

**SOCIAL SECURITY, THE GOLDEN RULE AND THE OPTIMAL ALLOCATION  
OF RESOURCES: THE CASE OF ENDOGENOUS RETIREMENT AND  
A STRATEGIC BEQUEST MOTIVE**

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No.473

**WARWICK ECONOMIC RESEARCH PAPERS**



DEPARTMENT OF ECONOMICS

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should be considered preliminary.

**Social security, the Golden Rule and the optimal allocation of resources :**  
**The case of endogenous retirement and a strategic bequest motive**

Graham Aylott  
University of Warwick  
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Abstract

Following the seminal work of Samuelson (1975), a theoretical literature has grown examining the macroeconomic relationship between social security, aggregate saving and the allocation of resources within an overlapping generations economy. One such paper by Hu (1979) suggests *inter alia* that a “pay-as-you-go” (PAYG) pension scheme cannot secure the optimal allocation of resources in the presence of either endogenous retirement or a bequest motive. This paper aims to extend the analysis of this issue, by showing that a suitably designed *two tier* PAYG pension scheme can in fact secure the first best outcome in the presence of endogenous retirement, provided either that no bequest motive is present, or that it takes the strategic form introduced by Bernheim et al. (1984, 1985). Under such a scheme the total pension benefit paid is positively related to the working lifetime of the individual.

Address for correspondence :  
Department of Economics  
University of Warwick  
Coventry CV4 7AL  
United Kingdom  
E-mail : g.j.aylott@warwick.ac.uk

I would like to thank Martin Cripps, Clive Fraser and, in particular, Norman Ireland for their helpful guidance and comments, and also the ESRC for financial support.

## 1. Introduction

General reasoning suggests the possibility of a trade-off between (aggregate) current consumption and the investment required to sustain and indeed enhance future productive capacity, and hence future consumption. This trade-off is formally encapsulated within the "Golden Rule", which broadly defines the optimal level of capital for a dynamic economy to be that which maximises the steady state lifetime utility of each generation. A theoretical literature has accordingly arisen which investigates both the feasibility of obtaining this optimum in particular, and the more general issue of the efficiency of aggregate resource allocation within the dynamic economy.

One means by which the optimal level of the capital stock might be secured, and hence dynamic inefficiency eliminated, is via the introduction of a compulsory social security scheme - though there are, of course, many other motives for its introduction. More generally, an unfunded or "pay-as-you-go" (PAYG) pension scheme (which we refer to hereafter as simply social security) can be used to influence the level of saving, and hence the aggregate capital stock, within the economy.

An important aspect of the analysis of the relationship between social security and dynamic efficiency therefore focuses on the feasibility or otherwise of thus securing the optimal or Golden Rule level of the capital-labour ratio. It is generally conducted within a simple two sector, two period overlapping generations (closed) economy, with the emphasis being placed upon the steady state. Amongst the papers in this field - a number of which are reviewed by Atkinson (1987), Blanchard and Fischer (1989), Felderer (1993) and Myles (1995) - the seminal work is perhaps that by Samuelson (1975).

Samuelson characterises the optimal (steady state) social security programme, and establishes that a *less than fully funded* state pension scheme can be used to manipulate the level of aggregate saving, and hence capital accumulation, in the economy, thereby securing attainment of the Golden Rule capital-labour ratio where it would otherwise not prevail. The "second best" policy instrument provided by social security can thus secure the "first best" outcome. Indeed, he finds a continuum of such schemes, characterised by appropriate combinations of the social security tax rate and the social security capital per unit of labour,  $k^s$  (if  $k^s = 0$ , then the scheme is PAYG).

This result arises because the introduction of a social security programme reduces the necessity for individuals to save for their retirement. It can therefore be used to reduce aggregate saving to the Golden Rule level in an otherwise decentralised economy - though a Pareto improvement can, of course, be effected only if the initial state is one of dynamic inefficiency (i.e. too much and *not* too little capital). In this regard, a fully funded scheme has a neutral effect only upon aggregate saving, as it merely displaces private saving on a "one for one" basis - assuming that the contributions to it do not exceed the level of saving that would otherwise occur. Consequently it cannot restore dynamic efficiency to an inefficient economy.

Samuelson's paper has formed the basis of much subsequent work, with later authors investigating the impact of relaxing various of the assumptions - both explicit and implicit - made therein. Amongst the more noteworthy of these are the absence of both altruism and myopia, and the assumptions of certainty (with respect to output and the length of lifetimes), a steady state, closed economy and an inelastic or exogenous supply of labour.

The last of these assumptions relates to the exogeneity of the labour supply, both within the working lifetime and also with respect to the retirement decision itself. The potential disincentive effects of social security and the associated taxes, and the consequent static inefficiencies thereby introduced into the economy, are thus ignored in Samuelson's model - as indeed is the resulting impact upon the equilibrium level of the aggregate capital-labour ratio, with which we are concerned here.

Amongst the subsequent theoretical papers<sup>1</sup> which allow for an endogenous supply of labour is that by Hu (1979), who investigates the impact of a compulsory PAYG pension scheme upon the supply of labour and the accumulation of capital. In addition to a "terminal consumption" or "bequest for their own sake" bequest motive, Hu's model allows for the possibility of endogenous retirement, and it is via its impact on this latter, that the introduction of a PAYG pension scheme is shown to influence the supply of labour. Similar continuous time models are developed by Hu (1978) and Sheshinski (1978).

Hu (1979) also considers the welfare implications of a PAYG pension scheme. He demonstrates that in his model of an otherwise decentralised economy, although social security can be used to secure the Golden Rule allocation of resources, in the presence of either endogenous retirement *or* a terminal consumption type bequest motive, it is not desirable to do so. Consequently, the "second best" (and non Golden Rule) outcome - that yielded by the "optimal" PAYG pension scheme - is suboptimal (as compared to the "first best" outcome attainable in a command economy).

This result reflects two separate factors. The first of these is the distortion of the individual's retirement decision induced by the PAYG pension scheme, and the consequent static inefficiency introduced into the economy. This distortion in turn arises from the reduction in the effective "cost" of retirement caused by the combined effect of the (non lump sum) social security tax and benefit programme. Additionally, the terminal consumption specification of the bequest motive assumed by Hu, whereby the level of bequests granted enters directly into the individual's objective function, introduces an intergenerational externality such that the individual's optimal bequest choice differs from the socially optimal outcome. It turns out that the optimal, second best PAYG pension scheme is unable to correct for this externality.

In contrast to Hu's result, this paper demonstrates the existence of a class of optimal (non lump sum) PAYG pension schemes that do not distort the retirement decision. A consequence of this is that the second best outcome thereby induced implies the Golden Rule allocation of resources. Moreover, in the absence of a bequest motive, this second best outcome is identical to the first best outcome. Alternatively, if a "strategic" bequest motive of the form introduced by Bernheim et al. (1984, 1985) operates, then this same class of PAYG pension schemes can likewise secure attainment of the first best outcome (including the Golden Rule allocation) in an otherwise decentralised economy.

Section two of the paper therefore outlines the model employed, before the main result of the paper is developed in section three. Section four concludes the paper. Note that throughout this paper we are concerned specifically with PAYG pension schemes based upon *non lump sum* taxes and benefits.

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<sup>1</sup> Reviews of the literature discussing the link between social security and retirement behaviour are provided by Danziger et al. (1981), Aaron (1982) and Lazear (1986).

## 2. The basic model

### 2.1 The strategic bequest motive

In this section we outline a variation upon Hu's (1979) model, based upon a slightly modified version thereof which appears in Myles (1995, pages 452-55). This models a deterministic, steady state and closed economy, in which the government implicitly enjoys access to complete information. In doing so, however, we amend Hu's model to allow for a strategic bequest motive, as developed by Bernheim et al. (1984, 1985), and later applied by Cremer et al. (1992). Our analysis in fact more closely follows that of the latter authors, who likewise consider a model without altruism. An important consequence of this change is to remove the intergenerational externality associated with the terminal consumption bequest motive, assumed by Hu, with the result that (second best) PAYG social security *can* be used to secure the first best allocation of resources. Note that because we deal predominantly with steady state situations, we eschew time and generational subscripts wherever it is possible to do so without loss of clarity.

The "strategic" bequest motive developed by Bernheim et al. (1984, 1985) is one of a number of competing bequest motives discussed in the literature<sup>2</sup>. According to this hypothesis, aged parents seek to manipulate the behaviour of their children with respect to the provision of care and attention, via the threat of disinheritance (should they not comply). In practice, no one formulation seems to offer an entirely convincing explanation of bequest behaviour, and we might expect a combination of motives to be present. Against this background, Bernheim and his co-authors discuss evidence which casts doubt upon some of the other contenders, whilst being not inconsistent with their own formulation, thereby suggesting that the latter may have an important role to play.

The strategic bequest game that Bernheim et al. describe is essentially a variation upon the classic principal-agent theme, in which the bequeather or parent is the principal, who tries to induce a particular action (level of attention) from the beneficiary or child, as agent. By designing an appropriate bequest rule, the bequeather thus tries to maximise his<sup>3</sup> utility, which depends on the level of attention received in old age, subject both to :

- i. A participation constraint - namely that each beneficiary / child chooses to play the bequest game.
- ii. An incentive compatibility constraint - such that the level of attention provided represents the optimal choice of the beneficiary / child.

The former constraint requires that the beneficiary receives at least as high a level of utility from playing the bequest game, than can be obtained from not doing so (and consequently being disinherited). The latter requires that the bequest rule be chosen so that given participation, the beneficiaries' utility level is maximised by choosing the level of attention preferred by the bequeather. In practice it is the former constraint that is binding as, given the stationary nature of the game, the action that maximises the utility of the principal in the current period, is also that which maximises that of the agents, each of whom is himself the principal in the following period. A consequence of this latter property is that the bequeather as principal effectively (albeit indirectly) chooses the beneficiaries' actions via his choice of bequest rule.

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<sup>2</sup> See, for example, Feldstein (1988).

<sup>3</sup> In order to simplify presentation, we assume throughout that all individuals are male.

Given this scenario, Bernheim et al. show that non strategic bequest behaviour (whereby bequests are independent of the actions of the beneficiaries) generally induces a suboptimal outcome, compared to which strategic behaviour can effect a Pareto improvement - the inefficiency arising from the externality associated with the action of the beneficiary, which appears in the utility function of both bequeather and beneficiary. Additionally, the bequeather is able to devise a bequest rule, which allocates a given level of total bequests between beneficiaries in such a way as to appropriate for himself the full value of the rent thus generated. This is because the bequeather can, as a last resort, threaten each potential beneficiary with disinheritance, should they fail to take the preferred action. Hence, provided that each beneficiary receives at least the minimal level of utility associated with disinheritance, then he participates in the bequest game, and any surplus is enjoyed by the bequeather.

## 2.2 The household sector

Hu considers a standard overlapping generations economy, in which two generations (young and old) of otherwise identical individuals coexist at any time. In marked contrast to most other similar models, however, he does not assume a fixed date of retirement for all individuals. Instead, they are assumed to both work and possibly care for their retired parent(s) throughout the first or "young" period of life, after which they choose when to retire. The retirement decision is thus endogenised within the model, and the individual's lifetime utility function,  $U$ , is consequently augmented to include a variable ( $\alpha$ ) representing the proportion of the second period spent in retirement. It is via their influence on this retirement decision that socially provided pensions are assumed to affect the labour supply.

Hu also considers their impact on the accumulation of capital. In his model, there are two sources of capital for production purposes, namely :

- i. saving by the young generation to provide for their old age (in addition to both any income from earnings when old, and a pension).
- ii. bequests from the "old" generation to their children upon death.

The first of these is motivated by the inclusion in  $U$  of a term reflecting consumption in old age ( $c_2$ ). The second reflects the (strategic) bequest motive discussed above, in which bequests are employed by parents as a means of persuading their children to take a particular action, which, following the previous authors, we can interpret as filial attention - looking after the parent in their old age. Hence, the lifetime utility function of a representative individual (of which each generation is assumed to consist) is therefore of the form :

$$U = U(c_1, c_2, \alpha, y') \quad (1)$$

where :

$c_1$	=	consumption when young
$c_2$	=	consumption when old <sup>4</sup>
$\alpha$	=	fraction of period when old spent in retirement
$y'$	=	filial attention received when old

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<sup>4</sup> This refers strictly to *expected* consumption when old (in the next period) and hence  $U$  strictly represents an expected lifetime utility function.

We make the usual concavity assumptions implied by the Inada conditions (namely that  $\forall$  arguments  $i$ ,  $U_i > 0$ ,  $U_{ii} < 0$ ,  $U_i \rightarrow 0$  as  $i \rightarrow \infty$  and  $U_i \rightarrow \infty$  as  $i \rightarrow 0$ ).

Each young individual is assumed to provide attention to his parent(s) for a proportion  $y$  of the first period of life and correspondingly to work for the remaining  $1-y$  of that period, earning income at the wage rate  $w$ , and paying tax at the *rate* of  $T$  per period to finance the PAYG pension scheme - i.e. the pension payments to the then retired. The scheme considered therefore involves strictly *non* lump sum taxes. His net income when young is divided between consumption  $c_1$  and saving  $s$ , which is carried over to the second period of life, and in doing so earns interest at the rate  $r$  (discussed in §2.3). At the end of this period, he also receives an inheritance,  $h$ , on the death of his parent (see (4) below). When old, the individual chooses to work for a fraction  $(1-\alpha)$  of the second period of life, again earning income at the wage rate  $w$ , and paying tax on earnings at rate  $T$ . He then retires for the remaining fraction,  $\alpha$ , of the latter period of life, during which a total (non lump sum) pension of  $P(\alpha,y,T)$  is received. At the end of this period he dies leaving total bequests of  $b$  to his children.

The budget constraints faced in each period of life are therefore, in the presence of social security :

$$\begin{aligned} c_1 &= (1-y).(w-T) - s \\ c_2 &= (h+s).(1+r) + (1-\alpha).(w-T) + P(\alpha,y,T) - b \end{aligned} \quad (2)$$

where :

$$y = \text{filial attention provided when young}$$

An important point to note is the dependence of the PAYG pension benefit on the total period of employment and hence on the length of retirement (and of course the social security tax  $T$ ), a feature to which we return in §2.4. Also, assuming that each child provides an equal share of the attention enjoyed by his parent :

$$y' = (1+n).y, \quad (3)$$

where  $n$  (which is positive and for analytical purposes continuous) is the percentage rate of population growth between generations. Hence, in any period the younger generation is  $1+n$  times larger than the older one<sup>5</sup>. One consequence of this is that the bequest of each member of generation  $t$  say, will be divided (we assume equally) amongst the corresponding  $1+n$  members of generation  $t+1$ . Hence :

$$h = b/(1+n) \quad (4)$$

The aim of each individual is thus to maximise lifetime utility as represented by (1) subject to the income constraints (2). In so doing :

- i. His choice variables are saving  $s$ , length of retirement  $\alpha$ , total amount of bequests  $b$  to leave and level of attention provided  $y$ .
- ii. He takes as given  $w$  and  $r$ , both of which are determined elsewhere in the model (see the following subsection),  $h$ ,  $T$ , and the benefit structure of the PAYG scheme  $P(\alpha,y,T)$ .

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<sup>5</sup> Note that operation of the strategic bequest motive requires that each parent has more than one "credible beneficiary", if he is to credibly threaten each with disinheritance.

### 2.3 The Production Sector

As is typical in such models, a competitive economy is assumed in which the total output in each period,  $F$ , depends upon the total inputs of capital,  $K$ , and labour,  $L$  in that period, so that :

$$F = F(K, L) \quad (5a)$$

Assuming that  $F(K,L)$  satisfies constant returns to scale and hence is homogeneous of degree one, it can be manipulated to give :

$$f = f(k, 1) = f(k) \quad (5b)$$

where :

$$k = K/L,$$

the capital-labour ratio. In addition, we make the usual concavity assumptions :  $f'(k) > 0$ ,  $f''(k) < 0$ . Hence, the equilibrium wage rate and rate of return on capital are equal to the respective marginal products of labour and capital :

$$r = f'(k) \quad (6a)$$

$$w = f(k) - f'(k).k \quad (6b)$$

As is customary, a one-good economy is assumed. In any period, the output of this good is either consumed or saved - as either private saving ( $s$ ) by the young, or bequests ( $b$ ) by the old - thereby forming the capital stock available for productive use in the following period - when it is "consumed" as part of the production process. Hence, for all periods  $t$  and  $t+1$  the following "capital constraint" applies :

$$K(t+1) = s(t).(1+n).N(t) + b(t).N(t) \quad (7)$$

where :

$K(t+1)$	=	aggregate capital stock in period $t+1$
$s(t)$	=	individual saving (by young) in period $t$
$N(t)$	=	elderly population in period $t$ .
$b(t)$	=	individual bequests (by old) at end of period $t$

The right hand side of (7) thus represents the total of income in period  $t$  that is carried over to provide the source of capital for production in the following period. Given that the total labour supply in period  $t$  is equal to  $[(1+n)(1-y)+(1-\alpha)].N(t)$ , substituting  $K_t = k_t.L_t$  into (7) gives (in the steady state) :

$$(1+n).k. [(1+n)(1-y)+(1-\alpha)].N = s.(1+n).N + b.N$$

from which :

$$(1+n).k. [(1+n)(1-y)+(1-\alpha)] = s.(1+n) + b \quad (8)$$

## 2.4 The PAYG pension scheme

An unfunded or PAYG pension scheme is one in which pension payments to the retired are paid directly out of the current contributions of those working. Hence at all times it must be the case that benefits and contributions are equal (i.e. the scheme is in "balance"). In this model the contributions in any period amount to :

$$C(\alpha, y, T) = [(1+n)(1-y) + (1-\alpha)].T, \quad (9)$$

whilst the (total) benefits paid are denoted  $P(\alpha, y, T)$ . Any *feasible* PAYG scheme must therefore satisfy the "feasibility constraint" :

$$P(\alpha, y, T) = [(1+n)(1-y) + (1-\alpha)].T \quad (10)$$

Hu considers a "*flat rate*" PAYG scheme in which pension is paid to the retiree at a flat *rate* of  $\beta$  per period subsequent to retirement. In other words, the pension rate is independent of the term of retirement (and by implication employment), and the *total* pension benefit received is :

$$P(\alpha, T) = \beta.\alpha. \quad (11)$$

By equating (10) and (11) it is a simple step to determine  $\beta$  such that the flat rate scheme is in balance and hence feasible. It is this value of  $\beta$  that each individual takes as given (i.e. independent of his own retirement decision) in maximising lifetime utility. Specifically, although the individual is aware of the benefit structure (11), he does *not* recognise the link between the value of  $\beta$  and the length of retirement  $\alpha$  if the flat rate PAYG scheme is to be in balance. This is the case, for example, if each individual takes the retirement decisions of everybody else in the same generation as given when making his own. Of course, in a large population of heterogeneous individuals retiring at different times, the pension benefit rate payable is to all intents and purposes independent of any individual retirement decision.

In contrast, we consider below a "*two tier*" PAYG scheme, in which benefits depend directly upon the total period worked. Consequently, the retiree receives both a fixed, basic pension irrespective of the date of retirement ( $q$ ), and a supplement proportional to the total term in work (at a rate  $p$ , say). Thus :

$$P(\alpha, y, T) = (2-y-\alpha).p + q, \quad p, q > 0, \text{ both constant} \quad (12)$$

Again, a feasible scheme is one such that the right hand sides of (10) and (12) are equal. In this respect, one obvious scheme - which yields immediate equality - involves :

$$\begin{aligned} p &= T \\ q &= n.(1-y).T \end{aligned} \quad (13)$$

In practice, we might consider this to be a loose approximation to the structure of the current provision of pension benefits facing the majority of individuals in the United Kingdom, who typically receive an essentially fixed level of (state) basic pension, plus a second tier of occupational and / or state earnings related pension related inter alia to their total period of employment. Though we should note that in most industrialised economies whilst state pension benefits are provided entirely on a PAYG basis, private pension benefits are in general funded. Finally, remember that in each of the above schemes

both taxes and benefits depend explicitly upon the actions of the individual - specifically his choice of retirement date. Hence, neither is a lump sum scheme.

### 3. PAYG social security and the allocation of resources

#### 3.1 The Golden Rule and the first best outcome

As stated above, the Golden Rule level of  $k$ , the capital-labour ratio, is that which maximises utility per capita in the steady state. It is straightforward to show that in the above economy - as is generally the case - the Golden Rule level of  $k$  is given by :

$$f'(k) = n \quad (14)$$

This can be shown by considering the case of a command economy in which the state enjoys both complete information concerning, and full dictatorial control of, all elements within the economy, and wishes to maximise the steady state lifetime utility of the representative individual of each generation. Under such a scenario (and in the absence of social security), the problem facing the state is to maximise (1) above, subject to :

- i. the income constraints facing the individual (2)
- ii. the filial attention relationship (3)
- iii. the bequest / inheritance relation (4)<sup>6</sup>
- iv. the competitive factor price conditions (6)
- v. the capital constraint identity (8)

Substituting these into (1) yields the unconstrained maximisation problem :

$$\begin{aligned} \max. \quad & U\{[f(k)-f'(k).k](1-y)-k[(1+n)(1-y)+(1-\alpha)]+b/(1+n), \\ k, \alpha, b, y \quad & k[(1+n)(1-y)+(1-\alpha)](1+f'(k))+ (1-\alpha)(f(k)-f'(k).k)-b, \alpha, y(1+n)\} \end{aligned} \quad (P1)$$

The necessary first order conditions reduce to (using  $U_i$  to denote the partial derivative with respect to the  $i$ -th argument of  $U$ ) :

$$k : U_2.[(1+n)(1-y)+(1-\alpha)].(r-n) = 0 \quad (15a)$$

$$\alpha : U_2.(-w) + U_3 = 0 \quad (15b)$$

$$b : U_1/(1+n) - U_2 = 0 \quad (15c)$$

$$y : U_1.(-w) + U_4.(1+n) = 0 \quad (15d)$$

Conditions (15), the solutions to which we can denote by  $k^*$ ,  $\alpha^*$ ,  $b^*$  and  $y^*$ , therefore characterise the "first best" steady state outcome - that which maximises the lifetime utility of the representative individual. Given that  $r = f'(k)$  ((6a) above), and also the concavity assumptions on both  $U(\cdot)$  and  $f(\cdot)$ , condition (15a) determining the optimal capital-labour ratio  $k^*$  implies (14) above. In other words, (first best) utility maximisation implies the Golden Rule allocation of resources.

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<sup>6</sup> Note that whereas the individual takes his inheritance as effectively given, the state recognises and duly allows for the equivalence of total bequests and inheritances between generations in any period as given by this relation.

As regards the other conditions, (15b) equates the marginal rate of substitution (MRS) between consumption when old and retirement to the effective cost of retirement - the wage rate  $w$  thereby foregone. (15c) equates the marginal cost of bequests in terms of reduced consumption when old ( $U_2$ ) to the marginal benefit of bequests to the recipient ( $U_1/(1+n)$ ). Finally, the attention condition (15d) equates the marginal rate of substitution between attention when old and consumption when young to the effective cost of attention,  $w/(1+n)$ .

### 3.2 Dynamic efficiency and the Golden Rule in the absence of social security

Typically, however, the state does not enjoy complete control of the resources available to the economy. Consequently a natural question to consider is whether or not it is possible to achieve the optimum outcome in a decentralised economy. It is straightforward to show that this is not the case (except by coincidence). In doing so, however, we must also allow for the strategic nature of the bequest motive assumed. Following Cremer et al. (1992), we utilise the result of Bernheim et al. that, because the parent can always threaten the children with disinheritance, the whole of the surplus arising from the bequest game is appropriated by the parent. We can therefore derive the equilibrium of the bequest game by considering the principal-agent problem in which the parent (principal) effectively maximises his utility subject to each child (agent) receiving the reservation level of utility associated with disinheritance (the participation constraint).

The problem facing the individual of generation  $t$  - who aims to maximise his lifetime utility subject to his individual budget constraints - can consequently be characterised as :

$$\begin{aligned} \max. \quad & U_t \{ w - s_t, s_t(1+r) + (1-\alpha_t)w - b_t, \alpha_t, y_{t+1}(1+n) \} & (P2) \\ s_t, \alpha_t, b_t, y_{t+1} \quad & + \lambda_t [U_{t+1} \{ (w(1-y_{t+1}) - s_{t+1}), (b_t/(1+n) + s_{t+1})(1+r) + (1-\alpha_{t+1})w - b_{t+1}, \alpha_{t+1}, y_{t+2}(1+n) \} - U^*] , \end{aligned}$$

where  $U^*$  is the reservation level of utility associated with disinheritance. Two points to note about this problem are first that because it involves two generations of individuals, we have included *generational* subscripts (whilst assuming that  $w$ ,  $r$  and  $n$  are all constant). Additionally, it assumes that the parent  $t$  effectively chooses the level of attention provided by each child  $y_{t+1}$  - which, given participation and in equilibrium, he effectively does (indirectly) via his choice of bequest rule.

The necessary first order conditions are :

$$s_t : \quad -U_{1t} + U_{2t}(1+r) \quad = \quad 0 \quad (16a)$$

$$\alpha_t : \quad U_{2t}(-w) + U_{3t} \quad = \quad 0 \quad (16b)$$

$$b_t : \quad -U_{2t} + U_{2t+1} \cdot \lambda_t \cdot (1+r)/(1+n) \quad = \quad 0 \quad (16c)$$

$$y_{t+1} : \quad U_{4t} \cdot (1+n) - U_{1t+1} \cdot \lambda_t \cdot w \quad = \quad 0 \quad (16d)$$

$$\lambda_t : \quad U_{t+1} \{ \cdot \} - U^* \quad = \quad 0 \quad (16e)$$

These clearly differ from their counterparts of the first best outcome (15). A valid comparison, however, requires use of the steady state equivalents of (16), namely :

$$s : \quad -U_1 + U_2 \cdot (1+r) \quad = \quad 0 \quad (17a)$$

$$\alpha : \quad U_2 \cdot (-w) + U_3 \quad = \quad 0 \quad (17b)$$

$$b : \quad U_2 \cdot \{ \lambda \cdot (1+r)/(1+n) - 1 \} \quad = \quad 0 \quad (17c)$$

$$y : \quad U_4 \cdot (1+n) - U_1 \cdot \lambda \cdot w \quad = \quad 0 \quad (17d)$$

$$\lambda : \quad U \{ \cdot \} - U^* \quad = \quad 0 \quad (17e)$$

Comparison of (17) with (15) implies that :

$$\lambda = 1 \quad \text{and} \quad n = r ,$$

are sufficient conditions for the decentralised outcome to be first best - each does in fact imply the other (from (17c)). (17) does not however imply that these conditions are satisfied (though they may hold *by coincidence*). In this respect, we can note that in the *non* steady state of (16), the solution for  $\lambda_t$ ,  $\lambda_t^*$ , represents the sensitivity of the parent's utility  $U_t$  to a change in the childrens' reservation utility  $U'$  (at the optimum). An increase in the latter (resulting from an improved outside offer) implies a decrease in the former, as the parent must offer a higher reward in order to induce participation in the bequest game (the participation constraint becomes more binding). The necessary increase in the childrens' participation utility  $U_{t+1}\{.\}$  in order to ensure participation, must be induced by reducing the level of attention provided ( $y_{t+1}$ ) and / or increasing the level of bequest received ( $b_t$ ). As each of these arguments appears *asymmetrically* in the respective utility functions of parent and children, however, the two opposing changes in utility will not be equal (except by coincidence). Hence, in general :

$$\lambda_t^* = -dU_t\{.\}/dU' \neq 1$$

The disparity between (15) and (17) means that in the absence of any state intervention a non Golden Rule and suboptimal outcome prevails, in which steady state lifetime utility is not maximised. Indeed, it is impossible to say whether or not  $k'$  is greater or less than  $k^*$ , and hence whether or not the economy is dynamically efficient. It is with this result in mind that Hu (1979) demonstrates that although a PAYG pension system influences the relative levels of capital and labour employed within the economy, and is thus able to secure the Golden Rule outcome in the presence of endogenous retirement and a bequest motive, it is not desirable to do so. Moreover, it is unable to secure the first best outcome.

### 3.3 The Golden Rule in the presence of social security

In this subsection we demonstrate the existence of a class of feasible and non lump sum PAYG schemes for which the opposite result holds, and it does in fact prove possible to secure the optimum steady state outcome of (15) - if retirement is endogenous and a bequest motive is either absent or strategic. In other words, a suitably designed two tier scheme can ensure that  $k^*$ ,  $\alpha^*$ ,  $b^*$  and  $y^*$ , from (15), prevail. We first consider how the optimisation problem of the representative individual is modified by the introduction of social security. Once more substituting (2) into (1), but this time allowing for the social security tax (rate)  $T$ , and benefits  $P(\alpha, y, T)$  - both of which the individual takes as given - yields the following necessary first order conditions (in the steady state) :

$$s : -U_1 + U_2 \cdot (1+r) = 0 \quad (18a)$$

$$\alpha : -U_2 \cdot (w-T-P_{\alpha i}) + U_3 = 0 \quad (18b)$$

$$b : U_2 \cdot \{\lambda \cdot (1+r)/(1+n) - 1\} = 0 \quad (18c)$$

$$y : +U_4 \cdot (1+n) - U_1 \cdot \lambda(w-T) + U_2 \cdot P_{y i} = 0 \quad (18d)$$

$$\lambda : U\{.\} - U' = 0 \quad (18e)$$

Note that  $P_{\alpha i}$  and  $P_{y i}$  are the partial derivatives of total pension benefit  $P$  with respect to the retirement fraction  $\alpha$  and attention variable  $y$  respectively as *perceived by the individual*. Recall that whilst each individual is assumed to be fully conversant with the benefit structure of the PAYG scheme, he assumes the level of benefits to be independent of his own actions, as will almost certainly be the case in

practice. More specifically, and unlike the state, he does not recognise the link between his own actions - if replicated by all members of his generation - and the level of benefits if the scheme is to be in balance.

In this instance, comparison of (18) with (15) reveals that even if both  $\lambda = 1$  and  $n = r$ , attainment of the first best outcome is not immediate, but instead depends upon the benefit structure of the PAYG scheme (and in particular  $P_{oi}$  and  $P_{yi}$ ). There is in any case no reason for either or both of these conditions to be satisfied (other than by coincidence). Consequently, introduction of social security into an otherwise decentralised economy generally does not induce the first best outcome. The *optimal* choice of (second best) two tier PAYG social security can, however, secure the first best outcome.

### Proposition

Assume that each individual knows the (non lump sum) benefit structure  $P(\alpha, y, T)$  of the PAYG pension scheme in operation. Then there exist a set of feasible PAYG pension schemes (with associated non lump sum taxes on earned income), which enable the state to secure the Golden Rule outcome in the steady state of an overlapping generations economy, as part of the first best outcome, in the presence of both endogenous retirement and a (strategic) bequest motive.

### Proof

Suppose the aim of the state is to maximise the steady state lifetime utility of the representative individual of each generation  $t$ , for which purpose a (feasible) PAYG pension scheme  $P(\alpha, y, T)$  financed by a proportional tax on income at a rate  $T$  is available. It therefore maximises (1) subject to (2), (3), (4), (6), (8), and (10). Substituting all of these bar the feasibility constraint (10) into (1) yields the unconstrained maximisation problem :

$$\max_{T, L} U\{[f(k)-f'(k).k-T](1-y)-k.[L]+b/(1+n), \quad (P3)$$

$$k.[L](1+f'(k))+ (1-\alpha)(f(k)-f'(k).k-T)+P(\alpha, y, T)-b, \alpha, y(1+n)\}$$

where :

$$L = (1+n)(1-y) + (1-\alpha)$$

The resulting, necessary first order condition<sup>7</sup> is then :

$$\begin{aligned} dU/dT = & dk/dT.[-U_1.\{k.f''.(1-y)+(L)\}+U_2.\{(L)(1+f')+k.f''.(1+n)(1-y)\}] \quad (19a) \\ & + d\alpha/dT.[U_1.k+U_2.\{-k-f+P_{\alpha s}+T\}+U_3] \\ & + db/dT.[U_1/(1+n)-U_2] \\ & + dy/dT.[U_1.(T-w-k.(1+n))-U_2.\{k(1+f')(1+n)-P_{ys}\}+U_4(1+n)] \\ & + U_2.\{\alpha-1+P_T\} - U_1(1-y) = 0 \end{aligned}$$

Again  $P_{\alpha s}$  and  $P_{ys}$  are the ("true") partial derivatives of the total pension benefit  $P$  with respect to the retirement fraction  $\alpha$  and attention variable  $y$  as *perceived by the state* (and allowing for the link between contributions and benefits implied by the feasibility constraint (10)), which generally differ from  $P_{oi}$  and  $P_{yi}$  in (18) above.

<sup>7</sup> Suppressing the "k's" in  $f(k)$  and its derivatives for the sake of concision.

When choosing the optimal social security scheme, the state allows for the subsequent (utility maximising) actions of individuals, as represented by the first order conditions (18). We can therefore substitute (18) into (19a). Additionally differentiating (10) enables us to substitute for  $P_T$ . The first order condition which characterises the optimal, “second best” PAYG scheme thus becomes :

$$\begin{aligned} dU/dT = & \quad dk/dT.\{U_2.(n-r)(1-y).kf''\} & (19b) \\ & + d\alpha/dT.\{U_2.(P_{as}-P_{ai})\} \\ & + db/dT.\{U_2.(r-n)\} \\ & + dy/dT.\{U_2.[(w-T)(n-r)+((1+r)P_{ys}-(1+n)P_{yi})/(1+r)]\} \\ & + U_2.(n-r)(1-y) & = \quad 0 \end{aligned}$$

Now, consider a two tier PAYG pension scheme such as that characterised by (12) and (13), in which :

$$P_{ai} = -T, \text{ and} \quad (20a)$$

$$P_{yi} = -(1+n).T^8 \quad (20b)$$

Substituting these values into (19b), reduces it to :

$$dU/dT = [dk/dT.\{(1-y).kf''\} + db/dT + dy/dT.\{w-T.(n-r)/(1+r)\} + (1-y)].U_2.(n-r) = 0 \quad (19c)$$

Hence the optimal two tier PAYG pension scheme with benefit structure given by (20a) and (20b) implies that  $n = r = f'(k)$ , thereby yielding the first best outcome for the capital-labour ratio,  $k^*$ , in accordance with the Golden Rule. In addition given that  $k = k^*$ , (20a) implies that (18b) and (15b) are not only identical but also have the same solution. Consequently, the retirement choice induced by the two tier scheme is also first best ( $\alpha = \alpha^*$ ). Substitution of  $n = r$  into the bequest condition (18c) next implies that  $\lambda = 1$ , which in turn means that (18c) yields the first best solution  $b^*$ . Finally, a substitution of (20b) and  $\lambda = 1$  into (18d) produces a similar result for the attention variable  $y$ .

Hence, if the PAYG pension scheme benefit structure satisfies (20a) and (20b), then the optimal, and second best, two tier scheme characterised by (19c) yields the first best and Golden Rule outcome for the capital-labour ratio  $k$ , and consequently from (18) the first best outcomes for each of the other choice variables  $\alpha$ ,  $b$  and  $y$ . Given (20), therefore, the second best outcome yielded by PAYG social security (19c) is equal to the first best steady state outcome of (15). In other words, and ignoring the (remote) possibility that the terms in (19b) coincidentally sum to zero, the optimal (second best) PAYG induces the *first best* steady state outcome in the presence of both endogenous retirement *and* a (strategic) bequest motive.  $\square$

This result therefore contrasts with that of Hu findings in respect of his flat rate scheme (where recall that  $P_{ai} = \beta \neq -T$  and  $P_{yi} = \beta \neq -(1+n).T$ ). It reflects the both the non-distortionary structure of the two tier PAYG pension scheme, in which (total) pension benefits are directly related to the (total) period of employment, and the absence of any externality when the bequest motive is strategic in nature.

### Corollary to the Proposition

A similar result applies in the absence of any bequest motive.

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<sup>8</sup> These expressions are both obtained from partial differentiation of the benefit structure implied by (24) and (24a), and they are crucially equal to the expressions for  $P_{as}$  and  $P_{ys}$  obtained from differentiation of the feasibility constraint (22).

## Proof

In the absence of a bequest motive there is no strategic bequest game, and hence the bequest and attention variables  $b$  and  $y$  do not appear in the lifetime utility of the individual. Consequently, the relevant first order conditions simplify to the following, from which an identical argument to that above yields the result.

First best outcome:

$$k : U_2 \cdot [(1+n) + (1-\alpha)] \cdot (r-n) = 0 \quad (15a)$$

$$\alpha : U_2 \cdot (-w) + U_3 = 0 \quad (15b)$$

Individual utility maximisation in the absence of social security :

$$s : -U_1 + U_2 \cdot (1+r) = 0 \quad (17a)$$

$$\alpha : U_2 \cdot (-w) + U_3 = 0 \quad (17b)$$

Individual utility maximisation in the presence of social security :

$$s : -U_1 + U_2 \cdot (1+r) = 0 \quad (18a)$$

$$\alpha : -U_2 \cdot (w - T - P_{ai}) + U_3 = 0 \quad (18b)$$

State maximisation of lifetime utility :

$$dU/dT = [dk/dT \cdot \{(1-y) \cdot kf''\} + ] \cdot U_2 \cdot (n-r) = 0 \quad (19c)$$

## 4. Conclusion

The impact of social security upon the macroeconomic allocation of resources in a dynamic economy was first investigated by Samuelson (1975), whose work was subsequently extended *inter alia* by Hu (1979). Whereas the former found that a suitably designed (less than fully funded) social security scheme enabled the state to secure the optimum allocation of resources, including the Golden Rule, Hu showed this not to be so in a model with a both endogenous retirement and a (terminal consumption) bequest motive. The current paper has extended the analysis of this topic, by showing that a suitably designed *two tier* pay-as-you-go pension scheme can in fact secure the first best outcome in the presence of endogenous retirement, provided either that no bequest motive is present, or that it takes the strategic form advocated by Bernheim et al. (1984, 1985). Under such a scheme the total pension benefit paid is positively related to the working lifetime of the individual. The paper therefore illustrates *inter alia* the sensitivity of the results obtained in this area of research to the assumptions employed.

Although generalising previous models, the assumptions employed by the current model remain rather restrictive. Future research might therefore be directed towards investigating the impact of their relaxation, by perhaps allowing for uncertainty with regard to income and length of life, or by relaxing the assumption of identical individuals within each generation, so as to allow for intragenerational distributional and free rider issues.

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