

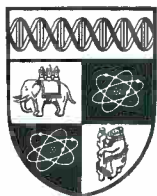
EFFICIENCY VERSUS EQUALITY : THE CASE FOR AGGREGATE COMPENSATING  
VARIATIONS IN COST-BENEFIT ANALYSIS\*

By

Yew-Kwang Ng  
Monash University, Australia 3168

No. 384

**WARWICK ECONOMIC RESEARCH PAPERS**



DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK  
COVENTRY

EFFICIENCY VERSUS EQUALITY : THE CASE FOR AGGREGATE COMPENSATING  
VARIATIONS IN COST-BENEFIT ANALYSIS\*

Yew-Kwang Ng

Monash University, Australia 3168

No. 384

Abstract

Despite its failure to perfectly ensure a potential Pareto improvement, the use of the unweighted sum of compensating variations can be justified as a practically acceptable approximation. The pursuit of equality is better achieved through taxes/transfers and the actual equality-efficiency trade-off is probably regarded by most economists as excessive than inadequate. The use of distributional weights or inequality-averse criteria is thus questionable.

\*A paper presented at the ESRC(UK) and SPES(EC)-sponsored Warwick Economics Summer Research Workshop on The Role of the Public Sector in a Mixed Economy in July 1991.

(ESRC Contract No.W10026-1035-01)

In a recent review article, Blackorby and Donaldson (1990) provide an apparently very powerful case against the use of the sum of compensating variations ( $\Sigma CV$ ) in cost-benefit analysis. Although the supporting "arguments are not new, many economists continue to use consumers' surpluses" (p.491). The present note defends this practice of many economists (in fact most, if not all of those who need to).

The arguments mustered by Blackorby and Donaldson may be divided into two groups. First, a positive  $\Sigma CV$  may occur without even a potential Pareto improvement and may thus lead to social preference reversal, unless all households have almost identical quasi-homothetic preferences. This is the question of purely positive validity and logical consistency. Secondly, even ignoring the first difficulty, the use of  $\Sigma CV$  is not ethically defensible since it treats a dollar as a dollar no matter who receives it. Let us consider these two issues in turn.

#### I. THE EFFICIENCY ISSUE

I have no quarrel with the very competent analysis of Blackorby and Donaldson based on the Boadway (1974) paradox and its extension. Acknowledging that a positive  $\Sigma CV$  does not ensure a potential Pareto improvement in the presence of price changes (due to compensation or the project), I (Ng 1979/1983, p.98) nevertheless question Boadway's (1974, p.938) claim that the magnitude and sign of  $\Sigma CV$  "are not related to the ability to compensate losers" or to the possibility of a potential Pareto improvement. Blackorby and Donaldson (1990, p.491) cite my argument in defence of the use of  $\Sigma CV$  in practice but dismiss it as "misguided".

My point is that if a project generates an aggregate benefit as measured by  $\Sigma CV$  of say \$50 million at an aggregate costs of say \$10 million with only minor price effects, then it must, in all probabilities, lead to a potential Pareto improvement, despite the logical possibility of the Boadway paradox. Thus the sign of  $\Sigma CV$ , especially if supplemented by a big magnitude, is related, though not perfectly, to the possibility of a potential Pareto improvement. Despite the obvious conceptual importance of the Boadway paradox, the use of  $\Sigma CV$  may be acceptable as a reasonable approximation to a potential Pareto improvement in most practical cases.

One could counter-argue that, if one can use something that perfectly correlates with potential Pareto improvements, or better still, with actual welfare levels, why use some imperfect substitutes? The answer is that such perfect measures are not available in most practical cases. For example, consider the proposal by Blackorby and Donaldson (1987) of using ratios of household incomes to the household's poverty lines as indexes of welfare.

For one thing, these proposed welfare ratios, as recognised by Blackorby and Donaldson (1987, p.276) themselves, "are not, in general, consistent with household preferences. Rather, they are inexact indexes of household welfare. .. the welfare ratio is an exact index of welfare for household member if and only if preferences are (individually) homothetic". Thus the superiority of using welfare ratios over the use of  $\Sigma CV$  (which has problems also when preferences are not quasi-homothetic) is rather suspect. This is consistent with my argument that "since the relation between units of any external

yardstick of welfare such as money (or welfare ratios) and internal units of welfare ... is in general not a constant, such objective measures can be, by their very nature, no more than an approximate measure of welfare, even abstracting from the problem of inaccuracies in practical data collection" (Ng 1979/1983, pp.98-9). The inexactness of both  $\Sigma CV$  and the welfare ratios must be understood in this light.<sup>1</sup>

Secondly, though Blackorby and Donaldson (1987, p.284) claim that "welfare ratios are easy to compute and interpret", they are really only easy to compute in principle, given the availability of the relevant cost or expenditure functions and the poverty line functions. In practice, they are very difficult to compute and are likely to be unavailable for most if not virtually all cases. To see that it is much more difficult to obtain approximate values of welfare ratios than CVs, note the following.

Consider a fairly typical cost-benefit analysis of building an airport, a freeway, or something like that. The traditional approach can proceed by estimating : (i) the total construction costs, (ii) total costs in terms of  $\Sigma CV$  of those significantly adversely affected (e.g. by the noise), (iii) total benefits in terms of  $\Sigma CV$  beneficiaries (e.g. travellers, commuters). While these estimates are not without difficulties, they can be done by a number of methods including survey questionnaires, estimation of time saved and the marginal valuation of time, comparisons of house prices (for houses with and without similar noises), etc.

In contrast, to estimate the relevant welfare ratios, we

cannot rely on these more direct and practicable methods. Rather, we also have to estimate how the project affect prices and incomes and how these changes affect the poverty lines for households of various types. Among others, these estimates require knowledge of the relevant expenditure functions which are unavailable for the practical cases of thousands of commodities. A reasonably qualified cost-benefit analyst knows how to proceed to obtain the traditional estimates of  $\Sigma CV$  but is most likely to be (I myself certainly would be) totally at a loss as to how to proceed with the estimation of the welfare ratios in practice.

One of the (by no means the only) reasons why it is difficult to estimate the welfare ratios in practice is that some variables (e.g. level of noise, degree of pollution) are usually involved that are not normally traded in the market. Blackorby and Donaldson's (1990, p.273) expression for determining the welfare ratio depends (apart from the household income and characteristics) only on the poverty reference utility and the set of prices. Changes in the noise and/or pollution level or some other non-market but utility-relevant variables are thus difficult to handle in such a framework.

The fact that welfare measures such as Blackorby and Donaldson's are difficult to compute in practice does not make them useless. Most practising economists are justified in continuing to use such practically more estimatable  $\Sigma CV$ . But this does not mean that we do not want our competent theorists to continue searching for superior measures. With both theoretical advances and better availability of data, practising economists may move to adopt superior measures in time. However, as the theorists are exploring better

measures, while they may note the advantages of their new proposed measures, it is important that they should not condemn the use of existing measures such as  $\Sigma CV$  as unjustifiable. Abstract theorists with no need for actual quantitative measures should not deride consumers' surpluses as "superfluous" (Samuelson 1947, p.198) or "a totally useless theoretical toy" (Little 1957, p.180) since practising economists do have to use them, and with justification. The marked contrast between theoretical and practising economists fosters a view of economists as contradicting each other --- "six economists, seven different answers to the same question!" It also fosters a view of theoretical economists (by practising economists) as a group of logicians/mathematicians in an ivory tower.

## II. THE DISTRIBUTIONAL ISSUE

A potential superiority of using the welfare ratios over  $\Sigma CV$  is that the former method "can be distributionally sensitive as well. There is no need ... to simply add welfare ratios. It may exhibit any degree of inequality aversion that the analyst chooses to impose" (Blackorby and Donaldson 1987, p.275). However, this is only the advantage (if any) over the summation of unweighted CVs, not the use of CVs as such, since one could use some distributional weights in the summation of the CVs if desired. However, since the CVs or some other measures of relevant gains and losses are usually values exclusive of disincentive effects, the use of weighted instead of unweighted sum is open to the following objection.

I have argued elsewhere that, for "any alternative ... using a system .. of purely equality-oriented preferential treatment between

the rich and the poor, there exists another alternative .. which does not use preferential treatment, that makes no one worse off, achieves the same degree of equality (of real income, or utility) and raises more government revenue, which could be used to make everyone better off" (Ng 1984, p.1038). The use of equality-oriented distortive policies produces disincentive effects on top of their specific distortive effects. Shifting to taxes/transfers is thus superior. This is true even in the presence of second-best complications, since purely equality-oriented policies (popularly used in the real world) are not based on second-best or other efficiency considerations which are also informationally very demanding and may require preferential treatment against the poor in half of the cases (See Ng 1984 for details.)

It is true we cannot and are not going to adjust the income tax/transfer system in a way to make everyone better off each time we adopt a project (A) with a higher aggregate unweighted net benefits against one (B) with better distributional effects. Some economists may thus be in favour of using distributional weights or criteria with inequality aversion so that project B can be adopted instead of project A. Similarly, other preferential treatment of the poor and purely equality-oriented policies (such as "equal access" or "first come first served" instead of price allocation) may similarly be sanctioned. Such an approach may appear to be welfare-improving, if their favourable distributional effects welfare-dominates their distortive effects. However, they are usually really welfare-reducing when their disincentive effects are taken into account. Many countries (Australia, Britain, Canada included) have in fact been using such policies for many decades. Together with the the rather



highly progressive tax/transfer system, the disincentive costs have been widely recognised (in the past two decades or so) as exceeding the distributional gain. This recognition led to drastic reductions in the progressivity of income tax/transfer systems of these countries. However, many purely equality-oriented policies remain. This is a very unfavourable situation. Had these countries not been using those purely equality-oriented policies, the progressivity of their income tax/transfer systems may not need to be reduced by such a big extent, still keeping a good degree of incentives. This would be better from the viewpoint of both the rich and the poor.

It is true that governments may not actively and optimally pursue distributive justice through income taxes and transfers. However, in the long run, some degrees of distributive balance are maintained. Even if this does not lead to an optimal trade-off between equality and efficiency, it is incorrect to ignore that some degree of balance is being maintained. As far as I know, all those in favour of using distributional weights or inequality-averse criteria effectively ignore any degree of such a balance, or, at least they have not shown their awareness of the implication of such a balance on the appropriate distributional weights. Of course, they are aware of the existence of such a balance. However, this balance implies that the distributional weights should be less unequal (e.g. a dollar to the poor counted as only \$1.20 instead of \$2) than the case of any balance. In the presence of an optimal balance, no weights (or only equal weights) should be used. In the presence of an excessive balance (i.e. equality pursued at excessively high incentive costs), the distributional weights should be reversed (i.e. more weight to the rich than the poor). While a general function such as one of welfare

ratios in expression (38) of Blackorby and Donaldson (1987, p.275) can be interpreted to be consistent with the above requirement, no advocate (as far as I know) of distributional weights or inequality-adverse criteria has explicitly shown an awareness in this respect. Many people probably believe that the distributional weights should be proportional to the social marginal utility of a dollar. However, this can only be justified if the use of distributional weights have no disincentive effects (which are efficiency costs over and above their direct distortive costs). I strongly suspect that many of those in favour of distributional weights simply ignore these disincentive effects.

It is possible that, despite some degree of balance between equality and efficiency, an economist may regard the balance as being far too inadequate. She may think that much more equality should be achieved despite the inefficiency costs involved. While this is certainly possible, the reverse case that an economist may believe that too much efficiency costs are being incurred to achieve equality should be even more likely, or at least as likely. As a group, economists are unlikely to be more equality-inclined than either politicians, bureaucrats, or voters generally whose inclinations influence government policies. Secondly, economists are more aware of the efficiency costs of pursuing equality, including administrative, compliance, policing, and disincentive costs. Thus, one should expect more economists to find the actual trade-off between equality and efficiency being excessively in favour of equality than the other way round. So, one should expect more economists in favour of using reversed distributional weights than those in favour of using the

normal distributional weights. However, to my knowledge, not a single economist has come out in favour of reversed weighting. In fact, the most radical in this respect is probably Posner (1981) who is in favour of wealth maximisation. (My argument of treating a dollar as a dollar differs from Posner's as I allow for the achievement of equality through taxes/transfers beyond wealth maximisation.) These considerations support my suspicion that many economists in favour of distributional weights simply ignore the disincentive effects involved, not just because they are more egalitarian.

What about those minority of economists who are genuinely more egalitarian than the prevailing policy? In my view, they, as economists, should not be in favour of using distributional weights and purely equality-oriented policies. However, as citizens, they should campaign to move government policies towards more equality (preferably by using more efficient methods), while the majority of their fellow economists, as citizens, should campaign for less equality if, as I argue above, they view the equality-efficiency trade-off as having been pursued to an excess.

Appendix APositive (though imperfect) Relationship Between Aggregate  
Compensating Variations and Potential Pareto Improvements

This appendix provides a specific benchmark example of a simple exchange economy with Cobb-Douglas utility functions to show the positive (though imperfect) relationship between aggregate compensating variations ( $\Sigma CV$ ) and potential Pareto improvements.

Consider only two (or rather two types of atomistic) individuals  $a$  and  $b$  with utility functions

$$(2) \quad U_i = x_i^j y_i^{1-j} ; i = a, b ; j = \alpha, \beta$$

respectively where  $x_i$  is the amount of good  $x$  consumed by individual  $i$ , etc. The budget constraint is

$$(3) \quad px_i + qy_i = M_i , i = a , b$$

where  $p$  and  $q$  are the money prices of the two goods and  $M_i$  the money income of individual  $i$ . Let  $M_a \equiv A$ ,  $M_b \equiv B$  for notational convenience.

Maximising (1) subject to (2), we have the familiar results:

$$\begin{aligned}
x_a &= \alpha A/p ; & Y_a &= (1 - \alpha)A/q \\
x_b &= \alpha B/p ; & Y_b &= (1 - \beta)B/q \\
(4) \quad x_a + x_b &= X = (\alpha A + \beta B)/p \\
Y_a + Y_b &= Y = \{(1 - \alpha)A + (1 - \beta)B\}/q
\end{aligned}$$

where  $X$  and  $Y$  are the total quantities of the two goods.

Substituting the results in (3) back into the utility function (1), we have

$$\begin{aligned}
(5) \quad U_a &= (\alpha A/p)^\alpha \{(1 - \alpha)A/q\}^{1-\alpha} \\
&= A(\alpha X/S)^\alpha \{(1 - \alpha)Y/T\}^{1-\alpha} \\
U_b &= (\beta B/p)^\beta \{(1 - \beta)B/q\}^{1-\beta} \\
&= B(\beta X/S)^\beta \{(1 - \beta)Y/T\}^{1-\beta}
\end{aligned}$$

where  $S \equiv \alpha A + \beta B$ ,  $T \equiv (1 - \alpha)A + (1 - \beta)B$  are used for notational simplicity.

In our simple economy, confining to free exchange equilibria, a situation is defined if the values of  $X$ ,  $Y$ ,  $A$ ,  $B$  are known. All other variables ( $x_a$ ,  $x_b$ ,  $Y_a$ ,  $Y_b$ ,  $p$ ,  $q$ ) can be derived from the above results and the given  $\alpha$  and  $\beta$ . We may thus define the pre-change situation 1 as given by  $X_1$ ,  $Y_1$ ,  $A_1$ ,  $B_1$  and the post-change situation 2 as given by  $X_2$ ,  $Y_2$ ,  $A_2$ ,  $B_2$ , where the 1 and 2 subscript denotes the situation. Noting that CVs are measured given the prices and incomes of the new situation, we may thus calculate the CV for individual  $a$  for the change from situation 1 to situation 2 from the equation

$$(6) \quad (A_2 - CV_a) (\alpha/p_2)^\alpha \{(1 - \alpha)/q_2\}^{1-\alpha} = A_1(\alpha/p_1)^\alpha \{(1 - \alpha)/q_1\}^{1-\alpha}$$

which follows from the definition of  $CV_a$  and the first equation in (4). Substituting the solutions for  $p_1$ ,  $p_2$ ,  $q_1$ , and  $q_2$  from (3) into (5), we may solve for  $CV_a$  as where  $S_1 \equiv \alpha A_1 + \beta B_1$ , etc. (See the definitions of  $S$  and  $T$  under Equation 4.) Similarly, we may calculate the CV for the same change from situation 1 to situation 2 for individual  $b$  as

$$(7) \quad CV_b = B_2 - \frac{B_1 X_1^\beta Y_1^{1-\beta} S_2^\beta T_2^{1-\beta}}{X_2^\beta Y_2^{1-\beta} S_1^\beta T_1^{1-\beta}}$$

From (6) and (7), we have

$$(8) \Sigma CV = CV_a + CV_b = A_2 + B_2 - \frac{A_1 X_1^\alpha Y_1^{1-\alpha} S_2^\alpha T_2^{1-\alpha}}{X_2^\alpha Y_2^{1-\alpha} S_1^\alpha T_1^{1-\alpha}} - \frac{B_1 X_1^\beta Y_1^{1-\beta} S_2^\beta T_2^{1-\beta}}{X_2^\beta Y_2^{1-\beta} S_1^\beta T_1^{1-\beta}}$$

Since a change in  $A + B$  only has nominal effects, we lose no generality by confining to cases where  $A_1 + B_1 = A_2 + B_2$ . The first thing that may be noted about (8) is that, if  $X_1 = X_2$ ,  $Y_1 = Y_2$  (i.e. if we are confined to a pure distributional change), we have  $\Sigma CV = 0$  (i.e. correctly reflecting no possibility of a PPI) if we also have  $\alpha = \beta$  (i.e. identical preferences), since we then have  $S_1 = S_2$  and  $T_1 = T_2$ . However, if  $\alpha \neq \beta$ ,  $\Sigma CV$  needs not equal zero even for a

pure distributional change, confirming the Boadway's paradox.

Now, consider non-purely distributional changes (where the values of  $X$  and  $Y$  may change). From (8), it can be seen that  $\Sigma CV$  increases with the values of  $X_2, Y_2$  relative to  $X_1, Y_1$ . That means that, if there is a sizeable increase in the quantity of one or both goods,  $\Sigma CV$  will be positive. There are cases where one good increases and the other falls such that the value of  $\Sigma CV$  does not necessarily reflect the possibility of PPI, so that the Boadway paradox generalises to non-purely distributional changes. However, if the increase is sufficiently big,  $\Sigma CV$  will be positive and a PPI is also possible. Thus, there is definitely a positive correlation between  $\Sigma CV$  and PPI; both being increasing in  $X_2, Y_2$  relative to  $X_1, Y_1$ .

Cost-benefit analysis is not concerned with purely distributional changes but with structural changes. In the simple model above, the values of  $X$  and  $Y$  change. Does a change satisfy PPI? The use of  $\Sigma CV$  does not give an exact answer to this but a big  $\Sigma CV$  is obtained when  $X_2, Y_2$  are big relative to  $X_1, Y_1$ . This obviously means that a PPI is more likely to be possible when  $\Sigma CV$  is positive and big.

References

- Blackorby, Charles and Donaldson, David (1990). A review article :  
The case against the use of the sum of compensating  
variations in cost-benefit analysis. Canadian Journal of  
Economics. 23, No.3, 471-494.
- Blackorby, Charles and Donaldson, David (1987). Welfare ratios and  
distributionally sensitive cost-benefit analysis. Journal  
of Public Economics. 34 : 265-290.
- Boadway, Robin W. (1974). The welfare foundations of cost-benefit  
analysis. Economic Journal, 84 : 926-939.
- Little, Ian M. D. (1957) . A Critique of Welfare Economics. 2nd  
edition. Oxford University Press.
- Ng, Yew-Kwang (1979/1983). Welfare Economics : Introduction and  
Development of Basic Concepts. London : Macmillan.
- Ng, Yew-Kwang (1984). Quasi-Pareto Social Improvements. American  
Economic Review. December, 74, No.5, 1033-1050.
- Posner, Richard (1981). The Economics of Justice. Cambridge, Mass. :  
Harvard University Press.
- Samuelson, Paul A. (1947). Foundations of Economic Analysis.  
Cambridge, Mass. : Harvard University Press.



Notes

1. Similarly, since all monetary measures of gains and losses must be approximations, the significance of the following superiority of equivalent variations (and surpluses) over compensating variations (and surpluses) has been overstated. As shown by Hause (1975, pp.1150-1), the compensating measures may not rank alternatives correctly when surpluses from the same initial point are compared (even for a single household) while the equivalent measures rank correctly. However, as I have shown elsewhere (Ng 1979/1983, pp.99-100), the equivalent measures may rank incorrectly for the same end point while the compensating measures rank correctly.