Training and Contracts:

Some Dynamic Aspects of Human Capital Formation.

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Abstract: A contracts-based model of the provision of training and employment with overlapping generations of workers is developed. Attention is focused on the role of imperfect capital markets in the determination of contracts and on intertemporal variations in the levels of training and employment, these may exhibit cyclical or chaotic behaviour.

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1. Introduction

Since the seminal work of Becker (1975), the distinction between specific and general training has been emphasised as the determinant of the level and structure of returns to workers. Becker (1975) reasons that the gains to general training will be appropriated by the workers and those from specific training by the firm. In addition, it is implicit in the literature that the intertemporal development of the stock of human capital will follow some stable adjustment path. Appealing as these conclusions may be, recent theoretical developments have exposed some of the weaknesses of their foundations.

The literature on contract theory, particularly Holmstrom (1983), has highlighted that long-term relationships between firms and workers are typically characterised by the development of implicit contracts. The provision of training and its future impact in increased productivity fall naturally into a contract framework: by entering into contracts, firms can guarantee their investment in human capital will be realised and workers can ensure low returns during training will be compensated in the future. The analysis below will emphasise this aspect and both the level of training, and the returns from it, will be determined contractually. Secondly, since Stiglitz and Weiss (1981), the existence of perfect capital markets, an implicit assumption in much of human capital theory, has been extensively questioned. Consequently, the relationship between the relative values of discount factors and the structure of contracts is also analysed. When workers and firms face different market rates of interest, there are possibilities for income to be transferred between periods, via the contract, at rates not available on the market. As demonstrated below, the structure of agreed contracts is determined jointly by the level and form of training and the functioning of the capital market. A similar argument to this, but in a model with a different structure, has also been made recently by Aziariadis (1988).

The provision of training is not something that the firm undertakes once. Invariably a new generation of workers enters the firm each year, undergos training and are then productively employed. In addition, the level of training given to one generation influences the cost of training the next: there may be fixed investment in training schools, in providing the teaching staff and economies in apprenticeships. These intertemporal features are modeled below by adopting an "overlapping generations" framework that allows the levels of training and employment to be charted over time. The analysis is conducted under the assumption of bounded rationality which is interpreted as the firm ignoring some of the impact of present training upon future profits. The value of this analysis is to demonstrate the variety of forms that the intertemporal human capital formation process may take and to isolate the features that determine which form is taken. The literature surveyed by Rosen (1987) is essentially static in nature and implicitly suggests that investments in human capital will, in the absence of any changes in the underlying economic environment, remain constant over time. As demonstrated below however, the natural intertemporal linkages involved with the training process can potentially lead to the level of human capital displaying complicated dynamic behaviour.

Section 2 introduces the model and focuses on the determination of contracts in a two-period model with a single generation of workers. The importance of discount factors is highlighted. The model is then generalised to an overlapping generations framework in section 3, with the firm assumed to have bounded rationality. Section 4 presents the conclusions.

2. Static analysis

This section presents an analysis of a static model in which a single generation of workers are trained in the first period and are productively employed in the second. Beyond these two periods there is no history and no future. This analysis serves several purposes: it introduces the model and notation, highlights the importance of discount rates and outside opportunities in the determination of contracts and provides a key result that will be employed in the following section.

I concentrate upon a single firm acting in competitive product and labour markets. The firm produces a single output, in quantity y, using labour alone (or has a fixed stock of physical capital) subject to the production function

$$y = f[\, n, t \,], \tag{1}$$

where n represents the labour employed and t the training given to each worker. The form of f[n, t] will be restricted further below but it will be taken throughout to be C^r, $r \geq 4$. Training costs are given by a cost function

$$c[\, n, t \,], \tag{2}$$

c[n, t] is also assumed to be C^r.

To maximise profits the firm offers n workers a wage and training contract described by the triple $\{ t, w_1, w_2 \}$ where w_1 is the wage received in the first period during training and w_2 is the wage received after training. The contract is considered to be binding for the firm and once offered it must stand by its terms. In contrast, the workers have freedom to break the contract after the first period; they cannot be forced to work for the firm.

The choice of contract by the firm is constrained by the opportunities available for the workers on the competitive outside labour market. I assume that for untrained labour there is available employment that pays a wage $\overline{w}$. Workers who undergo training will typically receive a higher wage on the outside market, the rate at which this wage increases with the level of training is determined by how 'general' is the training. To permit the discussion of alternative possibilities, let the outside value of a trained worker in period 2 be determined as a function g of t, where

$$g[\, t \,] \geq \overline{w}, \;\; g[\, 0 \,] = \overline{w}, \; g'[\, t \,] \geq 0. \tag{3}$$

If $g'[t] = 0$ for all t, the training can be viewed as entirely firm specific. Furthermore, I adopt the convention that $0 \leq g'[t] \leq 1$ with $g'[t] = 1$ representing an increase in training which is purely general.

The constraints upon the contract can now be derived. Firstly, since trained workers always have the option of leaving the firm, the second period wage must be at least equal to that attainable outside. Hence

$$w_2 \geq g[t] \geq \overline{w}. \tag{4}$$

Secondly, a worker will only take on the contract if its discounted value is at least equal to what can be obtained without undertaking training:

$$w_1 + \rho w_2 \geq (1 + \rho)\overline{w}, \tag{5}$$

where ρ is the discount factor for workers.

Writing δ for the discount factor of the firm and normalising the price of its output at 1, the contract offered by the firm is the solution to:

$$\max_{\{n,\, t, w_1, w_2\}} \pi = \delta.f[n, t] - c[n, t] - nw_1 - \delta n w_2, \tag{6}$$

subject to

$$w_2 \geq g[t] \geq \overline{w}, \;\; w_1 + \rho w_2 \geq (1 + \rho)\overline{w}, \; w_1 \geq 0.$$

The structure of this problem is discussed in the appendix.

To understand the solution to (6), first consider the form of the constraint set. For a given choice of t, w_1 and w_2 are chosen to minimise $w_1 + \delta w_2$ subject to the constraints being satisfied. If the chosen t satisfies

$$g[\,t\,] < \frac{\overline{w}.(1+\rho)}{\rho}, \tag{7}$$

(which occurs with certainty if the training is entirely specific) then three situations can arise as shown in figure 1.

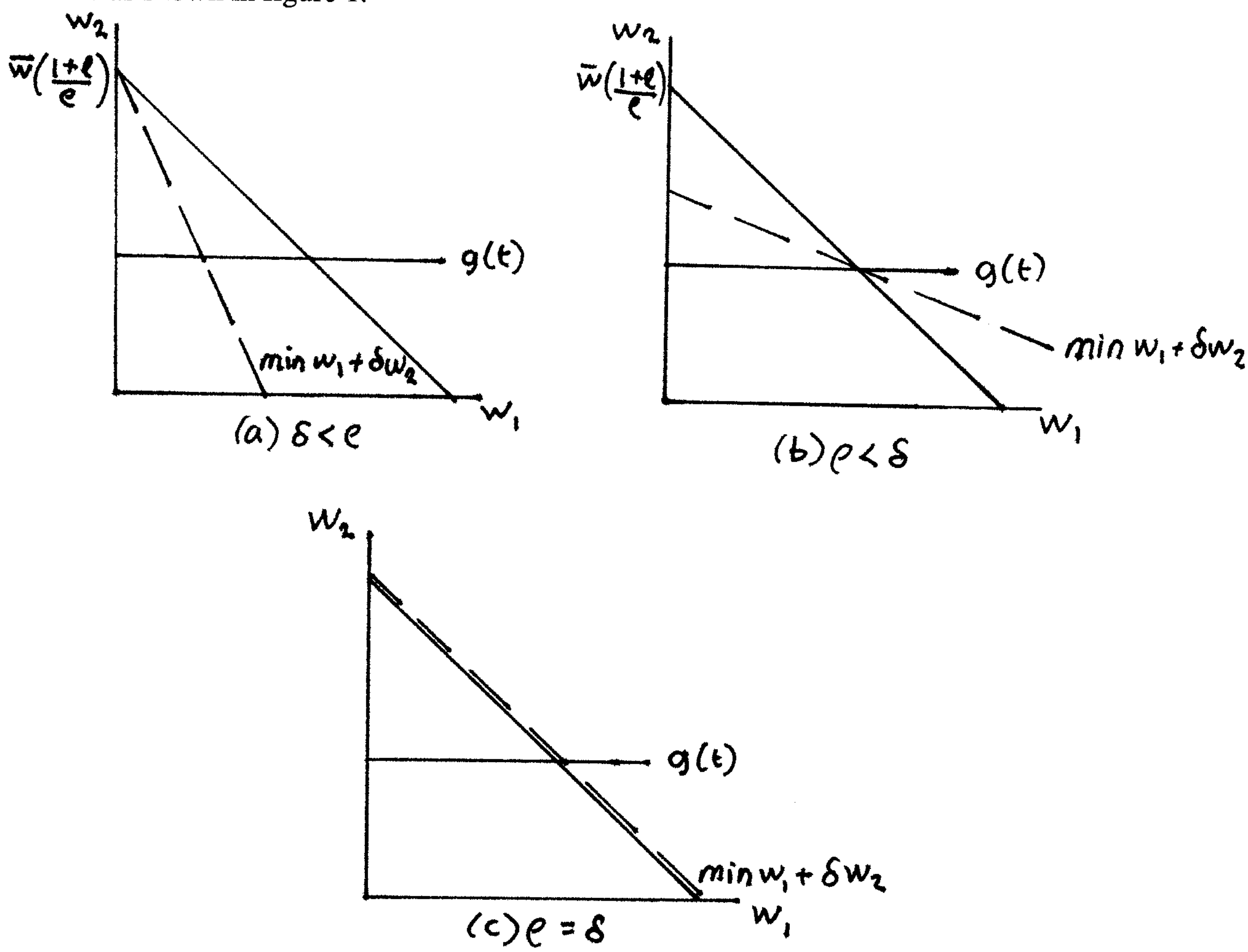

Figure 1.

In case (a) the firm discounts future receipts more highly than the workers and prefers to pay the full discounted value of the contract in the second period. It should be noted that the contract wage in the second period is strictly greater than the market clearing wage so even with purely specific training the firm may pay trained workers strictly more than their outside value. This occurs as the workers can transfer earnings forward via the firm at a rate preferable to that offered on the outside market. The converse occurs in (b): the firm would prefer to pay the workers entirely in the first period but is constrained from doing so by the second-period restriction. When $\rho = \delta$, the precise timing of payments is indeterminate. Finally, note that in all three cases the total value of the contract is equal to the discounted value of the outside option.

When

$$g[t] > \frac{\overline{w}.(1+\rho)}{\rho},$$

the second-period constraint always bites and $w_1 = 0$. See figure 2.

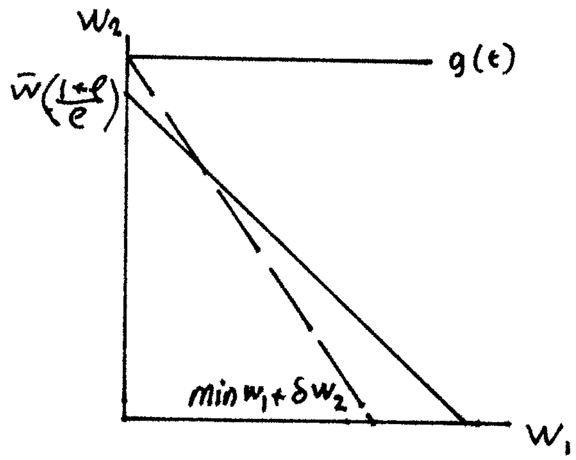

Figure 2.

Introducing multipliers μ and λ for the two constraints, the Kuhn-Tucker conditions for the maximisation are

$$\begin{gathered}
\delta f_n = c_n + w_1 + \delta w_2, \\
\delta f_t = c_t + \mu g', \\
\lambda - n \leq 0,\ w_1(\lambda - n) = 0, \\
\mu + \lambda\rho = \delta n, \\
\mu(w_2 - g) = 0,\ \lambda(w_1 + \rho w_2 - (1+\rho)\overline{w}) = 0, \\
\mu \geq 0,\ \lambda \geq 0,\ w_1 \geq 0.
\end{gathered}$$

When the optimal choice of t is such that (7) is satisfied, if $\rho > \delta$ (case (a)) these provide the characterisation

$$\begin{gathered}
w_1 = 0,\ w_2 = \frac{(1+\rho)}{\rho}\overline{w}, \\
\delta f_n = c_n + \frac{\delta}{\rho}(1+\rho)\overline{w}, \\
\delta f_t = c_t.
\end{gathered}$$

All wages are paid in the second period, the marginal revenue product of labour is equated to the sum of training and wage costs and that of training to the cost of training alone. It should be noted that the second period wage is strictly above the value of the outside option.

If $\rho < \delta$ then $\mu > 0$ and $\lambda = n > 0$. The solution is described by

$$\begin{gathered} w_1 = (1+\rho)\overline{w} - \rho g,\ w_2 = g, \\ \delta f_n = c_n + (1+\rho)\overline{w} + (\delta-\rho)g, \\ \delta f_t = c_t + (\delta-\rho)ng'. \end{gathered}$$

These conditions embody the cost of equipping the trained workers with a marketable skill: the requirement that the second-period constraint must be satisfied adds to the marginal cost of both training and employment. Note that if all training is specific, $g = \overline{w}$, $g' = 0$, these conditions reduce to

$$\begin{gathered} w_1 = \overline{w}\ ,\ w_2 = \overline{w}, \\ \delta f_n = c_n + (1+\delta)\overline{w}, \\ \delta f_t = c_t. \end{gathered}$$

Hence with specific training the workers, when they are more impatient than firms, receive their outside value in both periods. If $\rho = \delta$, the constraint and the objective are coincident and the contract is at some point on their intersection.

When the return to training, and the level of training chosen, are such that (7) is violated the solution becomes

$$\begin{gathered} w_1 = 0\ ,\ w_2 = g > \frac{(1+\delta)}{\delta}\overline{w} > (1+\delta)\overline{w}, \\ \delta f_n = c_n + \delta g, \\ \delta f_t = c_t + \delta n g'. \end{gathered}$$

In these circumstances the marginal revenue products are equated to a marginal cost that incorporates the cost of the second-period constraint. Workers are paid more than the discounted value of the outside option and hence there is a net gain to them from the training. For these conditions to be satisfied, there must be some element of generality about the training.

These results illustrate the importance of discount factors in determining the timing of payments. When the discount factor of the workers is greater than that of the firm, the workers will be paid a second period wage strictly greater than their outside value regardless of the degree of generality of the training, provided their outside value is less than the total discounted value of the no-training option. In this circumstance, the existence of the contract prevents outside competition from eliminating the differential. The total return to the workers of the contract is only greater than the value of working with no training when the level of training given is sufficient to raise the value of the trained worker above the discounted value of the no-training option. In all other cases, regardless of whether the training is specific or general, the firm will appropriate the return from providing the training.

3. A first-order dynamical system

One of the major features of training is that it is not a single event in the life of the firm but is repeated through time. Each generation of trained workers eventually leaves the firm and its replacement must be given the necessary training. As such, the provision of training and employment is best understood by viewing it as a dynamic process in which the present is linked to the future by the stock of skills that exist within the firm and the cost at which they can be passed on from one generation of the workforce to the next.

Once intertemporal links are admitted, the objective of the firm becomes the maximisation of discounted profits over its remaining (and possibly infinite) lifetime. However, as the generations are linked through time, in a manner made precise below, the maximisation involved is not intertemporally separable and cannot be analysed as a sequence of single-period problems. To cut through some of the linkages, but without losing the important intertemporal features, this section assumes that the firm chooses its workforce, and its provision of training in each period, conditional on past choices but without taking account of the effect of this choice upon future periods. This reflects bounded rationality on the part of the firm and, arguably, is probably not too unrepresentative of reality.

The intertemporal links are introduced by assuming that the greater the number of existing trained workers, the lower is the cost of providing training for the next generation. This can reflect either that, for a given number of trainees, skills can be passed to them from the trained workers with less cost the greater the number of existing workers or that previous training has lead to durable investments in training schools or programs which can be employed in future training. Specifically, the training cost function becomes

$$c[\ n_s,\ t_s,\ n_{s-1}\ ]\ , \tag{8}$$

where s represents the time period and

$$c_N \equiv \frac{\partial c}{\partial n_{s-1}} < 0. \tag{9}$$

To provide the analysis with further structure, it is also assumed that $\delta \geq \rho$, a reflection of the fact that workers typically face market rates of interest at least as great as those faced by the firms, and that there is no t_s such that $g[\ t_s\ ] \geq \left(\frac{1+\rho}{\rho}\right)\overline{w}$. As demonstrated above, under these conditions the second period wage of each generation is equal to their outside value and the present value of the contract is $(1+\rho)\overline{w}$. Hence

$$w_2^s = g[\ t_s\ ],$$
$$w_1^s = (1+\rho)\overline{w} - \rho g[\ t_s\ ],$$

with the superscript denoting the date of entry into the firm.

In any period, s, the objective of the firm is

$$\max_{\{\ n_s,\ t_s\ \}}\ \pi = \delta f[\ n_s,\ t_s\ ] - c[\ n_s,\ t_s,\ n_{s-1}\ ] - n_s((1+\rho)\overline{w} + (\delta-\rho)g[\ t_s\ ]).$$

It will be assumed throughout that $\pi[\, n_S, t_S, n_{S-1} \,]$ is strictly concave in $\{n_S, t_S\}$. The maximisation generates the first-order conditions

$$\delta f_n[\, n_S, t_S \,] - c_n[\, n_S, t_S, n_{S-1} \,] - ((1+\rho)\overline{w} + (\delta-\rho)g[\, t_S \,]) = 0\,, \tag{10}$$

$$\delta f_t[\, n_S, t_S \,] - c_t[\, n_S, t_S, n_{S-1} \,] - n_S(\delta-\rho)g'[\, t_S \,]) = 0. \tag{11}$$

The next step is to solve (11) for t_s as a function of n_s and n_{s-1}. The following lemma provides conditions for when this is possible and describes the solution.

Lemma 1.

If

i) $\lim_{t_s \to 0} f_t[\, n_s, 0 \,] = \infty$ all n_s and $\lim_{t_s \to 0} c_t[\, n_s, 0, n_{s-1} \,]$, $\lim_{t_s \to 0} g'[\, 0 \,]$ are finite,

ii) $\lim_{t_s \to \infty} c_t[\, n_s, t_s, n_{s-1}] = \infty$ and $\lim_{t_s \to \infty} f_t[\, n_s, t_s \,]$, $\lim_{t_s \to \infty} g'[\, t_s \,]$ are finite,

iii) $c_{tt} > 0$, $f_{tt} < 0$ and $g'' \geq 0$,

then (11) implies

$$t_s = k[\, n_s, n_{s-1}]$$

where k is C^r.

Proof.

See appendix.

In what follows, the derivatives of k with respect to its first and second arguments respectively will be denoted by k_n and k_N.

Substituting for t_s in (10), the choice of n_s conditional upon n_{s-1} is determined by

$$\delta f_n[n_S, k[n_S, n_{S-1}]\,] - c_n[n_S, k[n_S, n_{S-1}\,], n_{S-1}] - ((1+\rho)\overline{w} + (\delta-\rho)g[k[n_S, n_{S-1}]\,]) = 0$$

or

$$b[\, n_S, n_{S-1} \,] = 0 \,. \tag{12}$$

Lemma 2 states conditions under which (12) can be solved to express n_s as a differentiable function of n_{s-1}.

Lemma 2.

If

iv) $\lim_{n_s \to 0} f_n[\, 0, t_s \,] = \infty$ and $\lim_{n_s \to 0} c_n[\, 0, t_s, n_{s-1} \,]$ is finite all t_s, n_{s-1},

v) $\lim_{n_s \to \infty} c_n[\, n_s, t_s, n_{s-1}] = \infty$ and $\lim_{n_s \to \infty} f_n[\, n_s, t_s \,]$, is finite, all t_s, n_{s-1},

vi) $c_{nn} > 0$, $f_{nn} < 0$,

then (12) implies

$$n_s = v[\, n_{s-1}]$$

where v is C^r.

Proof.

See appendix.

Combining $k[\,\cdot\,]$ and $v[\,\cdot\,]$, the final characterisation of equilibrium is given by

$$t_S = k[\, n_S, n_{S-1} \,] = k[\, v[n_{S-1}], n_{S-1} \,] = w[\, n_{S-1} \,] \tag{13}$$

and

$$n_S = v[\, n_{S-1} \,] \,. \tag{14}$$

From (14), the evolution of n_S is generated by a first-order dynamical system and each realisation of n_{S-1} implies a value of t_S. It is also evident that (i) and (iv) imply

$w[\,0\,] > 0, \;\; v[\,0\,] > 0.$

To understand the behaviour away from 0 it is helpful to calculate the derivatives of v and w. These are given by

$$\frac{\partial t_s}{\partial n_{s-1}} \equiv w' = \frac{c_{tN}(\delta f_{nn} - c_{nn}) - c_{nN}(\delta f_{tn} - c_{tn} - (\delta\text{-}\rho)g')}{(\delta f_{nn} - c_{nn})(\delta f_{tt} - c_{tt} - n_s(\delta\text{-}\rho)g'') - (\delta f_{tn} - c_{tn} - (\delta\text{-}\rho)g')^2}, \tag{15}$$

and

$$\frac{\partial n_s}{\partial n_{s-1}} \equiv v' = \frac{c_{nN}(\delta f_{tt} - c_{tt} - n_s(\delta\text{-}\rho)g'') - c_{tN}(\delta f_{tn} - c_{tn} - (\delta\text{-}\rho)g')}{(\delta f_{nn} - c_{nn})(\delta f_{tt} - c_{tt} - n_s(\delta\text{-}\rho)g'') - (\delta f_{tn} - c_{tn} - (\delta\text{-}\rho)g')^2}. \tag{16}$$

The denominator of (15) and (16) is negative due to the maintained assumption of strict concavity of the profit function. In addition, a further natural restriction on costs is to assume

vii) $c_{nN} < 0, \quad c_{tN} < 0.$

Hence increased training in the previous period lowers marginal cost of increasing the number of trainees and the level of training in the next period. It is also assumed that

viii) $f_{tn} > 0, \quad c_{tn} \leq 0.$

These restrictions now permit analysis of the evolution of t_s and n_s.

It can be seen from (15) and (16) that the important variable is the value of g'. When this is close to zero both derivatives are unambiguously positive, as g' increases they may eventually become negative. This suggests treating the two possibilities g' = 0 and g' > 0 separately.

Firstly, consider the case where $g' \equiv 0$, that is, all training is firm specific. It follows that $w' \geq 0$ and $v' \geq 0$. In these circumstances, the system will converge to an equilibrium only if v' is eventually bounded below 1. If this is satisfied, the time evolution is as shown below. There is convergence to the unique fixed point of the mapping v and this point is stable. In addition, t_s also converges to its equilibrium value. The equilibrium value of n is determined implicitly by $n^* = v[n^*]$ and that of t by $t = w[n^*]$.

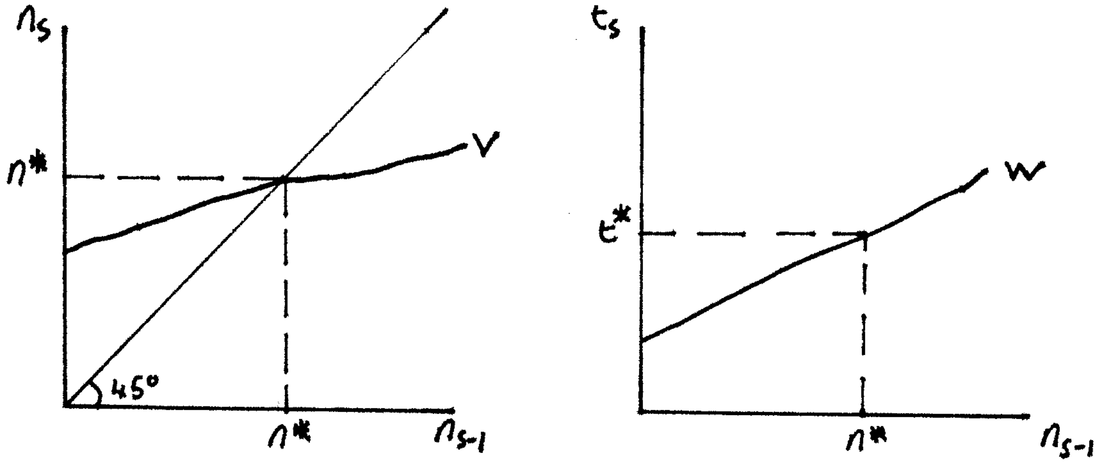

Figure 3.

Now consider $g' \neq 0$ and assume that the right-derivative of v and w is positive at $n_{s-1} = 0$. Four possible cases can occur as below:

a) w and v are monotonically increasing, $w' \geq 0$, $v' \geq 0$.

This occurs when g' never becomes sufficiently large to reverse the sign of the derivatives. The analysis of this case is the same as that for $g' = 0$ above.

b) v monotonic, $w' > 0$ for $n_{s-1} < n_{s-1}'$, $w' < 0$ for $n_{s-1} > n_{s-1}'$.

Here w has a single maximum. As for (a), if $v' < 1$ then n_s will converge to the unique and stable fixed point n^*. If $n_{s-1}' > n^*$, t_s will also increase monotonically. With $n_{s-1}' < n^*$, t_s will first increase and then decrease.

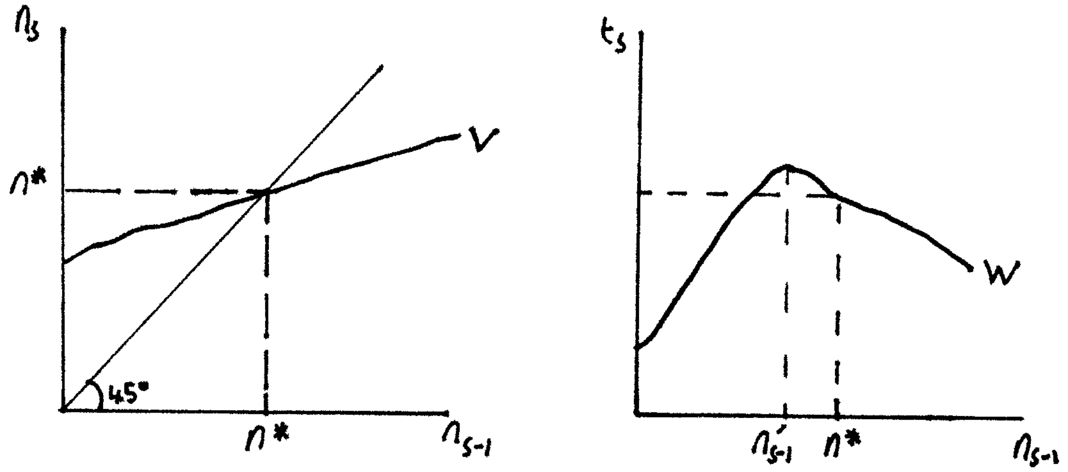

Figure 4.

c) w monotonic, $v' > 0$ for $n_{s-1} < n_{s-1}''$, $v' < 0$ for $n_{s-1} > n_{s-1}''$.

Define n^{max} by $n^{max} = v[\, n_{s-1}''\,]$. v can now be viewed as a mapping from $[0,n^{max}]$ to $[0,n^{max}]$. It will thus have at least one fixed point (which may be unstable). Furthermore, there may be periodic cycles in t_s and n_s or chaotic behaviour (see Baumol and Benhabib (1989) and Collet and Eckmann (1984)).

If the possible forms of behaviour of this system are to be restricted it would be necessary to demonstrate that it has further properties such as possessing a negative Schwarzian derivative

$$\frac{v'''}{v'} - \frac{3}{2}\left(\frac{v''}{v'}\right)^2 < 0.$$

However, in the present context assuming v to have a negative Schwarzian derivative would place restrictions upon the fourth derivatives of f, c and g. Although the assumptions guarantee that they exist, the conditions certainly have no economic content. Therefore, despite the number of strong regularity assumptions that have been invoked in proceeding this far, there is no guarantee that the system will not display chaotic or periodic behaviour as opposed to smooth convergence to equilibrium. Finally, t_s will always be positively correlated to n_s.

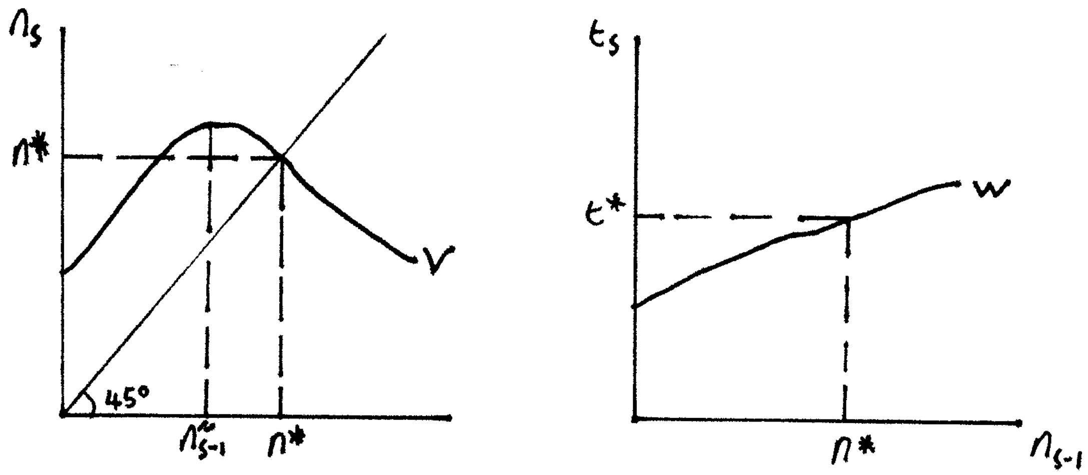

Figure 5.

d) v, w both have turning points.

This case can be ruled out by further assumptions but, if it does occur, it will lead to behaviour similar to that of (c) except that, if the turning points occur at different values of n_{s-1}, there will be regions where t_s and n_s.will be negatively correlated.

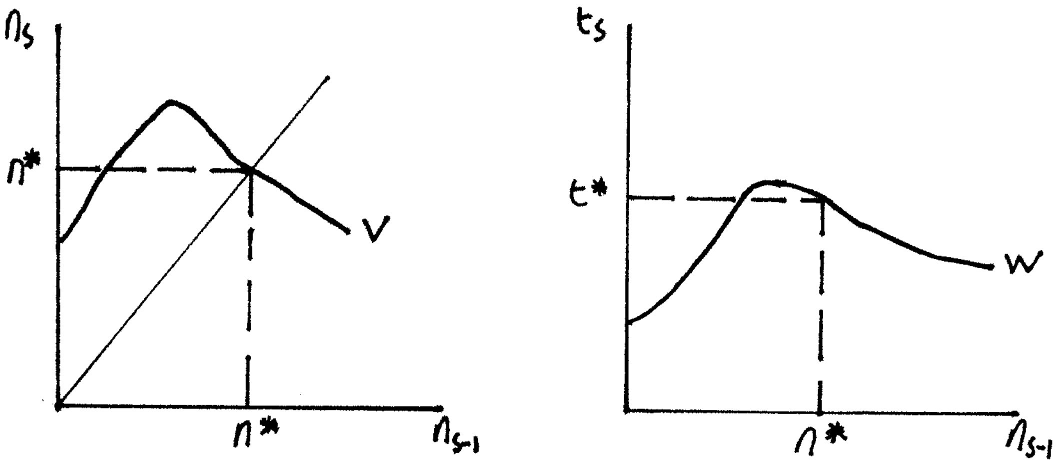

Figure 6.

This section has challenged the presumption that investment in human capital will remain constant in the absence of shocks or changes to the economy. If training is entirely specific there should be convergence to steady state values. In contrast, general training may lead to cyclical or chaotic behaviour and both negative and positive correlations may be observed between the levels of employment and training at different points in the cycle. This distinction can be explained by recalling that with specific training an increase in training in one period reduces costs in the next and hence a steady convergence occurs towards equilibrium but with general training there is both this cost saving and a cost increase due to raised wages on outside markets. This external cost increase can eventually lead to further training reducing the marginal profitability of increased employment (and vice versa), if this effect becomes sufficiently strong the downturn that generates the cycles will then occur. None of these features are apparent in static models.

4. Conclusions

This paper has considered some modifications of the standard assumptions of the human capital literature. The introduction of contracts permits trades that are not feasible without contracts, in some cases these allow wages to be set above the levels determined on competitive "outside" labour markets. Imperfect capital markets have been demonstrated to have implications for the structure of contracts, especially the timing of payments. The intergenerational linkages involved in the training process generate first-order dynamical systems that govern the intertemporal evolution of training and employment levels. This illustrates some of the weaknesses of using static models to analyse the formation of human capital, a process that is essentially dynamic and intertemporal.

Appendix.

There are two difficulties involved in the maximisation described by (6): a) the objective function is not concave and b) the constraint set need not be convex. However, these problems can be overcome by exploiting the inherent separability.

For any choice of t, w_1 and w_2 will be chosen to minimise the cost of satisfying the contractual constraints. Noting this, it is possible to proceed as follows:
Define h[t] to be the cost of satisfying the contract where
1) For $\delta \geq \rho$

$$h[\,t\,] = (1+\rho)\overline{w} + (\delta\text{-}\rho)g[\,t\,],\ \ g[\,t\,] \leq \overline{w}\left(\frac{1+\rho}{\rho}\right)$$

$$h[\,t\,] = \delta g[\,t\,],\ g[\,t\,] > \overline{w}\left(\frac{1+\rho}{\rho}\right)$$

2) For $\delta < \rho$

$$h[\,t\,] = \delta\left(\frac{1+\rho}{\rho}\right)\overline{w},\ \ g[\,t\,] \leq \overline{w}\left(\frac{1+\rho}{\rho}\right)$$

$$h[\,t\,] = \delta g[\,t\,],\ g[\,t\,] > \overline{w}\left(\frac{1+\rho}{\rho}\right)$$

h[t] is continuous and is differentiable everywhere except at t such that $g[\,t\,] = \overline{w}\left(\frac{1+\rho}{\rho}\right)$ but, since this is a set of measure zero, it should not cause any difficulties

The maximisation can now be redefined as:

$$\max_{\{\,n,\,t\,\}}\ \pi = \delta f[\,n,t\,] - c[\,n,t\,] - n.h[\,t\,] \qquad \text{(A1)}$$

By suitable assumptions on the value of h' relative to f_{nt} and c_{nt}, this is easily restricted to be a concave (unrestricted) maximisation. Furthermore, differentiation of (A1) and use of the alternative definitions of h[t] give the characterisations obtained by the use of Kuhn - Tucker in the text.

Proof of Lemma 1.

Write (11) as $d[\,n_s, t_s, n_{s-1}]$, then for all n_s, n_{s-1} (i) implies that $d \to \infty$ as $t_s \to 0$ and (ii) that $d \to -\infty$ as $t_s \to \infty$. $\therefore \exists\ t_s^*$ such that $d[\,n_s, t_s, n_{s-1}] = 0$. t_s^* is also unique since if both t_s' and t_s'' satisfy (11), with $t_s' < t_s''$, then from (iii) $f_t[\,n_s, t_s'] > f_t[\,n_s, t_s'']$, $c_t[\,n_s, t_s', n_{s-1}] < c_t[\,n_s, t_s'', n_{s-1}]$ and $g'[\,t_s'\,] \leq g'[\,t_s''\,]$, so $d[\,n_s, t_s'', n_{s-1}] < d[\,n_s, t_s'', n_{s-1}]$ contradicting the claim that they were both solutions. $\therefore t_s = k[\,n_s, n_{s-1}]$, with k one-to-one. The continuity of k is implied by the differentiability of f, c and g. Moreover, from Theorem C.3.2. of Mas-Collel (1985), as $\frac{\partial g}{\partial t_s} = \delta f_{tt} - c_{tt} - n_s(\rho-\delta)\,g'' \neq 0$ by (iii), k is C^r. ●

Proof of lemma 2.

From (iv) and (v) it follows that $b[\,0, n_{s-1}] > 0\ \forall\ n_{s-1}$ and that $\lim_{n_s\to\infty} b[\,n_s, n_{s-1}] < 0\ \forall\ n_{s-1}$. In addition, $\frac{\partial b}{\partial n_s} = [\,\delta f_{nn} - c_{nn}] + k_n[\,\delta f_{nt} - c_{nt} - (\delta\text{-}\rho)g'] < 0$ by using (vi), the strict concavity of the profit function and the fact that $k_n = -\frac{[\,\delta f_{nt} - c_{nt} - (\delta\text{-}\rho)g']}{[\,\delta f_{tt} - c_{tt} - n_s(\delta\text{-}\rho)g'']}$. $\therefore \forall\ n_{s-1}$, $b[\,0, n_{s-1}] = 0$ has a unique solution. By the argument of lemma 1, this solution can be written $n_s = v[n_{s-1}]$ with v being C^r. ●

Full rationality

To complement the analysis of bounded rationality in the main text a brief note is now presented on the model with full rationality. To simplify the notation define, as above,

$$h[t_s] = (1+\rho)\overline{w} + (\delta-\rho)g[t_s]$$

With this notation, the maximisation facing the firm is

$$\max_{\{n_s,t_s\},s=1,\ldots} \pi = \sum_{s=1}^{\infty} \left(\delta^s f[n_s,t_s] - \delta^{s-1}c[n_s,t_s,n_{s-1}] - \delta^{s-1}n_s h[t_s]\right),$$

with $n_0 = 0$.

As written, this is simply a discounted maximisation problem. However, it is preferable to place this into a dynamic programming framework so that a standard theorem can be appealed to. Let n_s and t_s be the control variables and let x_s be the state variable. The behaviour of x_s is governed by the transition equation

$$x_s \le n_{s-1},\ s = 1,\ldots$$

The programming problem then becomes

$$\max_{\{n_s,t_s\},s=1,\ldots} \pi = \sum_{s=1}^{\infty} \left(\delta^s f[n_s,t_s] - \delta^{s-1}c[n_s,t_s,x_s] - \delta^{s-1}n_s h[t_s]\right),$$

subject to $x_s \le n_{s-1},\ s = 1,\ldots$ and $n_0 = 0$.

Introducing multipliers $\lambda_0, \lambda_1,\ldots$ for the constraints, the Lagrangean is

$$\mathbf{L} = \sum_{s=1}^{\infty} \left(\delta^s f[n_s,t_s] - \delta^{s-1}c[n_s,t_s,x_s] - \delta^{s-1}n_s h[t_s]\right) + \sum_{s=1}^{\infty} \lambda_{s-1}(n_{s-1} - x_s)\,. \tag{A2}$$

Hence

$$\frac{\partial \mathbf{L}}{\partial n_s} = \delta^s f_n - \delta^{s-1}c_n - \delta^{s-1}h + \lambda_s = 0,\ s = 1,\ldots \tag{A3}$$

$$\frac{\partial \mathbf{L}}{\partial t_s} = \delta^s f_t - \delta^{s-1}c_t - \delta^{s-1}n_s h' = 0,\ \ s = 1,\ldots \tag{A4}$$

$$\frac{\partial \mathbf{L}}{\partial x_s} = -\delta^{s-1}c_N + \lambda_{s-1} = 0,\ s = 1,\ldots \tag{A5}$$

As $c_N \neq 0$, (A5) implies $\lambda_{s-1} > 0$ and, as expected given the structure of the payoff function, the constraint is met with equality in each period. Stepping (A5) one period ahead

$$\lambda_s = \delta^s c_N,$$

which, substituted into (A4) gives

$$\delta^s f_n - \delta^{s-1}c_n - \delta^{s-1}h + \delta^s c_N = 0, \tag{A6}$$

where it should be noted that c_N is evaluated at n_{s+1}, t_{s+1}, n_s. (A4) and (A6) characterise the solution to the original maximisation problem.

The dynamic programming framework provides the following result. If it is assumed that:

B. $f[n_s,t_s] - \delta c[n_s,t_s,n_{s-1}] - \delta n_s h[t_s]$ is bounded,

then it is possible to state:

Lemma 3.

The solution of (A2) is given by a time invariant policy rule

$n_s = \mu_1[n_{s-1}]$,

$t_s = \mu_2[n_{s-1}]$.

Proof.

Directly from proposition 2 of Bertsekas (1976, p.229).●

Lemma 3 characterises the form of the solution but no further results have been obtained on its properties.

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