Exchange Rate Risk and Imperfect Capital Mobility in an Optimising Macromodel

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Abstract

A stochastic two-period small open economy model with optimising consumption and portfolio choice is constructed. Exchange rate risk means domestic-currency bonds are imperfect substitutes for foreign-currency bonds. Expectations are rational, i.e. subjective probability distributions equal the true distributions resulting from the exogenous sources of uncertainty, which are the foreign inflation rate and either the future money supply or government spending. With the former, no real risk premium exists, but increased monetary variance reduces current output, which nominal wage rigidity makes responsive to aggregate demand With the latter a premium exists, but increased spending variance affects neither it nor output.

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1. Introduction

The degree of capital mobility is acknowledged to be critical to the macroeconomic behaviour of an open economy, but most theoretical models still assume it to be determined exogenously, and most commonly to be either perfect or zero. This paper builds a very simple macromodel of a small open economy in which the degree of capital mobility, which is to say the substitutability in portfolios of domestic and foreign bonds, is endogenous. There are two requirements for such a model: consumption and portfolio demands must be based on explicit optimisation under uncertainty by agents; and rational expectations in at least second moments must be imposed, i.e. subjective probability distributions must be endogenised by equating them to the objective probability distributions generated by the model. Imperfect substitutability of domestic and foreign bonds arises in our framework because of exchange rate risk. The real return on domestic bonds is stochastic because inflation is stochastic; the real return on foreign bonds is stochastic for the additional reason that exchange rate de- or appreciation is stochastic, implying that the two bonds' real returns are not in general perfectly correlated. Risk-averse investors may therefore wish to diversify their asset portfolios.

Amongst the questions which such a model can be used to answer are ones about the effects of changes in the degree of uncertainty over future government policy. A casual argument which is sometimes heard is that if future policy becomes more uncertain, then in an open economy with a floating exchange rate there will be an increase in perceived exchange rate risk, causing domestic bonds to become less substitutable for foreign ones, and raising the domestic interest rate to provide a risk premium to foreign investors. This may have undesirable effects, for example by depressing domestic demand. Indeed, such an argument might be interpreted as an analysis of the effects of a loss of "credibility" by the government, an alternative to recent game-theoretic analyses of this.

The traditional literature on the macroeconomics of imperfect capital mobility is grounded in the approach of direct postulation of portfolio and other demands, as surveyed by Branson and Henderson (1985). Later work deriving demands from explicit optimisation under uncertainty, for example by Kouri (1977, 1983), Frankel (1979) and Fama and Farber (1979), has provided important insights by linking open-economy macroeconomics to the theory of finance. However only fairly recently have attempts been made to endogenise subjective probability distributions in optimising models. Work by Lucas (1982), Stulz (1984), Svensson (1985) is in the tradition of the general equilibrium asset-pricing literature, assuming permanent market-clearing and full employment, and large numbers of assets. Applicability to short-run macroeconomics is achieved in the models with unemployment by Svensson (1987) and Svensson and Van Wijnbergen (1989), but these are still not very explicit about implications for interest rates and capital mobility.

The present model is an extension to a stochastic setting of the two-period perfect foresight model constructed in Rankin (1989). Amongst existing stochastic models its closest resemblance is to Persson and Svensson (1989), but with several notable differences. First, unemployment is permitted, by the assumption of temporary wage stickiness. Second, it does not use a mean-variance-equivalent version of expected utility maximisation, a departure which has several advantages explained below. Third, money demand is modelled by assuming real balances provide utility rather than by the currently popular cash-in-advance method, which removes a restrictive feature of the latter.

The most significant finding is that alternative assumptions as to the exogenous domestic source of uncertainty are critical to the properties of the model. When the future money supply is the provider of noise, expected real interest rates on domestic and on foreign bonds are equal, i.e. the uncovered interest parity condition holds in real terms, despite bonds being imperfect substitutes in portfolios. An increase in money supply variance thus causes no real risk premium, but it nevertheless has the real effect of lowering current output. An increase in the variance of foreign inflation, the other source of exogenous uncertainty in the model, has no effect on domestic output on the other hand. When instead future real government spending is the source of noise, a real risk premium does exist. However it is not necessarily positively associated with the country's indebtedness, and an increase in the variance of spending affects neither it nor the level of domestic output. In this regime an increase in the variance of foreign inflation does affect domestic output.

The layout of the paper is as follows. Section 2 details the microeconomic behaviour of individual agents. In Section 3, we solve for equilibrium under the assumption of unit-elastic expectations of future variables. This provides a stepping-stone to the full rational expectations solution presented in Section 4, and also reveals which properties of the model are directly attributable to the assumption of rational expectations and which are not, a considerable help in understanding it. Section 5 concludes.

2. Individual Behaviour

The economy lasts for two periods, and produces and consumes a single, internationally-traded, good. The world goods market is perfectly competitive with flexible prices. Thus the purchasing power parity condition, $p_t = E_t p_t^*$ holds for t =1,2. (As conventional, * denotes a foreign variable, and E_t is the exchange rate measured as the domestic currency price of foreign currency.) Since the country is "small", p_t^* is exogenous to it. This very simple structure for goods markets is maintained, despite its lack of realism, to permit the analysis to focus on asset markets. Labour is the only variable input to production. In the second period the wage is flexible, guaranteeing full employment of households' fixed labour supply (there is no utility of leisure), so that second-period output may be treated as exogenous. In the first period the money wage is fixed at a level creating excess labour supply, so that a positive aggregate supply relation between price and output obtains. This set-up, taken from Rankin (1989) but used elsewhere, roughly captures the notion of short-run nominal wage rigidity.

There are four assets - domestic and foreign currency, and domestic and foreign currency-denominated bonds. Letting i denote the gross (i.e. one plus the) nominal interest rate, and $d \equiv p_1/p_2$ the gross deflation rate, the (gross) real interest rate on domestic bonds is $r \equiv id$. Likewise $r^* \equiv i^*d^*$ for foreign bonds. Purchasing power parity implies that the real interest rate on a bond is the same whether calculated as above or by converting into the other currency and deflating at the other country's inflation rate: $r = i[E_1/E_2]d^*$, $r^* = i^*[E_2/E_1]d$. Nominal interest rates (i,i^*) are known and non-stochastic, so the uncertainty over the real return offered by a bond from the viewpoint of (say) a domestic investor, results from (a) uncertainty over the inflation rate, i.e. d, and (b) additionally in the case of foreign bonds, uncertainty over the rate of exchange rate de- or appreciation, i.e. E_2/E_1. Stochastic variables are thus (indicated by $\tilde{}$, for emphasis) $(\tilde{p}_2,\tilde{p}_2^*,\tilde{E}_2,\tilde{d},\tilde{d}^*,\tilde{r},\tilde{r}^*)$. To the extent that $\tilde{r}$ and $\tilde{r}^*$ are differently distributed, bonds are therefore imperfect substitutes.

The agents in the domestic economy consist of a representative household, which consumes goods, supplies labour and saves by holding a portfolio of assets; a representative firm; and the government. Corresponding agents exist for the foreign economy. Although the domestic economy is "small" in all markets, we shall picture the world as consisting of two countries which are qualitatively identical, and consider the consequences of letting the size of the foreign country tend to infinity.

Household behaviour

In its most succinct form, the domestic household's optimisation problem may be written:

Maximise with respect to (s,ω_F,ω_M),

$$\ln(a_1-s) + \gamma E(\ln(d\omega_M s)) + \delta E(\ln(y_2-\tau_2 + [[r^*-r]\omega_F + r + [d-r]\omega_M]s)) \qquad (1)$$

Here and below we use the definitions:

$a_1 = M_0/p_1 + y_1 - \tau_1$ = initial real balances plus period-1 disposable income (y_1 = real output and income in period 1, τ_1 = a lump-sum tax)

$s = a_1 - c_1$ = "savings" = period-1 resources less consumption

$(\omega_F,\omega_H,\omega_M)$ = portfolio shares in, respectively, foreign bonds, home bonds, and domestic currency, where $\omega_F+\omega_H+\omega_M \equiv 1$.

$(f,h,m_1) = (\omega_F s,\omega_H s,\omega_M s)$ = absolute real (using y_1 as numeraire) asset holdings

$y_2 - \tau_2$ = period-2 disposable income

$a_2 = y_2 - \tau_2 + [r^*\omega_F + r\omega_H + d\omega_M]s$ = real resources available in period 2

The first argument in the utility function (1) is period-1 consumption. The second argument is end-of-period-1 real money balances, M_1/p_2. Note that domestic currency is dominated by domestic bonds as a store of value, since bonds pay interest and money does not. The real return per unit of real money balances, $m_1 = M_1/p_1$, is d, whereas the real return per real unit of domestic bonds, h, is id. Thus provided $i > 1$, the household receives a higher real return from bonds whatever the realisation of d. Given this, money will not be held (or, in the absence of non-negativity constraints, an infinitely negative amount will be held, as the household arbitrages between money and bonds), unless it also has a transactions role. This is captured by assuming real balances provide utility. Since money held at the end of period 1 must be to assist transactions made in period 2, we use M_1/p_2 as the real balance variable, which is stochastic because p_2 is a random variable. This method of deriving money demand in a stochastic model is also employed by, for example, Kouri (1977), Fama and Farber (1979) and Stulz (1984). An alternative used by Lucas (1982), Kouri (1983), Svensson (1985) and Persson and Svensson (1989), to name but a few, is the cash-in-advance approach of imposing the constraint that all purchases must be made with money. Except in Svensson's (1985) sophisticated version, this imposes the simple quantity equation on the model and trivialises the determination of output, so that the more flexible utility-of-real-balances approach is essential given our desire to preserve the model's relevance to the macroeconomic question of output determination.

The third argument of the utility function is a_2, the household's total real resources available in period 2. This is stochastic since $(y_2\text{-}\tau_2, r^*, r, d)$ are stochastic. Its presence is due to the fact that (1) has been written in indirect form, having already solved the household's period-2 optimisation problem of maximising $[1-\beta]\ln c_2 + \beta \ln m_2$ $(0 < \beta < 1)$ subject to $a_2 = c_2 + m_2$. The inclusion of end-of-period-2 real balances in the utility function $m_2 = M_2/p_2$ represents the liquidity services they provide in a putative later period, which, however, is not modelled explicitly. As is well known, without a positive demand for terminal money balances, money would "unravel" from the model. Plugging the solutions $c_2 = [1-\beta]a_2$, $m_2 = \beta a_2$ back into the general log-linear lifetime utility function then yields (1).

The use of a log-linear form for the utility function has the valuable advantage (subject to a restriction mentioned below) in the present context, that the portfolio decisions are separable from the consumption decision in the sense that portfolio shares depend only on the joint distribution of asset returns and not on the level of savings. Moreover, preferences over consumption and real balances are homothetic, so that consumption and savings are directly proportional to initial wealth. Compared to the quadratic utility function, used for example by Frankel (1979) and Kouri (1983), and the CARA (constant absolute risk aversion) utility function combined with the normal distribution, used by Persson and Svensson (1989), it also avoids the very undesirable implications that risky assets are (respectively) "inferior" or of zero wealth elasticity, in the presence of a riskless alternative. This is because it belongs to the class of CRRA (constant relative risk aversion) functions, which guarantee "normality" of risky assets by Arrow's (1970) result that normality is associated with decreasing absolute risk aversion. The two alternatives mentioned are commonly chosen since they imply that utility depends only on the mean and variance of future wealth, i.e. that the expected utility framework becomes equivalent to the mean-variance framework. However, this latter has a serious defect when applied to situations in which one asset dominates another in the sense explained earlier. This is that bonds do not dominate money in mean-standard deviation space: the feasible set is a straight line through the origin with positive slope. Consequently a positive finite demand for money can exist even when it has no transactions role. This paradox is a result of negative marginal utility beyond a certain point (in the quadratic case) or of negative gross returns in certain states (in the CARA plus normality case).

The necessary restriction referred to above is that the household's future disposable income, y_2-τ_2, must be imposed to be zero. This is undoubtedly a serious cost in terms of the generality of the model, but the presence of non-diversifiable future income is a well-known obstacle to making progress with the CRRA family of utility functions, and to omit it is a logical first step. It will be seen that it permits us to obtain surprisingly explicit solutions, and does not deprive the model of the ability to answer most of the interesting questions. By contrast, the quadratic and CARA functional forms when employed in the same framework lead only to analytical intractability.

The three first-order conditions for solving the problem (1) may now be obtained as, after slight rearrangement:

$$E\left(\frac{r}{\omega_F[r^*-r] + r + [d-r]\omega_M}\right) = E\left(\frac{r^*}{\omega_F[r^*-r] + r + [d-r]\omega_M}\right) = \frac{\gamma i}{\delta[i-1]\omega_M} \qquad (2)$$

$$s = \frac{\gamma i}{\gamma i + [i-1]\omega_M} a_1 \tag{3}$$

(2) determines the portfolio shares (ω_F,ω_M) as functions only of the joint distribution of asset returns $(i,\tilde{d},\tilde{r}^*)$, and independently of savings, s, as just noted. (3) shows that savings (and thus also period-1 consumption $c_1 = a_1 - s$ and the absolute real asset demands (f,h,m_1)) are directly proportional to initial resources, a_1.

Symmetrically with the domestic household's optimisation problem, we may define the foreign household's problem. In solving for equilibrium below, we shall make use of the foreign household's first-order conditions for asset holding, which, analogously to (2), may be derived as:

$$E\left(\frac{r^*}{\omega_F^*[r-r^*] + r^* + [d^*-r^*]\omega_M^*}\right) = E\left(\frac{r}{\omega_F^*[r-r^*] + r^* + [d^*-r^*]\omega_M^*}\right) = \frac{\gamma^* i^*}{\delta^*[i^*-1]\omega_M^*} \tag{4}$$

(ω_F^*,ω_M^*) denote the shares of the foreign portfolio held in (respectively) the domestic country's bonds, and in the foreign currency.

Firm behaviour

The firm produces output using labour as the sole input, and so in both periods solves the purely atemporal problem of maximising profits by equating the real wage to the marginal product of labour. In period 1, the money wage is assumed fixed (or imperfectly indexed to the price level), causing unemployment in the labour market. Hence an increase in p_1 depresses the real wage and increases output and employment. This we represent by the familiar increasing aggregate supply function, $y(p_1)$, with $y_p \equiv \partial y_1/\partial p_1 \geq 0$. It is easy to consider the classical special case in which there is permanent full employment in period 1 by taking $y_p = 0$. In period 2, perfect money wage flexibility is assumed, resulting in permanent full employment. Since households obtain no utility from leisure, they supply their time endowments exogenously to the labour market, and these in combination with firms' production technologies result in an exogenous level of period-2 output, y_2. y_2 is stochastic from the perspective of period 1, as a result of an assumed randomness in either labour supply endowments or firms' technologies. In both periods, the firm's profits are instantaneously distributed to households, whence there is no need formally to distinguish labour and profit income: both are exogenous to the household and so may be aggregated as y_t in its budget constraint.

Government behaviour

The government makes real purchases of output g_t financed by a lump-sum tax on households and money issues. Its budget constraints are therefore:

$$g_1 - \tau_1 = M_1/p_1 - M_0/p_1, \qquad g_2 - \tau_2 = M_2/p_2 - M_1/p_1 \tag{5}$$

Bond issues by the government in either currency will not be regarded as net wealth by households, since households will have to bear the future taxation necessary to redeem current debt issues, and since they can undo any attempted intertemporal transfers by the government by virtue of their equal access to capital markets. There is thus no loss of generality in the assumption made in (5) that the government issues no debt. This "Ricardian equivalence" property of the model does of course have important implications, which will be pointed out below.

Two alternative exogenous domestic sources of uncertainty will be considered. In the first, M_2 is taken to have some exogenous probability distribution. Coupled with the restriction that $\tau_2 = y_2$, this leaves g_2 as the "passive" instrument of future government policy which must adjust to satisfy $g_2 - y_2 = M_2/p_2 - M_1/p_2$. The distribution of y_2 is unimportant because the only way it affects the economy is through the government budget constraint, and with M_2 treated as exogenous the financing implications of y_2 are buffered by g_2. There is in fact no loss of generality in subsuming y_2 into g_2 by normalising $y_2 = \tau_2 = 0$ and interpreting g_2 as the surplus (positive or negative) of government spending over output, provided negative values of g_2 are permitted. (The period-2 situation could equivalently and more simply be pictured as one in which there is no domestic output or taxation, but the government either makes a money-financed purchase g_2 on the world goods market (if $g_2 > 0$), or sells off an amount g_2 from a pre-existing goods stock in exchange for domestic currency (if $g_2 < 0$).) In the alternative specification, it is g_2 whose probability distribution is exogenous, and M_2 which acts as the passive instrument. The main difference between these is that in the first it is a nominal, and in the second a real, variable whose probability distribution is specified. For example fixing the distribution of M_2 is equivalent to fixing that of nominal period-2 spending G_2, up to a translation of its mean by the predetermined amount M_1, since the period-2 budget constraint can be written as $M_2 = M_1 + G_2$.

3. Equilibrium with Unit-Elastic Expectations

In this section we solve for equilibrium under the assumption that agents' subjective expectations of the period-2 variables which affect their period-1 behaviour are unit-elastic in the corresponding period-1 variables. This is an assumption of "adaptive" expectations in the broad sense. As well as providing a stepping-stone to the full rational expectations solution which is considered in the next section, and for which we simply need to add a further equilibrium condition to endogenise the subjective probability distributions, this case is of some interest in itself, and is implicitly the one considered in many optimising models already existing in the literature. Specifically, we assume that the subjective joint distribution of $(\tilde{p}_2, \tilde{p}_2^*)$ (and of the implied $\tilde{E}_2 = \tilde{p}_2/\tilde{p}_2^*$) is such that any change in p_1 causes a proportional change in p_2 for each state of nature, and likewise for p_1^* and p_2^*. Unit-elastic expectations provide the most convenient benchmark, since they imply that the subjective distribution of *intertemporal* relative prices, i.e. of the deflation rates $(\tilde{d}, \tilde{d}^*)$,which are the variables important for the portfolio decisions, is exogenous. Note we are also assuming that domestic and foreign households have the *same* subjective distributions.

Consider first the clearing conditions for the world markets in domestic and foreign bonds, respectively:

$$\omega_H s + \omega_F^* s^* = 0 \qquad (6)$$

$$\omega_F s + \omega_H^* s^* = 0 \qquad (7)$$

Since governments issue no debt, all demands or supplies are those of the domestic and foreign private sectors. In equilibrium, if one country is a creditor in terms of a particular type of bond, the other must be a debtor. Under the assumption that the domestic country is "small" and the foreign country "large", we let s* tend to infinity. (6) and (7) then imply that the foreign country's portfolio shares invested in bonds must be negligible in equilibrium: $\omega_F^* = \omega_H^* = 0$. The foreign portfolio consists (almost) entirely of foreign currency: $\omega_M^* = 1$.

Using these portfolio shares in the first-order conditions for the foreign portfolio, (4), we obtain:

$$E(r^*/d^*) \;=\; E(r/d^*) \;=\; \gamma^* i^*/\delta^*[i^*-1] \qquad (8)$$

or, dividing through by i* (recall that (i,i*) are non-stochastic):

$$1 \;=\; E(r/r^*) \;=\; \gamma^*/\delta^*[i^*-1] \qquad (9)$$

From (9), the foreign nominal interest rate takes the exogenous value $i^* = [\gamma^*+\delta^*]/\delta^*$. We also have that the domestic nominal interest rate is given by:

$$i \;=\; i^*/E(d/d^*) \qquad (10)$$

Since under our expectations assumption E(d/d*) is exogenous, being a subjective constant, so too is the domestic nominal interest rate, i. Under unit-elastic expectations, the domestic nominal interest rate, and likewise the domestic expected real interest rate, E(r) = iE(d), are therefore determined abroad, despite the fact that domestic and foreign bonds are not perfect substitutes.

Does the familiar uncovered interest parity (UIP) condition hold for this version of the model? Note that if it does, then the same is true under rational expectations, since (9) follows purely from bond-market clearing. We now encounter the complication that there is more than one version of UIP which can be considered. In nominal terms, we can use either the domestic or the foreign currency as the numeraire. In the former case it takes the form:

$$i \;=\; i^*E(E_2/E_1), \qquad (11)$$

and in the latter,

$$i^* = iE(E_1/E_2) \tag{12}$$

These are not equivalent, since by Jensen's inequality, $E(E_2/E_1) \neq 1/E(E_1/E_2)$, an apparent oddity first noted by Siegel (1972). Since purchasing power parity implies $E_1/E_2 = d/d^*$, (10) shows that nominal UIP in the form of (12) does hold in the model, and by the same token, fails to hold in the form of (11). We would argue, however, that the most interesting question is whether UIP holds in real terms, i.e. whether $E(r) = E(r^*)$. Such an implication is not contained in (9), and indeed in Section 4 we will look at a case in which it is explicit that $E(r) \neq E(r^*)$ despite the fact that (9) holds.

The condition that $E(r/r^*) = 1$ results in a very simple solution to the domestic household's portfolio choice problem, the first-order conditions for which were given in (2). It is easy to verify that the following portfolio shares satisfy (2), given that $E(r/r^*) = 1$:

$$\omega_F = \frac{\delta}{\gamma+\delta}, \quad \omega_H = -\frac{\gamma}{\gamma+\delta}\frac{1}{i-1}, \quad \omega_M = \frac{\gamma}{\gamma+\delta}\frac{i}{i-1} \tag{13}$$

The share in foreign bonds is simply an exogenous fraction between zero and one, depending only on the utility function weights (γ,δ). The shares in home bonds and money depend positively and negatively (respectively) on the domestic nominal interest rate and the same utility function parameters. Note that the share in home bonds is negative, indicating that the household is always a debtor in terms of home bonds (since s is always positive - see below). More than this, $i\omega_H = -\omega_M$, i.e. the household sells home bonds of an amount such that their redemption value exactly equals the value of its currency holdings. Home bonds are thus used exclusively as a hedge against the inflation risk inherent in holding money: the household "sells forward" the unwanted purchasing power risk which it is forced to take on when it holds money for transactions purposes. Fama and Farber (1979).identify this as the key role for bonds in international portfolios.

Using the solution for ω_M in (3), we obtain the following expression for savings:

$$s = \frac{\gamma + \delta}{1 + \gamma + \delta} a_1 \tag{14}$$

Savings (and hence consumption too) are just a constant fraction of initial resources, the fraction being the share of the weights on real balances and future wealth in the utility function (1). In this the model is identical to its deterministic counterpart.

The money market clearing condition for period 1 is now all that is needed to define the full macroeconomic equilibrium. No condition for goods market equilibrium is necessary, since the country is small in the world goods market: the assumption that p_1^* is exogenous effectively substitutes for this. Equating the supply of real balances M_1/p_1 to the demand $\omega_M s$, and using the expressions from (13) and (14) in the latter we have:

$$\frac{M_1}{p_1} = \frac{\gamma}{1 + \gamma + \delta} \frac{i}{i - 1} a_1$$

Finally substitute in the definition of a_1 and the aggregate supply function $y_1 = y(p_1)$:

$$\frac{M_1}{p_1} = \frac{\gamma}{1 + \gamma + \delta} \frac{i}{i - 1} \left[\frac{M_0}{p_1} + y(p_1) - \tau_1\right] \tag{15}$$

With i determined exogenously by (10), (15) defines the equilibrium value of p_1, and thus also of $E_1 = p_1/p_1^*$ and $y_1 = y(p_1)$. Since M_1 and τ_1 are treated as exogenous in (15), it is implicit that g_1 is the passive instrument of government policy in period 1; to obtain a more standard formulation in which instead τ_1 has this role, substitute out τ_1 as $g_1 - M_1/p_1 + M_0/p_1$ using the government budget constraint :

$$\frac{M_1}{p_1} = \frac{\gamma}{1 + \gamma + \delta} \frac{i}{i - 1} \left[\frac{M_1}{p_1} + y(p_1) - g_1\right] \tag{16}$$

A number of comparative static results may be derived by direct inspection of (10) and (16). First note that an increase in p_1 causes excess demand in (16), i.e. LHS < RHS, since M_1/p_1 falls proportionately more than $M_1/p_1 + y(p_1) - g_1$. Therefore an increase in g_1 (which, with M_1 unchanged is a balanced-budget increase, since τ_1 must adjust passively) raises the price level and in doing so depreciates the exchange rate and raises output. Fiscal policy is not ineffective here as it is in the textbook Mundell-Fleming model, because the increase in taxation lowers initial wealth and thus money demand, which in the textbook model is not allowed to depend on taxation. (This difference is discussed in more depth in Rankin (1990).) It also depreciates rather than appreciates the exchange rate because of the different goods market assumptions: rather than nominal prices being fixed and the real exchange rate free to change, here nominal prices are flexible and the real exchange rate is constrained to unity by the homogeneous output assumption. (See also Rankin (1989).) An increase in M_1 has similar effects, since it raises the LHS of (16) proportionately more than the RHS and so again creates excess money supply. It is clear that the stochastic nature of the model, and the consequent imperfect capital mobility, has no bearing on these results,. since just as in the perfect mobility case the country's nominal and real interest rates are determined abroad. Although domestic and foreign bonds are not perfect substitutes, their relative price is fixed independently of current monetary and fiscal policy. Taking an analogy with goods markets, there is little point in distinguishing "importables" and "exportables" for a small country - we can more conveniently aggregate them into a single composite so long as their relative price remains fixed. This feature of the model, however, is dependent on our assumption of unit-elastic expectations, as will be seen in the next section.

The stochastic nature of the model nevertheless permits us to consider certain other effects which are absent in the deterministic version. Suppose there is an increase in (subjective expectations of) the variance of domestic inflation. If domestic and foreign inflation rates are independently distributed (which is not in general true, as shown in the next section), then $E(d/d^*) = E(d)E(1/d^*)$. If the (gross) domestic inflation rate, $1/d$, is distributed lognormally, we can write $E(d) = [1+c(1/d)^2]/E(1/d)$, where $c(1/d)$ is the coefficient of variation of the inflation rate. Thus an increase in the variance of inflation with an unchanged mean causes $E(d/d^*)$ to increase and, by (10), i to fall. This in turn causes excess demand in (16), so p_1 falls, and with it, y_1 and E_1. Rather than pursue further the comparative statics of the model under arbitrary subjective distributions for (d,d^*), however, we move on to take guidance as to what are likely distributions by turning to the case of rational expectations.

4. Equilibrium with Rational Expectations

To solve for the true period-2 domestic price level p_2, and thus for the true deflation rate d, we appeal to the period-2 money market equilibrium condition:

$$M_2/p_2 = \beta a_2$$

$$= \beta[[r^*-r]\omega_F + r + [d-r]\omega_M]s \qquad (17)$$

(17) is appropriate when the exogenous domestic source of noise is taken to be M_2; when instead g_2 is treated as the source, we use the government budget constraint to replace M_2/p_2 by $M_1/p_1 + g_2$ (normalising $y_2 = \tau_2 = 0$, as discussed in Section 2):

$$M_1/p_1 + g_2 = \beta[[r^*-r]\omega_F + r + [d-r]\omega_M]s \qquad (18)$$

From the perspective of period 2, in which $(p_1,i,s,\omega_F,\omega_M)$ are predetermined - being the outcome of agents' choices already made in period 1 - (17) and (18) provide alternative equations for determining p_2, or equivalently d, as functions of the exogenous "noise" variables $(\tilde{M}_2,\tilde{r}^*)$ or $(\tilde{g}_2,\tilde{r}^*)$. Rearranging them, we get:

$$\text{Risky } M_2 \text{ case:} \quad \tilde{d} = \frac{\omega_F\tilde{r}^*}{\beta^{-1}\tilde{M}_2/p_1s - i[1-\omega_F] + [i-1]\omega_M} \qquad (19)$$

$$\text{Risky } g_2 \text{ case:} \quad \tilde{d} = \frac{\omega_F\tilde{r}^* - \beta^{-1}\tilde{g}_2/s}{\beta^{-1}M_1/p_1s - i[1-\omega_F] + [i-1]\omega_M} \qquad (20)$$

Substituting out (ω_F,ω_M) using (13) and s using (14) and simplifying:

$$\text{Risky } M_2 \text{ case:} \quad \tilde{d} = \frac{\beta\delta}{1+\gamma+\delta}p_1a_1\frac{\tilde{r}^*}{\tilde{M}_2} \qquad (21)$$

$$\text{Risky } g_2 \text{ case:} \quad \tilde{d} = \frac{\beta\delta}{1+\gamma+\delta}\frac{p_1a_1}{M_1}\tilde{r}^* - \frac{p_1}{M_1}\tilde{g}_2 \qquad (22)$$

These expressions make clear that d* (equivalently, r*) and d are in general positively correlated. The domestic deflation rate depends on the foreign deflation rate because some of the domestic portfolio is invested in foreign bonds, part of the stochastic real return on which, from the domestic viewpoint, comes from the exchange rate risk. Since, under purchasing power parity, the exchange rate is just the ratio of the two countries' price levels, exchange rate risk in turn partly reflects the risk in the foreign deflation rate. Period-2 domestic wealth therefore picks up some of this risk, and this is passed into the domestic deflation rate by the determination of the latter from money market clearing and the dependence of period-2 money demand on period-2 wealth. Note that different correlation of d and d* in the two cases arises from differences in the way the exogenous noise variables enter: in (21) as a ratio, but in (22) linearly.

We may now exploit i = i*/E(d/d*), from (10), to obtain conditional solutions for i. Dividing (21) and (22) through by d* and taking expectations gives E(d/d*), which may then be divided into i* to yield:

$$\text{Risky } M_2 \text{ case:} \quad i = \frac{1+\gamma+\delta}{\beta\delta}\,\frac{1}{p_1 a_1}\,\frac{1}{E(\widetilde{M}_2^{-1})} \tag{23}$$

$$\text{Risky } g_2 \text{ case:} \quad i = \frac{M_1}{p_1}\,\frac{1+\gamma+\delta}{\beta\delta a_1 - [1+\gamma+\delta]E(\tilde{g}_2/\tilde{r}^*)} \tag{24}$$

It is already clear from the presence of the domestic variable $p_1 a_1$ in these that i is no longer exogenously determined abroad. To complete the rational expectations solution, we now use (23) and (24) to substitute out i from the period-1 money market clearing condition (15), which defined equilibrium in the unit-elastic expectations case. We obtain after some rearrangement:

$$\text{Risky } M_2 \text{ case:} \quad \frac{M_1}{p_1} = \frac{\gamma+\beta\delta M_1 E(\widetilde{M}_2^{-1})}{1+\gamma+\delta}\left[y(p_1)+\frac{M_1}{p_1}-g_1\right] \tag{25}$$

$$\text{Risky } g_2 \text{ case:} \quad \frac{M_1}{p_1}+E\left(\frac{\tilde{g}_2}{\tilde{r}^*}\right) = \frac{\gamma+\beta\delta}{1+\gamma+\delta}\left[y(p_1)+\frac{M_1}{p_1}-g_1\right] \tag{26}$$

(25) and (26) define the equilibrium value of p_1, and thus as before of E_1 and y_1. (15) may also be used to substitute out p_1a_1 from (23) and (24) and so to complete the solutions for i:

$$\text{Risky } M_2 \text{ case:} \quad i = 1 + \frac{\gamma}{\beta\delta} \frac{1}{M_1 E(\tilde{M}_2^{-1})} \tag{27}$$

$$\text{Risky } g_2 \text{ case:} \quad i = \frac{\gamma + \beta\delta}{\beta\delta - \gamma[p_1/M_1]E(\tilde{g}_2/\tilde{r}^*)} \tag{28}$$

((28) is still not strictly a complete solution, since p_1 remains present.)

From the way in which (25) and (26), defining p_1, were derived, it is clear that any differences in the comparative static properties of (p_1,E_1,y_1) under rational compared to unit-elastic expectations arise through changes in i. When M_2 is the source of risk, i depends only on the product of the period-1 money supply and the expectation of the reciprocal of the period-2 money supply. Thus it is entirely tied down by domestic monetary policy, in stark contrast to the previous section where, given subjective expectations, it was entirely determined abroad. This is a generalisation of a similar finding for the deterministic case (Rankin (1989)), where i depends only on the *ratio* of the two money stocks. When g_2 is the source of risk, the determination of i is less dichotomised.

What is the effect of an increase in future policy uncertainty? In the risky M_2 case, suppose the exogenous distribution of M_2 is lognormal. Then $E(M_2^{-1}) = [1+c(M_2)^2]/E(M_2)$, where $c(M_2)$ is the coefficient of variation of M_2. An increase in the variance of M_2 with an unchanged mean therefore lowers i, from (27), and lowers p_1, y_1 and E_1, from (25) (note that as before an increase in p_1 causes excess demand in (25)). Uncertainty over future monetary policy is hence a bad thing from the point of view of current activity. However the channel through which it affects activity is not, as in the casual argument raised in the Introduction, through raising the domestic interest rate, but rather through lowering it. In the risky g_2 case, suppose, as would seem reasonable, that the noise sources g_2 and r^* are independent. Then $E(g_2/r^*) = E(g_2)E(1/r^*)$. From (26) and (28), an increase in the variance of g_2 with an unchanged mean then has *no* effect on p_1 or i. This shows the importance of the source of policy uncertainty to the question of its effects on the economy: starkly different results are obtained in the two cases.

A second major difference is observable with regard to the "insulation" of the economy from events abroad. In the risky M_2 case, note that no term in r* enters (25), whence neither the mean nor the variance of the foreign real interest rate matter for domestic activity and the current exchange rate. Insulation (in real terms) is complete. In the risky g_2 case, this is no longer true. With g_2 and r* independent and lognormal, an increase in the variance of r* at a constant mean, for example, increases E(1/r*). Note that under the reinterpretation of g_2 as the excess of government spending over domestic output (see Section 2), a negative as well as a positive value for $E(g_2)$ could be contemplated. If $E(g_2)$ is negative, or positive but not too large, a rise in p_1 continues to cause excess money demand in (26). With a positive $E(g_2)$, an increase in E(1/r*) raises the LHS of (26) and so causes a rise in p_1; with a negative $E(g_2)$ this is reversed. Thus the variance (and likewise the mean) of the foreign real interest rate now matter for domestic activity.

We noted in Section 3 that the nominal version of the UIP condition always holds when expressed in foreign currency as the numeraire, and by the same token always fails to hold when domestic currency is the numeraire, but suggested that the more interesting question was whether it holds for real interest rates. Expressions for the domestic real interest rate can be obtained by multiplying (21) and (23) (risky M_2 case) and (22) and (24) (risky g_2 case), and simplifying:

$$\text{Risky } M_2 \text{ case:} \quad \tilde{r} = \tilde{r}^* \frac{1}{E(\tilde{M}_2^{-1})\tilde{M}_2} \tag{29}$$

$$\text{Risky } g_2 \text{ case:} \quad \tilde{r} = \tilde{r}^* \frac{\beta\delta a_1 - [1+\gamma+\delta]\tilde{g}_2/\tilde{r}^*}{\beta\delta a_1 - [1+\gamma+\delta]E(\tilde{g}_2/\tilde{r}^*)} \tag{30}$$

If we now assume that the exogenous noise sources M_2 and r* are independently distributed, and likewise g_2 and r*, then taking expectations through (29) and (30) gives:

$$\text{Risky } M_2 \text{ case:} \quad E(\tilde{r}) = E(\tilde{r}^*) \tag{31}$$

$$\text{Risky } g_2 \text{ case:} \quad E(\tilde{r}) = E(\tilde{r}^*)\frac{\beta\delta a_1 - [1+\gamma+\delta]E(\tilde{g}_2)/E(\tilde{r}^*)}{\beta\delta a_1 - [1+\gamma+\delta]E(\tilde{g}_2)E(1/\tilde{r}^*)} \tag{32}$$

(31) shows that when M_2 is the source of noise there is no (positive or negative) real risk premium. This nevertheless does not mean that domestic and foreign bonds are perfect substitutes: (29) makes clear that r and r* are not identically distributed, and are imperfectly correlated. Only when the variance of M_2 is squeezed to zero, so that there is no domestic source of uncertainty, does r = r* in all states of nature, and "perfect" capital mobility hold. (32) shows that when g_2 is the source of noise there *is* a real risk premium, since E(1/r*) > 1/E(r*) by Jensen's inequality. There are two special cases in which no premium exists: when r* is nonstochastic, implying E(1/r*) = 1/E(r*); and when $E(g_2) = 0$. Unlike the risky M_2 case, squeezing the variance of g_2 to zero does not make domestic and foreign bonds perfect substitutes or eliminate the risk premium: this is because, as (30) shows, although r and r* are then linearly related and so perfectly correlated, they are not directly proportional unless $g_2 = 0$.

When a real risk premium exists, is it increasing in the country's indebtedness in terms of domestic bonds? From (13) and (14), this indebtedness is increasing in a_1. In (32), if $E(g_2) >$ 0 the denominator is smaller than the numerator and so E(r) *falls* as a_1 and thus domestic-currency indebtedness increases. However if $E(g_2) < 0$, then the expected positive relationship is observed. This ambiguity, together with the absence of a premium at all in the risky M_2 case, suggests that what the model is capturing is not whatever is the mechanism lying behind the conventional view of the risk premium. It is noteworthy that in the case in which increased policy uncertainty affects current activity, i.e. the risky M_2 case, there is no real risk premium, whereas in the case where it does not, i.e. the risky g_2 case, there is. Clearly then the transmission mechanism for the increased uncertainty is not through variation in the premium, in contradistinction to the argument floated in the Introduction.

The above findings are consistent with the result in Frankel (1979) (also Kouri (1983)), that to obtain the "conventional presumption" of a positive relationship between a country's indebtedness and the risk premium, there must be positive supplies of "outside" bonds in the world capital market, i.e. government bonds which are regarded as net wealth. The conventional view that what matters for the risk premium is the *country's* domestic bond debt is in fact wrong; it is only the *government's* debt which is important. As noted, such debt is absent in our model since Ricardian equivalence holds - all debt is inside debt between the domestic and foreign private sectors.

Further insight may be obtained by attempting to apply the familiar capital asset pricing formula to the model. This predicts:

$$E(r) = E(r_0) + \phi cov(r,r_m), \qquad E(r^*) = E(r_0) + \phi cov(r^*,r_m)$$

where r_m is the real return on the world market portfolio, ϕ is the "market price of risk", and r_0 is the return on an asset with zero covariance with the market. Thus the real risk premium is:

$$E(r) - E(r^*) = \phi[cov(r,r_m) - cov(r^*,r_m)] \tag{33}$$

What is the "market portfolio" in our model? Since aggregate holdings of each type of bond sum to zero, as just noted, it consists only of the two countries' currencies. Because currency is dominated as a store of value and only held for transactions reasons, the correct valuation of real currency balances, as Fama and Farber (1979) argue, is their value "sold forward" by taking a negative position in bonds of the same currency, which implies they should be discounted to obtain $[M_1/p_1i],[M_1^*/p_1^*i^*]$. The real returns on a unit of such holdings, are thus (r,r^*).. With this, $r_m = \omega r + [1-\omega]r^*$, where ω is the share of domestic currency real balances in the world total. Calculating $cov(r,r_m)$, $cov(r^*,r_m)$ and using in (33) gives, after some rearrangement:

$$E(r) - E(r^*) = \phi[cov(r,r^*) - var(r^*) + \omega var(r-r^*)] \tag{34}$$

A similar formula is derived by Dornbusch (1983) though under somewhat different assumptions.

Suppose first that there are exogenous subjective expectations of the second moments in (34) (a slight variation on our earlier unit-elastic expectations assumption). Then variation of the risk premium can occur, but (34) shows that (a) the country needs to be "large" for this to happen, i.e. ω must be non-negligible; and (b) the variation is with the level of the country's real balances, not with its (nominal interest-bearing) debt, via increases in ω as the share of domestic currency real balances held in the world portfolio increases. When the country is "small", $\omega = 0$, and (34) shows that under exogenous expectations though a risk premium can occur it will be fixed, as we found in Section 3. To permit variation of the risk premium in a small country we thus need to endogenise cov(r,r*) by moving to rational expectations. The significance of the results in (31) and (32) (which can be cross-checked from (34) by using (29) and (30) to calculate cov(r,r*) - var(r*) for our model) is that the existence and functional dependence of the risk premium in a small country is very sensitive to the source of the uncertainty and to parameter assumptions such as whether $E(g_2) >$ or < 0.

Tables 1 and 2 provide a complete summary of the comparative static properties of the rational expectations version of the model. They also contain results on the effects on savings and the trade balance, of which brevity prevents a detailed discussion. All these results can be obtained without the need for calculus: all that is required is careful inspection of the solutions obtained above.

5. Conclusions

The model suggests that casual arguments about the macroeconomic effects of increased future policy uncertainty in a small open economy should be treated with caution: any effects depend, especially, on the source of uncertainty. This conclusion is similar in spirit to that in Frankel (1979): certain "conventional presumptions" in open-economy monetary economics are easily overturned when the microeconomic basis for behaviour is spelled out. It is also akin to the findings of Persson and Svensson (1989) - their interest being in the pattern of trade in assets in a full-employment two-country world - who, like us, include rational expectations: the stochastic properties of future policy are the key to major differences in the current equilibrium.

One possible reaction to the failure to confirm conventional presumptions is to explore other reasons for imperfect capital mobility than the exchange rate risk reason investigated here. Default risk, transactions costs, asymmetric information and credit rationing, legal exchange

	p_1 (y_1,E_1)	i	s (c_1,a_1)	b	ω_M	ω_H	ω_F	$E(E_2/E_1)$
$\sigma(M_2)$	-	-	+*	-	+	-	0	+
g_1 $[\tau_1]$	+	0	-	-	0	0	0	0
M_1 $[g_1]$	+	-	-*	?	+	-	0	-
M_1 $[\tau_1]$	?	-	?	?	+	-	0	-
$E(M_2)$ $[g_2]$	+	+	-*	+	-	+	0	+
$M_1 = E(M_2)$	+	0	-*	+	0	0	0	0

Table 1 Comparative static effects when source of noise is M_2

	p_1 (y_1,E_1)	i	s (c_1,a_1)	b	ω_M	ω_H	ω_F	$E(r)$ $-E(r^*)$
$\sigma(g_2)$	0	0	0	0	0	0	0	0
$\sigma(r^*)$	+/-	+/-	-/+*	+/-	-/+	+/-	0	+/?
g_1 $[\tau_1]$	+	+/-	-	-	-/+	+/-	0	+/-
M_1 $[\tau_1]$	+	-/+	+	+	+/-	-/+	0	-/+
M_1 $[g_1]$	+	+/-*	-*	-*	-	+	0	+/-*
$E(g_2)$ $[M_2]$	+	+/?	-*	+	-/?	+/?	0	+/?
$E(r^*)$	-/+	-/+	+/-*	-/+	+/-	-/+	0	-

Table 2 Comparative static effects when source of noise is g_2

Signs are of the effect of an increase in a row variable on a column variable. (Some important cases of zero effects are omitted - see text.)

(.) indicates variables which are similarly affected
[.] indicates the budget-balancing policy variable
$M_1 = E(M_2)$ indicates an equal increase, with initial equality
X/Y: X = case where $E(g_2) > 0$; Y = case where $E(g_2) < 0$
* assumes y_p "small"
? ambiguous without further conditions

controls, all have potentially important roles. However most models of these would have to contain the ingredients already included here in some form, so that the present analysis can be seen as attempting to clarify how far we can advance without these extra factors. More immediate work which suggests itself is, first, to consider a two-country version of the present analysis. A two-country framework is already present in the background of the present model so this would be a natural extension, though it would almost certainly entail losing the ability to solve explicitly for the domestic portfolio, which is the most appealing feature of the small country model. Relaxing the restriction on future disposable income is unfortunately also likely to cause serious complications, desirable as this is in order to free future taxation as a policy instrument. More far-reaching modifications would be to investigate more general utility functions, for example CRRA, or the ordinal certainty equivalence approach which permits risk aversion and intertemporal substitutability of consumption to be disentangled; to incorporate extra assets such as indexed bonds or equity (though the *absence* of indexed bonds is a point in favour of the realism of our model compared to several others); to introduce non-Ricardian equivalence; and to permit deviations from purchasing power parity by assuming countries produce differentiated products.

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