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Abstract
The role of asymmetric information and the incentive to acquire information is considered for a monopolistically competitive economy. To focus on nominal rigidities, the money stock is the only state variable, and it is shown how informational problems can cause nominal price rigidities. Under an asymmetric information structure it is found that uninformed firms have a disproportionate large effect on aggregate prices, reinforcing the nominal rigidities caused by some firms being uninformed. Endogenizing the information structure shows as expected that for sufficiently small information costs all firms acquire information while for sufficiently large costs all firms stay uninformed. More interesting it is also found that strategic complementarities in price-setting may cause either multiple equilibria for the information acquisition problem or preclude existence of equilibrium.

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1. INTRODUCTION

The failure of prices to adjust plays a crucial role in Keynesian macroeconomics. In particular the failure of nominal prices to adjust instantaneously to nominal shocks is important for the role of demand shocks as a source of business-cycle fluctuations. Empirical evidence shows that nominal shocks contribute to business cycle fluctuations (see e.g. Andersen (1994)). Insufficient price adjustment may at a general level be caused by either adjustment being costly in terms of explicit or implicit costs or by price-setters lacking sufficient information to make the proper adjustment.

The most widespread model of price inflexibility is the so-called menu cost model assuming price adjustment to be costly. This model has recently been extensively analysed (see e.g. Ball, Romer and Mankiw (1988) and Andersen (1994) for introductions and references) and has provided a number of important insights on price adjustment. Still, the empirical relevance of price adjustment costs remains an open question, and it is not obvious whether price adjustment costs are more important than costs of adjusting quantities.

An alternative approach to the explanation of price rigidities is to consider informational problems arising in decentralized economies. Small departures from the benchmark case of full information are sufficient to cause adjustment failures especially if firms are differently informed or if there is confusion between permanent and transitory changes (see Andersen (1994)). In this paper we extend the analysis of informational problems by making information acquisition by actors in the economy explicit.

This has several purposes. First, it allows us to check whether menu costs can be interpreted as information costs and thus act as a simple modelling device. Secondly, it allows us to check whether information acquisition has other implications for price adjustment than the obvious one that if the value of information falls short of the costs firms do not acquire information and hence there will be price adjustment failures. Finally, the analysis is relevant for the more general question of the ability of decentralized market economies to aggregate and disseminate information. This issue plays a central role in economics but has hitherto only been analysed rigorously for competitive financial markets (see Grossman and Stiglitz
(1976/80), Radner (1981)). Clearly, the question is highly relevant in relation to product markets as analysed in this paper).

The framework adapted here is a model of monopolistic competition which has proved a useful vehicle for macroeconomic analysis. Firms set prices simultaneously given their private information, but price decisions interact since firms compete over market shares. Considering first an exogenous information structure with informed and uninformed firms, we find that the uninformed firms have a disproportionately large effect on the price level due to strategic complementarities in price-setting.

Endogenizing the information structure by assuming that information can be acquired at a cost, we find as expected that for sufficiently high information costs information acquisition is never worthwhile and nobody acquires information implying completely rigid nominal prices. However, there are more important results going beyond the obvious implications of information acquisition being costly. First, there may be multiple equilibria in information acquisition, due to a self-fulfilling property in information acquisition. The incentive for any single firm to acquire information depends on how many other firms are expected to acquire information. We show that at some cost levels, there exist equilibria in which all firms acquire information and others in which no firms acquire information. This implies that even for moderate information costs no acquisition may be an equilibrium outcome and moreover information acquisition and thus price adjustment may be path-dependent. Secondly, the interplay between firms may also preclude the existence of equilibrium in the information acquisition game. This arises due to a Grossman-Stiglitz (1976,1980) type of information paradox. Information is valuable to firms and if no other firms are acquiring information there is an incentive for each firm to do so. However, if all firms are acquiring information it is not worthwhile for any single firm to do so. Our setting differs from Grossman and Stiglitz as firms cannot infer the information held by informed firms directly, but their decisions are interrelated through the implications of information for the aggregate price level, which in turn affects the optimal price for each firm and the incentive to acquire information.

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1) See Andersen and Hviid (1993, 1994) for an analysis of these issues in the context of duopoly markets with sequential price setting.
Section 2 sets up a model with monopolistic competition. The adjustment of the aggregate price level to nominal shocks is considered in section 3 presuming an exogenous asymmetry in information between firms. Section 4 endogenizes information acquisition and considers equilibria to the information acquisition game. Section 5 summarizes the paper and appendices provide some technicalities.

2. PRICE DETERMINATION
Consider a monopolistically competitive economy where demand for goods produced by firm \( j \in J \) is given as\(^2\)

\[
d_j = \left( \frac{P}{P_j} \right)^\alpha \left( \frac{M}{P} \right) \quad \alpha > 1 ,
\]

(1)

where \( P_j \) is the price charged by firm \( j \), \( P \) the aggregate price index, and \( M \) the level of nominal demand (or money stock). The aggregate price level is defined as

\[
P = \prod_{j}^{J} P_j
\]

where \( J \) is the number of firms.

To focus on nominal adjustment, the money stock \( M \) is the only state variable and it is assumed to be a random variable. For tractability, it is assumed to be log-normally distributed, i.e.,

\[
\ln M \sim N(0, \sigma^2)
\]

(2)

To the extent that firms can condition their prices on the true value of \( M \), the general price index will also be a random variable. The specific form of the demand function is chosen to capture two essential variables affecting demand, namely relative prices and the aggregate demand level.

Firms produce output subject to the following decreasing returns to scale production function

\(^2\) See Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1989) for a derivation from a CES utility function.
\[ y_j = \frac{1}{\gamma} I_j^\gamma \quad 0 < \gamma < 1 \]  \hspace{1cm} (3)

where \( I_j \) is labour input.

As is usual for monopolistic competition models, each firm ignores its effect on the other firm’s prices which occur through the price index. Firms set prices simultaneously to maximize expected real profits \( E\Pi_j \) conditioned on their information set \( I_i \) which given (1) and (3) can be written as

\[
E\Pi_j = E \left[ \frac{P_j}{P} \left( \frac{P}{P_j} \right)^\alpha \left( \frac{M}{P} \right) - \frac{W}{P} \left[ \gamma \left( \frac{P}{P_j} \right)^\alpha \left( \frac{M}{P} \right) \right]^{\frac{1}{\gamma}} \right] |I_j| \]  \hspace{1cm} (4)

where \( W \) is nominal wage which is exogenously determined. To rule out nominal wage rigidities, it is simply assumed that the nominal wage level is proportional to the nominal shock variable, i.e.

\[ W = M \]

It is well-known that nominal wage rigidity causes nominal price rigidity. Hence, the above-mentioned assumption serves the purpose of showing that in the process of price-setting there may be reasons for nominal rigidities beyond those arising in wage-setting.

The first-order condition for profit maximization can be written as

\[
\frac{\partial E\Pi_j}{\partial P_j} = E \left[ (1 - \alpha)P_j^{-\alpha}P^{\alpha-2}M - \frac{1}{\gamma} M^{\frac{1}{\gamma}} P^{\frac{\alpha-1}{\gamma}} \left( \frac{-\alpha}{\gamma} \right) P_j^{-\frac{\alpha}{\gamma}} |I_j| \right] = 0 \]  \hspace{1cm} (5)

We have assumed above that \( M \) is log-normally distributed. Below we also show that \( P \) is log-normally distributed. This allows us to simplify (5). If \( x \) is random variable log-normally distributed, we have that (see e.g. Aitchison and Brown (1957))
\[ \ln E(x) = E(\ln x) + \frac{1}{2} \text{VAR}(\ln x) \] (6)

Using (6) on (5), the resulting price decision rule can be written as

\[ \ln P_j = \lambda_{0j} + \lambda_1 E(\ln P_j | I_j) + \lambda_2 E(\ln M_j | I_j) \] (7)

where expression for \( \lambda_{0j}, \lambda_1, \) and \( \lambda_2 \) are given in appendix A, and where

\[ \lambda_1 + \lambda_2 = 1 \]

reflecting that the nominal price quoted by firm \( j \) is homogenous of degree 1 in the two exogenous nominal variables \( P \) and \( M \).

3. ASYMMETRIC INFORMATION

We want to investigate the incentives of an individual firm to acquire information, but before we proceed with this task, it is useful to analyse the case of an exogenous information structure. We therefore first solve the model assuming two groups of firms, informed who knows the realisation of \( M \) and uninformed that only know the distribution of \( M \) as given in (2). Index these firms by \( I \) and \( U \) respectively. Let \( h \) be the fraction of informed firms and assume that \( h \) is known by all. Although this assumption in itself raises informational problems, it is made here to focus on the implications of lack of information concerning exogenous state variable\(^3\). Note that for \( h > 0 \), a fraction of firms can condition their price on the realization of \( M \) making the price index a random variable from the point of view of the uninformed firm.

Because of the symmetry, all informed firms will quote the same price and similarly for the uninformed. Denote the price quoted by an informed firm by \( P_I \) and by an uninformed firm \( P_U \). The aggregate price level becomes in this case

\[ P = P_I^h P_U^{1-h} \] (8)

where \( h \) is the fraction of firms being informed.

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\(^3\) This may be justified by thinking of this as a repeated event where firms from past observations can infer the fraction of informed firms or that the agency selling information uses \( h \) in its marketing strategy.
We shall prove the existence of an equilibrium to the model under this asymmetric information structure and provide a characterization of equilibrium prices by use of the so-called undetermined coefficients method. Conjecture that the equilibrium price is determined as

\[ \ln P = \rho_0 + \rho_1 \ln M \]  

(9)

Then

\[ E(\ln P | I_U) = \rho_0 + \rho_1 \ln M \]

\[ E(\ln P | I_U) = \rho_0 \]

and we find

\[ \ln P_i = \lambda_{0i} + \lambda_1 (\rho_0 + \rho_1 \ln M) + \lambda_2 \ln M \]  

(10)

\[ \ln P_U = \lambda_{0U} + \lambda_1 \rho_0 \]  

(11)

Using the definition of the aggregate price level given in (7), we find by inserting (10) and (11) that the resulting aggregate price level can be written as

\[ \ln P = h(\lambda_{0i} + \lambda_1 \rho_0) + (1-h)(\lambda_{0U} + \lambda_1 \rho_0) + h(\lambda_2 + \lambda_1 \rho_1) \ln M \]  

(12)

For (12) to be consistent with (9), we require

\[ \rho_0 = h\lambda_{0i} + (1-h)\lambda_{0U} + \rho_0(h\lambda_1 + (1-h)\lambda_1) \]

\[ \rho_1 = h(\lambda_2 + \lambda_1 \rho_1) \]

or

\[ \rho_0 = \frac{h\lambda_{0i} + (1-h)\lambda_{0U}}{1 - \lambda_1} \]  

(13)
\[ \rho_1 = \frac{h\lambda_2}{1-h\lambda_1} = \frac{(1-\lambda_i)h}{1-h\lambda_i} \]  

(14)

Consequently, the aggregate price level is determined in (9) with the coefficient given above in (13) and (14), i.e.

\[ \log P = \frac{h\lambda_{0i} + (1-h)\lambda_{0U}}{1-\lambda_1} + \frac{(1-\lambda_i)h}{1-h\lambda_1} \ln M \]  

(15)

Hence, we have found a unique rational expectations equilibrium to the model within the class of log-linear solutions. As assumed above, \( P \) is log-normally distributed with mean \( \rho_0 \) and variance \((\rho_1\sigma)^2\). Further, the covariance between \( \ln P \) and \( \ln M \) is \( \rho_1\sigma^2 \).

We note that the coefficient to the money stock is less than one (\( \rho_1 < 1 \)) reflecting that there is a nominal rigidity. This is no surprise given that a fraction \( 1-h \) of the firms is uninformed about the nominal shocks.

It is more interesting to note that the adjustment coefficient is actually less than the fraction of informed firms, i.e.

\[ \rho_1 < h \quad \text{for} \quad h < 1 \]

Hence, even though a fraction \( h \) of firms knows the true nominal shocks the aggregate price level is adjusted by less than this fraction. The reason is the strategic complementarity in price-setting (\( \lambda_i > 0 \)) implying that informed firms take into account the prices set by uninformed firms, and since these prices by definition cannot be adjusted to the nominal shock variable, it follows that the informed adjust their prices by less. This is a variant of the result proven by Haltiwanger and Waldman (1989) that with strategic complementarities the naive agents - here the uninformed firms - have a disproportionate large effect on the equilibrium outcome compared to the sophisticated agents - here the informed firms.

The importance of the interaction between differently informed price-setters is seen clearly by comparing the equilibrium price level (15) to the hypothetical price level given as a weighted average of the price level if all firms are either informed or uninformed, i.e.
\[ \ln \tilde{P} = h \ln \tilde{P}_1 + (1 - h) \ln \tilde{P}_u \] (16)

where \( \ln \tilde{P}_1 \) (\( \ln \tilde{P}_u \)) is the aggregate price level if all firms are informed (uninformed). The prices are weighted by the fraction of informed and uninformed agents respectively. The price level in (16) does not, therefore, take into account any interaction between differently informed price-setters.

We find that

\[ \ln P - \ln \tilde{P} = (\rho_1 - h) \ln M \]

Hence, the interaction between differently informed firms does not affect the average level of prices but only the adjustment to the state variable. The interaction implies that the price level becomes less sensitive to the state variable as is seen by noting that

\[ \chi(h) \equiv \rho_1 - h = \frac{(h - 1) h \lambda_1}{1 - h \lambda_1} < 0 \quad \text{for} \quad 0 < h < 1 \]

It turns out that the reduced sensitivity of the price level to the state is dependent on the fraction of informed firms

\[ \frac{\partial \chi}{\partial h} = \frac{\lambda_1 + 2 h \lambda_1 - h^2 \lambda_1^2}{(1 - h \lambda_1)^2} \geq 0 \quad \text{for} \quad h \leq 1 - \sqrt{1 - \lambda_1} \]

\[ \frac{\partial^2 \chi}{\partial h^2} = \frac{2 \lambda_1 (1 - \lambda_1)}{1 - h \lambda_1^3} > 0 \]

Hence, \( \chi \) is convex in \( h \), zero at the upper and lower bound on \( h \), negative elsewhere and achieves a minimum for an interior value of \( h \). It follows that the interaction between differently informed price-setters have important implications for the behaviour of aggregate prices, and we next turn to an analysis of how this affects incentives to acquire information.

4. ENDOGENOUS INFORMATION ACQUISITION

In the preceding section the information structure was exogenously imposed. In this section the information structure is endogenized. Assuming information acquisition to be costly, the
issue is whether firms find it worthwhile to acquire information thus removing all uncertainty that it faces and how this in turn affects the formation of prices.

The decision whether to acquire information is an ex ante decision where the firm must decide whether it will incur a fixed real cost $c$ of acquiring information on the state of the market\(^4\) or whether it will stay uninformed. Relevant for the information acquisition decision is thus the expected profit when informed and uninformed. Further we assume that price-setting occurs after the fraction of informed $h$ firms has become common knowledge.

Using the first-order condition in (5) - which must hold for all values of $h$ - in the expected profit expression (4), the equilibrium expected profits of a firm who has acquired information can be written as

$$E\Pi_I^*(h) = \left(1 - \frac{\gamma}{\alpha}(\alpha - 1)\right) \cdot E\left[\left(\frac{P}{P_I}\right)^{\alpha-1} \left(\frac{M}{P}\right)\right] \tag{17}$$

If the firms decides not to acquire information, expected profits are

$$E\Pi_U^*(h) = \left(1 - \frac{\gamma}{\alpha}(\alpha - 1)\right) \cdot E\left[\left(\frac{P}{P_U}\right)^{\alpha-1} \left(\frac{M}{P}\right)\right] \tag{18}$$

where the dependence of expected profits on the share of informed firms has been made explicit to stress the interrelationship between firms.

Clearly, information is acquired if, for a given fraction $h$, the net gain from becoming informed outweigh costs, i.e.,

$$E\Pi_I(h) - c \geq E\Pi_U(h)$$

or if

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\(^4\) The cost could be interpreted as the subscription fee for obtaining forecast from an agency selling business cycle information.
\[ E \Pi_U(h)[\Gamma(h) - 1] \geq c \]

where

\[ \Gamma(h) = \frac{E \Pi_1(h)}{E \Pi_U(h)} \]

measures the ratio of expected profit of informed firms to that of uninformed firms. In appendix B it is shown that

\[ \ln \Gamma(h) = (\alpha - 1)(\lambda_{ou} - \lambda_{ai}) + \frac{1}{2}(1 - \alpha) \left[ (1 - \alpha + h(\alpha - 2))(\rho_e h^{-1})^2 + \rho_e h^{-1} \right] \sigma^2 \]

We further show in appendix B that

\[ \Gamma(0) > 1 \]

and

\[ \Gamma(1) > 1 \]

implying that information is valuable to the firm no matter whether all other firms are informed or uninformed. The question is whether information is sufficiently valuable to justify costly acquisition of information.

Define the critical cost level which just makes information acquisition worthwhile as

\[ \bar{c}(h) = E \Pi_U(h)[\Gamma(h) - 1] \]  

(19)

Clearly, if \( c \leq \bar{c}(h) \), there is an incentive for uninformed firms to acquire information while if \( c > \bar{c}(h) \) there is no such incentive.

Our interest here is to consider Nash equilibria to the information game. Since we consider only pure strategies and all firms are ex ante identical, we consider the conditions under which no information acquisition (\( h^* = 0 \)) is a Nash equilibrium as well as whether information acquisition by all firms (\( h^* = 1 \)) is a Nash equilibrium.
No information acquisition \( h^* = 0 \) is Nash equilibrium if \( c > \tilde{c}(0) \), while information acquisition is a Nash equilibrium if \( c \leq \tilde{c}(1) \).

The existence of equilibrium is made non-trivial by the fact that

\[
\tilde{c}(0) \neq \tilde{c}(1)
\]

Due to interdependencies in information acquisition, the relationship \( \tilde{c}(0) \) and \( \tilde{c}(1) \) is in general ambiguous. Although we would normally expect \( \tilde{c}(1) > \tilde{c}(0) \), the converse is possible for \( \gamma \) close to unity, as can be seen from

\[
\tilde{c}(0) - \tilde{c}(1) = (\Gamma(0) - 1)E\Pi_U(0) - (\Gamma(1) - 1)E\Pi_U(1)
\]

because for \( \gamma \to 1 \), \( \ln\Gamma(1) \to 0 \) and hence the second term drops out. Thus we have to consider both \( \tilde{c}(0) < \tilde{c}(1) \) and \( \tilde{c}(0) > \tilde{c}(1) \).

Case I: \( \tilde{c}(0) < \tilde{c}(1) \)

Consider first the case where \( \tilde{c}(0) < \tilde{c}(1) \). For sufficiently small information costs, \( c \leq \tilde{c}(0) \) we find that there exists an equilibrium where information is acquired by all firms, i.e. \( h^* = 1 \). For sufficiently large information costs \( c > \tilde{c}(1) \), no information is acquired by any firm, i.e. \( h^* = 0 \). For the intermediary case where \( \tilde{c}(0) < c \leq \tilde{c}(1) \), we have that both information acquisition by all firms, i.e. \( h^* = 1 \) is an equilibrium as well as is the case where none of the firms acquire information, i.e. \( h^* = 0 \). That is, we have two equilibria to the information acquisition game.

Case II: \( \tilde{c}(0) > \tilde{c}(1) \)

Consider next the case where \( \tilde{c}(0) > \tilde{c}(1) \). For sufficiently small information costs, \( c \leq \tilde{c}(1) \), we find that there exists an equilibrium where information is acquired by all firms, i.e. \( h^* = 1 \). For sufficiently large information costs \( c > \tilde{c}(1) \), no information is acquired by any firm, i.e. \( h^* = 0 \). For the intermediary case where \( \tilde{c}(1) < c \leq \tilde{c}(0) \), we have that neither having all firms acquiring information nor having no one acquiring information is an equilibrium. That is, we have no equilibrium to the information acquisition game.
To understand the intuition underlying these results, it is useful to note that the following externalities are present in information acquisition. The larger the fraction of informed firms, the more the aggregate price level adjust to changes in the money stock, cf. section 1, and hence an increase in the fraction of informed firms has a positive externality to uninformed firms by stabilizing real balances (M/P). More variability in the aggregate price level following by a larger fraction of firms being informed also means that the relative price for uninformed firms becomes more variable (P_y/P), i.e. a negative externality. In case I the negative externality is dominating - if all others are acquiring information, each single firm is also more inclined to do so (\bar{c}(1) > \bar{c}(0)), while in case II the positive externality is dominating - if all others are acquiring information, the individual incentive to do so is less (\bar{c}(1) < \bar{c}(0)).

The non-existence result in case II is related to the so-called information paradox of Grossman and Stiglitz (1976,1980). If no firm is acquiring information it is worthwhile for each single firm to incur the information costs. However, if all firms are acquiring information it is not optimal for any single firm to acquire information. The fact that all others have acquired information therefore has a negative externality on the incentive of a firm to acquire information.

Important differences between Grossman and Stiglitz (1976,1980) and the present analysis should be noted. In the former agents can infer information instantaneously from the prices called by the auctioneer and modify their plans accordingly, while this is not possible here since prices are preset by firms. Prices therefore serves no signalling role, but information is useful in predicting both the state of nature and the behaviour (prices) of other firms. In the Grossman and Stiglitz framework observation of market prices is a substitute for information acquisition, and this creates a free-rider problem in information acquisition which may lead to non-existence of equilibrium. A free-rider problem is also present here but arises via the stabilizing role price adjustment has for aggregate demand - the more firms acquiring information the more stable is aggregate demand and this has a positive externality for other

---

5) We have that \( \frac{\partial^2 \pi}{\partial (P_y/P)^2} < 0 \) from the second order condition assumed to be fulfilled, and it is easily verified that \( \frac{\partial^2 \pi}{\partial (M/P)^2} < 0 \). Hence, the firm dislikes variation in both its relative price and real demands.
firms. The positive externality need not be dominating as revealed by case I driven by the negative externality.

The preceding argument has considered the implications of variations in the information costs. Precisely, the same argumentation could be followed by assuming an invariable cost and then consider how the incentives to acquire information depend on in the variability ($\sigma^2$) of the nominal state variable. It is easily seen that this case will yield the same qualitative implications.

5. CONCLUDING REMARKS

The importance of information for price-setting and thus for the incentive to acquire information has been considered in the case where firms make the information acquisition decision before knowing the price set by others firms. This precludes that firms may infer information from the prices set by competing firms and the public value of information is therefore not at the centre of the present discussion. Crucial here is the interrelationship between price decision of firms. There is a strategic complementarity in price-setting since the price decision of a single firm is increasing in the prices set by other firms as captured by the aggregate price level. Information is thus of relevance not only for predicting the state variables but also for inferring the decisions taken by competing firms.

This interrelationship turns out to have important implications for the incentive to acquire information. It is especially interesting to note that there may be multiple equilibria or non existence to the information acquisition game depending on whether negative or positive externalities in information acquisition are dominating.

We have not commented on the welfare consequences of nominal rigidities. Using the reasoning of Mankiw (1985) and Benassy (1987) it can be concluded that expansionary shocks in combination with nominal rigidities are potentially welfare improving by expanding activity. The reason being that nominal rigidities in combination with positive nominal shocks mitigate the consequences of imperfections in the product market. Oppositely for negative shocks. Hence, "small" information costs may have "large" consequences for welfare.
Adopting a sequential decision structure implies that firms may infer information from the prices set by competing firms. This raises new aspects in relation to the use and incentive to acquire information as firms may try to affect the information competitors extract from prices (see Andersen and Hviid 1993, 1994)
APPENDIX A

To derive (7), use (6) on (5) to get

\[ \lambda_{o_0} = \left( \frac{\alpha}{\gamma} + 1 - \alpha \right)^{-1} \left[ \ln \left( \frac{1}{\alpha \gamma^\gamma} \right) + \frac{1}{2} \left[ \frac{1}{\gamma (\alpha - 1)} \right]^2 \right. \]

\[ + \frac{1}{2} \left( \frac{1}{\gamma} \right)^2 \left( \frac{1}{\gamma} \right)^2 \right] \text{VAR} \left[ \ln P | I_j \right] \]

\[ + \left( \left( 1 + \frac{1}{\gamma} \right) \left( \frac{1}{\gamma} \right) \right) \text{COV} \left[ \ln M, \ln P | I_j \right] \]

\[ \lambda_1 = \left( \frac{\alpha}{\gamma} + 1 - \alpha \right)^{-1} (\alpha - 1) \left( \frac{1}{\gamma} - 1 \right) \]

\[ \lambda_2 = \left( \frac{\alpha}{\gamma} + 1 - \alpha \right)^{-1} \frac{1}{\gamma} \]

Note that only \( \lambda_{o_0} \) depends on the parameter \( h \). For completeness we write down the equilibrium values of \( \lambda_{o_1} \) and \( \lambda_{oU} \). Using from (10) and (11) that given the information of the informed, \( M \) is not a random variable, we find

\[ \lambda_{o_1} = \left( \frac{\alpha}{\gamma} + 1 - \alpha \right)^{-1} \ln \left( \frac{1}{\alpha \gamma^\gamma} \right) \]

\[ \frac{1}{\gamma (\alpha - 1)} \]
\[ \lambda_{\text{OU}} = \lambda_{\text{ol}} + \left( \frac{\alpha}{\gamma} + 1 - \alpha \right)^{-1} \left[ \frac{1}{2} \left( \left( 1 + \frac{1}{\gamma} \right)^2 - 1 \right) + \frac{1}{2} \left( \frac{\alpha - 1 - \gamma}{\gamma} \right)^2 \right] \rho_{\text{I}}^2 \]

\[ + \left( 1 + \frac{1}{\gamma} \right) \left( \frac{\alpha - 1 - \gamma}{\gamma} \right) (\alpha - 2) \rho_{\text{I}} \sigma^2 \]

\section*{APPENDIX B}

Expected profits can be written

\[ E \Pi_j = \left( 1 - \frac{\gamma}{\alpha} (\alpha - 1) \right) E \left( MP^{\alpha - 2} P_j^{1 - \alpha} \right) \]

implying that

\[ \ln E \Pi_j = \ln \left( 1 - \frac{\gamma}{\alpha} (\alpha - 1) \right) + (\alpha - 2) \rho_0 + (1 - \alpha) E(\ln P_j) \]

\[ + \frac{1}{2} (1 + \rho_1 (\alpha - 2))^2 \sigma^2 + \frac{1}{2} (1 - \alpha) \text{Var}(\ln P_j) \]

\[ + (1 - \alpha) (1 + \rho_1 (\alpha - 2)) \text{Cov}(\ln M, \ln P_j) \]

when it has been used that \( \ln P = \rho_0 + \rho_1 \ln M \).

The value of information can now be expressed as
\[\ln \Gamma(h) = \ln E \Pi_1 - \ln E \Pi_U\]

\[= (1 - \alpha)(E(\ln P_1) - E(\ln P_U))\]

\[+ \frac{1}{2} (1 - \alpha)^2 \text{Var}(\ln P_1) + (1 - \alpha)(1 + \rho_1 (\alpha - 2)) \text{Cov}(\ln M, \ln P)\]

Using that

\[\ln P_U = \lambda_{0u} + \lambda_1 \rho_0\]

\[\ln P_1 = \lambda_{0l} + \lambda_1 \rho_0 + (\lambda_1 \rho_1 + \lambda_2) \ln M\]

and

\[\lambda_2 + \lambda_1 \rho_1 = \rho_1 h^{-1} \equiv \rho_3\]

we get

\[\ln \Gamma(h) = (\alpha - 1)(\lambda_{0u} - \lambda_{0l}) + \frac{1}{2} (1 - \alpha)\left[ (1 - \alpha + h(\alpha - 2)) \rho_3^2 + 2 \rho_3 \right] \sigma^2 \quad (B-1)\]

Next we consider the signs of \(\Gamma(0)\) and \(\Gamma(1)\). Using that \(\rho_3 = 1 - \lambda_1 = \lambda_2\) for \(h = 0\) in (B-1), we have

\[\ln \Gamma(0) = \frac{1}{2} \frac{\alpha - 1}{\alpha + 1 - \alpha} \left( \left( 1 + \frac{1}{\gamma} \right)^2 - 1 \right) \sigma^2\]

\[+ \frac{1}{2} (1 - \alpha)\left( (1 - \alpha) \lambda_2^2 + 2 \lambda_2 \right) \sigma^2\]

Hence,
\ln \Gamma(0) = \frac{1}{2} (\alpha - 1) \frac{\alpha}{\gamma^3} \sigma^2 \left( \frac{\alpha}{\gamma + 1 - \alpha} \right)^2 > 0 \quad \text{(B-2)}

Using that \( \rho_1 = 1 \) and \( \rho_3 = 1 \) for \( h = 1 \), it follows from (B-1) that

\ln \Gamma(1) = (\alpha - 1) \frac{1}{2} \left( \frac{\alpha}{\gamma + \alpha - 1} \right)^{-1} \left[ \left( 1 + \frac{1}{\gamma} \right)^2 - 1 + \left( \frac{\alpha - 1 - \gamma}{\gamma} \right)^2 - (\alpha - 2)^2 \right.

+ 2 \left( 1 + \frac{1}{\gamma} \right) \left( \frac{\alpha - 1 - \gamma}{\gamma} \right) - 2(\alpha - 2) + \frac{1}{2} (1 - \alpha) \right] \sigma^2 \quad \text{(B-3)}

\quad = \frac{1}{2} (\alpha - 1) \left( \frac{\alpha}{\gamma} - \alpha \right) \sigma^2 > 0.

Comparing (B-2) and (B-3) we note that for \( \gamma \) close to unity, \( \ln \Gamma(0) > \ln \Gamma(1) \). Further the limit of \( \ln \Gamma(h) \) as \( \gamma \) approaches unity is zero for \( h = 1 \), and positive for \( h = 0 \).
REFERENCES


