HOW HIGH CAN INFLATION GET DURING HYPERINFLATION? A LIQUIDITY COSTS DEMAND FOR MONEY APPROACH*

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HOW HIGH CAN INFLATION GET DURING HYPERINFLATION?
A liquidity costs demand for money approach. *

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Two micro-founded demand functions for money are derived. One of them is Cagan's demand for money which implies the possibility of dual steady states and a high-inflation trap. Around the high-inflation steady state real money balances and inflation change slowly. The other money demand function which is obtained by assuming liquidity costs of the type in the Baumol-Tobin model, implies that there is a single steady state, therefore there is no possibility of a high-inflation trap. The steady state is unstable. Along the hyperinflationary path real money balances decrease and inflation increases, both at an increasing rate. By imposing a lower bound for per capita consumption allowing the hyperinflationary path to be feasible, we show that the inflation rate reaches a higher upper bound when the country is less financially developed and government expenditure is smaller.

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1. INTRODUCTION

Most of the inflationary finance models developed in the literature (Evans and Yarrow (1981), Kiguel (1989), Bruno (1989), Bruno and Fischer (1990) among others) are built on Cagan's demand for money. Under perfect foresight those models imply the possibility of dual equilibria and the existence of a high-inflation trap. This trap appears because the low-inflation steady state associated with a given level of government expenditure is unstable whereas the high-inflation associated with the same government expenditure is stable. The existence of a high-inflation trap offers economic policy a relevant role in removing the attractors to the high-inflation steady state. This task is important because the economy is paying a high-inflation tax when the same government expenditure can be financed with a lower inflation tax.

A different approach to the study of hyperinflation is proposed by Casella and Feinstein (1990) by focusing on 'individual trading patterns and on the exchange process.' Their model emphasizes the liquidity costs (time) associated with exchanging depreciating nominal money for goods.

In this paper we derive a demand for money from a model with a simple liquidity costs transaction technology of the type suggested by Baumol-Tobin, trying to bridge the gap between the approach developed by Casella and Feinstein (1990) and the inflationary finance literature. This paper shows that the possibility of dual equilibria (and then a high-inflation trap) under perfect foresight found in the inflationary finance models can be due to the common assumption of Cagan's type of demand for money. In this sense, the demand for money suggested in this paper implies a single steady state whose dynamic properties depend on how fast agents change their expectations to forecasting errors. The steady state is stable when individuals revise their expectations slowly, and becomes unstable when they adjust their expectations fast. Moreover, this model bears an interesting feature under perfect foresight: one of the unstable paths leads to hyperdeflation (but this path can be ruled out because violates the transversality condition) and the other leads to hyperinflation[1].

The aim of this paper is to explain the dynamics of hyperinflation. During such episodes real money balances decrease and inflation speeds up. We show the ability of the model with liquidity costs, in contrast to Cagan's model, in order to characterize these stylized facts. For the hyperinflationary path to be feasible it is necessary to impose a lower bound for per capita real money balances because otherwise this unstable path ends up implying negative real money balances which are not feasible. This restriction is imposed on the grounds that there is a lower bound for per capita consumption which is given by the *lowest per capita consumption socially acceptable.* This lower bound for per capita consumption implies a lower bound for per capita real money balances and an upper bound for inflation.

[INSERT FIGURE 1]

This paper assumes that a hyperinflationary process ends up with an *economic reform* which is adopted when the economy reaches the lowest per capita consumption socially acceptable. As in LaHaye (1985) the currency reform[2] is triggered when the growth rate of nominal money reaches an upper bound, although in this paper the existence of such an upper bound for money growth is implied by the restriction of a minimum level of per capita consumption. As pointed out by Webb (1989, p. 83) 'the quality of *consumption* statistics does not match its importance.' Nevertheless, Webb's Table 5.4 shows evidence that consumption fell dramatically during German hyperinflation, specially during the last quarter of 1923, just before the economic reform took place. Further

[1] This result also appears in Cagan's model when the money market does not clear instantaneously as has been shown by Kiguel (1989).

[2] In this paper we use the terms *economic reform* and *currency reform* as synonymous. Following LaHaye (1985) we understand a currency reform as being characterized by the implementation of tax revenues and spending policies adjustments *in order to reduce inflation to the optimum level.*

evidence is found in Bresciani-Turroni (1937). Figure 1 summarizes his Table XIII, showing for instance that consumption of meat during the fall of 1923 was around 40% below the level reached during 1913 and the last quarter of 1927. We show that, for a given minimum level of per capita consumption, the inflation rate gets higher before economic reform takes place when the country is less financially developed. The intuition is that less financially developed countries will need more money than more financially developed ones in order to make the same volume of transactions, which implies a higher rate of inflation since money and inflation are inversely related. This hypothesis is partially supported by the data on past hyperinflationary episodes collected by Cagan (1956). For instance, as shown in Figure 2[3], Germany at the end of 1923 suffered higher inflation rates than less financially developed countries such as Poland or Russia at the end of their hyperinflationary periods. However, Figure 3 shows that countries such as Greece and Hungary (after the World War II), presumably less financially developed than Germany, suffered higher inflation rates. However, there is another determinant of the upper bound of the inflation rate: the size of government expenditure. When government expenditure is large for a given level of output, private consumption (consumption plus liquidity costs) must be small, then the upper bounds for liquidity costs and inflation are small. This result is supported by the data reported by Webb (1989, pp.49-50) which show that real government debt fell dramatically during German hyperinflation. This falling of government debt would allow an increase of private resources, implying a higher upper bound for inflation.

[INSERT FIGURE 2]

[INSERT FIGURE 3]

[3] Figures 2 and 3 plot the last 16 observations of five hyperinflationary episodes.

The strategy used to compare both inflationary finance models has been to derive the demands for money from first principles of private agents intertemporal optimization. We show that Cagan's demand for money is obtained by assuming that private agents maximize a separable utility function on consumption and real money balances. On the other hand, the alternative demand for money is derived from a logarithmic utility function on consumption[4] and by adding liquidity costs associated with the management of money.

The rest of the paper is organized as follows. Section 2 derives Cagan's demand for money and summarizes the dynamic properties around the steady states. In Section 3 an alternative demand for money is obtained by introducing liquidity costs. Section 4 compares the performance of both models in order to characterize the accelerating inflation during a hyperinflationary episode. Section 5 concludes.

2. A MICRO-FOUNDATION OF CAGAN'S MODEL

We assume a continuous time model where the economy consists of a large number of identical infinitely lived agents and the government. There is no uncertainty. A constant per capita government consumption g is financed by issuing high-powered money

$$\frac{\dot{M}(t)}{P(t)} = g. \tag{1}$$

On the other hand, each agent has a non-produced endowment $y(t) > 0$ of the non-storable consumption good per unit of time. The representative consumer's utility at time 0 is

$$\int_0^{\infty} e^{-\beta t}[U(x(t)) + V(m(t))]dt, \tag{2}$$

[4] We assume a logarithmic utility function for two reasons. First, it is an isoelastic utility function. Second, it allows the optimization problem to be written down as a money-in-the-utility-function optimization problem which is convenient for comparing the microeconomic foundations of the money demands under consideration.

where $\beta > 0$ denotes the instantaneous discount factor, $x(t)$ and $m(t)$ are per capita consumption and real money balances, respectively; $U(.)$ and $V(.)$ are increasing and concave functions. Money is introduced in the agents's utility function as an easy way of modeling transaction costs[5].

We define financial wealth and nominal interest rate as

$$w(t) = m(t) + b(t),$$

$$i(t) = r(t) + \pi(t),$$

respectively, where $r(t)$ is the real interest rate, $\pi(t)$ is the rate of inflation and $b(t)$ is per capita real government debt. The agent's budget constraint is

$$\dot{w}(t) = r(t)w(t) + y(t) - \tau(t) - [x(t) + i(t)m(t)], \tag{3}$$

where $\tau(t)$ is a lump-sum tax and the term in brackets denotes full consumption, i.e. the sum of consumption and costs of holding money. For the sake of simplicity, we are assuming that the other components of the government budget constraint, equation (1), cancel out, i.e.

$$\frac{\dot{B}(t)}{P(t)} + \tau(t) = r(t)b(t).$$

The necessary and sufficient conditions for an optimum are

$$\dot{\lambda}(t) = \lambda(t)[\beta - r(t)], \tag{4}$$

$$\dot{w}(t) = r(t)w(t) + y(t) - \tau(t) - x(t) - i(t)m(t), \tag{5}$$

$$U'(x(t)) = \lambda(t), \tag{6}$$

$$V'(m(t)) = \lambda(t)i(t), \tag{7}$$

$$\lim_{t \to \infty} e^{-\beta t}\lambda(t)w(t) = 0. \tag{8}$$

[5] Feenstra (1986) studies the functional equivalence between introducing real money balances as an argument in the utility function and entering liquidity costs in the budget constraint.

Equation (7) is the demand for money. Next, we specialize $V(m(t))$ in order to get the familiar Cagan's demand for money. Let $V(m(t))$ be defined by[6]

$$V(m(t)) = m(t)[k + 1 - ln(m(t))], \tag{9}$$

where k is a constant[7]. k might be viewed as a measure of how important domestic currency is in the economy. The larger k is the larger the utility is which is derived from holding an additional unit of money, which is a signal that the economy is not financially well developed. Then, (7) can be written as

$$ln(m(t)) = k - \lambda(t)i(t). \tag{10}$$

Cagan (1956) argues that during hyperinflationary periods the variation of real interest rate and real income is negligible compared to the variation of inflation. In our model, this argument implies that $r(t)$ and $y(t)$ can be considered constants. Moreover, if the per capita government consumption g is constant, from the equilibrium condition in the goods market, $y(t) = x(t) + g$, we have that agent's consumption is a constant. Hence, according to (6), $\lambda(t)$ is also a constant. Therefore, equation (10) becomes

$$ln(m(t)) = \gamma - \lambda\pi(t), \tag{11}$$

where $\gamma = k - \lambda r$. Equation (11) is a continuous time non-stochastic version of Cagan's demand for money, where the semielasticity of the demand for money with respect to inflation, λ, according to (6), is the marginal utility of consumption.

The dynamics of the inflationary finance model based on Cagan's demand for money under perfect foresight are well known (Evans and Yarrow (1981), Calvo (1989), Bruno and Fischer (1990)). We summarize their results here. For the sake of simplicity we drop the index t from now on. The definition of

[6] Calvo and Leiderman (1992) use a similar utility function for money.

[7] k has to satisfy $m(t) < e^k$ for all t in order for $V(m(t))$ to be increasing.

real money balances implies that $\dot{m} = g - m\pi$. Substituting (11) in this equation we have the following first-order differential equation

$$\dot{m} = g + \frac{m}{\lambda}[ln(m) - \gamma]. \tag{12}$$

Figure 4 shows that dual steady states may co-exist for a given level of government consumption whenever this level is not too high. This figure also shows that the high-inflation steady state, m_1, is stable and the low-inflation steady state, m_2, is unstable. Therefore, this analysis suggests the possibility that an economy is stuck into a high-inflation trap when the same government consumption can be financed through a lower inflation tax. A surprising result of this model is that to the right of m_2, along the unstable path, real money balances increase leading to hyperdeflation. However, this path can be ruled out since it does not satisfy the transversality condition.

[INSERT Figure 4]

Proposition 1. At the high-inflation steady state the larger k is the larger the inflation rate associated with that steady state is.

Proof: See the appendix.

According to our interpretation of k, this proposition establishes that the less financially developed the economy is (the larger k is) the higher the inflation associated with the high-inflation steady state is. The intuition is that k is directly related to the autonomous money demand γ. Then, for any given money supply, the larger the autonomous money demand is the higher inflation is in order to maintain the money market equilibrium. In the next section we develop an inflationary finance model similar to the model described above, but instead of considering an ad-hoc utility function for real money balances we assume the existence of liquidity costs of the type suggested by Baumol (1952) and Tobin (1956). We will see that the result described in Proposition 1 is reversed for this alternative model.

3. LIQUIDITY COSTS AND DEMAND FOR MONEY

We assume that liquidity costs are characterized as in the Baumol-Tobin model by $\frac{kc(t)}{2m(t)}$, where $c(t)$ is per-capita consumption and k is a positive constant characterizing how well developed the financial system of the economy is. High (low) k indicates a less (more) developed financial system because for a given level of consumption and real money balances, the liquidity costs are high (low). This liquidity costs function is implied by a transaction technology whose associated costs increase with the volume of consumption and decrease with increasing holdings of real money balances.

The optimization problem that the representative consumer faces is given by

$$\max_{c(t)} \int_0^\infty e^{-\beta t} ln(c(t))dt,$$

subject to

$$\dot{w}(t) = r(t)w(t) + y(t) - \tau(t) - [c(t) + \frac{kc(t)}{2m(t)} + i(t)m(t)].$$

As shown by Feenstra (1986 pp. 283) this problem can be written as

$$\max \int_0^\infty e^{-\beta t}[ln(x(t)) - ln(1 + \frac{k}{2m(t)})]dt, \tag{13}$$

subject to

$$\dot{w}(t) = r(t)w(t) + y(t) - \tau(t) - [x(t) + i(t)m(t)], \tag{14}$$

where $x(t) = c(t) + kc(t)/2m(t)$ denotes a composite good formed by the sum of the consumption good and liquidity services.

There are two major differences between the model introduced in this section and the one described in the previous section. First, the assumption made about the instantaneous utility function in each case: in this economy

$U(.)$ and $V(.)$ are both logarithmic functions. In particular $V(m(t)) = -ln(1 + \frac{k}{2m(t)})$. In this case, the conditions (6) and (7) are[8]

$$\frac{1}{x} = \lambda, \tag{15}$$

$$\frac{k}{m(2m+k)} = \lambda i. \tag{16}$$

Equation (16) represents the per capita demand for money which also implies that real money balances and nominal interest rate are inversely related.

Second, the assumptions that the real interest rate, r, and the endowment, y, are constants imply that per capita consumption of the composite good x holds constant whereas per capita consumption, c, changes with real money balances, m, in order to keep consumption of the composite good constant.

Assuming instantaneous clearing in the money market we have

$$\frac{k}{m(2m+k)} = \lambda r + \lambda \pi. \tag{17}$$

This condition shows that inflation and real money balances are inversely related as in equation (11). Using this last equation we have that

$$\dot{m} = g - \frac{k}{(2m+k)\lambda} + rm. \tag{18}$$

[INSERT Figure 5]

Figure 5 describes the dynamics of real money balances. The curve $\dot{m}$ is increasing for all m which implies that a single unstable steady state exists. Therefore, there is no possibility of dual equilibria. Note that to the right of the unstable steady state m^*, the economy will be moving along an unstable path where real money balances go to infinity violating the transversality condition. This feature is sufficient to preclude this hyperdeflationary path because it is not optimal for agents to follow it. On the other hand, to the left of m^* the

[8] In this case $V(m)$ is also an increasing concave function.

economy will be moving along an unstable path leading to hyperinflation and real money balances going to zero and eventually becoming negative[9]. Such path are infeasible unless there exists a restriction in the model which prevents real money balances becoming negative. In this model liquidity costs ($\frac{kc}{2m}$) go to infinity as real money balances approach zero. Since by assumption the sum of consumption good and liquidity costs x is a finite constant, it is reasonable to assume that there is a lower bound for per capita consumption that is socially acceptable. Let us denote this lower bound by $\hat{c}$. The lower bound $\hat{c}$ implies also a lower bound for real money balances, denoted by $\hat{m}$, that according to the definition of the composite consumption good is given by[10]

$$\hat{m} = \frac{k}{2}(\frac{x}{\hat{c}} - 1)^{-1}. \tag{19}$$

As stated by this assumption the economy might be moving along an unstable path with real money balances decreasing and inflation exploding, as it occurs during a hyperinflationary episode, until the real money balances reach a certain level $\hat{m}$ associated with the lowest level of per capita consumption socially acceptable, $\hat{c}$. What happens in the aftermath? Based on the experience of past hyperinflationary episodes our hypothesis is that a drastic economic reform takes place when the economy reaches some level of deterioration, say $\hat{c}$. In order to simplify the analysis we assume that private agents are unaware of the economic reform[11].

Webb (1989, p. 83, Table 5.4) brings forward evidence that consumption decreased dramatically during German hyperinflation, specially during the

[9] This type of conduct is common in both maximizing models (Obstfeld and Rogoff (1983)) like the one in this paper and ad-hoc models like Kiguel (1989) (note that in his Figure 4 the $\dot{m}$ curves tend to cross the vertical axis at finite negative $\dot{m}$'s) where along the hyperinflationary path real money balances tend to zero, but for $m = 0$ his function $\dot{m}$ and Cagan's demand for money are not defined for real numbers.

[10] Note that these lower bounds imply also an upper bound for the liquidity costs given by $\frac{k\hat{c}}{2\hat{m}}$.

[11] La Haye (1985) studies the role of expectations of currency reform to explain the evolution of inflation and real money balances during the final months of some European hyperinflationary episodes.

last quarter of this hyperinflation. For instance, the consumption of butter, meat and sugar fell down to 5%, 39% and 3% of the levels of consumption in 1923, respectively. Further evidence is reported by Bresciani-Turroni (1937) (see Figure 1). The following paragraph reveals anecdotal evidence of how poverty hit certain classes during German hyperinflation: 'The poverty was revealed by many symptoms, some of which are measurable by statistics: the condition of children (underweight, spread of tuberculosis and rickets); lack of clothing; The lowered feeding standards (fall in the consumption of cereals, meat, butter, milk, eggs, etc., and the substitution of poorer foods, . . . The statistics of meat consumption reveal some curious details which throw an interesting light on social conditions in Germany in 1922 and 1923. While the consumption of the better quality meats (bullocks, calves, pigs, and sheep) declined, the consumption of horseflesh and, still more, of dogs increased . . .' (Bresciani-Turroni (1937, p.329)).

According to the hypothesis stated above the timing of the economic reform is determined by the lowest level of per capita consumption socially acceptable. This assumption is equivalent to LaHaye's assumption that a currency reform will take place when the growth rate of nominal money reaches some upper bound $\hat{\mu}$ because, according to equation (1), $\hat{\mu} = \frac{g}{\hat{m}}$. This hypothesis helps to understand why different hyperinflations can finish at very different inflation rates. Let us denote by $\hat{\pi}$ the inflation rate associated with $\hat{m}$ and $\hat{c}$, that is, the highest inflation rate socially acceptable. Substituting equation (19) into (17) we obtain that

$$\hat{\pi} = \frac{2(x - \hat{c})^2}{\lambda x k \hat{c}} - r. \tag{20}$$

Proposition 2. (i) The larger k ceteris paribus is the smaller $\hat{\pi}$ is. (ii) The larger x ceteris paribus is the larger $\hat{\pi}$ is.

Proof: See the appendix.

The first part of Proposition 2 establishes that less financially developed countries will reach a smaller inflation rate at the end of a hyperinflationary period than more financially developed countries. The intuition is that countries with underdeveloped financial systems will need more money than countries with developed financial systems for the same volume of consumption.

The hypothesis of a currency reform driven by a low per capita consumption explains why Germany during its hyperinflation (a financially developed country at the time, with a foreign exchange market working until few months before hyperinflation ended) suffered higher inflation rates before the economic reform took place at the end of 1923 than those borne by Poland or Russia (less financially developed countries) at the end of their respective hyperinflationary episodes[12].However, this hypothesis fails to explain why Greece and Hungary, presumably less financially developed countries than Germany, reached higher inflation rates than Germany during the last two months of their respective hyperinflationary experiences after World War II.

The second part of Proposition 2 says that where the private sector has many resources (x large which implies, for a given level of output, a small level of government spending) liquidity costs and inflation could become very large before $\hat{c}$ is reached. According to our hypotheses, private resources depend upon real income and government expenditure. Data on these variables is scarce during the hyperinflationary episodes. Although the production indexes reported by Cagan (1956) show that Germany and Russia suffered big losses in production while Hungary enjoyed stable production. Since stable production, for a given government expenditure, implies stability in private resources, this evidence supports the second part of Proposition 2, which explains why Hungary might reach higher inflation rates than Germany and Russia.

[12] See Figures 2 and 3 for a comparison of the inflation rates at the end of different european hyperinflationary episodes in the 1920's.

Real money balance dynamics with adaptive expectations

The adaptive expectations assumption might be reasonable when the economy is at a low inflation rate and information is costly. In this case agents might want to use simple rules to forecast inflation because prediction errors will be small and presumably costless. However, when inflation gets higher agents have incentives to take into account additional relevant information, for instance government expenditure, to forecast inflation. The model with adaptive expectations can be simply described by the following pair of differential equations

$$\dot{m} = g - m\pi, \tag{21}$$

$$\dot{\pi}^e = \delta(g)(\pi - \pi^e), \tag{22}$$

where π is the current inflation rate, π^e denotes expected inflation, and $\delta(g)$ denotes the adjustment speed of expected change of inflation to inflation forecast errors. We assume that $\delta'(g) > 0$ capturing the idea that agents see large government expenditures as a primary cause of inflation, then, when government expenditure increases, they increase the adjustment speed $\delta(g)$ because higher government spending is a sign of higher inflation rates in the near future. Substituting equation (21) into (22)

$$\dot{\pi}^e = \delta(g)(\frac{g}{m} - \frac{\dot{m}}{m} - \pi^e). \tag{23}$$

Given that $i = r + \pi^e$, the demand for money equation (16) implies that

$$\pi^e = \frac{k}{\lambda m(2m + k)} - r.$$

Deriving this expression with respect to time

$$\dot{\pi}^e = -\frac{k}{\lambda}[\frac{(4m + k)\dot{m}}{m^2(2m + k)^2}]. \tag{24}$$

Equating (23) and (24), and after a little calculation we obtain

$$\dot{m} = \delta(g)[g - \frac{k}{\lambda(2m + k)} + rm][\delta(g) - \frac{k(4m + k)}{\lambda m^2(2m + k)^2}]^{-1}.$$

The sign of the slope of $\dot{m}$ at the steady state m^* is given by

$$sign(\frac{\partial \dot{m}}{\partial m}|_{ss}) = sign[\delta(g) - \frac{k(4m+k)}{\lambda m^2(2m+k)^2}] = sign[\delta(g) + \frac{V''(m)}{U'(x)}].$$

Therefore, the monetary steady state is locally stable (the slope of $\dot{m}$ is negative) when agents revise their expectations slowly (that is, when $\delta(g)$ is small) as shown in Figure 6. However, the steady state becomes unstable when government spending is sufficiently large because agents, identifying an expansive fiscal policy (large g) with high inflation in the near future, will adjust their expected change of inflation more rapidly in the current period, that is, $\delta(g)$ becomes large.

[INSERT Figure 6]

4. ACCELERATING INFLATION AND PERFORMANCE OF THE MODELS

During hyperinflationary episodes real money balances decrease and inflation speeds up. As an illustrative example Figures 7 and 8 show the evolution of inflation and real money balances during German hyperinflation.

[INSERT FIGURE 7]

[INSERT FIGURE 8]

Exploding inflation is a typical feature during hyperinflation. This feature is not well characterized in the inflationary finance model based on Cagan's model (reviewed in Section 2). Figure 4 describes the dynamics of inflation for that model. Along the line connecting both steady states (m_2 with m_1) real money balances decrease and inflation increases, but it converges to the high-inflation steady state value $\frac{\gamma - ln(m_1)}{\lambda}$, instead of exploding. On the other hand, the model developed in Section 3 is capable of characterizing the dynamics of hyperinflation quite well. In Figure 5, we observe that along the hyperinflationary path, to the left of m_1, real money balances are decreasing and inflation increases, but as $\dot{m} < 0$ increases in absolute value, inflation grows faster and faster as time goes by.

In order to clarify the dynamics of inflation and real money balances in each model we simulate them. The evolution of real money balances is fully characterized by the differential equation (12) in Cagan's model. Once we have integrated (12), from equation (11) we can solve for the inflation rate[13]. On the other hand, the evolution of inflation and real money balances in the model with liquidity costs are obtained from equations (17) and (18) using the same method as in Cagan's model. Table 1 summarizes the parameter values used to carried out the simulations.

[INSERT TABLE 1]

In both models the per capita endowment (y) is distributed between private consumption (x) and government consumption (g). Normalizing y to be one, we assume that the government expenditure is 37% of the endowment. This value is close to the average of the share of net social product government spending during the German hyperinflation as reported by Webb (1989 pp. 83). Therefore, in our simple models that figure implies that the share of private consumption in the net social product during German hyperinflation was around 63%. Assuming that the utility of consumption is logarithmic in both models we have that $\lambda = \frac{1}{0.63} = 1.59$. The real interest rate (r) is assumed to be 5%. Finally, the constant k is chosen so that the daily steady state rate of inflation is 0.14 in the liquidity costs model. This value is roughly the value taken by inflation during the initial periods of German hyperinflation, from

[13] The differential equation (12) has been solved numerically using the software package SIMGAUSS V2.0 written in GAUSS V3.01.

June to December of 1921 (see Cagan (1956 pp.102))[14]. Figure 9 shows that the simulated inflation path implied by the model with liquidity costs explodes, as indeed it did during German hyperinflation. Figure 10 shows the simulated real money balances path approaching zero, as it seems to have done during that hyperinflationary episode.

[INSERT FIGURE 9]

[INSERT FIGURE 10]

However, Figure 11 shows that the simulated inflation path suggested by Cagan's model does not explode, instead it converges to a stable steady state. Moreover, Figure 12 shows that the simulated path of real money balances implied by Cagan's model converges to a steady positive value instead of going to zero. As pointed out by Buiter (1987 pp.117) those features implied by Cagan's model do not seem to match those observed during hyperinflation.

[INSERT FIGURE 11]

[INSERT FIGURE 12]

5. CONCLUSIONS

The aim of this paper is two-fold. First, we investigate how robust the high-inflation trap result found in the inflationary finance literature is. This study is relevant because many policy prescriptions are made on the basis of the existence of a high-inflation trap. The paper shows that a money demand derived from a model which introduces liquidity costs has very different implications for the dynamic properties of the steady states than those found in

[14] The inflation time series are multiplied by $\frac{1}{log_{10}e}$ in order to obtain the monthly rate.

a model based on Cagan's demand for money. In particular, Cagan's model implies the possibility of a high-inflation trap, that is, for a given level of government spending the high-inflation steady state is stable whereas the low-inflation steady state is unstable. However, a single unstable steady state exists when the demand for money based on liquidity costs is considered, implying that there is no possibility for a high-inflation trap. Another important difference between both models is that whereas in Cagan's model the stable path converges to a steady inflation rate, in the model with liquidity costs the single relevant unstable path (the other unstable path leads to hyperdeflation, which violates the transversality condition) leads to hyperinflation in the sense that real money balances fall down and inflation explodes. This feature of the model with liquidity costs matches the explosive behavior of inflation observed during hyperinflationary periods. To allow the feasibility of hyperinflationary paths, however, we need to impose the assumption that there exists a lower bound for per capita consumption. This assumption precludes real money balances becoming negative.

Second, we study the determinants of the timing of an economic reform in the liquidity costs model under the hypothesis that an economic reform to stabilize prices is triggered by the fact that the economy reaches the lowest level of per capita consumption instead of a particular rate of inflation. Fixing the lower bound for per capita consumption we show that the upper bound level of inflation reached at the end a hyperinflationary episode before an economic reform takes place depends on how well the financial sector is developed and how large the ratio government expenditure/output is. Inflation is higher when the financial sector is more developed and government expenditure is relatively smaller in comparison to output.

Our main hypothesis states that a stabilizing economic reform is triggered when the economy reaches the lowest level of per capita consumption socially acceptable. When this minimum level of consumption is reached we understand that the different social groups playing in the economy, which are not explicitly described in the paper, are ready to reach an agreement that allows stabilization, in the sense that the welfare cost of stabilization is smaller than the welfare cost of living with hyperinflation. Casella and Feinstein (1990) have found, in their model, that for sufficiently high rates of inflation all agents (buyers and sellers or workers and businessmen) suffer a welfare loss. This result shows that the incentives for an economic reform would appear at high enough rates of inflation. This result also supports the idea that stabilization is more successful during hyperinflation than during moderate inflation episodes.

Our hypothesis is difficult to test because few consumption statistics are available in the case of German hyperinflation and inexistent in other hyperinflationary episodes. We believe that finding new and more precise consumption statistics would help to understand the ending of hyperinflationary episodes.

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APPENDIX

Proof of Proposition 1: Note that $\gamma = k - \lambda r$, then we have only to show that $\frac{\partial \pi_1}{\partial \gamma} > 0$, where m_1 and π_1 denote the real money balances and the inflation rate associated with the high-inflation steady state, respectively. From (11) – (12) we have that (m_1, π_1) is determined by the following pair of equations:

$$ln(m_1) + \lambda \pi_1 = \gamma,$$

$$\lambda g = m_1 ln(m_1) - \gamma m_1.$$

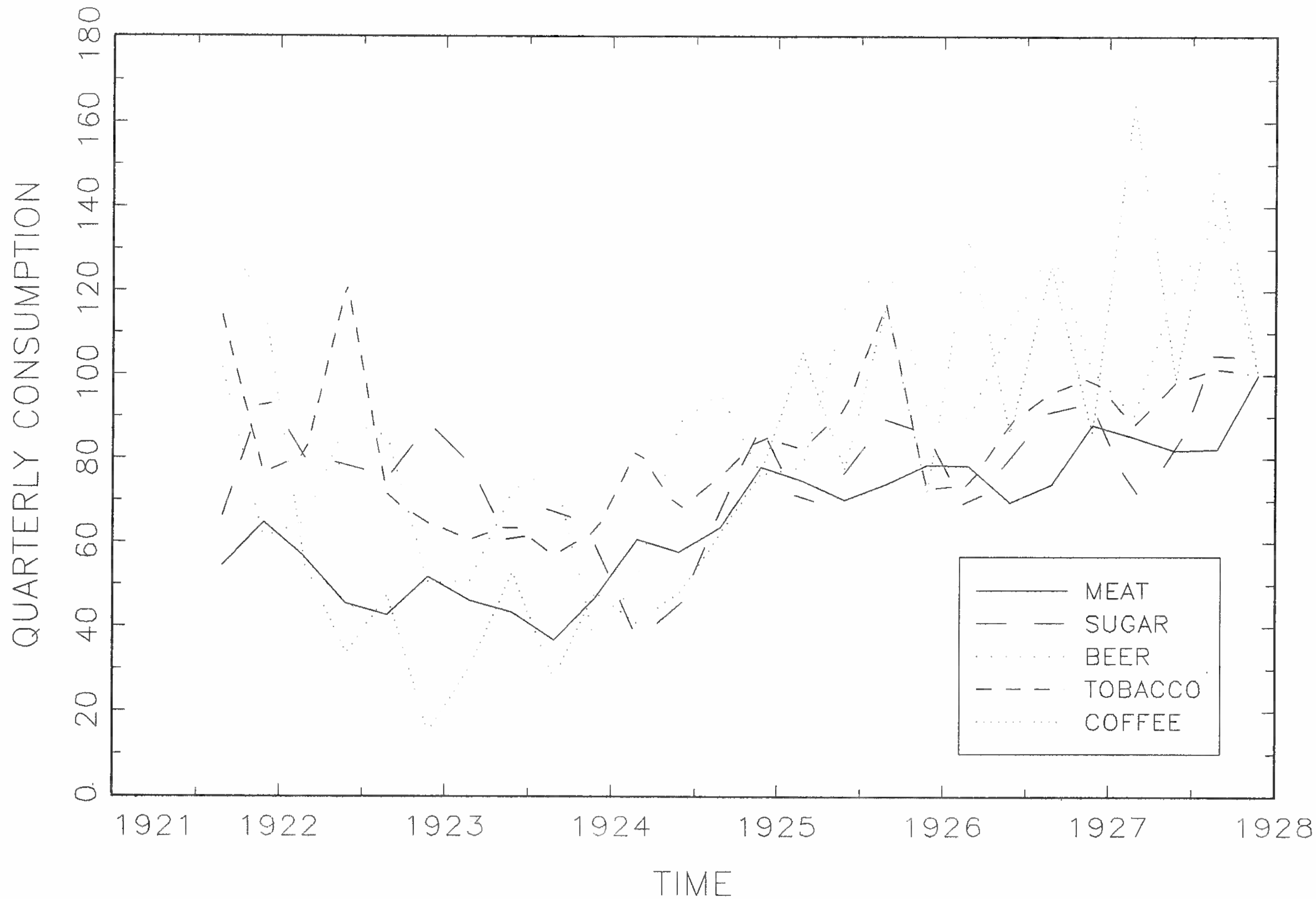
After a little calculation we have that $\frac{\partial \pi_1}{\partial \gamma} = \frac{\gamma - ln(m_1)}{\Delta}$, where $\Delta = -\lambda[ln(m_1)+1-\gamma]$. Note that $\frac{\partial \dot{m}}{\partial m}|_{m_1} = \frac{1}{\lambda}[ln(m_1)+1-\gamma]$. From Figure 4 we know that $\frac{\partial \dot{m}}{\partial m}|_{m_1} < 0$. Then $\Delta > 0$ and $\frac{\partial \pi_1}{\partial \gamma} > 0$ which completes the proof.

Proof of Proposition 2: The first part is straightforward from equation (20). Also, from (20) we have that $\frac{\partial \hat{\pi}}{\partial x} = \frac{2\lambda k\hat{c}(x^2-\hat{c}^2)}{(\lambda kx\hat{c})^2} > 0$, which proves the second part.

Table 1

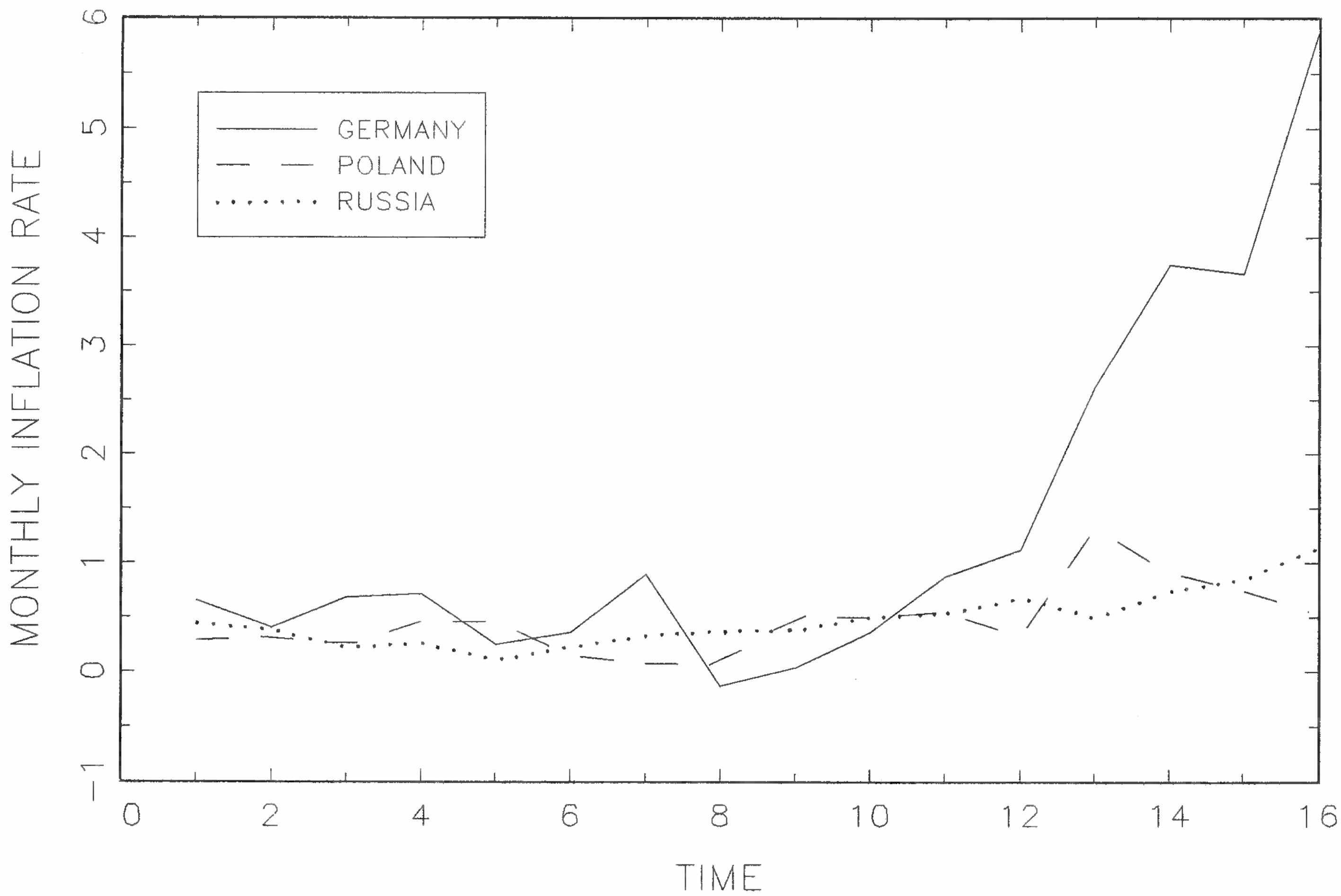
Liquidity Costs Model	Cagan's Model
λ=1.59	$\lambda = 1.59$
g=0.37	g=0.37
r=0.05	r=0.05
k=20	γ=0.5

Figure 1: GERMAN CONSUMPTION (1921,3–1927,4) (1927,4=100)



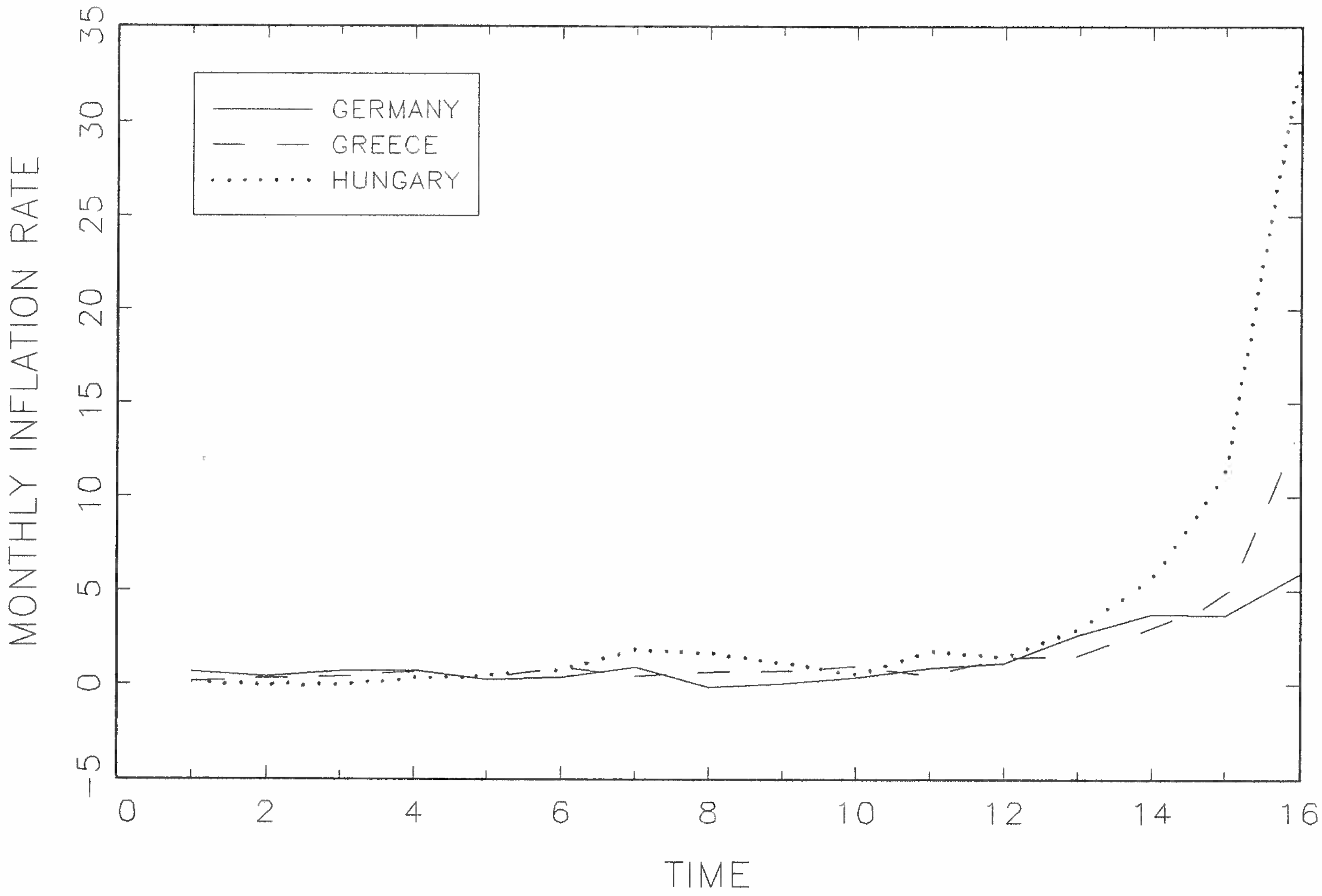
Source: Bresciani–Turroni (1937, pp.455–456)

Figure 2: INFLATION RATES FOR HISTORIC HYPERINFLATIONS



Source: Cagan (1956, pp. 102, 112, 115)

Figure 3: INFLATION RATES FOR HISTORIC HYPERINFLATIONS



Source: Cagan (1956, pp. 102, 107, 110)

FIGURE 4

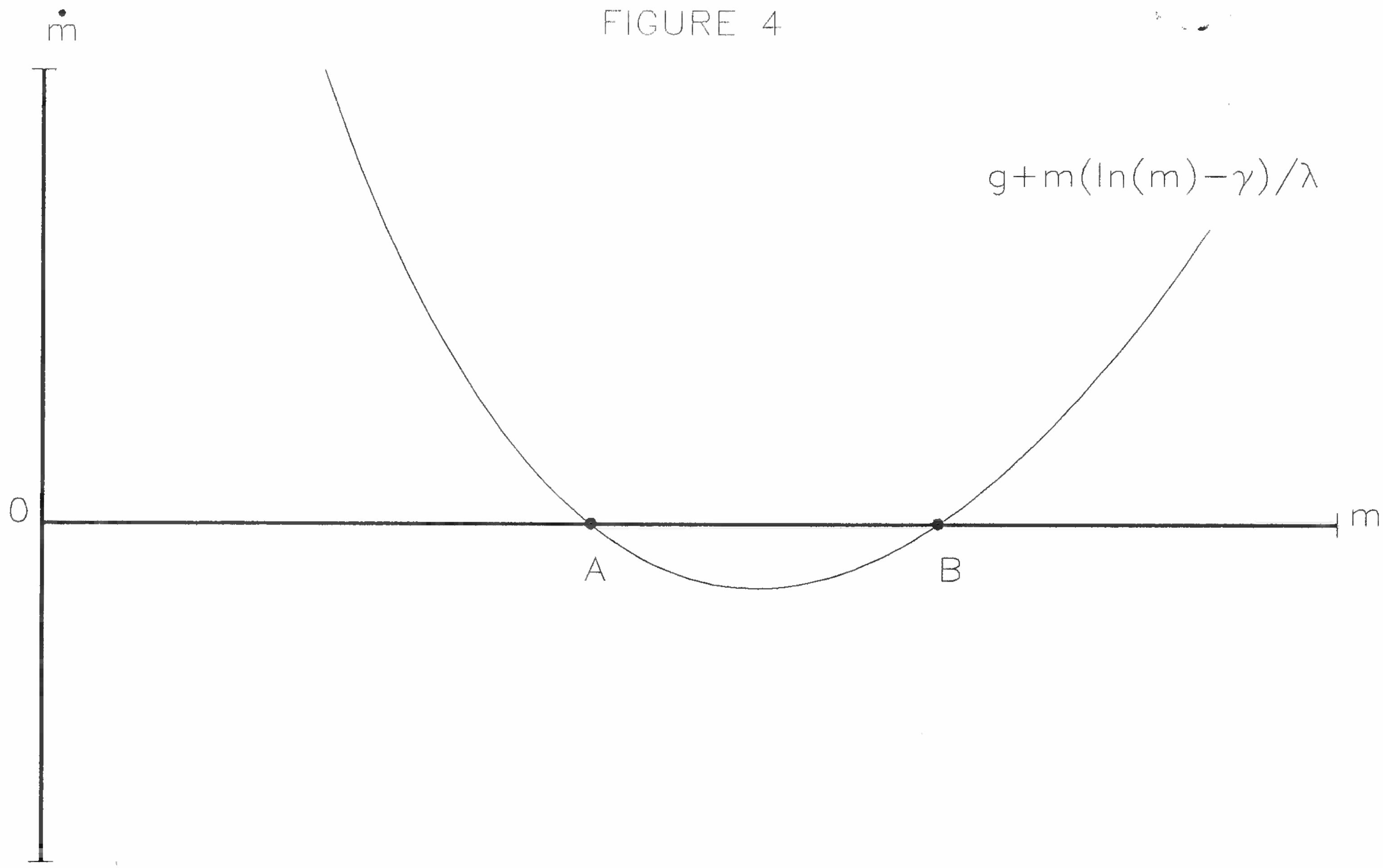


FIGURE 5

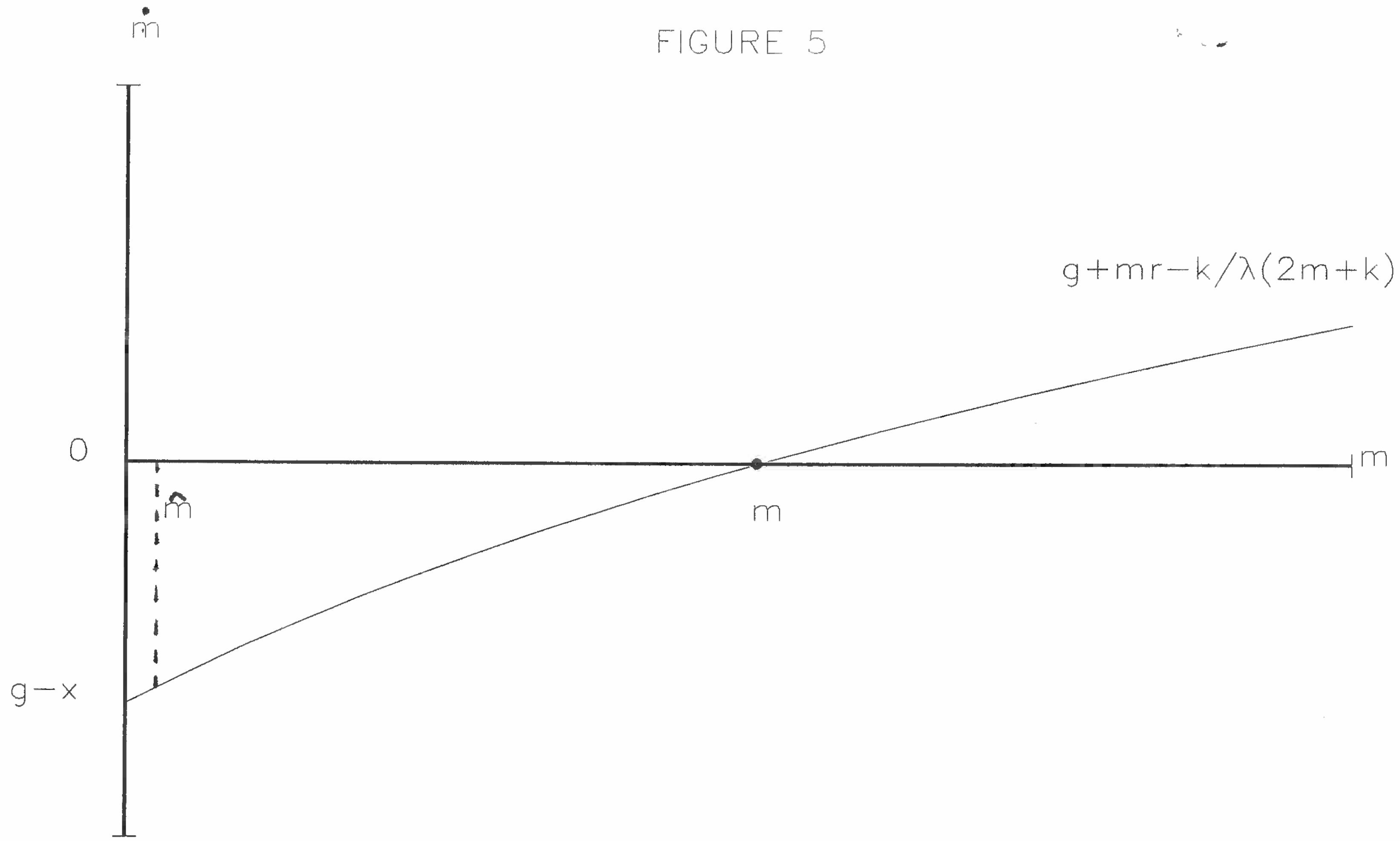
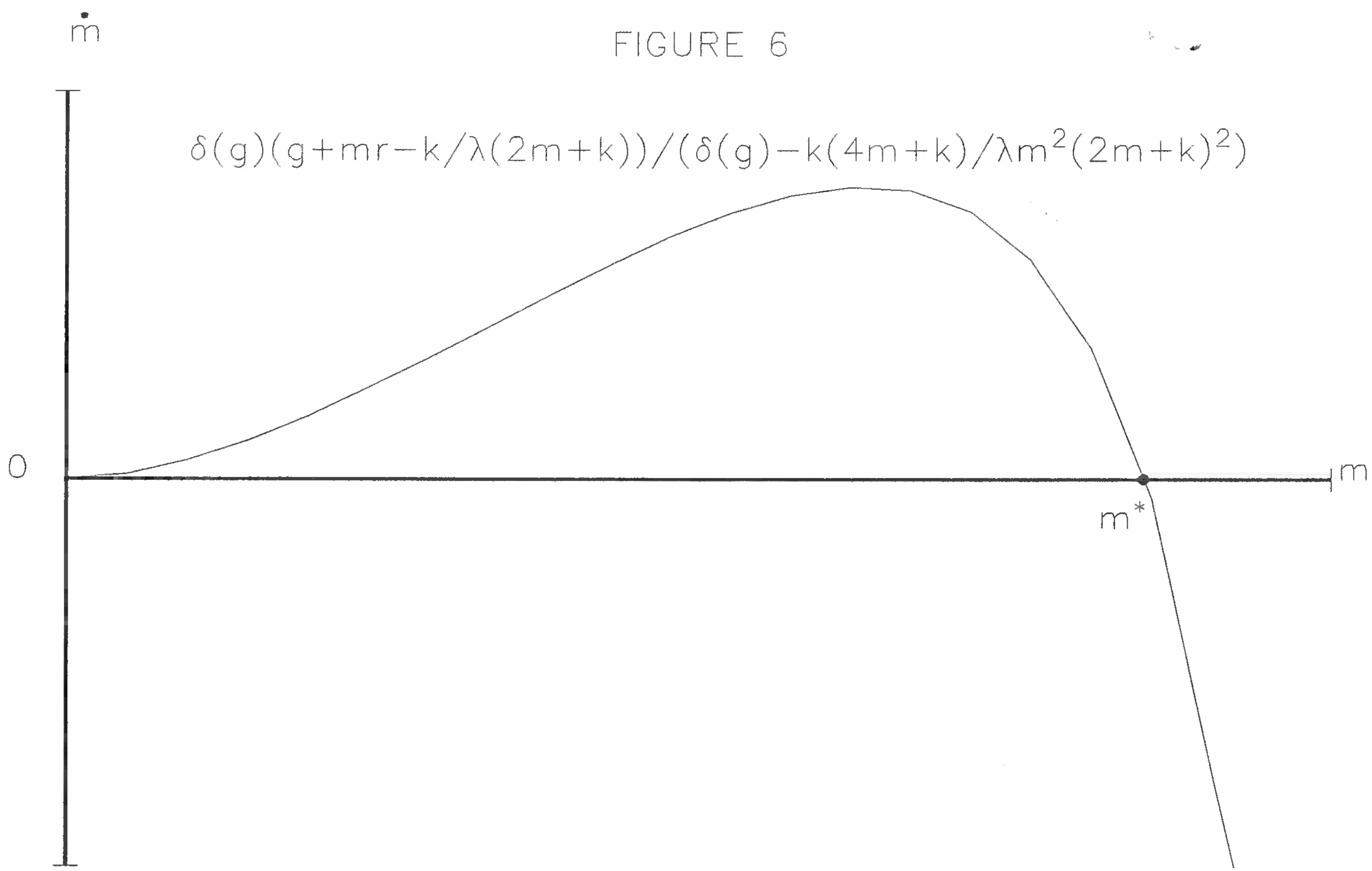
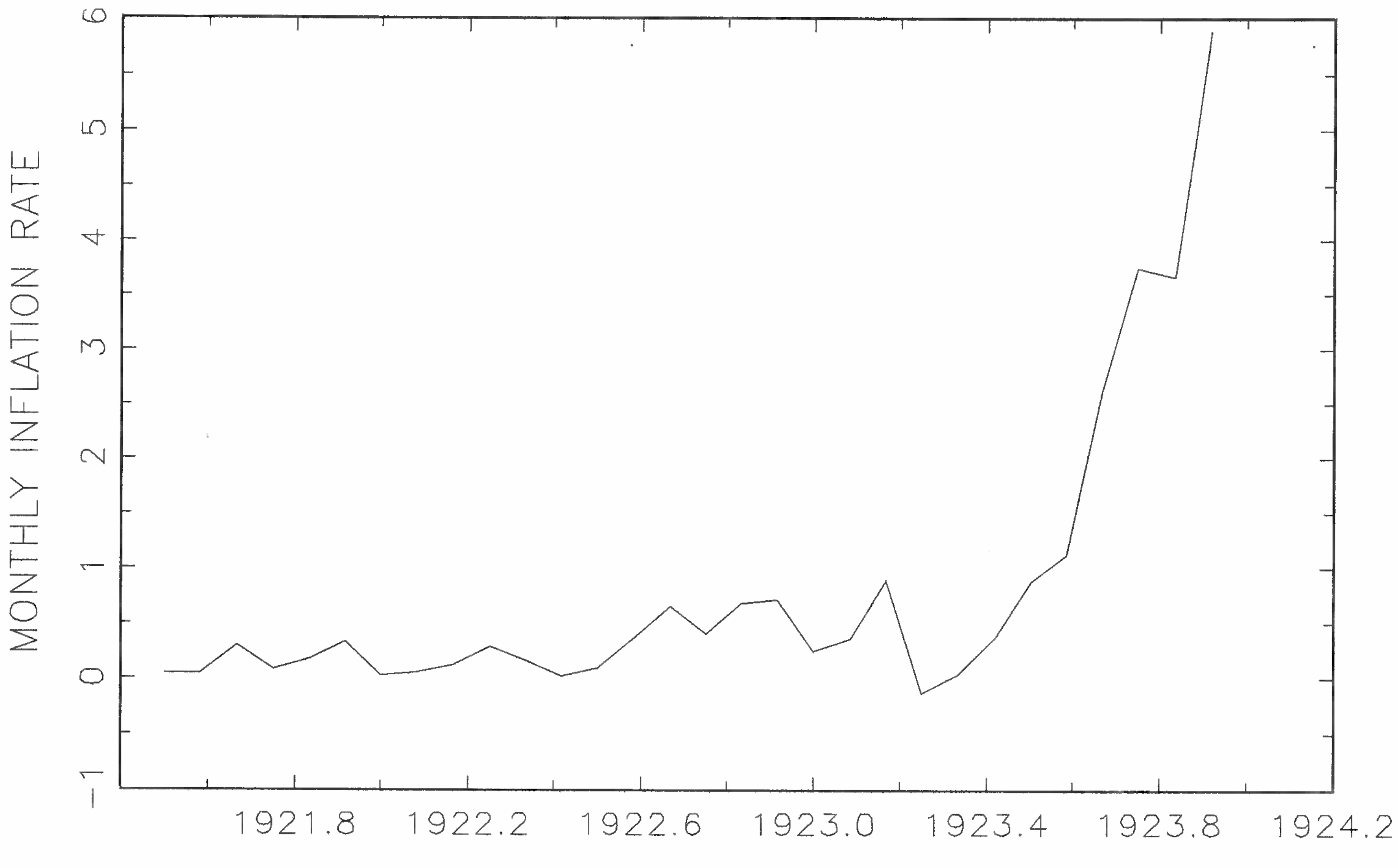


FIGURE 6



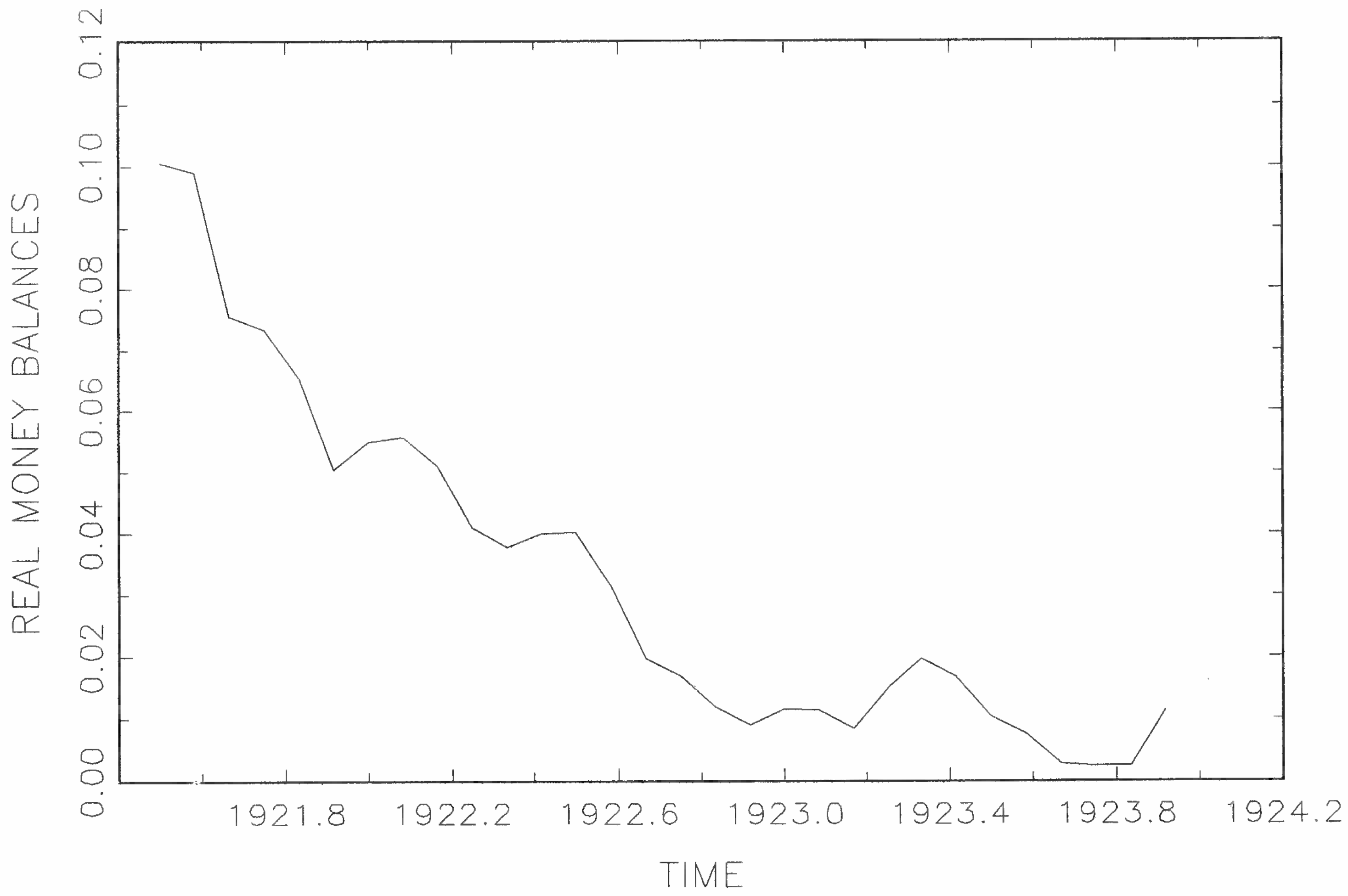
δ(g) small

Figure 7: EVOLUTION OF INFLATION



Source: Cagan (1956, p.102)

Figure 8: EVOLUTION OF REAL MONEY BALANCES



Source: Cagan (1956, p.102)

Figure 9: EVOLUTION OF SIMULATED INFLATION

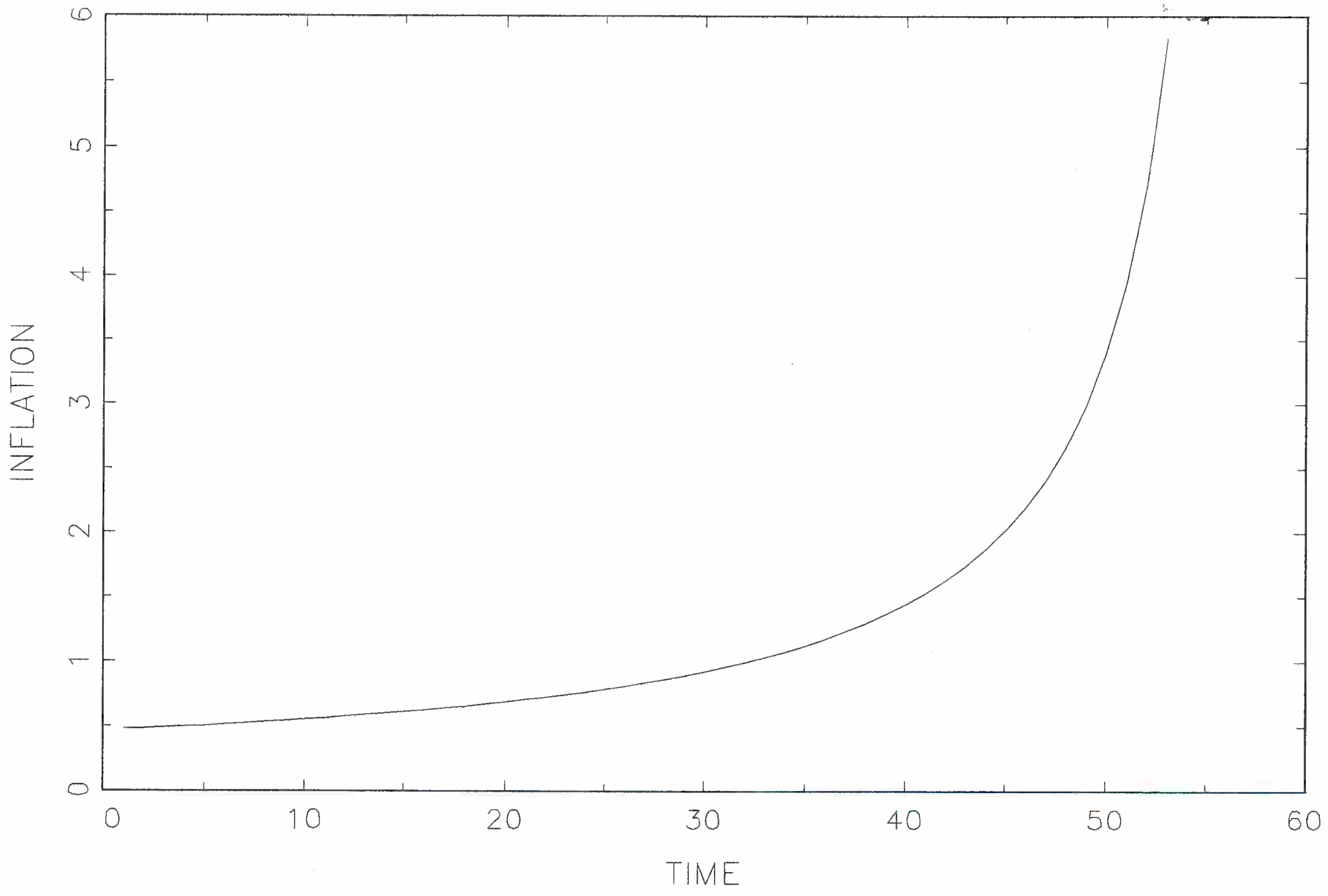


Figure 10: EVOLUTION OF SIMULATED REAL MONEY BALANCES

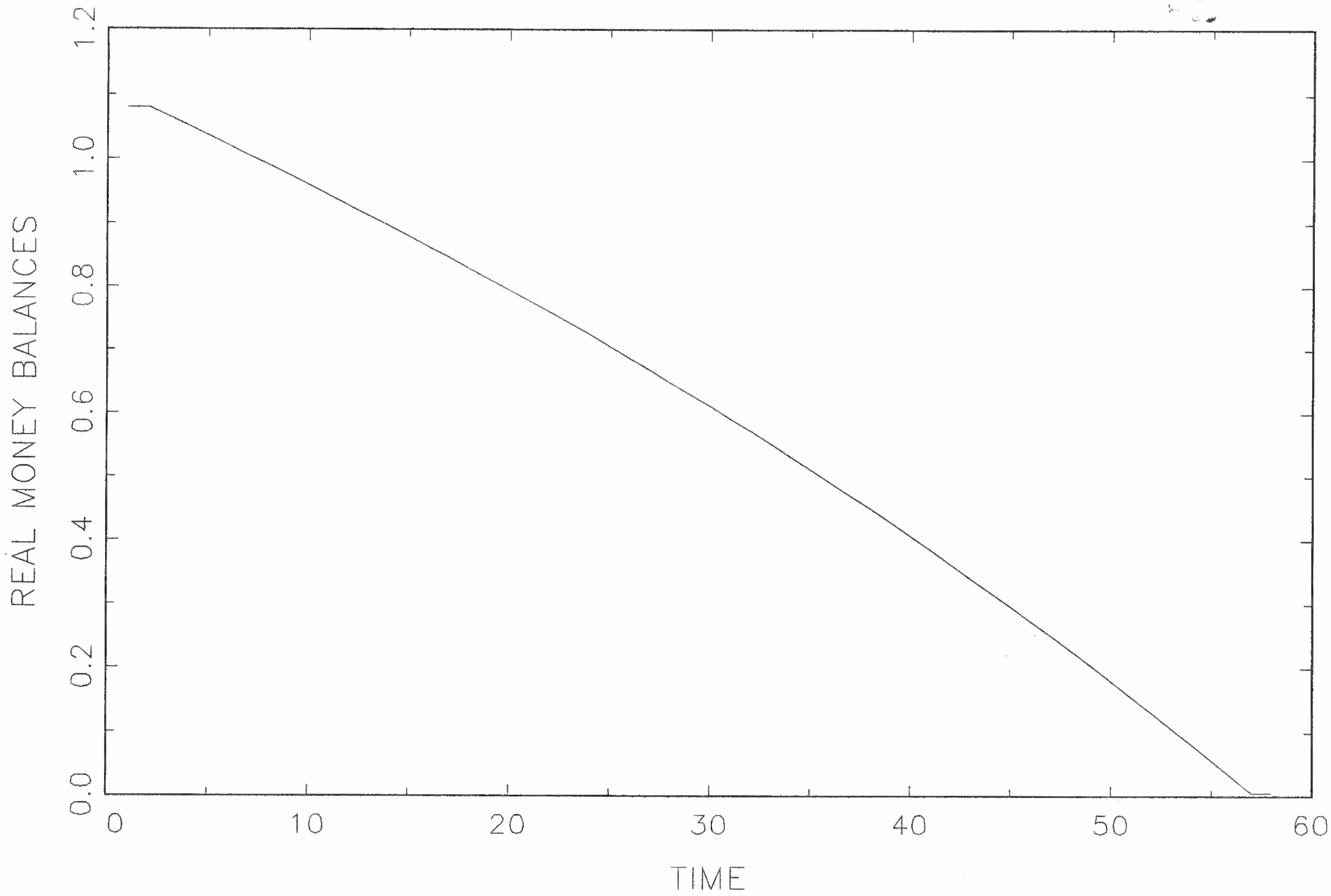


Figure 11: SIMULATED INFLATION IN CAGAN'S MODEL

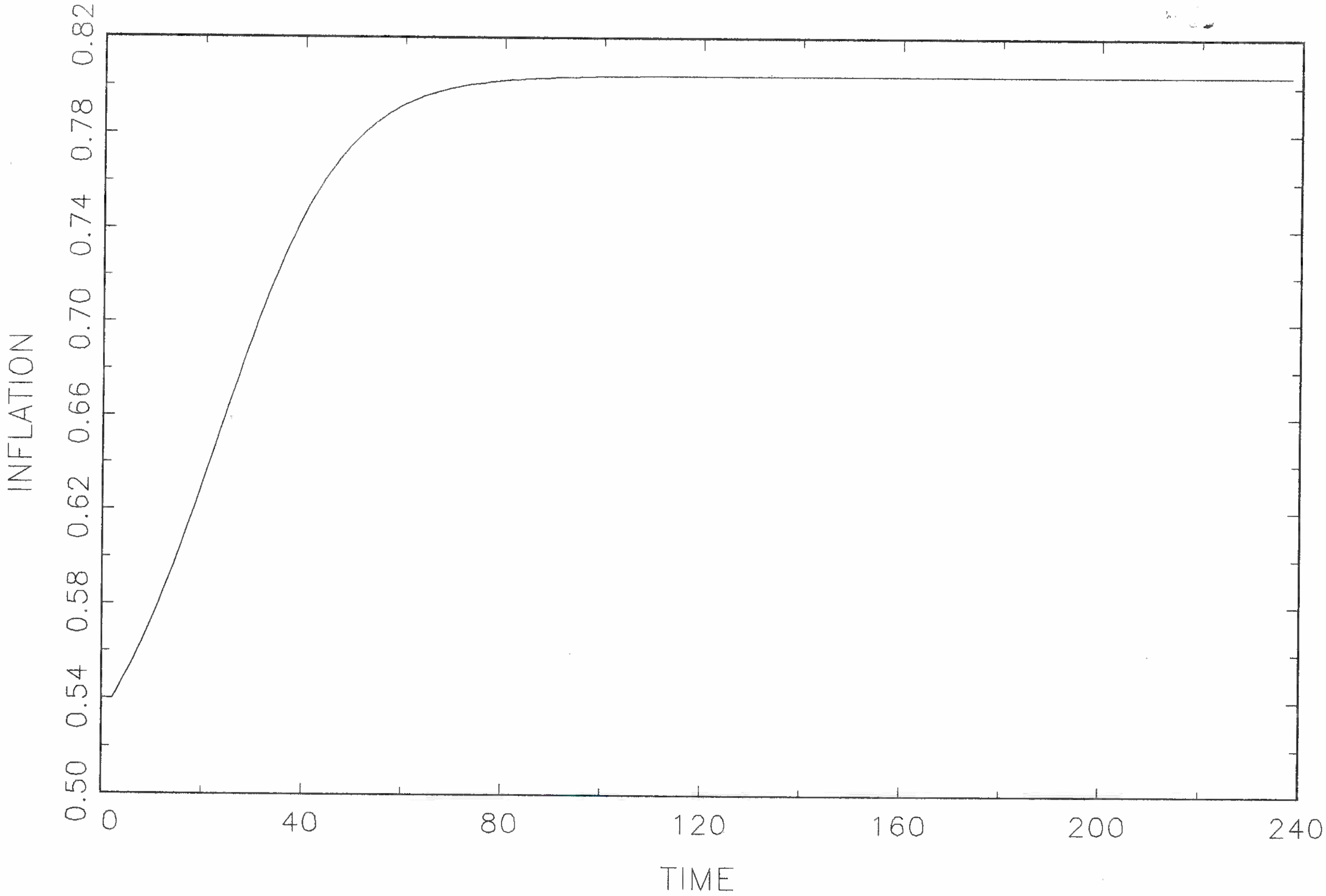


Figure 12: SIMULATED REAL MONEY BALANCES IN CAGAN'S MODEL