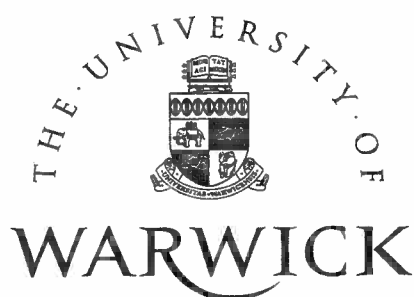


**A BANKRUPTCY PROCEDURE FOR SOVEREIGN STATES**

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DEPARTMENT OF ECONOMICS

# A BANKRUPTCY PROCEDURE FOR SOVEREIGN STATES

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

# A Bankruptcy Procedure for Sovereign States\*

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## Abstract

Do emerging economies need a bankruptcy procedure to handle potential debt defaults? Jeff Sachs and John Williamson, for example, say yes. But others, including notably the two Working Groups who issued reports on Crisis Resolution (on behalf of G10 and the Institute of International Finance) say no — mainly on account of “moral hazard” ascribed to **debtors**. But could the replacement of syndicated bank lending with widely held bond debt under the Brady plan have posed a problem of **inter-creditor** conflict sufficiently pressing to have tipped the balance in favour of having an orderly procedure?

To investigate this, we use the basic tools of finance, starting with the valuation of corporate debt and then going on to sovereign debt. What we find is that, without “water-tight” sovereign immunity, creditors face a serious Prisoner’s Dilemma in the absence of a code. Though it may be collectively inefficient, individual creditors may see it in their self-interest to grab what they can of the available collateral in a “race of the vultures”.

Avoiding inter-creditor conflicts of this sort is the primary reason for having a bankruptcy code. Our simulations also suggest that the case is fairly robust in the face of “moral hazard” problems among debtor countries.

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*“I wonder whether those who have opposed any reforms to facilitate the orderly restructuring of sovereign debt have asked themselves what will happen if and when another emergent borrower runs into trouble on a comparable scale to Mexico and is not bailed out, as it will probably not be.”* Williamson (1996).

## I INTRODUCTION

Soon after the Mexican currency crisis of 1994/5, the leaders of the Group of Seven major industrial countries, meeting in Halifax in June, 1995, made proposals to strengthen the international financial system. To help prevent crises the IMF was urged to increase its monitoring of national policies and of capital flows and also to set up benchmarks for the standard of statistical reporting by member governments. To help cope with a crisis, it was proposed that Group of Ten countries and others double the facilities available to the Fund under the General Arrangements to Borrow (GAB) and that the IMF provide faster access to credit disbursements in emergencies. They also asked the finance ministers and central-bank governors of the Group of Ten countries to look at further measures to help resolve liquidity crises.

Jeffrey Sachs (1995) had already thrown down a challenge, charging that the international financial system failed to provide the basic protection available to corporate borrowers. Just as domestic bankruptcy procedures protect Macy’s department store from asset stripping by fearful creditors, so he proposed that sovereign debtors be afforded similar protection through the aegis of an international bankruptcy court (IBC). (See also Williamson (1992) on the need for a legal mechanism for the revision of debt contracts — the sovereign equivalent of Chapter XI proceedings under US bankruptcy law.) Less enamoured of the analogy of corporate law but more influenced by the history of sovereign default, Eichengreen and Portes (1995) came forward with another proposal: the creation of a Bondholders Council able to negotiate the reconstruction of bond debt, together with changes in future bond covenants to permit a majority of creditors to alter the terms of payment. (The trigger for such renegotiation would be a stay on debt service authorized by the IMF.)

In writing their report, the working party for the Group of Ten surveyed the views of market participants on these and other mechanisms to prevent and resolve crises: and found very little support for anything that smacked of “no-fault default”! The final report on “The resolution of sovereign liquidity crises” (G-10, 1996) focused on sovereign bonds and affirmed a market-based approach. It rejected the “laissez faire” approach of letting creditors and debtors work things out without official involvement. But it also rejected the idea of a bankruptcy court, as both impractical and based on a flawed analogy. As Summers (1996,

p4) puts it, the analogy was flawed because “the decision of a state to suspend its debt service is at least partly volitional” and because “the safeguards against moral hazard built into domestic bankruptcy codes cannot be applied to sovereign debtors.” (He quotes the G-10 as saying “it would neither be appropriate nor possible to replace the authorities responsible for economic policies of a sovereign state with a new management, or to take possession of a state’s non-commercial property”.) Instead the working party adopted what has been described as a “middle ground” approach (not unlike that of Eichengreen and Portes (1995)). Specifically they recommended (a) that there be changes to the provisions covering sovereign debt (so as to allow for the rescheduling with non unanimous voting, to include sharing clauses and to encourage collective representation of bondholders); and (b) that the IMF should “lend into arrears” for borrowers in default — providing that they have undertaken to adopt the policies necessary to return to credit worthiness.

While ministers and governors in the G-10 countries endorsed the report, the Institute of International Finance, speaking for many of the principal creditors, issued its own alternative document (IIF, 1996). This criticised prearranged, officially based mechanisms for crisis management in general, and “the emphasis in the G-10 report on payment standstills with at least informal official approval” in particular. In the words of the IIF:

“Under conditions in which capital markets are broadly ‘normal’ and defaults isolated and sporadic, officially elaborated plans for crisis management run the risk of doing more harm than good. The central reason can be summarized as the distorting effect of such an approach on the behaviour of lenders and borrowers, or “moral hazard”... the more investors perceive that institutional arrangements are trending towards ‘no-fault default’, with minimal pain for the borrower and substantial risk of the politicization of debt, the less willing they will be to supply capital to the emerging markets.” (Annex A, p29).

While contingency plans may well pose some risk of “moral hazard”, one has to ask what will happen in the absence of prepositioned plans. One has to imagine another crisis in another Mexico (but without a land border with the USA!), and think through the implications. Far easier, perhaps, to leave it to fate and market forces.

Nevertheless, in this paper, we plan to go through the intellectual exercise, using the basic tools of asset valuation developed for corporate finance. Section II begins with debt which *is covered* by a bankruptcy code, namely corporate debt, and ask what might happen if the code were removed. By analogy to the corporate case, we then look at sovereign debt in the absence of the “moral hazard” problem and ask what might happen if default threatens but there is a waiver of sovereign immunity, as is very common.<sup>1</sup>

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<sup>1</sup>For valuing corporate debt we adopt the basic formulation of Lambrecht and Perraudin (1995). Be-

Recent case history suggests that the waiver of immunity gives litigious creditors (“vultures”) the right to seize or attach external financial assets owned by the sovereign power. Since external assets include inflows of new funds borrowed abroad — and may in certain cases include existing official reserves of foreign currency — this means that they can in principle impose something of a financial stranglehold on a debtor in default. By incorporating this feature, we can value the sovereign debt of emerging economies where the threat of attack by “vultures” poses a substantial downside risk. (In this respect it resembles characterizations of the US stock market before the crash of 1987 by Genotte and Leland (1990) and Krugman (1987)). Reducing the returns to litigious creditors by a payments “standstill” procedure could check this downside risk, however; and we provide quantitative estimates of the benefit to other creditors obtained by numerical simulations based on Brady bonds.

These potential gains from a standstill provide a substantial counterweight to the losses from moral hazard examined in Section III where first we show how removing the threat of the creditor race could indeed have the consequences predicted by the IIF, as debtors reduce their efforts to honour their obligations. But we go on to show how replacing this threat by a payment standstill combined with appropriate IMF conditionality could be Pareto improving (as it avoids the deadweight loss of the creditor race).

In conclusion, we suggest that the analogy between corporate and sovereign debt may be closer than the G-10 report was ready to acknowledge. For if sovereign immunity is waived and creditors learn how to apply the sanctions available, creditor grab races may occur in sovereign debt markets too. And the sovereign’s right to manage can be effectively abrogated by payment standstills subject to strict conditionality. As Williamson (1996) puts it, if the IMF can maintain or withdraw the standstill at its discretion, this “would take the IMF as close to assuming the role of a bankruptcy court as it is possible to imagine.”

The IIF was right to be suspicious that there is more in the G-10s proposal than meets the eye. But they are wrong to criticise it as departing from a market-based approach. As sovereign debt becomes more like commercial debt, it is only to be expected that rules governing these international bond markets will imitate the best features of commercial bond markets!

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cause this is similar to formulae for sovereign bond valuation developed by Bartolini and Dixit (1991) and Cohen (1993), it is a small step to move from corporate to sovereign bond valuation.

## II THE THEORETICAL CASE FOR A BANKRUPTCY PROCEDURE

Why have a bankruptcy procedure? Why let people off paying their debts? The short answer is that it could be worse for creditors if you don't!

### *II.1 A cautionary tale; or A parable from Poundstone (1992)*

Take corporate bankruptcy for an example. Consider the case of two creditors, A and B, with identical holdings of equal seniority, who bought their bonds when the company was doing well but are now informed that the company is unable to pay the full interest due. Imagine how each might reason in the absence of an orderly bankruptcy procedure, beginning with creditor A, thinking aloud as follows:

"I could just hang on to the bonds accepting reduced interest payments — or a rollup of arrears — and hoping for a recovery of profits and payments. Or I could sell them. But, if the word has got around, I can expect the same return in or out — assuming the bonds are priced 'fairly' — so there's no particular attraction in selling out, as compared to holding on.

Now what about grabbing the firms assets and selling them in the market for scrap? This will mean putting an end to the operations of the company, it's true. But if the scrap value is above the value of *my* bond holdings it seems like a better alternative.

Hold on. If this is true for me, then why won't it be true for B? And what then? Will grabbing the assets still be my best strategy if B also decides to go for them? Yes, of course. The assets may have to split, but that's a lot better than being taken for a sucker by B who gets everything!

So whether B is expected to hang on or not, the best thing for me to do is grab the assets. But wait. Maybe I can persuade B to hang on too! He's a smart fellow. But I'll have to move quickly."

(Rings B. Gets busy signal — could it be B on the line, trying to get in first? Ruefully, puts phone down. Picks it up. Rings her lawyer...)

So the company will collapse, as both creditors try to attach the assets, even when they would have been better off hanging on and waiting for a recovery.

In short, the creditors face a prisoner's dilemma. This can be shown more succinctly as in the payoff matrix, Table 1, where the first figure in each cell gives the payoff to A, the

second to B, measured, let us say, in thousands of dollars.

	B hangs on	B grabs assets
A hangs on	31,31	0, 40
A grabs assets	40, 0	20,20

Table 1: The prisoner’s dilemma.

Suppose the total face value of the bonds is 100 which is equally split between creditors A and B who have the equal seniority status, and the scrap value of the firm is worth 40. Assuming the current market value of the bonds is 62, the payoff when *both* hang on is 31 each, shown in the top left cell. But if B is to hang on, A will be tempted to do better by going for the assets, worth 40 ( i.e., A will chooses bottom of column 1). Even if B were also to go for the exit, it would be better for A to get a half share than be taken for a sucker. (A will choose bottom of column 2). So it always pays A not to wait. By symmetry, the same goes for B: so the firm goes into liquidation.

As Jackson (1986) suggests, reducing the incentives for such grab races is the primary motivation of bankruptcy law. It attempts to do this by giving equal treatment to creditors of the same seniority, and by overturning any concessions that may have been made in the period prior to the bankruptcy. How this works is easy to see in the context of our example. As the “equal treatment” principle rules out the off-diagonal outcomes, this leaves the bondholders effectively choosing between *both* hanging on, (worth 31 each), or *both* splitting the collateral (worth 20 each). So the firm stays in business.

Our simple tale makes a point but leaves many questions unanswered: such as

- when is it socially efficient for the firm to close down?
- will this not involve some debt restructuring?
- what if the bankruptcy code is imperfect, so that not all pre-emptive actions are nullified?

These and other issues are considered in the next section as a preliminary to applying the same logic to sovereign debt.



## II.2 Bankruptcy and bond prices: a numerical example

To see how the prospect of a grab race can lead to early closure and undermine bond values we use an explicit model of a levered firm where profits vary stochastically, and may fall far enough to trigger loan default. The formal model spelt out in the Technical Appendix is based on Lambrecht and Perraudin (1996). The key results, given in the Tables and text that follow, can be illustrated as in Figure 1, where the product price is measured along the horizontal axis and asset values are measured vertically.

### II.2.1 BOND VALUES WITH A CODE

To value bonds in the presence of a bankruptcy code, we consider undated corporate debt paying a coupon  $b$  issued by a firm with constant wage costs  $w$  and a product price  $P$  which follows a random walk. It is assumed that the firm can lay off workers and costlessly “mothball” capital to avoid making losses. Shareholders are assumed to have no funds to inject, so payments to bondholders are constrained by current cash flow. So bond holders can receive the coupon  $b$  in full for prices greater than or equal to  $w + b$ , and in part for prices between  $w + b$  and  $w$ ; but they can expect no payment at all for prices below  $P_m = w$ , when the firm is mothballed. In this circumstances, bond values will surely increase with the product price. At first blush one might guess that bond values would be given by the three connected line segments  $OW$ ,  $WB$ ,  $BF$  in Figure 1, where each segment reflects the current level of interest payment capitalised at the interest rate  $r$ . But that would be to neglect (i) the “stochastic smoothing” induced by price variability, and (ii) the possibility of seizing collateral assets. Taking these into account gives the smooth function labelled  $LL$  in Figure 1. Because bond payments become increasingly more secure as prices increase, the schedule  $LL$  slopes up to the right, approaching asymptotically the par value  $b/r$ , shown by the schedule  $FF$ . To the left  $LL$  is tangent to the line  $SS$  showing the disposal value of the physical capital. This is because when prices are low and coupon payments are not being made in full, bankruptcy may occur. Note that the closure decision is effectively delegated to the bondholders who can seize the assets and sell them for scrap whenever the firm is in default on interest payments. It is true that bondholders will not be inclined to do this as soon as interest payments are first interrupted just below  $P_d = w + b$ , because of the likelihood of a return to full interest payment. But when the product price falls substantially below  $P_m$ , the mothballing point, they may give up waiting for prices to recover and choose to exercise their option to realize the residual value of the firm, by foreclosing and selling the capital.

Let  $FF$ , the par value of bonds on issue, be 100, and so — with interest rates of 4% —  $b$  equal 4; and let wage costs be of the same magnitude. Given one more parameter, namely

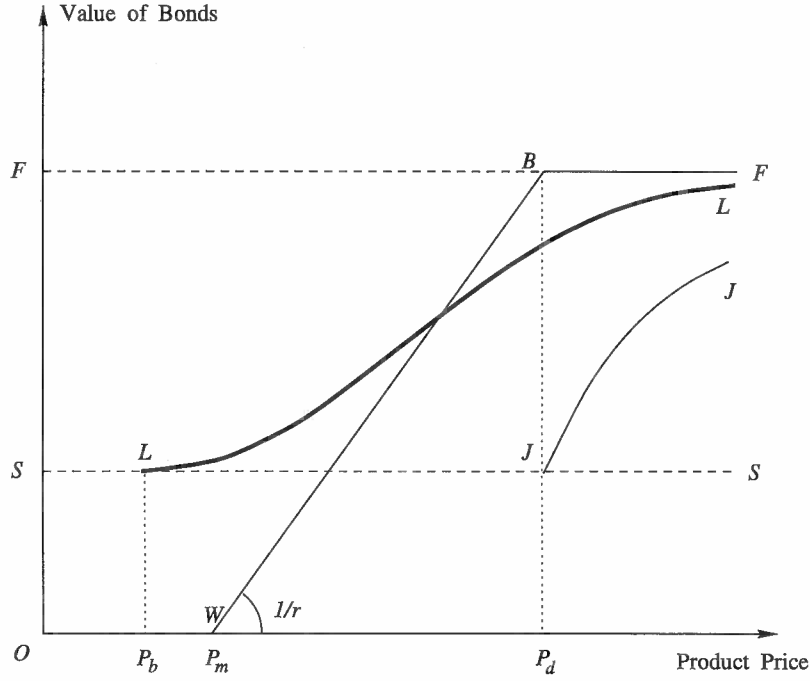


Figure 1: Value of bonds with and without a bankruptcy code.

the variability of the product price measured by  $\sigma$ , we can compute the schedule  $\bar{L}\bar{L}$  and find the closure point chosen by the bond holders, see Table 2.

Closure Prices and Bond Values	Levered Firm	Bond Values	Unlevered Firm
	$P_b$	$L(P_d)$	$P_X$
$\sigma = 0$	5.6	100	4.0
$\sigma = 0.1$	4.2	74	3.9
$\sigma = 0.2$	3.6	62	2.5

Table 2: Bond values, closure, and product price variability.

The column headed  $P_b$  reports the price level at which any bondholder would choose to quit. Note that the incentive to wait for a recovery of debt service increases with product price variability,  $\sigma$ . With no variability waiting is pointless, and bondholders would claim the residual value of the firm if ever the present discounted value of profits fell to 40, i.e., at a product price of 5.6. When  $\sigma$  is 0.1, bondholders are willing to hang on longer, almost to the point where the firm would be mothballed ( $P_m = 4$ ). With high variability, the quit point  $P_b$  falls to 3.6, i.e., well after the firm has been mothballed.

In the second column, we see how price variability (and the associated quitting decisions) impacts on bond values. Consider specifically  $L(P_d)$ , the value of bonds at the price  $P_d$ , where the firm is on the point of defaulting on interest payments. If prices were constant at that level, there would be zero profits, but all would be well for bondholders as coupons could just be paid in full and there would be no danger of quitting. So bonds would stand at par, i.e., 100, as shown in the top row. With high variability of prices and profits, when default on interest payments is likely and bankruptcy possible, bond values fall to 62, only 22 more than the residual value of capital.

In fact, the bond value appearing at the bottom of Table 2 provides the illustrative case already discussed as the Prisoner's Dilemma in Table 1. For when the firm is on the point of defaulting, each of two equal bondholders will get a payoff of 31 for "hanging on" — so long as the other does likewise (summing to the total bond value of 62 shown in the Table).

But what about the prisoner's dilemma? In calculating the value of waiting and in assessing bond values, we have been ignoring the incentive facing individual bondholders to improve their position by grabbing the assets and forcing early closure. The justification is that corporate debt is covered by a bankruptcy code designed to remove the incentive for such pre-emptive behaviour, by overturning any preferential deals made in the period before formal bankruptcy in favour of an equitable allocation of assets among creditors. But it may be worth considering what would happen to bond values otherwise.

Lambrecht and Perraudin (1995) analyse the incentive for pre-emption that remains when the code is poorly written or loosely implemented (so that not all preferences are overturned): and in the next section we look at results of having no code at all.

## II.2.2 BOND VALUES WHEN THERE IS NO CODE

Absent a bankruptcy code, there will surely be a "race for the exit" as in the anecdote above. But when will the race begin? Legally it can only begin after the firm is in default, i.e., when  $P$  falls below  $P_d$ . Backward induction leads to the conclusion that the race cannot be delayed beyond this point. So the firm will be closed down at  $P_d$ . This implies that one can value bonds by using as a "boundary condition" the restriction that there will be no hanging on beyond  $P_d$ , i.e.,  $L(P_d)=40$ . Basic arbitrage arguments suggest that this boundary condition will change the value of bonds for prices *greater* than  $P_d$  as well, as shown graphically in Figure 1 where the schedule  $JJ$  represents the value of bonds constrained to "value match" the scrap value function  $SS$  at  $P_d$ .

The numerical solutions for bond values in this case are shown in the middle row of Table 3, for the high variability case where  $\sigma$  is 0.2. For purposes of comparison bond

values with a bankruptcy code are reported in the top row.

	$L(P_d)$	$L(1.25P_d)$	$L(1.5P_d)$	$L(2P_d)$
With a code	62	70	75	81
With a race	40	52	60	70
% Fall	35	26	20	14

Table 3: Early exit and the value of bonds.

So a race for the exit not only destroys value for bondholders when it takes place. Bond prices weaken ahead in anticipation. (Mention models of stock market crash here?) The bottom row of the Table shows how much lower bond values are when early exit is expected. Even when the product price is twice the level at which technical default occurs, the prospect of a creditor race will knock 14% off the price, adding about 0.8 of a percentage point to the corporate bond rate.

What about shareholders? It probably goes without saying that the early collapse triggered by a credit race is not in the shareholders interest. But even when the bondholders hang on till  $P_b$ , there is still a potential loss of welfare — the so-called “agency cost” of delegating the closure decision to bondholders. (One motivation for debt/equity swaps under Chapter 11 is to mitigate this agency cost.) To see this we calculate when the shareholders would have chosen to quit in the absence of leverage.

### II.2.3 EQUITY VALUES AND SHAREHOLDER PREFERENCES FOR CLOSURE

Assume the firm is unlevered, i.e.  $b=0$ . To value equity, we begin with the kinked schedule  $UU$  which represents the present discounted value of profits, on the assumption losses are ruled out by mothballing, and that prices do not vary. (The kink is at the price  $P_m = w$ , where profits are zero and the firm is about to be moth-balled.) Random variation of prices will smooth out this kink, as shown by the schedule  $OW$ , because even when current profits are zero, there is some chance of returning to production and positive profits.

Schedule  $OW$  would represent the value of the unlevered firm if residual or scrap value of the physical assets was zero — in which case the firm would never be liquidated unless prices fell to zero. But allowing for a positive scrap value function  $SS$  implies that there is an opportunity cost to mothballing. Adding in the economic value of the option to sell for scrap yields both the equity value of the unlevered firm, represented by the schedule  $QQ$ , and the optimal liquidation price  $P_X$  where  $QQ$  is tangent to the scrap value function  $SS$ . For the same parameter values as before, the figures in the last column of Table 2 show that  $P_X$ ,

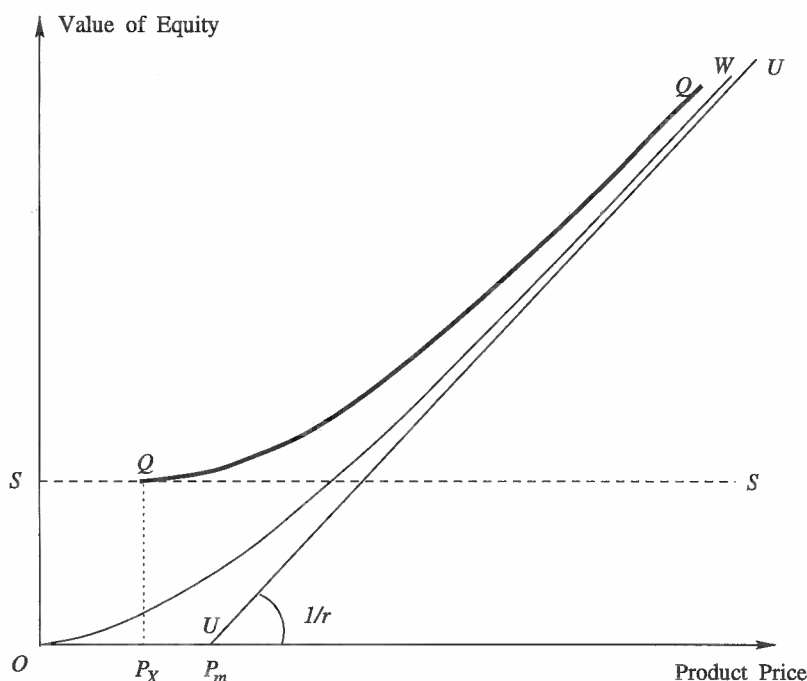


Figure 2: Equity value and optimal liquidation.

the closure price which the shareholders would prefer, is below  $P_b$  selected by bondholders under a bankruptcy code, and *a fortiori* less than the closure point without a code,  $P_d = 8$ . Shareholders would clearly like avoid a creditor race, which sharply exacerbates the agency problem of the firm.

### II.3 Sovereign debt

How, if at all, does this logic apply to sovereign debt? First for some obvious differences. How for example to define solvency when the revenues to be applied to debt service can vary with the wide power of the state to tax economic activity. (So the provisions for municipal rather than corporate bankruptcy are more likely to be relevant for sovereign states.) We have been assuming that bankruptcy is triggered by cash flow rather than net worth considerations so the question of defining solvency may not be too important *per se*. But the **endogeneity of cash flows** will be important.

Perhaps more important is the doctrine of **sovereign immunity** — the principle that the assets of the government are not subject to commercial law: so they cannot be seized for non payment of debt for example. But note that in the 1970s the US and the UK

formally restricted this immunity to governmental activities *sui generis* — embassies, for example, and made other government activities, e.g., trading, subject to commercial law. In addition, borrowers can choose to waive sovereign immunity. “Most developing-country government debt contracts after 1976 have contained explicit waivers... (which) have made it more difficult for sovereigns that repudiate their debt to engage in international trade, and their existence supports the assumption that creditors can impose direct sanctions on a reneging sovereign debtor.” Obstfeld and Rogoff (1996, p353).

How can the approach to bond valuation outlined above be modified to take account of both these factors: first that the revenues available to service debt are not exogenous (as action can be taken to boost them when there is a risk of default), and second that sovereign immunity will reduce the value of the assets to be grabbed. We take these points in reverse order, using a stylized example.

### II.3.1 BOND VALUES WITH “WATER-TIGHT” IMMUNITY

Consider the illustrative example in Figure 3, where the variable on the horizontal axis is  $X$ , a measure of “capacity to service debt”. Assuming, as in Cohen (1993) and Bartolini and Dixit (1991), that this follows a random walk, then we can apply the same techniques as for corporate bond valuation. The bond coupon is  $b$ , which is paid in full for  $X$  greater than or equal to  $b$ : otherwise coupon payments are simply  $X$ . Absent any variability in  $X$ , the value of sovereign debt in the two payment “regimes” would be represented by the two connected line segments  $OA$ , and  $AF$ . Note that, with water tight immunity, there is no question of seizing collateral: so bond values sink all the way to zero as  $X$  declines. Variability in  $X$  induces some “stochastic smoothing” and the resulting bond valuation function is the smooth schedule labelled  $OS$  in Figure 3. (This is similar, but not identical, to the solutions obtained by Cohen and Bartolini and Dixit: the essential difference being that for simplicity we have ignored the rolling up of arrears which they include. In Appendix 2, however, we repeat the analysis including roll-up)

By setting  $\sigma$ , the measure of variability in  $X$ , equal to 0.2, one can calculate the value of bonds, relative to par of 100, as in the bottom row of Table 4 (where the subscript  $S$  signifies that the debt has sovereign immunity). Notice in particular that at  $X = b = X_d$  the country is on the point of technical default but the value of bonds, is a reasonably healthy 67% of par. Bond values rise to 83% of par for  $X = 2b$  i.e. when debt service capacity is twice the default trigger. ( Lowering  $\sigma$  gives the results shown in the other rows). There is no need to calculate bond values with a creditor race as, with no prizes, there is no incentive to participate.

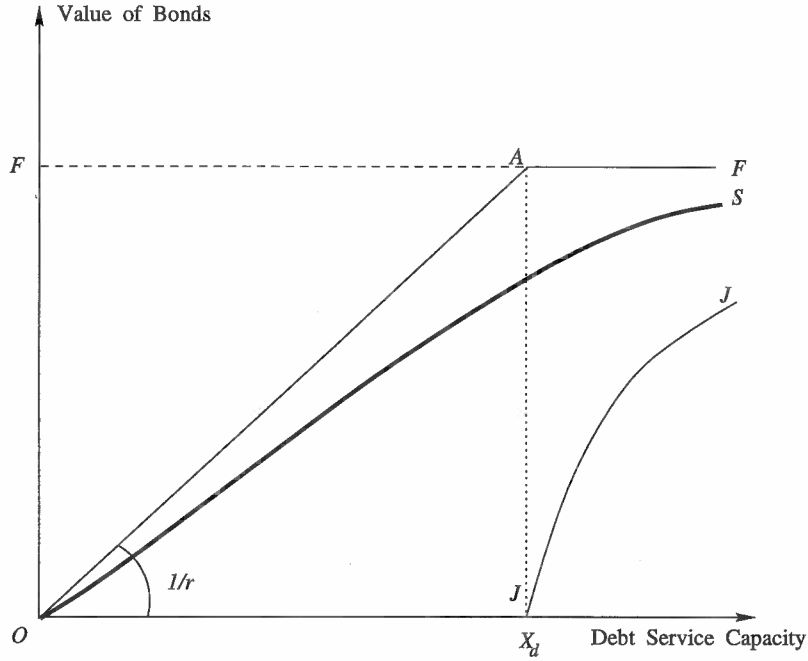


Figure 3: Value of debt with “water-tight” immunity.

	$L_S(b)$	$L_S(1.5b)$	$L_S(2b)$
$\sigma = 0$	100	100	100
$\sigma = 0.1$	83	93	97
$\sigma = 0.2$	67	78	83

Table 4: Value of sovereign debt — with “water-tight” immunity.

### II.3.2 EFFECT OF WAIVING IMMUNITY

But what if the country was to waive sovereign immunity, as is commonly the case in emerging country bond markets? How will this change bond values? The first effect, what may be termed “collateral enhancement”, tends to raise the value of the bonds. But there is a second effect which tends to work in the opposite direction — namely the Prisoner’s Dilemma that arises once there is positive collateral without a bankruptcy code.

By analogy with corporate debt, the prospect of a creditor race provides an economic incentive to grab assets at the earliest moment legally possible, namely  $X = b = X_d$  when the country is on the point of technical default on its interest payments; and this in turn provides a boundary condition for bond values of the form  $L_N(b) = C$ , where  $C$  is the

collateral that can be seized at the point of technical default (and the subscript  $N$  signifies that there is no code in place). Note that this gives a very simple way of checking whether bondholders gain on net from the waiver — does the value of the collateral exceed the value of debt with sovereign immunity at  $X = b$ ? Or does  $C$  exceed  $L_S(b)$ ?

Consider the case where  $\sigma$  is 0.2, the figures in Table 4 imply — to a close approximation — that the enhancement effect will more than offset the consequences of the prisoner’s dilemma, so bond holders will gain from the waiver of immunity, *so long as the collateral is worth more than two thirds of the face value of the bonds*. For smaller levels of collateral, the downside effect will dominate.

This is a pretty stringent test to set. For consider the not-too-implausible case of a country with widely dispersed holdings of its debt but few assets abroad worth seizing. This will fail the test in spades. For the “enhancement” effect of waiving immunity will be negligible, but the “dilemma” will be acute — particularly if the bond holders are thought to include a “vulture” or two!<sup>2</sup> The impact of waiving sovereignty in this case is shown by the line  $JJ$  in Figure 3, which starts from (almost) zero at  $X_d$  and rises asymptotically to  $FF$  as  $X_d$  increases. This is admittedly an extreme case, but if we assume instead that the collateral assets are 10% of the face value of bonds the downside risk remains substantial.

	$L(P_d)$	$L(1.5P_d)$	$L(2P_d)$
With immunity	67	78	83
No immunity, no code	<b>10</b>	44	55
Fall in value (%)	85	44	34

Table 5: How waiving immunity can reduce values — assuming 10% collateralization.

This is illustrated by the figures in Table 5 which show the negative impact of the waiver on bond values in this case. Underneath the values for sovereign debt with immunity, taken from the previous table (for  $\sigma$  of 0.2), we report bond values after a waiver which is expected to lead to a race for the exit at  $X_d = b$ . At  $X = b$  the value of bonds will be 10, the “boundary condition” being imposed in this case. But the waiver causes a sharp fall in bond values even when debt service is well above crisis level. Let us consider a case in point, Brady debt.

Under the Brady Plan the face value of debt is collateralized in the form of US “deep discount” bonds — discounted back from the due date. So the face value is not fully

<sup>2</sup>If there are  $N$  creditors with equal seniority and equal holding of the debt, as long as the collateral value  $C$  is greater than the value of debt while waiting,  $L(P_d)/N$ , the grab race turns out to be the unique subgame perfect equilibrium in the absence of a bankruptcy code.



collateralized before then. It is true that interest payments are guaranteed out of an escrow account for 15 months, but non-payment still constitutes technical default. So it seems that the debt of a Brady “backslider” might run the risk of a creditor race if it fails to meet the coupon payment. How might this affect bond values?

Let us consider this in the case where the official deep discounted US Treasury collateral is 40% of the face value. With a waiver of sovereign immunity and no expectation of a credit race, bond values would pretty healthy particularly if debt capacity is well above the crisis level,  $X_d = b$ . For example, in the top row of Table 6 we see that bond values are more than twice the value of collateral for  $X$  twice the crisis level. Let this represent the current position of a potential “backslider”. What if debt service capacity  $X$  falls from  $2b$  to  $1.5b$ ? The top row suggests a modest 5% fall in bond values from 85 to 81 is the appropriate revaluation.

But what if “vultures” begin to circle? Is there any danger of a market break? Now it is true that the official collateral enhancement behind Brady bonds cannot be grabbed, so in the absence of any other collateral there will be no race. But, as we have seen above, any (additional) assets which can be seized may trigger a creditor race so long as they exceed the value of the vultures holding by enough to cover the costs of litigation.

Assuming that there are enough other assets to tempt a litigious bondholder, but they are in total negligibly small relative to the size of the market as a whole, we can, to a reasonable approximation, assume that the Brady bonds will fall to the value of the 40% collateral at the point of technical default. Imposing this boundary condition gives the values shown in the second row of the table. For  $X = 1.5b$  (and  $\sigma = 0.2$ ) the threat of a grab race knocks 25% or so off bond values, so bonds would go to 60. From the assumed starting value of 85, this represents a drop of almost 30%: and this is well before debt service capacity has fallen to crisis level!<sup>3</sup>

	$L(b)$	$L(1.5b)$	$L(2b)$
No run expected: “as if”			
there was a code	71	81	85
Run expected	<b>40</b>	60	70

Table 6: A Backsliding Brady Graduate?

What about the country itself, the debtor. In the absence of a bankruptcy code and

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<sup>3</sup>Because the US bond collateral cannot be seized, one can split the value of the Brady into two parts, that of the “deep discount” backing and that of the interest rate “strip”. In the circumstances described, the former will be unaffected, but the latter will tend towards zero, cf. Figure 4 above. Adding them together gives the result described in the text.

any way of improving its debt service, things will be fairly bleak. Faced with a grab race, it will have to unilaterally suspend its debt service which could severely damage its trading prospects and access to capital markets. In particular the efforts of creditors to attach overseas assets could prevent access to fresh inflows of funds. The transfer of a limited quantity of assets to creditors is one thing. But if in addition the suspension of debt service triggers a severe punishment of this sort, one which is costly to the borrower without benefiting the creditor see Thomas (1996), it will be rather like the company that goes bankrupt.

How plausible is it that international bond markets might be subject to markets breaks as described above? There are as yet no technical defaults on Brady bonds, but litigation triggered by defaults on sovereign debt suggests that it does not take much to tempt vultures. One creditor has pursued a claim of \$120 million dollars through five different jurisdictions, while another is trying to attach sovereign assets held in London in pursuit of a claim of only \$20 million. (So it looks as if expected gains of say \$50 million is enough to expose a country to the risk of being taken to court if it defaults on its interest payments). Given the billions of dollars of Brady bonds outstanding, many at a hefty discount against their face value, the risk that someone could see and seize the opportunity to make a quick killing does not look so remote.

### III ALLOWING FOR MORAL HAZARD

We have shown how imposing a bankruptcy code could avoid a creditor race and so improve the bond value, but we have not explicitly considered the incentive effects on the sovereign. In fact, removing the threat of a creditor race reduces the incentive for the sovereign to maintain payments and so increases the default probability. This is the “moral hazard” problem emphasised by the IIF (1996). In what follows, we illustrate the issues involved using the two state diagram common in insurance literature and we go on to show how the incentive effect of a creditor race can be preserved by when IMF conditionality is imposed as the price of a debt standstill.

#### *III.1 Endogenous effort: a diagrammatic treatment*

We assume that the sovereign is risk averse, creditors are risk neutral and the credit market is competitive. The sovereign can issue bonds but is not able to buy insurance. The following two diagrams show various outcomes in the absence of the insurance market.

The horizontal and vertical axes in Figures 4 and 5 show the payoffs to the sovereign in good and bad states respectively, and the 45<sup>0</sup> line provides the measure of the utility of

the sovereign — the higher the points on this line the higher the utility. Along any curve in the diagram, the expected utility for the sovereign is constant for given probability of each state. Since the expected utility of the sovereign can be expressed as  $pU(B) + (1 - p)U(G)$ , the slope of the iso-utility curve when intersecting the  $45^\circ$  line is simply the negative of the ratio between probability in good state and that in bad state, i.e.,  $-(1 - p)/p$ . Thus the “steeper” curves reflect lower default probability.

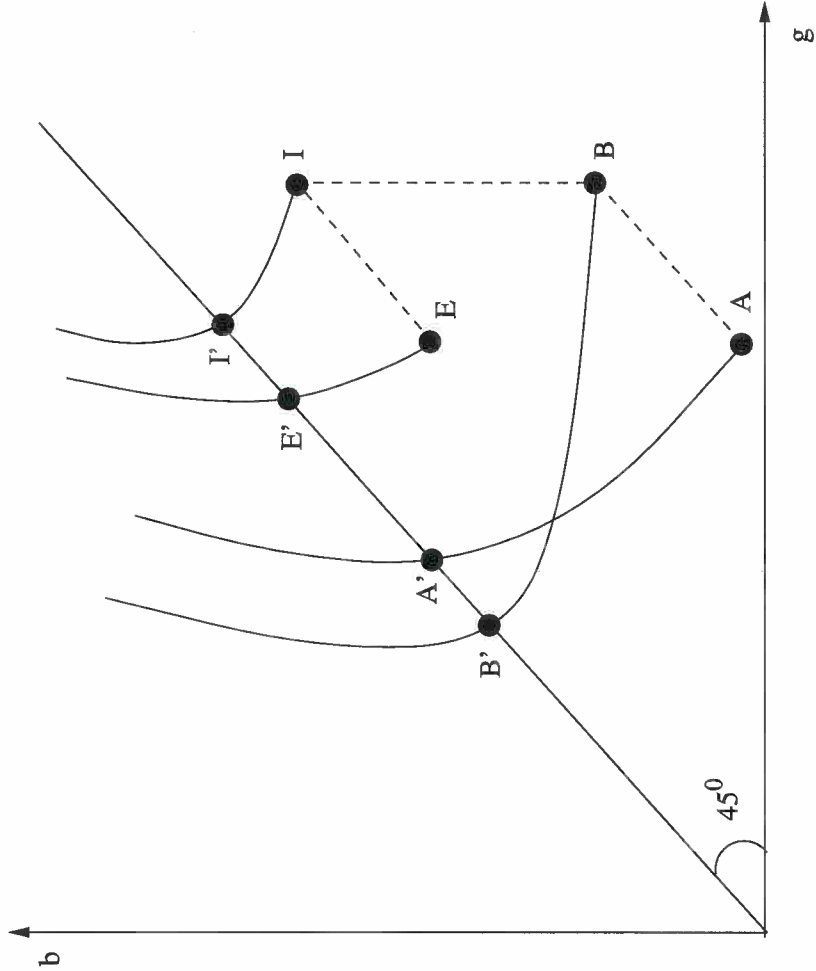


Figure 4: Avoiding moral hazard by creditor race.

We first show in Figure 4 how the threat of creditor race can avoid the moral hazard problem. In the absence of a creditor race the payoffs to the sovereign in each state are given by point  $I$ . The presence of the creditor race, however, imposes a large cost  $C$  in bad state. This moves point  $I$  vertically down to  $B$ . Assume the effort level chosen by the

sovereign is either 0 or 1. With zero effort to avoid bankruptcy, the probability of default will be high and the indifference curves correspondingly less steep, so the expected utility of the sovereign at point  $B$  is given by the point  $B'$  on the  $45^\circ$  line. Assume that by making an effort the sovereign can reduce the probability of bankruptcy and so shift to the steeper indifference curves. If the effort reduces sovereign's payoffs as shown by the move from  $B$  to  $A$  (i.e., the sovereign has to pay costs in both states) the incentive for the sovereign to put in effort is clear. For the expected utility at point  $A$  (measured by  $A'$  on the  $45^\circ$ ) exceeds that at point  $B$  (measured by  $B'$ ). But as we see in next paragraph removing the threat of the creditor race can remove the incentive to put in the effort.

As the cost  $C$  is a deadweight loss to the society, it is tempting to avoid it by introducing a bankruptcy code. But this also reduces the effort made by the sovereign, as shown in the diagram. With no threat of the creditor race the sovereign will have payoffs at  $I$  in the absence of effort. If the sovereign exerts effort (moving from  $I$  to  $E$ ), the expected utility would be given at point  $E'$ . Here the sovereign is clearly better off without exerting any effort as the expected utility at  $I$  is given at  $I'$  which is above  $E'$ . Absent the threat of a creditor race, the borrower puts in less effort to avoid default: this is the moral hazard problem posed by the bankruptcy code.

One way to avoid this problem is to introduce IMF monitoring to ensure that appropriate efforts are made to honour debt payments, i.e., to make the standstill subject to appropriate IMF conditionality. This solution is shown in Figure 5. In the event that the bad state is realised, and successful application is made for a standstill, the sovereign is monitored by IMF programme at a cost  $m$  ( $< C$ ). (Assume the IMF monitoring ensures the sovereign puts in the effort to reduce the default probability because the IMF can maintain or withdraw the standstill at its discretion.) In this case the costs of monitoring plus the effort it induces shifts the payoffs from  $I$  to  $G$  with the expected utility of the sovereign given at  $G'$ . As point  $G'$  lies above  $A'$ , the bankruptcy code plus IMF monitoring improves upon the outcome with a creditor race.

### *III.2 Bond valuation in the presence of 'moral hazard'*

Can the effort of the sovereign to avoid default be incorporated in our model of bond valuation? Assume for example that by putting in effort, the sovereign can ensure that  $X$  never falls below  $b$ . (So policy involves putting a "reflecting barrier" for  $X$  at  $b$ .) This would render bonds completely "safe" so their value would go to par! This would amply justify those who oppose a bankruptcy code; but it is surely overstates their case.

To do the issue justice would surely involve considering political and economic costs and benefits in a country specific model (Sachs, 1995). But for present purposes we make

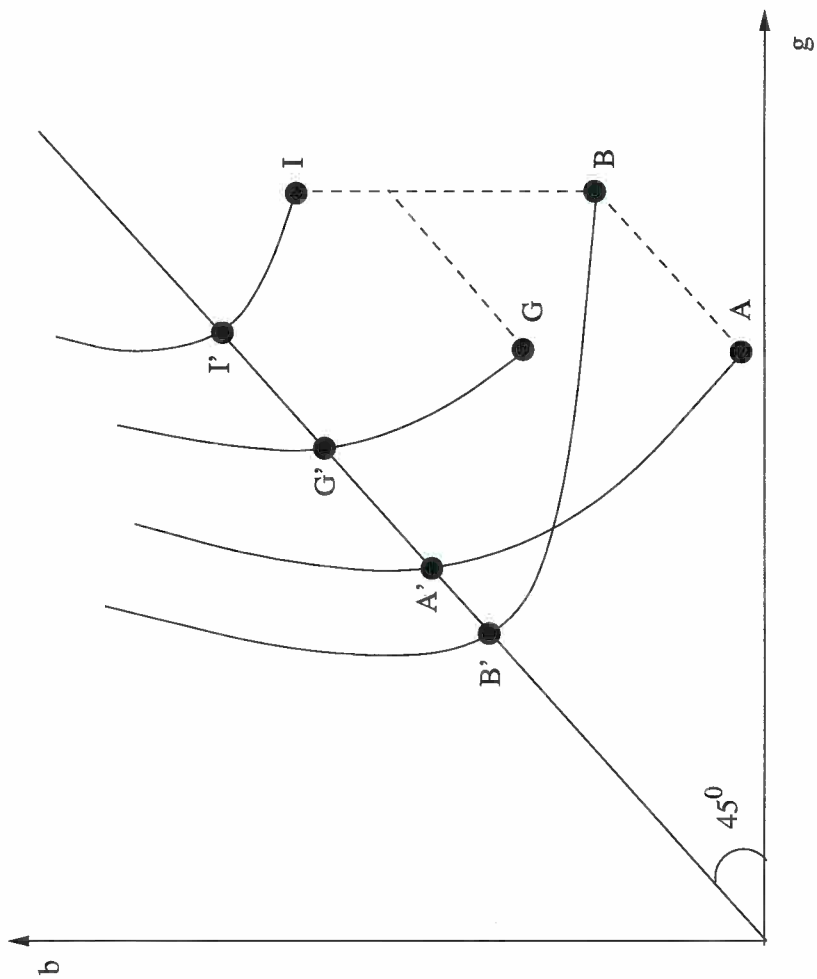


Figure 5: Avoiding moral hazard by IMF conditionality.

a rough and ready allowance for endogenous repayment by simply attaching a probability weight to the success of the effort made to avoid default, i.e., of keeping  $X$  from falling below  $X_d$ . Say this weight was 50%. What would that mean for bond values? In the absence of a bankruptcy code, it implies that when  $X$  falls to  $X_d$ , one of two things can happen with equal probability: either the policy succeeds and no run takes place; or the policy fails so coupons are interrupted, there's a race and bonds are realized for their collateral value. The implications for bond values are shown in Table III.2, where the third row is the simple average of the par values in row 1 and those with the creditor race in row 2.

Bond values	$L(b)$	$L(1.5b)$	$L(2b)$
(1) $X$ prevented from ever falling below $b$	100	100	100
(2) With a creditor race and zero effort	40	60	70
(3) With a creditor race and positive effort	70	80	85
(4) With a bankruptcy code and zero effort	71	81	85
(5) With a bankruptcy code and positive effort	86	91	93

Table 7: Taking account of “moral hazard”.

Clearly the incentive to avoid bankruptcy has benefited bondholders lifting bond values halfway towards par. Line 4 shows what would happen if there is a bankruptcy code which prevents the creditor race but suffers from the problem of moral hazard so the effort to avoid bankruptcy falls to zero. (It is simply the valuation given earlier in Table 6.) It is fascinating to see that despite of this lack of effort bond values in row 4 are virtually the same as those in row 3. The reason why the input of effort fails to add value here is that “no code” doesn’t just make debtors work harder. It makes creditors run faster too!<sup>4</sup>

Finally the bottom line shows the results of combining a bankruptcy code with continued effort. It is obtained as a simple average of line 4 and par. This, our punch line, is designed to illustrate how, in principle, a debt standstill conditional on satisfying an IMF “program”

<sup>4</sup>We note that in corporate law, it seems that avoiding the Prisoner’s Dilemma weighs more heavily than moral hazard issues: “Bankruptcy, at first glance, may be thought of as a procedure geared principally toward relieving an overburdened debtor from ‘oppressive’ debt. Yet this discharge-centered view of bankruptcy is correct neither from an historical perspective nor from a realistic appraisal of the presence and operation of most of the provisions the federal bankruptcy law over the years.” Jackson (1982).

can avoid the losses of a creditor race without adverse incentives for effort!

#### IV INSTITUTIONAL IMPLICATIONS

In their paper on orderly workouts for sovereign debtors, Eichengreen and Portes (1995) proposed creating a Bondholders Council responsible for restructuring bonded debts. What about dissident creditors who seek to challenge a restructuring endorsed by others? They are sceptical of the possibility of closing the courts to such “rogue” creditors by an international treaty. Instead they proposed changes in bond covenants to permit a majority of creditors to alter the terms of payment.

As regards the problem of “vultures” who might precipitate a bond market collapse by starting a grab race, they discuss the possibility of “foreclosing the litigation option through reinterpretation or amendment of the IMF’s Articles of Agreement” Specifically they suggest that the IMF Executive Board could provide a new, definitive interpretation to Article VIII(2)(b) broadening its coverage: or, alternatively, that the article could be formally amended to make it applicable to defaults as well as exchange controls. (“If a government were to suspend repayment of its debt with the approval of the IMF, a debt contract which required payment in foreign currency might be rendered unenforceable in the courts of any IMF member state under the terms of Article VIII(2)(b).”)

But they go on to stress how very difficult it is to amend IMF articles (85% of the votes are needed and 50% of the member countries must approve) and how problematic it would be to get member states to implement provisions overriding domestic law. They invite the reader to “imagine the resistance in the national parliaments and congresses of the major creditor countries to legislation that would limit the rights of their investors overseas”: but they do not mention the losses that those same investors might suffer in creditor grab races without a code. So one is left with the impression that any change to the IMF articles is an optional extra and not an imperative.

The temptation to participate in a grab race seems increasingly likely to overcome current legal impediments. Even now some players are entering when the rewards are reasonably low. But what if “learning by suing” leads to a fall in costs? and bad luck strikes a large scale Brady bond debtor with substantial external assets? Without any legal changes and with no bailout, the result could be a creditor race leading to financial strangulation (the sovereign equivalent of early liquidation for a company). For sovereign states as for companies and individuals, it may be imperative rather than optional to implement legal changes to blunt the incentives to grab assets.

If the establishing an international Bankruptcy Court is not feasible, then legal changes

which will permit the IMF to implement an automatic stay may prove necessary. Indeed, as Williamson has noted, the ability to impose and withdraw a standstill at its discretion “would take the IMF as close to assuming the role of a bankruptcy court as it possible to imagine”.

## V CONCLUSIONS

There is a substantial volume of emerging-country bond debt in existence (much of it not covered by sovereign immunity); and unlike the syndicated lending there is no rule forcing a creditor who gains a favorable settlement to share the benefits. Maybe some smart financiers (“vultures”) will decide that it’s worth buying nonperforming debt on the cheap and suing for the face value. (Note that passing up an interest payment is technically default so capital becomes due, as well as the coupon.) This could be serious for the country concerned which could be forced into unilateral debt repudiation, with tangible assets being sequestered, and access to world markets in goods and capital severely impaired. But it could also be serious for other creditors, faced with the loss of bond coupons and collateral assets unless they too indulge in a “race of the vultures”. Indeed, it could that it will be the creditors themselves who insist on a bankruptcy code — for protection against other creditors!

This is the case explored in this paper. In an explicit model of sovereign debt valuation, we quantify the downside risk posed by this “free rider” problem, where individual bondholders have a substantial private incentive to precipitate action against countries facing liquidity crises. We argue in particular that this risk could affect Brady bonds.

Our findings run contrary to the conclusions of two recent reports on Resolving Sovereign Liquidity Crises. The first reason for this is because, in their focus on moral hazard, they seem to have overlooked the risks of intercreditor conflicts and their unhappy consequences. The second reason is their neglect of the incentive effects of IMF conditionality.

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## Appendices

### A Corporate Debt

#### A.1 Corporate debt with a bankruptcy code

Suppose that the firm is to use one unit of labour input to produce one unit of output with the price characterised by the following geometric Brownian motion

$$dP_t = \mu P_t dt + \sigma P_t dW_t \quad (\text{A1})$$

where  $P_t$  is the product price,  $\mu$  is the percentage trend,  $W_t$  is a standard Brownian motion and  $\sigma$  is its instantaneous variance.

Following Lambrecht and Perraudin (1995) we assume that the firm can costlessly switch off production when the cash flow turns negative, then the cash flow is given by

$$F_t = \max\{P_t - w, 0\}. \quad (\text{A2})$$

Suppose this cash flow is the only source of income available to the firm, if the firm has issued some infinite maturity debt  $D$  with a fixed coupon payment  $b = rD$ , then the actual coupon paid by the firm is

$$c_t = \min\{b, \max\{P_t - w, 0\}\}. \quad (\text{A3})$$

As long as  $c_t < b$ , the firm is in default.

Before preceding to determine the optimal trigger for the bondholder to file the bankruptcy, we first calculate the optimal bankruptcy trigger for the firm in the absence of debt. Let the value of equity be  $V(P)$ , then  $V(P)$  has to satisfy the following arbitrage condition

$$rV(P) = \begin{cases} P - w + \mu PV'(P) + (\sigma^2/2)P^2V''(P) & \text{for } P > w, \\ \mu PV'(P) + (\sigma^2/2)P^2V''(P) & \text{for } P \leq w. \end{cases} \quad (\text{A4})$$

For  $P > w$  the firm is able to pay the wage so the cash flow is  $P - w$ . However, when  $P \leq w$ , the firm simply mothballs production so the cash flow is zero.

The boundary conditions to (A4) are given as follows. If the current product price is sufficiently high, then the value of equity is simply the present discounted value of the cash flow  $P - w$ , i.e.,

$$\lim_{P \rightarrow \infty} V(P) = \lim_{P \rightarrow \infty} \left( \frac{P}{r - \mu} - \frac{w}{r} \right). \quad (\text{A5})$$

Let  $P_X$  be the optimal bankruptcy point for the firm, then

$$V(P_X) = S, \quad (\text{A6})$$

$$V'(P_X) = 0, \quad (\text{A7})$$

where  $S$  is the scrap value of the firm. As the firm can switch freely from production to mothballing, value match and smooth pasting conditions must apply at mothballing point  $P_m = w$ .

The solution to (A4) subject to (A5) is

$$V(P) = \begin{cases} P/(r - \mu) - w/r + A_- P^{\xi_-} & \text{for } P > w, \\ B_+ P^{\xi_+} + B_- P^{\xi_-} & \text{for } P \leq w. \end{cases} \quad (\text{A8})$$

Applying the other boundary conditions implies the optimal bankruptcy trigger

$$P_X^{\xi_+} = -\frac{(r - \mu)\xi_- w^{\xi_+ - 1} S}{1 - \xi_- \mu/r}. \quad (\text{A9})$$

How does the introduction of the debt alter this bankruptcy trigger? Suppose that in the event of bankruptcy it is the bondholder who can claim the residual value of the firm,  $S$ . Assume  $S < D$ , if then bankruptcy trigger  $P_B$  is less than the mothballing trigger  $P_m = w$ , the value of debt,  $L(P)$ , must satisfy the following arbitrage conditions

$$rL(P) = \begin{cases} \mu PL'(P) + (\sigma^2/2)P^2 L''(P) & \text{for } P \in (0, w] \\ P - w + \mu PL'(P) + (\sigma^2/2)P^2 L''(P) & \text{for } P \in (w, w + b] \\ b + \mu PL'(P) + (\sigma^2/2)P^2 L''(P) & \text{for } P \in (w + b, \infty) \end{cases} \quad (\text{A10})$$

As the firm can switch freely between regimes at points  $w$  and  $w + b$ , the boundary conditions at these two points are characterised by value matching and smooth pasting. When  $P_t$  is very large ( $P_t \rightarrow \infty$ ), coupon  $b$  is almost always paid, so  $\lim_{P \rightarrow \infty} L(P) = b/r$ . As the bankruptcy trigger  $P_b$  can be chosen optimally, the following value matching and smooth pasting conditions have to be satisfied

$$\begin{aligned} L(P_b) &= C, \\ L'(P_b) &= 0. \end{aligned}$$

Solving (A10) subject to asymptotic condition yields the following solution

$$L(P) = \begin{cases} C_+ P^{\xi_+} + C_- P^{\xi_-} & \text{for } P \in (0, w] \\ P/(r - \mu) - w/r + B_+ P^{\xi_+} + B_- P^{\xi_-} & \text{for } P \in (w, w + b] \\ b/r + A_- P^{\xi_-} & \text{for } P \in (w + b, \infty) \end{cases} \quad (\text{A11})$$

where  $\xi_{\pm}$  are positive and negative roots of

$$(\sigma^2/2)\xi(\xi - 1) + \mu\xi = r.$$

Using boundary conditions one can derive the optimal bankruptcy trigger

$$P_b^{\xi_+} = \frac{(r - \mu)\xi_- S}{(1 - \xi_- \mu/r)[(w + b)^{1 - \xi_+} - w^{1 - \xi_+}]}, \quad (\text{A12})$$

and the coefficient  $A_-$  is given by

$$A_- = -\frac{\xi_+ P_b^{-\xi_-} S}{\xi_- - \xi_+} - \frac{(\xi_+ \mu/r - 1)[(w+b)^{1-\xi_-} - w^{1-\xi_-}]}{(r-\mu)(\xi_- - \xi_+)}. \quad (\text{A13})$$

Equations (A12) and (A13) are what we use to carry out the simulations.

For  $P_b > w$ , following similar procedure, one obtains

$$P_b = \frac{(1 - \xi_- \mu/r)(w+b)^{1-\xi_+}}{1 - \xi_-} P_b^{\xi_+} - \frac{(r-\mu)\xi_- (S + w/r)}{1 - \xi_-}, \quad (\text{A14})$$

$$A_- = -\frac{\xi_+ P_b^{-\xi_-}}{\xi_- - \xi_+} \left( S + \frac{w}{r} - \frac{(\xi_- - 1)P_b}{\xi_- (r-\mu)} \right) - \frac{(\xi_+ \mu/r - 1)(w+b)^{1-\xi_-}}{(r-\mu)(\xi_- - \xi_+)}. \quad (\text{A15})$$

## A.2 Creditor races

Assume for simplicity that there are two creditors each lends half of the total face value of debt  $D$  to the firm. The solution described above is the cooperative equilibrium if both creditors act for their collective interests. However, in the absence of this cooperation and formal bankruptcy procedure, this equilibrium may unravel. In what follows we show how to obtain the new equilibrium using backward induction.

Let us assume that the creditor who act first will be able to grab part of the scrap value of the firm which is equal to the face value of the creditor's claim, then the new outcome will be the Nash equilibrium described in the following sense. Let the two creditors take preemptive actions in turn and suppose it is the creditor A who takes the preemptive action first. Taking the bankruptcy trigger  $P_b$  determined in the last section as given, A will grab the scrap value of the firm when  $L(P)/2 = S$ . This equation can be used to solve for the action trigger for A, say,  $P_A(S; 1)$ . It is obvious that  $P_A(S; 1) > P_b$ . If  $P_A(S; 1) \geq w + b$ , then this implies that the bankruptcy will occur at  $P = w + b$  which is the earliest time that creditors can take any action. If  $P_A(S; 1) < w + b$ , then it is B's turn to take the preemptive action against A. Taking the bankruptcy trigger induced by A's preemptive action as given, B will grab firm's assets when  $L(P; P_A)/2 = S$ . Since  $L(P; P_A) < L(P)$ , this yields  $P_B(S; 1) > P_A(S; 1)$ . Repeating this iterative process, one will finally arrive at the preemptive trigger at  $P = w + b$ , which is the Nash equilibrium in the absence of cooperation.

What will be the market value of the total debt in this case? Note that for  $P > w + b$  firm always pays the required coupon  $b$ , so the value of debt is given by (A11) for  $P > w + b$ . As the the bankruptcy occurred at  $w + b$  is irreversible, the boundary conditions needed are

$$L(w+b) = S, \quad (\text{A16})$$

$$\lim_{P \rightarrow \infty} L(P) = b/r. \quad (\text{A17})$$

This yields the value of debt

$$L(P) = \begin{cases} \frac{b}{r} - \left( \frac{b}{r} - S \right) \left( \frac{P}{w+b} \right)^{\xi_-} & \text{for } P > w+b, \\ S & \text{for } P \leq w+b. \end{cases} \quad (\text{A18})$$

## B Sovereign Debt

### B.1 Valuation of sovereign debt with “water-tight” immunity

Following Cohen (1993) and Bartolini and Dixit (1991) we assume that the capacity to service debt,  $X$ , is governed by a geometric Brownian motion with drift

$$dX_t = \mu X_t dt + \sigma X_t dW_t. \quad (\text{B1})$$

Let the fixed coupon payment of debt be  $b$ . When  $X_t > b$  full coupon is paid and otherwise  $X_t$  is paid.

In the absence of arbitrage the equilibrium condition for the debt must satisfy the following condition

$$rL(X) = \begin{cases} b + \mu XL'(X) + (\sigma^2/2)X^2L''(X), & \text{if } X > b, \\ X + \mu XL'(X) + (\sigma^2/2)X^2L''(X), & \text{if } X \leq b. \end{cases} \quad (\text{B2})$$

The boundary conditions to (A2) are given as follows. When  $X$  is sufficiently high, the value of debt is almost at par, i.e.,  $\lim_{X \rightarrow \infty} L(X) = b/r$ ; when the capacity falls to zero, nothing is paid, so the value of debt is zero ( $X = 0$  is an absorbing point). As the switch between the full and partial payments is reversible, value matching and smooth pasting conditions apply.

Using these boundary conditions one can solve the value of debt as

$$L(X) = \begin{cases} \frac{b}{r} + \frac{1}{\xi_+ - \xi_-} \left[ (\xi_+ - 1) \frac{b}{r - \mu} - \xi_+ \frac{b}{r} \right] \left( \frac{X}{b} \right)^{\xi_-} & \text{if } X > b, \\ \frac{X}{r - \mu} + \frac{1}{\xi_+ - \xi_-} \left[ (\xi_- - 1) \frac{b}{r - \mu} - \xi_- \frac{b}{r} \right] \left( \frac{X}{b} \right)^{\xi_+} & \text{if } X \leq b. \end{cases} \quad (\text{B3})$$

### B.2 Collateralized debt with a bankruptcy code

In the above case, creditors cannot seize the assets of the sovereign. In what follows, we consider what happens if the debt is collateralized, i.e., the creditors can seize the assets of the sovereign when bankruptcy occurs.

With a strictly positive collateral value,  $C$ , the value of debt will eventually value match and smooth paste to  $C$  at the point of bankruptcy,  $X_b$ . So the only boundary conditions which are different from the above case are

$$L(X_b) = C, \quad (\text{B4})$$

$$L'(X_b) = 0. \quad (\text{B5})$$

These imply that the solution for the value of the debt is given by

$$L(X) = \begin{cases} b/r + A_- X^{\xi_-}, & \text{if } X > b, \\ X/(r - \mu) + B_+ X^{\xi_+} + B_- X^{\xi_-} & \text{if } X \leq b. \end{cases} \quad (\text{B6})$$

Using boundary conditions one can solve for the bankruptcy trigger  $X_b$  and these coefficients. Specifically, we find

$$X_b = -\frac{(1 - \xi_- \mu/r)b^{1-\xi_+}}{\xi_- - 1} X_b^{\xi_+} + \frac{\xi_- (r - \mu)}{\xi_- - 1} C, \quad (\text{B7})$$

$$A_- = -\frac{X_b^{-\xi_-}}{\xi_- - \xi_+} \left[ (1 - \xi_+) \frac{X_b}{r - \mu} + \xi_+ C \right] - \frac{(\xi_+ \mu/r - 1)b^{1-\xi_-}}{(r - \mu)(\xi_- - \xi_+)}. \quad (\text{B8})$$

These equations are used to generate numerical examples.

### B.3 Collateralized debt without bankruptcy code

As we have argued in Appendix A, in the absence of a bankruptcy procedure, creditor races occurs. So the value of debt is equal to the collateral value whenever the sovereign is in default. This implies that the value of the debt will value match the collateral value  $C$  at default trigger  $X_d = b$ . Since this switch is irreversible, smooth pasting condition is not applicable. Given this the value of debt can be easily solved to yield

$$L(X) = \begin{cases} b/r - (b/r - C)(X/b)^{\xi_-}, & \text{if } X > b, \\ C, & \text{if } X \leq b. \end{cases} \quad (\text{B9})$$

## C The Pricing of the Brady Bond

The Brady bond is essentially composed of two parts: the deep discounted US treasury bond and the country's promise to service the debt. In what follows, we first evaluate the Brady bond when there is "water tight" sovereign immunity, and then we look at how the value of the bond is affected when there is a potential creditor race.

### C.1 Value of the Brady Bond without creditor race

As the deep discounted US treasury bond does not have the interest payment, let its maturity be  $T$  and terminal payment be  $B_T$ . Let the country's capacity to service the debt be  $X_t$  which is given by (B1), as  $X_t$  varies stochastically, the promised coupon payment must be contingent on  $X_t$ . Let the regular coupon payment be  $b$ , then the state contingent coupon payment is given by

$$\tilde{b}_t = \begin{cases} b, & \text{if } X_t \geq b; \\ X_t, & \text{if } X_t < b. \end{cases} \quad (\text{C10})$$

Let the riskless interest rate be  $r$ , under risk neutrality, the value of the Brady bond is simply

$$\begin{aligned} \tilde{v}(X_t, t) &= E_t \left[ \int_t^T \tilde{b}_s e^{-r(s-t)} ds + B_T e^{-r(T-t)} \right] \\ &= E_t \left[ \int_t^T \tilde{b}_s e^{-r(s-t)} ds \right] + B_T e^{-r(T-t)}. \end{aligned} \quad (\text{C11})$$

It is obvious from (C11) that the pricing of the Brady bond is separable, the first part is the present discounted value of the future coupon stream and the second part represents the present value of the US deep discount bond.

Rewrite (C11) as

$$v(X_t, t) \equiv \tilde{v}(X_t, t) - B_T e^{-r(T-t)} = E_t \left[ \int_t^T \tilde{b}_s e^{-r(s-t)} ds \right], \quad (\text{C12})$$

then using Feynman-Kac formula (Karatzas and Shreve, 1991, p366), (C12) is the solution of the following Cauchy problem

$$\begin{cases} -\frac{\partial v}{\partial t} + rv(X, t) = \frac{\sigma^2}{2} X^2 \frac{\partial^2 v}{\partial X^2} + \mu X \frac{\partial v}{\partial X} + \tilde{b}, & \text{on } (0, T) \times (0, +\infty); \\ v(X, T) = 0, & X > 0; \\ v(0, t) = 0, & 0 \leq t \leq T. \end{cases} \quad (\text{C13})$$

Let  $y = \ln X$  and  $s = T - t$ , then  $v(y, s) = v[X(y), t(s)]$ . Equation (C13) can be transformed to

$$\begin{cases} \frac{\partial v}{\partial s} + rv(y, s) = \frac{\sigma^2}{2} \frac{\partial^2 v}{\partial y^2} + (\mu - \sigma^2/2) \frac{\partial v}{\partial y} + \tilde{b}, & \text{on } (0, T) \times R^1; \\ v(y, 0) = 0, & y \in R^1; \\ v(-\infty, s) = 0, & 0 \leq s \leq T. \end{cases} \quad (\text{C14})$$

Let  $v(y, s) = e^{\alpha y - \beta s} u(y, s)$ ,  $\alpha = 1 - 2\mu/\sigma^2$ ,  $a^2 = \sigma^2/2$  and  $\beta = r + \alpha^2 a^2$ , then (C14) is transformed to

$$\begin{cases} \frac{\partial u}{\partial s} + ru(y, s) = a^2 \frac{\partial^2 u}{\partial y^2} + \tilde{b} e^{\beta s - \alpha y}, & \text{on } (0, T) \times R^1; \\ u(y, 0) = 0, & y \in R^1. \end{cases} \quad (\text{C15})$$

The variable coupon payment under these transformations becomes

$$\tilde{b}(y) = \begin{cases} b & \text{if } y \geq \ln b, \\ e^y & \text{if } y < \ln b. \end{cases} \quad (\text{C16})$$

Equation (C15) is a non-homogeneous heat equation in an infinite line. From Koshlyakov *et al* (1964, p490), (C15) has the following solution

$$u(y, s) = \int_0^s \int_{-\infty}^{+\infty} \tilde{b}(\xi) e^{\beta\tau - \alpha\xi} \frac{\exp\{-(\xi - y)^2/[4a^2(s - \tau)]\}}{2a\sqrt{\pi(s - \tau)}} d\xi d\tau. \quad (\text{C17})$$

Inverse the transformations one obtains

$$v(X, t) = b \int_0^{T-t} e^{-r\tau} \left\{ 1 - \Phi \left[ \frac{\ln(b/X) + (\sigma^2 - 2\mu)\tau}{\sigma\sqrt{2\tau}} \right] \right\} d\tau + X \int_0^{T-t} e^{-(r-2\mu+\sigma^2/2)\tau} \Phi \left[ \frac{\ln(b/X) - \mu\tau}{\sigma\sqrt{2\tau}} \right] d\tau, \quad (\text{C18})$$

where  $\Phi(\cdot)$  is a standard error function. Using (C12), one obtains the pricing for the Brady bond.