MARKET INSURANCE, SELF-PROTECTION AND THE FAMILY: A BECKERIAN ANALYSIS

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No.437

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
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(Preliminary - Comments welcome!)

ABSTRACT.

We study parents' demand for insurance and protection in a Beckerian context. Parents derive utility from the household's material living standard and number of children and there is a trade-off between the two. Several important new results emerge. These include: first, a duality between how an increase in an exogenous child mortality risk affects the demand for children and how an exogenous increase in the number of children affects the demand for physical safety for a given child; second, a distinction between and the different implications of endogenous safety as a private good and as a local public good for the household; third, the important interactions between the parents' demand for insurance and personal and household safety and the presence and nature of their bequest functions.

JEL Classifications: D1, D18, D81, K13

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I. Introduction

Many of the major insurance decisions which households make concern physical risks with important pecuniary consequences. For example, the death of a parent, whether an income-earner or not, is likely to result in a substantial fall in the household's material standard of living if there are dependent children who require care. Conversely, the death of a dependent child who is not subsequently replaced is likely to result in an increase in the household's material standard of living. These effects are separate from the perhaps transient impact on the household's welfare of the remaining members' feelings of bereavement and the more enduring impact on the utility which household members derive from each other's company.

If we ignore time inconsistency in the determination of optimal family size and imperfections in contraception and conception, it is clear that parents' choice of their family's size involves a trade-off between the utility derived from this size, per se, and the material standard of living. Nevertheless, this is not reflected in conventional analyses of the household's insurance and safety decisions. These are based on households with single argument, utility-of-wealth-or-income functions which, sometimes, allow for event-dependence via a conditional expected utility approach or a bequest function[1].

Neglect of parents' trade-off between the household's size and its material standard of living can have important consequences. In two companion papers (Fraser, 1995b, c), we show that incorporating such a trade-off in a context where parents have the opportunity to buy fair insurance against child mortality risk enables us to overturn some of the most important results in the economics of insurance. These include the finding that agents equalise the marginal utility of income across state when given the opportunity to purchase fair insurance and Cook and Graham's (1977) "incomplete insurance theorem" for state-dependent preferences. The latter states that consumers will not equate income across states when they can purchase fair insurance against a loss which, other things equal, reduces the marginal utility of money as compared with the no-loss state. This means that if a loss is purely non-pecuniary but reduces the marginal utility as indicated, then consumers would purchase no insurance at all against such a loss.

Fraser (1995b) considered a model in which parents have a Beckerian utility function which generates utility from the household's material standard of living (surrogated by its per capita income or income per equivalent adult) and the size of the household. Among other things, we showed that: (a) the availability of fair insurance against child mortality risk resulted in parents opting for a situation where the marginal utility of consumption decreased as the number of children lost increased; (b) despite the fact that loss of a child increased the household's per capita income, reduced the marginal utility of consumption, other things equal, and consumption and children were complements, plausible examples could be constructed in which parents would choose to purchase fair insurance against the loss of a child, thereby reducing further the household's material standard of living in the event of the child's survival and augmenting it further in the event of the child's death. These findings are completely at variance with the results, based on single-argument utility functions, which underlie the analysis of liability in the law and economics literature, for example[2]. Nevertheless, they accord with the empirical evidence which we reviewed.

In this paper, we will extend the analysis of the household's optimal insurance purchase decision from a Beckerian perspective to incorporate insurance against the parent's mortality risk and the possibility that physical risks which household members confront are endogenous. Thus there is scope for the household to engage in "self-protection", along the lines first analysed by Ehrlich and Becker (1972)[3]. Empirically important examples of such self-protection include parents' choice of a more reputable and expensive physician with a lower mortality rate in surgery, expenditure on smoke alarms and carbon monoxide detectors as well as acceptance of less hazardous but lower wage jobs. To allow for the possibility that the death of either income-earning parent can be of equal significance to the household, we sometimes consider the implications of the availability of "joint-life, first-death insurance." Empirically, neglecting one-off flight insurance, this is now perhaps the most important form of life insurance for parents.

Our more interesting findings include the following. First, there is a "duality" between how an exogenous increase in child mortality risk affects the demand for children and how an exogenous increase in the number of children affects expenditure on protecting children . This is in the sense that the conditions for ensuring that an increased child mortality risk increases the demand for children are precisely those which ensure that an increase in the number of children results in a reduced expenditure on protecting a child. Indeed, we show that when both the number of children and the self-protection of children are endogenous, parents will spend more on protecting their children and have fewer children. This adds another component to the explanation of the well-known secular decline in fertility which has occurred in advanced economies. Second, the existence or otherwise or a parental bequest function, and its sign when its exists - hence the precise cardinalisation of utility - plays a crucial role in determining how much insurance parents purchase, the amount of self-protection which they undertake and the scope for public policy to influence their behaviour. Third, with fair parental life insurance, parents choose a situation in which the marginal utility of consumption if both survive shortfalls that if only one survives, if they have no bequest function. This is parallel to our earlier result that fair insurance against child mortality risk results in parents choosing a situation where the marginal utility of consumption is decreasing in the number of children lost. If parents have a bequest function both with respect to each other and to their children, the relative sizes of the state-contingent marginal utilities of consumption is unclear, a priori, but parents will definitely purchase more joint-life, first-death insurance than if without a bequest function if insurance purchase occurs in both situations. Fourth, if the household's investment in protection creates a local public rather than private good, the incentive to engage in protection increases, resulting, most plausibly, in more protection occurring.

The rest of the paper is structured as follows. Section II outlines the basic model and considers, separately, parents' optimal purchase of insurance on, and protection of, their own lives. Section III considers parents' protection of their children in the absence of the opportunity to insure them. Safety is considered first as a private good and then as a local public good. We also discuss the simultaneous determination of the number of children and the protection of children. Section IV then considers the protection of the household and how this is influenced by whether or not the parents have a bequest motive and the presence or absence of social security for orphaned children. V discusses the relationship between the insurance and protection of children. Section VI concludes.

II. The Basic Model with Parental Life Insurance and Self-Protection

Suppose each parent in a nuclear family confronts an identical and independent mortality risk p and has exogenous labour income, M, in the event of survival. In this introductory analysis we follow conventional practice by assuming that the risk is atemporal or once-for all. Thus we will conduct the analysis in a single-period setting. Each parent is then assumed to have an identical utility function, $U^i(x,\bar{n})$, defined over an agreed measure of the household's material standard of living, x, and the household's size, $\bar{n}$. This utility function is assumed to be strictly concave in its arguments and is taken to subsume the parents' evaluations of their children's utility[4]. Moreover, we take it to exhibit Edgeworth-Pareto complementarity between x and $\bar{n}$ - i.e., $U^i_{x\bar{n}} \geq 0$. Depending on the context, the index i on utility functions will refer to the number of parents and/or children who have died in a particular state. Subscripts usually refer to partial derivatives.

To allow for a parental bequest motive, we assume each has a bequest function, $\bar{U}^i(x)$, $i = 1,2$, representing their ex ante evaluation of household utility in the event of their own demise. Reasonably enough, we assume that each parent derives bequest utility from the standard of living that his or her surviving heirs will enjoy but derives none from their companionship (and hence from the number of surviving members), as they will not be present to enjoy that companionship[5]. The index i, $i = 1,2$, on a bequest function reflects the fact that the utility of bequest will depend on whether one or both parents die and each of these possibilities will be accommodated in the parents' expectational calculations. It will also be convenient to index surviving parents' utility functions by

0 and 1, respectively, to reflect the fact that their partner has or has not survived. (State 0, the no death one, is always the best state in this paper.) We also assume that all bequest functions are strictly concave and strictly increasing in the survivors' standard of living.

We will assume the simplest possible specification of the household's material standard of living, namely the per capita consumption of the single composite consumption good[6]. Thus, if $\overline{M}$ is the household's aggregate income (which might include social security payments as well as the parental labour income), we assume:

(A.1) (i) $x = \overline{M} / \overline{n}$; (ii) $U^i\left[\overline{M} / \overline{n}, \overline{n}\right]$ is strictly concave in $\overline{n}$.

Let n be the number of children in the household. If both parents survive, then $\overline{n} = n+2$. If one dies, $\overline{n} = n+1$; if both die, $\overline{n} = n$. Likewise, in subsequent sections of the paper, n is reduced by 1 for each child that dies. (A.1)(ii) can be justified by noting that, in circumstances without risk, it ensures that there is an unique optimal number of children for the parents. (A.1) does not require that there is always positive marginal utility, U_n, associated with an extra child. However, the optimal choice of an endogenous n will occur where $U_n > 0$ and we will assume this throughout.

Neglecting social security benefits for the moment and assuming that p is exogenous (thus parents cannot engage in *self-protection* in Ehrlich and Becker's terminology), in the absence of life insurance purchases the parents will evaluate the household's expected utility as

$$(1) \qquad (1-p)^2 2U^0\left[\frac{2M}{n+2}, n+2\right] + 2p(1-p)\left\{U^1\left[\frac{M}{n+1}, n+1\right] + \overline{U}^1\left(\frac{M}{n+1}\right)\right\} + p^2 2\overline{U}^2(0)$$

In (1), we allow for the fact that the parents maximise their joint utility but the 2 could be omitted throughout without affecting anything. The first term refers to utility when neither parent dies, the second to utility when one or other dies (including the anticipated bequest utility of the deceased) and the third to bequest utility for both when both dies. Analogous interpretations can be developed for all the other conditional expected utility expressions which will occur in different contexts below. Thus we will not interpret such expressions term by term subsequently.

(a) Parental life insurance

If social security is provided for destitute children, then the 0 in $\overline{U}^2(0)$ would be replaced by the per capita child benefit, b, say. Clearly, however, providing financially for children as well as a bereaved spouse is a major motivation for parental life insurance. To allow for this and the possibility that either or both parents might die prematurely, we will consider more general insurance contracts than that usually analysed, namely "joint-life, first death" insurance policies. Under these, as the name suggests, the insuree(s) pay a certain premium, here denoted π, in return for cover, here denoted s, to be paid on the death of one or the other or both. Noting that the insurance payout will occur with probability $1-(1-p)^2 = p(2-p)$ and assuming that insurance is actuarially fair - i.e., makes zero expected profits for the insurer - the relationship between the premium and gross payout satisfies:

(2) $$\pi = p(2-p)s$$

Hence the net payout is $s - \pi = \pi\left[(1-p)^2 / p(2-p)\right]$.

For parents with no bequest (NB) motive except to each other, the household's expected utility in the event of a joint-life, first-death (jlfd) insurance purchase is given by

(3) $$EU^{NB} = 2(1-p)^2 U^0\left[\frac{2M-\pi}{n+2}, n+2\right] + 2p(1-p)U^1\left[\frac{Mp(2-p)+\pi(1-p)^2}{p(2-p)(n+1)}, n+1\right]$$

With jlfd insurance, insurers still pay the policy proceeds to the children if both parents die when the parents have no bequest function. Parents simply do not assign any anticipated utility to this.

Letting NB denote optimal magnitudes in this environment, positive optimal jlfd insurance purchase which maximises (3) is characterised by the following necessary and sufficient condition:

$$(4)\quad \left(\frac{1}{n+2}\right)U_x^0\left[\frac{2M-\pi^{NB}}{n+2},n+2\right]=\left(\frac{1}{n+1}\right)U_x^1\left[\frac{Mp(2-p)+\pi^{NB}(1-p)^2}{p(2-p)(n+1)},n+1\right]\frac{(1-p)}{(2-p)}$$

or, after suppressing functional arguments and rearranging,

$$(5)\qquad \left(\frac{U_x^0}{U_x^1}\right)^{NB}=\frac{(1-p)(n+2)}{(2-p)(n+1)}<1$$

Clearly parents with children but without a bequest motive need not necessarily purchase life insurance if the drop in the household's income on the death of one parent is not too great. Note also that the inequality in (5) does not result from the insurance policy being a jlfd one. If there were a policy on the life of a named parent, then the expected utility maximisation problem becomes:

$$(6)\ \underset{\pi}{Max.}\left\{EU=2(1-p)^2U^0\left[\frac{2M-\pi}{n+2},n+2\right]+p(1-p)U^1\left[\frac{M-\pi}{n+1},n+1\right]+\right.$$
$$\left.+p(1-p)U^1\left[\frac{pM+\pi(1-p)}{p(n+1)},n+1\right]\right\}$$

with a first-order condition (FOC) characterising optimal positive insurance purchase of:

$$(7)\qquad -2(1-p)[U_x^0/(n+2)]-p[U_x^1/(n+1)]+(1-p)[U_x^{1P}/(n+1)]=0$$

Here, the superscript "P" on the marginal utility U_x^{1P} indicates that this is the state in which the payout occurs on the death of a parent.

Rearranging (7), we have the relationship between the marginal utility of consumption in the no-death state and the payout state now given by

$$(8) \qquad \left(\frac{U_x^0}{U_x^{1P}}\right)^{\substack{single\ life\\insurance}} = \left(\frac{1}{2}\right)\left(\frac{n+2}{n+1}\right)\left[1-\frac{pU_x^1}{(1-p)U_x^{1P}}\right]$$

Comparing (8) and (5) we have

$$(9) \qquad \left(\frac{U_x^0}{U_x^1}\right)^{\substack{NB,\ jlfd\\insurance}} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \left(\frac{U_x^0}{U_x^{1P}}\right)^{\substack{NB,\ single\ life\\insurance}} \quad \text{as} \left(\frac{1-p}{2-p}\right)\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}\left(\frac{1}{2}\right)\left[1-\frac{pU_x^1}{(1-p)U_x^{1P}}\right]$$

$$\Leftrightarrow \text{ as } 2(1-p)U_x^{1P} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} (2-p)\left[(1-p)U_x^{1P} - pU_x^1\right]$$

$$\Leftrightarrow \text{ as } 0\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} - p(1-p)U_x^{1P} - (2-p)pU_x^1$$

But $-p(1-p)U_x^{1P} - (2-p)pU_x^1 < 0$ (where here, as in the previous two lines, U_x^1 refers to the relevant magnitude in the single-life insurance, no bequest case). Hence,

$$(10) \qquad 1 > \left(U_x^0 / U_x^1\right)^{\substack{NB,\ jlfd\\insurance}} > \left(U_x^0 / U_x^{1P}\right)^{\substack{NB,\ single\ life\\insurance}}.$$

Equation (10) does not tell us whether parents would purchase more of jlfd or single-life insurance. Because a given expenditure purchases less cover in the jlfd case, any relationship between the purchases under the two regimes is consistent with (10).

The results above are couched in terms of parents with the opportunity to purchase fair insurance not equalising the marginal utility of *consumption* across states of the world. The conventional insurance model speaks of equalisation of the marginal utility of *income* across states. However, we can show, along the lines of Fraser (1995c), that parents will not equalise the (derived) marginal utility of *income* across states, except fortuitously, here either. Thus, once family composition is taken into account, optimal life insurance purchase involving households equalising marginal utilities across states of the world would be exceptional. This finding arises from the assumed income- and costsharing within the household and the natural dependence of the material standard of living on the number of surviving household members.

If parents have a bequest motive, we would anticipate that their incentive to insure would increase. It is easily shown that this is indeed the case. With bequest functions as specified above and jlfd insurance, parents maximise expected utility given by

$$(11)\ EU^B \equiv (1-p)^2 2U^0\left[\frac{2M-\pi}{n+2}, n+2\right] + 2p(1-p)\left\{U^1\left[\frac{M(2-p)p+\pi(1-p)^2}{(n+1)(2-p)p}, n+1\right] + \right.$$
$$\left. + \overline{U}^1\left[\frac{M(2-p)p+\pi(1-p)^2}{(n+1)(2-p)p}\right]\right\} + 2p^2\overline{U}^2\left[\frac{\pi(1-p)^2}{n(2-p)p}\right]$$

Here, the superscript "B" denotes magnitudes with a parental bequest motive. Suppressing functional arguments, positive optimal insurance purchase now satisfies:

$$(12)\quad \left(\frac{U_x^0}{U_x^1}\right)^B = \left(\frac{n+2}{n+1}\right)\left(\frac{1-p}{2-p}\right)\left\{1+\left(\frac{\overline{U}_x^1}{U_x^1}\right)+\left(\frac{\overline{U}_x^2}{U_x^1}\right)\frac{p}{(1-p)}\right\}$$

Thus, comparing (5) and (12), we see that $(U_x^0 / U_x^1)^B > (U_x^0 / U_x^1)^{NB}$. As (U_x^0 / U_x^1) is monotonically increasing in π with jlfd insurance, this argument proves the following:

Proposition 1. Parents with a bequest motive will purchase more jlfd insurance than those without such a motive (i.e., $\pi^B > \pi^{NB}$ if both are positive).

Proposition 1 is fairly obvious. The argument underlying it shows, however, that with a bequest motive the ratio of the parents' marginal utilities of consumption in the states where one or both of them survives is indeterminate. The presence of the positive second and third terms in (12), capturing the expected marginal utility of bequests per household survivor in terms of the marginal utility of consumption when only one parent survives, means that the ratio of these marginal utilities can be less than or exceed 1[7].

(b) Parental self-protection

In their seminal analysis of market insurance, self-insurance and self-protection, Ehrlich and Becker (1972) noted that self-protection - involving the expenditure of money and/or effort to reduce the probability of occurrence of the unfavourable event - is often a substitute for market insurance. Their analysis was confined to that of pecuniary losses but numerous authors have subsequently considered self-protection against physical risks, often in the context of the valuation of life-saving[8]. Clearly, one of the most important functions of parents is the protection of their offsprings. This will be a major focus of much of what we do below. However, one way in which parents can protect their children is by ensuring their own continued presence in the household by protecting themselves. Thus, we will consider parents' incentive to engage in self-protection first, focusing on how this depends on the presence or otherwise of a bequest motive and, where this motive exists, on the nature of the bequest function which represents it. To keep the analysis uncluttered, we will consider parental self-protection in the absence of life-insurance[9].

Suppose now that the common and independent risk that each parent faces is endogenous. Let this risk be denoted by the function $p(e)$, satisfying (A.2):

(A.2) $1 > p(e) > 0,\ p'(e) \leq 0,\ p''(e) \geq 0$

Here, e is the level of protective expenditure, a choice variable. This might refer to an amount of effort or money but, in this paper, is taken to be money. The first and second derivatives of $p(e)$ indicate that there are non-negative and non-increasing marginal returns to e. We suppose e is a local public good for the parents, so the same e reduces the risk each faces equally. Clearly, self-protective expenditure might be purely private, as for example when the partners each exclusively drive their own Volvos instead of Skodas (or whatever other car Department of Transport figures reveal to be least safe), or when the expenditure refers to choice between different surgical procedures with different effectiveness and costs for a given partner. We return to purely private protective expenditures in the context of risks to children below.

The objective of the parents is again taken to be expected utility maximisation. Without a bequest motive either to each other or to their children, they face the problem:

$$(13) \qquad \underset{e}{Max.}\left\{ EU^P(e) \equiv (1-p(e))^2 2U^0\left[\frac{2M-e}{n+2}, n+2\right] + 2p(e)(1-p(e))U^1\left[\frac{M-e}{n+1}, n+1\right]\right\}$$

Unlike the earlier expected utility maximisation problems involving (3), (6) and (11), (13) is not generally a concave problem, as many authors (including Ehrlich and Becker) have noted[10]. The source of the difficulty here is that, although the utilities are concave in e, the terms involving the probability $p(e)$ are not necessarily so. Moreover, even were the utility and probability terms each concave in e, their products are not necessarily so. In the absence of this concavity, we need to check the second-order conditions (SOC) to see whether outcomes satisfying the first-order conditions are local maxima rather then minima. Nevertheless we will, for the most part, assume that SOCs are satisfied and focus on FOCs in this introductory analysis.

The FOC for an interior solution to (13) is now[11]:

$$(14) \; EU_e^P(e) = -2p'\left[(1-p)2U^0 - (1-2p)U^1\right] - 2(1-p)\left[((1-p)/(n+2))U_x^0 + (p/(n+1))U_x^1\right] = 0$$

Thus, as is to be expected, the increment in utility which a marginal increase in e purchases should be equated to the marginal cost of an increase in e in terms of the expected marginal utility of per capita consumption foregone.

When parents possess bequest functions, they maximise an expression for expected utility identical to that given in (11) except that the terms in p are endogenous, $\pi = 0$ throughout, and gross household wealth in each state is reduced by e. The last observation highlights a minor difficulty with an atemporal analysis of self-protection when the risk is to the lives of those making the self-protective expenditure. If the risk is resolved simultaneously with the expenditure being made, then if the death of both parents occurs there will be no household income to cover e. To avoid this problem, we must assume either that there is some social security transfer to bereaved children, who then cover the committed expenditure e, or that the parents have some residual estate on death (e.g., the value of their house) which is independent of labour income. In the case of social security, we consider a pure transfer without regard to the sources of finance for it, purely for simplicity in this paper. Then the FOC for an interior solution to the parents' expected utility maximisation can be shown to be:

$$(15) \qquad EU_e^P(e) + 2p'[(1-2p)\overline{U}^1 + p\overline{U}^2] - 2p[(1-p)\left(\overline{U}_x^1 / (n+1)\right) + p\overline{U}_x^2 / n] = 0$$

The signs of the terms beside $EU_e^P(e)$ in (15) are ambiguous because they depend, *inter alia*, on the sign(s) of the bequest functions. What signs these functions should take seems largely a philosophical issue. However, a little thought suggests that the state-contingent utility of wealth functions should be cardinalised so that bequest functions have negative signs. Intuitively, neglecting any "warm glow" which parents might derive from thinking that they are considerate, one would argue that, other things equal, concern for their spouse and dependants' welfare in the event of their own death should reduce a parent's welfare as compared with a situation where this concern does not exist. For example, if the absence of a bequest motive means that parents get zero utility from their spouse and offsprings' welfare in anticipation of their own death, then when they care about their heirs' welfare, they should get negative utility from the knowledge that the latter will be left to fend for themselves[12]. In that event, however, the sign of the extra terms in (15) is ambiguous. Thus, letting e^{NB} and e^B be the levels of protective expenditures which solve (14) and (15), respectively, we have the following result.

Proposition 2 Suppose expected utility with parental self-protection is concave in a neighbourhood containing both e^{NB} and e^B. Then: (i) the relative magnitude of these two levels of expenditure is generally ambiguous; (ii) if $1 > 2p$ and the utility of bequests is positive, $e^{NB} > e^B$.

Proof. (i) Given the concavity assumption and noting that (14) holds at e^{NB} and (15) at e^B, if $p'[(1-2p)\overline{U}^1+2p\overline{U}^2]-p\left[(1-p)\left(\overline{U}^1_x/(n+1)\right)+p\overline{U}^0_x/n\right]\Big|_{e^B}<0$, then the like terms in (15) exceed those in (14) and $e^{NB}>e^B$ would hold. Similarly, if $p'[(1-2p)\overline{U}^1+2p\overline{U}^2]-p\left[(1-p)\left(\overline{U}^1_x/(n+1)\right)+p\overline{U}^0_x/n\right]\Big|_{e^B}>0$, then $e^B>e^{NB}$ would hold. However, neither of these possibilities can be ruled out a priori without restricting the signs of the state-contingent bequest functions. (ii) Under the stated conditions, we would have $p'[(1-2p)\overline{U}^1+2p\overline{U}^2]-p\left[(1-p)\left(\overline{U}^1_x/(n+1)\right)+p\overline{U}^0_x/n\right]\Big|_{e^B}<0$. Then, using (15) and the argument of part (i), $e^{NB}>e^B$ would follow. Q.E.D.

The intuition for Proposition 2 is simple. If concern for the spouse and children means that each parent assigns extra disutility to dying, each has an added incentive to incur self-protective expenditure to avoid that state. But, this extra incentive must be set against an added disincentive arising from the fact that parents now care that more protective expenditure diminishes the estate to be left to dependants. If, conversely, a bequest motive makes death less undesirable, the incentive to spend to try to avoid that state diminishes. Similar considerations apply if, instead of comparing the case where parents have no bequest motive whatsoever with that where each cares about the fortunes of both the spouse and children, we compared the no bequest motive case with that where parents felt concern only for children or only for the spouse, and so on.

III. Protecting One's Children

(a) Safety as a private good

Suppose we now focus on parents' incentive to protect their children. Let the number of children in the absence of a mortality risk be exogenously given as n. Initially we will consider the risk, again denoted $p(e)$, to one and only one child but assume all children are valued equally by the parents. Then, any protective expenditure which the parents incur is a purely private good for the protected child but it impacts on the household's material standard of living from consumption. Protective expenditure, e, in this case could, e.g., involve choice of a better reputed and more expensive physician or operative procedure for the child.

Let us use the same notation as before to indicate a parent's utility function on the death of a household member or otherwise. We will then assume that each parent wants the level of e to be that which maximises conditional expected utility according to

$$(16) \qquad \underset{e}{Max.}\left\{ EU^{PC}(e) = (1-p(e))U^{0}\left[\frac{\overline{M}-e}{n+2}, n+2\right] + p(e)U^{1}\left[\frac{\overline{M}-e}{n+1}, n+1\right]\right\}$$

This yields as the first-order condition for an interior "protective equilibrium:"

$$(17) \qquad EU_{e}^{PC} = p'\left(U^{1}-U^{0}\right) - \left[(1-p)\frac{U_{x}^{0}}{(n+2)} + p\frac{U_{x}^{1}}{(n+1)}\right] = 0$$

Equation (17) is virtually identical to the conventional condition characterising a self-protective equilibrium in the absence either of market insurance or of market insurance with a price which is responsive to the household's risk-reducing expenditure[13]. The only difference, and an important one, is that the state-contingent marginal utility costs of protective expenditure now reflect the varying household composition.

The first thing which we wish to know is how an (exogenous) increase in the number of children affects parents' incentive to protect a given child. Thus, treating the optimal e solving (17) as an implicit function of n, denoted $e^{1PC}(n)$, *and ignoring the integer issue w.r.t. n here as elsewhere,* standard comparative statics techniques show that

$$(18) \qquad e^{1PC\prime}(n) = -EU_{en}^{PC} \,/\, EU_{ee}^{PC}$$

Here

$$(19)\quad EU_{en}^{PC} = p'\left[\frac{(\overline{M}-e)}{(n+2)^2}U_x^0 - U_n^0 - \frac{(\overline{M}-e)}{(n+1)^2}U_x^1 + U_n^1\right] + \left[\frac{(1-p)}{(n+2)^2}U_x^0 + \right.$$

$$\left. + \frac{(1-p)(\overline{M}-e)}{(n+2)^3}U_{xx}^0 + \frac{p}{(n+1)^2}U_x^1 + \frac{p(\overline{M}-e)}{(n+1)^3}U_{xx}^1 - \frac{(1-p)}{(n+2)}U_{xn}^0 - \frac{p}{(n+1)}U_{xn}^1\right]$$

and, by the SOC for the self-protective optimum, $EU_{ee}^{PC} < 0$. Hence $e^{1PC}{}'(n)$ has the sign of EU_{en}^{PC}. To sign this requires restrictions on the state-contingent utility functions.

First, we will assume utility can be parameterised in the following fashion:

$$(A.3)\quad (i)\ U^i(x^i,\overline{n}^i) \equiv U(x^i,\overline{n}^i,i),\ i = 0,1,2,\ldots;\ (ii)\ \partial\left[\partial U(\overline{M}/\overline{n}^i,\overline{n}^i,i)/\partial\overline{n}\right]/\partial i \geq 0$$

The interpretation of (A.3)(i) is that the degree of bereavement that parents feel acts as a parameter to alter the shape of their utility function; (A.3)(ii) tells us that an increase in bereavement does not decreases the total impact on utility from additional children, other things equal - in a sense, children become more valuable if one has lost a child. Given (A.3), we can show the following:

Proposition 3. If: (i) (A.3) holds; (ii) the state-contingent utilities satisfy Arrow's hypothesis that relative risk aversion for consumption gambles is greater than or equal to unity and the same is true for "relative risk aversion" w.r.t. the number of children; (iii) children and per capita consumption are Edgeworth-Pareto complements (i.e., $U_{nx} \geq 0$), then $e^{1PC}{}'(n) < 0$.

Proof. Recalling that the contingent indices of relative risk aversion w.r.t. consumption gambles are defined by $RA^i \equiv -x^iU_{xx}^i / U_x^i$, $i = 0,1,\ldots$, if Arrow's hypothesis holds then $RA^i \geq 1$, $i = 0,1,\ldots$. Also, let $RA^{in} \equiv -nU_{nn}^i / U_n^i \geq 1$. Using these with part (iii) of the Proposition, the second square-bracketed term in (19) is non-positive. Thus, it remains to sign the first square-bracketed term. As

$$\left((\overline{M}-e)/(n+2)^2\right)U_x^0 - U_n^0 - \left((\overline{M}-e)/(n+1)^2\right)U_x^1 + U_n^1 = d\left[U^1 - U^0\right]/dn$$

we simply need to see which of dU^0 / dn or dU^1 / dn is larger.

Now, each of the $dU^i / dn,\ i = 0,1$, is of the form

(20) $$dU^i / dn = \{\bar{n}^i U_n[x^i, \bar{n}^i, i] - x^i U_x[x^i, \bar{n}^i, i]\} / \bar{n}^i \equiv N_1 / \bar{n}^i,\ i = 0,1$$

The denominator $\bar{n}^i$ in (20) is decreasing in i (as $\bar{n}^0 = n+2 > n+1 = \bar{n}^1$). Therefore, if N_1 is increasing or constant in i, then dU^i / dn will be increasing in i and we will be done. Totally differentiating,

$$dN_1 = dx^i[\bar{n}^i U^i_{nx} - U^i_x - x^i U^i_{xx}] + d\bar{n}^i[\bar{n}^i U^i_{nn} + U^i_n - x^i U^i_{nx}] + di[\bar{n}^i U^i_{ni} - x^i U^i_{ni}]$$

Now, as i increases from 0 to 1,
$dx^i = (\overline{M} - e)/(n+1) - (\overline{M} - e)/(n+2) = (\overline{M} - e)/(n+1)(n+2)$ and $di = -dn = 1$.
Substituting these into the expression for dN_1 evaluated at $i = 0$ yields

$$\begin{aligned} dN_1 = {} & [x^0 + x^1]U^0_{nx} - \{(\overline{M} - e)[x^0 U^0_{xx} + U^0_x] / (n+2)(n+1)\} \\ & - [(n+2)U^0_{nn} + U^0_n] + [(n+2)U^0_{ni} - x^0 U^0_{xi}] \end{aligned}$$

Given (A.3)(ii), the last term of dN_1 is non-negative while the first term is non-negative by the assumed complementarity of children and consumption. The two middle terms are both non-negative by part (ii) of the Proposition. Under these conditions, $dN_1 \geq 0$. But then $d[U^1 - U^0] / dn > 0$, hence $p' \, d[U^1 - U^0] / dn < 0$, holds. Using this in (19) with our earlier observation on the sign of the second right hand term means that, under Proposition 3's conditions, $e^{1PC}{}'(n) < 0$. Q.E.D.

An immediate corollary of Proposition 3 is:

Corollary 1. When the parents' utility function is state-independent (i.e., bereavement does not alter the function used for evaluating children and consumption), if:

(i) the state-contingent utilities satisfy Arrow's hypothesis that relative risk aversion for consumption gambles is greater than or equal to unity; (ii) children and per capita consumption are Edgeworth-Pareto complements , then $e^{1PC\prime}(n) < 0$.

Proof. In (19) now, the term equivalent to $d\left[U^1 - U^0\right] / dn$ is positive as the assumed concavity of $U\left[\overline{M} / n, n\right]$ in n means that $dU\left[\overline{M} / (n+1), n+1\right] / dn > dU\left[\overline{M} / (n+2), n+2\right] / dn$. Hence $p'\, d\left[U^1 - U^0\right] / dn < 0$ there. That the other terms in EU_{en}^{PC} in (19) are non-positive follows from (i) and (ii) of Corollary 1. Thus $e^{1PC\prime}(n) < 0$. Q.E.D.

Thus, under Proposition 3's or even weaker conditions, an increase in the number of their children induces parents to take less care of any given child. Equivalently, they would accept greater risk for any given child. This establishes an important duality with a result in a companion paper (Fraser, 1995b). There, we showed that the conditions of Proposition 3 or Corollary 1 were precisely those which ensured that introducing or increasing an exogenous child mortality risk resulted in an increased demand for children if the number of children was endogenous. Together, these results suggest that parents regard the number of children and the protection of children as substitutes.

To investigate this possibility further, we will consider briefly the implications of a model in which both the number of children and the protection of one and only one child are endogenous. (Implicitly, we assume that producing children is not in itself risky for the mother.) Accordingly, the parents are taken to maximise their common expected utility by solving the following problem

$$(21)\ \underset{e,n}{Max.}\left\{EU^{CP}(e,n) \equiv (1-p(e))U^0\left[(\overline{M}-e)/(n+2), n+2\right] + p(e)U^1\left[(\overline{M}-e)/(n+1), n+1\right]\right\}$$

This is the same as the expected utility maximisation problem just considered with the added complication that n is also endogenous. Again noting that $EU^{CP}(e,n)$ is not necessarily concave in e, we nevertheless assume that $EU^{CP}(e,n)$ is locally concave in an neighbourhood of an optimum and thus an interior solution to (21) can be characterised by the following first-order conditions:

$$(22)\qquad EU_e^{CP} = p'\left[U^1 - U^0\right] - \left[\frac{(1-p)}{(n+2)}U_x^0 + \frac{p}{(n+1)}U_x^1\right] = 0;$$

$$(23)\qquad EU_n^{CP} = (1-p)\left[U_n^0 - \frac{(\overline{M}-e)}{(n+2)^2}U_x^0\right] + p\left[U_n^1 - \frac{(\overline{M}-e)}{(n+1)^2}U_x^1\right] = 0$$

Now we know, from the arguments underlying Proposition 3, that $EU_{en}^{CP} < 0$ under the conditions stated there. Further, we know that when n alone is endogenous, (23) holds if $n > 0$ and, if e alone is endogenous, (22) holds if $e > 0$. Suppose that prior to any protective expenditure the level of risk is at some endowed level satisfying $p = p(0) \equiv \overline{p}$. Starting from this level of risk and with only n endogenous and satisfying $EU_n^{CP} = 0$, let us go to a position where both n and e are endogenous , satisfying $EU_n^{CP} = 0 = EU_e^{CP}$ with $e > 0$. As compared with the initial situation, if e increases from zero, then EU_n^{CP} decreases, ceteris paribus (i.e., at unchanged n), given $EU_{en}^{CP} < 0$. By the assumed local concavity of $EU^{CP}(e,n)$, this requires a decrease in n to restore $EU_n^{CP} = 0$ as compared with the outcome when p is exogenous. This establishes the following proposition:

Proposition 4. Under the conditions of Proposition 3, if both the number of children and the expenditure on protecting any given child are endogenous, parents will have fewer children as compared with a situation where they cannot protect a child.

This proposition establishes the extent to which we can say that the ability to reduce the risk to children can be substituted for having more children. It adds a small fragment to the overall mosaic of formal explanations of the secular decline in the number of offsprings which parents choose to have. If the opportunity for protecting offsprings has improved through time, parents will rationally tend to choose to have fewer while reducing the bereavement risk by greater protective efforts.

(b) Safety as a local public good

Because e protected one and only one child in the previous two exercises, it was like a pure private good as far as the safety aspect was concerned. However, as with parents' self-protective expenditure considered earlier, expenditure to protect children might be a local public good. To investigate the implications of this, we will examine the incentive to engage in such expenditure when it protects *two* children confronting identical and independent risks at the same level as the one considered above. Intuitively, as the marginal benefit of any given protective expenditure is now greater, we would expect that the incentive to engage in it increases and thus the equilibrium level would increase as compared with the case where it reduces the risk to only one child.

Of course, there is another way in which protective expenditure could be treated as a local public good here. That is by assuming that the expenditure protected all the children in the household and then examining what happens as the number of children increases. However, we have just seen in Proposition 3 that an increase in the number of children is likely to reduce the incentive to protect any given child. Thus, while parents might wish to take advantage of the increased productivity of any given protective expenditure and increase such expenditure as the number protected increase, simultaneously the diminished value placed on children as their number increases reduces parents' incentive to sacrifice consumption to protect them the more they have. Hence, the overall outcome will be ambiguous. Thus, we abstract from the latter effect by fixing the number of children. For simplicity we also confine attention to state-independent preferences.

When parents with n (≥ 2) children can use a given expenditure e to protect two and only two of them, they have the expected utility maximisation problem

$$(24) \qquad \underset{e}{Max.}\left\{ EU^{2PC}(e) \equiv (1-p(e))^2 U^0\left[\frac{\overline{M}-e}{n+2}, n+2\right] + 2(1-p(e))p(e)U^1\left[\frac{\overline{M}-e}{n+1}, n+1\right] + \right.$$
$$\left. + + p(e)^2 U^2\left[\frac{\overline{M}-e}{n}, n\right]\right\}$$

Again assuming that an interior protective equilibrium occurs, this will now be characterised by the FOC:

$$(25) \qquad -2p'\left[(1-p)U^0-(1-2p)U^1-pU^2\right]-\left[\frac{(1-p)^2}{n+2}U_x^0+\frac{2(1-p)p}{n+1}U_x^1+\frac{p^2}{n}U_x^2\right]=0$$

Denote the e which solves (25) by e^{2PC}. Comparing (25) with the equivalent condition (17) when the parents' expenditure protects only one child, we can show:

Proposition 5. (a) If: (i) the parents' expected utility $EU^{2PC}(e)$ is concave in an neighbourhood containing both e^{1PC} and e^{2PC}; (ii) their utility function is state-independent; (iii) utility satisfies Arrow's hypothesis that relative risk aversion for consumption gambles is greater than or equal to unity and (iv) children and per capita consumption are Edgeworth-Pareto complements, then $e^{2PC} > e^{1PC}$. (b) If: (i), (iii) and (iv) hold; (v) utility is state-dependent; (vi) satisfies (A.3(i)) and the marginal utility of consumption is non-increasing as the degree of bereavement increases (i.e.,$U_{xi} \leq 0$) and, with household income and expenditure fixed, (vii) utility decreases at an increasing rate as the degree of bereavement increases, then $e^{2PC} > e^{1PC}$.

Proof. (a) Using (25), the first derivative of EU^{2PC} w.r.t. e can be rewritten as

$$(26) \qquad U_e^{2PC}=2p'\left(U^1-U^0\right)+2p'\,p\left(U^2-2U^1+U^0\right)-\left[\frac{(1-p)}{n+2}U_x^0+\frac{p}{n+1}U_x^1\right]+$$
$$+\,p(1-p)\left[\frac{U_x^0}{n+2}-\frac{U_x^1}{n+1}\right]+p^2\left[\frac{U_x^1}{n+1}-\frac{U_x^2}{n}\right]$$

Evaluating this derivative at e^{1PC} and using (17) we see that

$$(27) \qquad EU_e^{2PC}(e^{1PC})=2p'\,p\left(U^2-2U^1+U^0\right)+p'\left(U^1-U^0\right)$$
$$+\,p(1-p)\left[\frac{U_x^0}{n+2}-\frac{U_x^1}{n+1}\right]+p^2\left[\frac{U_x^1}{n+1}-\frac{U_x^2}{n}\right]$$

Now, with state-independent utility,

$$\partial\left\{U_x\left[(\overline{M}-e)/\overline{n},\overline{n}\right]/\overline{n}\right\}/\partial\overline{n}=-(1/\overline{n})^2\left[\left((\overline{M}-e)/\overline{n}\right)U_{xx}+U_x\right]+U_{xn}/\overline{n}\geq 0$$

if (iii) and (iv) of the Proposition are satisfied. Then, $U_x^0/(n+2)-U_x^1/(n+1)\geq 0$ and $U_x^1/(n+1)-U_x^2/n\geq 0$. Using these, in (27)

$$p(1-p)\left[U_x^0/(n+2)-U_x^1/(n+1)\right]+p^2\left[U_x^1/(n+1)-U_x^0/n\right]>0$$

Hence, as $p'(U^1-U^0)>0$ by (17), in (27) we have

$$\text{(28)}\qquad p'(U^1-U^0)+p(1-p)\left[\frac{U_x^0}{n+2}-\frac{U_x^1}{n+1}\right]+p^2\left[\frac{U_x^1}{n+1}-\frac{U_x^2}{n}\right]>0$$

evaluated at e^{1PC}. For the remaining term on the right of (27), we note from the assumed concavity of $U\left[\overline{M}/\overline{n},\overline{n}\right]$ in $\overline{n}$ (i.e., (A.1)(ii)) that

$$\begin{aligned}U^1&=U\left[(\overline{M}-e)/(n+1),n+1\right]\\&\geq(1/2)\left\{U\left[(\overline{M}-e)/(n+2),n+2\right]+U\left[(\overline{M}-e)/n,n\right]\right\}=(1/2)\left\{U^2+U^0\right\}\\&\Leftrightarrow 2U^1-\left(U^2+U^0\right)\geq 0\end{aligned}$$

Thus

$$\text{(29)}\qquad 2pp'\left[\left(U^2+U^0\right)-2U^1\right]\geq 0$$

Combining (28) and (29) in (27), we see that $EU_e^{2PC}(e^{1PC})>0$. Thus, by (i) in the Proposition, $e^{2PC}>e^{1PC}$. (b)When the parents' utility function is state-dependent, (28) remains true if (vi) holds. Provided (vii) in the Proposition is satisfied, so does (29). The result then follows as per part (a). Q.E.D.

Proposition 5 accords with intuitive notions about the provision of a public good. "The more bangs per buck," in the sense of the greater the number of beneficiaries receiving benefit at a particular level from a given expenditure, the greater will be the incentive to make that expenditure. Moreover, the sufficient (but not necessary) conditions for the Proposition to hold, notably clause (vii), are not at all implausible[14].

Indeed, Proposition 5 is so appealing, it is tempting to believe that: (a) it would hold without the qualifications in the Proposition; (b) it could readily be extended to state that the protective expenditure would always increase the more children it protected in the household. Unfortunately, we have been unable to ensure either of these outcomes so far within our current approach. Moreover, it is not difficult to see why some of the qualifications need to be made. For example, when utility is state-dependent, suppose, albeit somewhat implausibly, that the marginal utility of consumption increases with the degree of bereavement. Then, the incentive to protect against the loss of two children is less than the incentive to protect against the loss of one, other things equal.

IV. Protecting the Household

When parents choose to purchase the latest Volvo estate car rather than a more stylish and/or affordable Mercedes or Audi equivalent, perhaps the dominant factor which motivates their choice is the Volvo's reputation for safety. In this case, unlike in some which we have considered hitherto, the protective expenditure can be taken to protect all household members and might be termed a "pure local public good" within the household. We will consider how parents' incentives to make such expenditures depend on the presence or not of a bequest motive and the fact that, in seeking to protect their children, the lives that they save might well be their own.

For simplicity, we will assume that all household members face a common but independent mortality risk, $p(e)$. This is affected by the protective expenditure, e, according to (A.2). Clearly, with a Volvo estate, the individual risks effected by this protective choice might be correlated while, for the fitting of stair handrails, say, the individual risks faced are more likely to be independent.

We will use the following notation: $U^r[x, n+2-r-i, i]$, $r = 0,1$; $i = 0,1,\ldots,n$ is a parent's anticipated utility depending on whether the partner has not ($r = 0$) or has died ($r = 1$), if i children have died and household per capita income is x; $\overline{U}^r[(\overline{M}-e)/(n+2-r-i)]$, $r = 1,2$; $i = 0,1,\ldots,n$, is a parent's bequest utility, again depending on whether ($r = 2$) or not ($r = 1$) the partner also dies, if heirs and dependants' per capita consumption is $(\overline{M}-e)/(n+2-r-i)$. Also, we will use the abbreviations $U^{ri} \equiv U^r[(\overline{M}-e)/(n+2-r-i), n+2-r-i], r = 0,1,2; i = 0,1,\ldots,n$ and $\overline{U}^{ri} \equiv \overline{U}^r[(\overline{M}-e)/(n+2-r-i)]$, $r = 1,2$; $i = 0,1,\ldots,n$. Of course, $\overline{U}^{2n} \equiv 0 \equiv \overline{U}_x^{2n}$: whatever the level of bequests might happen to be, each parent anticipates deriving no utility of bequest if they, their partner and all their children die. Otherwise, all the bequest functions are taken to be concave and increasing in per capita bequests, as before. The derivatives of utility and bequest functions are denoted conformably in the abbreviated notation.

In this environment, parents have a complicated expected utility maximisation problem in which they have to take account of the possibility of both their own and their partner's death and that of their children. Allowing for these, their problem is:

$$(30)\ \underset{e}{Max.}\Big\{EU^A(e) \equiv \sum_{i=0}^{n}\tbinom{n}{i}p(e)^i(1-p(e))^{n-i}\Big[2(1-p(e))^2U^0\Big(\frac{2M-e}{2+n-i}, 2+n-i, i\Big) \\ +2(1-p(e))p(e)\left(U^1\Big(\frac{M-e}{1+n-i}, 1+n-i, i\Big) + \overline{U}^1\Big(\frac{M-e}{1+n-i}\Big)\right) + 2p(e)^2\overline{U}^2\Big(b - \frac{e}{n-i}\Big)\Big]\Big\}$$

Throughout, $\tbinom{n}{i}$ is the binomial coefficient giving the number of ways i children can be chosen from n.

In (30), to allow for the fact that if both parents die their labour income will be lost, we assume as before that there is some social security for children which allows them a per capita consumption of b gross of their share of the protective expenditure. This share for a survivor is $e/(n-i)$ if i children also die. We will see presently that results are much sharper if we assume that surviving children do not have to foot the bill for their parents' protective expenditure if both parents die. However, the notion that surviving children pay for their parents' committed protective expenditure from their social security payments should not be taken too literally. Rather, our formulation represents the more plausible idea that jointly-deceased parents are likely to have some residual estate. This estate will be reduced by protective expenditures which the parents make and will be lower per capita the more children that survive them.

An interior optimum for parents' protective expenditures now satisfies the FOC:

$$
\begin{aligned}
(31)\ EU_e^A = {} & \sum_{i=o}^{n}\binom{n}{i}p'\,p^{i-1}(1-p)^{n-i-1}(i-np)\left[2(1-p)^2U^{0i}+2(1-p)p\left(U^{1i}+\overline{U}^{1i}\right)+2p^2\overline{U}^{2i}\right] \\
& +\sum_{i=0}^{n}\binom{n}{i}p^i(1-p)^{n-i}2p'\left[(1-2p)\left(U^{1i}+\overline{U}^{1i}\right)-2(1-p)U^{0i}+2p\overline{U}^{2i}\right] \\
& -\sum_{i=0}^{n}\binom{n}{i}p^i(1-p)^{n-i}2\left[\frac{(1-p)^2}{n+2-i}U_x^{0i}+\frac{(1-p)p}{n+1-i}\left(U_x^{1i}+\overline{U}_x^{1i}\right)+\frac{p^2}{n-i}\overline{U}_x^{2i}\right]=0
\end{aligned}
$$

If children receiving social security in the event of both parents' deaths do not have to cover any protective expenditure that the parents were committed to, then the $\frac{p^2}{n-i}\overline{U}_x^{2i}$ term will be missing from (31) and the argument of the $\overline{U}^{2i}$ terms will simply be b.

How does the presence or absence of a bequest motive now affect the parents' incentive to invest in protection? Following our familiar route, we will examine the extra marginal benefit and marginal cost terms which appear in (31) because the parents have a bequest motive. These terms are:

$$
\begin{aligned}
(32)\quad & \sum_{i=o}^{n}\binom{n}{i}p'\,p^{i-1}(1-p)^{n-i-1}(i-np)2\left[(1-p)p\overline{U}^{1i}+p^2\overline{U}^{2i}\right] \\
& +\sum_{i=o}^{n}\binom{n}{i}p'\,p^i(1-p)^{n-i}2\left[(1-2p)\overline{U}^{1i}+2p\overline{U}^{2i}\right] \\
& -\sum_{i=o}^{n}\binom{n}{i}p^i(1-p)^{n-i}2\left[\frac{(1-p)p}{n+1-i}\overline{U}_x^{1i}+\frac{p^2}{n-i}\overline{U}_x^{2i}\right]
\end{aligned}
$$

In (32), the first two terms capture the additional marginal benefit parents perceive from protective expenditure when they have a bequest motive. The third gives the additional expected cost in terms of dependants' forgone consumption at the margin. As np is the expected value of i, it follows that

$$(33) \quad \sum_{i=o}^{n} \binom{n}{i} p' p^{i-1}(1-p)^{n-i-1}(i-np)2(1-p)p\overline{U}^{1i} = 2p' Cov\left[i, \overline{U}^{1i}\right]$$

and

$$(34) \quad \sum_{i=o}^{n} \binom{n}{i} p' p^{i-1}(1-p)^{n-i-1}(i-np)2p^2\overline{U}^{2i} = 2pp' Cov\left[i, \overline{U}^{2i}\right] / (1-p)$$

where $Cov(y, z)$ denotes the covariance between the magnitudes y and z.

The signs of these terms involving covariances depend on our hypotheses about how the utility of bequests behave as the number of dependants change. If, as assumed thus far, the utility of bequests depends only on the dependants' per capita income, then it will be increasing in i if one parent survives. The covariance in (33) is then positive and the expression in (33) is negative. If both parents die and surviving children have to cover the cost of committed protective expenditure from their social security or the residual estate, their per capita net income and $\overline{U}^{2i}$ will decrease as i increases. Then $Cov(i, \overline{U}^{2i}) < 0$ and the term in (34) is positive. If surviving children do not have to cover protective expenditure, then $Cov(i, \overline{U}^{2i}) = 0$ and the term in (34) is also zero.

The third term in (32) is negative. If the parents' utility from bequests is positive in the sense discussed already - i.e., concern for dependants gives them a "warm glow" rather than anxiety - then the second term in (32) will be negative also. Let "NB" and "B" again denote magnitudes in the no bequest and bequest cases, respectively, and e^{AB} and e^{ANB} be the corresponding optimum protective expenditure. These observations indicate that when surviving children do not have to cover protective expenditure if both parents die, then $EU_e^{ANB}\left(e^{AB}\right) > EU_e^{AB}\left(e^{AB}\right)$. If surviving children do have to cover this expenditure, the relationship between $EU_e^{ANB}(e^{AB})$ and $EU_e^{AB}(e^{AB})$ is ambiguous. This proves:

Proposition 6. If parents' utility of bequest functions are positive and depend only on the material standard of living of survivors, surviving children do not have to cover protective expenditure and $EU^A(e)$ is concave in the neighbourhood containing both

e^{ANB} and e^{AB}, then $e^{ANB} > e^{AB}$. Otherwise, the relationship between e^{ANB} and e^{AB} is ambiguous but anxiety about dependants' welfare makes $e^{AB} > e^{ANB}$ more likely.

Perhaps the most important thing which Proposition 6 and its underlying argument illustrate is the importance of the cardinalisation of utility, especially bequest utility. This finding is in the same vein as Proposition 2. The additional complication here, as compared with the latter, derives from the behaviour of the covariances between the number of children who die and the utility of bequests. This is seen by comparing the extra terms in (15) with those in (32). These covariances do not depend on the sign of the utility of bequests, only upon whether or not, "on average", the utility of bequests moves in the same direction as the number of children lost. Despite our specification calling for one of these covariances to be positive, a reasonable argument might be made that, other things (including household income) equal, the parents' utility of bequest should decrease as the number of dependants decreases, and thus as i increases[15].

Social security and self-protection

Would the state's provision of social security for children's upkeep in the event of both parents' deaths induce the parents to take less care? Such a reaction might be expected because the effect of social security as modelled here is to make the possibility of their joint death less intolerable to parents with a bequest motive. We will investigate this question with our admittedly rudimentary specification of social security by examining the impact of an increase in b on e^{AB}.

Treating e^{AB} solving (31) as an implicit function of b, we know that $e^{AB\prime}(b)$ $= -EU_{eb}^{AB}(e^{AB}) / EU_{ee}^{AB}(e^{AB})$ and that $-EU_{ee}^{AB}(e^{AB}) \geq 0$ from the second-order conditions. Thus the sign of $e^{AB\prime}(b)$ is that of $EU_{eb}^{AB}(e^{AB})$. Now,

$$(35)\qquad EU_{eb}^{AB} = \sum_{i=0}^{n}\binom{n}{i}\Big\{p' p^{i-1}(1-p)^{n-i-1}(i-np)2p^2\overline{U}_x^{2i} + p' p^i(1-p)^{n-i}4p\overline{U}_x^{2i} - p^i(1-p)^{n-i}2p^2\overline{U}_{xx}^{2i} / (n-i)\Big\}$$

and, in (35), by our earlier argument,

(36) $$\sum_{i=o}^{n}\binom{n}{i}p'\,p^{i-1}(1-p)^{n-i-1}(i-np)2p^2\overline{U}_x^{2i} = 2pp'\,Cov(i,\overline{U}_x^{2i})\,/\,(1-p)$$

In signing EU_{eb}^{AB} via (35), the simplest case to consider is that where surviving children do not have to cover any protective expenditure when both parents die. Then, as their per capita income is b, however many of them survive, $\overline{U}_x^{2i}$ is constant and $Cov(i,\overline{U}_x^{2i}) = 0$. Moreover, by examining (31) to find the source of the term in $\overline{U}_{xx}^{2i}\,/\,(n-i)$ in (35), we see that this would not be present in this case as well. Thus the sign of EU_{eb}^{AB} is then simply that of $\sum_{i=o}^{n}\binom{n}{i}p'\,p^i(1-p)^{n-i}4p\overline{U}_x^{2i} < 0$. In this simplest of case, therefore, as one would expect, an increase in social security provision for orphaned children will induce parents with a bequest motive to take less care to protect themselves.

This intuitively plausible outcome only holds unambiguously in this simplest of cases however. If surviving children's net per capita consumption is decreasing in i, then $\overline{U}_x^{2i}$ will be increasing in i and we will have $Cov(i,\overline{U}_x^{2i}) > 0$ and $2pp'\,Cov(i,\overline{U}_x^{2i})\,/\,(1-p) < 0$. While this reinforces the earlier result, the final term in (35) is now positive, making the overall sign of $e^{AB\prime}(b)$ ambiguous. Thus we cannot conclude that social provision here would necessarily create "moral hazard" for parents in the sense of inducing behaviour which makes their loss states more likely.

V. Market Insurance and Self-Protection

Ehrlich and Becker's thorough seminal analysis of market insurance and self-protection remains relevant here. However, the discussion in Section II above and Fraser (1995b) suggest the obvious modification that optimal actuarially fair insurance purchase does not call for parents to equate the marginal utility of consumption (or income) across states. For brevity, we will consider just one aspect of the link between market insurance and protection here. This involves parental purchase of insurance on their children's' lives when the risk facing children is endogenous. As we have noted elsewhere, the conventional wisdom is that parents would not find it optimal to purchase such insurance, but this is true neither empirically nor theoretically if family composition is taken into account and a trade-off is allowed between the household's standard of living and size[16].

For the greatest brevity and simplicity here, we will examine the link between insurance and protection when the insurance policy which parents purchase is a mandatory one which, e.g., might be implicit in the price of a product or service which they purchase. E.g., house rebuilding and contents insurance in the UK is often mandatory but its price will often depend on other precautionary expenditures that the parents make which have the incidental effect of reducing the physical risks to household members. Moreover, such policies usually provide "free" cover for public liability. This includes the opportunity for children to sue their parents' insurers for serious injury, though not usually for death. Alternatively, under strict liability, the choice of an expensive, reputable paediatrician who is likely to be well-insured both reduces the risks to children in their care and provides an implicit insurance policy should anything go wrong in an operation. In such a setting, we will consider a risk to one and only one child (one requiring medical care, say) and compare the incentive to protect this child when insurance is available implicitly or explicitly at a price which is either fair and responds to protective expenditure or does not.

Suppose first that child mortality insurance is available at a fair price of $1/p(e)$ per unit of gross cover. This is identical to the situation in Section II except that now the price is endogenous because it responds to protective expenditure[17]. Suppose that the premium paid, π, hence the amount of cover which it can purchase at a given e, is fixed. We will subsequently study the implications of varying the premium and hence the cover. The parents then solve:

(37)

$$\underset{e}{Max.}\left\{EU^{IP}(e) \equiv 2(1-p(e))U^0\left[\frac{\overline{M}-\pi-e}{n+2}, n+2\right] + 2p(e)U^1\left[\frac{p(e)\overline{M}+(1-p(e))\pi-p(e)e}{p(e)(n+1)}, n+1\right]\right\}$$

An interior solution to (35) satisfies, after some manipulation,

$$(37) \qquad EU_e^{IP} = p'\left(U^1 - U^0\right) - \left[\frac{(1-p)}{n+2}U_x^0 + \frac{p}{n+1}U_x^1\right] - \frac{p'\,\pi}{p(n+1)}U_x^1 = 0$$

The final term gives the utility value of the extra cover which can be purchased for a given premium if protective expenditure increases by a marginal unit and thereby reduces the price of cover.

To see the effect of an increase in the mandated premium on the optimal amount of protection, denote the latter by $e^{IP}(\pi)$. We know that if the second-order condition for this protective equilibrium are satisfied, then $e^{IP}{}'(\pi)$ has the sign of $EU_{e\pi}^{IP}$. Now,

$$(38) \qquad EU_{e\pi}^{IP} = p'\left(\frac{U_x^0}{n+2} - \frac{U_x^1}{n+1}\right) + \frac{(1-p)}{(n+2)^2}U_{xx}^0 - \frac{(1-p)}{(n+1)^2}\left[1 + \frac{p'\,\pi}{p^2}\right]U_{xx}^1$$

While the sign of this expression is ambiguous in general, it is easy to see what determines its behaviour, as Proposition 7 indicates.

Proposition 7. If: (i) the parents are optimally- or over-insured against child mortality risk, and (ii) an increase in protective expenditure increases the net income in the death state, then an increase in the mandated level of the insurance premium will lead parents to reduce their protective expenditure.

Proof. By concavity, $\left((1\text{ - p})U_{xx}^0 / (n+2)^2\right) < 0$. Next, note that

$$U_x^0 / (n+2) - U_x^1 / (n+1) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \text{ as parents are } \begin{Bmatrix} \text{over} \\ \text{optimaly} \\ \text{under} \end{Bmatrix} \text{ insured against the child mortality risk}$$

Thus, if the mandated level of cover is optimal or is excessive, the first term on the right of (38) is non-positive. Finally, note that $-\left[1 + \left(p'\,\pi / p^2\right)\right]$ in the last term in (38) gives the change in net income in the loss state, at an unchanged π, from increasing e. If this is positive, then $-\left[1+\left(p'\pi/p^2\right)\right](1-p)U_{xx}^1/(n+1)^2<0$ and, with (i), (38) would imply $EU_{e\pi}^{IP}<0$ and $e^{IP}{}'(\pi)<0$. Q.E.D.

Intuitively, this result follows because if the mandated level of insurance is already optimal or excessive, were a mandated increase in it to be reinforced by extra protective expenditure, the level of cover would become excessive or even more excessive. Both the increase in π and in e would decrease x^0 while increasing x^1. Thus, to avoid this, the parents are induced to substitute away from protection. It is also clear that conditions (i) and (ii) of Proposition 7 are only sufficient, not necessary for this outcome, given the presence of the middle term of $EU_{e\pi}^{IP}$ in (38). Thus, parents would need to be under-insured or be in a position where a decrease in protective expenditure increases the net income in the death state for it to be possible that an increase in mandated cover would be complemented by increased protective expenditure.

Suppose, next, the price of insurance does not respond to the amount of protection undertaken. I.e., suppose the situation is exactly as in Section II insofar as there is a fixed price per unit of cover. However, we allow "true" risks as perceived by the parents to be endogenous as before. Thus, let gross cover be purchasable at a price of λ per unit for sure. Thus, if cover s is purchased, the premium is λs and the net payoff in the loss state is $s(1-\lambda)$. Alternatively, if a premium of δ is paid for certain, this purchases net cover of $\delta(1-\lambda)/\lambda$.

Now, the parents' optimisation with mandated insurance in this environment involves:

$$(39)\quad \underset{e}{Max.}\left\{EU^{IPF}(e)\equiv 2(1-p(e))U^0\left[\frac{\overline{M}-\delta-e}{n+2},n+2\right]+2p(e)U^1\left[\frac{\overline{M}-e}{n+1}+\frac{\delta(1-\lambda)}{\lambda(n+1)},n+1\right]\right\}$$

with a first-order condition for an interior optimum given by:

$$(40)\qquad EU_e^{IPF}=p'\left(U^1-U^0\right)-\left[\frac{(1-p)}{n+2}U_x^0+\frac{p}{n+1}U_x^1\right]=0$$

Rather than consider the issue of the complementarity or otherwise of protective expenditure and insurance in this setting, we will discuss the relative incentives for protection in the two environments considered in this section. Here it is very easy to see, by comparison of (37) and (40), that we get the familiar "moral hazard" outcome, summarised in Proposition 8:

Proposition 8. When the price of insurance responds to protective expenditure, parents offered fair insurance against child mortality risk will incur more protective expenditure than if they are offered an identical policy but without the opportunity to affect its price by protective measures.

Proof. Let e^{IPF} solve (40). Suppose $\delta = \pi$ and $\lambda = p\left(e^{IPF}\right)$, thus the insurance policy which is offered in the fixed price case happens to be fair. Comparing (37) and (40) and evaluating EU_e^{IP} at e^{IPF}, we have

(41) $$EU_e^{IP}(e^{IPF}) = -p'\left(e^{IPF}\right)\pi U_x^1 / p\left(e^{IPF}\right)(n+1) > 0$$

Thus, repeating our earlier arguments, $e^{IP} > e^{IPF}$. Q.E.D.

Proposition 8 shows that when the price of insurance responds to the household's protective activities, the household is likely to engage in greater risk reduction. Thus, in this respect at least, focusing on the household's composition makes no qualitative difference to the textbook analysis.

VI. Conclusions

We have given a fairly comprehensive introductory treatment of the demand for parental and child mortality insurance and protection in a Beckerian context where household composition matters. Among other things, we showed that parents might or might not purchase more joint-life, first-death than single-life insurance for themselves but they would certainly not equalise the marginal utility of consumption across states with either. With both their own life insurance and self-protection, parents decisions depended crucially on the presence or otherwise of a bequest motive and on the precise nature of this when it exists.

Given the opportunity to protect a given child, we showed that parents' incentive to do so decreases as the number of children they have increases. This establishes a duality between the incentive to protect a child when their number increases exogenously and the demand for children as their exogenous mortality risk increases. Moreover, when the number of children and the risk one faces are both endogenous, parents will usually substitute protection for the number of children. When protection is a local public good in that a given expenditure protects more than one child equally, parents will plausibly incur more protective expenditure than when protection is a purely private good with respect to the safety of just one child. Some rudimentary analysis of the introduction of social security for orphans in the model when protection is a "pure local public good" in the household indicates that such social provision will not necessary lead to moral hazard, in the sense of inducing parents to take less care to ensure their own survival. However, classical moral hazard manifests itself when parents purchase mandatory insurance against child mortality risk, such as that implicit in the price of some goods and service under strict liability laws: parents will take less care to ensure a child's survival when the price of this insurance is not responsive to their protective activities than when it is.

Overall, while some of our results are elaborations of those which can be obtained in the context of single-argument utility functions, the most important are not. These include the following. First, that relating to the duality between how increased exogenous child mortality risk influences the demand for children and how an (exogenously) increased number of children influences the demand for children's safety; second, the implications of the interactions between the bequest function and the composition of the household; third, the differing consequences of safety as a local public good or a private good within the household.

Footnotes and Acknowledgement

*I have benefited from a useful conversation with Chris Skeels, another with Carlo Perroni, several with Martin Judge, suggestions by Mike Waterson and John Whalley, and from comments on a related paper by participants at the Warwick Public Finance Weekend in February 1995. I am most indebted to Melrose Stewart (Fraser) for clarifying some of my inchoate thoughts on the family. Remaining errors are mine.

1. See Luce and Krantz on the concept of conditional expected utility and, e.g., Cook and Graham (1977), Fraser (1984, 1995a,b), Frech III (1994), Jones-Lee (1974), Spence (1977), Viscusi (1979), Viscusi and Evans (1990), among many others, for applications including to the valuation of life-saving.

2. E.g., cf. Calfee and Rubin (1992), and Frech III (1994).

3. Becker and Ehrlich did not consider self-protection with state-dependent preferences, hence explicitly physical risks. Several authors, such as those mentioned in footnote 1, have done so subsequently, notably in the context of the value of life-saving. Arnott and Stiglitz (1988) also indirectly considered this issue in the context of insurance and moral hazard. However, none of these authors studied explicitly the impact of mortality on family composition, hence neither did they consider the trade-off between the standard of living and the household size which we noted above.

4. John Solow (1994) gives a very recent discussion of paternalistic preferences and the different ways in which parents' preferences might subsume the interests of their children.

5. This assumption might be regarded as objectionable as it means that, in some sense, children are not valued in their own right for their independent existence but simply as consumption goods.

6. Nerlove, Razin and Sadka (1987, 65) use the same measure of the standard of living with somewhat different utility functions in an overlapping generations context. Related but more complicated measures based on the number of "equivalent adults" in the household, could be employed without altering our main results. Perhaps a more important observation to make is that such measures implicitly assumes that resources are pooled within the household. Although this is a defensible assumption in our introductory treatment, several recent analyses have indicated that it is more realistic to recognise the

existence of intra-household inequality and to examine bargaining over the distribution of resources within the household. See, e.g., Chiappori (1992), Apps and Rees (1994), Alderman et al (1995), and references therein.

7. This does suggest a weak test for the presence and strength of the bequest motive by comparing state-dependent utilities of consumption along the lines of Viscusi and Evans (1990). For the set of parents who purchase jlfd insurance, those with $U_x^0 / U_x^1 > 1$ must definitely have bequest functions. Our analysis implicitly assumes that parents make conscious bequests although, empirically, few make wills. Another test which might be undertaken would be to investigate whether, other things equal, those parents who purchased relatively large amounts of jlfd insurance were represented disproportionately among those who made wills. The making of a will might be interpreted as the expression of a conscious bequest motive. I am indebted to Mike Waterson for this idea.

8. E.g., see Jones-Lee (1974) Fraser (1984), Drèze (1962), Drèze and Dehez (1982), Fromm (1968) and Viscusi (1979).

9. We will briefly study protection alongside child mortality insurance in Section V.

10. Arnott and Stiglitz's (1988) analysis is perhaps the most complete treatment of the implications of this non-concavity, especially focusing on insurance and the problem of moral hazard.

11. Despite what we have just said in the text, it is worth remarking on the SOC here:

$$\text{(F1)} \quad -p''[(1-2p)2U^0 - (1-2p)U^1] + p'[\left(4(1-p)U_x^0 / (n+2)\right) - \left((2-4p)U_x^1 / (n+1)\right)]$$
$$+(p')^2[2U^0 - 2U^1] + (1-p)\left[\left((1-p)U_{xx}^0 / (n+2)^2\right) + \left(pU_{xx}^1 / (n+1)^2\right)\right] \leq 0$$

Note that the FOC does not guarantee $U^0 > U^1$ nor does satisfaction of the SOC require it. But, $U^0[(2M-e)/(n+2), n+2] > U^1[(M-e)/(n+1), n+1]$ if $U^0(x,n) > U^1(x,n)$ and $U_n^o \geq 0$ because $(2M-e)/(n+2) > (M-e)/(n+1)$.

12. However, the late Professor Jack Wiseman of York University used to tell me that the principal reason that Yorkshiremen were so concerned about increasing the size of their estates was because of the pleasure they derived, during life, from knowing how envious their neighbours would feel when their wills were declared.

13. See, e.g., Erhlich and Becker and other references in footnote 9.

14. Utility as a function of the degree of bereavement need not decrease uniformly; it only needs to be concave. As far as the other sufficient conditions are concerned, in both the state-independent and -dependent utility cases they all reduce to requiring that
$p\left\{(1-p)\left[\left(U_x^0/(n+2)\right)-\left(U_x^1/(n+1)\right)\right]+p\left[\left(U_x^1/(n+1)\right)-\left(U_x^2/n\right)\right]\right\}>$
$-\left\{(1-p)\left(U_x^0/(n+2)\right)+p\left(U_x^1/(n+1)\right)\right\}$ - i.e., after simplification,
$(1-p)(1+p)\left(U_x^0/(n+2)\right)+p^2\left(U_x^1/(n+1)-U_x^2/n\right)>0$ - evaluated at e^{1PC}. While (iii), (iv) and (vi) in Proposition 5 are sufficient to ensure this, clearly they are unnecessary.

15. This would require the utility of bequests to depend on more than the per capita income of heirs, in particular on the number of survivors - something which we argued against earlier.

16. Compare, e.g., Becker and Murphy, Calfee and Rubin, Calfee and Winston, Frech III and Fraser (1995b, c).

17. Again we stress that this might simply be the choice of a more reputable and expensive paediatrician who can, however, purchase third-party insurance at a cheaper, experience-rated price.

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