

Tariff Policy and Imperfect Competition

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1. INTRODUCTION

It has become a commonly observed fact that much of international trade involves imperfectly competitive firms, particularly with regard to markets for high unit cost consumer goods (Grubel and Lloyd 1975). Many aspects of trade theory, such as cross-shipping of identical goods (Neven and Philips 1984), can best be explained by the presence of imperfect competition. In contrast, the theory of international trade has, until recently, been concerned predominantly with models of perfect competition, for a summary see Dixit (1986). This, of course, reduces its applicability.

To make progress into the analysis of tariff policy in the presence of imperfect competition, this paper builds on recent work in trade theory (Dixit 1984, Venables 1983, 1985, Brander and Spencer 1984) and optimal tax theory (Myles 1987) to present an analysis of tariff policy in a general equilibrium model of international trade with imperfect competition.

Obvious problems exist with partial equilibrium models of trade; they simply cannot capture all the repercussions of any policy action and the range of effects that are relevant. Conversely, most previous attempts at trade and general equilibrium have been of the "small country" against "large world" type so that the general equilibrium relates only to the small country. This is clearly not satisfactory when considerable trade takes place between similarly sized countries, as it does within the E.C. and, for example, between the U.S.A. and Japan.

Specific failures of existing models can be traced to the usual restrictions that are made for the sake of simplicity. Linear demand

functions are commonly used but these restrict the range of effects that can occur in imperfectly competitive markets in response to policy variations; these effects being one of the most interesting aspects of imperfect competition, see Seade (1986). Consumer surplus often appears as a measure of welfare despite its inherent weaknesses; it would seem apparent that any worthwhile approach must be based on a stronger concept of welfare. Furthermore, partial equilibrium models that use a cost function to characterise production possibilities implicitly assume that factor prices are constant. There are important reasons, which are discussed fully below, for rejecting this as an appropriate assumption. Indeed, variations in factor prices are an important determinant of the optimal policy. Profits are another area of difficulty, they sometimes enter measures of surplus and are sometimes ignored, any satisfactory treatment must take them fully into account. Also, the feedback of profits into demand (Cripps and Myles 1988) has received no attention in the trade literature. Finally, equilibrium is never proven except for those cases where it is obvious. If the imperfectly competitive model of trade is to be taken seriously, the nature and existence of equilibrium needs to be carefully established.

Section 2 of the paper describes the structure of the model, characterises equilibrium and presents a proof of the existence of equilibrium. Four aspects of tariff policy are analysed in section 3. The results of Brander and Spencer (1984) are first generalised to accommodate arbitrary differentiable demand functions but holding factor prices constant and with a 100% profits tax. The exercise is then repeated allowing for factor price variations and the two sets of results contrasted. Returning to a fixed factor price model, the interrelation of tariff policy and profit income is considered. Finally, the optimal tariffs resulting from a Nash equilibrium in

tariff-setting between the two countries and a collusive equilibrium are compared; the focus is placed on determining when collusive tariff levels are lower. Conclusions are given in section 4.

2. THE MODEL AND THE EXISTENCE OF EQUILIBRIUM

i) General Structure.

The model consists of two equally sized countries, labelled A and B, each of which has some members of an oligopolistic industry, those in A are indexed $i = 1, \dots, n$ and in B $j = 1, \dots, m$. The output of this industry, good X, is traded between the two countries. Each country also has a perfectly competitive industry producing good Y, which is not traded, with constant returns to scale. Both goods are produced using labour alone. Pre-tax (and no-tax) prices for the two goods are p_x^a and p_y^a in country A and p_x^b , p_y^b in B. Post-tax prices are q_x^a , q_y^a , q_x^b and q_y^b . The two country's wage rates are w^a and w^b .

There is a household in each country which receives the profits, π , of the firms located in that country, supplies labour, consumes the two goods produced and, where appropriate, also consumes a publicly provided good, G. The behaviour of this household is characterised by an indirect utility function, $V^i = V^i(q_x^i, q_y^i, w^i, \pi^i, G^i)$, $i = a, b$, which also acts as a measure of social welfare.

Each country has a government that levies a tariff, denoted τ^i , $i = a, b$, taxes good Y at rate t^i and provides the public good. The aim of the government is always to maximise the welfare of the household while maintaining a balanced budget.

In defining equilibrium it is necessary to distinguish carefully how imperfectly competitive firms treat profits. In contrast to much of the previous literature, see Hart (1985) for a survey, it is assumed here that the firms take into account the dependence of their demand upon the profits they make. In effect, it is assumed that the firms maximise profits subject to an objective demand function. The relationship between models of subjective and objective demand is discussed in Nikaido (1975) and Cripps and Myles (1988), the assumption is not unreasonable.

There is also a distinction between the model below and that developed to analyse optimal commodity taxation (Myles 1987). Both models have labour as the only input into production and it is natural to use its price, the wage rate, as numeraire and build the price system up from this. With labour as the only input, once the wage is determined the cost functions are fixed for all firms as are the prices of goods produced competitively with constant returns to scale. However, in this trade model with two countries there are two wage rates but only one of these can be normalised. Furthermore, the factor price equalisation theorem cannot be appealed to. Hence one wage rate must be determined endogenously by the model and this introduces factor price effects into policy analysis, a point that has been ignored by previous authors. The factor price effect is also crucial in the existence proof.

ii) Characterisation of Equilibrium.

The model has six markets: in each country there is an oligopolistic market, a perfectly competitive market and a labour market. To prove the existence of an equilibrium it is necessary, by Walras' law, only to

demonstrate the existence of equilibrium in five of these markets. However, as is now explained, the existence of equilibrium can be proved by considering only one market.

If the price of the competitive good in each country is set equal to the cost of production, the two competitive markets will be in equilibrium. These are also the only prices at which they will be in equilibrium. Providing profit-maximising choices exist for the oligopolists, and the existence proof below will establish that they do, the oligopolistic markets must be in equilibrium as the oligopolists always produce on their demand curves. Consequently this leaves only the two labour markets but, from Walras' law, only one of these requires analysis; the analysis will focus on the labour market in country A.

The equilibrating mechanism is as follows: the wage rate of country A, w^a , is taken as numeraire and remains fixed. This determines q_y^a and the cost structure for the oligopolists in country A. In contrast, w^b is to be determined endogenously by the system; q_y^b , q_x^a and q_x^b can all be determined conditionally upon w^b . The state of the labour market in A is dependent upon the entire vector w^a , q_y^a , q_y^b , q_x^a , q_x^b , w^b and hence, given w^a , upon w^b alone. Consequently, existence of equilibrium is proven by demonstrating that there exists a w^b such that all the oligopolistic firms have mutually compatible profit-maximising choices that also generate equilibrium in the labour market of A.

Having described the structure of the problem it is now necessary to analyse labour market equilibrium. This is achieved by first placing the equilibrium equation into a form suitable for the existence proof.

As each unit of Y production requires one unit of labour, labour demand from the competitive industry, $L_d^a(Y)$, is equal to the demand for

Y , Y_d^a ; thus $L_d^a(Y^a) = Y_d^a$. Letting $L_d^a(X_p^a)$ represent labour demand from the oligopolists located in A, where X_p^a is their total production, labour market equilibrium occurs when

$$Y_d^a + L_d^a(X_p^a) = L_s^a \quad (1)$$

with L_s^a representing labour supply. The consumer's budget constraint is

$$q_y^a Y_d^a + q_x^a X_d^a = w^a L_s^a + \pi^a \quad (2)$$

From constant returns to scale, $q_y^a = w^a + t^a$; t^a the commodity tax levied upon Y . Hence

$$Y_d^a - L_s^a = (\pi^a - q_x^a X_d^a - t^a Y_d^a) / w^a \quad (3)$$

Substituting into (1)

$$w^a L_d^a(X_p^a) + \pi^a - q_x^a X_d^a - t^a Y_d^a = 0 \quad (4)$$

This paper will consider only balanced-budget tax and tariff schemes, so writing τ^a as the tariff levied on imports of X , X_b^a , from B into A, budget-balance implies $\tau^a X_b^a = -t^a Y_d^a$. Using this in (4), labour market equilibrium occurs when

$$w^a L_d^a(X_p^a) + \pi^a - q_x^a X_d^a + \tau^a X_b^a = 0 \quad (5)$$

For a model of a closed economy with $X_b^a = 0$ (or with τ^a a commodity tax and X_b^a replaced by X_d^a), (5) would describe the budget constraint for the oligopolistic industry and would be satisfied definitionally. It follows that a closed economy must have an equilibrium provided the imperfectly competitive firms can find profit-maximising decisions, a point explored by Cripps and Myles (1988).

In contrast, for the trade model under consideration (5) need not be satisfied and indeed will only be met if an equilibrium exists.

However, (5) is not yet in the most appropriate form. From the budget constraints of the oligopolists located in A

$$\pi^a = \sum_{i=1}^n \pi_i = \sum_{i=1}^n q_x^a x_{i1}^a + \sum_{i=1}^n q_x^b x_{i1}^b - \sum_{i=1}^n w^a L_{i1}^a (x_{i1}^a + x_{i1}^b) - \sum_{i=1}^n \tau^b x_{i1}^b$$

Substituting for π^a in (5) and noting $\sum_i w^a L_{i1}^a (x_{i1}^a + x_{i1}^b) = w^a L_d^a (X_p^a)$

$$\sum_{i=1}^n q_x^a x_{i1}^a + \sum_{i=1}^n q_x^b x_{i1}^b - \sum_{i=1}^n \tau^b x_{i1}^b + \sum_{j=1}^m \tau^a x_{j1}^a - q_x^a X_d^a = 0$$

where $\sum_j x_{j1}^a = X_b^a$ by definition, or, as $\sum_i q_x^a x_{i1}^a - q_x^a X_d^a = -\sum_j q_x^a x_{j1}^a$,

$$\sum_{j=1}^m (q_x^a - \tau^a) x_{j1}^a = \sum_{i=1}^n (q_x^b - \tau^b) x_{i1}^b \quad (6)$$

Eq. (6) is the final description of equilibrium and provides a convenient form for analysis. It states that general equilibrium is equivalent to trade balance between the two countries. Recalling the previous discussion, both sides of (6) are functionally dependent upon w^b which gives one variable to solve a single equation. The next section derives sufficient conditions for there to be an equilibrium value of w^b .

iii) Existence.

The existence proof will be concerned only with $t^a = \tau^a = t^b = \tau^b = 0$ for notational simplicity so all prices will be represented by p 's. It can easily be extended to other cases.

To proceed, first define the two demand functions for the oligopolists' output

$$X^a = X^a(p_y^a, p_x^a, w^a, \pi^a)$$

$$X^b = X^b(p_y^b, p_x^b, w^b, \pi^b)$$

Defining an index $l = 1, \dots, k, \dots, n+m$, for which the first n components refer to firms $i = 1, \dots, n$ located in A and the remaining components to the firms located in B, and assuming invertibility of direct demand, inverse demand can be written

$$p_x^a = \Gamma^a(\sum_l x_l^a, p_y^a, w^a, \pi^a)$$

$$p_x^b = \Gamma^b(\sum_l x_l^b, p_y^b, w^b, \pi^b)$$

Alternatively, using the notation X_{-k} for the aggregate less the k 'th term,

$$p_x^a = \Gamma^a(X_{-k}^a + x_k^a, p_y^a, w^a, \pi^a) \quad (7)$$

$$p_x^b = \Gamma^b(X_{-k}^b + x_k^b, p_y^b, w^b, \pi^b) \quad (8)$$

For $l = 1, \dots, k, \dots, n$, the profits of the k 'th firm are

$$\begin{aligned} \pi_k &= x_k^a \cdot \Gamma^a(X_{-k}^a + x_k^a, p_y^a, w^a, \pi^a) + x_k^b \cdot \Gamma^b(X_{-k}^b + x_k^b, p_y^b, w^b, \pi^b) \\ &\quad - C^k(x_k^a + x_k^b; w^a) \\ &\equiv \pi^k(X_{-k}^a + x_k^a, p_y^a, w^a, \pi^a, X_{-k}^b + x_k^b, p_y^b, w^b, \pi^b) \end{aligned} \quad (9)$$

and for $l = n+1, \dots, k, \dots, m$

$$\begin{aligned} \pi_k &= x_k^a \cdot \Gamma^a(X_{-k}^a + x_k^a, p_y^a, w^a, \pi^a) + x_k^b \cdot \Gamma^b(X_{-k}^b + x_k^b, p_y^b, w^b, \pi^b) \\ &\quad - C^k(x_k^a + x_k^b; w^b) \\ &\equiv \pi^k(X_{-k}^a + x_k^a, p_y^a, w^a, \pi^a, X_{-k}^b + x_k^b, p_y^b, w^b, \pi^b) \end{aligned} \quad (10)$$

Since w^a is taken as numeraire throughout and as p_y^a and p_y^b depend directly on the respective wage rates, for all $l = 1, \dots, n+m$ the general expression for profit may be written

$$\pi_k = \pi^k(X_{-k}^a + x_k^a, \pi^a, X_{-k}^b + x_k^b, w^b, \pi^b) \quad (11)$$

Now note that π^a and π^b , given w^a , are dependent upon the vector of outputs supplied to country A, x^a , $x^a = x_1^a, \dots, x_{n+m}^a$, the vector of outputs x^b , $x^b = x_1^b, \dots, x_{n+m}^b$, supplied to B and w^b ,

$$\pi^a = \pi^a(x^a, x^b, w^b),$$

$$\pi^b = \pi^b(x^a, x^b, w^b)$$

Substituting these into (11)

$$\pi_k = \pi^k(X_{-k}^a + x_k^a, X_{-k}^b + x_k^b, x_k^a, x_k^b, w^b).$$

Using the notation x_{-k}^a for the vector less its k 'th term, the final expression for profits is

$$\begin{aligned} \pi_k = \pi^k(X_{-k}^a + x_k^a, X_{-k}^b + x_k^b, x_k^a, x_{-k}^a, x_k^b, x_{-k}^b, w^b), \\ l = 1, \dots, k, \dots, m+n \end{aligned} \quad (12)$$

From (12) it can be seen that there are two effects of an output change: a direct effect of quantity upon price and an income effect working through the consumers' profit incomes.

Noting that knowledge of x_{-k}^a and x_{-k}^b is sufficient to also determine X_{-k}^a and X_{-k}^b , the following assumptions are maintained throughout:

- A1. For all values of x_{-k}^a , x_{-k}^b , and w^b , $\pi^k(\cdot)$ is strictly concave in x_k^a and x_k^b and twice differentiable.
- A2. $\pi^k(\cdot)$ is continuous with respect to all arguments.
- A3. $x_k^a, x_k^b \in \Omega_k$, $\Omega_k \subset \mathbb{R}_+^2$, contains the origin, and is compact with $\max \{x_k^a + x_k^b\} < K < \infty$.

The first step in the existence proof is to demonstrate that assumptions A1. - A3. guarantee an equilibrium exists for the imperfectly competitive industry, given a value of w^b , and that the

equilibrium is continuously dependent upon w^b . This is stated as theorem 1.

Theorem 1. Assuming A1. - A3. a Cournot equilibrium exists for the imperfectly competitive industry and this equilibrium is continuously dependent upon w^b .

Proof.

The first step is to show an equilibrium exists. For given values of x_{-k}^a , x_{-k}^b and w^b , A1. and A3. imply that there is a unique profit-maximising output choice x_k^a , x_k^b for all firms $k = 1, \dots, m+n$. Write this choice as

$$(x_k^a, x_k^b) = h^k(x_{-k}^a, x_{-k}^b; w^b), \quad h^k: \prod_{l=1, l \neq k}^{m+n} Q_l \times w^b \rightarrow Q_k$$

By standard arguments, for example Okuguchi (1976), each $h^k(\cdot)$ is a continuous, point-valued function of x_{-k}^a and x_{-k}^b .

Forming the composite function

$$H(x^a, x^b; w^b) = (h^1(x_{-1}^a, x_{-1}^b; w^b), \dots, h^{m+n}(x_{-m+n}^a, x_{-m+n}^b; w^b)),$$

$$H: \prod_{l=1}^{m+n} Q_l \times w^b \rightarrow \prod_{l=1}^{m+n} Q_l \times w^b$$

H is a continuous point-valued function from a compact set, the product of the Q_l 's with the single point w^b , into itself. Hence, by Brouwer's Theorem, H has a fixed point which, by construction, is the Cournot equilibrium. This establishes the existence of an equilibrium for a given value of w^b ; it remains to investigate the continuity of this equilibrium with respect to w^b .

Take fixed values of x_{-k}^a and x_{-k}^b and consider a sequence $\{w^r\}_{r=1}^{\infty}$ and let $\lim_{r \rightarrow \infty} w^r = w^0$. Given this sequence, a sequence $\{h^k(x_{-k}^a, x_{-k}^b; w^r)\}$, each $h^k(\cdot) \in \Omega_k$, is obtained. As Ω_k is compact, this sequence has a convergent subsequence. Let

$$\{h^k(x_{-k}^a, x_{-k}^b; w^t)\}_{t=1}^{\infty}$$

be such a subsequence.

By definition

$$\pi^k(h^k(x_{-k}^a, x_{-k}^b; w^t), x_{-k}^a, x_{-k}^b; w^t) \geq \pi^k(x_k^a, x_k^b, x_{-k}^a, x_{-k}^b; w^t),$$

all $x_k^a, x_k^b \in \Omega_k$.

and, by continuity of the profit function,

$$\pi^k(\lim_{t \rightarrow \infty} h^k(x_{-k}^a, x_{-k}^b; w^t), x_{-k}^a, x_{-k}^b; w^t) \geq \pi^k(x_k^a, x_k^b, x_{-k}^a, x_{-k}^b; w^0),$$

but, as $h^k(x_{-k}^a, x_{-k}^b; w^t)$ is single-valued,

$$\lim_{t \rightarrow \infty} h^k(x_{-k}^a, x_{-k}^b; w^t) = h^k(x_{-k}^a, x_{-k}^b; w^0)$$

so that $h^k(\cdot)$ is continuous with respect to w^b .

Under A1. - A3. the fixed point of $H(\cdot)$ is unique (Okuguchi 1976) and by definition it satisfies

$$(x_1^a, x_1^b) = h^1(x_{-1}^a, x_{-1}^b; w^b),$$

⋮

$$(x_{m+n}^a, x_{m+n}^b) = h^{m+n}(x_{-m+n}^a, x_{-m+n}^b; w^b)$$

Now consider the sequence $\{w^r\}$, for each value of w^r there is a fixed point such that

$$(x_1^a, x_1^b)^r = h^1((x_{-1}^a, x_{-1}^b)^r; w^r),$$

⋮

$$(x_{m+n}^a, x_{m+n}^b)^\tau = h^{m+n}((x_{-m+n}^a, x_{-m+n}^b)^\tau; w^\tau)$$

which generates the sequence $\{(x_1^a, x_1^b)^\tau, \dots, (x_{m+n}^a, x_{m+n}^b)^\tau\}_{\tau=1}^\infty$. Each member of this sequence belongs to the compact set $\pi_{l=1}^{m+n} Q_1$ and therefore it has a convergent subsequence. Take this to be the sequence itself with $\lim_{\tau \rightarrow \infty} ((x_1^a, x_1^b)^\tau, \dots, (x_{m+n}^a, x_{m+n}^b)^\tau) = ((x_1^a, x_1^b)^0, \dots, (x_{m+n}^a, x_{m+n}^b)^0)$.

So, by continuity of the $h^k(\cdot)$'s,

$$\begin{aligned} & \lim_{\tau \rightarrow \infty} (h^1((x_{-1}^a, x_{-1}^b)^\tau; w^\tau), \dots, h^{m+n}((x_{-m+n}^a, x_{-m+n}^b)^\tau; w^\tau)) = \\ & (h^1(\lim_{\tau \rightarrow \infty} (x_{-1}^a, x_{-1}^b)^\tau; \lim_{\tau \rightarrow \infty} w^\tau), \dots, \\ & \quad \dots, h^{m+n}(\lim_{\tau \rightarrow \infty} (x_{-m+n}^a, x_{-m+n}^b)^\tau; \lim_{\tau \rightarrow \infty} w^\tau)) \\ & = (h^1((x_{-1}^a, x_{-1}^b)^0; w^0), \dots, h^{m+n}((x_{-m+n}^a, x_{-m+n}^b)^0; w^0)) \\ & = \lim_{\tau \rightarrow \infty} ((x_1^a, x_1^b)^\tau, \dots, (x_{m+n}^a, x_{m+n}^b)^\tau) \end{aligned}$$

which demonstrates continuity of the fixed point with respect to w^b . ■

The idea lying behind this proof has a close formal resemblance to those used in proving continuity of the Walras correspondence (Hildenbrand and Mertens 1972) and methods for relaxing some of the strong assumptions may be found in that literature.

Recalling (6) and expressing functional dependence, equilibrium occurs when

$$\begin{aligned} & \sum_{j=1}^m (\Gamma^a(\sum_{l=1}^{m+n} x_{1l}^a(w^b), \pi^a(x^a(w^b), x^b(w^b), w^b)) - \tau^a) \cdot x_{1j}^a(w^b) = \\ & \sum_{i=1}^n (\Gamma^b(\sum_{l=1}^{m+n} x_{1l}^a(w^b), \pi^b(x^a(w^b), x^b(w^b), w^b)) - \tau^b) \cdot x_{1i}^b(w^b) \quad (13) \end{aligned}$$

or

$$\Phi(w^b) = \Theta(w^b) \quad (14)$$

Theorem 1 has shown, given continuity of the inverse demand functions, that both sides of (14) are continuously dependent upon w^b . It now remains to demonstrate that a value of w^b exists that satisfies (14).

Before proceeding with the main proof it will be helpful to analyse the comparative statics, with respect to changes in w^b , of a simplified variant of the model. This serves two purposes: it illustrates the workings of the above proof and indicates the form that assumptions to guarantee the existence of a solution to (14) should take.

Consider a duopoly model with firm 1 located in country A and firm 2 in B. The firms' profit levels, taking w^a as fixed, can be expressed as

$$\pi_1 = x_1^a \cdot \Gamma^a(x_1^a + x_2^a, \pi^a(x_1^a, x_2^a, x_1^b, x_2^b; w^b)) + x_1^b \cdot \Gamma^b(x_1^b + x_2^b, p_y^b, w^b, \pi^b(x_1^a, x_2^a, x_1^b, x_2^b; w^b)) - C^1(x_1^a + x_1^b)$$

and

$$\pi_2 = x_2^a \cdot \Gamma^a(x_1^a + x_2^a, \pi^a(x_1^a, x_2^a, x_1^b, x_2^b; w^b)) + x_2^b \cdot \Gamma^b(x_1^b + x_2^b, p_y^b, w^b, \pi^b(x_1^a, x_2^a, x_1^b, x_2^b; w^b)) - C^2(x_2^a + x_2^b; w^b)$$

The profit-maximising choices for the firms' are described by

$$\delta\pi_1/\delta x_1^a = \Gamma^a + x_1^a \cdot (\delta\Gamma^a/\delta X^a + \delta\Gamma^a/\delta\pi^a \cdot \delta\pi^a/\delta x_1^a) + x_1^b \cdot (\delta\Gamma^b/\delta\pi^b \cdot \delta\pi^b/\delta x_1^a) - C^1_0 = 0$$

$$\delta\pi_1/\delta x_1^b = \Gamma^b + x_1^b \cdot (\delta\Gamma^b/\delta X^b + \delta\Gamma^b/\delta\pi^b \cdot \delta\pi^b/\delta x_1^b) + x_1^a \cdot (\delta\Gamma^a/\delta\pi^a \cdot \delta\pi^a/\delta x_1^b) - C^1_0 = 0$$

$$\delta\pi_2/\delta x_2^a = \Gamma^a + x_2^a \cdot (\delta\Gamma^a/\delta X^a + \delta\Gamma^a/\delta\pi^a \cdot \delta\pi^a/\delta x_2^a) + x_2^b \cdot (\delta\Gamma^b/\delta\pi^b \cdot \delta\pi^b/\delta x_2^a) - C^2_0 = 0$$

and

$$\begin{aligned}\delta\pi_2/\delta x_2^b &= \Gamma^b + x_2^b.(\delta\Gamma^b/\delta X^b + \delta\Gamma^b/\delta\pi^b.\delta\pi^b/\delta x_2^b) \\ &+ x_2^a.(\delta\Gamma^a/\delta\pi^a.\delta\pi^a/\delta x_2^b) - C_2^0 = 0\end{aligned}$$

As these are assumed to be the only firms in each country earning positive profits, it follows that at the maximising values

$$\delta\pi_1/\delta x_1^a = \delta\pi^a/\delta x_1^a = 0, \quad \delta\pi_1/\delta x_1^b = \delta\pi^a/\delta x_1^b = 0$$

and

$$\delta\pi_2/\delta x_2^a = \delta\pi^b/\delta x_2^a = 0, \quad \delta\pi_2/\delta x_2^b = \delta\pi^b/\delta x_2^b = 0$$

Obviously these restrictions cannot be included once more than a single firm is located in each country.

Substituting the restrictions into the first-order conditions

$$\Gamma^a + x_1^a.(\delta\Gamma^a/\delta X^a) + x_1^b.(\delta\Gamma^b/\delta\pi^b.\delta\pi^b/\delta x_1^a) - C_1^0 = 0 \quad (15)$$

$$\Gamma^b + x_1^b.(\delta\Gamma^b/\delta X^b + \delta\Gamma^b/\delta\pi^b.\delta\pi^b/\delta x_1^b) - C_1^0 = 0 \quad (16)$$

$$\Gamma^a + x_2^a.(\delta\Gamma^a/\delta X^a + \delta\Gamma^a/\delta\pi^a.\delta\pi^a/\delta x_2^a) - C_2^0 = 0 \quad (17)$$

and

$$\Gamma^b + x_2^b.(\delta\Gamma^b/\delta X^b) + x_2^a.(\delta\Gamma^a/\delta\pi^a.\delta\pi^a/\delta x_2^b) - C_2^0 = 0 \quad (18)$$

The next step is to calculate the effect of variations in w^b upon the solution to these equations. To make the analysis tractable two additional assumptions are made: the profit terms $\delta\Gamma^a/\delta\pi^a.\delta\pi^a/\delta x_2^b$ etc. will be set equal to zero, this is equivalent to either a zero income effect in demand or a 100% profit tax, and the cost functions will be assumed linear. Aside from their dependence upon w^b , and in general equilibrium their joint determination of w^b , these two assumptions are sufficient for the two markets to be treated separately, given an assumed value of w^b .

For country A, differentiating (15) and (17) and solving

$$\frac{dx_1^a}{dw^b} = - \frac{C^{201} \left(\frac{\delta \Gamma^a}{\delta X^a} + x_1^a \cdot \frac{\delta^2 \Gamma^a}{\delta X^{a2}} \right)}{\frac{\delta \Gamma^a}{\delta X^a} \left(2 \cdot \frac{\delta \Gamma^a}{\delta X^a} + x_1^a \cdot \frac{\delta^2 \Gamma^a}{\delta X^{a2}} \right) + \frac{\delta \Gamma^a}{\delta X^a} \left(\frac{\delta \Gamma^a}{\delta X^a} + x_2^a \cdot \frac{\delta^2 \Gamma^a}{\delta X^{a2}} \right)} \quad (19)$$

and

$$\frac{dx_2^a}{dw^b} = \frac{C^{201} \left(2 \cdot \frac{\delta \Gamma^a}{\delta X^a} + x_1^a \cdot \frac{\delta^2 \Gamma^a}{\delta X^{a2}} \right)}{\frac{\delta \Gamma^a}{\delta X^a} \left(2 \cdot \frac{\delta \Gamma^a}{\delta X^a} + x_1^a \cdot \frac{\delta^2 \Gamma^a}{\delta X^{a2}} \right) + \frac{\delta \Gamma^a}{\delta X^a} \left(\frac{\delta \Gamma^a}{\delta X^a} + x_2^a \cdot \frac{\delta^2 \Gamma^a}{\delta X^{a2}} \right)} \quad (20)$$

where $C^{201} = \delta C^{20}/\delta w^b$. Assuming the demand function to be concave, $\delta^2 \Gamma^a / \delta X^{a2} < 0$, it follows that $dx_1^a / dw^b > 0$ and $dx_2^a / dw^b < 0$. Also $|dx_1^a / dw^b| < |dx_2^a / dw^b|$.

For the existence proof it is the relation of the term $p_x^a \cdot x_2^a$ to w^b that is of most interest. As

$$d(p_x^a \cdot x_2^a) / dw^b = x_2^a \cdot \left(\delta \Gamma^a / \delta X^a \left(dx_1^a / dw^b + dx_2^a / dw^b \right) \right) + \Gamma^a \cdot dx_2^a / dw^b$$

and using (17)

$$d(p_x^a \cdot x_2^a) / dw^b = x_2^a \cdot \delta \Gamma^a / \delta X^a \cdot dx_1^a / dw^b + C^{20} \cdot dx_2^a / dw^b < 0.$$

Consequently as w^b increases, the value of imports into country A, valued at p_x^a , falls.

The mechanism behind this result can be illustrated diagrammatically. Eqs. (19) and (20) are implicit representations of the firms' reaction functions, these are also the $h^k(\cdot)$ functions in Theorem 1. For firm 1, $x_1^a = h^1(x_2^a)$ and is independent of w^b . In contrast, the reaction function of firm 2 is $x_2^a = h^2(x_1^a, w^b)$. An increase in w^b then has the effect of shifting $h^2(x_1^a, w^b)$ uniformly downwards, as in fig. 1, while $h^1(x_2^a)$ is unchanged. This increases x_1^a and reduces x_2^a and the concavity implies that their sum also decreases. This causes an increase in p_x^a but this is more than offset by the reduction in x_2^a .

Furthermore, as w^b continues to increase x_2^a will tend to zero and the equilibrium will tend to point C.

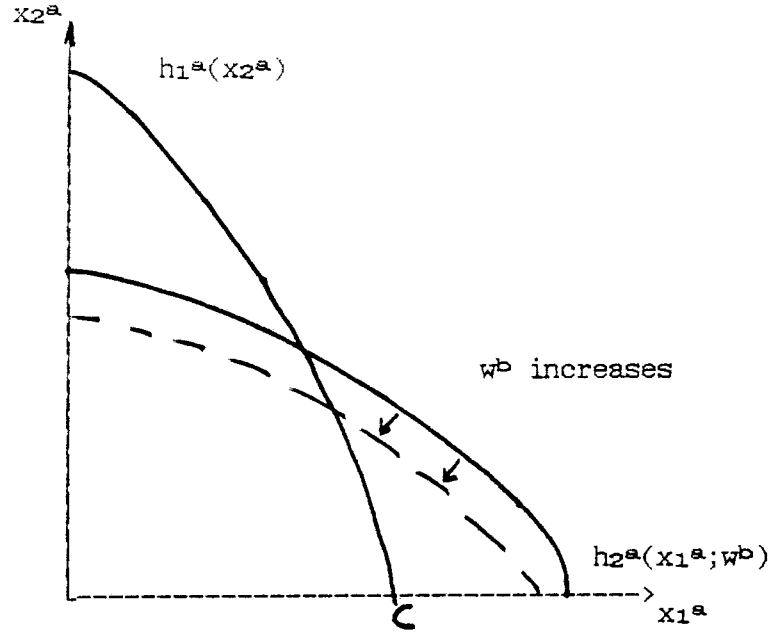


Fig. 1

Repeating this analysis for the market in country B and, in order to simplify expressions, making the additional restriction that $x_1^b = x_2^b$ at the initial equilibrium, differentiation of (16) and (18) gives

$$\frac{dx_1^b}{dw^b} = - \frac{\frac{\partial \Gamma^b}{\partial X^b} \frac{d\Gamma^b}{dw^b} + x_1^b \cdot \frac{\partial \Gamma^b}{\partial X^b} \cdot \frac{d^2 \Gamma^b}{dw^{b2}} + C^2_{01} \left(\frac{\partial \Gamma^b}{\partial X^b} + x_1^b \cdot \frac{\partial^2 \Gamma^b}{\partial X^{b2}} \right)}{\frac{\partial \Gamma^b}{\partial X^b} \left(2 \cdot \frac{\partial \Gamma^b}{\partial X^b} + x_1^b \cdot \frac{\partial^2 \Gamma^b}{\partial X^{b2}} \right) + \frac{\partial \Gamma^b}{\partial X^b} \left(\frac{\partial \Gamma^b}{\partial X^b} + x_2^b \cdot \frac{\partial^2 \Gamma^b}{\partial X^{b2}} \right)} \quad (21)$$

and

$$\frac{dx_2^b}{dw^b} = \frac{C^2_{01} \left(2 \frac{\partial \Gamma^b}{\partial X^b} + x_1^b \cdot \frac{\partial^2 \Gamma^b}{\partial X^{b2}} \right) - \frac{\partial \Gamma^b}{\partial X^b} \frac{d\Gamma^b}{dw^b} - x_1^b \cdot \frac{\partial \Gamma^b}{\partial X^b} \cdot \frac{d^2 \Gamma^b}{dw^{b2}}}{\frac{\partial \Gamma^b}{\partial X^b} \left(2 \cdot \frac{\partial \Gamma^b}{\partial X^b} + x_1^b \cdot \frac{\partial^2 \Gamma^b}{\partial X^{b2}} \right) + \frac{\partial \Gamma^b}{\partial X^b} \left(\frac{\partial \Gamma^b}{\partial X^b} + x_2^b \cdot \frac{\partial^2 \Gamma^b}{\partial X^{b2}} \right)} \quad (22)$$

where $d\Gamma^b/dw^b = \partial \Gamma^b / \partial p_y^b + \partial \Gamma^b / \partial w^b$ and $d^2 \Gamma^b / dw^{b2} = \partial^2 \Gamma^b / \partial p_y^b \partial w^b + \partial^2 \Gamma^b / \partial w^{b2}$. From the concavity of demand, the denominator of (21) and (22) is positive and, since C^2_{01} and $d\Gamma^b/dw^b > 0$, a sufficient condition for $dx_1^b/dw^b > 0$ is that $d^2 \Gamma^b / dw^{b2} \geq 0$; the necessary condition would allow some negative values. In addition, $d^2 \Gamma^b / dw^{b2} \geq 0$

also implies that $|dx_1^b/dw^b| > |dx_2^b/dw^b|$ but does not sign dx_2^b/dw^b ; it is assumed below to be negative.

Finally, it is necessary to determine $d(p_x^b x_1^b)/dw^b$. Differentiating,

$$\begin{aligned} d(p_x^b x_1^b)/dw^b &= \Gamma^b dx_1^b/dw^b + x_1^b \left(\frac{\partial \Gamma^b}{\partial X^b} \left(dx_1^b/dw^b + dx_2^b/dw^b \right) \right) \\ &\quad + x_1^b \left(\frac{\partial \Gamma^b}{\partial p_y^b} + \frac{\partial \Gamma^b}{\partial w^b} \right) \end{aligned}$$

and using (16)

$$d(p_x^b x_1^b)/dw^b = C^1_0 dx_1^b/dw^b + x_1^b \frac{\partial \Gamma^b}{\partial X^b} dx_2^b/dw^b + x_1^b d\Gamma^b/dw^b$$

Hence, $d(p_x^b x_1^b)/dw^b > 0$ when $dx_1^b/dw^b > 0$ and $dx_2^b/dw^b < 0$ which, as noted above, are ensured by $d^2\Gamma^b/dw^{b2} \geq 0$. Consequently, assuming $d^2\Gamma^b/dw^{b2} \geq 0$, $p_x^b x_1^b$ will increase as w^b increases. This solution is illustrated in the reaction function diagram below.

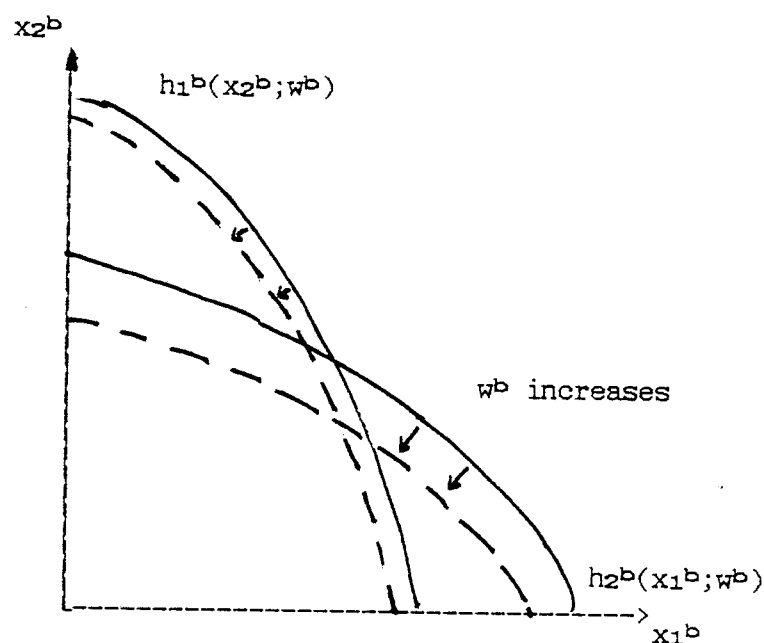


Fig. 2

Returning to the existence of general equilibrium, equilibrium for this example will occur when

$$p_x^a x_2^a = p_x^b x_1^b.$$

The analysis has concluded that the left-hand side of this equation is a decreasing continuous function of w^b and the right-hand side is an increasing continuous function. An equilibrium must then exist provided

$$\lim_{w^b \rightarrow 0} p_x^a x_2^a > \lim_{w^b \rightarrow 0} p_x^b x_1^b$$

and

$$\lim_{w^b \rightarrow \infty} p_x^a x_2^a < \lim_{w^b \rightarrow \infty} p_x^b x_1^b$$

These limits now form the basis of an existence proof for the initial model.

The following assumptions are made:

A4. For $w^b \leq w^{b1}$ and for all x_j^b , $j = 1, \dots, m$, x_1^a , x_1^b , $i = 1, \dots, n$, and π^b , $\Gamma^b(0 + \sum_j x_j^b, w^b, \pi^b) < C^b_0(x_1^a + x_1^b)$.

A5. For $w^b \leq w^{b1}$, $\exists x_j^a > 0$, some $j = 1, \dots, m$.

A6. For $w^b \geq w^{bn}$, $\Gamma^a(\sum_i x_1^a + 0, \pi^a) < C^b_0(x_j^a + x_j^b; w^b)$, $j = 1, \dots, m$, for all $\sum_i x_1^a$, x_j^a , x_j^b and π^a .

A7. For $w^b \geq w^{bn}$, $\exists x_1^b > 0$, some $i = 1, \dots, n$.

These assumptions are fairly reasonable. Taken together A4. and A5. impose the condition that as the wage rate in B tends to zero, and consequently the costs for firms located in B also tend to zero, it is no longer economic for firms in A, whose costs remain fixed, to compete in B while firms in B can still benefit from supplying to A. Referring back to fig. 2, this is equivalent to the reaction function of 2, h_2^b , moving outwards until it intersects h_1^b at the point h_1^b meets the vertical axis. At this point x_1^b becomes zero. A6. and A7. guarantee similar behaviour as w^b becomes large; eventually there will be reached a point where it is no longer feasible for firms located in B to export

to A, while exports to B can be profitable. In terms of fig. 1, h_2^a moves inwards and eventually intersects h_1^a at point C and x_2^a becomes zero.

These assumptions are now used to prove the following:

Theorem 2. Under assumptions A1. - A7. the model of trade has an equilibrium.

Proof.

From (14), equilibrium occurs when $\Phi(w^b) - \theta(w^b) = 0$ or, writing $G(w^b) = \Phi(w^b) - \theta(w^b)$, when $G(w^b) = 0$. Theorem 1 has already shown that $G(w^b)$ is a continuous function of w^b , it remains to show that there exists a value of $w^b > 0$ for which it is zero.

From A4., for $w^b \leq w^{b1}$ the return to a firm located in A of supplying to B is always less than the cost of production; it follows that $x_1^b = 0$ for all $i = 1, \dots, n$. Hence $\sum_i p_x^b x_1^b \equiv \theta(w^b) = 0$. As A5. guarantees the existence of $x_j^a > 0$, it must be that $p_x^a > 0$ and $\sum_j p_x^a x_j^a > 0$. Consequently for $w^b \leq w^{b1}$, $\theta(w^b) < \Phi(w^b)$ so $G(w^b) > 0$.

In a similar manner, for $w^b \geq w^{bn}$ A6. ensures that the return to a firm located in B of supplying to A is always less than the cost of production; hence $x_j^a = 0$ for all $j = 1, \dots, m$ and $\sum_j p_x^a x_j^a \equiv \Phi(w^b) = 0$. As A7. assumes the existence of $x_1^b > 0$, it must follow that $p_x^b > 0$ and that $\sum_i p_x^b x_1^b > 0$. Consequently for $w^b \geq w^{bn}$, $\theta(w^b) > \Phi(w^b)$ so $G(w^b) < 0$.

From Theorem 1, $G(w^b)$ is continuous and point-valued so the set $\{w^b \mid w^b \leq w^{b1}\}$ is disjoint from $\{w^b \mid w^b \geq w^{bn}\}$ and $w^{b1} < w^{bn}$. By the intermediate value theorem there must exist w^{b*} such that $w^{b1} < w^{b*} < w^{bn}$ and $G(w^{b*}) = 0$. w^{b*} is the equilibrium wage and the theorem is proved. ■

This existence proof could easily be extended to take explicit account of alternative conjectural assumptions and, of more importance, could also incorporate product differentiation by basing the analysis upon Friedman's (1977) model of price-setting oligopoly. In both cases only a minor rephrasing of Theorem 1 would be required.

3.Aspects of Tariff Policy.

Four analyses of tariff policy will be conducted in this section within the context of the model set out above. Various additional assumptions will be made in order to highlight certain issues and to simplify wherever possible. In particular, it is maintained that within each country the oligopolistic firms are identical and produce with constant marginal costs.

The first analysis assumes a 100% profit tax and a constant value of w^B ; the results are direct generalisations of those presented by Brander and Spencer (1984) and Dixit (1984). These results are then extended to incorporate the equilibrating response of country B's factor price to changes in taxes and tariffs. Contrasting the two sets of results indicates the biases that occur when factor prices are assumed constant. Following this, factor prices are again fixed and the assumption of a 100% profits tax is relaxed. Finally, the profits tax is re-imposed and the level of collusive and Nash-equilibrium tariffs compared.

1) Optimal Tariffs with Constant Factor Prices.

Assuming w^b and marginal costs to be constant allows the markets in A and B to be treated in isolation. Concentrating on A, the policy problem to be studied is the choice of a tariff on imports from B in conjunction with a commodity tax upon Y; the budget must also remain balanced. The simplest case of this problem, and the one that provides most insight into the determinants of the solution, is to consider the direction of welfare-improving taxes beginning from an initial zero-tax position. In particular, what conditions will guarantee the optimality of a move in the direction of positive tariffs?

The formal statement of the problem is:

WI 1. Find $d\tau^a$, dt^a s.t. $dV^a > 0$, $dR = 0$

where

$V^a = V^a(q_x^a, q_y^a)$, $R = \tau^a X_b^a + t^a Y^a$ and $\tau^a = t^a = 0$ initially.

Differentiating the indirect utility function

$$dV^a = \frac{\partial V^a}{\partial q_x^a} (\frac{\partial q_x^a}{\partial \tau^a}) d\tau^a + (\frac{\partial V^a}{\partial q_x^a} \cdot \frac{\partial q_x^a}{\partial q_y^a} \cdot \frac{\partial q_y^a}{\partial t^a} + \frac{\partial V^a}{\partial q_y^a} \cdot \frac{\partial q_y^a}{\partial t^a}) dt^a$$

where the derivative $\frac{\partial q_x^a}{\partial \tau^a}$ reflects the adjustment of the oligopolistic market to the change in costs of the importing firms, or the degree of forward-shifting of the tariff, and $\frac{\partial q_x^a}{\partial q_y^a}$ captures the effect of the change in the price of Y working through the demand function (7). These terms are calculated and analysed below. Writing x_2^a for the quantity each of the m identical firms in B export to A, the budget constraint gives

$$dR = 0 = m x_2^a d\tau^a + Y dt^a$$

so

$$d\tau^a = -mx_2^a d\tau^a / Y$$

Using this expression and Roy's identity, where α^a is the marginal utility of income and x_1^a the supply of each of the n firms located in A , the change in utility can be written

$$dV^a = [\alpha^a(n x_1^a + m x_2^a)((m x_2^a / Y) \delta q_x^a / \delta q_y^a - \delta q_x^a / \delta \tau^a) + \alpha^a m x_2^a] d\tau^a$$

Hence $d\tau^a > 0$ when

$$Y[1 - ((n x_1^a + m x_2^a) / m x_2^a) \cdot \delta q_x^a / \delta \tau^a] + (n x_1^a + m x_2^a) \cdot \delta q_x^a / \delta q_y^a > 0 \quad (23)$$

From (23), a sufficient condition for the tariff to be positive is

$$\delta q_x^a / \delta \tau^a < m x_2^a / (n x_1^a + m x_2^a) \text{ and } \delta q_x^a / \delta q_y^a > 0. \quad (24)$$

When $x_1^a = 0$, this reduces to $\delta q_x^a / \delta \tau^a < 1$ and $\delta q_x^a / \delta q_y^a > 0$ which requires the tariff to be under-shifted and the price of X to rise in response to a tax on Y . The reasoning behind this result is straightforward: with undershifting the tariff can raise revenue from the importers at the expense of a price increase less than the value of the tariff, the revenue raised can then subsidise good Y the price of which falls by an amount equal to the level of subsidy due to the assumption of constant returns. With $\delta q_x^a / \delta q_y^a > 0$, the reduction in the price of Y further reduces that of X . For small tax changes this modification of the relative prices is welfare-improving.

Reversing (23), the tariff should be negative when

$$Y[1 - ((n x_1^a + m x_2^a) / m x_2^a) \cdot \delta q_x^a / \delta \tau^a] + (n x_1^a + m x_2^a) \cdot \delta q_x^a / \delta q_y^a < 0 \quad (25)$$

In general terms, this will occur when the tariff is over-shifted and the price of X falls in response to a tax upon Y . To proceed further it is necessary to evaluate $\delta q_x^a / \delta \tau^a$ and $\delta q_x^a / \delta q_y^a$, this delineates the circumstances that will lead to inequalities (23) and (25).

Under the maintained assumptions, equilibrium in the oligopolistic market in country A satisfies

$$\Gamma^a(nx_1^a + mx_2^a, q_y^a) + x_1^a \Gamma_{ax}^a(nx_1^a + mx_2^a, q_y^a) - C_0^1 = 0 \quad (26)$$

and

$$\Gamma^a(nx_1^a + mx_2^a, q_y^a) + x_2^a \Gamma_{ax}^a(nx_1^a + mx_2^a, q_y^a) - \tau^a - C_0^2 = 0 \quad (27)$$

which are the necessary conditions for profit-maximisation of typical firms located in A and B. Differentiating these equations

$$\begin{aligned} ((n+1)\Gamma_{ax}^a + nx_1^a \Gamma_{axx}^a)dx_1^a + (m\Gamma_{ax}^a + mx_1^a \Gamma_{axx}^a)dx_2^a \\ = - (\Gamma_{ay}^a + x_1^a \Gamma_{axy}^a)dq_y^a \end{aligned} \quad (28)$$

$$\begin{aligned} (n\Gamma_{ax}^a + nx_2^a \Gamma_{axx}^a)dx_1^a + ((m+1)\Gamma_{ax}^a + mx_2^a \Gamma_{axx}^a)dx_2^a \\ = d\tau^a - (\Gamma_{ay}^a + x_2^a \Gamma_{axy}^a)dq_y^a \end{aligned} \quad (29)$$

Solving (28) and (29)

$$\frac{dq_x^a}{d\tau^a} = \frac{\Gamma_{ax}^a}{(n+m+1)\Gamma_{ax}^a + (nx_1^a + mx_2^a)\Gamma_{axx}^a} \quad (30)$$

Noting that the second-order condition for profit maximisation is

$$2\Gamma_{ax}^a + x_i^a \Gamma_{axx}^a < 0, \quad i = 1, 2$$

it is reasonable to assume that the denominator of (30) is negative. Moreover, from Seade (1980), negativity is also a sufficient condition for stability of the equilibrium. It follows that $dq_x^a/d\tau^a > 0$. For the analysis of tariff reform, (23) and (25) indicate that it is the possibility of tariff over-shifting, $dq_x^a/d\tau^a > 1$, that is of most interest. From (30), $dq_x^a/d\tau^a > 1$ when $(n+m)\Gamma_{ax}^a + (nx_1^a + mx_2^a)\Gamma_{axx}^a > 0$, or

$$X^a \Gamma_{axx}^a / \Gamma_{ax}^a < -(n+m) \quad (31)$$

Following Seade (1986), $X^a \Gamma_{x\alpha}^a / \Gamma_{xx}^a$ is the elasticity of the gradient of the inverse demand function and, for over-shifting, this must be less than minus the number of firms. This is a considerable strengthening of the condition for over-shifting of commodity taxes identified by Seade; this is due to the asymmetry of the tariff in affecting only one group of firms within the industry. For (31) to be satisfied it is necessary that the inverse demand function be convex in the locality of the equilibrium, hence under-shifting will always occur with linear demands.

Returning to (26) and (27) and differentiating with respect to q_y^a , x_1^a and x_2^a

$$\frac{dq_x^a}{dq_y^a} = \frac{\Gamma_{xy}^a \Gamma_{yy}^a + (nx_1^a + mx_2^a) [\Gamma_{xy}^a \Gamma_{xy}^a - \Gamma_{yx}^a \Gamma_{yy}^a]}{(n + m + 1) \Gamma_{xx}^a + (nx_1^a + mx_2^a) \Gamma_{x\alpha}^a} \quad (32)$$

where $\Gamma_{yy}^a = \delta \Gamma^a / \delta q_y^a$ and $\Gamma_{xy}^a = \delta^2 \Gamma^a / \delta X^a \delta q_y^a$. There can be no presumption as to the sign of (31). A sufficient condition for it to be positive is that X and Y are substitutes ($\Gamma_{xy}^a > 0$), demand is concave ($\Gamma_{x\alpha}^a < 0$) and $\Gamma_{xy}^a > 0$. However, it can be appreciated that there are no a priori reasons for these to be expected.

To summarise this section: a condition was derived that indicated the direction a welfare-improving tax/tariff policy should take. This indicated that positive tariffs would be the solution except when over-shifting could occur and the price of the oligopolists' output fall in response to taxes on the competitive good. Analysis of the comparative statics of the model concluded that over-shifting would occur only when a stringent condition on the elasticity of the inverse demand curve was satisfied and could therefore be viewed as unlikely. In the light of this, it is likely that positive tariffs will generally be welfare-improving. Finally, a sufficient condition for positive tariffs is: $\Gamma_{xy}^a > 0$, $\Gamma_{x\alpha}^a < 0$ and $\Gamma_{xy}^a > 0$.

ii) Effect of Factor Price Variations.

The analysis of tariffs in the above section was based on the assumption of a fixed wage rate in country B. Relaxing this assumption raises the question of whether variations in w^b , in response to the tariff policy in A, will reinforce or weaken the previous argument. A possible answer is the following: the tariff in A reduces x_2^a which, in turn, reduces the demand for labour in B. This results in w^b falling which lowers the costs of the oligopolists in B and increases their competitiveness. The increase in competitiveness then offsets the initial reduction in x_2^a , resulting in a smaller increase in q_x^a for any given level of tariff. From (23), this would reinforce the argument for positive tariffs.

To develop a formal argument to support this viewpoint it is necessary to consider the effect of a differential change dx_2^a upon the equilibrium of B and "solve" for dw^b in terms of dx_2^a . This is achieved by first analysing the labour market; any changes must leave this in equilibrium. From the labour market a linear relationship between dw^b , dx_2^a , dx_2^b and dq_x^b is derived. Next, the demand function Γ^b provides a linear relationship between dw^b , dx_1^b , dx_2^b and dq_x^b ; dq_x^b can then be eliminated between these two equations. Analysis of the comparative statics of the market for X in B provides two further equations: one relates dw^b and dx_1^b , the other dw^b and dx_2^b . These relations then provide a final equation concerning only dw^b and dx_2^a . A comparative statics exercise for the oligopolistic market in A, allowing for variations in w^b , can then have dw^b replaced by terms in dx_2^a and the solution obtained. This is now presented formally.

Equilibrium in the labour market is represented by the use of the consumer's indirect utility function and the cost function for the oligopolists. As labour demand from the Y industry is equal to Y demand, using Roy's identity labour market equilibrium satisfies

$$V_{b_w}/V_{b_\pi} = - V_{b_y}/V_{b_\pi} + mC_1^2 \quad (33)$$

where $V_{b_w} = \delta V^b/\delta w^b$, $V_{b_y} = \delta V^b/\delta q_y^b$, $V_{b_\pi} = \delta V^b/\delta \pi^b$ and $C_1^2 = \delta C^2/\delta(x_2^a + x_2^b)$. The assumption of 100% profit tax is retained so that derivatives of indirect utility are all evaluated at $\pi = 0$. Totally differentiating (33), and noting $dw^b = dq_y^b$, gives

$$0 = a_1 dw^b + a_2 dq_x^b + a_3 dx_2^a + a_4 dx_2^b \quad (34)$$

with

$$a_1 = - 2V_{b_{yw}} + mC_1^2 V_{b_{\pi w}} - V_{b_{ww}} - V_{b_{yy}} + mC_1^2 V_{b_{\pi y}}$$

$$a_2 = - V_{b_{yx}} - V_{b_{wx}} + mC_1^2 V_{b_{\pi x}}$$

$$a_3 = V_{b_{\pi m}} C_1^2 > 0$$

$$a_4 = V_{b_{\pi m}} C_1^2 > 0$$

It is further assumed that $a_1 < 0$ and $a_2 > 0$; these place mild restrictions upon the indirect utility function.

From the inverse demand function $q_x^b = \Gamma^b(n x_1^b + m x_2^b, q_y^b, w^b)$,

$$dq_x^b = b_1 dx_1^b + b_2 dx_2^b + b_3 dw^b \quad (35)$$

$b_1 = \Gamma_{b_x n}^b < 0$, $b_2 = \Gamma_{b_x m}^b < 0$, $b_3 = \Gamma_{b_y}^b + \Gamma_{b_w}^b > 0$. Eliminating dq_x^b from (34) and (35),

$$0 = c_1 dw^b + c_2 dx_1^b + c_3 dx_2^a + a_4 dx_2^b \quad (36)$$

$c_2 = a_2b_1 < 0$, $c_3 = a_3 > 0$ and, by assumption, $c_1 = a_1 + a_2b_3 < 0$, $c_4 = a_2b_2 + a_4 < 0$. Next, from the comparative statics of equilibrium in the oligopolistic market in B,

$$dx_1^b = d_1dw^b \quad (37)$$

$$d_1 = [-\Gamma^b_x((\Gamma^b_y + \Gamma^b_w) + (mx_2^b - (m+1)x_1^b)(\Gamma^b_{xy} + \Gamma^b_{xw})) + (mx_1^b - mx_2^b)\Gamma^b_{xx}(\Gamma^b_y + \Gamma^b_w) - mC_01^2(\Gamma^b_x + x_1^b\Gamma^b_{xx})]/|A^2|$$

$$dx_2^b = e_1dw^b \quad (38)$$

$$e_1 = [-\Gamma^b_x((\Gamma^b_y + \Gamma^b_w) + (nx_1^b - (n+1)x_2^b)(\Gamma^b_{xy} + \Gamma^b_{xw})) + (nx_2^b - nx_1^b)\Gamma^b_{xx}(\Gamma^b_y + \Gamma^b_w)]/|A^2|$$

with

$$|A^2| = \Gamma^b_x[(n + m + 1)\Gamma^b_x + (nx_1^b + mx_2^b)\Gamma^b_{xx}] > 0.$$

It is further assumed that d_1 and e_1 are greater than zero, this is guaranteed with a linear demand function and is not too restrictive in other cases. dx_1^b and dx_2^b can now be eliminated from (36) using (37) and (38), hence

$$dw^b = f_1dx_2^a \quad (39)$$

$f_1 = -c_3/(c_1 + c_2d_1 + c_4d_2) > 0$. With the assumption of constant marginal cost, w^b is the link between the two markets and (39) captures the specific nature of this link: an increase in x_2^a will raise wages in country B when the assumptions made above are satisfied. Alternative assumptions can be followed through the analysis in the same manner. This expression can now be employed in the analysis of the comparative statics of the oligopolistic market in A.

Returning to (26) and (27) and allowing for the dependence of C^2 upon w^b , total differentiation gives

$$dx_1^a = g_1 dq_y^a + g_2 d\tau^a + g_3 dw^b \quad (40)$$

where

$$|A^1| = \Gamma_{ax}^a[(n+m+1)\Gamma_{ax}^a + (nx_1^a + mx_2^a)\Gamma_{axx}^a] > 0$$

$$g_1 = -\Gamma_{ax}^a\Gamma_{ay}^a + (mx_2^a - (m+1)x_1^a)[\Gamma_{axx}^a\Gamma_{axy}^a - \Gamma_{axx}^a\Gamma_{ay}^a]/|A^1|$$

$$g_2 = -m\Gamma_{ax}^a - mx_1^a\Gamma_{axx}^a/|A^1|$$

$$g_3 = (-m\Gamma_{ax}^a - mx_1^a\Gamma_{axx}^a)C^2_{01}/|A^1|$$

and

$$dx_2^a = h_1 dq_y^a + h_2 d\tau^a + h_3 dw^b \quad (41)$$

$$h_1 = -\Gamma_{ax}^a\Gamma_{ay}^a + (nx_1^a - (n+1)x_2^a)[\Gamma_{axx}^a\Gamma_{axy}^a - \Gamma_{axx}^a\Gamma_{ay}^a]/|A^1|$$

$$h_2 = -(n+1)\Gamma_{ax}^a - nx_1^a\Gamma_{axx}^a/|A^1|$$

$$h_3 = (-(n+1)\Gamma_{ax}^a - nx_1^a\Gamma_{axx}^a)/|A^1|$$

Using (39) to eliminate dw^b and solving the two resulting equations simultaneously,

$$dx_2^a = (h_1/1-h_3f_1)dq_y^a + (h_2/1-h_3f_1)d\tau^a \quad (42)$$

As $h_3f_1 < 0$, contrasting (42) with (29) demonstrates that the effect of changes in w^b is to reduce the response of x_2^a to variations in the level of the tariff and commodity tax. Solving for dx_1^a

$$dx_1^a = (g_1+g_3f_1(h_1/1-h_3f_1))dq_y^a + (g_2+g_3f_1(h_1/1-h_3f_1))d\tau^a \quad (43)$$

Since $g_3f_1 < 0$, $g_2+g_3f_1(h_1/1-h_3f_1) < g_2$ and, contrasting (43) and (30), the effect of the wage variation reduces the effect that the tariff has upon x_1^a . Combining these results, it is apparent that

$$\left. \frac{dq_x^a}{d\tau^a} \right|_{w^b = \text{constant}} > \left. \frac{dq_x^a}{d\tau^a} \right|_{w^b \text{ variable}}$$

This inequality, in relation to (23), demonstrates that the allowance for variation in w^b strengthens the argument for a positive tariff, for a given value of dq_x^a/dq_y^a . Analysis of the wage effect upon dq_x^a/dq_y^a leads to an indeterminate conclusion. For example, in the linear case, $0 < dx_2^a/dq_y^a|w^b \text{ varies} < dx_2^a/dq_y^a|w^b \text{ constant}$ but $0 < dx_1^a/dq_y^a|w^b \text{ constant} < dx_1^a/dq_y^a|w^b \text{ varies}$; the aggregate effect is reduced if $m(n+1)\Gamma_x^a < nm\Gamma_x^a C_{01}^2$ and increased otherwise.

In conclusion, by following through the effects upon equilibrium in B of a change in x_2^a it has been possible to relate x_2^a and w^b ; under the assumptions made they are positively related. Taking account of the variation of w^b when analysing the comparative statics of the oligopolistic market in A, it was demonstrated that the effect of the tariff on the market price was reduced; this factor works in favour of positive tariffs. In contrast, the relation of the wage effect to the response of output, and price, to the commodity tax was indeterminate even for the linear demand case; in general the effects upon x_1^a and x_2^a would tend to be offsetting. This line of reasoning suggests that the total effect of allowing variations in country B's wage is to reinforce arguments for positive tariffs. Obviously this conclusion is dependent upon the assumptions made and in some sense was illustrative; the methodology permits the implications of alternative sets of restrictions to be calculated.

iii) Profits.

To model the effect of taking profit income into account it is helpful to take the following approach: the 100% profit tax remains in place but the revenue from this tax is used to provide a public good, in quantity

G^a , which is additively separable in the indirect utility function; hence welfare is dependent upon profit levels although in a restricted manner. The benefit of this approach is to ensure that demand is not directly dependent upon profit so that the comparative statics of price remain as for (i) above and the effects of policy on profits may be treated in isolation. If this procedure were not adopted it would be necessary to invert a 6×6 matrix in order to solve the system and the resulting expressions would be unwieldly and, most likely, impenetrable.

With this background, welfare-improving tariffs solve:

WI 2. Find $d\tau^a$, dt_y^a s.t. $dV^a > 0$, $dR = 0$

where

$V^a = V^a(q_x^a, q_y^a) + G^a$, $G^a = \pi^a$, $R = \tau^a X_B^a + t_y^a Y^a$ and $\tau^a = t^a = 0$ initially.

The effect on welfare of the changes is

$$dV^a = [\delta V^a / \delta q_x^a \cdot \delta q_x^a / \delta \tau^a + \delta \pi^a / \delta \tau^a] d\tau^a \\ + [\delta V^a / \delta q_x^a \cdot \delta q_x^a / \delta q_y^a + \delta V^a / \delta q_y^a + \delta \pi^a / \delta q_y^a] dt_y^a$$

Using the budget constraint to eliminate dt_y^a , $d\tau^a > 0$ if

$$Y^a [1 - ((nx_1^a + mx_2^a) / mx_2^a) \cdot \delta q_x^a / \delta \tau^a] + (nx_1^a + mx_2^a) \cdot \delta q_x^a / \delta q_y^a \\ + (Y^a / mx_2^a) \delta \pi^a / \delta \tau^a - \delta \pi^a / \delta q_y^a > 0 \quad (44)$$

As (44) illustrates, the profit effect acts in favour of positive tariffs if $(Y^a / mx_2^a) \delta \pi^a / \delta \tau^a - \delta \pi^a / \delta q_y^a > 0$. The first term of this, $(Y^a / mx_2^a) \delta \pi^a / \delta \tau^a$, is likely to be positive as the tariff is detrimental to the competitiveness of importers and increases in the profits of home firms. There does not appear to be a simple argument to determine the expected sign of $\delta \pi^a / \delta q_y^a$.

As

$$d\pi^a = n([\Gamma^a + nx_1^a\Gamma_{xx}^a - C_{10}^a]dx_1^a + [x_1^am\Gamma_{xx}^a]dx_2^a - [x_1^a\Gamma_{xy}^a]dq_y^a)$$

substitution from (28) and (29) gives

$$\frac{d\pi^a}{d\tau^a} = \frac{mx_1^a(2\Gamma_{xx}^a + x_1^a\Gamma_{xxx}^a)}{(n+m+1)\Gamma_{xx}^a + (nx_1^a + mx_2^a)\Gamma_{xxx}^a} \quad (45)$$

Hence $d\pi^a/d\tau^a > 0$ as suggested above. Repeating the procedure to derive $d\pi^a/dq_y^a$

$$\frac{d\pi^a}{dq_y^a} = - \frac{x_1^a[2\Gamma_{xy}^a(n+m)(2\Gamma_{xx}^a + x_1^a\Gamma_{xxx}^a) + \Gamma_{xy}^a\Gamma_{xyx}^a(x_1^a(n-(m+1))+2mx_2^a)]}{(n+m+1)\Gamma_{xx}^a + (nx_1^a + mx_2^a)\Gamma_{xxx}^a} \quad (46)$$

Although (46) may be of either sign, it is negative when demand is linear, when $\Gamma_{xxx}^a < 0$ and $\Gamma_{xyx}^a = 0$ and when $\Gamma_{xxx}^a < 0$ and $\Gamma_{xyx}^a(x_1^a(n-(m+1))+2mx_2^a) \geq 0$. These include a number of important possibilities.

In conclusion, since $d\pi^a/d\tau^a > 0$, and assuming $d\pi^a/dt^a < 0$, (44) demonstrates that the effect of profit income entering the measure of social welfare is to increase the likelihood of a move in the direction of positive tariffs being welfare-improving. This result has been derived on the basis of a particular representation of utility but, leaving complications in the analysis of the comparative statics to one side, it seems reasonable to propose that it would extend to all cases for which welfare was an increasing function of profits.

iv) Nash and Collusive Equilibria in Tariff-Setting.

It has been implicit in the previous sections that the two countries set their tariffs and taxes independently, effectively optimal policies would form a Nash equilibrium. Applying standard arguments to this Nash equilibrium (Friedman 1977), welfare in both countries could invariably be raised if they were to act collusively and jointly determine tariff

policy. Such gains are possible as the presence of imperfect competition in trade reduces welfare in comparison to the competitive level. The approach taken below is to characterise the Nash equilibrium level of tariffs and to contrast this with the collusive level. Interest is focussed on whether collusion raises or lowers optimal tariffs.

As the focus of the model is centred on tariffs alone, it is assumed that the commodity tax in both countries is zero and that the revenue raised from the tariff is used to fund the provision of a public good. Concentrating on country A (the argument is symmetric for B), indirect utility is:

$$V^a = V^a(q_x^a, p_y^a, G^a) \quad (47)$$

where G^a is the level of public good provision. By definition $G^a = \tau^a X_b^a$ so the maximisation faced by the government, taking B's tariff as given, becomes

$$\text{NT. } \max_{\tau^a} V^a = V^a(q_x^a(\tau^a), p_y^a, \tau^a X_b^a(\tau^a))$$

The necessary condition for NT. is

$$\delta V^a / \delta q_x^a \cdot \delta q_x^a / \delta \tau^a + \delta V^a / \delta G^a \cdot [X_b^a + \tau^a \delta X_b^a / \delta \tau^a] = 0$$

which can be re-arranged to give an implicit expression for τ^a

$$\tau^a = - \frac{\left[\frac{\delta V^a}{\delta q_x^a} \cdot \frac{\delta q_x^a}{\delta \tau^a} + X_b^a \cdot \frac{\delta V^a}{\delta G^a} \right]}{\frac{\delta V^a}{\delta G^a} \cdot \frac{\delta X_b^a}{\delta \tau^a}} \quad (48)$$

Inspection of (48) reveals that τ^a need not be positive, a negative tariff occurs when the forward-shifting of the tariff is large in relation to the marginal utility of government expenditure. To rule out this possibility the following assumption is made:

A8. $\delta V^a / \delta G^a = \infty$ when $G^a = 0$.

This assumption guarantees $\tau^a > 0$.

When the countries act collusively it is taken that they choose a pair of tariffs τ^a, τ^b to maximise the sum of their welfare levels. Their joint maximisation problem is

$$CT. \max \tau^a, \tau^b \quad V^a(q_x^a, p_y^a, \tau^a X_b^a) + V^b(q_x^b, p_y^b, \tau^b X_a^b)$$

The necessary conditions for this maximisation are

$$\delta V^a / \delta q_x^a \cdot \delta q_x^a / \delta \tau^a + \delta V^a / \delta G^a \cdot [X_b^a + \tau^a \delta X_b^a / \delta \tau^a] + \delta V^b / \delta q_x^b \cdot \delta q_x^b / \delta \tau^a + \delta V^b / \delta G^b \cdot \tau^b \delta X_a^b / \delta \tau^a = 0$$

and

$$\delta V^a / \delta q_x^a \cdot \delta q_x^a / \delta \tau^b + \delta V^a / \delta G^a \cdot \tau^a \delta X_b^a / \delta \tau^b + \delta V^b / \delta q_x^b \cdot \delta q_x^b / \delta \tau^b + \delta V^b / \delta G^b \cdot [X_a^b + \tau^b \delta X_a^b / \delta \tau^b] = 0$$

Re-arranging these to give an implicit expression for τ^a

$$\tau^a = \frac{-\left[\frac{\delta V^a}{\delta q_x^a} \frac{\delta q_x^a}{\delta \tau^a} + X_b^a \cdot \frac{\delta V^a}{\delta G^a} + \frac{\delta V^b}{\delta q_x^b} \frac{\delta q_x^b}{\delta \tau^a}\right] + \left[\frac{\delta X_b^a}{\delta \tau^a}\right] \left[\frac{\delta V^a}{\delta q_x^a} \frac{\delta q_x^a}{\delta \tau^b} + X_a^b \cdot \frac{\delta V^b}{\delta G^b} + \frac{\delta V^b}{\delta q_x^b} \frac{\delta q_x^b}{\delta \tau^b}\right]}{\frac{\delta V^a}{\delta G^a} \frac{\delta X_b^a}{\delta \tau^a} - \left[\frac{\delta X_b^a}{\delta \tau^a}\right] \left[\frac{\delta V^a}{\delta G^a} \frac{\delta X_b^a}{\delta \tau^b}\right]} \quad (49)$$

The focus here is whether τ^a determined by (49) is greater than or less than that determined by (48). One result is immediately apparent: if costs are linear, so that the markets may be treated as distinct, $\delta X_b^a / \delta \tau^b = \delta X_a^b / \delta \tau^a = \delta q_x^b / \delta \tau^a = 0$ and the two solutions are equivalent, identical tariffs being set in both the Nash and collusive equilibria. The reasoning behind this result is that the linearity of costs, and hence the distinctness of the two markets, means that the tariff policy of one country does not affect the welfare of the other so that there is

nothing to be gained from collusion. Note that this result does depend on taking w^b to be fixed, if it varied there would be interdependence even in the linear costs case: a tariff in B would reduce x_1^b and cause a trade deficit for A, w^b would rise to remove this deficit reducing x_2^a and increasing q_x^a . Alternatively, a tariff in A raises q_x^a and reduces w^b and q_x^b . There are obviously opportunities for collusive tariff policy to exploit these trade-offs.

To move beyond the linear case assume instead that the values of the derivatives in the two expressions are constant and that the equilibrium is symmetric. The symmetry implies $\delta X^a_b / \delta \tau^b = \delta X^b_a / \delta \tau^a$ and, if the tariff is to be smaller in the collusive equilibrium, the inequality

$$\frac{\delta V^b}{\delta q_x^b} \cdot \frac{\delta q_x^b}{\delta \tau^a} + \frac{\delta V^a}{\delta q_x^a} \cdot \frac{\delta q_x^a}{\delta \tau^a} + \frac{\delta V^b}{\delta q_x^b} \cdot \frac{\delta q_x^b}{\delta \tau^b} + X_a^b \cdot \frac{\delta V^b}{\delta G^b} < 0 \quad (50)$$

must be satisfied. This represents a trade-off between the value of the provision of the public good which is reduced as the level of tariffs falls and the level of prices in the two countries. Note that when $\delta V^b / \delta G^b = 0$, the tariffs will always be smaller provided the effect of the tariff in A does not reduce the price of X in B (ie. $\delta q_x^b / \delta \tau^a < 0$). Hence, without the public good effect the only factor that could lead to higher tariffs in the collusive equilibrium is a sufficiently large negative effect of tariffs on prices linking the two countries.

4.CONCLUSIONS.

The primary interest of this paper was to provide a formal proof of the existence of equilibrium in a model of trade with imperfect competition and to analyse tariff policy given lessons learnt from the existence proof. The equilibrating mechanism specified by the model highlighted the importance of factor price movements, the wage rate in this labour-only model, for the achievement of equilibrium. This leads to the viewpoint that partial equilibrium analyses with constant factor prices can be interpreted as short-run models. In them, any change in tariff policy invariably leads to a disequilibrium situation and the characterisations of tariff policy derived are valid only as long as this disequilibrium persists. The equilibrating process must modify the rules for optimal tariffs.

Under the assumptions made in section 2.ii, it was demonstrated that the variations in the wage rate strengthened the arguments for positive tariffs by reducing the change in price level for any change in tariffs. For negative tariffs to be welfare-improving, it had to be the case that the tariff was over-shifted to a considerable degree which, given the strong condition that had to be satisfied for over-shifting to occur at all, may be deemed unlikely. Similarly emphasising the importance of profit income further weighted the argument in favour of positive tariffs; the tariff made home firms relatively more competitive and increased their profits.

With two countries setting tariffs competitively, a Nash equilibrium in tariffs will be reached and it is likely that, for the countries in aggregate, this will not be an efficient equilibrium. Analysis of a collusive equilibrium, in which tariff policy was designed

to maximise joint welfare, revealed that tariffs will be lower than the Nash levels provided that the tariffs in one country do not have a negative effect upon the price level in the other.

It can be appreciated that the analysis of tariff policy with imperfect competition rarely leads to clear-cut results; those given above all rely on fairly precise sets of assumptions. What has been emphasised throughout are the factors that determine the direction a result will take and the methodology that can be used to tackle these issues.

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