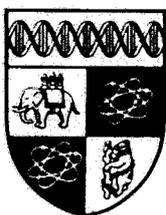


IMPERFECT COMPETITION AND THE TAXATION OF
INTERMEDIATE GOODS

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

Imperfect Competition and the Taxation of Intermediate Goods.

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Abstract: It is an implication of the productive efficiency lemma of Diamond and Mirrlees that intermediate goods should not be taxed in a world of constant returns to scale and perfect competition. Three simple models are analysed to examine whether this conclusion can be extended to accommodate imperfect competition. The importance of returns to scale and the form of the production function are emphasised and, where applicable, welfare-improving and optimal tax schemes are described that include taxes on intermediate goods. If all technologies are Leontief, productive efficiency remains desirable.

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1. INTRODUCTION.

In their seminal paper on commodity taxation, Diamond and Mirrlees (1971) established the production efficiency lemma. As this states that private and public producers should face the same shadow prices, it carries the implication that intermediate goods should not be taxed. The belief that this result embodies a general principal is seemingly widespread, for instance Kay and King (1980) view non-taxation of intermediate goods as the "first principle" of commodity taxation. However, before such emphasis is placed upon a result it seems appropriate to analyse its robustness to changes in its underlying assumptions.

The initial result in Diamond and Mirrlees (1971) is proved only for a competitive economy producing with constant returns to scale; in the absence of pure profits consumers are indifferent to the price vector faced by firms, a major factor in supporting the result. When decreasing returns are permitted, Dasgupta and Stiglitz (1972) conclude that production efficiency is only desirable if the range of government instruments is sufficiently great, in effect, only if profits can be taxed at appropriate rates. Mirrlees (1972) provides further clarification of the relation of profits and production efficiency. These findings indicate that one of the theory's major assumptions, that of constant returns to scale, can be relaxed.

However, there remains one aspect that has not yet been pursued and which is the subject of this paper: does the removal of the perfect competition assumption invalidate the production efficiency lemma? Rather than answer this question directly, I will consider instead whether the taxation of intermediate goods is implied by imperfect competition; this is of course the point of immediate policy relevance.

To bring out most clearly the factors at work I will analyse welfare-improving and optimal taxes for three simple models. In the first, the competitive industry's entire output is sold to the monopolist whose output constitutes the model's final good. The roles are reversed in the second model. The third model, with two monopolists

producing final goods, allows for discriminatory intermediate good taxation. Attention will be focussed upon efficiency arguments by assuming the existence of a single consumer who consumes the final goods, receives the monopolies' profits and supplies labour.

Three major results emerge from the analysis. Firstly, if all production processes are Leontief production efficiency remains desirable. This suggests that linear models of imperfect competition, such as Wilner (1983), although analytically tractable are too specialised to provide a basis for further analysis of taxation. Secondly, the presumption that non-taxation of intermediate goods is always optimal must be dismissed. In each of the models analysed circumstances do exist for which welfare maximisation requires such taxes. Finally, it is apparent that the form of an optimal tax scheme will be closely linked to the returns-to-scale of the industries in the conomy.

Section 2 introduces the model and studies competitive production of the intermediate good. In section 3 the monopolist becomes the intermediate good producer. The model is extended to include two producers of final goods in section 4. Conclusions are given in section 5.

2. COMPETITIVE FACTOR PRODUCTION.

Each firm in the competitive industry, which in this section produces the model's intermediate good, is assumed to have a fixed coefficient production function and units are normalised so that each unit of output requires one unit of labour. Writing w for the wage rate, the post-tax price of the competitive industry's good, which is labelled y , is

$$(1) \quad q_y = w + i_y$$

where i_y is the intermediate good tax. Labour acts as the numeraire and the wage rate remains constant at w throughout the paper. Directly from (1)

$$(2) \quad \frac{\partial q_y}{\partial i_y} = 1$$

The monopoly produces with costs given by

$$(3) \quad C(q_y, w; x) + xt_x$$

where x is the firm's output level, t_x is the commodity tax and $C(\cdot)$ is the cost function. The price q_x is chosen to maximise profit, π , where

$$(4) \quad \pi = [q_x - t_x].f(q_x, w, \pi) - C(q_y, w; f(q_x, w, \pi))$$

$f(\cdot)$ being the demand function. The presence of π on both sides of (4) captures the income effects that occur in a general equilibrium model. Sufficient conditions for the maximisation of implicit functions of the form of (4) are derived in Cripps and Myles (1988), effectively these require the income effect

$$\frac{\partial}{\partial \pi} [[q_x - t_x].f(q_x, w, \pi) - C(q_y, w; f(q_x, w, \pi))]$$

to be bounded below 1. In any case, the profit maximising choice is characterised by

$$(5) \quad \frac{\partial \pi}{\partial q_x} = \frac{f + f_1.[q_x - t_x - C_0]}{1 - f_3.[q_x - t_x - C_0]} = 0$$

where $f_1 < 0$ and $f_3 > 0$ are the partial derivatives of f with respect to its first and third arguments and $C_0 \equiv \frac{\partial C}{\partial x}$. Assuming the constraint on the income effect is satisfied

implies that $1 - f_3.[q_x - t_x - C_0] > 0$, hence from (5)

$$(6) \quad f(q_x, w, \pi) + [q_x - t_x - C_0].f_1(q_x, w, \pi) = 0$$

The second-order condition, found by differentiating (5), is

$$(7) \quad \frac{\partial^2 \pi}{\partial q_x^2} = \frac{2f_1 + f_{11}.[q_x - t_x - C_0] - f_1^2 C_{00}}{1 - f_3.[q_x - t_x - C_0]} < 0$$

In addition, the equilibrium must also satisfy the profit identity

$$(8) \quad \pi - [q_x - t_x].f(q_x, w, \pi) - C(q_y, w; f(q_x, w, \pi)) = 0$$

The important step in the analysis is the determination of how the taxes t_x and i_y affect the monopolist's profit maximising price and level of profit. Equations (6) and (8) are a two equation system that determine q_x and π . Differentiating and solving,

$$(9) \quad \frac{\partial \pi}{\partial t_x} = \frac{-f}{1 - f_3[q_x - t_x - C_0]} < 0$$

$$(10) \quad \frac{\partial \pi}{\partial i_y} = \frac{-C_1}{1 - f_3[q_x - t_x - C_0]} < 0$$

where $C_1 = \partial C / \partial q_y$.

$$(11) \quad \frac{\partial q_x}{\partial t_x} = \frac{f_1}{[2f_1 + f_{11}[q_x - t_x - C_0] - f_1^2 C_{00}]} + \frac{f.[f_3 + f_{13}[q_x - t_x - C_0] - f_3 f_1 C_{00}]}{[1 - f_3[q_x - t_x - C_0]].[2f_1 + f_{11}[q_x - t_x - C_0] - f_1^2 C_{00}]}$$

and

$$(12) \quad \frac{\partial q_x}{\partial i_y} = \frac{f_1 C_{01}}{[2f_1 + f_{11}[q_x - t_x - C_0] - f_1^2 C_{00}]} + \frac{C_1.[f_3 + f_{13}[q_x - t_x - C_0] - f_3 f_1 C_{00}]}{[1 - f_3[q_x - t_x - C_0]].[2f_1 + f_{11}[q_x - t_x - C_0] - f_1^2 C_{00}]}$$

The first term of (11) is the direct effect of the change in tax upon price and, as shown by Seade (1985), it may be greater than 1. When marginal costs are constant, $C_{00} = 0$, it is greater than 1 if $f_{11}f/f_1f_1 > 1$ where $f_{11}f/f_1f_1$ is the elasticity of the slope of the demand function. Assuming $f_{13} > 0$, the second term will certainly be negative if $C_{00} \leq 0$. This captures the fact that the tax lowers profits, which then leads to a reduction in demand. Hence the value of $\frac{\partial q_x}{\partial t_x}$ will tend to be less than the values reported in the partial equilibrium simulations of Myles (1987). The same interpretation applies to (12).

I now wish to analyse whether circumstances exist for which the maximisation of social welfare will imply values of t_x and i_y different from zero. In other words, should the intermediate good in the above model be taxed? The answer to this question

is best understood by first considering a simpler problem: starting from an initial position of zero taxation ($t_x = i_y = 0$) can a pair of tax changes dt_x, di_y be found that increase welfare while retaining a balanced government budget. This is stated as problem W11

W11: Starting from an initial position with $t_x = i_y = 0$, find dt_x, di_y such that $dV > 0$ and $dR = 0$

where $V = V(q_x, w, \pi)$ is the single consumer's indirect utility function, which acts as the measure of social welfare, and $R = t_x x + i_y y$.

To characterise the solution to W11, first differentiate the utility function and the revenue constraint

$$(13) \quad dV = \left[\frac{\partial V}{\partial q_x} \cdot \frac{\partial q_x}{\partial t_x} + \frac{\partial V}{\partial \pi} \cdot \frac{\partial \pi}{\partial t_x} \right] dt_x + \left[\frac{\partial V}{\partial q_x} \cdot \frac{\partial q_x}{\partial i_y} + \frac{\partial V}{\partial \pi} \cdot \frac{\partial \pi}{\partial i_y} \right] di_y$$

and

$$(14) \quad dR = x dt_x + y di_y = 0$$

From Shephard's lemma, $y = C_1$ so the budget constraint gives $dt_x = - (C_1/f) \cdot di_y$.

Substituting this and (9) to (12) into (13)

$$(15) \quad dV = \frac{\partial V}{\partial q_x} \left[\frac{f_1 C_{01} - \frac{C_1 f_1}{f}}{2f_1 + f_{11} \cdot [q_x - C_0] - C_{00} f_1^2} \right] di_y$$

As $\partial V / \partial q_x, f_1$ and $2f_1 + f_{11} \cdot [q_x - C_0] - C_{00} f_1^2 < 0$, proposition 1 follows

Proposition 1.

$di_y \neq 0$ if $fC_{01} - C_1 \neq 0$. In particular, $di_y > 0$ if $fC_{01} - C_1 < 0$ and $di_y < 0$ if $fC_{01} - C_1 > 0$.

This can be seen by inspection of (15).

Assuming that $\text{sign}\{fC_{01} - C_1\} = \text{sign}\{fC_{02} - C_2\}$, so the production function is "well-behaved", and noting that if the monopolist produces with constant returns to scale, $C(q_y, w; x) = c(q_y, w)x$ and $fC_{01} - C_1 = 0$, proposition 1 can be restated:

Proposition 2.

If the monopolist produces with constant returns to scale, $di_y = 0$. Increasing returns imply $di_y > 0$ and decreasing returns that $di_y < 0$.

As a consequence of proposition 2, the intermediate goods tax would be zero if the monopolist produced with a Leontief technology.

The analysis of perfect competition with decreasing returns has demonstrated that production efficiency is desirable when profits are correctly taxed. To investigate whether this is also true for imperfect competition a profits tax is now introduced. Writing ζ (< 1) for the rate of profit taxation, the demand function now becomes

$$(16) \quad x = f(q_x, w, (1-\zeta)\pi)$$

Repeating the comparative statics analysis

$$(17) \quad \frac{\partial \pi}{\partial t_x} = \frac{-f}{1 - f_3[1 - \zeta] \cdot [q_x - t_x - C_0]} < 0$$

$$(18) \quad \frac{\partial \pi}{\partial i_y} = \frac{-C_1}{1 - f_3[1 - \zeta] \cdot [q_x - t_x - C_0]} < 0$$

$$(19) \quad \frac{\partial q_x}{\partial t_x} = \frac{f_1}{[2f_1 + f_{11} \cdot [q_x - t_x - C_0] - f_1^2 C_{0d}]}$$

$$+ \frac{f \cdot [1 - \zeta] \cdot [f_3 + f_{13} \cdot [q_x - t_x - C_0] - f_3 f_1 C_{0d}]}{[1 - f_3[1 - \zeta] \cdot [q_x - t_x - C_0]] \cdot [2f_1 + f_{11} \cdot [q_x - t_x - C_0] - f_1^2 C_{0d}]}$$

and

$$(20) \quad \frac{\partial q_x}{\partial i_y} = \frac{f_1 C_{01}}{[2f_1 + f_{11} \cdot [q_x - t_x - C_0] - f_1^2 C_{0d}]}$$

$$+ \frac{C_1[1 - \zeta] \cdot [f_3 + f_{13}[q_x - t_x - C_0] - f_3 f_1 C_{0d}]}{[1 - f_3[1 - \zeta]] \cdot [q_x - t_x - C_0] \cdot [2f_1 + f_{11}[q_x - t_x - C_0] - f_1^2 C_{0d}]}$$

Including the revenue from the profits tax, the new revenue constraint is

$$(21) \quad \zeta \pi + x t_x + y i_y = 0$$

and, for a given value of ζ , the differential of this is

$$(22) \quad \left[\zeta \frac{\partial \pi}{\partial t_x} + x \right] dt_x + \left[\zeta \frac{\partial \pi}{\partial i_y} + y \right] di_y = 0$$

Differentiating the utility function and substituting from the above, leads to the result that

$$(23) \quad dV = \frac{\partial V}{\partial q_x} \left[\frac{f_1 C_{01} - \frac{C_1 f_1}{f}}{2f_1 + f_{11}[q_x - C_0] - C_{00} f_1^2} \right] di_y$$

which is precisely (15) once more. Consequently, the characterisation of policy given in proposition 1 is still valid independently of the value of the profit tax. This result can also be established, although by a more circuitous route, by considering the optimal choice of ζ , t_x and i_y : t_x and i_y will be non-zero whenever $f C_{01} - C_1 \neq 0$ whatever the optimal value of ζ . Therefore, if there are decreasing returns, $f C_{01} - C_1 > 0$ a subsidy on intermediate goods will be welfare-improving even if an optimal profits tax can be levied. This is in contrast to the Dasgupta and Stiglitz (1972) result for competitive economies.

The above discussion has illustrated the major factors that will influence the form of welfare-improving tax policy. It is now demonstrated that the same factors are present in the determination of optimal taxes and that a very simple characterisation of optimal taxes can be given.

Requiring, as above, that the government budget should be balanced the optimal tax problem may be written

WM1. Choose i_y, t_x to maximise $V = V(q_x, w, \pi)$ subject to $t_x x + i_y y = 0$

Assuming that the equilibrium values of π and q_x can be treated as differentiable functions of i_y, t_x the necessary conditions for WM1 are

$$(24) \quad \frac{\partial V}{\partial q_x} \cdot \frac{\partial q_x}{\partial t_x} + \frac{\partial V}{\partial \pi} \cdot \frac{\partial \pi}{\partial t_x} - \lambda \left[x + (t_x + i_y \frac{\partial y}{\partial x}) \left(\frac{\partial x}{\partial \pi} \cdot \frac{\partial \pi}{\partial t_x} + \frac{\partial x}{\partial q_x} \cdot \frac{\partial q_x}{\partial t_x} \right) \right] = 0$$

and

$$(25) \quad \frac{\partial V}{\partial q_x} \cdot \frac{\partial q_x}{\partial i_y} + \frac{\partial V}{\partial \pi} \cdot \frac{\partial \pi}{\partial i_y} - \lambda \left[y + (t_x + i_y \frac{\partial y}{\partial x}) \left(\frac{\partial x}{\partial \pi} \cdot \frac{\partial \pi}{\partial i_y} + \frac{\partial x}{\partial q_x} \cdot \frac{\partial q_x}{\partial i_y} \right) \right] = 0$$

Employing the budget constraint, these can be solved to provide an implicit expression for i_y ,

$$(26) \quad i_y = \frac{f^2}{f_1 \cdot [fC_{01} - C_1]}$$

From (26), it can be seen that the characterisation of the sign of i_y presented in proposition 1 also applies to the optimal tax except at $fC_{01} - C_1 = 0$. The characterisation in (26) is invalid at $fC_{01} - C_1 = 0$ as its derivation would involve manipulating expressions that were identically zero. The optimal i_y is thus a discontinuous function of $fC_{01} - C_1$, with the discontinuity occurring at zero. As $fC_{01} - C_1$ tends to zero from above or below the absolute value of i_y will tend to infinity, indicating that "large" taxes are necessary in order to obtain any increase in welfare.

3. MONOPOLISTIC FACTOR PRODUCTION.

In this section the roles of the two industries are reversed. The competitive industry now produces the final good using labour and the monopolists output, subject to a constant returns to scale technology. The monopolist employs labour alone.

Under these assumptions the total costs of the competitive industry, whose output is again labelled y , are

$$(27) \quad C(q_x, w; y) + t_y y = C(q_x, w).y + t_y y$$

As the industry is competitive, equilibrium price must equal marginal cost and demand must equal supply. Hence

$$(28) \quad q_y = C(q_x, w) + t_y$$

and

$$(29) \quad y = f(q_y, w, \pi) = f(C(q_x, w) + t_y, w, \pi)$$

where π is the monopolist's profit. The demand facing the monopolist can be found by applying Shephard's lemma to (27). Thus

$$(30) \quad x = \frac{\partial(C(q_x, w).y + t_y y)}{\partial q_x} = C_1(q_x, w).y = C_1(q_x, w).f(C(q_x, w) + t_y, w, \pi)$$

With demand determined by (30), the monopolist will maximise profits

$$(31) \quad \pi = [q_x - i_x].C_1(q_x, w).f(C(q_x, w) + t_y, w, \pi) - C^m(w; C_1(q_x, w).f(C(q_x, w) + t_y, w, \pi))$$

where $C^m(\cdot)$ is the monopolist's cost function. Proceeding as with (4), the comparative statics are derived from

$$(32) \quad C_1.f + [q_x - i_x - C_0^m].[C_1.f + C_1^2.f_1] = 0$$

$$(33) \quad \pi - [q_x - i_x].C_1.f - C^m = 0$$

Before developing the general case it is worth analysing the model for Leontief technology in the production of the final good. Assuming all production function coefficients to be unity, the cost function of the competitive firm becomes

$$(34) \quad C(q_x, w; y) + t_y y = [q_x + w + t_y].y$$

Using (32) and (33), it can be calculated that

$$(35) \quad \frac{\partial \pi}{\partial t_y} = \frac{f_1 \cdot [q_x - i_x - C_0^m]}{1 - f_3 \cdot [q_x - i_x - C_0^m]} < 0$$

$$(36) \quad \frac{\partial \pi}{\partial i_x} = \frac{-f}{1 - f_3 \cdot [q_x - i_x - C_0^m]} < 0$$

$$(37) \quad \frac{\partial q_x}{\partial t_y} = \frac{-[f_1 + f_{11} \cdot [q_x - i_x - C_0^m] - C_0^m f_1^2]}{[2f_1 + f_{11} \cdot [q_x - i_x - C_0^m] - f_1^2 C_0^m]} \\ - \frac{[f_1 \cdot [q_x - i_x - C_0^m] \cdot [f_3 + f_{13} \cdot [q_x - i_x - C_0^m] - f_3 f_1 C_0^m]}{[1 - f_3 \cdot [q_x - i_x - C_0^m]] \cdot [2f_1 + f_{11} \cdot [q_x - i_x - C_0^m] - f_1^2 C_0^m]}$$

and

$$(38) \quad \frac{\partial q_x}{\partial i_x} = \frac{f_1}{[2f_1 + f_{11} \cdot [q_x - i_x - C_0^m] - f_1^2 C_0^m]} \\ + \frac{f \cdot [f_3 + f_{13} \cdot [q_x - i_x - C_0^m] - f_3 f_1 C_0^m]}{[1 - f_3 \cdot [q_x - i_x - C_0^m]] \cdot [2f_1 + f_{11} \cdot [q_x - i_x - C_0^m] - f_1^2 C_0^m]}$$

Differentiating the indirect utility function,

$$(39) \quad dV = \left[\frac{\partial V}{\partial q_y} \cdot \left[\frac{\partial q_x}{\partial t_y} + 1 \right] + \frac{\partial V}{\partial \pi} \cdot \frac{\partial \pi}{\partial t_y} \right] dt_y + \left[\frac{\partial V}{\partial q_y} \cdot \frac{\partial q_x}{\partial i_x} + \frac{\partial V}{\partial \pi} \cdot \frac{\partial \pi}{\partial i_x} \right] di_x$$

where the fact that $\frac{\partial q_y}{\partial q_x} = 1$ has been used. Noting that the assumptions on technology

technology imply $x = y$ and substituting into (39) from (35) to (38) demonstrates that

$$(40) \quad \left[\frac{\partial V}{\partial q_y} \cdot \left[\frac{\partial q_x}{\partial i_x} - \frac{\partial q_x}{\partial t_y} - 1 \right] + \frac{\partial V}{\partial \pi} \cdot \left[\frac{\partial \pi}{\partial i_x} - \frac{\partial \pi}{\partial t_y} \right] \right] = 0$$

Hence

Proposition 3.

If the competitive final goods industry has Leontief technology, the tax upon intermediate goods will be zero whatever the technology of the monopolistic intermediate good producer.

For the general case it follows that

$$(41) \quad dV = \alpha f \left[B^{-1} [q_x - i_x - C_0^m] C_{11} - A^{-1} \left[C_{00}^m [C_1^2 C_1 f f_1 + (f C_{11})^2] \right. \right. \\ \left. \left. - [q_x - i_x - C_0^m] [f C_{111} + C_1 C_1 f_1 - C_1 f^2 C_1^{-1} + E B^{-1} f C_{11}] \right] \right]$$

where

$$A = 2[C_1 f + C_1^2 f] + [q_x - i_x - C_0^m] \cdot [f C_{111} + 3C_1 f C_1 f_1 + f C_1 f^3] - C_{00}^m [C_1 f + C_1^2 f_1]^2 < 0$$

$$B = 1 + [q_x - i_x - C_0^m] \cdot C_1 f_3 > 0$$

and

$$E = C_1 f_3 + [q_x - i_x - C_0^m] \cdot [C_1 f_3 + f_1 C_1^3] - C_{00}^m [C_1 f + C_1^2 f_1] C_1 f_3$$

From these it is evident that if $C_{11} < 0$, increasing returns to scale for the monopolist, $C_{00}^m < 0$, tend to lead to the intermediate good being subsidised. To simplify further, let $C_{00}^m = 0$. Then

Proposition 4.

If $C_{11} < 0$, $C_{111} > 0$ and i) $A - f C_{11} E < 0$ ii) $(f C_{111} / C_{11}) + C_1 f_1 - (C_{11} f / C_1) < 0$ then $di_x < 0$.

The conditions $C_{11} < 0$, $C_{111} > 0$ are satisfied, for instance, by the Cobb - Douglas technology, condition (i) will be met if E , the profit effect, is small and (ii) when C_1 is large relative to C_{11} . Hence, provided there is some scope for input substitution, the inclusion of intermediate goods in the tax system may be welfare-improving even when all industries have constant returns to scale.

Although this model does not lead to the precise results of the first, the major points are clear. Leontief technologies imply zero taxation, in other cases it is likely that intermediate goods should be brought into the tax system.

4. AN EXTENSION: TWO FINAL GOODS.

The models above have demonstrated that circumstances exist in which taxes should be levied upon intermediate goods. However it has not been demonstrated that such taxes should be discriminatory. This section now seeks to answer the question: should different intermediate goods be taxed at different rates? In order to provide a satisfactory answer to this it is necessary to recall that the Diamond-Mirrlees efficiency lemma is only applicable when optimal commodity taxes are levied and that Newbery (1986) has shown that intermediate goods taxes are desirable when a complete set of commodity taxes are not available. To reflect these facts the model described below also incorporates optimal commodity taxes.

It is now assumed that there are two intermediate goods, each of which is produced by a competitive industry. Each intermediate good is used by one of the two monopolists producing final goods. All production requires labour. The demand facing the first monopolist is $x = f(q, w)$ and, for the second, $y = g(p, w)$; q and p are the two firm's prices. These demands are constructed under the assumptions that the consumer's utility function is additively separable and that the monopolies' profits are not returned to the consumer; the firms may be viewed as being under foreign ownership with the profits being remitted abroad.

The first monopolist seeks to maximise

$$(42) \quad \pi^1 = [q - t].f(q, w) - C^1(\varphi, w; f(q, w))$$

and the second

$$(43) \quad \pi^2 = [p - r].g(p, w) - C^2(\rho, w; g(p, w))$$

where t and r are the commodity taxes and φ and ρ are the prices of the intermediate goods. From (42)

$$(44) \quad \frac{\partial q}{\partial t} = \frac{f_1}{2f_1 + [q - t - C_{01}^1].f_{11} - C_{00}^1.f_1^2} > 0$$

and, from (43),

$$(45) \quad \frac{\partial p}{\partial r} = \frac{g_1}{2g_1 + [p - r - C_{01}^2] \cdot g_{11} - C_{00}^2 g_1^2} > 0$$

Assuming that the choices of q and p can be written as differentiable functions, $q(t)$, $p(r)$, of the tax rates, the optimal commodity tax problem is:

$$CT1: \quad \text{Max}_{r, t} V(q(t), p(r), w) \text{ subject to } xt + yr = 0.$$

The solution to CT1 can be characterised by

$$(46) \quad r = \frac{\alpha xy \left[\frac{\partial q}{\partial t} - \frac{\partial p}{\partial r} \right]}{\left[\frac{\partial V}{\partial q} \cdot \frac{\partial q}{\partial t} \cdot \frac{\partial y}{\partial p} \cdot \frac{\partial p}{\partial r} + \frac{x}{y} \cdot \frac{\partial V}{\partial p} \cdot \frac{\partial p}{\partial r} \cdot \frac{\partial x}{\partial q} \cdot \frac{\partial q}{\partial t} \right]}$$

Provided $f_1 < 0$ and $g_1 < 0$, the numerator is non-zero and r will be zero when $\frac{\partial q}{\partial t} = \frac{\partial p}{\partial r}$;

equality of tax shifting implying that optimal commodity taxes are both zero. This is the situation that is concentrated upon below, in particular it is assumed that at the commodity tax optimum with $t = r = 0$

$$(47) \quad f_1 = g_1, \quad 2f_1 + [q - t - C_{01}^1] \cdot f_{11} - C_{00}^1 f_1^2 = 2g_1 + [p - r - C_{01}^2] \cdot g_{11} - C_{00}^2 g_1^2$$

Now writing i for the tax on the intermediate good used by the first monopolist and j for that on the input of the second, it follows that

$$(48) \quad \frac{\partial q}{\partial i} = \frac{C_{01}^1 f_1}{2f_1 + [q - t - C_{01}^1] \cdot f_{11} - C_{00}^1 f_1^2}$$

and

$$(49) \quad \frac{\partial p}{\partial j} = \frac{C_{01}^2 g_1}{2g_1 + [p - r - C_{01}^2] \cdot g_{11} - C_{00}^2 g_1^2}$$

It is now considered whether any welfare-improving changes in the taxes on intermediate goods can be found starting from the commodity tax optimum. Let any possible pair of changes di , dj satisfy $C_{11}^1 di + C_{11}^2 dj = 0$, then

$$(50) \quad dV = \alpha \left[\frac{C_1^1}{C_1^2} \frac{y \cdot C_0^2 g_1}{2g_1 + [p - r - C_0^1] \cdot g_{11} - C_0^2 g_1^2} - \frac{x \cdot C_0^1 f_1}{2f_1 + [q - t - C_0^1] \cdot f_{11} - C_0^1 f_1^2} \right] di$$

However, employing (47)

$$(51) \quad dV = \alpha S \left[\frac{C_1^1}{C_1^2} y \cdot C_0^2 - x \cdot C_0^1 \right] di$$

where $S = \frac{\partial q}{\partial t} = \frac{\partial p}{\partial r}$. It can be seen from (51) that it may be possible to find welfare-

improving discriminatory intermediate goods taxes starting from the commodity tax optimum. This is stated as proposition 5.

Proposition 5.

If $D = \frac{y \cdot C_0^2}{C_1^2} - \frac{x \cdot C_0^1}{C_1^1} \neq 0$ it is possible to increase welfare by the use of discriminatory

intermediate goods taxes.

$\frac{x \cdot C_0^1}{C_1^1}$ can be interpreted as the elasticity of input demand with respect to output

for firm 1 (and respectively for 2). If both monopolists have constant returns to scale then the elasticity is equal to unity for both and $D = 0$: no welfare-improving changes will exist. If a single firm deviates from constant returns, then changes will exist. Furthermore, the input to the firm with the greater elasticity of input demand or, equivalently for well-behaved technologies, the lower returns to scale, should be subsidised.

5. CONCLUSIONS.

It has become common in the literature to assume that intermediate goods should remain untaxed. The analysis above was addressed to questioning the validity of this assumption in the presence of imperfect competition. The major result of the paper has been to establish that there is a strong case for including intermediate goods in the tax

system. The only general exception to this rule appears to be the case of Leontief technology.

The analysis of this paper was intended only to demonstrate how taxes on intermediate goods could be motivated and, as such, was illustrative rather than definitive. Some of the difficulties in reaching a broader set of conclusions, particularly the lack of a general theory of imperfect competition with intermediate goods, are discussed in Myles (1989).

The precise characterisations of taxes derived for the three models are all closely dependent upon the precise organisation of production and the nature of technology, this limits the scope for drawing general conclusions. However it has been demonstrated that when intermediate good production is monopolised, taxation can improve welfare even with constant returns to scale in all industries provided some input substitution can occur. In addition, the results have also indicated that inputs to the firms with lower returns to scale should bear lower taxes.

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