions than those outlined above and then use this to establish the existence and homogeneity properties of the general equilibrium and to discuss the role of price normalisations.

The detailed structure of the model is as follows: there are three goods X,Y,L which are monopoly output, perfectly competitive output, and labour respectively. There are three types of agent: a monopolist, a perfectly competitive sector, and a set of consumers. The consumers' behaviour is characterised by an aggregate labour supply function $L^s(q,p,w,\pi)$ and two demand functions $X(q,p,w,\pi)$, $Y(q,p,w,\pi)$, where p and w are the prices of the competitive good and labour. (In writing these we are assuming that the distribution of profit income remains constant.) The monopolist and the perfectly competitive sector have only one input: labour. The technology is constant returns to scale in the competitive sector, using σ units of labour to produce one unit of output, and the monopolist's technology is described by the cost function $C[X(q,p,w,\pi),w]$.

The monopolist will choose a profit level and a price q to solve the constrained optimisation problem $\max_{\pi,q} \pi$; subject to, $\pi \geq 0$, $q \geq 0$; and $\pi = qX(q,p,w,\pi) - C(X(q,p,w,\pi);w)$. Using homogeneity of the demand and cost functions and assuming the perfectly competitive industry to be in equilibrium, so that $p = \sigma w$, this problem can be re-written;

$$\max_{a,b} wb; \quad \text{s.t. } b \geq 0,\ a \geq 0;\ b = af(a,b) - c(f(a,b)). \qquad (I)$$

Where $a \equiv q/w$, $b \equiv \pi/w$, $c(.) \equiv C(f(a,b);1)$ and $f(a,b) \equiv X(q/w,\sigma,\pi/w,1)$. We will make the following assumptions:

A.2: $f(a,b)$ and $c(.) > 0$ are continuously differentiable.

A.3: $\lim_{a\to\infty} af(a,b) = 0$, for all b.

A.4: Let $L^s(a,b)$ be continuously differentiable and $\lim_{b\to\infty} L^s(a,b) = 0$ for all a and $\lim_{a,b\to\infty} L^s(a,b) = 0$.

A.5: There exists $a \geq 0$ such that $0 = af(a,0) - C[f(a,0)]$.

Assumption A.3 assumes that the demand function generates zero revenue as the relative price of the monopolist's output becomes large. This requires either demands that eventually intersect the price axis or that tend to zero sufficiently quickly. Assumption A.4 says that as consumers profit income becomes large consumers' labour supply tends to zero uniformly in wages. A.3 and A.4 are essentially generalisations of eq. (2). Assumption A.5 ensures that there is a zero profit equilibrium with $q > 0$.

We now establish a more general result on the existence of of a solution to the monopolist's optimisation problem.

Theorem 2. Under A.2,A.3,A.4,A.5 there exists a well-defined finite solution to problem I.

Proof. See appendix. ∎

Having established the existence of a solution to the monopolist's optimisation, the existence of a general equilibrium is a trivial matter and can be verified by an accounting exercise. The monopolist is always producing on his demand curve so that his market is always in equilibrium. The monopolist's budget constraint is satisfied by construction and the budget constraint of the competitive sector will also be satisfied. Furthermore, there will be a zero-profit equilibrium in the competitive market when $p = \sigma w$. Equilibrium in the labour market then follows from the consumer's budget constraint by Walras' Law. This proves the existence of a general equilibrium.

2.3 HOMOGENEITY AND NORMALISATION RULES.

We now analyse the homogeneity properties of the equilibrium in this model and show that there is the usual first order homogeneity in prices and that there is a natural normalisation rule which does not affect the real equilibrium. Before proceeding to demonstrate this, it is useful to outline the underlying reasoning. The monopolist's price and profit level are both functions of the wage rate and the price of the competitively produced good, which in turn is a constant multiple of the wage rate. In effect, all prices and profits are determined by the level of the wage rate hence any normalisation procedure must amount to a choice of the wage rate or, equivalently, of the price of the competitive good. We show below that the functions determining other prices and the level of profits are homogenenous of degree one in the wage rate, so the value of the wage rate will not affect relative prices. Hence the equilibrium will be unaffected by a normalisation rule that consists of selecting a value for the "absolute" wage rate.

From the maximisation of profits, the monopolist will condition his optimal price upon the price of the competitively produced good and upon the wage rate. We will write this choice as $q = \Phi(p, w)$. Similarly, maximised profits are written $\pi = \Omega(p, w)$. The competitive sector produces with constant returns to scale so that its labour demand is a constant fraction of output. Writing demand as $Y(q, p, w, \pi)$, labour demand from this sector, $L^d{}_Y$, is $\tau Y(q, p, w, \pi)$, $\tau = 1/\sigma$. The demand for labour from the monopolist is given, by Shephard's lemma, as the first derivative of the cost function $L^d{}_X = \delta/\delta w\ (C(X; w))$. Finally, labour supply is determined by utility maximisation of the private sector, hence; $L^s = L(q, p, w, \pi)$, and equilibrium requires $L^d{}_X + L^d{}_Y = L^s$. The following two lemmas are required.

Lemma 2. Profit π is homogeneous of degree one in q, p and w.

Proof. Defining profits and multiplying q, p and w by μ gives

$$\pi(\mu) = \mu q.X(\ \mu q,\ \mu p,\ \mu w,\ \pi(\mu)\ ) - C(\ X(\ \mu q,\ \mu p,\ \mu w,\ \pi(\mu)\ );\ \mu w\ ).$$

As X(μq, μp, μw, $\pi(\mu)$) is homogeneous of degree zero, and the cost function is homogeneous of degree one in w

$$\pi(\mu)/\mu\ = q.X(\ q,\ p,\ w,\ \pi(\mu)/\mu\ ) - C(\ X(\ q,\ p,\ w,\ \pi(\mu)/\mu\ );\ w\ ).$$

But, as π was the unique maximum profit $\pi(\mu)/\mu = \pi$, or $\pi(\mu) = \mu\pi$. ∎

Lemma 3. The profit maximising price, q^*, is homogeneous of degree one with respect to p and w.

Proof. Let q* maximise profits for price p, wage rate w and write the maximised value of profits as π^*. Consequently, assume for μp and μw that profits are maximised by $q' \neq \mu q^*$. As profits are homogeneous of degree one in q, p and w , $q' = \max_q \{\ \pi(\mu)/\mu\ \}$, where

$$\pi(\mu)/\mu = q/\mu.X(q/\mu,\ p,\ w,\ \pi(\mu)/\mu) - C(\ X(q/\mu,\ p,\ w,\ \pi(\mu)/\mu);\ w\ )\}.$$

But q* was the solution of this problem, so $q' = \mu q^*$ and the profit maximising price is homogeneous of degree one in p and w. ∎

These prove the claim that the equilibrium satisfies standard homogeneity properties. The important features of this model are summarised in theorem 3 where it should be recalled that general equilibrium is synonymous with equilibrium in the labour market.

Theorem 3. Labour market equilibrium holds, it satisfies standard homogeneity properties, and it is continuously dependent upon the transformation ratio, r, in the competitive industry.

Proof. The first two parts of the statement have been demonstrated above, to validate the remainder of the theorem, labour market equilibrium requires $\tau Y(q,p,w,\pi) + \delta/\delta w(C(X(q,p,w,\pi);w)) = L(q,p,w,\pi)$. This can be written in terms of the monopolist's behaviour.

$$\tau Y(\Phi(p,w),p,w,\Omega(p,w)) + \delta/\delta w(C(X(\Phi(p,w),p,w,\Omega(p,w));w)) = L(\Phi(p,w),p,w,\Omega(p,w)).$$

The assumption of zero profits in the competitive sector implies that $p = \sigma w$, where $\sigma = 1/\tau$. Using this and employing Lemmas 2 and 3, the homogeneity allows this to be expressed as

$$(1/\sigma)Y(w\Phi(\sigma,1),\sigma w,w,w\Omega(\sigma,1)) + \delta/\delta w(C(X(w\Phi(\sigma,1)\sigma w,w,w\Omega(\sigma,1));w)) = L(w\Phi(\sigma,1),\sigma w,w,w\Omega(\sigma,1)).$$

As $Y(.)$, $X(.)$ and $L(.)$ are derived from utility maximisation they are homogeneous of degree zero in their arguments so

$$(1/\sigma)Y(\Phi(\sigma,1),\sigma,1,\Omega(\sigma,1)) + \delta/\delta w(C(X(\Phi(\sigma,1)\sigma,1,\Omega(\sigma,1));w)) = L(\Phi(\sigma,1),\sigma,1,\Omega(\sigma,1)).$$

Finally, as $C(X(\Phi(\sigma,1)\sigma,1,\Omega(\sigma,1));w)$ is homogeneous of degree one in w, its derivative is homogeneous of degree zero. The equilibrium equation becomes:

$$(1/\sigma)Y(\Phi(\sigma,1),\sigma,1,\Omega(\sigma,1)) + \delta/\delta w(C(X(\Phi(\sigma,1)\sigma,1,\Omega(\sigma,1));1))$$

$$= L(\Phi(\sigma,1),\sigma,1,\Omega(\sigma,1)).$$

This is continuously dependent on σ, hence τ, given continuity of consumer demands and of the monopolist's labour demand. ∎

The conclusion to be drawn from Theorem 3 is that starting from any initial wage rate w, we can move to any other, say w', without affecting the equilibrium. In particular, if we choose equilibrium w, q, p to be such that w + q + p = 1, so these are "relative" prices, we can transform to absolute prices using $w' = \beta(w).w$, $q' = \beta(w).q$ and $p' = \beta(w).p$, $\beta(w) > 0$ but otherwise arbitrary. Here $\beta(w)$ represents our "normalisation rule" and the equilibrium of the economy is invariant with respect to choice of $\beta(w)$. Section 3 investigates this form of normalisation in a formal context.

2.4 COMPARISON WITH STANDARD MODEL.

In this sub-section we contrast the solution in our model with the solution obtained when there are no feedback effects. The equilibrium concept we have in mind for our comparison is one in which the monopolist chooses his output to maximise profits conditional on the observed demand function but does not take account of how profits affect demand. The requirement that the monopolist's profit is equal to non-wage incomes is imposed as an equilibrium condition. Hence, equilibrium is described by:

(i) $q(1) \in \text{argmax}_q\, qX(q,p,w,\pi) - C[X(q,p,w,\pi)]$; s.t. $q > 0$;

(ii) $L^d_X + L^d_Y = L^s$;

(II)

(iii) $p = w/\tau$;

(iv) $I = \pi$.

This notion of equilibrium resembles as closely as possible that used in the literature (for example Hart (1982)), where objective demand functions without profit effects are considered.

It is obvious that in general the two different equilibrium concepts will generate two different behavioural patterns. However, we have already outlined sufficient conditions for the two outcomes to be equivalent. The partial equilibrium approach generated a solution to the monopolist's problem by calculating a function q(I) which solved (i), and then ensuring there was a unique solution to (iv). This unique solution to the monopolist's profit optimisation problem is also a unique solution to the above, hence under the conditions for Theorem 1 the two notions of equilibrium are identical.

Without the strong restrictions imposed by Theorem 1 there will be many possible equilibria which satisfy the condition (II) and a typical situation is illustrated by fig. I. Both A and C satisfy the first- and second-order conditions for (II) but only C satifies (I).

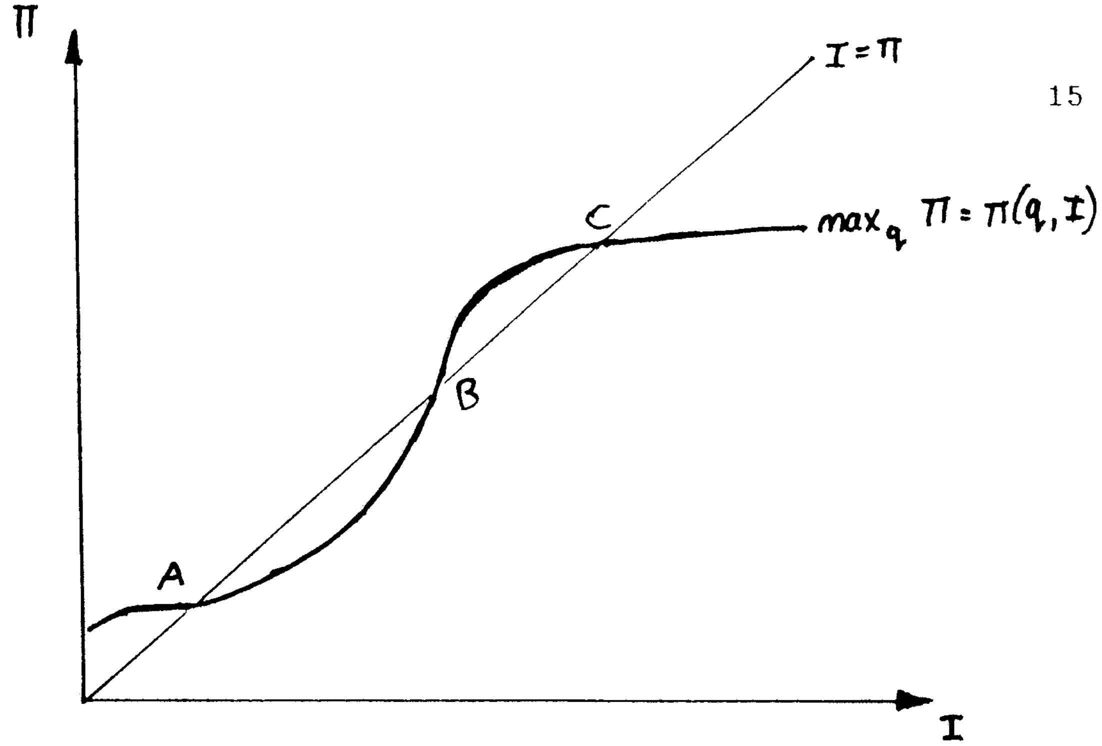

FIG. I

Thus recognition of the feedback profit effect results in the monopolist choosing amongst the various equilibria of the economy to find the one which ensures maximum profit.

3. PRICE NORMALISATIONS AND IMPERFECT COMPETITION.

In this section we outline a full general equilibrium model with quantity setting oligopolists and with profit feedback effects. We show that the merits of the normalisation procedure employed in Section 2.3 also apply in this more general framework. That is provided one normalises with respect to the prices of goods produced in *perfectly competitive industries with constant returns to scale*, there is no problem with the equilibrium being dependent on the particular form of the normalisation. This is contrary to the result of Dierker and Grodal (1986), who argue that every normalisation will affect the observed equilibrium behaviour of imperfect competitors. This is only true if there are no perfect competitors in the model, when these are present one can normalise with respect to the prices in these industries without the outcome being dependent on the form of the normalisation chosen. When there are *only* imperfect competitors in the model any normalisation, other than multiplying all prices by some common factor, must affect the relative profits of these agents and hence the real variables. However, provided one normalises in the perfectly competitive sector there need be no such effects.

Two results are given below. The first shows that the set of general equilibria of an oligopolistic economy with feedback is invariant with respect to the normalisation rule, provided one normalises with respect to the prices of the perfectly competitive constant returns firms. The second shows that this is, in general, the only normalisation rule with this property.

We begin by outlining the model. There are l goods indexed $k = 1,2,\ldots,l$, the first l_1 are produced by competitive industries, and goods $(l_1+1,\ldots,l)$ are produced by oligopolistic industries. The oligopolist firms are indexed $j = 1,2,\ldots,m$ and have production sets G_j which are compact and convex subsets of $\mathbb{R}^l$ and produce outputs $y_j \in G_j$. We assume the production sets satisfy the usual requirements and these firms only produce the goods indexed $(l_1+1,\ldots,l)$. The competitive firms are indexed $i=1,2,\ldots,n$, have production plans z_i from production sets G_i in IR^l. These firms are all assumed to experience constant returns to scale and only produce goods $(1,2,\ldots,l_1)$.

The consumers have strictly quasi-concave continuous preferences $U^h(x_h)$, $h=1,2,\ldots,H$, defined on the consumption set IR_+^l, endowments e_h and shares θ_{hj} in the relative profits π_j of the j^{th} firm. We will define the following arrays: $x = (x_1,\ldots,x_h,\ldots,x_H)$, $y = (y_1,\ldots,y_j,\ldots,y_m)$, $z = (z_1,\ldots,z_i,\ldots,z_n)$, $\theta = (\theta_{11},\ldots,\theta_{1m},\ldots,\theta_{Hm})$, and $\pi = (\pi_1,\ldots,\pi_j,\ldots,\pi_m)$ (relative profits). Also the vector of *relative* prices p is contained in the set $P := \{q \in IR_+^l \mid \Sigma q_k=1\}$.

Definition: A Competitive Equilibrium relative to (y,π) consists of a triple (p,x,z) s.t.

(I) x_h max. $U^h(x)$ s.t. $p.x_h \leq p.e_h + \Sigma_j\theta_{hj}\pi_j$ $(h = 1,\ldots,H)$

(II) z_i max. $p.z_i$ s.t. $z_i \in G_i$ $(i = 1,\ldots,n)$

(III) $\Sigma(x_h - e_h) = \Sigma_i z_i + \Sigma_j y_j$

The conditions above are sufficient for the existence of a competitive equilibrium relative to (y,π) for every (y,π), by standard arguments. It is also important to note that the perfectly competitive relative prices $(p_1,p_2,\ldots,p_l)$, are fixed by constant returns to scale and are independent of y_j or π_j.

We now define the notion of a price normalisation using the approach of Grodal and Dierker (1986). In essence a normalisation is a scaling of the relative prices p into absolute prices $q \in IR_+^1$. This can be described by a function α taking relative prices to absolute prices.

$$\alpha: P \to IR_+^1 \qquad \text{s.t. } \alpha(p)/\alpha(p).e = p$$
$$e' = (1,1,\ldots,1)$$

We now describe a full equilibrium for this model with quantity setting oligopolists and feedback effects. This is very much in the spirit of Gabszewicz and Vial (1972) apart from the explicit recognition of the feedback effects.

Definition: A *Cournot Feedback Equilibrium* consists of (y^*,π^*,p^*,x^*,z^*) and a normalisation rule $\alpha: p \to q$ s.t.

1) p^*,x^*,z^* is a competitive equilibrium relative to (y^*,π^*);
2) $p^*.y_j^* = \pi_j^*$, $(j = 1,\ldots,m)$;
3) $\alpha(p^*).y_j^* \geq \alpha(p').y_j'$ $\quad y_j' \in G_j \quad (j = 1,\ldots,m)$
   s.t. (p',x',z') is a C.E. relative to (y',π'),
   $y' = y_1^*,\ldots,y_{j-1}^*,y_j',y_{j+1}^*,\ldots,y_m^*)$
   $\pi'_j = p'.y_j'$ $(j = 1,\ldots,m)$.

This definition requires that the oligopolists choose their outputs y_j^* to maximise absolute profits $\alpha(p)y_j$; given the outputs of other firms; and subject to the constraints that the alternatives considered are on the objective demand function *and* the oligopolist recognises the profit feedback effect. The existence of this equilibrium requires first, strictly quasi-concave profit functions in (3) and, second, a unique C.E. associated with every (y,π) with a continuous normalisation, see Gabszewicz and Vial (1972). This is sufficient for the existence of an equilibrium without feedback, with these effects equilibrium must be proved using an extension of Theorem 2.

We now must describe the class of normalisations that are to be considered in the proof of the main result of this paper

DEFINITION: A normalisation $\alpha: p \to q$ is said to be *admissible* if it satisfies

$$\alpha(p_1,\ldots,p_l) = \beta(p_1,\ldots,p_{l_1})p$$

where $\beta: IR_+^{l_1} \to IR_+$.

To prove the theorem we will split the vector of relative prices into competitive and imperfectly competitive prices $p = (p^C, p^I)$.

Theorem 4. Let the normalisation rule be of the form $q = \beta(p^C)p$, then the set of feedback equilibria is invariant with respect to the function β.

Proof. Let (y^*,π^*,p^*,x^*,z^*) be a C.F.E. when the normalisation is β. This implies:

(1) x_h^* max. $U_h(x)$ s.t. $p^*x_h \leq p^*e_h + \Sigma_j\theta_{hj}\pi_j^*$ $(h = 1,\ldots,H)$

(2) z_i^* max. p^*z_i s.t. $z_i \in G_i$ $(i = 1,\ldots,n)$

(3) $\Sigma_h(x_h^* - e_h) = \Sigma_i z_i^* + \Sigma_j y_j^*$

(4) $p^*y_j^* = \pi_j^*$, $(j = 1,\ldots,m)$

(5) $\beta(p^{C*})p^*y_j^* \geq \beta(p^{C'})p'y_j' \quad y_j' \in G_j \quad (j = 1,\ldots,m)$

Notice that provided y_j^* satisfies (5) for an alternative normalisation $\Phi(p^C)$, 1,2,3,4 must be satisfied for $\Phi(p^C)$ as these conditions are independent of $\Phi(p^C)$. Hence it is sufficient to establish that the outcome of condition (5) is independent of the function Φ. Thus we must establish that if

$$\beta(p^{C*})p^*.y_j^* \geq \beta(p^{C'})p'.y_j' \qquad y_j' \in G_j$$

it is also true that

$$\Phi(p^{C*})p^*.y_j^* \geq \Phi(p^{C'})p'.y_j' \qquad y_j' \in G_j$$

for any function Φ. The constraint set in both these optimisations consists of a set of competitive equilibria relative to (π,y). In general a normalisation will influence an oligopolist's choice amongst these pairs, because it determines the absolute profit level associated with every relative profit level. The normalisation may also have a direct effect on the constraint set itself because it effects the relative size of p^C and p^I. However, by normalising with respect to the prices of competitive goods the determination of the relative prices p given (π,y) is independent of the form of normalisation. Also, by constant returns to scale the vector p^C is independent of y_j'. That is, the competitive equilibrium prices for goods produced under constant returns to scale are independent of other outputs, they are purely determined by the conditions for equilibrium in the perfectly competitive market. It follows that

$\beta(p^c)$ and $\Phi(p^c)$ are independent of the oligopolist's choice of y_j, hence we can divide the inequalities above by these constants. It remains to establish

$$p^* y_j^* \geq p' y_j', \quad y_j' \in G_j; \quad \Rightarrow \quad p^* y_j^* \geq p' y_j', \qquad y_j' \in G_j.$$

This is not as trivial as it appears because in general re-normalisation may affect the constraint set faced by the oligopolists. This is not a problem here because, as noted above, the map $(y,\pi) \rightarrow (x,z,p)$ is invariant with respect to β. ∎

It should now be clear why the condition of constant returns is necessary in the perfectly competitive industries. Under more general types of normalisation the oligopolists have incentives to adjust their outputs to optimally exploit the way relative prices are converted into absolute profits. This does not hold when there is normalisation in the constant returns perfectly competitive industries for two reasons. First the feasible set of competitive equilibria is independent of the normalisation when this only depends on p^c. Second, there is no incentive for the oligopolists to exploit the relative size of prices in the perfectly competitive sector, because the relative size of these is determined by constant returns.

Theorem 4 has demonstrated that the equilibrium is invariant to normalisation rules of the form $p = \beta(p^c)$. We now wish to establish that this is the only form of normalisation rule that has this property. We place two conditions on the normalisation rule: C1. requires relative prices to remain constant, C2. specifies that real behaviour taken here to be synonymous with profit maximising output choice, must remain unaffected.

C1. $p_i q_j = q_i p_j$ for all $i,j \leq 1$

C2. $y_j^* \equiv \text{argmax } \pi_j = y_j^* p(y_j^*, \pi_j) = y_j \equiv \text{argmax } \pi_j = y_j q(y_j, \pi_j)$

$(j=1,2,\ldots,m)$

where $p(y_i,\pi_i)$ and $q(y_i,\pi_i)$ are the Walras Correspondances for relative and absolute prices respectively.

We make three assumptions:

A6. The normalisation rule is continuous and differentiable.

A7. For each $i = 1,\ldots,m$ $\pi_i = y_i q(y_i,\pi_i)$ has a unique maximiser y_i^*.

To motivate the next assumption consider choosing an initial vector p^c. This will lead to the choice of profit-maximising outputs for the imperfectly competitive firms and, ultimately, a vector of prices for their outputs p^I. We can then view p^I as dependent upon p^C or $p^I = \theta(p^C)$. The assumption only requires that $\theta(p^C)$ is non-constant.

A8. $\exists\ \Phi \subset P$ s.t. for p^{C*} and $p^{C'} \in \Phi$, $p^I(p^{C*}) \neq p^I(p^{C'})$.

Theorem 5. For the CFE defined above, the only normalisation rules that satisfy C1. and C2. and A6,A7,A8 are of the form $q = \beta(p^C).p$.

Proof. This is broken down into two Lemmas.

Lemma 4. C1 $\Rightarrow q_i = \alpha(p)p_i$ *for all i.*

Proof. Define an intial normalisation rule $q_i = \alpha^i(p)$. From C1 we have $p_i q_j = p_j q_i$, summing both sides of this with respect to j gives $q_i = p_i \Sigma q_j$. Substitution from the normalisation gives

$$q_i = p_i \Sigma \alpha^j(p) \equiv p_i \alpha(p).$$

Lemma 5. $\alpha(p)$ is independent of p^I.

Proof. From the structure of the model, the first l_1 components of $p(y_i,\pi_i)$ are independent of y_i for all i=1,2,..m. Conmsider the maximisation of firm i, to satisfy C2. argmax $y_i p(y_i,\pi_i)$ = argmax $y_i \alpha(p(y_i,\pi_i))p(y_i,\pi_i)$ and, from Theorem 1, any maximiser must satisfy the first order condition

$$\delta Y_i P(Y_i,\pi_i)/\delta Y_i = \delta Y_i \alpha(P(Y_i,\pi_i))P(Y_i,\pi_i)/\delta Y_i = 0.$$

Carrying out these differentiations

$$y_i \delta p(y_i,\pi_i)/\delta y_i + p(y_i,\pi_i) =$$

$$\alpha(p(y_i,\pi_i))p(y_i,\pi_i) + y_i \alpha(p(y_i,\pi_i))\delta p(y_i,\pi_i)/\delta y_i +$$

$$y_i p(y_i,\pi_i)\delta\alpha(p(y_i,\pi_i))/\delta p(y_i,\pi_i)\delta p(y_i,\pi_i)\delta y_i = 0.$$

A8 then implies that for these to hold for all firms i=1,2,..m and all vectors p^C, $\delta\alpha(p(y_i,\pi_i))/\delta p(y_i,\pi_i) \equiv 0$ for all k = l_1+1,.,l.■

Combining Lemmas 4 and 5, $\alpha^i(p)$ must be of the form $\beta(p^C).p_i$ which proves Theorem 5. ■

Finally note that this normalisation rule also keeps the relative values of the profits of imperfectly competitive firms constant as we move from relative to absolute prices, another desirable property for a normalisation rule to possess.

4. SUMMARY AND CONCLUSIONS.

The example in Section 2 illustrated two features of the analysis of general equilibrium models with imperfect competition: the treatment of feedback effects and the issue of price normalisations. Both of these potential problems were remarkably simple to solve in that context. The feedback effect can be interpreted as the monopolist choosing amongst the potential equilibria, and we showed that there are reasonable and intuitive conditions for the existence of a well defined solution to this optimisation problem. The treatment of normalisation was also straightforward because the general equilibrium of the model was homogeneous degree zero with respect to the wage rate, which we assumed was determined in a competitive market. Thus any acceptable normalisation rule was equivalent to a choice of wage rate.

We investigated this form of normalisation in greater depth in Section 3, and showed that once one sector with constant returns to scale and perfect competition was included in a model of imperfect competition then there is a natural way of normalisisng prices and preserving all the usual homogeneity properties. The real behaviour in the economy was invariant with respect to normalisation rules that were functions of prices in the perfectly competitive sector and these are the only acceptable rules. Thus a slight generalisation of the usual general equilibrium model with imperfect competition makes apparent the class of normalisation rules that can be employed.

APPENDIX

EXTENSION OF THEOREM 1 TO A MODEL OF OLIGOPOLY.

Consider an n-firm oligopoly producing an homogeneous good in which each firm chooses their output level to maximise profits taking account of the effect of their distributed profits upon demand.

Indexing the firms by $i = 1,\ldots,i,\ldots,n$, a typical firm i chooses output to

$$\max \pi_i = F^i(x_i + X_{-i}, \pi_i + \pi_{-i})$$

where $X_{-i} = \Sigma^n_{j=1, j=i} x_j, \quad \pi_{-i} = \Sigma^n_{j=1, j=i} \pi_j$

Assume:

Ap 1: $F^i(x_i + X_{-i}, \pi_i + \pi_{-i})$ is continuous in both arguments.

Ap 2: $F^i(x_i + X_{-i}, \pi_i + \pi_{-i})$ is strictly concave with respect to its first argument.

Ap 3: X' and π' such that

$$F^i(X' + X_{-i}, \pi_i + \pi_{-i}) < 0, \text{ all } X_{-i}, \pi_i, \pi_{-i}$$

$$F^i(x_i + X_{-i}, \pi' + \pi_{-i}) < \pi', \text{ all } x, X_{-i}, \pi_{-i}$$

Ap 4: x_i such that $F^i(x_i + X_{-i}, 0 + \pi_{-i}) > 0$, all X_{-i}, π_{-i}

A Cournot Equilibrium for this model can be defined as:

An output vector $(x^c_1,\ldots,x^c_i,\ldots,x^c_n)$ and a profit vector $(\pi^c_1,\ldots,\pi^c_i,\ldots,\pi^c_n)$ such that

$$F^i(x^c_i + X^c_{-i},\ \pi^c_i + \pi^c_{-i}) > F^i(x_i + X^c_{-i},\ \pi_i + \pi^c_{-i})\quad i = 1,\ldots,n$$

all x_i, π_i s.t. $\pi_i = F^i(x_i + X^c_{-i},\ \pi_i + \pi^c_{-i})$, $0 < x_i < X'$

and x^c_i, π^c_i satisfy $\pi^c_i = F^i(x^c_i + X^c_{-i},\ \pi^c_i + \pi^c_{-i})$.

We prove the following theorem:

Theorem. Under Ap 1 - Ap 4, plus Ap 5 stated below, there exists a Cournot Equilibrium.

Proof.

First fix X_{-i}, and π_{-i} and set $\pi_i = I_i$. As $F^i(\ \ )$ is strictly concave in x_i there exists a unique maximiser of $F^i(\ \ )$, write this value as

$$x_i = m^i(X_{-i},\ \pi_{-i} + I_i).$$

Due to continuity of $F^i(\ \ )$, m^i is a continuous function of its arguments.

Writing

$$\pi^i(I_i) = F^i(m^i(X_{-i},\ \pi_{-i} + I_i) + X_{-i},\ I_i + \pi_{-i})$$

we make the further assumption

Ap 5: $\pi^i(I_i)$ is a contraction mapping.

Under Ap 5 there is a unique I_i' s.t.

$$l_i' = F^i(m^i(X_{-i}, \pi_{-i} + l_i') + X_{-i}, l_i' + \pi_{-i})$$

and this determines, given X_{-i} and π_{-i}, a unique profit-maximising output choice x_i', $x_i' = m^i(X_{-i}, \pi_{-i} + l_i')$. As $F^i(\ )$ is continuous, l_i' is continuously dependent upon X_{-i} and π_{-i}. Due to continuity of $m^i(\ )$, x_i' is also continuously dependent upon X_{-i} and π_{-i}.

Combining these we have constructed a continuous mapping from X_{-i}, π_{-i} to x_i', l_i, denote this as h^i where

$$h^i(X_{-i}, \pi_{-i}) = (x_i', l_i').$$

Now construct the composite mapping

$$H(X, \pi) = (h^1(X_{-1}, \pi_{-1}), \ldots, h^i(X_{-i}, \pi_{-i}), \ldots, h^n(X_{-n}, \pi_{-n})),$$

$$H(X, \pi): X \text{ X } \pi \ \to \ X \text{ X } \pi.$$

As each component $h^i(X_{-i}, \pi_{-i})$ is continuous, $H(X, \pi)$ is continuous. Ap 3 guarantees that each $h^i(X_{-i}, \pi_{-i})$ only takes values on bounded intervals of the real line, hence X X π is compact and convex. By applying Brouwer's theorem, $H(X, \pi)$ has a fixed point which, by construction, is the Cournot equilibrium. ∎

PROOF OF THEOREM 2.

To prove theorem 2 we need the following lemma.

Lemma 3. If A.4,A.5,A.6 hold, then the constraint set for A is closed and is generically a smooth manifold with boundary of dimension 1.

Proof. Closed is obvious by A.4. Let $g(a,b) = af(a,b) - c(f(a,b)) - b$. By Sard's theorem, 0 is generically a regular value for $g(a,b)$ and, by the implicit function theorem, $0 = g(a,b)$ defines a 1 dim. smooth manifold. The intersection of $0 = g(a,b)$ and $a,b \geq 0$ gives a smooth manifold with boundary. ∎

Proof of theorem 2. It now suffices to show that the constraint set for A is non-empty and compact. That it is non-empty follows from A.5, that it is closed follows from Lemma 1 so it must only be established that it is bounded. Suppose that there exists a sequence $\langle a^n, b^n \rangle$ in the constraint set s.t. either $a^n \to \infty$ or $b^n \to \infty$, or both. We must derive a contradiction for each of these cases.

$a^n \to \infty$, $b^n < K$: then by A.3. $a^n f(a^n,b^n) \to 0$

$$\Rightarrow \lim_{n\to\infty} \{a^n f(a^n,b^n) - c(f(a^n,b^n))\} < 0 \leq \lim b^n$$

Contradiction.

$b^n \to \infty$, $a^n < K$: then by consumer's budget constraint we have $Ls(a^n,b^n) + b^n \geq a^n f(a^n,b^n) + Y(a^n,b^n)/w$ where $Y(a^n,b^n)$ is demand for the competitively produced good.

By A.4. we have $\lim_{n\to\infty} \{Ls(a^n,b^n) + b^n - a^n f(a^n,b^n\}$

$$= \lim_{n\to\infty} \{b^n - a^n f(a^n,b^n\}$$

$$\geq \lim_{n\to\infty} Y(a^n,b^n)/w$$

$$> 0 \geq \lim_{n\to\infty} c(f(a^n,b^n))$$

This is a contradiction as we have

$$\lim_{n\to\infty} \{b^n > a^n f(a^n,b^n) - c(f(a^n,b^n))\}$$

$b^n \to \infty$, $a^n \to \infty$: This case contradicted by A.4 as well. ∎

REFERENCES

CORNWALL, R.R. (1977), "The Concept of General Equilibrium in a Market Economy with Imperfectly Competitive Producers", *Metroeconomica*, 29, 55-72

DIERKER, H. and B. GRODAL, (1986), "Non-existence of Cournot-Walras Equilibrium in a General Equilibrium Model with Two Oligopolists", in *Contributions to Mathematical Economics*, ed. Hildenbrand and Mas-Collel, North Holland.

GABSZEWICZ, J.J. and J.-P. VIAL, (1972), "Oligopoly "a la Cournot" in a General Equilibrium Analysis", *Journal of Economic Theory*, 4, 381-400.

HART,O.(1982),"A model of imperfect competition with Keynesian features." *Quarterly Journal of Economics 97; 109-138.*

NIKAIDO,M.(1975)"*Monopolistic Competition and Effective Demand.*" Princeton University Press, Princeton.