NORMATIVE ASPECTS OF STATE-CONTINGENT CAPITAL INCOME TAXATION

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Abstract

State contingent and asset specific capital income taxes are studied within the framework of a two-period, two-asset and two-state model. Distortions of savings and choice of portfolio in the presence of uncertainty are discussed. Important properties of the taxes are exposed. Conditions for having no portfolio distortion at the tax optimum are established.

1. Introduction

Analysis of taxation is an issue with several aspects. This is true under certainty and even more so when aspects of uncertainty are added. In general there is a positive approach, analysing the effects of taxes on economic behaviour and economic equilibria, and a normative approach, analysing welfare effects and choice of taxes according to a social objective. Most contributions to the literature on capital income taxation and risk-taking have been positive analyses of the effects of taxes on portfolio and savings decisions. Early modern articles were Mossin (1968), Stiglitz (1969) and Sandmo (1969). Within the welfare sphere a main distinction can be made between the study of taxes imposed for fiscal purposes and the study of tax-transfer devices as a possible remedy for market imperfections in the absence of such intervention (the Pigouvian aspect). In the presence of uncertainty instances of market failure may be attributed to the existence of risk. For instance the lack of insurance markets is frequently cited as an example of incomplete markets. An analysis of taxes and transfers as an insurance device was presented in Varian (1980). Examples of other relevant contributions which are less explicitly tax-oriented, but which discuss the (lack of) efficiency of markets under uncertainty are Stiglitz (1982) and Dixit (1986).

The focus of the present paper will be on welfare aspects of taxes imposed for fiscal reasons in the presence of uncertainty. The concern will be with tax induced distortions rather than market failure arising in the absence of taxation. This is an issue which has received little attention in the presence of uncertainty in spite of the large body of literature on optimum taxation in deterministic models. One aspect of this issue is the possible welfare effect of discrepancies between private and social risk-taking. Taxes may shift some of the risk (variability in consumption across states) from private to public consumption. This was pointed out by Atkinson and Stiglitz (1980, lecture 4), but optimum tax implications were not derived. An analysis which does establish an optimum tax rule for capital income taxation under uncertainty is provided by Richter (1988). This paper deals with asset specific but state independent income taxes in the special case where people are risk neutral with respect to publicly provided consumption, although taking a risk-averse attitude to private consumption.

In analysis of uncertainty it is vitally important to specify the nature of the uncertainty. We can imagine a whole range of cases in which the individual taxpayer, the government or society as a whole face some kind of uncertainty. There may be pure individual risk implying that each person faces a random outcome, while the average outcome is known with certainty. Or there may be pure aggregate risk implying that everybody will share the same luck or misfortune. Other cases are mixtures between these polar cases. Uncertainty may be more or less exogenous to the economy. In the present study the source of uncertainty is assumed to be exogenous to the economy, and only pure aggregate risk is considered. Individual risk seems to be less interesting in the assumed absence of market failure that can be improved by government policy.[2]

The tax policy to be analysed is differential, state-contingent capital income taxation. The taxes are assumed to affect savings behaviour as well as choice of portfolio (investment allocation). Differential taxation implies that the returns to different assets may be taxed at different rates. The purpose is to examine properties of these taxes. In particular their distortionary effects are exposed. A main task is to derive the class of preferences under which it is optimal to have no portfolio distortion.

The model used in the analysis is presented in section 2, and some comparative statics results are derived in section 3 of the paper. In section 4 I discuss the degrees of freedom available to the tax designer in the tax regime under survey. Section 5 introduces an alternative formulation of the model which proves useful in the subsequent analysis of tax distortions in section 6. The sections 7 and 8 are devoted to the case of no

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[2]It is important to note that market failure does not have policy implications if the cause of market failure also implies that there is government failure, so that no policy induced improvement is feasible.

portfolio distortion. Section 9 concludes the paper.

2. The Model

We shall establish a model which allows us to analyse some aspects of taxation and risk-taking. Two concerns govern our specific choice of model. Firstly, a simple model is wanted in order to isolate and bring out the uncertainty considerations. Secondly, it seems useful to stick to the modelling tradition of the positive analysis of taxation and risk-taking. Only one individual, or, equivalently, a representative individual of a population of homogeneous persons, is considered. The time horizon is limited to two periods. The uncertainty is associated with the state of the economy that will come into being in the second period. For simplicity, only two possible states are considered. In the first period the individual has an exogenous income or a fixed initial endowment of resources, which can be used for savings or consumption. Savings take place in two different assets. Each asset yields a return in the second period which depends on the state that actually occurs. The assets may be thought of as inputs into two different linear production activities which yield an amount of output in the second period, and with which some technological uncertainty is associated. We shall be concerned with taxes and public consumption only in the second period.

Let us refer to the two possible states as 1 and 2, occurring with probabilities p_1 and p_2. Let Y denote the initial resource endowment. A denotes total saving of which an amount m is allocated to asset 1, yielding a rate of return r_i in state i, and an amount a is allocated to asset 2, yielding a rate of return x_i in state i, where i=1,2. It may be useful to introduce exogenous incomes Y^1 and Y^2 in the respective states in period 2. Even if these variables are assumed to be zero, they may be useful for defining income effects. These incomes are not assumed to be taxable. The resources available in the second period after taking the return to savings into account, can be used for private and public consumption. The rate of transformation between the two is set equal to unity.

We shall consider a flexible tax regime, which allows us to have both asset specific and state contingent tax rates. Let $t^i{}_x$ and $t^i{}_r$ denote the tax rates on the returns to the respective assets in state i. Such a tax regime may appear to offer a scope for tax design which is unrealistic in practice. Even if this objection is accepted, the regime in question is a polar case which seems to be a useful benchmark against which other tax regimes can be judged. However, it is not so clear that such a tax regime is totally beyond practical feasibility. It is well known that different kinds of capital income are in fact taxed at different rates. Taxes that are formally state contingent are less easy to find. However, if tax rates are adjusted to the state that actually emerges, and if this adjustment is rationally foreseen by the taxpayers, state contingency is de facto built into the system. Moreover, tax schedules are often constructed in such a way that for instance particularly high income is taxed at a higher rate than lower income. If the high income can only be due to lucky circumstances we can interpret the tax <u>rates</u> as being state contingent, even though formally the tax schedule is not. Finally, economic analysis should not be confined to the existing legal and administrative framework.

The consumer is assumed to have preferences over consumption bundles, consisting of first period consumption and state contingent second period consumption. The Expected Utility Theorem is assumed to hold. If u denotes the utility function, the expected utility, E, is expressed as

$$(1)\qquad \begin{aligned} E &= p_1u(Y-m-a,(1+r_1(1-t^1{}_r))m+(1+x_1(1-t^1{}_x))a+Y^1) \\ &\quad + p_2u(Y-m-a,(1+r_2(1-t^2{}_r))m+(1+x_2(1-t^2{}_x))a+Y^2) \\ &= p_1u^1 + p_2u^2, \end{aligned}$$

where u^1 and u^2 denote the utilities of the respective states when the arguments of the functions are left out. The levels of public expenditure are assumed to be constant, and have been suppressed.

Subscripts are used to indicate partial derivatives with respect to the arguments that are being indicated.

The first order conditions for the consumer's choice of a and m are

(2) $$E_m = -p_1u^1{}_1 - p_2u^2{}_1 + p_1u^1{}_2(1+r_1(1-t^1{}_r)) +p_2u^2{}_2(1+r_2(1-t^2{}_r)) = 0$$

(3) $$E_a = -p_1u^1{}_1 - p_2u^2{}_1 + p_1u^1{}_2(1+x_1(1-t^1{}_x)) +p_2u^2{}_2(1+x_2(1-t^2{}_x)) = 0$$

The second order conditions are

$E_{mm} < 0$, $E_{aa} < 0$

and

$$D = \begin{vmatrix} E_{mm} & E_{ma} \\ E_{am} & E_{aa} \end{vmatrix} > 0$$

3. Comparative statics

The choice of m and a depends on the tax rates, and (2) and (3) implicitly define m and a as functions of the tax rates. Effects of changes in the tax parameters can in principle be found by implicitly differentiating the equation system (2) and (3). It is useful to decompose the effects of tax changes into compensated effects and income effects. By compensated effects we understand the effects that arise when a tax change is accompanied by a change in the exogenous income in the same period and state such that expected utility is kept unchanged. By income effects we mean effects of changes in exogenous income. The following notation is used to express the effects of marginal tax and income changes:

$$m_{ri} = \frac{\partial m}{\partial t^i_r},$$

$$m_{yi} = \frac{\partial m}{\partial Y^i},$$

that is, the income effect on m of an income change in state i in period 2. Let m^c_{ri} denote the compensated effect on m of a marginal change in t^i_r.

The following relationship holds:

$$m_{ri} = m^c_{ri} - m_{yi}\, mr_i.$$

Analogous effects can be defined for the other asset and for both assets with respect to the other rates of return. Similar notation is used for these effects.

We can easily derive the following expressions for the compensated effects:

(4) $$m^c_{r1} = p_1 u^1{}_2 r_1 E_{aa}/D$$

(5) $$a^c_{r1} = -p_1 u^1{}_2 r_1 E_{am}/D$$

(6) $$m^c_{r2} = p_2 u^2{}_2 r_2 E_{aa}/D$$

(7) $$a^c_{r2} = -p_2 u^2{}_2 r_2 E_{am}/D$$

(8) $$m^c_{x1} = -p_1 u^1{}_2 x_1 E_{ma}/D$$

(9) $$a^c_{x1} = p_1 u^1{}_2 x_1 E_{mm}/D$$

(10) $$m^c_{x2} = -p_2 u^2{}_2 x_2 E_{ma}/D$$

(11) $$a^c_{x2} = p_2 u^2{}_2 x_2 E_{mm}/D$$

It follows from the second order conditions that the compensated effects of the taxes on the own rates of return are negative. If for instance the after-tax rate of return on m in some state is reduced, less is invested in this asset. The lower rate of return may encourage a reallocation towards the other asset. On the other hand total savings may be discouraged. The net effect on the other asset is then ambiguous. Hence signing of cross effects is not possible without making further assumptions.

4. Degrees of freedom in the tax policy

The government is assumed to have a fixed tax revenue requirement in each state in period 2, denoted by R_1 and R_2, respectively. Hence the following constraints are imposed.

$$t^1{}_r r_1 m + t^1{}_x x_1 a = R_1 \tag{12}$$

$$t^2{}_r r_2 m + t^2{}_x x_2 a = R_2 \tag{13}$$

In the traditional optimum tax theory there is a single tax revenue requirement (government budget constraint). In the presence of uncertainty there is normally one for each state. Since in general publicly provided goods in different states are not perfect substitutes the tax level in each state is a matter of concern. As a consequence the optimization problem becomes more complicated.

In order to characterize the tax policy which maximizes the expected utility subject to the constraints (12) and (13), the following Lagrange expression is formulated

$$L=E+\beta_1(t^1{}_r r_1 m+t^1{}_x x_1 a-R_1)+\beta_2(t^2{}_r r_2 m+t^2{}_x x_2 a-R_2). \tag{14}$$

The following first order conditions must hold:

$$L_{t^1{}_r} = p_1 u^1{}_2(-r_1 m)+\beta_1(r_1 m+t^1{}_r r_1 m_{r1}+t^1{}_x x_1 a_{r1}) \tag{15}$$

$$+ \beta_2(t^2{}_r r_2 m_{r1}+t^2{}_x x_2 a_{r1}) = 0,$$

$$(16)\qquad L_{t^1_x} = p_1u^1_2(-x_1a)+\beta_1(x_1a+t^1_rr_1m_{x1}+t^1_xx_1a_{x1})$$

$$+\ \beta_2(t^2_rr_2m_{x1}+t^2_xx_2a_{x1}) = 0,$$

$$(17)\qquad L_{t^2_r} = p_2u^2_2(-mr_2)+\beta_1(t^1_rr_1m_{r2}+t^1_xx_1a_{r2})$$

$$+\beta_2(t^2_rr_2m_{r2}+t^2_xx_2a_{r2}+r_2m) = 0$$

$$(18)\qquad L_{t^2_x} = p_2u^2_2(-ax_2)+\ \beta_1(t^1_rr_1m_{x2}+t^1_xx_1a_{x2})$$

$$+\ \beta_2(t^2_rr_2m_{x2}+t^2_xx_2a_{x2}+ax_2) = 0$$

In the mathematical appendix to the paper it is shown that when three of the conditions in (15) - (18) are satisfied, the fourth one is automatically fulfilled. Hence (15) through (18) and the side conditions do not determine all the four tax rates. We have one degree of freedom. The reason for this appears from the mathematical derivations. Let the tax on r_1 be increased and the tax on x_1 be reduced to the extent that the income effects and hence the effects on utility just cancel out. This reform induces certain changes in the choice of m and a and affects the tax revenue. Then consider the case in which the tax on r_2 is increased and the tax on x_2 is reduced to the extent that the income effects and hence the effects on utility just cancel out. Then the choice of m and a is affected in exactly the same way as in the former case, and hence all the effects are identical.

In other words, one can always neutralize all the effects of changing one tax rate by adjusting the others. Suppose that one is at an optimum, and that, for instance, t^1_r is changed. Then the effects on utility can be offset by changing t^1_x in the opposite direction. The effects on tax revenue of the combined changes can be offset by simultaneous changes in t^2_r and t^2_x which leave utility unchanged. Hence it makes no difference whether one chooses the initial or the new value of t^1_r.

It seems appropriate to ask what is different from the traditional models of optimal taxation in which such a degree of freedom is not obtained. The difference lies in the number of tax parameters compared to the number of decision variables. In a traditional model, e.g., the Ramsey model, one has got one tax rate for each commodity that is being taxed. In the present model one has got four tax parameters, but only two decision variables, a and m. Each kind of savings is from the saver's point of view taxed by two rates. On the other hand there are two tax revenue requirements. If, for instance, one increases t^1_r and reduces t^1_x, the effect cet.par. is to make investment in asset a more profitable. But the comparative profitability of the two assets also depends on how they are taxed in state 2. By implementing opposite changes in the tax rates associated with the two assets in state 2, one can neutralize all effects of the former change in taxes.

An alternative approach is to consider the decision problem of the consumer in terms of three consumption goods, that is, first period consumption, c_1, second period consumption in state 1, c^1_2, and second period consumption in state 2, c^2_2. The social optimum can then be expressed in terms of real quantities as the optimal values of c_1, c^1_2, c^2_2, a and m. With c_1 as the numeraire good, two prices are required to support the optimal consumption bundle. These prices are determined by the choice of tax policy. In addition the taxes must be chosen so as to support the optimal value of a, and m then follows from the fact that $m = y - c_1 - a$. Hence the tax system is demanded to generate two prices and the optimal value of a (which is a linear function of c^1_2 and c^2_2). Since four tax parameters are available to satisfy these three requirements, there is one degree of freedom.

5. An alternative formulation

To provide an alternative description of exactly the same economic behaviour, we can assume that the consumer chooses total saving, $A = m+a$, and a, the share that is invested in the asset that yields a rate of return x_i in state i. The expected utility can then be expressed as

$$(19)\qquad E = p_1u(Y-A, A+r_1(1-t^1{}_r)(A-a)+x_1(1-t^1{}_x)a+Y^1) + p_2u(Y-A, A+r_2(1-t^2{}_r)(A-a)+x_2(1-t^2{}_x)a+Y^2)$$

Let us define

$$(20)\qquad s_1 = \frac{t^1{}_x x_1 - t^1{}_r r_1}{x_1 - r_1}$$

$$(21)\qquad s_2 = \frac{t^2{}_x x_2 - t^2{}_r r_2}{x_2 - r_2}$$

The parameters s_1 and s_2 can be interpreted as net taxes in the terminology of Sandmo (1985, 1988)[3]. Each tax rate is the rate of tax on the return to one asset over the other in a specific state. For instance, if one unit of wealth is reallocated from m to a, the additional return that is obtained is taxed at a rate s_1 in state 1. Let us now simplify the notation from $t^i{}_r$ to t_i. We can then write

$$(22)\qquad E = p_1u(Y-A, A(1+r_1(1-t_1))+a(x_1-r_1)(1-s_1)+Y^1) + p_2u(Y-A, A(1+r_2(1-t_2))+a(x_2-r_2)(1-s_2)+Y^2)$$

[3]A difference is that Sandmo only considers tax rates that are uniform across states.

The first order conditions for the individual's choice of a and A become

$$(23) \qquad E_a = p_1u^1{}_2(x_1-r_1)(1-s_1)+p_2u^2{}_2(x_2-r_2)(1-s_2) = 0$$

$$(24) \qquad E_A = -(p_1u^1{}_1+p_2u^2{}_1)+p_1u^1{}_2(1+r_1(1-t_1))+p_2u^2{}_2(1+r_2(1-t_2))=0$$

.

By convention we assume that

$$x_1 - r_1 > 0 > x_2 - r_2.$$

We can interpret (23) as stating that the risk-adjusted expected marginal net return to the asset a is zero at the optimum. Net return implies both that it is the return after allowing for the return foregone by not investing in the other asset, and that it is in this sense the after-tax return. There is risk adjustment since the net return in each state is weighted by the marginal utility of consumption. Since the marginal utility of consumpton is decreasing in the presence of risk aversion, the valuation of the vector of net returns is lower the more it contributes to variability in consumption and utility. Letting z denote the stochastic net return to a and $\bar{z}$ denote the expected net return, we can rewrite (23) as

$$(23') \qquad E_a = E(u_2\bar{z}) + \mathrm{cov}(u_2,z) = 0$$

where E is the expectations operator, and the covariance can be interpreted as a risk adjustment term.

The interpretation of (24) is that the optimum level of savings is characterized by the expected utility of the marginal unit of consumption foregone in period 1 being equal to the expected marginal utility of the consumption obtained in period 1 after taking the after-tax return to savings into account.

6. Tax distortions

The first best social optimum is achieved by applying state contingent lump sum taxes. At this optimum we would instead of (23) and (24) have

$$(25) \qquad p_1u_2^1(x_1-r_1)+p_2u_2^2(x_2-r_2) = 0$$

and

$$(26) \qquad -(p_1u_1^1+ p_2u_1^2) + p_1u_2^1(1+r_1)+p_2u_2^2(1+r_2) = 0.$$

For a fixed level of total saving (25) (or 23) is the condition for optimal choice of portfolio. A distortion of the choice of portfolio is attributed to the income taxes if condition (23) deviates from the first best condition (25). The absence of a portfolio distortion does not necessarily imply that the allocation of wealth between assets is the same as the first best allocation since the level of savings may be different. What is implied is that contingent on the level of savings the allocation is socially optimal.

To get a clearer picture of the portfolio distortion it may be useful to rewrite (23) as

$$(23'') \qquad p_1u_2^1(x_1-r_1) + p_2u_2^2(x_2-r_2) - [p_1u_2^1s_1(x_1-r_1) + p_2u_2^2s_2(x_2-r_2)] = 0$$

The expression in square brackets can be interpreted as the risk-adjusted expected marginal net tax on the return to a. This marginal tax acts as a tax wedge between the social returns to the two assets and distorts the allocation in one direction or the other depending on the sign of the wedge. If the marginal tax, as defined by the expression in square brackets, is zero, the first best condition (25) is obviously satisfied.

From (23) we see that no portfolio distortion requires that

(27) $s_1 = s_2$.

There must be uniform taxation of net returns which implies that there is neutral taxation of capital income. If the investor considers allocating an additional unit of wealth to the asset a, he will face a positive tax liability in one state, while there is (in expected utility terms) an offsetting subsidy in the other state. All in all there is no need to reconsider the allocation because taxes are taken into account.

We say that there is a savings distortion if (24) deviates from (26). This means that given the choice of portfolio, the private marginal return to savings deviates from the social one. In order to have no savings distortion the following condition must hold

$$t_1 r_1 p_1 u_2^1 + t_2 r_2 p_2 u_2^2 = 0,$$

which is equivalent to

$$\text{(28)} \qquad \frac{t_1 r_1}{x_1 - r_1} = \frac{t_2 r_2}{x_2 - r_2},$$

where (23) has been employed. The condition implies that one of the tax rates (t_1 or t_2) must be negative. This may not even be a feasible solution given the tax revenue requirements. To require the tax system to be non-distortive in this respect would be a most rigorous restriction. As in the non-stochastic framework it seems that a savings distortion must normally be accepted under income taxation. It seems to be a much more common belief, at least in the tax policy debate, that taxes could be and should be designed not to distort the allocation of savings. And the further analysis is devoted to this issue.

7. The undistorted portfolio case

Let us explore the interesting case of no portfolio distortion. Let us consider the case in which the tax policy has been determined such that $s_1 = s_2 = s$. Let us further assume that s is given a small increment ds while t_1 and t_2 are adjusted to keep tax revenues unchanged. We can then easily show that the equilibrium is maintained if the individual choice of A, m and a remain the same. That is, we can show that all the conditions governing behaviour then remain satisfied. As we recall from section (4) above the tax revenue requirements are

(29) $$t_1r_1A + (x_1-r_1)s_1a = R_1,$$

(30) $$t_2r_2A + (x_2-r_2)s_2a = R_2,$$

where (12) and (13) have been relabelled.

When A and a are kept constant, we see that the precondition that tax revenues do not change implies that

$$dt^1r_1A + (x_1-r_1)sa\ ds = 0,$$

that is,

(31) $$dt_1 = -\frac{(x_1-r_1)sa}{r_1A}ds,$$

and similarly

(32) $$dt_2 = -\frac{(x_2-r_2)sa}{r_2A}ds.$$

We easily see from (22) that the arguments of the utility function have not been changed. Since $s_1 = s_2$, (23) obviously remain satisfied. From (24) we find that

$$(33) \qquad dE_A = p_1u^1{}_2r_1 \frac{(x_1-r_1)sa}{r_1A} ds + p_2u^2{}_2r_2 \frac{(x_2-r_2)sa}{r_2A} ds$$

$$=(p_1u^1{}_2(x_1-r_1) + p_2u^2{}_2(x_2-r_2)) \frac{sa}{A} ds = 0,$$

due to (23). Then the expected utility also remains unchanged.

The tax revenue requirements implicitly define t_1 and t_2 as functions of s_1 and s_2:

$$(34) \qquad \begin{array}{l} t_1(s_1, s_2), \\ t_2(s_1, s_2). \end{array}$$

Taking these relations into account, expected utility, which depends on the tax parameters, can be expressed as a function of s_1 og s_2: $E(s_1, s_2)$. It follows from the analysis above that for $s_1 = s_2 = s$, $E(s,s)$ is constant. That is, the 45^o line in the s_1, s_2-diagram is a contour curve of expected utility. If this is also the highest contour curve of $E(s_1, s_2)$, it is optimal to have $s_1 = s_2$. Whether this is the case is a question to which we shall come back.

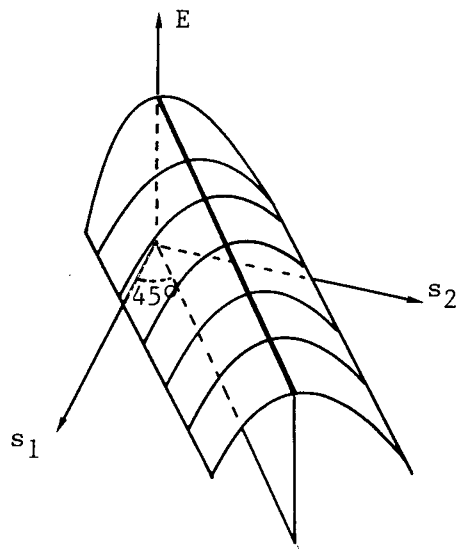

Figure 1

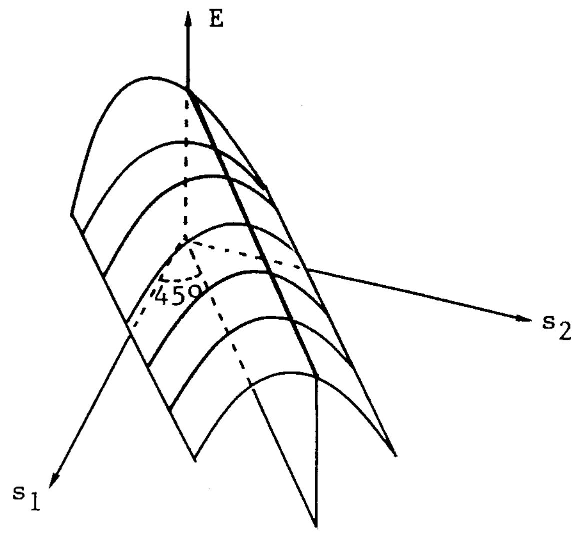

Figure 2
$s_1 = s_2$ is optimal

Figure 1 and figure 2 depict the graphs of the function $E(s_1,s_2)$ in two different cases. Each graph has the form of a hill above the s_1, s_2-plane. In figure 1 the points along the 45-degree line in the s_1, s_2-plane correspond to an E-value along the slope of the graph, and the E-function is not maximized by this configuration of s_1 and s_2. In figure 2 the ridge of the graph has the 45-degree line as its projection into the s_1, s_2-plane, and it is optimal to have uniform values of s_1 and s_2.

Since there is one degree of freedom in the optimum choice of tax policy, one has a certain scope for choosing combinations of tax parameters. Yet one is not entirely free to exploit the degree of freedom in any way. In particular, one cannot immediately set $s_1 = s_2$, since one has then restrained oneself to movements along a specific countour line of expected utility, and has then been barred from searching for the highest contour line.

An interesting question is then under what conditions, if any, it would in fact be optimal to have $s_1=s_2$. This question will be adressed in the next section.

8. Conditions for undistorted portfolio at the social optimum.

Let us consider a marginal tax reform (dt_1, dt_2, ds_1, ds_2) and the corresponding changes (dA, da) in economic behaviour.

Differentiating (29) and (30) yields

$$(35) \qquad r_1A\,dt_1 + t_1r_1dA + (x_1 - r_1)a\,ds_1 + (x_1 - r_1)s_1da = 0$$

$$(36) \qquad r_2A\,dt_2 + t_2r_2dA + (x_2 - r_2)a\,ds_2 + (x_2 - r_2)s_2da = 0$$

Differentiating (22) we get

$$\begin{aligned}
(37) \qquad dE &= p_1u_1^1(-dA) \\
&+ p_1u_2^1((1 + r_1)dA - r_1A\,dt_1 + (x_1 - r_1)da - r_1t_1dA \\
&- (x_1 - r_1)s_1da - a(x_1 - r_1)ds_1) + p_2u_1^2(-dA) \\
&+ p_2u_2^2((1 + r_2)dA - r_2t_2dA - r_2A\,dt_2 \\
&- a(x_2 - r_2)ds_2 + (x_2 - r_2)da - (x_2 - r_2)s_2da) \\
&= (t_1r_1p_1u_2^1 + t_2r_2p_2u_2^2)dA \\
&+ (s_1(x_1 - r_1)p_1u_2^1 + s_2(x_2 - r_2)p_2u_2^2)da
\end{aligned}$$

after making use of (35), (36), (23) and (24). By employing the first order conditions once more, we obtain

$$(38) \qquad dE = p_1u_2^1(x_1 - r_1)(s_1 - \frac{1-s_1}{1-s_2} s_2)da$$

$$+ (t_1r_1p_1u_2^1 + t_2r_2 \quad (- \frac{p_1u_2^1(x_1-r_1)(1-s_1)}{(x_2-r_2)(1-s_2)})dA =$$

$$p_1u_2^1(x_1-r_1) \; \{[s_1 - \frac{1-s_1}{1-s_2} s_2]da + [\frac{t_1r_1}{x_1-r_1} - \frac{t_2r_2}{x_2-r_2} \frac{1-s_1}{1-s_2}]dA\}$$

We observe that

$$s_1 - \frac{1-s_1}{1-s_2} s_2 = 0$$

is equivalent to $s_1 = s_2$, and

$$\frac{t_1r_1}{x_1-r_1} - \frac{t_2r_2}{x_2-r_2} \frac{1-s_1}{1-s_2} = 0$$

is equivalent to

$$t_1 = \frac{x_1-r_1}{x_2-r_2} \frac{r_2}{r_1} \frac{1-s_1}{1-s_2} t_2.$$

Recalling (27) and (28) we may note that a change of portfolio, defined as a partial change in a, has an impact on expected utility only if there is a distortion of the choice of portfolio. Similarly, a change of saving, defined as a partial change in A, has an impact on expected utility only if there is a savings distortion.

We shall now explore the class of utility functions which imply that there is no portfolio distortion at the social optimum as a general result, i.e. for any set of other exogenous features of the economy (initial resource endowment, distribution of returns, etc.). As we see from (38) the condition that it shall be optimal to have no portfolio distortion ($s_1=s_2$) is that a marginal tax reform from a point where $s_1=s_2$ does not affect total savings, dA=0. In the second part of the mathematical appendix to the paper it is demonstrated that this condition is equivalent to the following:

$$(39) \qquad \frac{A}{a}\left(\frac{u^1_{12}}{u^1_2} - \frac{u^2_{12}}{u^2_2}\right)$$

$$- \frac{-u^2_{22}}{au^2_2}\left((x_2-r_2)(1-s)a + (1+r_2(1-t_2))A\right)$$

$$- \frac{u^1_{22}}{au^1_2}\left((x_1-r_1)(1-s)a + (1+r_1(1-t_1))A\right) = 0$$

The last two terms can be interpreted as relative risk aversion measures. They cancel out if the relative risk aversion is state independent. The first term is zero if there is neither risk complementarity nor risk substitutability in the sense of Sandmo (1969). Let us apply the term <u>risk independence</u> to this case. As shown by Sandmo

$$\frac{\partial}{\partial c_2}\left(-\frac{u_{12}}{u_2}\right) \equiv \frac{\partial}{\partial c_1}\left(-\frac{u_{22}}{u_2}\right).$$

He defines $u_{22}(c_1,c_2)/u_2(c_1,c_2)$ as the risk aversion function,

the value of which is shown to reflect and move in the same direction as the risk premium. Sandmo writes that "the higher is present consumption, the higher is the consumer's risk premium for gambles on future consumption. It is tempting to call this risk complementarity and its opposite (risk premium decreasing in c_1) risk substitutability".

<u>Proposition</u>: In order to have no portfolio distortion at the optimum taxation as a general outcome, the utility function must belong to the class characterized by the relative risk aversion being state independent and by risk independence (absence of risk complementarity and risk substitutability in the sense of Sandmo).

Let us examine the implications of these properties somewhat further. Since first period consumption is independent of the state that emerges in period 2, the relative risk aversion is state independent if it only depends on first period consumption, i.e.,

$$(40) \qquad -\frac{u_{22}}{u_2} c_2 = f(c_1)$$

The absolute risk aversion is independent of c_1 (absence of risk complementarity and substitutability) if $f(c_1)$ is constant (independent of c_1). The implication is that the relative risk aversion is constant. That is, the utility function must belong to the class defined by

$$(41) \qquad u(c_1, c_2) = h(c_1)c_2^{\gamma} + g(c_1)$$

where γ is a constant. To complete the analysis we must also recall that the public expenditure level is an argument in the utility function even though it was suppressed at an earlier stage. In order to ensure that u_{12}/u_2 and the relative risk aversion are both state independent, these quantities must be independent of the public provision of goods or the latter must be constant, which is a very special case. In general the utility function must belong to the class defined by

$$(42) \qquad u(c_1,c_2,R) = v(R)h(c_1)c_2^{\gamma} + g(c_1,R)$$

to satisfy these conditions. Here R is used to denote the public expenditure level.

It is easy to show that this class of utility functions implies separability between the choice of total amount of savings and the choice of portfolio in the sense that the relative share of total savings allocated to each asset does not depend on the level of savings.

The utility function in question is a special one, but may not be totally unrealistic. From the general insight obtained from second best theory one would expect that an optimal policy would normally imply a mixture of all potential distortions. That utility functions implying such outcomes exist is not surprising. It is more interesting that there actually does exist a class of utility functions implying that it would be desirable to have <u>no portfolio distortion</u>.

It is useful to know that if one wants to study savings and portfolio behaviour as a basis for deriving tax implications, it is too restrictive to postulate a utility function of the form given by (42). The reason is that a fundamental property of the tax structure (the non-distortion of the portfolio) has already been imposed by the choice of functional form. The significance of being aware of special tax implications of assuming special functional forms has been emphasized by Angus Deaton. In Deaton (1981) he writes: "It is thus of central importance that empirical work directed towards providing parameters for evaluating optimal tax formulae should employ functional forms sufficiently general to allow measurement rather than assumption to determine the structure of taxes." If on the other hand one does believe that the class of utility functions (42) provides an adequate description of preferences, then a very strong conclusion about the tax structure can be drawn.

9. Conclusion

The paper has analysed some aspects of state contingent capital income taxes within the framework of a two-period, two-asset and two-state model. The model determines total savings and the allocation of savings between two risky assets. Taxes on the returns are assumed to be state contingent and asset specific. The representative consumer of the population maximizes expected utility with respect to the level and allocation of savings. The government decides the structure of taxation subject to state specific tax revenue requirements.

It has been shown that there is a degree of freedom in this tax design problem. The reason is that the returns to each asset are affected by two tax rates, one for each state. In the period when the allocation decision is made and the future state is unknown, changes in the two taxes can be offsetting from the point of view of the taxpayer. On the other hand taxes on the two different assets are alternative means of collecting revenue from the point of view of the government.

The taxes may distort the consumption-saving trade-off as well as the allocation of savings (the choice of portfolio). In general both distortions are expected to be present at the second best tax optimum. While the savings distortion is hard to escape, the portfolio distortion may be avoided under special conditions. Absence of a portfolio distortion at the optimum taxation in general requires that the preference ordering belongs to a special class of von Neumann-Morgenstern utility function. This class of preferences has been derived and characterized.

The present paper has been concerned with one case of taxation and uncertainty. It would be interesting to extend the normative analysis of (capital) taxation in the presence of uncertainty to different kinds of risk, tax regimes and economic structures.

References

Atkinson, A.B. and J.E. Stiglitz, Lectures on public economics (McGraw-Hill, London).

Deaton, A., 1981, Optimal taxes and the structure of preferences, Econometrica 49, 1245-1260.

Dixit, A.K., 1986, Trade and insurance with moral hazard, Journal of International Economics 23, 201-220.

Mossin, J., 1968, Taxation and risk-taking: An expected utility approach, Economica 35, 74-82.

Richter, W.F., 1988, The optimal taxation of risky capital income: An elasticity rule. Discussion paper, University of Dortmund.

Sandmo, A., 1969, Capital risk, consumption and portfolio choice, Econometrica 37, 586-599.

Sandmo, A., 1985, The effects on taxation on savings and risk-taking, in: A.J. Auerbach and M. Feldstein, eds., Handbook of Public Economics, vol. I (North-Holland).

Sandmo, A., 1988, Differential taxation and the encouragement of risk-taking, Discussion Paper no. 15, The Norwegian School of Economics and Business Administration.

Stiglitz, J.E., 1982, The inefficiency of the stock market equilibrium, Review of Economic Studies, 241-261.

Stiglitz, J.E., 1969, The effects of income, wealth and capital gains taxation on risk-taking, Quarterly Journal of Economics 83, 262-283.

Varian, H.R., 1980, Redistributive Taxation as a Social Insurance, Journal of Public Economics 14, 49-68.

Mathematical appendix

By decomposing the effects in (15) through (18) into compensated effect and income effects and multiplying both sides by the same expression in all equations, we can instead of (15)-(18) write the conditions as

(a1) $$L_{t^1_r} D/mr_1 p_1 u^1_2 = [-p_1 u^1_2 + \beta_1(1+t^1_r r_1 m^c_{r1}/r_1 m + t^1_x x_1 a^c_{r1}/mr_1 - t^1_r r_1 m_{y1} - t^1_x x_1 a_{y1}) + \beta_2(t^2_r r_2 m^c_{r1}/mr_1 + t^2_x x_2 a^c_{r1}/mr_1 - t^2_r r_2 m_{y1} - t^2_x x_2 a_{y1})]D/p_1 u^1_2 = 0$$

(a2) $$L_{t^1_x} D/ax_1 p_1 u^1_2 = [-p_1 u^1_2 + \beta_1(1+t^1_r r_1 m^c_{x1}/ax_1 + t^1_x x_1 a^c_{x1}/ax_1 - t^1_r r_1 m_{y1} - t^1_x x_1 a_{y1}) + \beta_2(t^2_r r_2 m^c_{x1}/ax_1 + t^2_x x_2 a^c_{x2}/ax_1 - t^2_r r_2 m_{y1} - t^2_x x_2 a_{y1})]D/p_1 u^1_2 = 0$$

(a3) $$L_{t^2_r} D/mr_2 p_2 u^2_2 = [-p_2 u^2_2 + \beta_1(t^1_r r_1 m^c_{r2}/mr_2 + t^1_x x_1 a^c_{r2}/mr_2 - t^1_r r_1 m_{y2} - t^1_x x_1 a_{y2}) + \beta_2(1+t^2_r r_2 m^c_{r2}/mr_2 + t^2_x x_2 a^c_{r2}/mr_2 - t^2_r r_2 m_{y2} - t^2_x x_2 a_{y2})]D/p_2 u^2_2 = 0$$

(a4) $$L_{t^2_x} D/ax_2p_2u^2_2 = [-p_2u^2_2 + \beta_1(t^1_rr_1m^c_{x2}/ax_2 + t^1_xx_1a^c_{x2}/ax_2 - t^1_rr_1m_{y2} - t^1_xx_1a_{y2}) +\beta_2(1+t^2_rr_2m^c_{x2}/ax_2 + t^2_xx_2a^c_{x2}/ax_2 - t^2_rr_2m_{y2} - t^2_xx_2a_{y2})]D/p_2u^2_2 = 0$$

Let us denote the expressions on the left hand sides of (a1)-(a4) by (α), (β), (γ) and (δ).

(a5) $$(\alpha)-(\beta) = [\beta_1(t^1_rr_1m^c_{r1}/r_1m + t^1_xx_1a^c_{r1}/r_1m - t^1_rr_1m^c_{x1}/ax_1 - t^1_xx_1a^c_{x1}/ax_1) + \beta_2(t^2_rr_2m^c_{r1}/mr_1 + t^2_xx_2a^c_{r1}/mr_1 - t^2_rr_2m^c_{x1}/ax_1 - t^2_xx_2a^c_{x1}/ax_1)]D/p_1u^1_2 = 0$$

(a6) $$(\gamma)-(\delta) = [\beta_1(t^1_rr_1m^c_{r2}/mr_2 + t^1_xx_1a^c_{r2}/mr_2 - t^1_rr_1m^c_{x2}/ax_2 - t^1_xx_1a^c_{x2}/ax_2) + \beta_2(t^2_rr_2m^c_{r2}/mr_2 + t^2_xx_2a^c_{r2}/mr_2 - t^2_rr_2m^c_{x2}/ax_2 - t^2_xx_2a^c_{x2}/ax_2)]D/p_2u^2_2 = 0$$

The differences can be rewritten as

(a7) $$(\alpha)-(\beta) =$$

$$\beta_1(t^1_rr_1(m^c_{r1}/r_1m - m^c_{x1}/ax_1) + t^1_xx_1(a^c_{r1}/r_1m - a^c_{x1}/ax_1))D/p_1u^1_2 + \beta_2(t^2_rr_2(m^c_{r1}/mr_1 - m^c_{x1}/ax_1)$$

$$+ t^2{}_x x_2(a^c{}_{r1}/mr_1 - a^c{}_{x1}/ax_1))D/p_1u^1{}_2$$

and

(a8) $$(\gamma)-(\delta) =$$

$$\beta_1(t^1{}_r r_1(m^c{}_{r2}/mr_2 - m^c{}_{x2}/ax_2)$$

$$+ t^1{}_x x_1(a^c{}_{r2}/mr_2 - a^c{}_{x2}/ax_2))D/p_2u^2{}_2$$

$$+\beta_2(t^2{}_r r_2(m^c{}_{r2}/mr_2 - m^c{}_{x2}/ax_2)$$

$$+ t^2{}_x x_2(a^c{}_{r2}/mr_2 - a^c{}_{x2}/ax_2))D/p_2u^2{}_2.$$

Employing the compensated terms in (4)-(11) of the main text, the differences can further be rewritten as

(a9) $$(\alpha)-(\beta) =$$

$$(\beta_1 t^1{}_r r_1 + \beta_2 t^2{}_r r_2)(E_{aa}/m + E_{ma}/a)$$

$$+ (\beta_1 t^1{}_x x_1 + \beta_2 t^2{}_x x_2)(-E_{am}/m - E_{mm}/a)$$

and

(a10) $$(\gamma)-(\delta) =$$

$$(\beta_1 t^1{}_r r_1 + \beta_2 t^2{}_r r_2)(E_{aa}/m + E_{ma}/a)$$

$$+ (\beta_1 t^1{}_x x_1 + \beta_2 t^2{}_x x_2)(-E_{am}/m - E_{mm}/a).$$

Then we see that

(a11) $$(\alpha)-(\beta) \equiv (\gamma)-(\delta).$$

This means that when three of the conditions in (a1)-(a4), and equivalently three of the conditions in (17)-(18), are satisfied, the fourth one is automatically fulfilled.

*

Differentiating the first order conditions (23) and (24) of the main text and exploiting (35) and (36), we obtain

(a12) $$p_1u^1_{21}(-dA)(x_1 - r_1)(1 - s_1)$$

$$+ p_1u^1_{22}(x_1-r_1)(1-s_1)((1+r_1)dA + (x_1-r_1)da)$$

$$- p_1u^1_2(x_1 - r_1)ds_1$$

$$+ p_2u^2_{21}(-dA)(x_2 - r_2)(1 - s_2)$$

$$+ p_2u^2_{22}(x_2-r_2)(1-s_2)((1+r_2)dA + (x_2-r_2)da)$$

$$- p_2u^2_2(x_2-r_2)ds_2= 0$$

and

(a13) $$- p_1u^1_{11}(-dA) - p_2u^2_{11}(-dA)$$

$$- p_1u^1_{12}((1+r_1)dA + (x_1-r_1)da)$$

$$- p_2u^2_{12}((1+r_2)dA + (x_2-r_2)da)$$

$$+ p_1(1+r_1(1-t_1))u^1_{21}(-dA) + p_2(1+r_2(1-t_2))u^2_{21}(-dA)$$

$$+ p_1(1+r_1(1-t_1))u^1_{22}((1+r_1)dA + (x_1-r_1)da)$$

$$+ p_2(1+r_2(1-t_2))u^2_{22}((1+r_2)dA + (x_2-r_2)da)$$

$$+ p_1u^1_2r_1[\frac{t_1}{A} dA + \frac{(x_1-r_1)a}{r_1A} ds_1 + \frac{(x_1-r_1)s_1}{r_1A} da]$$

$$+ p_2 u_2^2 r_2 [\frac{t_2}{A} dA + \frac{(x_2 - r_2)a}{r_2 A} ds_2 + \frac{(x_2 - r_2)s_2}{r_2 A} da] = 0$$

Equation (a12) is equivalent to

$$\text{(a14)} \quad \begin{aligned} & dA[-p_1 u_{21}^1 (x_1 - r_1)(1 - s_1) + p_1 u_{22}^1 (x_1 - r_1)(1 - s_1)(1 + r_1) \\ & - p_2 u_{21}^1 (x_2 - r_2)(1 - s_2) + p_2 u_{22}^2 (x_2 - r_2)(1 - s_2)(1 + r_2)] \\ & + [p_1 u_{22}^1 (x_1 - r_1)^2 (1 - s_1) + p_2 u_{22}^2 (x_2 - r_2)^2 (1 - s_2)] da \\ & = p_1 u_2^1 (x_1 - r_1) ds_1 + p_2 u_2^2 (x_2 - r_2) ds_2 \end{aligned}$$

Equation (a13) is equivalent to

$$\text{(a15)} \quad \begin{aligned} & dA[p_1 u_{11}^1 + p_2 u_{11}^2 - p_1 u_{12}^1 (1 + r_1) - p_2 u_{12}^2 (1 + r_2) \\ & - p_1 (1 + r_1 (1 - t_1)) u_{21}^1 - p_2 (1 + r_2 (1 - t_2)) u_{21}^2 \\ & + p_1 (1 + r_1 (1 - t_1)) u_{22}^1 (1 + r_1) + p_2 (1 + r_2 (1 - t_2)) u_{22}^2 (1 + r_2) \\ & + p_1 u_2^1 r_1 t_1 / A + p_2 u_2^2 r_2 t_2 / A] \\ & + [-p_1 u_{12}^1 (x_1 - r_1) - p_2 u_{12}^2 (x_2 - r_2) \\ & + p_1 (1 + r_1 (1 - t_1)) u_{22}^1 (x_1 - r_1) \\ & + p_2 (1 + r_2 (1 - t_2)) u_{22}^2 (x_2 - r_2) \\ & + p_1 u_2^1 (x_1 - r_1) s_1 / A + p_2 u_2^2 (x_2 - r_2) s_2 / A] da \\ & = - (p_1 u_2^1 (x_1 - r_1) a/A) ds_1 - (p_2 u_2^2 (x_2 - r_2) a/A) ds_2 . \end{aligned}$$

As we see from (38), the condition that it shall be optimal to have no portfolio distortion is that a marginal tax reform from a point where $s_1 = s_2 = s$ does not affect total savings ($dA=0$). Let us first consider a marginal change in s_1 for a fixed value of s_2. From (a14) and (a15) follows that the condition holds if and only if

$$\frac{p_1 u^1_{22}(x_1-r_1)^2(1-s)}{p_1 u^1_2(x_1-r_1)} + \frac{p_2 u^2_{22}(x_2-r_2)^2(1-s)}{p_1 u^1_2(x_1-r_1)}$$

$$= \frac{-p_1 u^1_{12}(x_1-r_1)}{-p_1 u^1_2(x_1-r_1)a/A} + \frac{p_2 u^2_{12}(x_2-r_2)}{p_1 u^1_2(x_1-r_1)a/A}$$

$$+ \frac{p_1(1+r_1(1-t_1))(x_1-r_1)u^1_{22}}{-p_1 u^1_2(x_1-r_1)a/A}$$

$$+ \frac{p_2(1+r_2(1-t_2))(x_2-r_2)u^2_{22}}{-p_1 u^1_2(x_1-r_1)a/A}$$

$$+ \frac{p_1 u^1_2(x_1-r_1)s/A}{-p_1 u^1_2(x_1-r_1)a/A} + \frac{p_2 u^2_2(x_2-r_2)s/A}{-p_1 u^1_2(x_1-r_1)a/A}$$

This is equivalent to

$$\text{(a16)} \quad - \frac{u^1_{22}}{u^1_2}(x_1-r_1)(1-s) - \frac{p_2(x_2-r_2)u^2_{22}u^2_2(x_2-r_2)(1-s)}{p_1 u^1_2(x_1-r_1)u^2_2}$$

$$+ \frac{u^1_{12}}{u^1_2}\frac{A}{a} + \frac{u^2_{12}p_2(x_2-r_2)u^2_2 A}{u^2_2 u^1_2 p_1(x_1-r_1)a} - \frac{(1+r_1(1-t_1))u^1_{22}A}{au^1_2}$$

$$- \frac{p_2(1+r_2(1-t_2))(x_2-r_2)u^2_{22}u^2_2 A}{p_1(x_1-r_1)a\ u^2_2\ u^1_2}$$

$$- \frac{s}{a} - \frac{p_2(x_2-r_2)su^2_2}{p_1(x_1-r_1)au^1_2} = 0$$

By invoking the first order conditions, this condition can be transformed to

$$- \frac{u^1_{22}(x_1-r_1)(1-s)}{u^1_2} + \frac{u^2_{22}(x_2-r_2)(1-s)}{u^2_2} + \frac{u^1_{12}A}{u^1_2 a} - \frac{u^2_{12}A}{u^2_2 a}$$

$$- \frac{(1+r_1(1-t_1))A\ u^1_{22}}{a\ u^1_2} + \frac{(1+r_2(1-t_2))u^2_{22}A}{u^2_2\ a} = 0$$

Similarly, we find that for a marginal change in s_2 for a fixed value of s_1 the condition that dA=0 implies that

$$\frac{u^2_{22}}{u^2_2}(x_2-r_2)(1-s) + \frac{p_1 u^1_{22}(x_1-r_1)(x_1-r_1)(1-s)}{p_2 u^2_2(x_2-r_2)}$$

$$- \frac{p_1 u^1_{12}(x_1-r_1)A}{p_2 u^2_2(x_2-r_2)a} - \frac{u^2_{12}A}{u^2_2 a}$$

$$+ \frac{p_1(1+r_1(1-t_1))u^1_{22}(x_1-r_1)A}{p_2 u^2_2(x_2-r_2)a} + \frac{(1+r_2(1-t_2))u^2_{22}A}{u^2_2\ a}$$

$$+ \frac{p_1 u_2^1 (x_1 - r_1) s}{p_2 u_2^2 (x_2 - r_2) a} + \frac{s}{a}$$

$$= \frac{u_{22}^2}{u_2^2} (x_2 - r_2)(1-s) - \frac{u_{22}^1}{u_2^1} (x_1 - r_1)(1-s)$$

$$+ \frac{u_{12}^1}{u_2^1} \frac{A}{a} - \frac{u_{12}^2}{u_2^2} \frac{A}{a}$$

$$- \frac{(1+r_1(1-t_1))u_{22}^1 A}{u_2^1 \; a} + \frac{(1+r_2(1-t_2)u_{22}^2 A}{u_2^2 \; a} = 0$$

which coincides with (a16).

By further manipulations we get

$$\text{(a17)} \qquad \frac{A}{a} \left(\frac{u_{12}^1}{u_2^1} - \frac{u_{12}^2}{u_2^2}\right)$$

$$- \frac{-u_{22}^2}{au_2^2} ((x_2 - r_2)(1-s)a + (1+r_2(1-t_2))A)$$

$$- \frac{u_{22}^1}{au_2^1} ((x_1 - r_1)(1-s)a + (1+r_1(1-t_1))A) = 0$$