International Trade and Cournot Equilibrium:

Existence, Uniqueness and Comparative Statics*

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Abstract

This paper proves the existence and uniqueness of Cournot equilibrium in models of international trade under oligopoly. The existence of Cournot equilibrium is established without the usual assumption that profit functions are concave. Instead the proof uses a weaker "aggregate concavity" condition. A simple proof is used to establish the uniqueness of the equilibrium. And, the paper considers the implications of the assumptions, used to prove the existence and uniqueness of the equilibrium, on the comparative static results.

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Introduction

Models of international trade under imperfect competition have frequently used the concept of Cournot equilibrium. They have been used to explain intra-industry trade, Brander (1981), Brander and Krugman (1983) etc, and to analyse trade policy, Brander and Spencer (1984), Dixit (1984) etc. These models usually have two countries, the domestic and the foreign country, and in each there are a number of firms that compete in both markets. It is assumed that marginal costs are constant and that markets are segmented, then firms are engaged in two independent games, one in the domestic market and one in the foreign market. Hence, the game in one market can be analysed separately from the game in the other market. Also, the models usually assume symmetry, in the sense that in each country all firms have identical costs. The question of the existence and uniqueness of equilibrium has largely been ignored.

This paper will prove the existence and uniqueness of Cournot equilibrium in models of international trade under oligopoly. The usual proof of the existence of a Cournot equilibrium assumes that each firm's profit function is concave in its own output, so that a standard existence proof for concave games can be applied. An alternative proof by McManus (1962, 1964) does not require profit functions to be concave, but assumes that all firms have identical cost functions. The proof developed here adapts the method used by McManus, and replaces the concavity assumption with a weaker "aggregate concavity" condition. This ensures that the industry reaction functions are continuous, and hence allows existence to be proved. A necessary and sufficient condition for uniqueness has been derived by Kolstad and Mathiesen (1987), and their condition will be employed in this paper. But, the proof of uniqueness developed here is much simpler than the proof used by Kolstad and Mathiesen. The implications of the assumptions, used to prove the existence and uniqueness of the equilibrium, for the comparative static results are also considered. The assumptions yield comparative static results with reasonable signs, and also allow the possibility that domestic output and foreign exports are strategic complements.

The Model

The model is similar to Dixit (1984). There are two countries: the domestic and the foreign country. In the domestic country there are n identical firms each with constant marginal cost c_1 and in the foreign country there are m identical firms each with constant marginal cost c_2. The domestic and foreign markets are segmented so there is no possibility of arbitrage between them, and so there can be price discrimination. Since marginal cost is constant and markets are segmented, the firms are involved in two independent games, one in the domestic market and one in the foreign market, which can be analysed separately. Consider the domestic market, where the inverse demand function is $P = P(Q)$. The output of the ith domestic firm, for domestic consumption, is y_i, the output of all domestic firms but firm i is Y_{-i} and total domestic industry output Y, so $Y = y_i + Y_{-i}$. Similarly, define the exports, to the domestic market, of the ith foreign firm as x_i, the exports of all foreign firms but firm i as X_{-i} and total foreign exports X, so $X = x_i + X_{-i}$. Total domestic consumption is $Q = X + Y$, foreign exports plus domestic production. The profits of domestic and foreign firms, from sales in the domestic market are

$$
\begin{aligned}
\pi_{1i}(\mathbf{y}, \mathbf{x}) &= (P - c_1)y_i \qquad && i = 1,..,n \\
\pi_{2i}(\mathbf{y}, \mathbf{x}) &= (P - c_2)x_i \qquad && i = 1,..,m
\end{aligned}
\tag{1}
$$

At a Cournot equilibrium, a Nash equilibrium in quantities, each firm's output is an optimal response to the output of all other firms. Therefore, a Cournot equilibrium is a vector of outputs $(\mathbf{y}^c, \mathbf{x}^c) = (y_1^c,..,y_n^c, x_1^c,..,x_m^c)$ such that

$$
\begin{aligned}
\pi_{1i}(\mathbf{y}^c, \mathbf{x}^c) &= \underset{y_i \geq 0}{\text{Max}}\ \pi_{1i}(y_1^c,..\ y_i,..\ y_n^c, \mathbf{x}^c) \qquad && i = 1,..,n \\
\pi_{2i}(\mathbf{y}^c, \mathbf{x}^c) &= \underset{x_i \geq 0}{\text{Max}}\ \pi_{2i}(\mathbf{y}^c, x_1^c,..\ x_i,..\ x_m^c) \qquad && i = 1,..,m
\end{aligned}
\tag{2}
$$

It will be assumed that the inverse demand function, and hence since marginal cost is constant, profits are twice continuously differentiable. Therefore, necessary conditions for a Cournot equilibrium are

$$\frac{\partial \pi_{1i}}{\partial y_i} = P + y_i P' - c_1 \le 0, \quad y_i \ge 0, \quad \frac{\partial \pi_{1i}}{\partial y_i} . y_i = 0 \qquad i = 1,..,n$$

$$\frac{\partial \pi_{2i}}{\partial x_i} = P + x_i P' - c_2 \le 0, \quad x_i \ge 0, \quad \frac{\partial \pi_{2i}}{\partial x_i} . x_i = 0 \qquad i = 1,..,m \tag{3}$$

For an interior solution, where all firms produce a positive output, these reduce to the usual first order conditions. Further assumptions will have to be made to ensure the existence and uniqueness of a Cournot equilibrium.

Existence and Uniqueness

A Cournot equilibrium is a pure strategy Nash equilibrium in quantities, so the existence problem is similar to that for any Nash equilibrium in pure strategies. According to Dasgupta and Maskin (1986) there are two reasons for the non-existence of pure strategy Nash equilibrium: if the payoff function is not continuous or not quasi-concave. For Cournot equilibrium the payoff, profit, function is continuous but may not be quasi-concave. Therefore, it is the profit function not being quasi-concave which maybe the cause of non-existence of the Cournot equilibrium. If a Cournot equilibrium does not exist it should not be concluded that the model has no equilibrium. Dasgupta and Maskin have shown that a mixed strategy equilibrium exists for most games, even if payoff functions are discontinuous.

There are three methods to prove the existence of a Cournot equilibrium each using different assumptions about demand and cost functions. One approach by Frank and Quandt (1963) is to assume that each firm's profit function is concave in its own output, so that reaction functions are continuous and a standard existence proof can be applied. This approach is used by Myles (1988). Szidarovszky and Yakowitz (1977) assume that the inverse demand function is concave, $P'' < 0$, and cost functions are convex, which yields concave profit functions. But, this assumption is stronger than required to obtain concave profit functions, from (1) the second derivatives of the profit functions are

$$\begin{aligned} \frac{\partial^2 \pi_{1i}}{\partial y_i} &= 2P' + y_i P'' \qquad i = 1,..,n \\ \frac{\partial^2 \pi_{2i}}{\partial x_i} &= 2P' + x_i P'' \qquad i = 1,..,m \end{aligned} \tag{4}$$

Therefore, each firm's profit function will be everywhere concave in its own output if $2P' + QP'' < 0$, so a Cournot equilibrium exists provided demand is not too convex, but this is still a fairly strong assumption. The assumption that profits are concave could be replaced with the assumption that they are quasi-concave, without affecting the proof, but it is not clear what this implies for the shape of demand functions.

The second approach, due to McManus (1962, 1964), does not require the profit functions to be concave. It assumes that all firms have identical and convex costs, but imposes no restrictions on demand functions, except that it is a non-increasing function and total revenue is bounded. Without the assumption of concavity of profit functions, the reaction functions need not be continuous so fixed point theorems cannot be applied in the normal way. McManus shows that the cumulative reaction function is non-decreasing, hence any discontinuities must be jumps upwards, and in this way is able to show that a symmetric equilibrium exists.

A more recent approach by Novshek (1985) does not require cost functions to be convex, in this way it is less restrictive than previous approaches, but it assumes that the demand function is such that $P' + QP'' < 0$. This assumption implies that each firm's marginal revenue be everywhere decreasing in the output of other firms, which is the same as the Hahn (1961-62) stability condition. For constant marginal cost, this proof is more restrictive than assuming that profit functions are concave.

Proofs that the Cournot equilibrium is unique have generally used the Gale-Nikaidô (1965) theorem for the univalence of mappings. A sufficient condition for uniqueness is that the Jacobian, derived from the first order conditions for profit maximisation, is a P-matrix, all the principal minors are positive. This is restricted to equilibrium in the interior of the strategy space, and does not apply to equilibrium where some firms produce zero output. This condition for uniqueness is related to the Seade (1980) stability condition. A necessary and sufficient condition for uniqueness has been obtained by Kolstad and Mathiesen (1987) using index analysis. If the Jacobian determinant is positive at all equilibrium then there is a unique Cournot equilibrium, and conversely if the equilibrium is unique then the Jacobian determinant is positive at the equilibrium.

The existence of the Cournot equilibrium will be proved here, without the assumption that profit functions are concave. The proof exploits the symmetry of the model, all firms in each country have identical costs, so although the proof by McManus (1962, 1964) is not directly applicable it can be used to show the existence of domestic and foreign industry reaction functions. Despite the fact that individual firms' reaction functions need not be continuous it can be shown that the industry reaction functions are continuous. And, since the industry reaction functions are continuous it is possible to show that a Cournot equilibrium exists using a fixed point theorem. The proof of uniqueness presented here is much simpler than the proof used by Kolstad and Mathiesen.

To prove the existence and uniqueness of the Cournot equilibrium the following assumptions are required:

(A1) The inverse demand function $P(Q)$ is decreasing, twice continuously differentiable and total revenue, $P(Q).Q$, is bounded.

(A2) The following conditions are satisfied:

$$(n + 1)\ P'(X+Y) + Y\ P''(X+Y) < 0 \qquad \forall\ X, Y$$

$$(m + 1)\ P'(X+Y) + X\ P''(X+Y) < 0 \qquad \forall\ X, Y$$

(A3) The following condition is satisfied:

$$(n + m + 1)\ P'(Q) + Q\ P''(Q) \ < \ 0 \qquad \forall\ Q$$

Assumption (A2) ensures that the demand function is not too convex, and it replaces the usual assumption that each firm's profit function is concave in its own output. For the domestic industry concavity of the profit function requires that $2P' + YP'' < 0$, whereas assumption (A2) requires that $(n+1)P' + YP'' < 0$. When there is a single domestic firm (A2) is equivalent to the concavity of the profit function, but when there is more than one domestic firm it is less restrictive than the concavity assumption. Obviously, to take advantage of the fact that all domestic firms are identical requires that there are at least two firms in the domestic industry. Assumption (A2) can be interpreted as an "aggregate concavity" condition which ensures that the industry reaction functions are continuous, in the same way that concavity ensures that the reaction functions of the firms are continuous. Assumption (A3) is the necessary and sufficient condition for uniqueness of Kolstad and Mathiesen (1987) and also the stability condition of Seade (1980).

Theorem: For a homogeneous product Cournot oligopoly with inverse demand function $P(Q)$ and n identical domestic firms each with constant marginal cost $c_1 > 0$ and m identical foreign firms each with constant marginal cost $c_2 > 0$. If assumptions (A1), (A2) and (A3) are satisfied, then there exists a unique and symmetric Cournot equilibrium.

Note that (A1) implies that there exists a $\bar{Q}$ such that price is below marginal cost for all firms if $Q \geq \bar{Q}$, this follows from the fact that total revenue is bounded and $P(Q)$ is decreasing. Let the strategy set of the firms be $[0, \bar{Q}]$, which is non-empty, convex and compact.

First it is necessary to show that a domestic industry equilibrium exists. That is, for any given X, there exists a $(y_1^*, \ldots y_n^*)$ such that each domestic firm is setting its output optimally. The proof closely follows McManus (1962, 1964). Define the cumulative reaction function, $Y = r(Y_{-i}, X)$, as the domestic industry output, Y, when firm i chooses optimally as a function of the output of all other domestic firms, Y_{-i}, for given foreign exports, X. The cumulative reaction function may not be continuous since profit functions need not be concave, so it is not possible to prove existence in the usual way. However, it can be shown that the cumulative reaction function is non-decreasing, so any discontinuities must be jumps upwards. For $Y_{-i} = 0$ then $Y \geq 0$ and for $Y_{-i} = \bar{Q}$ then $Y = \bar{Q}$, since the optimal output of firm i is zero, $y_i = 0$. Hence, it must intersect the line $Y = \frac{n}{n-1} Y_{-i}$, at say Y^* in figure one. This yields a symmetric domestic industry equilibrium where each domestic firm produces $y^* = Y^*/n$. Then $Y_{-i} = (n-1)y^*$ and the optimal response for each domestic firm is to produce y^* so that $Y = ny^*$.

To prove the cumulative reaction function is non-decreasing, let

y_i^A be the optimal output for firm i when the rest of the domestic industry produces Y_{-i}^A. Then domestic industry output is Y^A and the price is then $P^A(Y^A + X)$. Similarly, let y_i^B be the optimal output for Y_{-i}^B, then domestic industry output is Y^B and the price is $P^B(Y^B + X)$. If $y_i^A = 0$ then $Y^B > Y^A$ if $Y_{-i}^B > Y_{-i}^A$. For $y_i^A > 0$, let Y_{-i}^B be such that $Y_{-i}^A < Y_{-i}^B < Y^A$. In situation A the ith firm could produce $Y^B - Y_{-i}^A$ instead of output y_i^A and the price would be P^B. But since y_i^A is the profit maximising output

$$(P^A - c_1)\, y_i^A \;\geq\; (P^B - c_1)\,(Y^B - Y_{-i}^A) \tag{5}$$

Similarly in Situation B the ith firm could produce $Y^A - Y_{-i}^B$, in which case the price would be P^A, and since y_i^B is the profit maximising output

$$(P^B - c_1)\, y_i^B \;\geq\; (P^A - c_1)\,(Y^A - Y_{-i}^B) \tag{6}$$

Adding together (5) and (6) and then rearranging yields

$$(P^A - P^B)\,(Y_{-i}^B - Y_{-i}^A) \;\geq\; 0 \tag{7}$$

Since, by assumption, $Y_{-i}^B \geq Y_{-i}^A$ then $P^A \geq P^B$ therefore $Y^B \geq Y^A$, which proves that the cumulative reaction function is non-decreasing, so any discontinuities must be jumps upwards.

Define the domestic industry reaction function f(X) as the output of the domestic industry when each domestic firm sets its output optimally, given the foreign output X. Let $\bar{X}$ be such that $P(\bar{X}) = c_1$. Then if $X > \bar{X}$, price will be below domestic marginal cost for any level of domestic output, so the optimal output for all domestic firms is obviously zero. For $X \le \bar{X}$, a necessary condition for profit maximisation is

$$\frac{\partial \pi_{1i}}{\partial y_i} = P + y_i P' - c_1 = 0 \qquad i = 1,\dots n \tag{8}$$

It can be shown that any equilibrium is symmetric, all domestic firms produce the same output. Consider any two firms, say, i and j. Subtract the first order condition for profit maximisation for firm j from the first order condition for firm i yields

$$(y_i - y_j)P' = 0 \tag{9}$$

Since $P' < 0$, it follows that $y_i = y_j$, which holds for any i and j. Hence all domestic firms produce the same output in equilibrium.

Summing (8) over all domestic firms yields the following necessary condition for a domestic industry equilibrium

$$F(Y, X) = nP(Y+X) + YP'(Y+X) - nc_1 = 0 \tag{10}$$

Since $P(Q)$ is twice continuously differentiable, $F(Y, X)$ is continuously differentiable. For $X \le \bar{X}$, $F(0, X) \ge 0$ (since $P(X) \ge c_1$) and $F(\bar{Q}-X, X) < 0$ (since $P(\bar{Q}) < c_1$). It is also decreasing in Y since, by assumption (A2)

$$\frac{\partial F}{\partial Y} = (n+1)P' + YP'' < 0 \tag{11}$$

Therefore, for any given $X \le \bar{X}$, there is a unique Y which solves $F(Y, X) = 0$. Since it has already been shown that, for any X, there exists a symmetric domestic industry equilibrium and (10) which is a necessary condition for an equilibrium has a unique solution, then it follows that (10) must be a necessary and sufficient condition for an equilibrium. Hence $F(Y, X) = 0$ implicitly defines the domestic industry reaction function, $Y = f(X)$. Since $F(Y, X)$ is continuously differentiable, and $\partial F/\partial Y < 0$, then by the implicit function theorem $f(X)$ is continuous. Also by the implicit function theorem

$$f' = \frac{-\partial F/\partial X}{\partial F/\partial Y} = \frac{-(nP' + YP'')}{(n+1)P' + YP''} \tag{12}$$

The right hand side exists and is continuous, since $P(Q)$ is twice continuously differentiable and the denominator is non-zero by assumption (A2), hence $f(X)$ is continuously differentiable.

Hence, the domestic industry reaction function is given by

$$f(X) = \begin{cases} \{ Y \mid F(Y, X) = 0 \} & X \le \bar{X} \\ \{ 0 \} & X > \bar{X} \end{cases} \tag{13}$$

Which is defined for $X \in [0, \bar{Q}]$. The reaction function is shown in figure 2.

By similar arguments it can be shown that a foreign industry equilibrium exists and the foreign industry reaction function is

$$g(Y) = \begin{cases} \{ X \mid G(Y, X) = 0 \} & Y \leq \bar{Y} \\ \{ 0 \} & Y > \bar{Y} \end{cases} \tag{14}$$

Where $G(X, Y) \equiv mP(Y+X) + XP'(Y+X) - mc_2$, and $\bar{Y}$ is defined such that $P(\bar{Y}) = c_2$. In equilibrium all foreign firms export the same output, $x_i = x_j$ for all i and j. The reaction function g(Y) is continuously differentiable. For $Y \leq \bar{Y}$, by the implicit function theorem

$$g' = \frac{-\partial G/\partial Y}{\partial G/\partial X} = \frac{-(mP' + XP'')}{(m+1)P' + XP''} \tag{15}$$

At a Cournot equilibrium the domestic industry's output Y^c must be the optimal response to the foreign industry's exports X^c, which itself must be an optimal response to Y^c. That is $Y^c = f(X^c)$ and $X^c = g(Y^c)$, or equivalently

$$Y^c = f(g(Y^c)) = f \circ g(Y^c) \tag{16}$$

A Cournot equilibrium exists if $f \circ g(Y)$ has a fixed point. To prove that it does have a fixed point define the function $h(Y) \equiv f \circ g(Y) - Y$, for $Y \in [0, \bar{Q}]$. At a Cournot equilibrium

$f \circ g(Y^c) = Y^c$ so $h(Y^c) = 0$. The function $h(Y)$ is obviously continuous since a composite function of two continuous functions, f and g, is itself continuous. At $Y = 0$, $h(0) \geq 0$ and at $Y = \bar{Q}$, $h(\bar{Q}) < 0$. Therefore, by the intermediate value theorem, there must exist a Y^c, $0 \leq Y^c < \bar{Q}$, such that $h(Y^c) = 0$, which proves that a Cournot equilibrium exists.

The Cournot equilibrium will be unique if $h(Y)$ is decreasing in Y, $h'(Y) < 0$, since then $h(Y)$ is one-to-one. By the chain rule the derivative of $h(Y)$ is

$$h'(Y) = f'(g(Y)).g'(Y) - 1 \tag{17}$$

For $Y \leq \bar{Y}$ and $g(Y) \leq \bar{X}$, using (12) and (15) yields

$$h'(Y) = \frac{-P'((n+m+1)P' + QP'')}{((n+1)P' + YP'')((m+1)P' + XP'')} < 0 \tag{18}$$

This is negative by assumptions (A2) and (A3). For $Y > \bar{Y}$ then $g' = 0$ so $h' = -1 < 0$, and for $g(Y) > \bar{X}$ then $f' = 0$ so $h' = -1 < 0$. Therefore, $h(Y)$ is clearly decreasing in Y, and hence there is a unique Cournot equilibrium.

Comparative Statics

The assumptions required to prove the existence and uniqueness of the Cournot equilibrium can be used to sign the comparative static results. The effects of shifts in domestic and foreign marginal cost will be considered. These can be interpreted as the effects of trade taxes, such as a production tax which increases domestic marginal cost or a tariff which increases foreign marginal cost. For an interior solution where the market is supplied by both domestic production and foreign exports, the necessary and sufficient conditions for equilibrium are $F(Y, X) = 0$ and $G(Y, X) = 0$. The comparative static results are obtained by totally differentiating these equations.

$$\begin{bmatrix} (n+1)P' + YP'' & nP' + YP'' \\ mP' + XP'' & (m+1)P' + XP'' \end{bmatrix} \begin{bmatrix} dY \\ dX \end{bmatrix} = \begin{bmatrix} n\, dc_1 \\ m\, dc_2 \end{bmatrix}$$

Matrix inversion yields

$$\begin{bmatrix} dY \\ dX \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} (m+1)P' + XP'' & -(nP' + YP'') \\ -(mP' + XP'') & (n+1)P' + YP'' \end{bmatrix} \begin{bmatrix} n\, dc_1 \\ m\, dc_2 \end{bmatrix} \tag{19}$$

Where $\Delta = ((n+m+1)P' + QP'')P' > 0$ by assumption (A3), and the principal diagonal elements of the matrix are negative by assumption (A2). Therefore, an increase in domestic (foreign) marginal cost will reduce domestic (foreign) output. The signs of the off-diagonal elements determine whether domestic output and foreign exports are strategic substitutes or complements, as defined by Bulow *et al* (1985). They are strategic substitutes (complements) for the domestic country if $nP' + YP'' < (>) \ 0$, and for the foreign country if $mP' + XP'' < (>) \ 0$. An increase in domestic marginal cost will increase (decrease) foreign output if domestic output and foreign exports are strategic substitutes (complements) for the foreign country. The effects on price are

$$\frac{\partial P}{\partial c_1} = \frac{n(P')^2}{\Delta} > 0 \qquad\qquad \frac{\partial P}{\partial c_2} = \frac{m(P')^2}{\Delta} > 0 \qquad (20)$$

An increase in domestic or foreign marginal cost will result in an increase in price. The assumptions, used to prove the existence and uniqueness of equilibrium, yield reasonable comparative static results and allow the possibility that domestic output and foreign exports are strategic complements. In fact, any weaker assumptions would probably give perverse comparative static results.

Conclusions

A proof of the existence of Cournot equilibrium in models of international trade under oligopoly has been developed which does not make the usual assumption that profit functions are concave. A simple proof that the equilibrium is unique has also been presented. And, it has been shown that the assumptions, used to prove existence and uniqueness, yield reasonable comparative static results.

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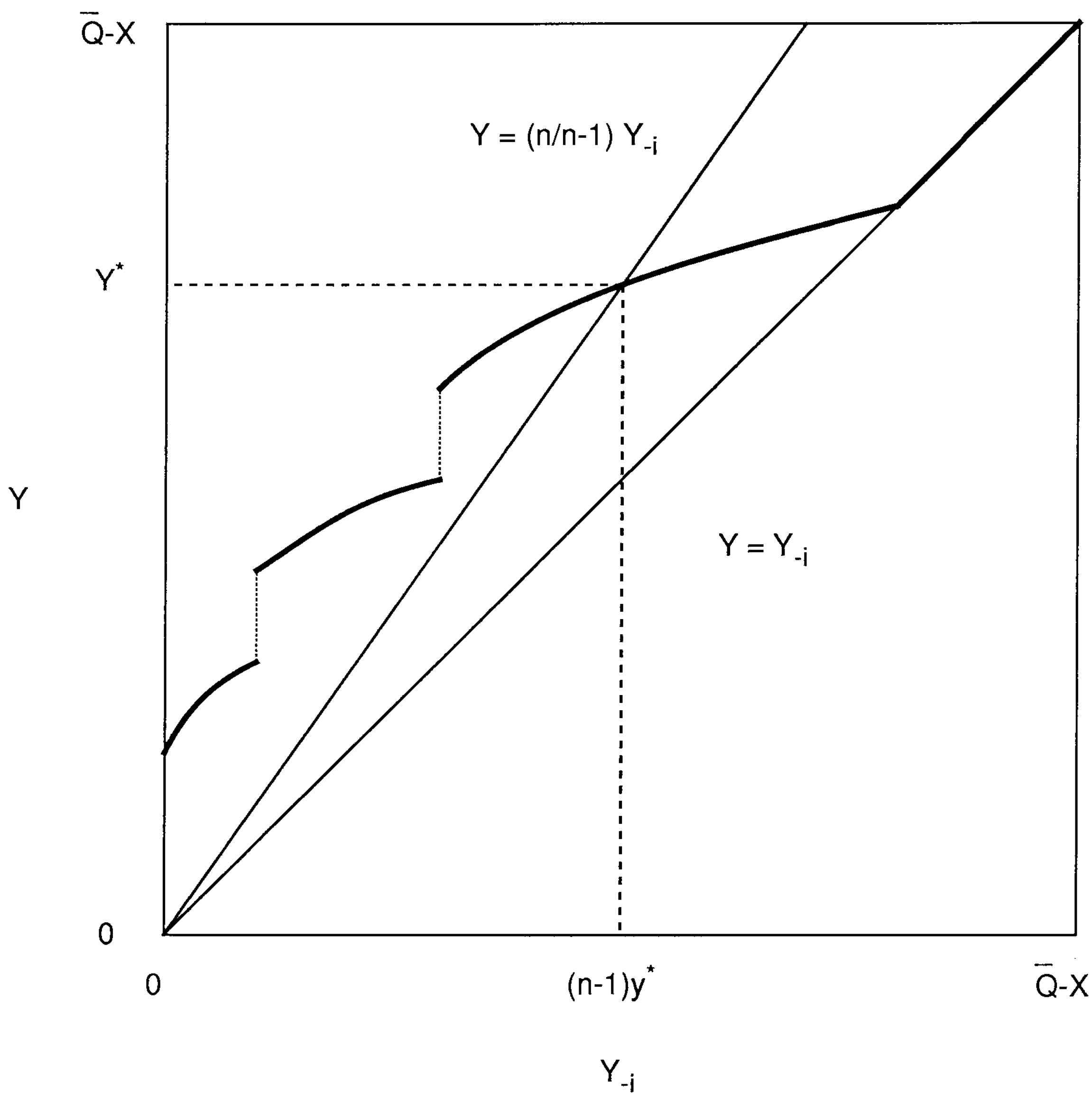

Figure 1: Cumulative Reaction Function

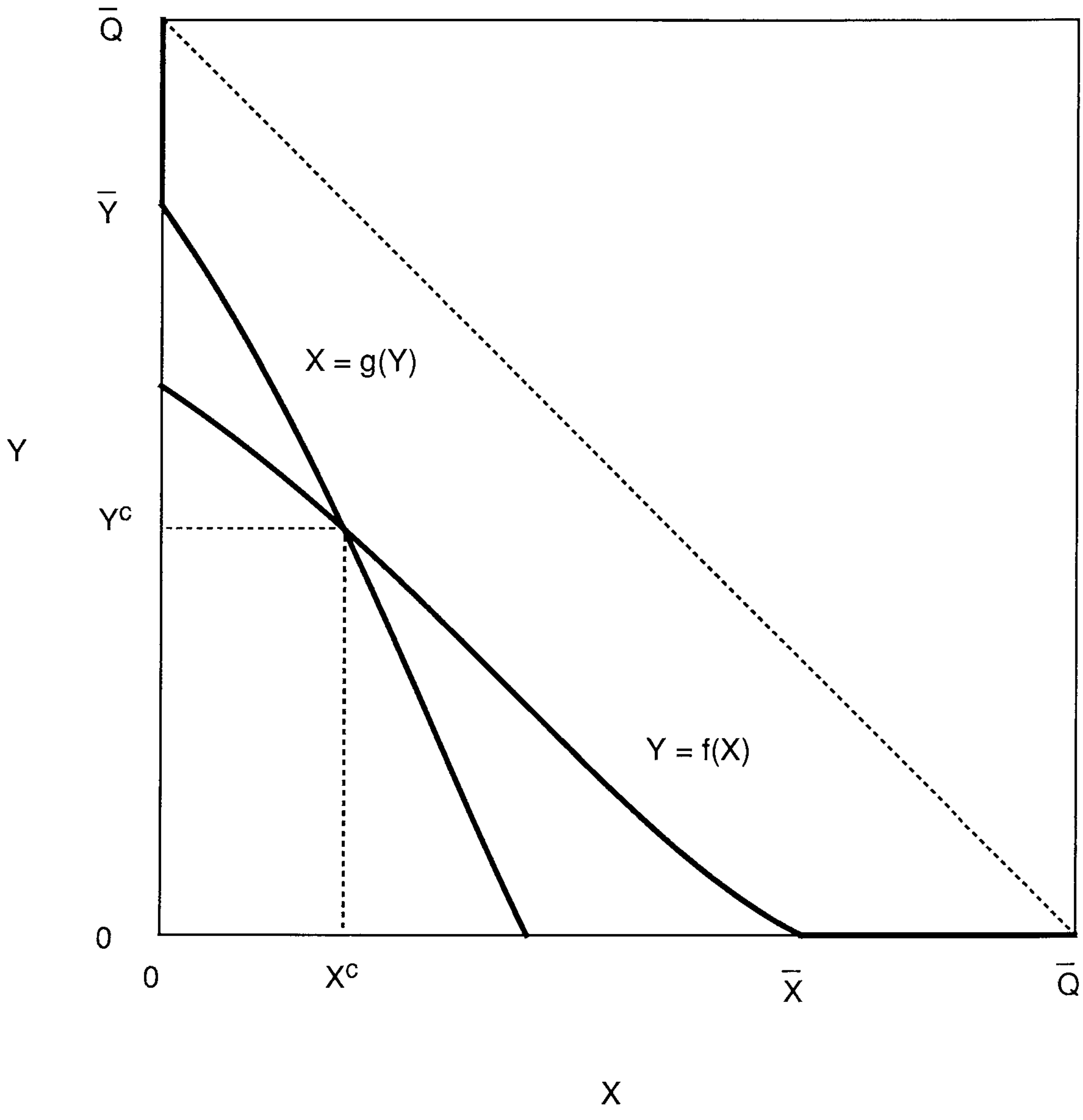

Figure 2: Industry Reaction Functions