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BEYOND THE "NO DISTORTION AT THE TOP" CASE

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NON-LINEAR PRICING IN A VERTICAL DIFFERENTIATION SETTING

Beyond the "no distortion at the top" case.*

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Abstract

This paper examines the relevance of the well known "no distortion at the top" result in a model of vertical differentiation. The analysis shows that the no crossing condition is a *sufficient but not necessary* condition in order to get no distortion at the top. Relaxing some of the canonical preferences' assumptions we generate non standard cases. We also extend the analysis allowing consumers to buy *more than one* unit of the products. In the presence of interactions between quality and quantity the occurrence of non standard cases become more likely, though the implications on the optimal customers' bundles are less obvious.

JEL: D40, D42, L12.

Keywords: Vertical differentiation, non-linear pricing, price and quality discrimination.

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I. Introduction

Quantity and quality discrimination are very similar in many aspects. As long as we can regard quality as a "fictitious quantity," at a formal level the two models are perfectly equivalent. The seminal paper of Mussa and Rosen (1978) showed how vertical differentiation leads to discriminate amongst buyers with different taste parameters. Their model predicts "no distortion at the top" equating high-demand customers' marginal price and marginal cost.¹ This outcome seems to be very different from what is observed in real situations. Examples taken from everyday life show us that the top quality good is not sold at its marginal price.

On the other hand models attempting to portray situations closer to reality are very special in many aspects. Specifically, in order to avoid the occurrence the no "distortion at the top" case, they introduce quite specific -and somewhat *ad hoc*- assumptions. For instance, some of them assume the variable cost of quality to rise "slowly" with quality.² These considerations leads us to explore in detail this issue, in order to solve some of the theoretical and empirical puzzles.

The aim of this paper is to examine the relevance of the well known no distortion at the top result in a model of "strong" vertical differentiation, where quality is the most relevant economic variable. Specifically, we investigate the conditions under which no distortion at the top holds in the case of a multi-quality monopolist. Relaxing the no crossing or Spence-Mirrlees (S-M) condition we generate non standard cases, in which high-demand customers marginal prices may differ from marginal cost. We also show how the S-M condition is *sufficient* but *not necessary* in order to get the no distortion at the top case. Finally we examine how the interaction between quality *and* quantity, allowing consumers to buy *more than one* unit of quantity of the product, makes more likely the realisation of these non standard cases.

¹ Models of quantity discrimination, starting from Spence's (1980) contribution, are discussed in Wilson (1993). See also Itoh (1983) and Ireland (1991) for welfare considerations.

² However, also these contributions, built upon the original quality model, kept standard assumptions on consumers' preferences. We can refer to the works of Gabszewicz and Thisse (1979, 1980) and Shaked and Sutton (1983, 1987). A comprehensive account on horizontal and vertical differentiation is given in Waterson (1989). These models are quite similar, as shown by Cremer and Thisse (1991) and Anglin (1992).

The paper is organised as follows. First of all, in section II, we consider in detail the crucial hypotheses that drive the no distortion at the top result in a canonical quality model. We offer an intuitive account and a graphical representation of the model, and we derive the standard result in analytical terms. Section III uses the previous framework to examine more closely Mussa and Rosen's model and to investigate into the progress made to go beyond their paradigm. Section IV examines the non standard cases in which: (i) consumption choices are not distorted or (ii) there is no distortion only at the bottom (a case which is the reverse of the standard one). Section V examines a model in which a monopolist can discriminate, apart from offering different qualities, also by selling different quantities of the same good. Section VI concludes the paper, sketching a simple case in which the optimal vertical product differentiation determines automatically the whole sequence of cases different from the standard one of no distortion at the top.

II. The "no distortion at the top" result in an enlarged framework

In a stylised setting, where a monopolist is unable to distinguish different types, the basic ingredients to get "no distortion at the top" are the existence of top (or high-demand) customers (i.e. an upper bound on the value of the taste parameter) and a binding incentive compatibility constraint for these customers.

The intuition behind the "no distortion at the top" result is easily explained. The monopolist would like to extract the high-demand consumers' surplus; however, doing this he faces the threat of their personal arbitrage. In fact, high-demand customers can consume the low-demand bundle if the latter generates more surplus (compared with theirs). Hence, a reduction of the quality offered to low-demand consumers is needed in order to relax their arbitrage (or incentive compatibility) constraint. This occurs because, due to a technical sorting condition (known as *no crossing* or S-M condition) high-demand customers suffer more from a reduction of the quality than low-demand customers do.

II.A The canonical assumptions on quality preferences and the M-S condition

In order to derive analytically the standard discrimination result we must specify the utility (and cost) functions in the presence of different product qualities. Specifically, we examine the quality and pricing policies of a multi-product firm producing goods of different quality x for an economy of a *finite* number of consumers. Consumers differ in tastes and

incomes (in a standard way) and purchase at most one unit of one of the goods.

We refer to x^m as the minimum quality standard and we will also set a maximum feasible quality level so that $x^M \geq x \geq x^m$. In this qualitative range the firm is characterised by a unitary production cost denoted by $m(x)$ which is positive, strictly increasing and convex in quality:

$$[A0] \quad m(x) > 0; \quad m'(x) > 0; \quad m''(x) \geq 0$$

We consider a quite general setting by specifying the utility as a function of income in the presence of different product qualities x_t , where the subscript t refers to a consumers' type characterised by an income Y_t (interpreted as a Hicksian composite commodity) and a taste parameter θ_t .³ Let P_t and $u_t = u(x_t, \theta_t, Y_t - P_t)$ represent respectively the price charged and the utility function of a consumer of type t , who consumes a unit of good of quality x_t and all his disposable income $y_t = Y_t - P_t$ on an Hicksian composite commodity.

In this enlarged setting the canonical preferences' assumptions are specified as below:

(1) Products of higher quality and lower prices give greater utility to each consumers' type:⁴

$$[A1] \quad \begin{aligned} \partial u_t(\cdot)/\partial x_t &= u_x(x_t, \theta_t, Y_t - P_t) > 0 & \forall x_t, \theta_t \text{ and } y_t \\ \partial u_t(\cdot)/\partial y_t &= u_y(x_t, \theta_t, Y_t - P_t) > 0 & \forall x_t, \theta_t \text{ and } y_t \end{aligned}$$

(2) For any type of customer the utility derived by the minimum quality level x^m , when the price charged equals the production cost $m(x^m)$, is greater than his reservation utility level u_t^* :

$$[A2] \quad u(x^m, \theta_t, Y_t - m(x^m)) > u_t^* \quad \forall Y_t \text{ and } \theta_t$$

Thus, with perfect discrimination the firm can extract a positive profit $\pi_t^m > 0$ offering the lowest quality, while leaving the reservation utility level $u(x^m, \theta_t, Y_t - m(x^m)) - \pi_t^m = u_t^* > 0$.

(3) Customers of higher income have higher taste parameter and valuations:⁵

$$[A3] \quad Y_t > Y_s \Rightarrow \theta_t \geq \theta_s \text{ and } \partial u_t(\cdot)/\partial \theta_t \geq 0 \quad \forall x_t, \theta_t \text{ and } y_t$$

³ Henceforth, often to have clear-cut results and simple graphical representations, we restrict the analysis to the two type case and to simplify notation use the subscript t ($= L, H$) as a shortcut for θ_t .

⁴ This simply means that the utility function of any type is increasing in its first argument (i.e. in the level of quality) and its third argument (the level of disposable income). Hence, for any given value of θ , we know that $u(z, \theta, Y - P) > u(x, \theta, Y - P)$ for $z > x$ and $u(x, \theta, Y - P_s) > u(x, \theta, Y - P_t)$ for $P_t > P_s$.

⁵ In practice, if consumer t is endowed by a higher income relative to consumer s he will also be characterised by a higher or equal taste parameter. Consequently he values equally or more highly the same level of quality.

Thus for a finite number of types we have $u_t \leq u_{t+1}$ (as well as $\pi_t^m \leq \pi_{t+1}^m$) for any level of quality. From $u(x_t, \theta_t, Y_t - m(x_t) - \pi_t) = u_t^*$ we may define the maximum per capita profit functions $\pi_t^* = \pi(x_t, \theta_t, u_t^*)$ and order in the same way $\pi_t^* \leq \pi_{t+1}^*$ for any level of quality.

(4) If consumer s prefers the bundle $\langle x_s, P_s \rangle$ to $\langle x_r, P_r \rangle$, with x_r less than x_s , the same holds for a customer t with a higher taste parameter value and hence income (i.e. $\theta_t > \theta_s$ and $Y_t > Y_s$):

$$[A4] \quad \frac{\partial}{\partial \theta_t} \left(\frac{\partial u_t(\cdot)/\partial x_t}{\partial u_t(\cdot)/\partial y_t} \right) > 0 \quad \forall x_t, \theta_t \text{ and } y_t$$

This is the *single crossing* or S-M condition. It implies that the marginal rates of substitution between quality and income can be ordered like total utilities, so that $u_t' < u_{t+1}'$ for any level of quality. The same is true for the maximum per capita marginal profit ($\pi_t' \leq \pi_{t+1}'$).

(5) Finally, the utility function is concave in all its arguments:

$$\begin{aligned} [A5] \quad \partial^2 u_t(\cdot)/(\partial x_t)^2 &\leq 0 & \forall x_t, \theta_t \text{ and } y_t \\ \partial^2 u_t(\cdot)/(\partial \theta_t)^2 &\leq 0 & \forall x_t, \theta_t \text{ and } y_t \\ \partial^2 u_t(\cdot)/(\partial y_t)^2 &\leq 0 & \forall x_t, \theta_t \text{ and } y_t \end{aligned}$$

Looking at these canonical assumptions it seems quite demanding and restrictive (to say the least) to assume, for any number of types, to have a complete ranking of types with respect to total *and* marginal utility, as implied by condition [A3]. In section III we are going to relax conditions [A3] and [A4]. Moreover, in section V we are going to relax another underlying assumption, which represents a relevant limit of the model, never being questioned; that is, the possibility to buy only one unit of the produced goods.

Before concluding this section let us describe the mechanism design which allows a multi-quality monopolist to separate customers of different types, whose preferences are specified as above.

II.B The derivation of the "no distortion at the top" result

The kind of equilibria we are interested in are the ones for which consumers of different types can be *separated* by offering different bundles. A separating mechanism designed for a *finite* number of types must satisfy the participation and incentive compatibility constraints specified below. The purchasing or participation condition [PC_t] tells us that customers can always get the reservation utility u_t^* purchasing nothing, whilst the incentive compatibility constraint [IC_t] ensures that customers always choose the bundles which give them a greater

net utility. The mechanism design in the case of a multi-quality monopolist derives from the solution of a constrained profit maximisation. If we interpret n_t as the probability of serving a consumer whose taste parameter is θ_t rather than the type's proportion we will talk in terms of expected profit maximisation.

$$\begin{aligned} \max \Pi &\equiv \sum_t n_t \pi_t && \text{subject to:} \\ [\text{PC}_t] &u(x_t, \theta_t, Y_t - m(x_t) - \pi_t) \geq u_t^* && \forall t \\ [\text{IC}_t] &u(x_t, \theta_t, Y_t - m(x_t) - \pi_t) \geq u(x_s, \theta_t, Y_t - m(x_s) - \pi_s) && \forall t \text{ and } s \neq t \end{aligned}$$

Notice how we express both the objective function and the constraints in terms of per capita profit $\pi_t = \Pi / n_t = P_t - m(x_t)$. This allows us to provide a nice graphical representation directly in terms of profits, as will be shown in what follows.⁶ For simplicity sake from now on we will restrict the analysis to the two type case. Henceforth the superscripts * and ° characterise respectively the full information and the asymmetric information cases.

Result 1

Assuming that assumptions [A0] -[A5] hold and considering only two qualities of goods the distortion in prices for a low quality good depends only on the proportion of types of consumers and the income effect. There is no distortion for high quality goods which will be sold at marginal costs.

The proof of this result is given in Appendix A.

Basically, setting the marginal tariffs p_{x_t} so as to satisfy the binding constraints specified above, and denoting by U_H the variable which captures the income effect we can write the first order condition as:

$$\begin{aligned} [\pi_L^\circ] \quad \pi_{x_L} &= p_{x_L} - m'(x_L) = n_H U_H [u_x(x_L, \theta_H, Y_H - P_L) / u_y(x_L, \theta_H, Y_H - P_L) - p_{x_L}] / n_L \\ [\pi_H^*] \quad \pi_{x_H} &= p_{x_H} - m'(x_H) = 0 \end{aligned}$$

These equations determines the departure from marginal cost pricing. We can also interpret these conditions in intuitive terms by defining the distortion in the pricing structures

⁶ Limiting the analysis to two types, under the usual assumptions (PC_L and IC_H being the only binding constraints) it is easy to show that (a) the high customer always enjoys a positive surplus if the low type does, (b) the low customer has never an incentive to buy the higher quality item purchased by the high type, (c) the first order conditions are the same as in the standard monopoly case (relative to quantity discrimination). For an analytical derivation of the no distortion of the top result see Appendix A.

by the ratio σ_{x_t} . The distortion is given by the ratio between the marginal profit obtained on the t type (selling the quality level t) and the marginal rent enjoyed by the other type from the bundle addressed to t . In analytical terms: $\sigma_{x_L} = \pi_{x_L} / [u_x(x_L, \theta_H, Y_H - P_L) / u_y(x_L, \theta_H, Y_H - P_L) - p_{x_L}]$

Using this new tool, equation $[\pi_L^\circ]$ tells us that the distortion (in prices) for low customers is equal to the population ratio n_H/n_L times U_H . The same ratio for high customers σ_{x_H} is instead equal to zero, since the marginal profit on the H quality is equal to zero (i.e. $\pi_{x_H} = 0$), as shown by equation $[\pi_H^*]$.

$$\sigma_{x_L} = n_H/n_L U_H$$

$$\sigma_{x_H} = 0$$

In fig. 1 below the maximum per capita profit functions for each type t $\pi_t^*(x, \theta_t, Y_t, u_t^*)$ implicitly defined by the purchasing conditions PC_L and PC_H are represented. They show the profits the monopolist enjoyed from each type if he were able to perfect discriminate.⁷

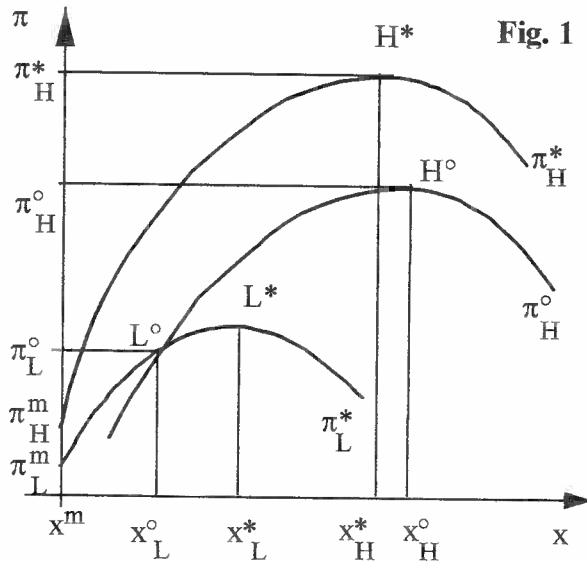


Fig. 1

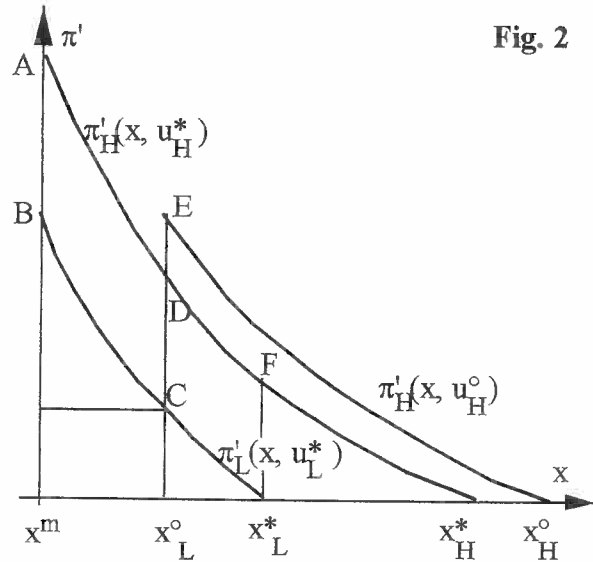


Fig. 2

Under *asymmetric information* the binding constraints PC_L and IC_H are represented by π_L^* and $\pi_H^\circ = \pi(x, \theta_H, Y_H, u_H^\circ)$, the latter being implicitly defined by the condition $u(x_H, \theta_H, Y_H -$

⁷ Since they represent the locus of points for which customers' utility is equal to the reservation level (and hence is constant), they can be interpreted as a sort of hicksian demand for quality when $u_t = u_t^*$. These curves do not pass through the origin as the usual participation constraints of Maskin and Riley (1984), since $0 < \pi_L^m \leq \pi_H^m$. Following the previous interpretation, it is easy to explain their shape. In the *perfect information benchmark* (where individual tastes θ_t and incomes Y_t are known and PC_L and PC_H represent the only binding constraints) the non distortionary contracts [according to which marginal profits are zero so that: $p_{x_t} = m'(x_t)$] offered to each type maximise profits. In other words, the monopolist is able: (1) to fully extract all the surplus from each type of customers; (2) to deliver both types the efficient level of quality (x_H^* and x_L^*).

$m(x) - \pi_H^o = u_L^o$, where $u_H^o \geq u_H^*$ denotes the higher level of utility reached by the high type. The high type must be given a net surplus (the rent $\pi_H^* - \pi_H^o$ in fig. 1) in order to satisfy his incentive compatibility constraint.

Clearly, in order to have an interior solution (so that the quality boundaries are irrelevant) we must also assume that: $x^M > x_H^o \geq x_L^o > x^m$. Notice also how, due to the income effect, the quality offered to the high demand customers is x_H^o greater than x_H^* . A more detailed graphical explanation can be given in the marginal profit space (π', x) in fig. 2 where the three constraints (PC_L , PC_H and IC_H) are denoted by $\pi'_L(x, u_L^*)$, $\pi'_H(x, u_H^*)$, and $\pi'_H(x, u_H^o)$ respectively. They represent the marginal willingness to pay (net of marginal cost) which allow the customers' to maintain utility constant at the reservation levels u_L^* , u_H^* and at the higher level u_H^o . The high type's net surplus is represented by the area ABCD. Profits π_L^o and π_H^o provide a measure of the net revenue per unit of customer: π_L^o is simply given by the area $Bx^m x_L^o C$ while to obtain π_H^o the area $Ex_L^o x_H^o$ should be added. The optimal non-linear tariffs and the couple of bundles $\langle \pi_t^o, x_t^o \rangle$ that maximise the monopolist's profit under the standard constraints show that the low demand customer is constrained to consume a lower quality $x_L^o < x_H^*$ and the high type's rent is reduced from $ABx_L^* F$ to ABCD.

Because of its simplicity, the previous framework is also quite useful to examine the main problems dealt with by the more recent theoretical literature. At the same time we can also assess the attempts made to go beyond the paradigm of the Mussa and Rosen model of vertical differentiation, avoiding the analytical difficulties present in more complex settings.

III. The standard case and beyond it

It seems useful to start the analysis from the two type example of Mussa and Rosen (1978) under their specification of *linear* utility curves $u_t = \theta_t x_t + y_t$ (with a quadratic cost function $m(x) = mx^2/2$). The constant taste parameter θ_t has sometimes been interpreted as a sort of marginal utility of income. It is straightforward to sketch the Mussa and Rosen example as a particular case in which in our framework: (i) the marginal profits $\pi'_L(x, u_L^*)$ $\pi'_H(x, u_H^*)$ are linear and (ii) there are no income effects (since $U_H = 1$) so that $\pi'_H(x, u_H^o) = \pi'_H(x, u_H^*)$.

The first order condition with respect to x_L determines the departure from marginal cost pricing on the low type [$p_{x_L} - m'(x_L)$] and can be rewritten as follows:

$$[\pi_L^\circ]' \quad \pi_{x_L} = p_{x_L} - m'(x_L) = n_H[u'(x_L, \theta_H) - m'(x_L)]$$

$$[\pi_L^\circ]'' \quad \pi_{x_L} = p_{x_L} - m'(x_L) = (n_H/n_L)[u'(x_L, \theta_H) - p_{x_L}]$$

Let us give an intuitive account of the impact of a marginal increase of the level of quality x_L on the profitability of the firm.

Referring to the left hand side of equation $[\pi_L^\circ]'$ the per capita profit on the marginal unit x_L is overall equal to Cx_L° as in fig. 2. In fact, the marginal tariff paid by both types is Cx_L° times their relative proportion. The segment Cx_L° represents the difference between the marginal utility and the marginal cost for the low type; that is, $u'(x_L, \theta_L) - m'(x_L)$. For the other type the same segment derives from the difference between Dx_L° [i.e. $u'(x_L, \theta_H) - m'(x_L)$] and DC, since the marginal rent DC must be given to respect the high type's incentive compatibility constraint. The right hand side shows the foregone profits (had the marginal unit x_L not increased), which are Dx_L° times the proportion of high demand customers n_H , since low-demand customers are not offered the marginal unit x_L . For instance, when the proportion of type L is twice as much the proportion of type H (i.e. $2 n_H = n_L = 2/3$) we immediately determine that the new marginal tariffs are just a third of the old ones $Cx_L^\circ = Dx_L^\circ / 3$. In this way in the absence of income effects it is quite easy to interpret the departure from marginal cost pricing.

Alternatively, from condition $[\pi_L^\circ]''$ (which sets σ_{x_L} equal to n_H/n_L) we can infer that the marginal distortion on the low type's tariff $Cx_L^\circ = p_{x_L} - m'(x_L)$ is optimal when the marginal profit associated to a marginal increase of the low type's quality level x_L (that is $Cx_L^\circ = p_{x_L} - m'(x_L)$ on the left hand side) equates the marginal loss due to the rent enjoyed by the high type customer DC [i.e. $u'(x_L, \theta_H) - p_{x_L}$] times the ratio n_H/n_L (on the right hand side). Hence, with $2 n_H = n_L$ we have $Cx_L^\circ = DC / 2$.

A growing dissatisfaction with the canonical assumptions and results is testified by a number of papers which tried to go beyond the standard framework.

Gabzewicz, Shaked, Sutton and Thisse (1986) consider a continuum of consumers identical in tastes and study the relationship between pricing and customers' income distribution, when the willingness to pay for quality improvements increase with income (being $u = x y$) while the unit variable cost rises "slowly" (since $m(x) = 0$, removing assumption [A0]) and the highest income is not twice as much the lowest one (since $Y^M <$

$2Y^m$). Cost and utility functions are specified so to give rise to a setting which is completely different from the one sketched before. Specifically, in graphical terms in fig. 1 the profit function π_L^* and π_H^* should always be increasing and in fig. 2 marginal profits $\pi'_L(x, u_L^*)$ and $\pi'_H(x, u_H^*)$ should never intersect the horizontal axes, since they are always positive (even if decreasing). Hence the optimal values of x_L^* and x_H^* go to infinity, so that the maximum feasible quality x^M is the socially efficient level, since it is preferred by all the customers (independently of the income level) at marginal costs. Clearly, since the utility functions are the same (apart from the income level), the flatness of the variable cost schedule causes bunching between customers with different incomes on the same product.⁸

Shitovitz, Spiegel and Weber (1989) examine a similar problem, with a finite number of consumers, i.e. they look for the conditions under which to offer only the 'top of the line' quality is the monopolist's optimal policy. This is always the case when utility and cost functions are linear, $u_t = \theta_t x_t + y_t$ and $m(x) = m x$. In fact, also in this special case the profit function π_L^* , π_H^* (in fig. 1) are always increasing and $\pi'_L(x, u_L^*)$, $\pi'_H(x, u_H^*)$ are positive constants, so that x_L^* and x_H^* go to infinity. Hence, for any value of maximum feasible quality level x^M , it is optimal either not to serve low type customers or to offer the top quality x^M . In the other cases the realisation of this condition depends on the customers' income and taste distribution. In the two type case for n_L small enough it is always optimal not to serve low type customers since x_L^o is equal to zero for any value of the top quality x_H^* (independently from $x^M > x_H^*$) at the high demand customers' price P_H . On the other hand, when the quality upper bound is relevant (since it is relatively small $x^M < x_L^o$) it turns out optimal to sell only the top quality x^M to each type at the low customers' price P_L . This is likely to be the case with a narrow range of income and taste distribution, and a large n_L number of low demand customers, i.e. when x_L^o , x_L^* and x_H^* are close enough (see also their remarks at pp. 1648-9).

However, avoiding standard pricing distortion through bunching (since for technical reasons either $m(x) = 0$ or $x^M < x_L^o$) or discontinuities of linear specifications is not particularly satisfactory. Champsaur and Rochet (1989), instead, excluding bunching, examine within a model of product differentiation the constraints to monopoly powers deriving from

⁸ In our previous framework -setting $x^M < x_L^*$ to get a similar setting- bunching implies selling quality x^M to both customers at the low customers' price P_L and implies a reduced range of income (Y^m , Y^M) (compared with the feasible quality range), so that x_L^o is sufficiently close to x_L^* and we have: $x^M < x_L^o$.

competition for the high (or low) customers given the presence of "outside goods". Specifically, the high type customers can purchase a close substitute enjoying a net surplus.⁹

The generalised framework has proved to be very useful to offer an overview of the more recent literature. In the next subsection we are going to relax some of the most restrictive hypotheses, first of all showing how we can get the no distortion at the top case with no recourse to the M-S condition.

III.A The standard case without recourse to the M-S condition

We believe that to operate on the consumer side (expanding the allowed preference patterns) is a relevant development, but this should not be necessarily tied to the presence of competitors and "outside goods". There has been no attempt yet to remove the more crucial and technical M-S condition [A4], even in models with a finite number of consumers.

Even in the one dimensional qualitative setting with quasi linear utilities, it seems quite demanding and restrictive (to say the least) to assume, for any number of types, to have a complete ranking of types with respect to total and marginal utility so that, with a finite number of groups, we have $u_t < u_{t+1}$ and $\partial u_t / \partial x < \partial u_{t+1} / \partial x$ for any level of quality x ([A3]). It is instead unquestionable that, given the unit cost function (and the relative assumption [A0]) groups of customers can be ordered by the quality levels that is efficient for them to buy.

In other words, excluding bunching, what we require is that [A5] the optimal quality purchased by customers x_t^* is an increasing function of the taste parameter θ_t and [A6] the profit functions π_L^* and π_H^* (in fig. 1) have a positive marginal value before quality reaches the levels x_L^* and x_H^* and a negative one afterwards. But once we remove assumptions [A3] and [A4] the no distortion at the top case is no longer the unique relevant case to consider, since PC_L and IC_H are not necessarily binding. Thus, we must examine the other possible combinations of the participation and incentive constraints, as will be done in section III. Let us first state a second result.

⁹ This may imply, in the two type case, that the firm can no longer extract a positive profit from high customers offering the minimum level of quality, since the reservation utility level has increased, removing assumptions [A2] and [A3]. Hence the profit level π_H^* (in fig. 1) is reduced, and, as shown in fig. 2, higher rents and eventually higher qualitative levels must be offered to high type customers (when the quality upper bound is not yet reached, i.e. $x^M > x_H^*$). In a quantitative framework, Laffont and Tirole (1990) examine a similar case with only two consumer types. For a discussion of their model see also Vagliasindi (1995).

Result 2

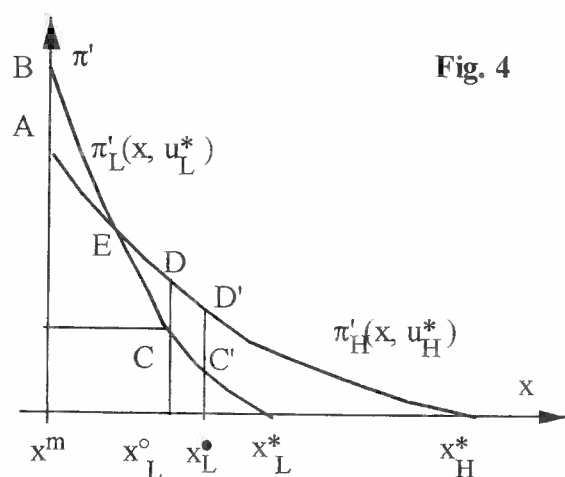
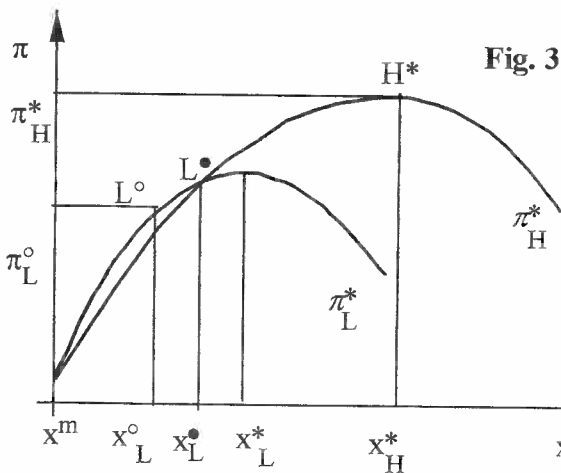
In a model in which assumptions [A3] - [A4] are replaced by the less demanding assumptions [A5] - [A6] specified above we can still get Result 1 if the binding constraints are the standard ones plus the participation constraint of the high type. This implies that the S-M condition is sufficient but not necessary in order to get no distortion at the top.

This represents an interesting extension of the standard result, in which the distortion at the bottom does not depend on the previous marginal conditions but is determined by the three binding constraints; PC_L , PC_H and IC_H . In fact, this time straight from the constraints we get:

$$[x_L^*] \quad u(x_H, \theta_H, Y_H - m(x_H) - \pi_H) = u(x_L, \theta_H, Y_H - m(x_L) - \pi(x_L, \theta_L, u_L^*)) = u_H^*$$

The discriminatory quality x_L of the low type is now determined by the previous equality corresponding to the point in which the two maximum per capita profit functions π_t^* intersect. In order to find x_H we can simply maximise the high type maximum per capita profit function π_H^* with respect to x_H and verify that the first order condition for type H is the same as before, that is p_{x_H} is equal to $m'(x_H)$. We have no distortion at the top (since PC_L and IC_H are binding) but no surplus is left to the H type since the constraint PC_H is also binding.

This implies that the distortion ratio σ_{x_L} is less than the optimal one derived in Result 1 (that is, $n_H U_H / n_L$) while σ_{x_H} is still equal to zero. Hence, the level of distortion x_L^o (derived from $[\pi_L]$) is not reached, since the incentive compatibility constraint π_L^o corresponding to contract L^o , is no longer feasible. In this case, represented in fig. 3 below, it is instead optimal a reduced distortion, corresponding to $x_L^* > x_L^o$ (where the purchasing constraint π_H^* is binding), since a greater distortion could not increase the rent taken away from high-demand customers.



At the same time it is optimal not to allow rent to the H type reducing distortion any

further, since the gains from a marginal increase of x_L are smaller than the losses $\pi_{x_L} < n_H U [u_x(x_L, \theta_H, Y_H - P_L) / u_y(x_L, \theta_H, Y_H - P_L) - m'(x_L)] / (n_L + n_H U)$. In order to combine the marginal and total utility spaces in the following graphical analysis we assume, without loss of generality, that the maximum profits π_L^m extracted offering the minimum level of quality x^m are equal. Also in the (x, π') space (fig. 4 above) the constraints $(PC_H$ and $IC_H)$ coincide and are represented by $\pi_L'(x, u_L^*)$, so that the high type no longer enjoy a net surplus to satisfy IC_H . Hence, given the equality between π_L^m and π_H^m for quality $x_L^* (> x_L^o)$ the areas BAE and EC'D' are equal and consequently a reduction in the low quality can decrease the marginal net rent C'D' of a high-demand customer but not total rent extracted from him (as in this case IC_H would not be satisfied). On the other hand, there is no reason to increase the low quality level, since the marginal net rent C'D' gained is greater than CD the one corresponding to which marginal profits equal marginal losses in condition $[\pi_L]$.

IV. The analysis of the other non standard cases

In what follows we will study the other combinations of participation and incentive compatibility constraints which give rise to the case in which: (i) there is *no distortion at the bottom*, which is exactly the reverse of the first standard one and (ii) consumption's choices are not distorted; that is, *no distortion both at the top and at the bottom*. We will also show how, in the former case, for a given proportion of the high demand type of customers it may become not profitable to serve them, which might seem at first slightly counterintuitive.

It is useful to briefly describe these possible cases depending on which constraints are binding. In the no distortion at the bottom case (i), the L type should be prevented from buying the bundle addressed to the H type. To achieve this aim high-demand consumers are charged marginal prices below marginal costs. High-demand customers have no rent, since the participation constraint of the H type is binding. Low-demand customers may enjoy a positive rent, if their participation constraint does not bind. In a limiting case the firm finds it convenient to serve only low-demand customers, taking all their surplus, with no distortion. In fact, with a low supply for high-demand consumers there is a relevant incentive to reduce rents of low-demand customers.

Instead, in case (ii), where the consumption of both types is not distorted, there must be no need to give any type of customers incentives to prevent mimicking the other type. Since

the incentive compatibility constraints are automatically satisfied, only the individual rationality constraint are binding.

IV.A. The no distortion at the bottom result

Let us now turn to the analysis of the *no distortion at the bottom* case by stating the following result.

Result 3

In the model specified above in which assumptions [A3] - [A4] are replaced by the less demanding conditions [A5] - [A6] we can get no distortion in the price of the low quality good if the binding constraints are the low type's participation constraint and the high type's incentive compatibility constraint. The distortion in the price of the high quality good is dependent on the types proportion and on the income effect. This result can be considered as the speculative image of Result 1.

Intuitively, for high qualities $x \leq x_H^*$ the surplus of the low type is greater the one of type H and hence the L type should be prevented from buying the high-demand customer's bundle (as his incentive compatibility constraint IC_L is binding as well as PC_H).

There are two subcases to be considered. When constraint PC_L is binding too the firm can extract all the surplus from low-demand customers. In this subcase the high type quality x_H^\bullet (and the distortion at the top) is directly determined by the three binding constraints as follows:

$$[x_H^\bullet] \quad u(x_L, \theta_L, Y_L - m(x_L) - \pi_L) = u(x_H, \theta_L, Y_L - m(x_H) - \pi(x_H, \theta_H, u_H^*)) = u_L^*$$

Specifically, for $x_H = x_H^\bullet$ the two gross surpluses are equated as in this point the maximum per capita profits π_t^* intersect. The value of x_L is determined simply by maximising the low type maximum per capita profit function π_L^* with respect to x_L ; hence, we verify that we have no distortion at the bottom as p_{x_L} is equal to $m'(x_L)$:

$$[x_L^*] \quad p_{x_L} = m'(x_L) \quad \text{No distortion at the bottom}$$

For high-demand customers the distortion ratio σ_{x_H} is positive but lower than the optimal level $n_L U_L / n_H$ determined in the case in which only IC_L and PC_H are binding analysed below [where $U_L = u_y(x_H, \theta_L, Y_L - P_H) / u_y(x_L, \theta_L, Y_L - P_L)$], while for low customers the distortion ratio σ_{x_L} is equal to zero. Thus, we have no distortion at the bottom (since IC_L is binding) but

no surplus is left to both types as the constraints PC_H and PC_L are binding as well.

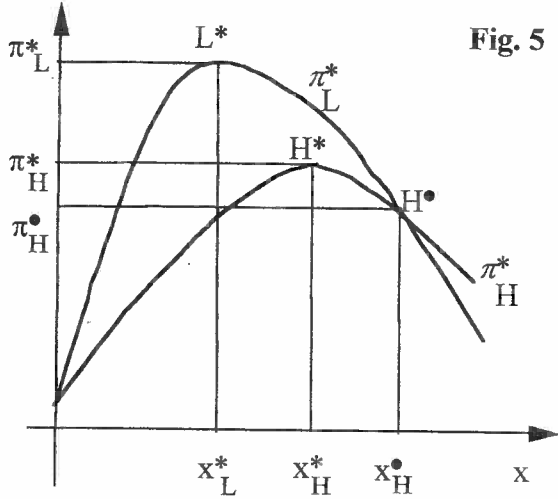


Fig. 5

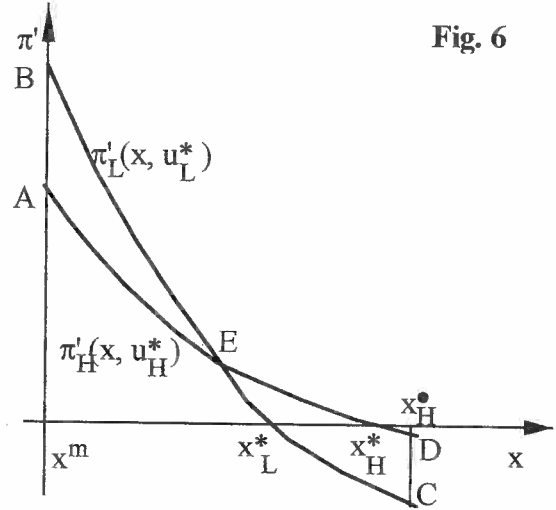


Fig. 6

While the L type marginal price stays equal to its marginal cost, the H type optimal discriminatory marginal price goes below $m'(x_H)$, as the quality level x_H (as well as the surplus P_H) is increased with respect to the efficient one x_H^* . As represented in fig. 5 above, it is optimal to set the high demand quality where the individual rationality constraint PC intersects, as a greater distortion would not increase profits. In fact, corresponding to x_H^\bullet the areas ABE and ECD are equal and no rent is left to the L type. Reducing distortion would not be optimal, since the cost of low demand customers' rents are greater than the marginal profits on the high demand customers.

Finally, if only IC_L and PC_H are binding, we get the new first order conditions.

$$[\pi_L^*] \quad \pi_{x_L} = p_{x_L} - m'(x_L) = 0$$

$$[\pi_H^0] \quad \pi_{x_H} = p_{x_H} - m'(x_H) = n_L U_L [u_{x_H}(x_H, \theta_L, Y_L - P_H) / u_{y_H}(x_H, \theta_L, Y_L - P_H) - p_{x_H}] / n_H$$

where $U_L = u_{y_H}(x_H, \theta_L, Y_L - P_H) / u_{y_L}(x_L, \theta_L, Y_L - P_L)$ and p_{x_H} is a weighted average between $m'(x_H)$ and $u'(x_H, \theta_L)$. The high type distortion ratio σ_{x_H} is set equal to the population ratio n_L/n_H times U_L , while σ_{x_L} is just equal to zero being $\pi_{x_L} = 0$. As before the firm charges marginal prices below marginal cost to high-demand consumers to prevent low-demand customers from buying the high-demand bundle.

Moreover, as shown in fig. 7 below, low-demand customers enjoy a positive net surplus $[P_L < u(x_L, \theta_L, Y_L - P_L)]$ -since their individual rationality constraint π_L^* is not binding- and due to the income effect are offered a higher quality x_H^0 . In fact, in fig. 8, increasing to x_H^0 the quality of the high demand customers reduce the low demand customers rents -from the area $EC'D'$ (equal to ABE) to ECD.

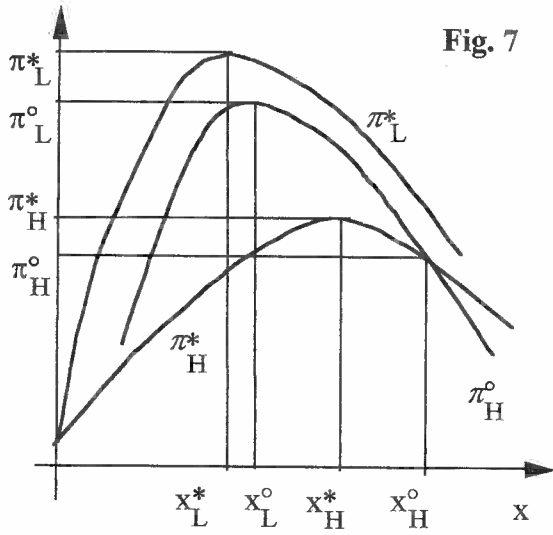


Fig. 7

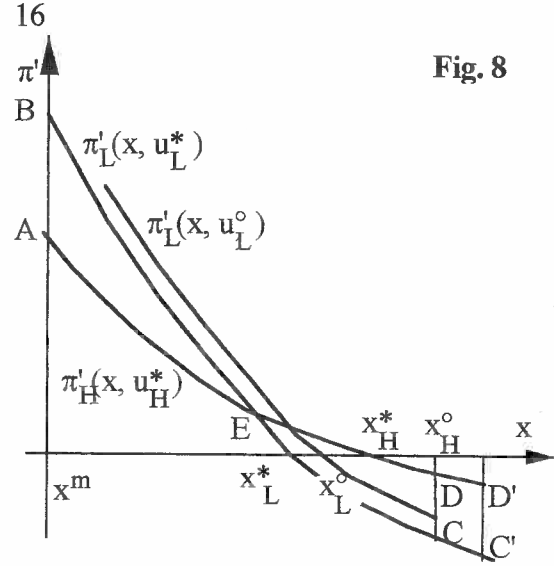


Fig. 8

Finally, if the high-demand consumers' demand disappears (since it is no longer profitable to serve them) and the firm serves only low-demand customers, all their surplus will be extracted by the firm. Let \hat{x}_H denote the level of quality necessary to extract all the surplus from low-demand consumers and assume $u(\hat{x}_H, \theta_H) - m(\hat{x}_H)$ to be negative. Clearly when the proportion of high-demand customers n_H is sufficiently low, x_H is sufficiently close to \hat{x}_H and $u(x_H, \theta_H)$ is strictly less than $m(x_H)$, so that it is no longer optimal for the monopolist to serve high-demand customers.

IV.B The non-distortionary case

Let us then proceed in the analysis of the case in which there is *no distortion* both at the top and at the bottom stating the following result.

Result 4

In the generalised model specified above we can get no distortion in the prices set for the two qualities if only the participation constraints are binding.

Substituting PC_L and PC_H -using of the two maximum per capita profit functions π_t^* - the profit function becomes:

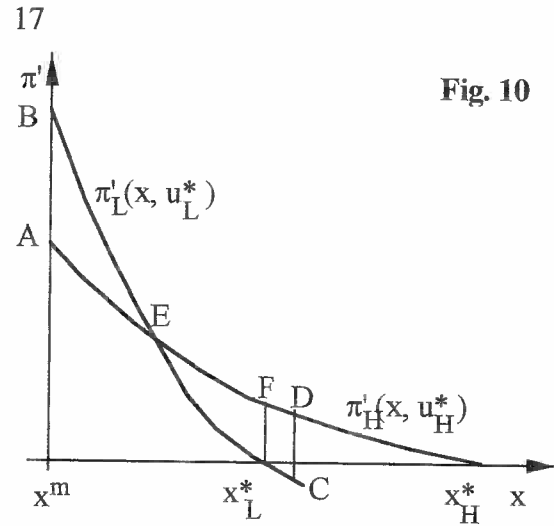
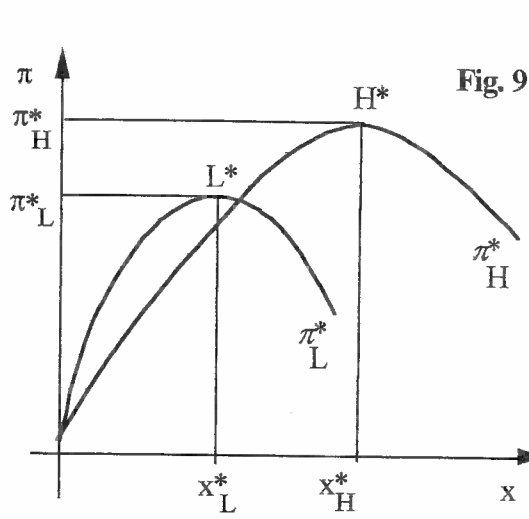
$$\Pi = n_L \pi(x_L, \theta_L, u_L^*) + n_H \pi(x_H, \theta_H, u_H^*)$$

and we can easily verify that in equilibrium marginal prices equal to marginal costs:

$$[\pi_L] \quad \pi_{x_L} = p_{x_L} - m'(x_L) = 0 \quad \text{No distortion at the bottom}$$

$$[\pi_H] \quad \pi_{x_H} = p_{x_H} - m'(x_H) = 0 \quad \text{No distortion at the top}$$

Hence marginal price p_{x_L} reaches marginal cost $m'(x_L)$ corresponding to the efficient quality x_L^* . In practice, while x_H remains constant, the distortion at the bottom disappears.



In fig. 10 above we represent in the profit space the participation constraints PC_t for each type π_t^* . Since for low and average qualities we have $x \leq x_L^*$ the surplus of the low type is greater than the one of type H the incentive constraint IC_H is not binding and the firm offers efficient quality levels to all customers attaining the maximum profit levels, i.e. leaving no rent to both types. In fig. 6, the marginal profit on the low demand type $\pi'_L(x, u_L^*)$ is steeper and the high-demand customer needs no longer incentives not to mimic the L type (BAE being equal to ECD and hence greater than Ex_L^*F). In this way, the monopolist can directly extract the entire rent (of low and high demand customers, i.e. respectively $A\pi^m x_L^*$ and $B\pi^m x_H^*$) with non-linear pricing, as in a first degree price discrimination setting.

So far, even within a very simple model we have derived non-standard cases, by relaxing the M-S condition and considering non standard binding constraints. In the next section we will generalise further the model allowing the consumer to buy more than one unit of the good.

V. The extension of the model to the multi-quantity case

Let us assume now that consumers can freely purchase any desired quantity q_t of the product of quality x_t and represent the preferences by the utility function $u(q_t, x_t, \theta_t, Y_t - P_t)$. In this new setting the previous *canonical preferences' assumptions* can be modified as follows.

(1') Utility increases purchasing more units, or products of higher quantity or at lower prices:

$$\begin{aligned}
 [A1'] \quad \partial u_t(\cdot) / \partial x_t &= u_x(q_t, x_t, \theta_t, Y_t - P_t) > 0 & \forall q_t, x_t, \theta_t \text{ and } y_t \\
 \partial u_t(\cdot) / \partial y_t &= u_y(q_t, x_t, \theta_t, Y_t - P_t) > 0 & \forall q_t, x_t, \theta_t \text{ and } y_t \\
 \partial u_t(\cdot) / \partial q_t &= u_q(q_t, x_t, \theta_t, Y_t - P_t) > 0 & \forall q_t, x_t, \theta_t \text{ and } y_t
 \end{aligned}$$

(2') The utility of one unit of lowest quality product x^m , charged $m(x^m)$, is greater than u_t^* .

$$[A2'] \quad u(1, x^m, \theta_t, Y_t - m(x^m)) > u_t^* \quad \forall Y_t \text{ and } \theta_t$$

(3') Consumers with a higher income have non-decreasing valuation of the same bundle:

$$[A3'] \quad \theta_t \geq \theta_s \text{ for } Y_t > Y_s \text{ and } \partial u_t(\cdot)/\partial \theta_t \geq 0 \quad \forall q_t, x_t, \theta_t \text{ and } y_t$$

(4') The *single crossing* or Spence-Mirrless condition implies also that "high" consumers gain more by purchasing not only products of higher quality but also more unit of a given product:

$$[A4'] \quad \frac{\partial}{\partial \theta_t} \left(\frac{\partial u_t(\cdot)/\partial q_t}{\partial u_t(\cdot)/\partial y_t} \right) > 0 \quad \forall q_t, x_t, \theta_t \text{ and } y_t$$

(5') Finally, the utility function is concave in all its arguments.

$$[A5'] \quad \begin{aligned} \partial^2 u_t(\cdot)/(\partial q_t)^2 &\leq 0 & \forall q_t, x_t, \theta_t \text{ and } y_t \\ \partial^2 u_t(\cdot)/(\partial x_t)^2 &\leq 0 & \forall q_t, x_t, \theta_t \text{ and } y_t \\ \partial^2 u_t(\cdot)/(\partial \theta_t)^2 &\leq 0 & \forall q_t, x_t, \theta_t \text{ and } y_t \\ \partial^2 u_t(\cdot)/(\partial y_t)^2 &\leq 0 & \forall q_t, x_t, \theta_t \text{ and } y_t \end{aligned}$$

As before, the monopolist maximises profits Π with respect to π_t and x_t subject to the purchasing conditions and incentive compatibility constraints (now related to contracts $\langle q_t, x_t, P_t \rangle$ which specify also the quantity offered at non-linear prices P_t):

$$\begin{aligned} \max \Pi &\equiv \sum_t n_t \pi_t & \text{subject to:} \\ [PC_t] &u(q_t, x_t, \theta_t, Y_t - q_t m(x_t) - \pi_t) \geq u_t^* & \forall t \\ [IC_t] &u(q_t, x_t, \theta_t, Y_t - q_t m(x_t) - \pi_t) \geq u(q_s, x_s, \theta_t, Y_t - q_s m(x_s) - \pi_s) & \forall t \text{ and } s \neq t \end{aligned}$$

In order to compare the results of the previous sections, (and for simplicity's sake), we will just examine the two types case (that is $t = L, H$). Following the canonical assumptions the constraints PC_L and IC_H are the only binding ones: therefore, as usual, the high customer always enjoys a positive surplus while the low type does not.

Result 5

In a model where consumers are allowed to buy more than one unit of the good and assumptions [A0] and [A1']-[A5'] hold, Result 1 is still valid so that the distortion in prices is exactly equal to the distortion in the quantities offered of the low quality good.

The first order conditions with respect to quality are the same as in section II.

Specifying the additional marginal tariffs as: $p_{q_L} = u_q(q_L, x_L, \theta_L, Y_L - P_L) / u_y(q_L, x_L, \theta_L, Y_L - P_L)$ and $p_{q_H} = u_q(q_H, x_H, \theta_H, Y_H - P_H) / u_y(q_H, x_H, \theta_H, Y_H - P_H)$, and denoting by $U_H = u_y(x_L, \theta_H, Y_H - P_L) / u_y(x_H, \theta_H, Y_H - P_H)$ we can write the first order condition as follows (see appendix C).

$$[\pi_{q_L}^o] \quad \pi_{q_L} = n_H U_H [u_q(q_L, x_L, \theta_H, Y_H - P_L) / u_y(q_L, x_L, \theta_H, Y_H - P_L) - p_{q_L}] / n_L$$

$$[\pi_{q_H}^*] \quad \pi_{q_H} = p_{q_H} - m(x_H) = 0$$

$$[\pi_{x_L}^o] \quad \pi_{x_L} = n_H U_H [u_x(q_L, x_L, \theta_H, Y_H - P_L) / u_y(q_L, x_L, \theta_H, Y_H - P_L) - p_{x_L}] / n_L$$

$$[\pi_{x_H}^*] \quad \pi_{x_H} = p_{x_H} - q_H m'(x_H) = 0$$

The first order conditions with respect to quantity q_t are the same as in the standard monopoly case: equation $[\pi_{q_H}^*]$ shows no *quantity distortion* at the top (since the high type distortion ratio $\sigma_{q_H} = \pi_{q_H} / [u_q(q_H, x_H, \theta_L, Y_L - P_H) / u_y(q_H, x_H, \theta_L, Y_L - P_H) - p_{q_H}]$ is equal to zero), while equation $[\pi_{q_L}^o]$ implies some *quantity distortion* at the bottom (the low type distortion ratio $\sigma_{q_L} = \pi_{q_L} / [u_q(q_L, x_L, \theta_H, Y_H - P_L) / u_y(q_L, x_L, \theta_H, Y_H - P_L) - p_{q_L}]$ being equal to $n_H U_H / n_L$). As in the previous model the marginal cost for quality p_{x_H} is equal the marginal cost $m(x_H)q_H$ (to supply q_H units of quality x_H) and p_{x_L} is just a weighted average between the marginal cost $m(x_L)q_L$ and the marginal evaluation of quality x_H by the high type. In this case too we have: no *quality distortion* at the top, as one can easily see from $[\pi_H^*]$, and some *quality distortion* at the bottom (as one can argue from $[\pi_L^o]$).

It seems interesting to notice how $p_{x_H} / q_H = \partial p_{q_H} / \partial x_H$ represents nothing else than the well known optimal choice of quality for the consumers of type H.

$$[x_H]' \quad p_{x_H} / q_H = [\partial u(q_H, x_H, \theta_H, Y_H - P_L) / \partial x_H] / q_H = \partial^2 u(q_H, x_H, \theta_H, Y_H - P_L) / \partial q_H \partial x_H = m'(x_H)$$

On the other hand, even within a *quasi linear* preference setting, for the low type $p_{x_L} / q_L = \partial p_{q_H} / \partial x_L + n_H [\partial^2 u(q_L, x_L, \theta_H) / \partial x_L \partial q_L - \partial u(q_L, x_L, \theta_H) / \partial x_L / q_L]$ and quality distortion can be decomposed in two components: (i) the one which is implied by the level of *quantity distortion* at the bottom and (ii) the *pure quality distortion*. Once we remove the single crossing condition, we can derive all the previous cases even in this more complex setting.

When also the high type participation constraint is binding we get an extension of the standard case, since the distortion at the bottom is no longer tied to the previous first order conditions. The optimal values q_L^* and x_L^* are found corresponding to the highest value of π_L^* for which the maximum per capita profit functions π_t^* intersect, i.e.:

$$u(q_H, x_H, \theta_H, Y_H - q_H m(x_H) - \pi_H) = u(q_L, x_L, \theta_H, Y_H - q_L m(x_L) - \pi(q_L, x_L, \theta_L, u_L^*)) = u_H^*$$

The first order condition for the low bundle implies, as in the previous case, an interesting equality between the quantitative and qualitative distortion ratios:

$$\begin{aligned}\sigma_{q_L} &= [p_{q_L} - m(x_L)]/[u_q(q_L, x_L, \theta_H, Y_H - P_L)/u_y(q_L, x_L, \theta_H, Y_H - P_L) - p_{q_L}] = \\ \sigma_{x_L} &= [p_{x_L} - q_L m'(x_L)]/[u_x(q_L, x_L, \theta_H, Y_H - P_L)/u_y(q_L, x_L, \theta_H, Y_H - P_L) - p_{x_L}]\end{aligned}$$

In words, these ratios express the difference between marginal tariffs and marginal costs of one type (here the low type) divided by the difference between marginal valuations of the bundle by the other type (i.e. the high type) and marginal tariffs. In practice, the marginal distortion on the low type's tariff [$\pi_{q_L} = p_{q_L} - m(x_L)$ and $\pi_{x_L} = p_{x_L} - q_L m'(x_L)$] are optimal when the ratio of marginal profits on low demand customers (associated to a marginal increase of the low type's quality or quantity levels) and marginal losses due to the marginal rent left to high type customers are equal to each other.

In the previous case (where PC_H is not binding) these ratios are both equal to $n_H U_H / n_L$ as we can easily see from conditions $[\pi_L^0]$ and $[\pi_H^*]$. On the other hand, when the high type's participation constraint is binding as well, the value of the quantitative and qualitative distortion ratios is reduced below this level. This shows again (if one was not yet convinced) how this case constitutes just an extension of the standard no distortion at the top.

The values of q_H and x_H are determined by maximising the high type maximum per capita profit function π_H^* with respect to x_H and q_H . Consequently, the first order conditions for type H (derived in Appendix C) are the same as in the case examined above (where the constraint PC_H was not binding), that is p_{x_H} is equal to $m'(x_H)$. We have no distortion at the top (since PC_L and IC_H are binding) but no surplus is left to the H type since the constraint PC_H is also binding.

Result 6

In the multi-quantity case we can derive the non standard cases specified in Results 3 and 4 under the same set of conditions.

In the *no distortion at the bottom* case, since the surplus of the low type is greater the one of the high type, he should be prevented from buying the high-demand customer's bundle. We get the usual *no distortion at the bottom* case when PC_L is no longer binding. Setting $U_L = u_y(x_H, \theta_L, Y_L - P_H)/u_y(x_L, \theta_L, Y_L - P_L)$ we get the new first order conditions:

$$[\pi_{q_L}^*] \quad \pi_{q_L} = p_{q_L} - m(x_L) = 0$$

$$[\pi_{q_H}^o] \quad \pi_{q_H} = n_L U_L [u_q(q_H, x_H, \theta_L, Y_L - P_H) / u_y(q_H, x_H, \theta_L, Y_L - P_H) - p_{q_H}] / n_H$$

$$[\pi_{x_L}^*] \quad \pi_{x_L} = p_{x_L} - q_L m'(x_L) = 0$$

$$[\pi_{x_H}^o] \quad \pi_{x_H} = n_L U_L [u_x(q_H, x_H, \theta_L, Y_L - P_H) / u_y(q_H, x_H, \theta_L, Y_L - P_H) - p_{x_H}] / n_H$$

Not only does the firm charge marginal prices below marginal cost to high-demand consumers to prevent low-demand customers from buying the high-demand bundle, but also low-demand customers enjoy a positive net surplus.

When constraints IC_L , PC_H and PC_L are binding the firm can extract all the surplus from low-demand customers. In this subcase quantity q_H^* and quality x_H^* are found in correspondence of the highest value of π_L^* for which the maximum per capita profit functions π_L^* intersect, i.e.:

$$u(q_L, x_L, \theta_L, Y_L - m(x_L) - \pi_L) = u(q_H, x_H, \theta_L, Y_L - m(x_H) - \pi(q_H, x_H, \theta_H, u_H^*)) = u_L^*$$

Specifically, the first order condition for the high bundle (derived in Appendix D) tells us that marginal distortions on the high customer's tariff [$\pi_{q_H} = p_{q_H} - m(x_H)$ and $\pi_{x_H} = p_{x_H} - q_H m'(x_H)$] are optimal when the ratio of marginal profits on high demand customers (associated to a marginal increase of the high type's quality or quantity levels) and marginal losses due to the marginal rent left to low demand customers are equal to each other.

$$\begin{aligned} \sigma_{q_H} &= [p_{q_H} - m(x_H)] / [u_q(q_H, x_H, \theta_L, Y_L - P_H) / u_y(q_H, x_H, \theta_L, Y_L - P_H) - p_{q_H}] = \\ \sigma_{x_H} &= [p_{x_H} - q_H m'(x_H)] / [u_x(q_H, x_H, \theta_L, Y_L - P_H) / u_y(q_H, x_H, \theta_L, Y_L - P_H) - p_{x_H}] \end{aligned}$$

The very same equality holds also in the previous case (where PC_L is not binding and the multiplier relative to this constraint is equal to zero, i.e. $\lambda_L' = 0$), with both ratios equal to $n_L U_L / n_H$ as it can be verified from conditions $[\pi_H^o]$ and $[\pi_L^*]$. In the extension of the no distortion at the bottom case (when the low type participation constraint is binding) the value of the quantitative and qualitative distortion ratios are reduced below the previous level.

The values of q_L and x_L are determined maximising the low type's maximum profit function π_L^* so that we get no distortion at the bottom, p_{q_L} being equal to $q_L m(x_L)$ and p_{x_L} to $m'(x_L)$. As in the previous section, in this case no surplus is left to both types, as the constraints PC_H and PC_L are binding as well.

As in the previous canonical model where quantity was kept constant there is *no*

distortion both at the top and at the bottom when only the two participation constraints are binding:

$$\Pi = n_L \pi(q_L, x_L, \theta_L, u_L^*) + n_H \pi(q_H, x_H, \theta_H, u_H^*)$$

and we can easily verify that in equilibrium marginal prices equal marginal costs and the marginal utility of quality equal its marginal cost. Therefore, as shown by the first order conditions specified below we get no quantity and quality distortion for all types of customers:

$$[\pi_{q_L}^*] \quad \pi_{q_L} = p_{q_L} - m(x_L) = 0$$

$$[\pi_{q_H}^*] \quad \pi_{q_H} = p_{q_H} - m(x_H) = 0$$

$$[\pi_{x_L}^*] \quad \pi_{x_L} = p_{x_L} - q_L m'(x_L) = 0$$

$$[\pi_{x_H}^*] \quad \pi_{x_H} = p_{x_H} - q_H m'(x_H) = 0$$

To conclude allowing interactions between quality and quantity discrimination our previous analysis can be easily extended, so that we get non-standard cases without recourse to ad hoc hypotheses.

VI. Concluding remarks on the relevance of non standard cases

The economic literature on vertical differentiation has kept canonical assumptions on consumers' preferences, closely related to the ones made in quantity discrimination models. As a consequence it treated quality as a "fictitious quantity", starting from the seminal paper of Mussa and Rosen. In this paper we have examined what drives the well known "no distortion at the top" result in a model where consumers differ in incomes and tastes and we modify the canonical assumptions on their preferences. Our analysis showed how this result can be derived without recourse to the no crossing (or Spence Mirrlees) condition.

By relaxing some of the canonical preferences' assumptions we also derived non standard distortionary cases. Specifically, in these cases either the monopolist is able to perfectly discriminate amongst buyers with different taste parameters (so that no distortion is needed), or the marginal price charged to high-demand customers differs from the marginal cost (i.e. there is distortion at the top).

Finally, allowing customers to consume *more than one* unit of the products, we have considered the interactions between quality *and* quantity. In general, the possibility to purchase more than a unit of the product considerably complicates the model, with relevant

implications on the optimal customers' bundles. Rather than attempting to provide a general taxonomy of all possible cases in what follows we conclude our analysis restricting attention to a simple example which strengthen the relevance of the non standard cases even in a very stylised model, where we endogenise product differentiation (expressed by a one dimensional variable).

Specifically, we sketch a relatively straightforward case in which the interactions between the quantity offered by the monopolist and his optimal vertical product differentiation choice give rise directly (in a second degree price discrimination setting) to cases different from the standard no distortion at the top. This alternative framework provided an additional justification for non standard cases in a vertical differentiation setting.

To simplify matters, we ignore interactions within the consumers' preference assuming a quasi linear separable utility function $u_t = V_t(q_t) + U_t(x_t) + Y_t - P_t$. Thus we write the utility functions in a multiplicative way as:

$$u_L = \theta v(q_L) + u(x_L) \quad \text{and} \quad u_H = v(q_H) + \theta u(x_H)$$

We normalise the measure of quality and quantity (e.g. setting $z_H^* = q_H^* = x_H^*$ and $z_L^* = q_L^* = x_L^*$) and assume that the marginal utility of quality increases more slowly than the marginal utility of quantity i.e. $v'(z) > u'(z)$. Moreover, these two functions are assumed to intersect for intermediate values, say the average between low and high quality (i.e. $z_A = q_A = x_A$). Clearly, as a consequence of the previous hypotheses we have: $v(q_H^*) > u(x_H^*)$ and $v(q_L^*) < u(x_L^*)$.

It is thus easy to show how when the quantity offered by the monopolist is set equal to q_L^* the case becomes similar to the one portrayed in fig. 1 (section II) where the high type must be given a rent in order to satisfy his binding incentive compatibility constraint. In fact, the high type reaches a higher level of utility choosing the low customer's bundle $u_H = v(q_L^*) + \theta u(x_H^*) \geq u_L = \theta v(q_L^*) + u(x_L^*)$, since $v(q_L^*)$ is less $u(x_L^*)$. On the other hand when the monopolist offers all customers the quantity q_H^* the graphical representation follows the lines of the case sketched in fig. 9 since now the low type that reaches a higher level of utility choosing the high customer's bundle, that is: $u_L = \theta v(q_H^*) + u(x_L^*) \geq u_H = v(q_H^*) + \theta u(x_H^*)$, since $v(q_H^*)$ is greater than $u(x_H^*)$. Now since IC_L is binding high-demand consumers' quality is increased to prevent low-demand customers from buying the high-demand bundle.

Naturally for the intermediate value z_A we have $v(q_A^*) < u(x_H^*)$ and $v(q_A^*) > u(x_L^*)$. In this

case the high type reaches a lower utility level choosing the low customer's bundle $u_H = v(q_A^*) + \theta u(x_H^*) < u_L = \theta v(q_A^*) + u(x_L^*)$ as well as the low type purchasing the high customer's bundle, that is: $u_L = \theta v(q_A^*) + u(x_L^*) < u_H = v(q_A^*) + \theta u(x_H^*)$. Hence, no distortion at the top and at the bottom occurs when only the purchasing constraints are binding. This is clearly also the case when the monopolist offers different quantities to the two customers, so that in the extended model the firm is able to perfectly discriminate customers.

The previous reasoning shows in a quite straightforward way how by varying of the quantity consumers purchase, even in the absence of internal interactions in the consumers' preference (i.e. assuming a quasi linear separable utility function), we can get the sequence of all the different cases examined previously in the benchmark case. In this way not only do we provide an additional justification for non standard cases in a one dimensional setting of vertical differentiation but also we show how the case in which consumption's choices are not distorted (i.e. there is no distortion both at the top and at the bottom) is likely to become the most relevant case once we go beyond the one dimensional quality differentiation models. Hence we believe it would be of great interest to examine further the interactions between quality and quantity discrimination and their impact on the pricing decisions of firms.

Appendix A: Derivation of the standard case

Substituting $\pi_t = P_t - m(x_t)$ we redefine the objective function that the multiquality monopolist maximises with respect to P_t and x_t and the constraints as:

$$\begin{aligned} \max \Pi &\equiv \sum_t n_t [P_t - m(x_t)] && \text{subject to:} \\ [\text{PC}_t] &u(x_t, \theta_t, Y_t - P_t) \geq u_t^* && \forall t \\ [\text{IC}_t] &u(x_t, \theta_t, Y_t - P_t) \geq u(x_s, \theta_t, Y_t - P_s) && \forall t \text{ and } s \neq t \end{aligned}$$

Once we define the Lagrangean function L where λ_L and λ_H are the Lagrangean multiplier of the two binding constraints PC_L and IC_H the profit function Π , can be easily maximised with respect to P_L , P_H , x_L and x_H .

$$L \equiv n_L P_L + n_H P_H - n_L m(x_L) - n_H m(x_H) + \lambda_L [u(x_L, \theta_L, Y_L - P_L) - u_L^*] + \lambda_H [u(x_H, \theta_H, Y_H - P_H) - u(x_L, \theta_H, Y_H - P_L)]$$

The first order condition are:

$$[\mathbf{x}_L] \quad -n_L m'(x_L) + \lambda_L u_x(x_L, \theta_L, Y_L - P_L) - \lambda_H u_x(x_L, \theta_H, Y_H - P_L) = 0$$

$$[\mathbf{x}_H] \quad -n_H m'(x_H) + \lambda_H u_x(x_H, \theta_H, Y_H - P_H) = 0$$

$$[\mathbf{P}_L] \quad n_L - \lambda_L u_y(x_L, \theta_L, Y_L - P_L) + \lambda_H u_y(x_L, \theta_H, Y_H - P_L) = 0$$

$$[\mathbf{P}_H] \quad n_H + \lambda_H u_y(x_H, \theta_H, Y_H - P_H) = 0$$

We can find the value of the multipliers from $[\mathbf{P}_H]$ and $[\mathbf{P}_L]$ $\lambda_H = n_H / u_y(x_H, \theta_H, Y_H - P_H)$ and $\lambda_L = (n_L + n_H U_H) / u_y(x_L, \theta_L, Y_L - P_L)$. Substituting λ_H and λ_L in $[\mathbf{x}_H]$ and $[\mathbf{x}_L]$ we can verify that the first order conditions are the same as in the standard monopoly case for quantity.

$$[\mathbf{x}_L^0] \quad p_{x_L} = [n_L m'(x_L) + n_H U_H u_x(x_L, \theta_H, Y_H - P_L) / u_y(x_L, \theta_H, Y_H - P_L)] / (n_L + n_H U_H)$$

$$[\mathbf{x}_H^*] \quad p_{x_H} = m'(x_H)$$

Notice how the marginal tariffs p_{x_t} must satisfy the binding constraints specified above, and U_H is just the ratio between the marginal valuation of net incomes by the H type (and hence represents the income effect):

$$p_{x_L} = u_x(x_L, \theta_L, Y_L - P_L) / u_y(x_L, \theta_L, Y_L - P_L) \text{ and } p_{x_H} = u_x(x_H, \theta_H, Y_H - P_H) / u_y(x_H, \theta_H, Y_H - P_H)$$

$$U_H = u_y(x_L, \theta_H, Y_H - P_L) / u_y(x_H, \theta_H, Y_H - P_H)$$

Specifically, the first order conditions tell us that p_{x_L} is just a weighted average between the marginal cost of the low quality $m'(x_L)$ and its marginal evaluation by the high type $u'(x_L, \theta_H)$, whereas p_{x_H} is set equal to the marginal cost of high quality $m'(x_H)$. We thus obtained the usual case of no distortion at the top since PC_L and IC_H are the only binding constraints.

Appendix B: Derivation of the no distortion at the bottom case

When PC_H and IC_L are the only the only binding constraints we define the Lagrangean function as:

$$L \equiv n_H P_H + n_L P_L - n_H m(x_H) - n_L m(x_L) + \lambda_H [u(x_H, \theta_H, Y_H - P_H) - u_H^*] + \lambda_L [u(x_L, \theta_L, Y_L - P_L) - u(x_H, \theta_L, Y_L - P_H)]$$

obtaining the following first order conditions:

$$[x_L] \quad -n_L m'(x_L) + \lambda_L u_x(x_L, \theta_L, Y_L - P_L) = 0$$

$$[x_H] \quad -n_H m'(x_H) + \lambda_H u_x(x_H, \theta_H, Y_H - P_H) - \lambda_L u_x(x_H, \theta_L, Y_L - P_H) = 0$$

$$[P_L] \quad n_L + \lambda_L u_y(x_L, \theta_L, Y_L - P_L) = 0$$

$$[P_H] \quad n_H - \lambda_H u_y(x_H, \theta_H, Y_H - P_H) + \lambda_L u_y(x_H, \theta_L, Y_L - P_H) = 0$$

We can find the value of the multipliers from $[P_L]$ and $[P_H]$ $\lambda_L = n_L / u_y(x_L, \theta_L, Y_L - P_L)$ and $\lambda_H = (n_H + n_L U_L) / u_y(x_H, \theta_H, Y_H - P_H)$. Substituting λ_L and λ_H in $[x_L]$ and $[x_H]$ we can verify that the first order conditions are the same as in the standard monopoly case for quantity:

$$[x_L^*] \quad p_{x_L} = m'(x_L)$$

$$[x_H^o] \quad p_{x_H} = [n_H m'(x_H) + n_L U_L u_x(x_H, \theta_L, Y_L - P_H) / u_y(x_H, \theta_L, Y_L - P_H)] / (n_H + n_L U_L)$$

Specifically, p_{x_L} is set equal to the marginal cost of low quality $m'(x_L)$, whereas p_{x_H} is just a weighted average between the marginal cost of the high quality $m'(x_H)$ and its marginal evaluation by the low type $u'(x_H, \theta_L)$. We thus obtained the case of no distortion at the bottom since PC_H and IC_L are the only binding constraints.

Appendix C: No distortion at the top in the multiquantity purchase model

Substituting $\pi_t = P_t - q_t m(x_t)$ we redefine the maximisation problem as:

$$\begin{aligned} \max \Pi &\equiv \sum_t n_t [P_t - q_t m(x_t)] && \text{subject to:} \\ [PC_t] &u(q_t, x_t, \theta_t, Y_t - P_t) \geq u_t^* && \forall t \\ [IC_t] &u(q_t, x_t, \theta_t, Y_t - P_t) \geq u(q_s, x_s, \theta_t, Y_t - P_s) && \forall t \text{ and } s \neq t \end{aligned}$$

Setting $\underline{x}_t = (q_t, x_t)$, $y'_H = Y_H - P_L$ and denoting by λ_L , λ_H and λ'_H the Lagrangean multiplier of the constraints PC_L , IC_H and PC_H (with $\lambda'_H = 0$ in the usual case) we define the Lagrangean as:

$$\begin{aligned} L &\equiv n_L P_L + n_H P_H - n_L q_L m(x_L) - n_H q_H m(x_H) + \lambda_L [u(\underline{x}_L, \theta_L, y'_L) - u_L^*] + \lambda_H [u(\underline{x}_H, \theta_H, y_H) - u(\underline{x}_L, \theta_H, y'_L)] \\ &\quad + \lambda'_H [u(\underline{x}_H, \theta_H, y_H) - u_H^*] \end{aligned}$$

The new first order conditions are:

$$\begin{aligned}
[q_L] \quad & -n_L m(x_L) + \lambda_L u_q(q_L, x_L, \theta_L, Y_L - P_L) - \lambda_H u_q(q_L, x_L, \theta_H, Y_H - P_L) = 0 \\
[q_H] \quad & -n_H m(x_H) + \lambda_H u_q(q_H, x_H, \theta_H, Y_H - P_H) + \lambda'_H u_q(q_H, x_H, \theta_H, Y_H - P_H) = 0 \\
[x_L] \quad & -n_L m'(x_L) + \lambda_L u_x(q_L, x_L, \theta_L, Y_L - P_L) - \lambda_H u_x(q_L, x_L, \theta_H, Y_H - P_L) = 0 \\
[x_H] \quad & -n_H m'(x_H) + \lambda_H u_x(q_H, x_H, \theta_H, Y_H - P_H) + \lambda'_H u_x(q_H, x_H, \theta_H, Y_H - P_H) = 0 \\
[P_L] \quad & n_L - \lambda_L u_y(q_L, x_L, \theta_L, Y_L - P_L) - \lambda_H u_y(q_L, x_L, \theta_H, Y_H - P_L) = 0 \\
[P_H] \quad & n_H - \lambda_H u_y(q_H, x_H, \theta_H, Y_H - P_H) - \lambda'_H u_y(q_H, x_H, \theta_H, Y_H - P_H) = 0
\end{aligned}$$

We drop the multipliers λ_H and λ'_H from $[P_H]$, $[x_H]$ and $[q_H]$ equating the values of $\lambda_H + \lambda'_H$ obtaining the following first order condition independently of the positive value of λ'_H .

$$\begin{aligned}
[q_H^*] \quad & p_{q_H} = m(x_H) \\
[x_H^*] \quad & p_{x_H} = q_H m'(x_H)
\end{aligned}$$

From conditions $[P_L]$, $[x_L]$ and $[q_L]$ we eliminate λ_L and λ_H getting the equality between the quantitative and qualitative distortion ratios.

$$\begin{aligned}
& [m(x_L) - u_q(\underline{x}_L, \theta_L, y_L) / u_y(\underline{x}_L, \theta_L, y_L)] / [u_q(\underline{x}_L, \theta_L, y_L) / u_y(\underline{x}_L, \theta_L, y_L) - u_q(\underline{x}_L, \theta_H, y'_H) / u_y(\underline{x}_L, \theta_H, y'_H)] = \\
& [q_L m'(x_L) - u_x(\underline{x}_L, \theta_L, y_L) / u_y(\underline{x}_L, \theta_L, y_L)] / [u_x(\underline{x}_L, \theta_L, y_L) / u_y(\underline{x}_L, \theta_L, y_L) - u_x(\underline{x}_L, \theta_H, y'_H) / u_y(\underline{x}_L, \theta_H, y'_H)]
\end{aligned}$$

When PC_L is not binding and $\lambda'_L = 0$ these ratios are equal to $n_H U_H / n_L$ and we get:

$$\begin{aligned}
[q_L^0] \quad & p_{q_L} = [n_L m(x_L) + n_H U_H u_q(q_L, x_L, \theta_H, Y_H - P_L) / u_y(q_L, x_L, \theta_H, Y_H - P_L) - m(x_L)] / (n_L + n_H U_H) \\
[x_L^0] \quad & p_{x_L} = [n_L q_L m'(x_L) + n_H U_H u_x(q_L, x_L, \theta_H, Y_H - P_L) / u_y(q_L, x_L, \theta_H, Y_H - P_L)] / (n_L + n_H U_H)
\end{aligned}$$

Appendix D: No distortion at the bottom in the multi-quantity purchase model

Setting $\underline{x}_t = (q_t, x_t)$, $y'_L = Y_L - P_H$ and denoting by λ_H , λ_L and λ'_L the Lagrangean multiplier of constraints PC_H , IC_L and PC_L (with $\lambda'_L = 0$ in the usual case) we define the Lagrangean as:

$$L \equiv n_L P_L + n_H P_H - n_L q_L m(x_L) - n_H q_H m(x_H) + \lambda_L [u(\underline{x}_L, \theta_L, y_L) - u^*] + \lambda_H [u(\underline{x}_H, \theta_H, y_H) - u(\underline{x}_L, \theta_H, y'_L)]$$

When the two binding constraints are PC_H and IC_L the Lagrangean function becomes:

$$\begin{aligned}
L \equiv & n_H P_H + n_L P_L - n_H q_L m(x_H) - n_L q_L m(x_L) + \lambda_H [u(\underline{x}_H, \theta_H, y_H) - u^*] + \lambda_L [u(\underline{x}_L, \theta_L, y_L) - u(\underline{x}_H, \theta_L, y'_L)] \\
& + \lambda'_L [u(\underline{x}_L, \theta_L, y_L) - u^*]
\end{aligned}$$

obtaining the following first order conditions:

$$\begin{aligned}
[q_L] \quad & -n_L m(x_L) + \lambda_L u_q(q_L, x_L, \theta_L, Y_L - P_L) + \lambda'_L u_q(q_L, x_L, \theta_L, Y_L - P_L) = 0 \\
[q_H] \quad & -n_H m(x_H) + \lambda_H u_q(q_H, x_H, \theta_H, Y_H - P_H) - \lambda_L u_q(q_H, x_H, \theta_L, Y_L - P_H) = 0
\end{aligned}$$

$$[x_L] \quad -n_L m'(x_L) + \lambda_L u_x(q_L, x_L, \theta_L, Y_L - P_L) + \lambda'_L u_x(q_L, x_L, \theta_L, Y_L - P_L) = 0$$

$$[x_H] \quad -n_H m'(x_H) + \lambda_H u_x(q_H, x_H, \theta_H, Y_H - P_H) - \lambda_L u_x(q_H, x_H, \theta_L, Y_L - P_H) = 0$$

$$[P_L] \quad n_L - \lambda_L u_y(q_L, x_L, \theta_L, Y_L - P_L) - \lambda'_L u_y(q_L, x_L, \theta_L, Y_L - P_L) = 0$$

$$[P_H] \quad n_H - \lambda_H u_y(q_H, x_H, \theta_H, Y_H - P_H) - \lambda_L u_y(q_H, x_H, \theta_L, Y_L - P_H) = 0$$

We eliminate the multipliers λ_L and λ'_L from $[P_L]$, $[x_L]$ and $[q_L]$ equating the values of $\lambda_L + \lambda'_L$ obtaining the following first order condition independently of the positive value of λ'_L .

$$[q_L^*] \quad p_{q_L} = m(x_L)$$

$$[x_L^*] \quad p_{x_L} = q_L m'(x_L)$$

Applying the same procedure as in Appendix C from conditions $[P_H]$, $[x_H]$ and $[q_H]$ we eliminate λ_H and λ_L getting the equality between the quantitative and qualitative distortion ratios:

$$\begin{aligned} & [u_q(\underline{x}_H, \theta_H, y_H) / u_y(\underline{x}_H, \theta_H, y_H) - m(x_H)] / [u_q(\underline{x}_H, \theta_H, y_H) / u_y(\underline{x}_H, \theta_H, y_H) - u_q(\underline{x}_H, \theta_L, y'_L) / u_y(\underline{x}_H, \theta_L, y'_L)] = \\ & = [u_x(\underline{x}_H, \theta_H, y_H) / u_y(\underline{x}_H, \theta_H, y_H) - q_H m'(x_H)] / [u_x(\underline{x}_H, \theta_H, y_H) / u_y(\underline{x}_H, \theta_H, y_H) - u_x(\underline{x}_H, \theta_L, y'_L) / u_y(\underline{x}_H, \theta_L, y'_L)] \end{aligned}$$

When PC_L is not binding and $\lambda'_L = 0$ these ratios are equal to $n_L U_L / n_H$ and we get:

$$[q_H^0] \quad p_{q_H} = [n_H m(x_H) + n_L U_L u_q(q_H, x_H, \theta_L, Y_L - P_H) / u_y(q_H, x_H, \theta_L, Y_L - P_H) - m(x_H)] / (n_H + n_L U_L)$$

$$[x_H^0] \quad p_{x_H} = [n_H q_H m'(x_H) + n_L U_L u_x(q_H, x_H, \theta_L, Y_L - P_H) / u_y(q_H, x_H, \theta_L, Y_L - P_H)] / (n_H + n_L U_L)$$

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