

EXCHANGE RATE BANDS AND REALIGNMENTS
IN A STATIONARY STOCHASTIC SETTING

by

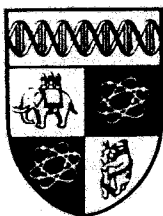
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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

ABSTRACT

Exchange Rate Bands and Realignment in a Stationary Stochastic Setting

The extent to which exchange rate management can coexist with an independent monetary policy is examined in the context of a model with exchange rate bands. Using a Dornbusch model in which stochastic stocks are added to the Phillips curve, we analyse the implications of assuming that the monetary authorities follow certain simple rules for realigning the band when fundamentals have drifted too far from equilibrium. Assuming that information about whether the band is to be defended or there is to be a realignment is revealed at the point when the exchange rate hits the edge of the band, we show how the path of the exchange rate can be completely characterised in terms of solution to a second order non linear differential equation -- together with jumps in the rate at the edge of the band, which satisfy a zero-profit arbitrage condition. When the realignment is expected with certainty, hysteresis is introduced into the behaviour of the exchange rate.

JEL Classification: 430, 431, 432

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Introduction

As financial capital became ever more mobile and U.S. monetary leadership progressively less credible (due, in part, to the exigencies of war finance), the Bretton Woods system of pegged-but-adjustable exchange rates became crisis-prone. Then, in the early 1970s, it finally collapsed.

In the aftermath, OECD countries were, by and large, content to allow their exchange rates to float — firmly anchored, it was hoped, by declared national money-supply targets. The U.K. and the U.S. were particularly enthusiastic advocates of national monetary autonomy and floating rates (together with substantial financial deregulation); and they bore with stoic indifference the enormous changes in international competitiveness that ensued.

After a while, however, disillusionment set in. Thus, under domestic pressures for protection, the U.S. began actively to "manage the dollar" in 1985, and was instrumental in arranging the Louvre Accord among the G6 in February 1987. According to Funabashi (1988), this involved bilateral exchange rate bands of $\pm 5\%$ against the U.S. dollar, although participants made no explicit surrender of national monetary autonomy. The U.K. was, of course, a participant, but was, at that time, closely shadowing the DM. (Currently, however, there are signs of a return to floating-plus-monetary targets.)

Concerned about their "internal market," Europeans (other than the U.K.) had never been enthusiastic for generalised floating and experimented instead with regional schemes for managing their cross-rates; first the Snake and then, in 1979, the EMS. The latter has proved relatively successful in preventing big shifts in competitiveness between the member countries, but has relied heavily on capital market controls. According to current plans, however, these controls are to be abolished; and the lessons of history and

economic theory have led observers such as Padio-Schioppa (1988) to warn those planning the next phase of the EMS that they are seeking the impossible if they try to achieve free trade in goods and capital and to manage exchange rates without ending national monetary autonomy.

In this paper we examine this issue — the compatibility of exchange rate management and monetary autonomy — using a model with free trade, perfect capital mobility, and (adjustable) exchange rate bands. In particular, we examine the implication for monetary policy and exchange rates of announcing rules for realigning the exchange rate bands when the rate hits the edge. In a setting where there are no inherent trends in the price level, Obstfeld (1988) has recently argued that accommodation of price changes may nevertheless cause devaluation spirals. But we get different, and less startling, results from applying Krugman's (1987,1988) analysis to a stochastic Dornbusch model. The model, and the effect of fixed exchange rate bands therein,¹ is given first, before considering the implications of prospects for realignment.

1. A Simple Stochastic Dornbusch Model

The model to be used has four equations, two static and two dynamic:

$$(1) \quad m - p = \kappa y - \lambda i \quad \text{(Money Market)}$$

$$(2) \quad y = -\gamma i - \eta v \quad \text{(Goods Market)}$$

$$(3) \quad dp = \phi y dt + \sigma dz \quad \text{or} \quad p = \int_{-\infty}^t y(s) ds + z \quad \text{(Stochastic Phillips Curve)}$$

$$(4) \quad E(dx) - (i^* - i)dt \quad \text{or} \quad x = E \int_t^{+\infty} (i(s) - i^*(s))ds + \hat{x} \quad (\text{Arbitrage})$$

where the symbols used are defined as follows:

- m -the domestic money stock (in logs).
- p -price index of domestic final product (in logs)
- p^* -price index of imported foreign product (in logs)
- y -level of domestic final production,
measured from its non-inflationary level (in logs)
- x -the exchange rate, defined as the
foreign currency price of domestic currency (in logs)
- $v = (x + p - p^*)$ -the "real" exchange rate
- i -(instantaneous) domestic nominal interest rate
- i^* -(instantaneous) foreign nominal interest rate
- df -the stochastic differential of the process f

z -a scalar Brownian motion process with unit variance,
so σdz is white noise

E -the expectations operator

\hat{x} -a hat ($\hat{\quad}$) above a variable denotes its long-run equilibrium value.

The equations are doubtless familiar (see Dornbusch (1976) Appendix), so we can be brief. The first defines equilibrium in the domestic money market, where the demand for real balances is associated positively with output and negatively with the opportunity cost of holding money (measured by i). For simplicity, the price deflator for the money stock is that of domestic product. So long as the exchange rate is inside the band, we assume that the nominal money stock is fixed at a "target" level: what happens at the edges of the band is either that the money supply is adjusted to ensure $i = i^*$, i.e., the band is defended; or there is a realignment and the money stock is adjusted to a new target level.

The level of output is demand-determined and, as shown by equation (2), depends (negatively) on the real exchange rate (the relative price of domestic output) and on the domestic interest rate. Although it would doubtless be more accurate to use the "real" interest rate here, we have stayed with Dornbusch's original formulation because it is simpler and because the fixed money supply rules out persistent inflation.

The next two equations show the contrast between the way the price of domestic production evolves and the way the exchange rate is determined.

(Foreign prices are assumed to be constant throughout.) The price of domestic output is "predetermined," i.e., it depends on its previous value except insofar as it is raised or lowered by movements in current production above or below the non-inflationary level or by white noise disturbances, σdz . Thus we have adhered to Dornbusch's formulation in ignoring terms reflecting past or future inflation, but have added a random disturbance.

The nominal exchange rate — here defined as the foreign currency price of domestic currency — is a "forward-looking" variable which depends not at all on its previous value. As indicated to the right of equation (4), the current exchange rate is the long-run equilibrium, \hat{x} , "discounted" back by the integral of expected future international interest differentials. So a future interest differential which is, on average, expected to be in favour of the domestic economy will lift the currency above equilibrium, and vice versa.

The evolution of these two "prices" can be expressed as two simultaneous stochastic differential equations (after eliminating y and i by substitution). Thus

$$(5) \quad \begin{bmatrix} dp \\ E(dx) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -\phi(\gamma+\lambda\eta) & -\phi\lambda\eta \\ \kappa\eta-1 & \kappa\eta \end{bmatrix} \begin{bmatrix} p dt \\ x dt \end{bmatrix} \\ + \frac{1}{\Delta} \begin{bmatrix} \phi\gamma & \phi\lambda\eta & 0 \\ 1 & -\kappa\eta & \Delta \end{bmatrix} \begin{bmatrix} m dt \\ p^* dt \\ i^* dt \end{bmatrix} \begin{bmatrix} \sigma dz \\ 0 \end{bmatrix}$$

where $\Delta = \kappa\gamma + \lambda$. Alternatively, defining variables x_0, P_0 , as deviations from equilibrium, measuring x and p from equilibrium (so $x_0 = x - \hat{x}$, $P_0 = p - \hat{p}$),

$$(6) \quad \begin{bmatrix} dp_0 \\ E(dx_0) \end{bmatrix} = A \begin{bmatrix} p_0 dt \\ x_0 dt \end{bmatrix} + \begin{bmatrix} \sigma dz \\ 0 \end{bmatrix}$$

where A is the matrix of coefficients on the right-hand side endogenous variables in equation (5). From now on, we will work with the formulation in (6), and to simplify notation will relabel $p = p_0$, $x = x_0$.

The determinant of the matrix A (namely, $\frac{-\phi\eta}{\Delta}$) is negative, indicating roots of opposite sign. One solution to the stochastic system is that it lies on the linear stable manifold associated with the negative root of A , ρ_s , so

$$(7) \quad x = \theta_s p$$

$$\text{where } \theta_s = \frac{1 - \kappa\eta}{\kappa\eta - \rho_s \Delta}.$$

Note that the sign of θ_s depends on that of $1 - \kappa\eta$. If $\kappa\eta < 1$, and θ_s is positive, then the exchange rate is high in response to positive price disequilibria; which is also the case where the exchange rate "overshoots" in response to monetary changes. (When $\kappa\eta > 1$ and $\theta_s < 0$, one has "under-shooting.")

But, where $\sigma^2 > 0$, there are many other functional relationships between x and p which satisfy equation (6); and as the problem is stationary, one

can obtain an ordinary differential equation characterising these solutions, as follows. Let $x = f(p)$ be a solution. Then, by Ito's lemma,

$$(8) \quad dx = f'(p) dt + \frac{\sigma^2}{2} f''(p) dt .$$

On taking expectations and substituting from (6), one obtains the required equation, namely

$$\begin{bmatrix} A_2 - f'(p) A_1 \end{bmatrix} \begin{bmatrix} p \\ f(p) \end{bmatrix} = \frac{\sigma^2}{2} f''(p)$$

By imposing the boundary condition $f(0) = 0$, we obtain solutions which satisfy the symmetry property $f(x) = -f(-x)$. These are the appropriate ones to consider for problems with boundary conditions symmetric about equilibrium.

For the case of exchange rate "undershooting" where the rate weakens as the price level rises relative to the money stock), we illustrate, in Figure 1, the qualitative nature of all these trajectories² and indicate how the boundary conditions implied by a symmetric, fully credible exchange rate band serve to pin down a particular solution. The appropriate boundary condition requires that the trajectory be tangent to the band (so-called "smooth pasting"³). So the desired solution consists of the bands themselves, and the backward S-shaped curve that links them, and is shown labelled as ABCD in the figure. The curvature of this trajectory from B to C relative to the manifold SOS' exhibits what Krugman has dubbed the "bias in the band." (But note that as σ^2 tends to zero, the section BC moves closer to SOS', the stable eigenvector of A, coinciding with it in the limit.) In 1(b) is shown

the stance of monetary policy necessary to support such a band, where m^* is the target level of the money supply and the "state contingent" shifts of the money stock from this target level are those required to keep international interest differentials at zero at the edges of the band.

2. Realignments

The boundary conditions considered above are those appropriate for fully credible currency bands where the temporary departures of the money stock from its target level reflect action taken by the monetary authorities to hold the rate at the edge of the band. But what if the consequence of the rate hitting the edge of the band was, instead, a realignment of the band itself and a simultaneous shift of the monetary target? And what if market participants were unsure whether there was to be a defence of the band or a realignment? In these circumstances, as Krugman has argued, different boundary conditions apply depending on the size of the possible realignment and how likely it is expected to be.

Consider, for example, the rule that, "if the rate hits the edge of the band, then the money stock target be adjusted exactly to accommodate the divergence of the price level from its equilibrium, and the centre of the exchange rate band be moved by the same percentage amount." And suppose that market participants believe that the rule will be followed with some probability π . Also, following Krugman (1988), let us make two additional, simplifying assumptions. First, if the band is ever defended, the authorities establish full credibility. Second, if a realignment does occur, the market does not change its perception of the probability of future realignments. Then the probability π assigned to realignment (as opposed to defense)

systematically affects both the size of the expected realignment itself and the trajectory for the exchange rate within the currency band.

To show this, we proceed geometrically, with the aid of Figure 2. In the top panel, the 45° line, PPP, is the locus of points which would preserve purchasing power parity and, as before, SOS' indicates the stable manifold of the deterministic system (which for suitable initial conditions is also a solution to the stochastic system). We concentrate for simplicity only on what happens at the bottom edge of the band. The argument is exactly symmetric at the top. The locus ODB describes the path of the exchange rate in the presence of a fully credible band. Then, where the market expects that a realignment will occur with some positive probability, the rate will follow a path such as OC. Its properties are, first, that it is a solution to (9), and so satisfies the arbitrage condition within the band; second, that when the rate hits the edge of the band at C, the expected change in the rate is zero; that is

$$(10) \quad \pi (\text{loss on realignment}) + (1 - \pi) (\text{gain if no realignment}) = 0.$$

Thus, if a realignment occurs, and the money stock is increased by \bar{p} in harmony, then the rate drops instantaneously from C to E, the new equilibrium lying in the middle of the realigned band. But, if no realignment occurs, and m is held at m^* , then the rate jumps upwards to point D.

Now, as π varies from zero to one, point C moves from B to F. For if $f(p)$ is the solution to (9) in the fully credible case, then using the relationship in (10) we see that for any π , \bar{p} must solve

$$(11) \quad \pi(b - p) + (1 - \pi)(f(p) + b) = 0$$

where b is equal to half the width of the band.

This produces a relationship between π and \bar{p} of the form

$$(12) \quad \pi(\bar{p}) = \frac{f(\bar{p}) + b}{f(\bar{p}) + \bar{p}}$$

which is illustrated in the lower panel of Figure 3.

There, by definition, $\pi(p_U) = 0$ and $\pi(p_L) = 1$, and it is straightforward to confirm that $\pi' < 0$, $\pi'' > 0$ for $p_L \leq p < p_U$, and that $\pi'(p_U) = 0$.

More generally, we can think of a family of solutions satisfying (10) for a class of realignment rules. If we identify the realignment rule with the associated change in money supply Δm , then the following relationship must hold:

$$(13) \quad \pi(\bar{p}) = \frac{f(\bar{p}) + b}{f(\bar{p}) - g(\bar{p} - \Delta m) + \Delta m}$$

where $g(p)$ is a solution to (9) which will depend upon π and Δm . A fully accommodating rule involves setting Δm equal to \bar{p} , and (13) reduces to (12) since $g(0) = 0$. (It is important to point out that the relationship in (13) is not valid for arbitrary Δm . This is best illustrated by observing that if Δm is "too large," then for some values of π there will exist no \bar{p} satisfying (13). In particular, if $\Delta m \geq 2p_U$, there is no value of \bar{p} that will satisfy (13) for any positive π .

An interesting case is the hitting point at M associated with the price $\bar{p} = \frac{-b}{\theta_s}$. The associated probability of realignment, π_M , will depend upon the realignment rule in the following way:

$$\pi_M = \frac{f(-b/\theta_s) + b}{f(-b/\theta_s) + b + (1 + \theta_s)\Delta m}, \quad \Delta m < -2b/\theta_s$$

This is the value of π which will exactly counteract Krugman's "bias in the band" and ensure that the equilibrium lies along the stable manifold SOS' (until any band is defended, at which time full credibility is established).

It may be easier to see the implications of the fully accommodating realignment rule with reference to Figure 3 which shows what happens when the rate hits the edge in the case where $\pi = 1$. On the assumption that the market fully expects such a realignment (and the corresponding shift in the monetary target), then from a band centred on E_0 the rate will lie along the solid line $E_H E_0 E_L$ which snakes around the 45° line of PPP. On reaching either end as a result of the random inflation shocks, there will be a realignment: from E_0 each of these possibilities has equal probability in the eyes of the market. If, for instance, there was a run of positive inflation shocks taking p to p_1 , then the band would move down by half its width so the rate would now lie at the centre of the new band. On the assumption that the market continues to assign the same probability to future realignments, the rate will lie on the dashed line through E_L .

At first sight this result appears counterintuitive. Indeed, for the semi-stable model he analyses, Krugman (1988) argues that setting $\pi = 1$ is

equivalent to a "free float." In our model this is not the case. If we interpret a free float as equivalent to an infinitely wide exchange rate band, the path for the exchange rate will be S_0S_0' . But as we show below, that is the solution when $\pi = \pi_M = 1$.

The reason why, even when $\pi = 1$, there is an effect upon the path of the exchange rate, is that we have in fact introduced thresholds for monetary accommodation. If fundamentals fluctuate within a certain range (here $-b < p < b$), no adjustment to the money stock occurs. When p hits the trigger value b , there is an immediate increase in m , by the same amount, to a new target level which remains unchanged — until the next realignment.

Because of the discontinuous nature of monetary adjustment, the behaviour of the exchange rate betrays an element of "hysteresis," in that its relationship to the price level is not unique, but depends on the money stock, which is changed from time to time (when shocks have cumulated sufficiently since the last adjustment). It is precisely this element of hysteresis which obviates the need for a "smooth pasting" or tangency condition between regimes. With a fully credible exchange rate band, switching between support and no support regimes is triggered by the price level passing through a single critical value. But in this case a realignment alters the trigger values for the fundamental, and the arbitrage equation simply imposes the restriction that there be no discontinuity in the path of the exchange rate.

Figure 4 illustrates the case where the probability of a realignment-with-complete-monetary-accommodation is such as to keep the exchange rate on S_0S_0' until it hits the edge of the band (i.e., $\pi = \pi_M$, in the notation used above). If this happens to be the lower edge then, if a realignment occurs, as shown, the rate jumps down to a new equilibrium on the PPP line, at E_L .

It then remains on the linear trajectory $S_L S_L$ until another realignment brings it back to E_0 or carries it further away.

3. Underalignment and Overalignment

As we have already observed, there is no necessity for a realignment to accommodate the price level exactly. To see what happens in the cases of "overalignment" ($\Delta m > \bar{p}$) and "underalignment" ($\Delta m < \bar{p}$), we shall continue to assume that π is unchanged after any realignment. In the former case (where the amount of monetary accommodation and the potential shift in the exchange rate when realignment takes place are both increased) it seems reasonably obvious that, for any given trigger price, there must be less of a risk of realigning for the arbitrage conditions to be satisfied at the edge of the band; and, conversely, for underalignments. The shifts in the relationship between the trigger price, \bar{p} , and the risk, π , implied by changing the degree of price accommodation in either direction are shown in the lower panel of Figure 5, where the line AB represents the relationship already derived for the case of 100% accommodation.

In the upper panel, we sketch the implied trajectory for the exchange rate path within the band, focusing on an overalignment, expected with certainty.

The requirement that the percentage change in the money stock (Δm) will be greater than the percentage deviation of prices (\bar{p}) in this case is indicated by the slope of the line $E_0 B$ which has a slope greater than that of the PPP line shown there. The trajectory implied by this particular degree of overalignment, anticipated for sure, is shown labelled as $E_0 L$ and cuts the lower edge of the band to the left of the PPP line. (At point L the

greater-than-proportionate shift in the money supply and the bands will for sure shift the centre of the band down to the level A, given a new equilibrium on the PPP line at E_L ; and, at point L, the exchange rate must also lie on the trajectory associated with the new equilibrium, see Figure, since the transition is expected with complete certainty.) The distance from L to the 45° line must be equal to the distance CA in the lower panel, if both measure the fall in the hitting price corresponding to the same rule for overalignment, expected for sure.

Two observations on this solution are in order. First (as has already been indicated), there is a limit to the degree of overalignment which may be anticipated with complete certainty and still generate zero expected profits. Second, we note that along trajectories which cross the PPP line in this type of model, the expected movement of the price level may switch away from the old equilibrium towards the new as the edge of the band is approached (see Miller and Weller (1988b) for further discussion).

These two caveats do not, however, apply to the case of underalignments (these are not shown in the top panel), but it is reasonably clear that the hitting point for $\pi = 1$ is to the left of the PPP line, so the function in the bottom panel is shifted to the right for such a case. Ultimately, of course, as the degree of underalignment approaches zero, one converges to the case where the existing exchange rate band is fully credible.

Conclusion

By using a stochastic approach, we have shown how it is possible to combine some measure of monetary autonomy and some management of the exchange rate while preserving free trade in both goods and financial assets. If the

specific realignment rules examined here were fully believed, then the money supply would usually be fixed and the currency would float freely, except when the rate hit the edges of the band — when the money supply would jump without shocking the current spot rate. (The spot exchange rate could jump too, if the probability of realignment were less than one.) We offer this as an alternative to the "speculative attack" treatment proposed by Obstfeld (1988) to help understand the economic implications of abolishing capital controls within the EMS. (Note, however, that there are additional parameter differences — other than in the stochastic specification — between the models used which will affect a detailed comparison of results.)

There are numerous ways in which the stochastic approach adopted here can be improved — thus the realignment probability might be revised systematically, for example (cf. the work of Backus and Driffil) and account taken of the presence of "noise traders" in the foreign exchange market (cf. DeLong et al. (1988)), to name but two. Past experience of the EMS and the pressures exacted upon it by further financial deregulation will doubtless suggest others.

Footnotes

1. As discussed earlier in Miller and Weller (1988a), to which the reader is referred for additional detail.
2. A formal derivation of these qualitative characteristics is provided in Miller and Weller (1988b).
3. A detailed discussion of the "smooth pasting" condition as it applies here is given in Miller and Weller (1988a).

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FIGURE 1a

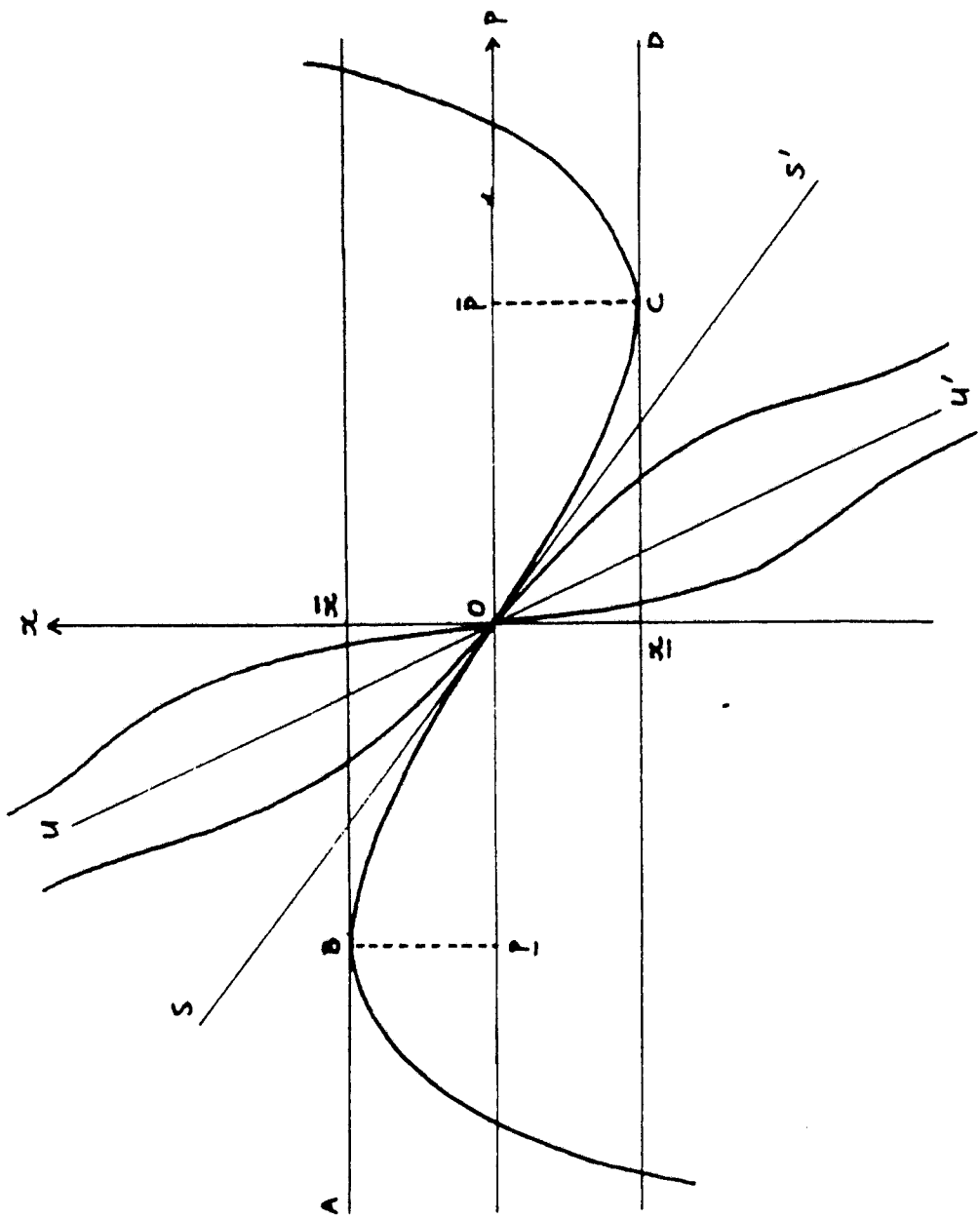


FIGURE 1b

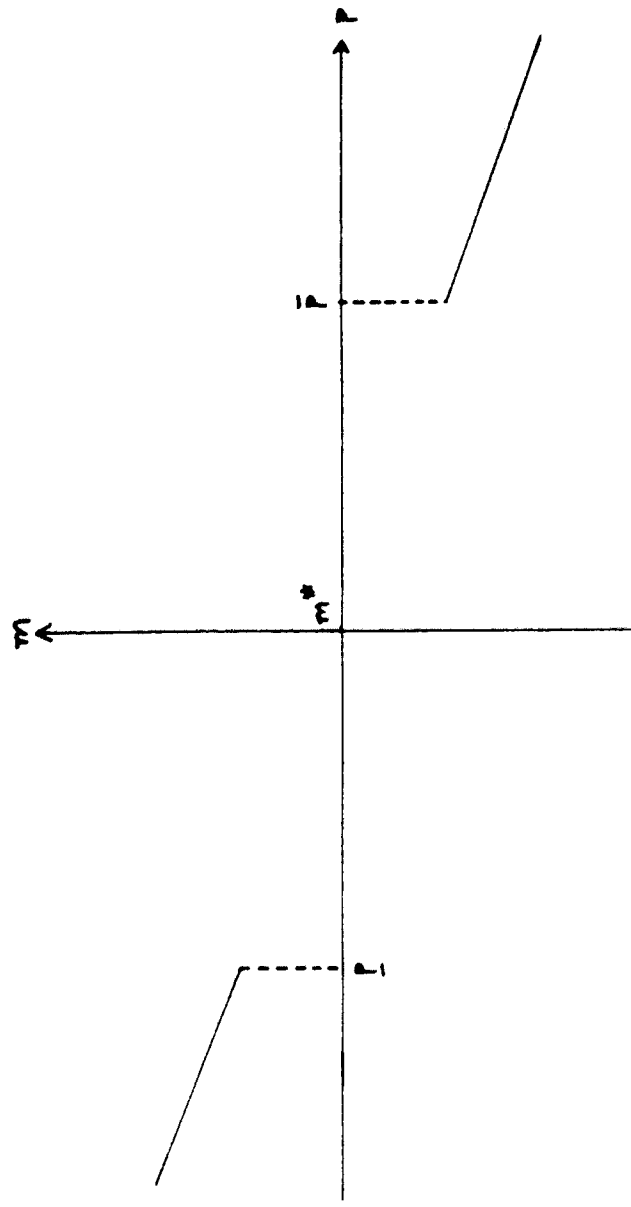


FIGURE 2

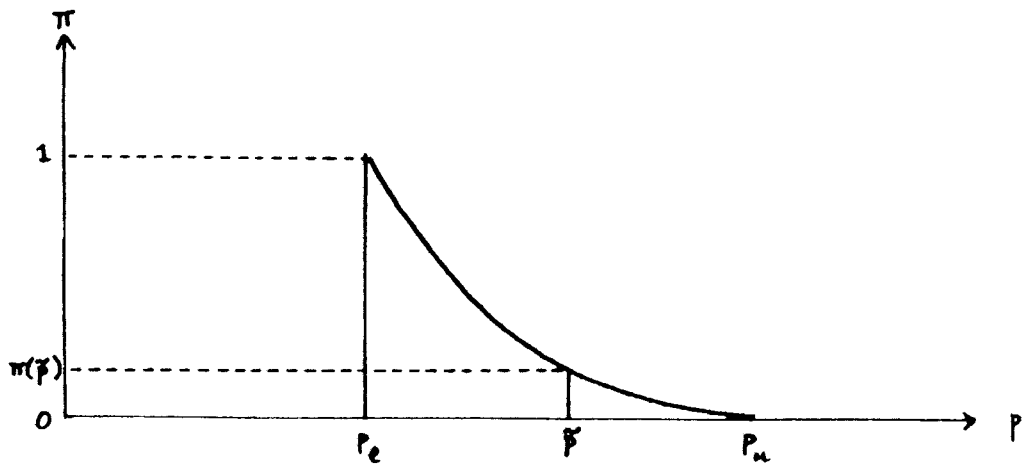
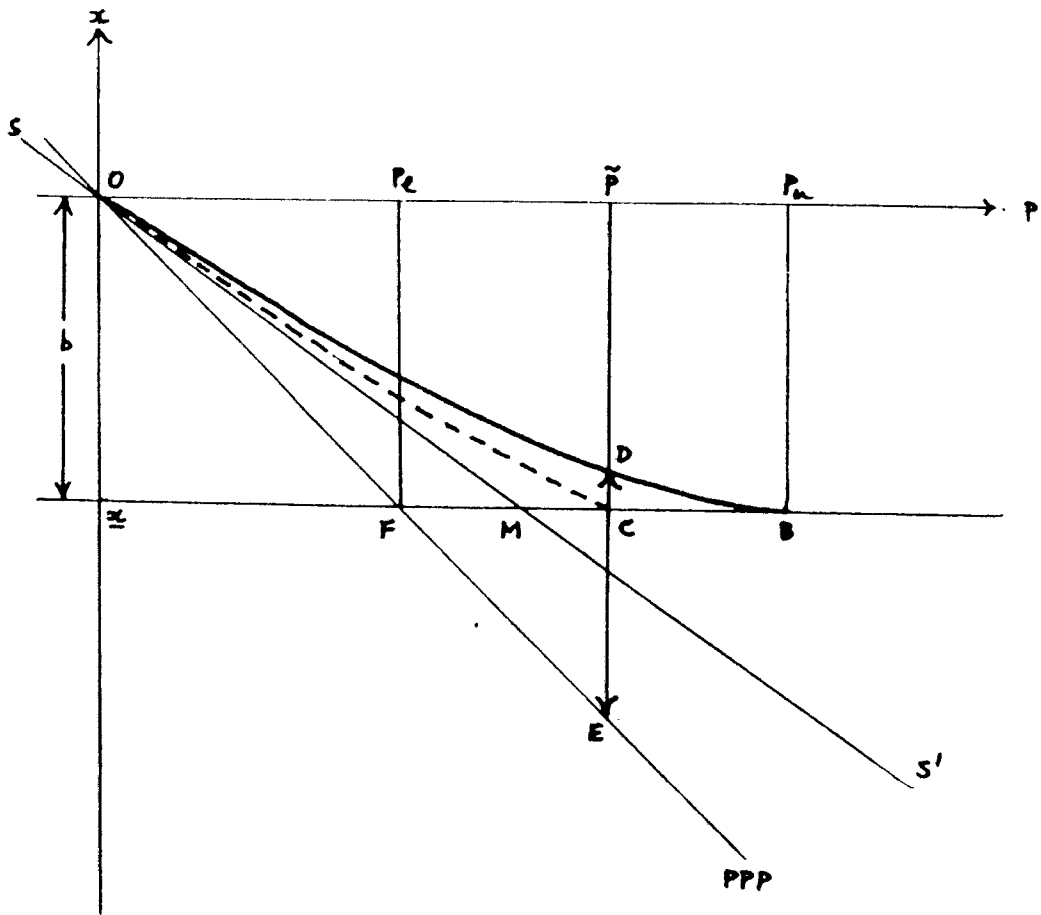


FIGURE 3

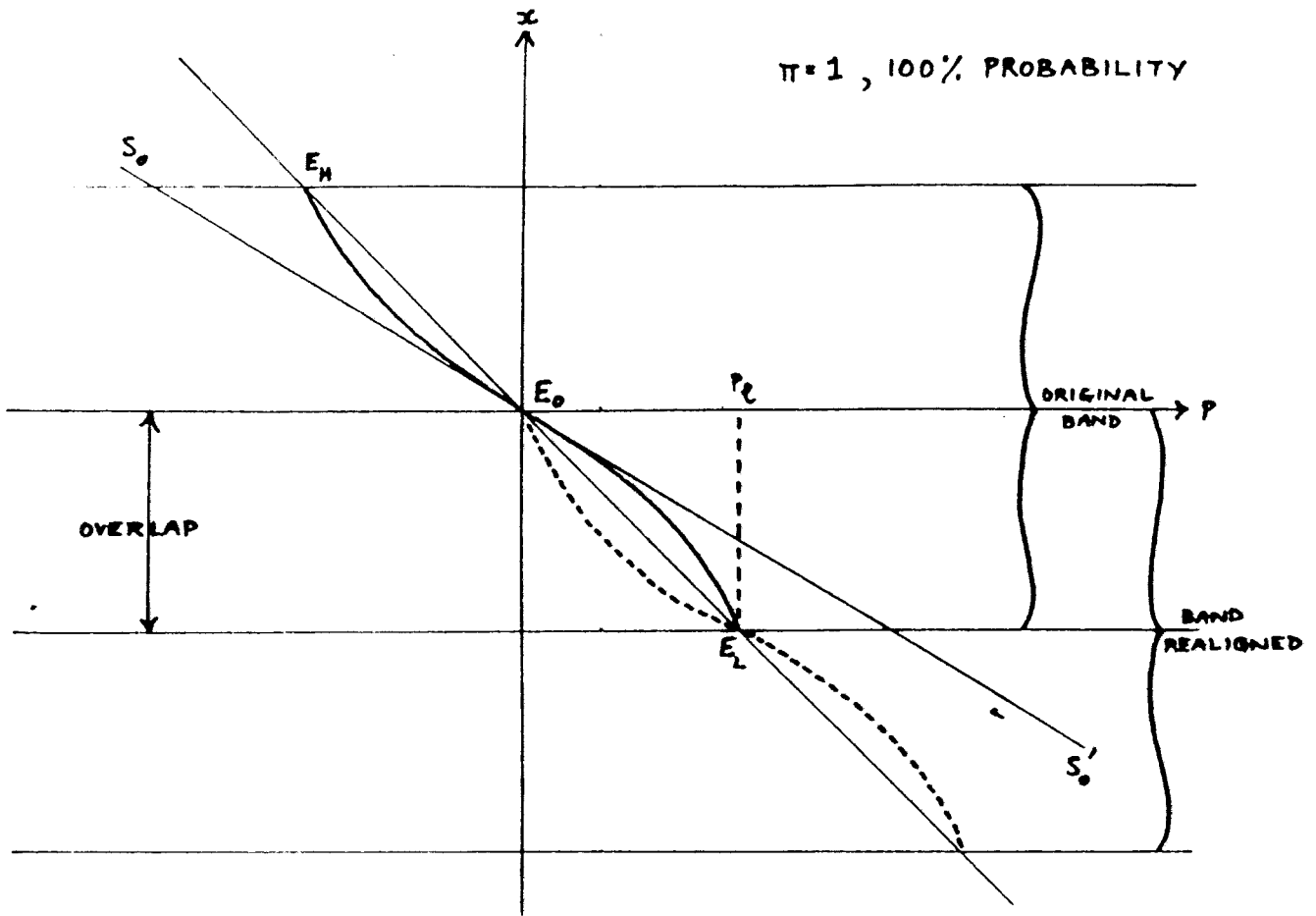


FIGURE 4

OFFSETTING THE BIAS IN THE BAND

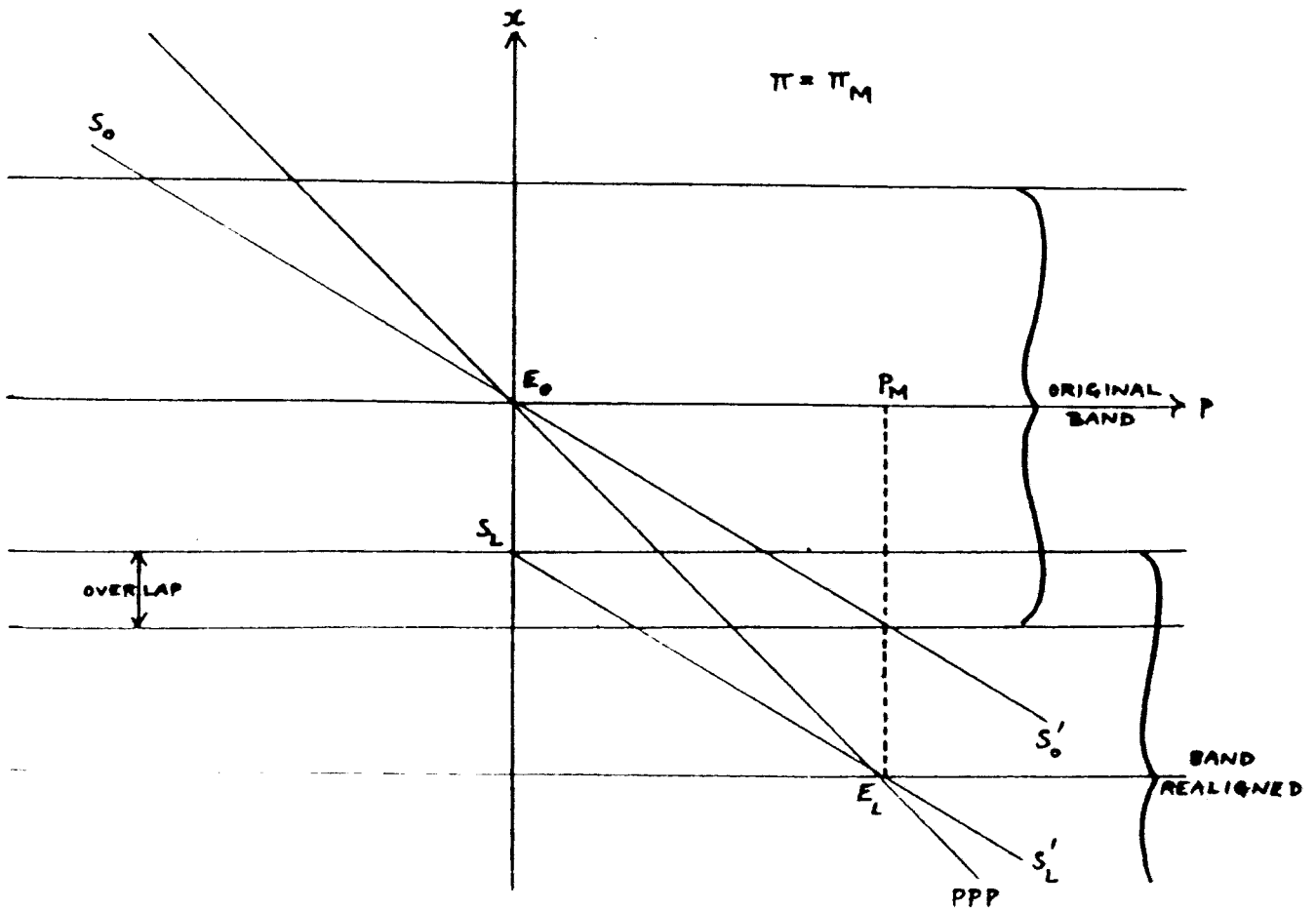
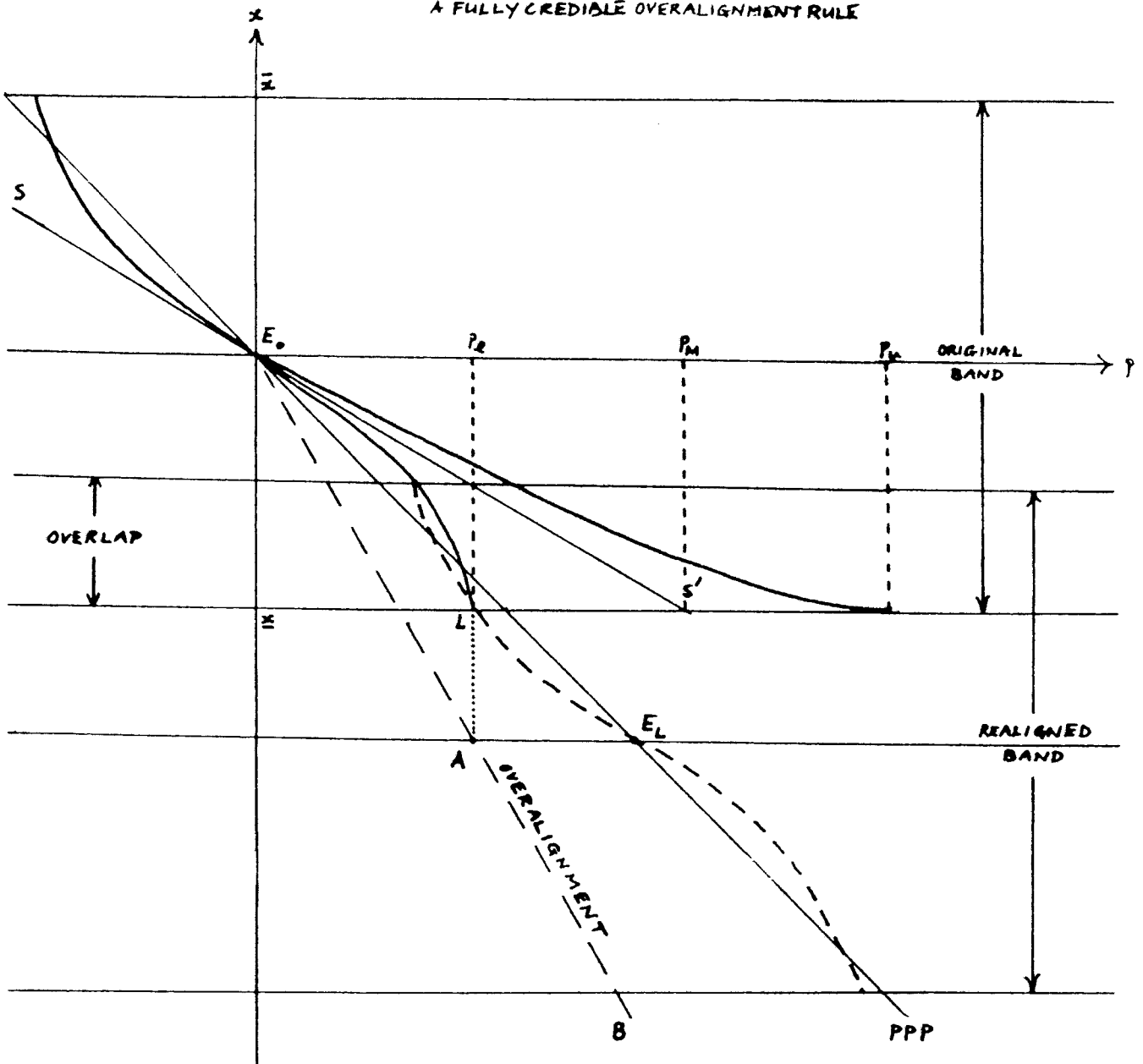


FIGURE 5

A FULLY CREDIBLE OVERALIGNMENT RULE



$\pi(p)$ IN
VARIOUS CASES

