On Membership in Clubs

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Abstract : Clubs are voluntarily-shared but excludable facilities prone to congestion. Examples include tolled trunk road, the telephone system and "gentlemen's" clubs. Despite 25 years of development, the clubs literature does not contain a satisfactory treatment of the membership of the sharing group which allows for differences in income and/or tastes and for agents' self-selection to club membership or non-membership. This paper provides such a treatment and applies it to profit-maximising and revenue-constrained welfare-maximising clubs and to the analysis of the social organisation of a facility of fixed size.

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ON MEMBERSHIP IN CLUBS*/

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Club goods are shared, but excludable, facilities prone to congestion. Examples include the telephone system, trunk roads and the eponymous "gentlemen's" clubs. The modern theory of clubs began twenty-five years ago with the seminal contributions of Buchanan (1965) and Olson (1965). However, club analysis has antecedents in the work of Pigou (1920), Knight (1924) and Wiseman (1957), among others.

Sandler and Tschirhart (1980, p.1482) define a club as

> "a voluntary group deriving mutual benefit from sharing one or more of the following: production costs, the members' characteristics, or a good characterized by excludable benefits".

Cornes and Sandler (1986, p.159) stress, "Clubs must be voluntary; members choose to belong because they anticipate a benefit from membership. [T]he utility jointly derived from membership and the consumption of other goods must exceed the utility associated with non-membership status. This voluntarism serves as one factor by which to distinguish between a pure public good and a club good".

Surprisingly, despite this widespread recognition of the importance of voluntarism or self-selection to club membership, none of the contributors to the club literature have successfully incorporated this feature in a general analysis in contexts where it matters crucially: when individuals differ.[1/] This paper provides a treatment of club membership incorporating individual differences and self-selection to membership.

The organisation is as follows. Section I considers in more detail the shortcomings of the existing treatment of membership in the literature. II discusses club membership where voluntarism is treated explicitly. We consider clubs both with fixed and with variable utilisation by members and in which potential members differ only in endowments, not tastes. We establish conditions for membership and utilisation to be positively related to income. In the fixed utilisation case, we illustrate explicitly how membership depends upon the level of provision of club facilities and the membership fee. In the variable utilisation case, we show how utilisation and membership are related to the fee - with the surprising possibility that an increase in the membership fee could result in an increase in membership. III indicates how club membership depends upon the institutional environment by considering profit-maximising monopoly clubs, revenue-constrained, welfare-maximising clubs, and the social organisation of a facility of fixed size. Section IV considers the case where population members differ in both tastes and incomes. In V we conclude by recapping some lessons to be drawn from our analysis and by discussing its relationship to the game-theoretic analysis of clubs incorporating the core, the one area of the existing literature in which voluntarism in principle plays a major role.

I. THE EXISTING TREATMENT OF MEMBERSHIP

In what Sandler and Tschirhart (1980) call the "general club model", analysis of membership proceeds as follows. Consider a population of households of fixed size, conveniently normalised to unity. Households are indexed h, $h \varepsilon [0,1]$, and distributed with density $f(h)$. Household h, if a member of the only club, has utility

$$U^h[x(h), v(h), c(y, V)] \tag{1}$$

Here, U^h is strictly concave in x and v and $x(h)$ is h's consumption of the solitary numeraire private commodity, $v(h)$ its utilisation of the club facility and $c(\cdot)$ an index of club quality. Club quality depends positively on y, the "size" of facility provision, and negatively on members' total utilisation (or "congestion"), V. Thus, $c_1 \equiv \partial c(y, V)/dy > 0$, $c_2 \equiv \partial c(\cdot)/\partial V < 0$, with subscripts denoting partial derivatives hereafter.

If not a club member, h has utility

$$U^h[\tilde{x}(h), 0, 0] \equiv \tilde{U}^h \tag{2}$$

the tilde signifying "non-member".

It is assumed households can be "rank-ordered" along $[0,1]$ "by their net willingness to pay for the club, as measured by their net gain from membership. This ordering is assumed [...] invariant to the level of use and the degree of congestion", (Cornes and Sandler, p.180). Thus non-members and members form distinct intervals, $[0,s]$, $[s,1]$, s being a marginal household.

The economy's production possibilities are represented by a strictly convex transformation function,

$$T\left(y, \int_0^s \tilde{x}(h)f(h)dh + \int_s^1 x(h)f(h)dh\right) \leqq 0 \tag{3}$$

The government maximises the utilitarian social welfare function

$$\int_0^s U^h[\tilde{x}(h),0,0]\ f(h)dh + \int_s^1 U^h[x(h),v(h),c(y,V)]f(h)dh \tag{4}$$

subject to (3) by choice of $x(h)$, $\tilde{x}(h)$, $v(h)$, y and s (where $V = \int_s^1 v(h)f(h)dy)$. As it is assumed the government is free to choose any distribution of income it desires, the possibility of lump-sum transfers is also implicit.

Forming the Lagrangean for this problem, differentiating w.r.t. $\tilde{x}(h), x(h), v(h)$, y and s, respectively, and rearranging the first-order conditions yields

$$\tilde{U}_x^s[\tilde{x}(s),0,0] = U_x^s[x(s),v(s),c(\cdot)] \tag{5}$$

$$x(s)-\tilde{x}(s)-v(s)\int_s^1 -c_2(U_c^h/U_x^h)f(h)dh = (U^s/U_x^s)-\tilde{U}^s/\tilde{U}_x^s \tag{6}$$

$$\int_s^1 (U_c^h/U_x^h)c_1 f(h)dh = \frac{T_y}{T_x} \tag{7}$$

and

$$-(U_v^h/U_x^h) = -c_2\int_s^1 -(U_c^h/U_x^h)f(h)dh \tag{8}$$

In (5) - (8) and in the sequel, we omit the obvious functional arguments unless to do so causes confusion.

The optimal membership, facility provision and toll conditions (respectively (6), (7) and (8)) are given in Cornes and Sandler and Sandler and Tschirhart. (7) is the familiar Samuelson condition for public good provision. However, the membership condition is the one of most interest.[2/] In this, Consider first $U^s/U_x^s-\tilde{U}^s/\tilde{U}_x^s$, the value of the utility increment arising from membership for the marginal household. Given (5), this has the sign of, and is proportional to, $U^s - \tilde{U}^s \equiv \Delta U^s$. If $\Delta U^s \neq 0$, then the marginal household is not indifferent between club membership and otherwise, despite the population being distributed along a continuum and the government's presumed ability to vary continuously all the magnitudes $x(h)$, $\tilde{x}(h)$, $v(h)$, y and s. In fact, if $\Delta U^s > 0$ (and there is nothing in the analysis to preclude this), if the marginal man is in the club, he must be there by coercion. This would violate the voluntarism ennunciated so eloquently by the quoted authors. If $\Delta U^s > 0$ and the marginal man were excluded (and, by continuity, so must be some of his neighbours in utility), then membership must be being subjected to rationing by some unspecified mechanism.

A more fundamental flaw than the above is the following: the government's unmodelled but assumed ability to choose s directly conflicts with the principle of voluntarism and self-selection to membership which the quoted authors rightly stress.

Other shortcomings of this approach include its inability to relate club membership and visitation to household characteristics such as skill endowments or income.3/ Of course, this is a feature shared with many "first best" models wherein lump-sum transfers are implicit. We therefore proceed to develop a model in which membership is via self-selection and it is possible to rank-order the heterogeneous population. This enables us to confront Sandler and Tschirhart's (1980, p.1490) claim that "in practice, populations cannot be ordered".

II. SELF-SELECTION TO CLUB MEMBERSHIP

In this Section we will develop what could be regarded as the "demand" side of the club model. Initially, let households differ in endowed incomes but not tastes. Incomes $M^h \varepsilon [\underline{M}, \bar{M}]$ are distributed as $f(M^h)$. Then h's utility is given by

$$U[x^h, v^h, c(y, \int_{h\epsilon\zeta} v^h f(M^h)dM)] \tag{9}$$

Here, ζ is the set of club members and, if $h \epsilon \zeta$, $x^h = M^h$ and $v^h = 0 = c$. The latter assumption means that, in this paper, we follow the literature in assuming that there are no externalities or spillovers from clubs onto non-members. This will be relaxed in subsequent work.

Suppose the only club levies a per "visit" fee p and offers facility size y. Alternative institutional arrangements for determining p and y, the "supply" side of the model, will be discussed in Section III. We assume a single "visit" entitles a household to club membership - i.e., there is no capitation fee or standing charge. "Membership fee" and "per visit fee" will be used synonymously. Then, acting non-cooperatively, households would choose v^h to maximise utilities (9) subject to budget constraints

$$M^h - pv^h = x^h \tag{10}$$

As in all the congestion and club literature, we assume households ignore the effects of own visits on congestion or club quality, $c(\cdot)$, and only consider the effect of congestion on themselves.

To ease notation, let $U[M^h,0,0] \equiv \bar{U}^h$ while $U[\cdot]$ continues to refer to utility if a club member. Club members are h's for whom maximised utility $\geq U[M^h,0,0] \equiv \bar{U}^h$. The marginal member would have maximised utility of $\bar{U}^h$ inside the club. Thus, with h taking $c(\cdot)$ as parametric, maximisation by choice of v^h yields

$$-pU_1[M^h-pv^h,v^h,c(\cdot)]+U_2[M^h-pv^h,v^h,c(\cdot)] = 0 \tag{11}$$

The marginal h then will be the one with income M and optimal club visitation v^* for whom

$$U[M, 0,0]-U[M-pv^*,v^*,c(y, \int_{h\varepsilon\zeta} v(h)f(M^h)dM)] = 0 \tag{12}$$

where v^*, generally a function of p,M and $c(\cdot)$ (denoted $v^*(p,M,c)$), satisfies

$$-pU_1[M-pv^*,v^*,c(y, \int_{h\varepsilon\zeta} v(h)f(M^h)dM)] + U_2[\cdot] = 0 \tag{13}$$

Our first result gives conditions under which we can say unambiguously that if a household with a given income is indifferent between club membership and non-membership, then all households with greater income would wish to be members and all those with lower income would not wish to join.

Proposition 1. If $U_{12} \geqq 0$, $U_{13} \geqq 0$, then if the h with $M^h = M$ is indifferent between club membership and otherwise (i.e., satisfies (12)), all h's with $M^h > M$ belong to the club while those with $M^h < M$ do not join.

Proof. Consider the impact of an increase in M^h from the M satisfying (12). Then, differentiating through (12), we have (suppressing inessential functional arguments hereafter),

$$U_1(\cdot)[1-pdv^*(M)/dM] + U_2(\cdot)dv^*(M)/dM - \bar{U}_1 = U_1 - U_2 + \{U_2 - pU_1 dv^*/dM\}$$

$$= U_1[M-pv^*(M), v^*(M), c(\cdot)] - U_1[M,0,0] \tag{14}$$

using (13).[4/] Hence, given strict concavity of U w.r.t. the private good, if $U_{12} \geqq 0$ and $U_{13} \geqq 0$, $U_1[M-pv^*(M), v^*(M), c] - U_1[M,0,0] > 0$. Then the increase in M^h from M means the h with $M^h > M$ obtains greater maximised utility in the club than out. A symmetric argument indicates that if $M^h < M$, then h would prefer non-membership to club membership. Q.E.D.

If all h with $M^h > M$ are club members and those with $M > M^h$ are non-members, then

$$\int_{h\varepsilon\zeta} v^h f(M^h)dM = \int_M^{\bar{M}} v^h f(M^h)dM \tag{15}$$

Thus the membership conditions becomes (12) and (13) with (15) substituted in.

Proposition 1 is most important because it gives conditions under which the population with identical tastes can be rank-ordered by the most natural index, income. The conditions of the proposition will be met if, for example, $U[\cdot]$ takes any separable form $U[\cdot] = U^1(x)+U^2(v,c)$, e.g. $U^1(x)+vc(\cdot)$,[5/] and $U^2(\cdot)$ is not subject to an Inada condition. Perhaps equally interesting is the possibility that, if $U_{12} < 0$ and/or $U_{13} < 0$, this natural ordering fails. Then we could have both some relatively rich and some relatively poor h's which belong to the club. We consider in due course how Proposition 1 can be extended to a population differing in both incomes and tastes.

The formulation so far immediately enables us to explore how club utilisation or visitation varies with income. As arbitrary member h's optimal utilisation must satisfy

$$-pU_1[M^h-pv^h(M^h),v^h(M^h),c(y,\int_{M}^{\bar{M}} v^h(M^h)f(M^h)dM)]+U_2[\cdot] = 0 \qquad (11')$$

we must have

$$-p[U_{11}(1-pdv^h/dM^h)+U_{12}dv^h/dM^h]+(1-pdv^h/dM^h)U_{21}+U_{22}dv^h/dM^h = 0 \qquad (16)$$

or $$dv^h/dM^h = (pU_{11}-U_{12})/[p^2U_{11}- 2pU_{12}+U_{22}] \equiv N_1/D_1$$

$D_1 < 0$ by the second-order condition for h's optimisation. $U_{12} \geqq 0$ is sufficient for $N_1 > 0$, hence for $dv^h/dM^h > 0$. It can also be shown that, under the conditions of Proposition 1, club membership will increase with club quality, c, though members' utilisation need not increase with club quality. If, however, utilisation does increase with club quality, then club quality increases with the level of facility provision y, even allowing for the induced increase in members' utilisation.

Of greater importance than these latter comparative statics effects are the impacts of a variation in the membership fee upon club membership and utilisation. In fact, these two effects are linked. At a given level of visitation, an increase in p makes membership less attractive to any given individual. This is because it reduces feasible private consumption. Thus, unless the club good were Giffen, non-marginal men might be expected to at least reduce their visits. However, if all members reduce their visits, this can make membership more attractive insofar as congestion is reduced, hence quality is increased. If the second effect dominates the first, membership can be increased by an increase in the membership fee. An empirical counterpart of this would be where an increased toll reduced each driver's intensity of use of a toll road but increased the total number of drivers using the road at various times, given the reduction in congestion each confronts on average.

The algebra confirms the above intuition. To see this, note from (12), (13) and (15) that the marginal man satisfies

$$U[M(p,y)-pv^*(M(p,y)),v^*(\cdot),c(y,\int_{M(p,y)}^{\bar{M}} v^h(M^h,p,y)f(M^h)dM)]$$

$$= U[M(\cdot),0,0] = \bar{U} \qquad (12')$$

and

$$-pU_1[\cdot]+U_2[\cdot] = 0 \qquad (13')$$

Then, differentiating through (12') w.r.t. p , using (13') and rearranging, we have

$$dM/dp = [v^*U_1-U_3c_2\int_M^{\bar{M}}(\partial v^h/\partial p)f(M^h)dM]\ \Delta_1^{-1}$$ 6/

(where $\Delta_1 \equiv U_1-\bar{U}_1-U_3c_2v^*f(M))$.

In this, Δ_1 is unambiguously positive provided $U_{12}, U_{13} \geqq 0$, as assumed, Thus dM/dp has the sign of the RHS numerator. This is ambiguous in general. However, if $\int_M^{\bar{M}}(\partial v^h/\partial p)f(M^h)dM$ is sufficiently negative, given $c_2 < 0$, this might result in this numerator being negative, hence $dM/dp < 0$ (i.e., a p increase increases membership).

Now, $\int_M^{\bar{M}}(\partial v^h/\partial p)f(M^h)dM$ captures the impact on overall club utilisation of an increase in p , ignoring any induced change in club membership. As we have modelled the club good, it is a luxury good, hence it cannot be a Giffen good. Hence $\int_{M}^{\bar{M}} (\partial v^h/\partial p) f(M^h) dM < 0$ must be expected to hold. However, there is the complication that as p increases, if aggregate utilisation of the club declines, the increased per visit price would be paying for visits of increased quality. Thus, the Giffen analogy is not exact. Nevertheless, we will show presently that $\int_{M}^{\bar{M}} \partial v^h/\partial p f(M^h) dM < 0$ will normally be true.

First, the separate influences on individuals' visits can be clarified by considering how an arbitrary member's visits vary with p. Thus, differentiating through (11') w.r.t. p and rearranging, then using the expression for dM/dp above,

$$D_1 \partial v^h/\partial p = -v^h[pU_{11}^h - U_{21}^h] + [U_{13}^h + U_{23}^h]c_2 v^* f(M) \cdot [U_1 - U_3 c_2$$

$$\int_{M}^{\bar{M}} (\partial v^h/\partial p) f(M^h) dM] \Delta_1^{-1} - (U_{13}^h + U_{23}^h) c_2 \int_{M}^{\bar{M}} \partial v^h/\partial p \; f(M^h) dM$$

(where $D_1 \equiv p^2 U_{11}^h - 2pU_{12}^h + U_{23}^h < 0$ by the second-order conditions)

$$= [-v^h[pU_{11}^h - U_{21}^h]\Delta_1 + (U_{13}^h + U_{23}^h) c_2 (v^{*2} f(M) U^1 - (U_1 - \bar{U}_1)) \int_{M}^{\bar{M}} (\partial v^h/\partial p) f(M^h) dM] \Delta_1^{-1} .$$

We use the superscript h to distinguish between the h-man and the marginal man's utility at this point.

The former of the two expressions for $\partial v^h/\partial p$ is perhaps more illuminating. The first RHS term of this, which is negative (when divided by D_1), gives the income effect of the p increase. The second term captures the effect arising from the impact of a p increase on membership and, in turn, the effect of the membership change on quality. The final term is the effect arising from the impact on quality of a change in members' aggregate club visits. These last two terms are effectively the substitution effect as they reflect the consequences for demands for club visits of the change in the relative price of obtaining club quality.

We can now use the expressions for $\partial v^h/\partial p$, $h\varepsilon\zeta$, just given, to show $\int_M^{\bar{M}} \partial v^h/\partial df(M^h)dM < 0$ will normally be true. To see this, note

$$\partial v^h/\partial p=\{-v^h[pU_{11}^h-U_{21}^h]\Delta_1+(U_{13}^h+U_{23}^h)(c_2v^{*2}f(M)U_1-c_2(U_1-\bar{U}_1))\int_M^{\bar{M}}\partial v^h/\partial pf(M^h)dM\}D_1^{-1}\Delta_1^{-1},$$

where D_1 is specific to the h-man. Then integrating over $[M, \bar{M}]$,

$$\int_M^{\bar{M}}\partial v^h/\partial pf(M^h)dM= -\int_M^{\bar{M}}v^h[pU_{11}^h-U_{21}^h]D_1^{-1}f(M^h)dM$$

$$+ \Delta_1^{-1} \int_M^{\bar{M}}\partial v^h/\partial pf(M^h)dM \int_M^{\bar{M}}(U_{13}^h+U_{23}^h)D_1^{-1} f(M^h)dM .$$

Thus

$$[1-\Delta_1^{-1} \int_M^{\bar{M}} (U_{13}^h+U_{23}^h)D_1^{-1}f(M^h)dM] \int_M^{\bar{M}} \partial v^h/dpf(M^h)dM = - \int_M^{\bar{M}} v^h[pU_{11}^h-U_{21}^h]D_1^{-1}f(M^h)dM \ .$$

Now, the coefficient of $\int_M^{\bar{M}} \partial v^h/\partial pf(M^h)dM$ in this expression is positive unambiguously if $U_{13}, U_{23} \geqq 0$, given $D_1 < 0$ by the second-order conditions. We have already presumed $U_{13} \geqq 0$ and $U_{23} \geqq 0$ also seems reasonable: it merely requires that an increase in the quality of the club increases the marginal utility from club consumption. Thus, as the RHS of the last equation is negative provided $U_{12} \geqq 0$, as assumed, $\int_M^{\bar{M}} (\partial v^h/\partial p)f(M^h)dM < 0$.

Despite the above observations, to progress further in developing a tractable supply and demand theory of membership, we need to consider the more manageable case of fixed utilisation of the club by members.7/ Then all members' utilisation, v^h , may be normalised at unity for much of the subsequent analysis. It can be shown that Proposition 1 remains valid except that the arbitrarity chosen common utilisation level, $\bar{v}$, obviously will not be the level that all members would choose freely.

To establish this, note that the only condition defining the marginal h is now (12) with v at some arbitrary level $\bar{v}$. If we then consider $M^h > M$ there , $U[M^h-p\bar{v},\bar{v},c(y, \int_{h\varepsilon C} vf(M^h)dM)] > \bar{U}^h$ unambiguously, given precisely the conditions of Proposition 1. A symmetric argument applies for $M^h < M$ as before. Henceforth, therefore, we assume that if a given h is indifferent between membership and non-membership, all others with higher income will belong to the club and those with lower income will not.

Applying (15), total club utilisation is now $\bar{v}\int_{M}^{\bar{M}} f(M^h)dM = \bar{v}[1-F(M(\bar{v},p,c(\cdot)))]$ where $M(\bar{v}, p, c)$ is defined implicitly by the condition identifying the marginal man,

$$U[M(\bar{v},p,c),0,0] = U[M(\cdot)-p\bar{v},\bar{v},c(y,\bar{v}(1-F(M(\bar{v},p,c))))] \qquad (17)$$

This indicates M, hence club membership, depends on the fixed common level of utilisation $(\bar{v})$, the per visit fee (p) and the level of provision in the club (y). The precise nature of this dependence is summarised in Proposition 2:

<u>Proposition 2</u>. (i) $dM/dp > 0$; (ii) $dM/dy < 0$; (iii) $dM/d\bar{v}$ is amgiguous, if $U_{12} \gtreqless 0$, $U_{13} \gtreqless 0$.

<u>Proof</u>. Ordinary derivatives are used in the statement of the proposition to indicate that we are considering the <u>total</u> effects of varying p or y or $\bar{v}$., i.e. the direct effects plus any induced effects operating via the impact upon club quality, c. Despite the apparent infinite regress of having M depend on c which depends on M, and so on, what we do is very much like performing Keynesian multiplier analysis.

(i) Differentiating through (17) w.r.t. p, we have

$$\bar{U}_1 dM/dp = U_1[(dM/dp)-\bar{v}]-\bar{v}U_3c_2f(M)dM/dP \ . \quad \text{Hence}$$

$$dM/dp = \bar{v}U_1/\{U_1-\bar{U}_1-\bar{v}c_2U_3f(M)\} \equiv \bar{v}U_1/D_2 > 0 \tag{18}$$

as $\quad D_2 > 0$ if $U_{12}, U_{13} \geqq 0$ and $c_2 < 0, U_3 > 0$.

(ii) Differentating through (17) w.r.t. y ,

$$\bar{U}_1 dM/dp = U_1 dM/dy + U_3[c_1-\bar{v}c_2f(M)\ dM/dy] \text{ . Thus}$$

$$dM/dy = -\ c_1U_3D_2^{-1} < 0 \tag{19}$$

as $c_1 > 0$. (iii) Differentiating through (17) w.r.t. $\bar{v}$ and rearranging,

$$dM/d\bar{v} = \{pU_1-U_2-c_2U_3(1-F(M))\}\ D_2^{-1} \equiv N_2D_2^{-1} = (?)(+) \tag{20}$$

Q.E.D.

A decrease in the marginal household's income means that the number of club members will have increased. Therefore, an increase in the visitation fee and the level of facility provision, respectively, will have the anticipated effects of decreasing membership and increasing membership, respectively.

It is worth contrasting dM/dp with fixed utilisation with dM/dp with variable utilisation of the club. We see immediately that it is the opportunity to substitute away from the now relatively more expensive visits to private consumption, and the effect of this in enhancing club quality via reducing congestion, which produced the earlier ambiguity.

The ambiguity of the response of membership to an increase in the level of visits to which members are rationed arises because of the threefold way $\bar{v}$ influences each member's utility: directly and positively via h's visitation; indirectly and negatively via the <u>ceteris paribus</u> impact of all member's increased visitation on congestion, hence club quality and, of course, directly and negatively via the impact on the budget for private consumption. Furthermore, $\bar{v}$ prior to the increase might have been already too high for some club members and too low for others. These two groups would generally respond differently to the $\bar{v}$ increase. Those for whom $\bar{v}$ was too low would be less likely to be induced to leave the club than their counterparts. However, despite this ambiguity, it is easy to find instances where an increase in $\bar{v}$ will decrease membership, e.g., when $U = U^1(M^h - p\bar{v}) + \bar{v}c(y,(1-F(M))\bar{v})$ for member h and $c(\cdot)$ is a specialisation of the form used throughout the congestion literature, $c(\cdot) = y/(1-F(M))\bar{v}$. 8/

In the special quasilinear utility case with the specified congestion function above, $U[\cdot] = U^1(M^h - p\bar{v}) + y/(1-F(M))$ for member h. Thus, the increased relative weight given to club consumption vis-a-vis private consumption as $\bar{v}$ increases exactly cancels with the effect of increased $\bar{v}$ on congestion at given y. What remains is only the negative effect on utility arising from the loss of private consumption due to the increased commitment to club expenditure for those remaining in the club. Hence the previously marginal man and some of his neighbours in utility must find it optimal to leave the club.

It is interesting to observe how pre-specifying a high level of visitation at a given p might act to curtail membership just as raising the price at a given level of visitation does. This is because the two policies, despite their differences, have a common element: they both raise members' unavoidable financial commitments to the club. Given the club good is modelled as a luxury, as this commitment increases, it must be expected to make non-membership more desirable to the relatively poor. Overall, this observation just reflects the fact that, in general, clubs could compete in either price, or permitted visitations, or both.

III. THE DETERMINATION OF p AND y

The membership equilibrium results from individual households self-selecting themselves for given p, y and $\bar{v}$. In the process, they determine $c(y,(1-F(M))\bar{v})$ by their joint actions. The actual equilibrium is likely to be attained iteratively. For example, suppose the club is a tolled high-speed trunk road between A and B. When initially opened, the road might well experience considerable congestion because "all-comers" will attempt to use it. But gradually those with the greatest aversion to congestion at the given toll will cease to use the road in the light of their previous experiences and public information about the usual levels of traffic. Those with relatively high willingness to pay will be the ones who remain.

The parameters p and y (and $\bar{v}$) which govern the self-selection process just outlined are themselves subject to choice. Who makes these choices depends upon the institutional arrangements for the provision of the club good. These institutional arrangements constitute the "supply" side of the club model.

We will consider three institutional environments. These are provision by a profit-maximising monopoly, by a revenue-constrained welfare-maximising government, and the social organisation of a given facility of fixed size.

(a) The profit-maximising monopoly club

Let $K(y)$ denote the convex cost of providing club size y in all environments. The monopoly club[9/] would seek to maximise profits, $\pi(p, y)$, given by

$$\max_{p,y} \pi(p,y) = p[1-F(M(\bar{v},p,c(y,(1-F(M))\bar{v})))]\bar{v} - K(y) \quad (21)$$

Normalise $\bar{v}$ at unity. The monopolist's optimum satisfies

$$\text{(i)}\ [1-F(M)]-pf(M)dM/dp = 0\ ;\ \text{(ii)}\ -pf(M)dM/dy - K'(y) = 0 \quad (22)$$

(second-order conditions here, as elsewhere, being presumed satisfied), dM/dp and dM/dy being given by (18) and (19) and we again use these ordinary derivatives to indicate total effects of p and y variations, as before. M satisfies (17).

Substituting (18) and (19) into (22) (i) and (ii) respectively,

$$1-F(M) = pf(M)U_1/D_2 \tag{23}$$

$$K'(y) = pf(M)U_3c_1/D_2 \tag{24}$$

(23) implies

$$p = (1-F(M))[U_1-\bar{U}_1-c_2U_3f(M)]/f(M)U_1 \tag{25}$$

while substituting (25) into (24) yields

$$K'(y) = (1-F(M))c_1U_3/U_1 \tag{26}$$

(25) is the fee condition, (26) the provision condition and (17) the membership condition. All three would have to be solved simultaneously to obtain the optimal p , y and M . Note that all the optimality conditions are expressed in terms of magnitudes specific to the marginal man. This is because the non-discriminating monopolist ignores differences in the valuation of congestion or club quality and considers only the decrease in price necessary to induce an additional club member to join.[10/]

(25), the fee condition, is actually the unitary elasticity condition for the marginal revenue arising from a variation in the membership fee to be zero at the optimum. Equivalently, it is the condition for the revenue derived from the marginal member to just equal the revenue lost from other members. But, to first order, the latter is simply the price reduction necessary to elicit the marginal h's membership, given c, times the existing membership. Thus the term $[U_1-\bar{U}_1-c_2f(M)/U_3]/U_1$, or dp/dM, which recurs constantly in the sequel, is merely the price reduction required to elicit an incremental h's membership.

The provision condition (26) is akin to the familiar Samuelsonian $\Sigma MRS = MRT$ condition for efficient supply of collectively consumed goods with the important proviso noted above: the monopolist does not take into account all club members' total willingness to pay for club facilities, which it is unable to extract, but, rather, acts as if all are identical as it is only able to charge them uniformly. Note that what the marginal household pays for membership is precisely what membership is worth to it as it obtains no surplus. Thus p is its compensating variation in income for membership. All other members must be earning positive surplus at the given p. However, the essence of the club congestion feature is that aggregate surplus need not increase with membership because quality is deteriorating.

(b) The Revenue-Constrained Welfare-Maximising Club

If monopoly rights are not vested in one individual, a utilitarian government might run the club in a manner which maximised overall welfare without subsidies or surpluses. In that event, it would seek to

$$\underset{p,y}{\text{Max}} \int_{M}^{\bar{M}} U[M^h-p,1,c]f(M^h)dM + \int_{\underline{M}}^{M} U[M^h,0,0]f(M^h)dM \tag{27}$$

subject to the breakeven or budget balance constraint,

$$p[1-F(M(p,1,c))] = K(y) \tag{28}$$

Formulating the Lagrangean and differentiating yields first-order conditions

$$-\int_{M}^{\bar{M}} \{U_1+c_2U_3f(M)dM/dp\}f(M^h)dM+\lambda[(1-F(M(p,1,c)))-pf(M)dM/dp] = 0 \tag{29}$$

$$\int_{M}^{\bar{M}} \{U_3(c_1-c_2f(M)dM/dy)\}f(M^h)dM+\lambda[-pf(M)\ dM/dY-K'(y)] = 0 \tag{30}$$

λ being the multiplier on the break-even constraint, together with (28) and (17). Eliminating λ from (29) and (30) gives

$$\frac{\int_{M}^{\bar{M}} U_1f(M^h)\ dM}{[c_1-c_2f(M)dM/dy]\ \int_{M}^{\bar{M}} U_3f(M^h)dM} + \frac{c_2f(M)\ dM/dp}{c_1-c_2f(M)dM/dy} = \frac{(1-F(M))-pf(M)dM/dp}{pf(M)dM/dy+K'(y)} \tag{31}$$

Now, $$\int_{M}^{\bar{M}} U_1 f(M^h)dM / \int_{M}^{\bar{M}} U_3 f(M^h)dM = \left[U \int_{M}^{\bar{M}} U_3(M^h)f(M^h)dM / \int_{M}^{\bar{M}} U_1 f(M^h)dM \right]^{-1}$$

$$= (1-F(M)) / \int_{M}^{\bar{M}} (\beta^h/\bar{\beta})\ MRS^h_{x,c} f(M^h)dM,$$

where $\beta^h \equiv U_1[M^h-p,1,c], \bar{\beta} \equiv (1-F(M))^{-1} \int_{M}^{\bar{M}} U_1 f(M^h)dM$

and $MRS^h_{x,c} \equiv U_3/U_1$. Thus, from (31),

$$\frac{1-F(M)+c_2 f(M)dM/dp \int_{M}^{\bar{M}} (\beta^h/\bar{\beta})MRS^h_{x,c} f(M^h)dM}{[c_1-c_2 f(M)dM/dy] \int_{M}^{\bar{M}} (\beta^h/\bar{\beta})\ MRS^h_{x,c} f(M^h)dM} = \frac{(1-F(M))-pf(M)dM/dp}{pf(M)dM/dy+K'(y)} \tag{34}$$

In (32), $c_1-c_2 f(M)\ dM/dy = c_1[U_1-\bar{U}_1]/D_2$ (> 0 provided $U_{12} \geqq 0$, $U_{13} \geqq 0$, as assumed) is the net impact upon club quality of an increase in the facility size and incorporates the degradation of quality arising from the induced effect upon membership. Thus, from (30), $\lambda[pf(M)\ dM/dy + K'(y)] > 0$. Hence, using (28), $\lambda[K(y)/y][(yf(M)dM/dy/(1-F(M)))+yK'(y)/K(y)] > 0$ and

$$\lambda \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \quad \text{as} \quad [-\eta_{M,y}+ S_c^{-1}] \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \tag{33}$$

where $\eta_{My} \equiv -yf(M)dM/dy/(1-F(M))$ is the elasticity of club membership with respect to facility provision and $S_c \equiv K(y)/yK'(y)$ is the usual measure of scale economy, here in the provision of y.

A positive social marginal utility of money $(\lambda > 0)$ implies and is implied by $-\eta_{M,y} + S_c^{-1} > 0$, i.e. by an increase in y resulting in a decrease in net revenue, p held constant. It seems reasonable to believe this would hold. Then, from (32),

$$(1-F(M))-(-c_2 f(M)dM/dp \int_M^{\bar{M}} (\beta^h/\bar{\beta})MRS^h_{x,c} f(M^h)dM)\{\gtreqless\}\ 0$$

as

$$(1-F(M))[1-pf(M)dM/dp/(1-F(M))] \equiv (1-F(M))[1+\eta_{M,p}]\{\gtreqless\}\ 0 \qquad (34)$$

where $\eta_{M,p}(< 0)$ is the elasticity of club membership with respect to the membership fee.

Now, in (34), $(1-F(M))[1 + \eta_{M,p}]$ is the impact upon the club's revenues from an increase in the fee, y held constant. Given this, (34) has an interesting interpretation. Consider, e.g., the case where there is positive marginal revenue from an increase in p at the optimum $(1+\eta_{M,p} > 0)$. Then (34) says the extra revenue from a p increase from the existing club members $((1-F(M)))$ is more than they would pay for the induced change in club quality arising solely from the <u>fall</u> in membership induced by the fee increase $((-c_2 f(M)dM/dp \int_M^{\bar{M}} (\beta^h/\bar{\beta})MRS^h_{x,c} f(M^h)dM))$. This is to be expected because, to satisfy the breakeven constraint, the extra revenue is used to pay for an increased y and the difference is accounted for by the value of the net change in quality induced by a y change.

There are four analytically distinct effects associated with a fee change at the optimum: (i) y must vary to maintain budget balance; (ii) the p change directly induces a membership change with associated effects on club congestion and quality; (iii) the y change directly induces a change in membership with associated congestion effects; (iv) the revenue change from the p change, when used to finance a y change, directly improves club quality. Overall, one would expect the optimum to be characterised as follows: when these four effects are taken into account, if p changes, the resulting change in club expenditure must just equal the social valuation of the change in club quality which it induces. One further modification to (32) enables us to obtain precisely this characterisation.

Cross multiplying through (32 yields, after simplification,

$$(1-F(M))[pf(M)dM/dy+K'(y)]/K'(y) = \{[(1-F(M))-pf(M)dM/dp]c_1/K'(y)$$

$$-(1-F(M))c_2f(M)dM/dy/K'(y)-c_2f(M)dM/dp\}\int_M^{\bar{M}}(\beta^h/\bar{\beta})MRS^h_{x,c}f(M^h)dM \qquad (35)$$

In this, as $c_1/K'(y)$ is the impact on club quality per unit of expenditure upon y, $[(1-F(M)) - pf(M)dM/dp]c_1/K'(y)$ is the direct impact on quality were p increased and the resulting increased revenue used to finance a y increase (effect (iv)). $-c_2f(M)dM/dp$ is the impact on quality of a p increase, arising from the induced change in club membership, hence congestion (effect (ii)). Next,

$-c_2 f(M)dM/dy$ is the impact on quality from a change in membership induced by a marginal change in y. Thus $-c_2 f(M)dM/dy/K'(y)$ $= c_2 f(M)(dM/dy)(dy/dK)$ is the impact on quality from the change in membership induced by the y change associated with an extra unit of expenditure on y. Hence, as a marginal change in p produces extra revenue (hence expenditure) of $(1-F(M))$ before taking account of any induced membership changes, $-(1-F(M))c_2 f(M)dM/dy/K'(y)$ is the impact on quality from the membership change induced by the extra expenditure on y following a p increase (effect (iii)). The sum of these quality changes, when multiplied by $\int_{M}^{\bar{M}} (\beta^h/\bar{\beta})MRS^h_{x,c} f(M^h)dM$, the club members' aggregate valuation of a marginal change in quality, gives the members' valuation of the total change in club quality induced by a p increase which generates expenditure on y to satisfy the breakeven constraint.

Finally, we have the LHS (35) term. Here, $[pf(M)dM/dy+K'(y)]$ is the absolute value of the impact on the club's revenue from an increase in facility size, y. Thus $[pf(M)dM/dy+K'(y)]/K'(y)$ $= [pf(M)dM/dy+K'(y)]dy/dK$ is the absolute change in revenue per unit of expenditure on y. Hence, given our interpretation of $(1-F(M))$ immediately above, $(1-F(M))[pf(M)dM/dy+K'(y)]/K'(y)$ is the total impact on club revenue arising from increased expenditure on y after the increase in p (effect (i)).

Overall, therefore, (35) tells us precisely what we expect: at the optimum a marginal change in p generates a level of expenditure on the club which is just equal to the members' valuation of the quality changes which it induces. (17), (28) and (35), which have to be solved simultaneously, are now the membership, toll/fee and provision conditions, respectively.

(c) The Social Organisation of a Facility of Fixed Size

If the government inherits a club of a given unalterable size, $\bar{y}$ (e.g., a trunk road), it might seek merely to constrain membership by levying an entrance fee. This is the standard setting of the congestion problem - except that we do not posit the presence of an alternative "club" for non-members or spillovers from club congestion onto non-members. Membership would be constrained to the level maximising the aggregate welfare of both members and non-members taking into account the congestion externality members impose on each other. In that event, the government chooses p without regard to budget balance according to

$$\underset{p}{\text{Max.}} \int_{\underline{M}}^{M} U[M^h,0,0]f(M^h)dM + \int_{M}^{\bar{M}} U[M^h-p,1,c(\bar{y},(1-F(M)))]f(M^h)dM \quad (36)$$

We assume that any revenues which the government derives from operating the club are disbursed in ways not affecting welfares in this model. Its problem then yields first-order condition (for $p > 0$)

$$- \int_{M}^{\bar{M}} \{U_1[M^h-p,1,c(\bar{y},(1-F(M)))] + U_3[M^h-p,1,c(\bar{y},(1-F(M)))]c_2f(M)dM/dp\}f(M^h)dM = 0 \quad (37)$$

By a similar route to that in deriving (35), we obtain

$$(1-F(M))[U_1-\bar{U}_1-c_2U_3f(M)]/U_1 = -c_2f(M)\int_M^{\bar{M}}(\beta^h/\bar{\beta})MRS^h_{x,c}f(M^h)dM \qquad (38)$$

where the symbols represent magnitude analogous to those used previously. Recalling (25) above, the LHS (38) is the extra revenue derived from the existing club members when price is increased sufficiently to induce the marginal man to leave the club. At the optimum, then, this must equal the club members' valuation of the increase in quality arising from the exclusion of the marginal man. I.e., what members collectively would pay to exclude the marginal man is what they would have to pay. This outcome is very much like a perfectly discriminating solution as it assumes the government is able to extract all of club members' willingness to pay for marginal quality improvements.

IV. HETEROGENEITY IN TASTES AND IN INCOMES

When considering the case of heterogeneity (in tastes or other unspecified dimensions) it has been usual in the clubs literature to consider the segregation issue. Thus, the question is posed: will population members congregate in mixed clubs or will they segregate according to type into homogeneous clubs with different characteristics?[11/] In this paper we will proceed as if the many clubs outcome is infeasible. For example, it might be environmentally, hence politically, difficult to have two or more trunk roads between locations A and B. While this could often result in the creation of other clubs - e.g., for scheduled flights and a railway between A and B - such additional clubs, themselves mixed, usually coexist with a mixed club of trunk road users. It is therefore of interest to see what can be said about the membership composition in such mixed clubs.

For simplicity, we will confine attention to a case with two types of individuals, G and F, types being demarcated according to tastes or utility functions. Individuals of a particular type are, however, heterogeneous w.r.t. income in the manner discussed already. It will be readily apparent that what we say below can be extended, in principle, to any number of types.

Suppose, in transparent notation, (F,f,U^F) and (G,g,U^G) represent the two types' distribution, density and utility functions, respectively. Let $M^i, i = F,G$, be the income of the marginal man of type i.

We will assume club members' utilisation of the single club is fixed and common to all at unit level so that club congestion depends only upon aggregate membership. Club quality at a given level of provision, y, and aggregate usage, V, is then given by $c(y, V)$ where, now, V = aggregate membership of the two types. In that event, M^i satisfies

$$U^i[M^i-p,1,c(y,V)] = U^i[M^i,0,0], i = F,G \tag{39}$$

Clearly, the same argument as in Proposition 1 goes through: if $U^i_{12}, U^i_{13} \geqq 0$, $i = F,G$, if someone of a particular type is marginal, then all men with incomes greater than his will be members of the mixed club and those with lower income will be non-members. Thus we may write

$$V = [1-F(M^F)]+[1-G(M^G)]$$ [12/]

The form that aggregate membership takes with heterogeneous tastes inevitably has implications for the ranking of members by income. Willingness to pay for membership clearly decreases with income at given p and $c(\cdot)$ (certainly if $U^i_{12}, U^i_{13} \geqq 0$, $i = F,G$). But at given income and p and c, individuals of different types will differ in their propensity to join the club as measured by their MRS's. Thus the range of incomes for club members from the two types will generally overlap. The lowest income for a club member of the type with the greater propensity to join the club will typically be much lower than that for the type with the lower propensity. This means that some individuals with relatively high incomes in the latter group will not be club members while some with relatively low (and lower) incomes in the former group will be.

With two types of individuals there would seem to be, also, many ways in which any given overall level of membership, V, might be attained: in fact, by all combination of $V^F = 1-F(M^F)$ and of $V^G = 1-G(M^G)$ satisfying $V^G+V^F = V$. However, the following proposition establishes that the composition of a membership of given size is unique under the conditions of p1.

<u>Proposition 3</u>. If $U^i_{12}, U^i_{13} \geqq 0$, $i = F,G$, the composition of a membership of given size is unique.

Proof. Suppose $V = (1-F(M^{F1}))+(1-G(M^{G1})) \equiv V^{F1}+V^{G1}$, where V^{i1} as defined, $i = F,G$, satisfy

$$U^i[M^{i1}-p,1,c(y,V)] = U^i[M^{i1}-p,1,c(y,1-F(M^{F1})+1-G(M^{G1}))]$$

$$= U^i[M^{i1},0,0], \; i = F,G \tag{40}$$

Suppose w.l.o.g. we consider a $V^{F2} > V^{F1}$ and $V^{G2} < V^{G1}$ so that $V^{F1}+V^{G1} = V^{F2}+V^{G2} = V$. If $(1-F(M^{F2}) = V^{F2}$, then the M^{F2} man must satisfy

$$U^F[M^{F2}-p,1,c(y,V)] = U^F[M^{F2},0,0] \tag{41}$$

But, given our assumptions, M^{i1} is the only one satisfying the outer equality in (40). Thus (41) cannot hold for $M^{F2} \neq M^{F1}$ at the arbitrary V . An analogous argument holds for $M^{G2} \neq M^{G1}$. Hence the composition of any given membership size is unique. Q.E.D.

Uniqueness of the membership composition at arbitrary y and p is particularly important because it enables us to obtain the comparative statics of changes in y and p when tastes are heterogeneous, just as when they are homogenous. Thus, in principle, the analysis of alternative institutional environments for the determination of p and y with heterogeneity in tastes could be conducted as in Section III. However, for brevity, we will only derive and discuss the comparative statics of heterogeneous tastes within an arbitrary environment (the "demand' side) here. Analysis of the determination of y and p in different institutional contexts will be deferred to another paper.

The comparative statics of membership with heterogeneous tastes are of special interest because they allow for the possibility of substitution between types in membership when, in particular, p increases. These comparative statics are presented as Proposition 4.

Proposition 4. Aggregate membership (i) increases unambiguously with an increase in y $(dV/dy > 0)$, (ii) decreases unambiguously with an increase in $p (dV/dp < 0)$ but (iii) the membership of neither type decreases unambiguously with a p increase. However, if both types are uniformly distributed on $[\underline{M}, \bar{M}]$ and the marginal type F values club facilties no more than the marginal type G (i.e., $U_3^F(M^F-p,.)/U_1^F(M^F-p,.) \leqq U_3^G(M^G-p,.)/U_1^G(M^G-p,.))$, then $dV^F/dp < 0$ unambiguously. Likewise, if the marginal type G values club facilities no more than the marginal type F, then $dV^G/dp < 0$ unambiguously.

Proof. (i) The marginal men satisfy (40). Differentiating through w.r.t. y,

$$U_1^F(M^F-p,.)dM^F/dy+U_3^F(M^F-p,.)[c_1-c_2(f(M^F)dM^f/dy+g(M^G)dM^G/dy)]$$

$$=U_1^F(M^F,0,0)dM^F/dy \tag{41}$$

$$U_1^G(M^G-p,.)dM^G/dy+U_3^G(M^G-p,.)[c_1-c_2(f(M^f)dM^F/dy+g(M^G)dM^G/dy)]$$

$$= U_1^G(M^G,0,0)dM^G/dy \tag{42}$$

Thus, following earlier notional conventions (e.g., $\bar{U}_1^F = \partial U^F(M^F,0,0)/\partial M$),

$$\frac{dM^F/dy\,[U_1^F - U_3^F c_2 f(M^F) - \bar{U}_1^F] + c_1 U_3^F}{U_3^F c_2 g(M^G)} = dM^G/dy \tag{43}$$

By symmetry,

$$\frac{dM^G/dy\,[U_1^G - U_3^G c_2 g(M^G) - \bar{U}_1^G] + c_1 U_3^G}{U_3^G c_2 f(M^F)} = dM^F/dy \tag{44}$$

Substituting (44) into (43) and simplifying,

$$dM^G/dy\{U_3^G U_3^F c_2^2 f(M^F) g(M^G) - [U_1^F - U_3^F c_2 f(M^F) - \bar{U}_1^F][U_1^G - U_3^G c_2 g(M^G) - \bar{U}_1^G]\}$$

$$\equiv dM^G/dy.D_3 = c_1 c_2 U_3^F U_3^G f(M^F) + c_1 U_3^G [U_1^F - U_3^F c_2 f(M^F) - \bar{U}_1^F] = c_1 U_3^G [U_1^F - \bar{U}_1^F] \tag{45}$$

By symmetry,

$$D_3 dM^F/dy = c_1 U_3^F [U_1^G - \bar{U}_1^G] \tag{46}$$

In (45), the coefficient of dM^G/dy is unambiguously negative while the RHS (45) is positive provided $U_{12}^i, U_{13}^i \geq 0$. Thus $dM^G/dy < 0$ and $dV^G/dy = -g(M^G) dM^G/dy > 0$. Similarly reasoning shows $dV^F/dy > 0$.

Hence $dV/dy = d[V^G+V^F)/dy > 0$.

(ii) Differentiating through (40) w.r.t. p and following an identical route to part (i), we obtain

$$D_3 dM^G/dp = U_1^F U_3^G c_2 f(M^F) + [U_1^F - \bar{U}_1^F - c_2 f(M^F) U_3^F] U_1^G \tag{47}$$

and

$$D_3 dM^F/dp = U_1^G U_3^F c_2 g(M^G) + [U_1^G - \bar{U}_1^G - c_2 g(M^G) U_3^G] U_1^F \tag{48}$$

Now, $dV/dp = -[f(M^f)(dM^F/dp)+g(M^G)(dM^G/dp)]$. Thus, using (47) and (48),

$$-dV/dp = g(M^G)D_3^{-1}\{U_1^F U_3^G c_2 f(M^F)+[U_1^F-\bar{U}_1^F-c_2 f(M^F)U_3^F]U_1^G\}+f(M^F)D_3^{-1}\{U_1^G U_3^F c_2 g(M^G)$$
$$+ [U_1^G-\bar{U}_1^G-U_3^G c_2 g(M^G)]U_1^F\}=\{g(M^G)[U_1^F-\bar{U}_1^F]+f(M^F)[U_1^G-\bar{U}_1^G]\}D_3^{-1} > 0$$

after simplification. Hence an increase in p decreases aggregate membership.

(iii) As $D_3 > 0$, dM^G/dp and dM^F/dp have the signs of the RHS (47) and RHS (48), respectively. These signs are ambiguous in general (although, from part (ii), at least one must be positive). Consider the RHS (47) first. This equals $c_2[U_1^F U_3^G f(M^F) - U_1^G U_3^F g(M^G)] + [U_1^F - \bar{U}_1^F)]\, U_1^G$. The second term of this is positive. Also,

$c_2\{U_1^F U_3^G f(M^F) - U_1^G U_3^F g(M^G)\}\{\gtreqless\}0$ as $MRS^G_{x,c}/MRS^F_{x,c}\{\lesseqgtr\}g(M^G)/f(M^F)$.
Thus $MRS^G_{x,c}/MRS^F_{x,c} \leq g(M^G)/g(M^F)$ is sufficient for $dM^G/dp > 0$,

hence for $dV^G/dp < 0$. Consider the case where both types' incomes are uniformly distributed on $[\underline{M}, \bar{M}]$, thus $g(M^G)/f(M^F) = 1$. Then $MRS^G_{x,c} \leqq MRS^F_{x,c}$ is sufficient for $dV^G/dp < 0$. An identical argument applied to (48) indicates $MRS^G_{x,c}/MRS^F_{x,c} \geqq g(M^G)/f(M^F)$ is sufficient for $dV^F/dp < 0$. Thus dV^F/dp and dV^G/dp cannot both be unambiguous unless $MRS^G_{x,c}/MRS^F_{x,c} = g(M^G)/f(M^F)$, in which case they would both be negative. Q.E.D.

P4's intuition can best be understood in the context of a concrete example. Suppose the club is a tolled trunk road. Then an increase in its size, (e.g., increased number of lanes and/or toll booths at each fee point) increase quality and increases the number of users. An increase in the toll at a given size of the road reduces overall use. However, some previously non-using individuals from the group with the greater marginal willingness to pay for quality or reduction in congestion might take advantage of the overall reduction in traffic to become users, despite the increased toll. This last, inter-group substitution effect is a feature absent from the analysis of just one type.

The policy relevance of our analysis is immediate and obvious in the context of transportation in the UK. There, at the time of writing, plans are being considered: (a) to build some "high-speed" inter-city toll roads to reduce motorway congestion; (b) to impose charges (additional to parking fees) on people driving into Central London. It seems clear that, as the plans' detractors argue, the "club" of fee-paying drivers in both cases generally will be from relatively high income groups. However, some of the relatively poor with a preference for driving over other transport forms might also be members of the respective "clubs".

V. CONCLUSIONS

Clubs are an important and pervasive phenomenon and voluntarism or self-selection is integral to the determination of membership. This paper has provided a treatment of membership in which voluntarism is of the essence and has been made operational.[13/] This contrasts with most if not all of the existing literature. In the latter, either homogeneity of the population of potential club members is assumed, effectively by-passing the self-selection issue, or ranking of heterogeneous potential members by some unexplained *deus ex machina* occurs. In this paper, ranking of individuals is by perhaps the most natural index, income, although this can be complicated slightly when both tastes and incomes are heterogeneous. This ranking is very important because it seems likely to provide a workable basis for empirical research. Cornes and Sandler (1986, p.273) note the dearth of empirical work on clubs when they state:

> "Very little empirical estimation has been applied to clubs;... For example, the relationship between members' income levels and the choice of provision and membership size has never been adequately tested."

We have provided a framework within which such relationships can be elucidated and also a set of comparative statics results which should be susceptible to empirical investigation.

There is perhaps one partial exception to the otherwise general neglect of an explicit treatment of voluntarism or self-selection in determining club membership. This is provided by the game-theoretic approach of Pauly (1967, 1970) and followers. These treat membership in terms of the game-theoretic concept of the core.

There are two essential differences between the game - theoretic and non-game formulations of membership. First, in the game formulation, membership, provision and (full-) finance conditions are solved simultaneously and the toll condition subsequently. Different person-specific tolls are compatible with a given solution to the other conditions. In the non-game approach, membership, provision, toll and finance are settled simultaneously. Second, the game formulation has not yet really addressed the membership heterogeneity issue adequately. Pauly and others established the existence of a core for a heterogeneous population on the presumption that the population can be partitioned into a set of homogenous clubs. However, this presumption seems a negation of the very issues, such as intergroup substitution possibilities, motivating interest in heterogeneous populations and mixed clubs. Furthermore, the core approach has not, as yet, afforded us the comparative statics insights so rightly prized by applied economists.

The institutional arrangement for delivering club facilities which appears closest in spirit to the concept of core allocations is that of member-managed clubs. Here, members would collectively and simultaneously choose y and p, given the requirements of full financing and self-selection to membership. In equilibrium, the characteristics and membership of the club would be such that no individual or group has the incentive to change the club specification or membership composition.

Clearly, there are timing problems associated with the membership choosing a level of provision and fee which, given full financing and self-selection, determines itself. These timing problems are very similar to those which lead to separation of provision and toll decisions in the game formulation. Evidently, a two-stage or sequential equilibrium analysis is required. However, the timing problems are attenuated when an external agent - the monopolist or the benevolent government - chooses p and y and lets the membership self-select itself, perhaps iteratively, over time. This is what we assumed in the main body of the paper. The special problems of member-managed clubs will be pursued in a subsequent paper.

There are many other directions in which our model can be extended. Some, such as competing clubs - e.g., for travel by train, plane or tolled trunk road - have been mentioned already. Of special interest, we feel, would be further analysis of the variable utilisation case. This would allow for incorporation of more general fee structures. In particular, we could then consider the case of two-part fees: a per annual membership fee alongside a per visit fee. Formally, this is equivalent to having members pay a per visit fee or toll and purchase an enabling good (e.g., a car in the event of the club being a tolled road). This and other extensions will be pursued elsewhere.

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FOOTNOTES

*/ This paper has benefitted from comments by seminar participants at Aarhus, Copenhagen and Warwick universities, especially Jonathan Cave, Birgit Grodal, Peter Law, Yew Kwang Ng and Karl Vind. The usual caveats apply.

1/ A number of authors - notably Edelson (1971), Freeman III and Haveman (1977), Berglas and Pines (1981), Hillman and Swan (1983) and Berglas (1984) - do, apparently, deal with situations incorporating individual differences. However, these treatments are characterised by one or other of two deficiencies. First is the presumption that the population can be ranked according to willingness to pay for the club good without regard to how this ranking is to be established (Edelson, Freeman and Haveman); second is the focus only upon Pareto-optimal club membership, again without regard to any institutional arrangements for achieving this outcome (Berglas and Pines (1981), Berglas (1984), Hillman and Swan (1983). Neither category of papers produces an operational analysis which enables us, for example, to relate club membership to objective characteristics such as potential members' incomes.

2/ This membership condition was actually first derived by Artle and Averous (1973). It was rediscovered by Helpman and Hillman (1977). See also Ng (1978).

3/ To be fair, it might be argued that the contributors to the club literature have had other concerns. Three issues in particular have received much attention. These are the integer problem (i.e., whether or not the population divides exactly between the available clubs), the relative efficiencies of homogenous and mixed clubs, and whether or not centralised provision of club facilities is superior to market provision. We have nothing to say on these issues here although we hope to return to them in subsequent work utilising our framework.

4/ Of course, this is an "envelope" result.

5/ These separable forms are cases of Edgeworth-Pareto independence between club and private goods. The latter quasilinear case has the desirable feature that if $v = 0$, $u[x,0,c] = U[x,0,0]$ automatically.

6/ The ordinary derivative is used for the <u>total</u> impact of p on M to indicate that we are considering not just direct effects but also indirect effects operating via the induced change in club quality, c.

7/ To motivate the fixed utilisation case, consider the cases of the club being a trunk road whose members make one return journey per day, or a classical concert.

8/ See Berglas and Pines (1981), in particular, for an extensive and illuminating discussion of the congestion function. To see how the result claimed arises with the given function, note that $U_1 = U^{1'}$, $U_2 = c$, $U_3 = \bar{v}$, $c_1 = [(1-F(M))\ \bar{v}]^{-1}$, $c_2 = -y[(1-F(M))\bar{v}]^{-2}$ now. Substituting these into (20), we have $N_2 = pU^{1'} - y[(1-F(M))\bar{v}]^{-1} + \bar{v}y(1-F(M))[(1-F(M)\bar{v}]^{-2} = pU^{1'} > 0$.

9/ This monopoly status could derive from some specific endowment - e.g., the ownership of the spa in a spa town or the sole toll road between locations A and B . Who is the monopolist, and the destination of monopoly rent, are issues ignored in this paper.

10/ See Edelson (1971, p.879) on this point.

11/ This is still an unsettled issue. See, e.g., Berglas and Pines (1984), Sandler and Tschirhart (1984) and Cornes and Sandler (1986) for elements of the controversy.

12/ For simplicity, but without loss of generality, we have normalised the population of both types to unity.

13/ The model which we have employed extends the one developed in Fraser (1989) to analyse voluntarily-consumed public goods such as visual broadcasting.

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