COOPERATIVE BARGAINING AND INTRAHOUSEHOLD ALLOCATIONS: DEMAND SYSTEMS AND COMPARATIVE STATICS
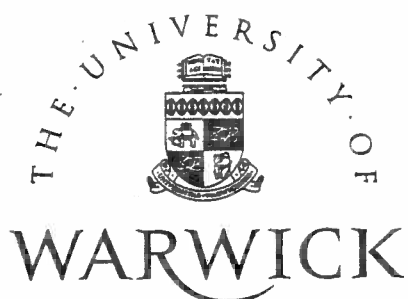
Cavelle Creightney

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Cooperative Bargaining and Intrahousehold Allocations: Demand Systems and Comparative Statics[1]

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Abstract

This paper examines the cooperative bargaining approach to intrahousehold distribution. The 2-person symmetric Nash model is extended to a generalized model for several household members. Also a model employing a generalized utilitarian solution is proposed and the intrahousehold models suggested by Chiappori (1992) are reviewed. It is argued that the traditional model is a special case of Chiappori's efficiency model, in turn a special case of the generalized utilitarian model which is in turn a special case of the generalized Nash model. The demand functions and comparative statics of these models are obtained and a structure of nested hypotheses is derived. These results extend those of McElroy (1990). It is also argued that the sharing rule and Pareto efficiency models proposed by Chiappori are not equivalent as has been suggested. The distinction between exogenous and endogenous household sharing rules is emphasized. Two further cooperative bargaining solutions, namely the dictatorial and the egalitarian solutions, are examined and applied to the intrahousehold bargaining problem. The demand functions implied by these solutions and the comparative statics of the models are obtained and discussed. The analyses in this paper show that different cooperative bargaining solutions yield different predictions for demand functions and for the response of individual and household demands to parameter changes. The choice of an appropriate bargaining solution cannot be settled theoretically and is therefore essentially an empirical question.

1 Introduction

Models of intrahousehold resource allocation rely primarily on the cooperative bargaining framework to analyze intrahousehold distribution. The main questions addressed are:

(i) What are the processes, mechanisms or rules by which resources are allocated among members of a household?

(ii) What intrahousehold distributions (in terms of consumption and leisure/labour supply outcomes, health outcomes, human capital investment outcomes, etc.) are predicted by different cooperative bargaining solution concepts?

(iii) Are there significant differences in the outcomes for different members of a household? Specifically are male members treated differently from female members, adults from children, the elderly from the young, relatives from non-relatives? How much intra-household inequality is there?

(iv) How do intrahousehold interactions affect the design of policies towards specific household members and the response of individual household members to a given policy?

The use of a cooperative bargaining approach presupposes the existence of an enforceable and binding agreement on the members of the household to accept some particular bargaining solution. How such an agreement might have come about is not considered.

This paper examines the demand systems and comparative statics for an individual household member predicted by a selection of cooperative bargaining solutions. Section 2 reviews intrahousehold distribution in the traditional model of the household. This model is well-known and the individual demands and comparative statics are summarized only so as to facilitate comparison with cooperative models. Section 3 examines the cooperative bargaining approach to intrahousehold distribution. The most commonly used bargaining solutions are summarized and extended. In particular, the 2-person symmetric Nash model is generalized so as to take account of asymmetries in the bargaining power of household members. Also, a model employing a generalized utilitarian solution is put forward. The Pareto efficiency model proposed by Chiappori (1992) is reviewed and it is shown that this model is identical to the generalized utilitarian model with the disagreement points set to zero. Finally the sharing rule model, also proposed by Chiappori (1992), is reviewed and is shown not to generate the same results as the efficiency model, as suggested by Chiappori (1992). In section 4, a whole structure of nested hypotheses is proposed: it is shown that the traditional model is nested within the efficiency model which is nested within the

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generalized utilitarian model which is in turn nested within the generalized Nash model. It is also shown that the sharing rule model is not nested within any of these preceeding models. The results of this section are an extension of those of McElroy (1990) in which the traditional model is shown to be nested within a symmetric Nash model. Section 5 argues that each of the models discussed so far imply an intrahousehold sharing rule. However the distinction is made between pre-determined sharing rules, as implied by the sharing rule model, and endogenously determined sharing rules, as implied by the remaining models. Sections 6 and 7 examine two further cooperative bargaining solutions, namely the dictatorial and the egalitarian solutions, and their application to the intrahousehold bargaining problem. The demand functions implied by these solutions and the comparative statics of the models are obtained and discussed.

The analyses in this paper illustrate that the choice of bargaining solution is important as they yield different predictions for demand functions and for the response of individual demands to parameter changes. Since theory does not enable us to choose one solution over another (except as required for analytical convenience), the choice of an appropriate bargaining solution must essentially be an empirical question.

2   The traditional household model

The traditional model of the household, including the restrictions on demands and empirical specifications, has been extensively analyzed and is now well-known (see Goldberger (1967), Barten (1977), Leuthold (1968), Ashenfelter and Heckman (1974) and Wales and Woodland (1976)). Its demands and empirical restrictions are summarized here only so that they can be compared with those of the cooperative bargaining household models.

The traditional model is commonly referred to as the classical model, the common preference model or the unitary model. Where there are H household members, $h = 1, ..., H$, each consuming a vector of n private goods, $\mathbf{X}^h = \left(X_1^h, ..., X_n^h \right)'$, and leisure, N^h, the traditional model implies that the household maximizes a well-behaved household welfare function

$$U \left(\mathbf{X}^1, ..., \mathbf{X}^H, N^1, ..., N^H \right)$$

subject to a household budget constraint

$$\mathbf{P}'\mathbf{X} + \mathbf{W}'\mathbf{N} = \mathbf{I}'\mathbf{i} + \mathbf{W}'\mathbf{T},$$

where $\mathbf{X} = \sum_{h=1}^{H} \mathbf{X}^h$ represents the n-vector of private goods consumed by the household, i.e. aggregated over each member of the household; $\mathbf{P}' = (P_1, \ldots\ldots, P_n)$ represents the n-vector of private goods prices common to each member of the household; $\mathbf{W}' = \left(W^1, \ldots\ldots, W^H \right)$ is the h-vector of wage rates of each household member; $\mathbf{N}' = \left(N^1, \ldots\ldots, N^H \right)$ is the h-vector of leisure hours enjoyed by each household member; $\mathbf{I}' = \left(I^1, \ldots\ldots, I^H \right)$ is the h-vector of nonwage incomes of each household member; T is the time endowment available to each household member for leisure, N^h, and for market work, l^h; and $\mathbf{T}'$ is the h-vector of T's.

In such a model, the opportunity cost of household membership is irrelevant since household members are not regarded as having the option of leaving the household. Also the question of intrahousehold bargaining does not arise. The familiar commodity and leisure demand functions of individual h are given by

$$\mathbf{X}_i^h = \mathbf{X}_i^h \left[\mathbf{P}, W^h, \sum_{h=1}^{H} I^h \right],$$

$$N^h = N^h \left[\mathbf{P}, W^h, \sum_{h=1}^{H} I^h \right],$$

$$\forall h = 1, \ldots\ldots, H,$$

$$i = 1, \ldots\ldots, n.$$

In this model, the total cross-price effect for person h consists of a substitution effect and an income effect:

$$\frac{\partial X_i^h}{\partial P_k} = \frac{\partial X_i^h}{\partial P_k}\bigg|_{\bar{U}} + \frac{1}{H} \sum_{j=1}^{H} \frac{\partial X_i^h}{\partial I^j} \left(-X_k \right),$$

where

$$-X_k = \frac{\partial \sum_{j=1}^{H} I^j}{\partial P_k}.$$

The effect of another member's wage on h's demands are given by

$$\frac{\partial X_i^h}{\partial W_j} = \frac{1}{H} \sum_{j=1}^{H} \frac{\partial X_i^h}{\partial I^j} \left(T - N^j \right),$$

where

$$\left(T - N^j \right) = \frac{\partial \sum_{j=1}^{H} I^j}{\partial W_j}.$$

The effect of a change in non-wage incomes on person h's demands are given by

$$\frac{\partial X_i^h}{\partial I^j} = -g_i \neq 0, \forall j = 1, ..., H,$$

where g_i is the i-th element in the $(n+1)$-vector $\mathbf{g}$ of the inverted bordered Hessian matrix

$$\begin{bmatrix} \underset{((n+1)\times(n+1))}{\mathbf{a}} & \underset{((n+1)\times 1)}{\mathbf{g}} \\ \underset{(1\times(n+1))}{\mathbf{g}'} & \underset{1\times 1}{q} \end{bmatrix}.$$

In this model, a change in person h's income has the same impact on h's demands as a change in person j's income:

$$\frac{\partial X_i^h}{\partial I^h} - \frac{\partial X_i^h}{\partial I^j} = -g_i - (-g_i) = 0.$$

This is referred to as income-pooling.

3 Cooperative bargaining approaches to intrahousehold distribution

Cooperative models of intrahousehold resource allocation employ the basic structure and concepts of cooperative bargaining theory. In an intrahousehold bargaining model with H household members, there are H agents who face a set of possible utility allocations,

$$\mathbf{U} \subset \mathbf{R}^H,$$

as long as they are members of the same household. Each agent's utility, $U^h(.)$, is defined over a n-vector of private consumption goods, $\mathbf{X}^h = \left(X_1^h, ..., X_n^h\right)'$ and own-leisure, N^h. Each agent faces a time and full income budget constraint given respectively by

$$T = N^h + l^h$$

and

$$\mathbf{P}'\mathbf{X}^h + W^h N^h = I^h + W^h T.$$

$U^h\left(\mathbf{X}^h, N^h\right)$ is taken to be quasiconcave, twice continuously differentiable and strictly increasing in all of its arguments. As long as the H agents are

members of the same household, then the set of possible utility allocations **U**, can be defined as

$$\mathbf{U} \equiv \left\{ U = \left(U^1 \left(\mathbf{X}^1, N^1 \right), \ldots\ldots, U^H \left(\mathbf{X}^H, N^H \right) \right) : \mathbf{P'X} + \mathbf{W'N} = \mathbf{I'i} + \mathbf{W'T} \right\}.$$

This guarantees that the utility possibility set is convex. Each agent may have different preferences over these alternative utility allocations. If they agree on a particular allocation, then that is what they get. Otherwise, they get a pre-specified alternative in the utility possibility set,

$$V = \left(V^1 \left(\mathbf{P}, I^1, W^1, \alpha^1 \right), \ldots\ldots, V^H \left(\mathbf{P}, I^H, W^H, \alpha^H \right), \right) \in \mathbf{U}$$

called the disagreement point. $V^h \left(\mathbf{P}, I^h, W^h, \alpha^h \right)$ represents the maximum utility available to h if not a member of the household (the opportunity cost of household membership). In such a case, h maximizes

$$U^h \left(\mathbf{X}^h, N^h \right)$$

subject to

$$T = N^h + l^h$$

and

$$\mathbf{P'X}^h + W^h N^h = I^h + W^h T.$$

This yields individual demand functions given by

$$\begin{aligned} \mathbf{X}^h &= X \left(\mathbf{P}, W^h, I^h \right), \\ N^h &= N \left(\mathbf{P}, W^h, I^h \right), \end{aligned}$$

and indirect utility functions given by

$$U^h \left(X \left(\mathbf{P}, W^h, I^h \right), N \left(\mathbf{P}, W^h, I^h \right) \right) = V^h \left(\mathbf{P}, I^h, W^h, \alpha^h \right).$$

The parameter α^h is included among the other exogenous parameters which affect the disagreement point. This parameter is regarded as a shift parameter (also called an efficiency parameter or extra-household environmental parameter) since it affects the level of utility obtained as a non-household member for any given price-income combination. A key example of a shift parameter would be any systematic differences in the prices and nonwage incomes of those living alone and those living as part of a group. This might

be the result of the tax and benefit system which treats single people differently from those living as a group. For instance married persons allowance which is not received when single would constitute a shift parameter. Likewise, benefits received by single mothers which are not payed when married is another example. Also, private income transfers from parents or other family members which are conditional on family status should be treated as a shift parameter. In general, any government or private transfer that is available in one state but not in another should be treated as a shift parameter. Another source of shifts in the utility level may be the psychic benefits or costs of living alone or with others. If a person values companionship in itself, then the utility from being single may be lower for any given price-income combination. On the other hand if a person is of a more solitary nature and values privacy quite highly, then the shift parameter may result in a higher level of utility for any given set of prices and incomes.

In a bargaining problem, the utility possibility set is bounded and closed (contains its boundary). The boundary is the utility possibility frontier which is the set of all $U = \left(U^1 \left(\mathbf{X}^1, N^1 \right), ..., U^H \left(\mathbf{X}^H, N^H \right) \right) \in \mathbf{U}$ which are Pareto optimal. $\left(U^1 \left(\mathbf{X}^1, N^1 \right), ..., U^H \left(\mathbf{X}^H, N^H \right) \right)$ is Pareto optimal if there exists no other $\left(U^1 \left(\mathbf{X}^1, N^1 \right), ..., U^H \left(\mathbf{X}^H, N^H \right) \right)'$ in $\mathbf{U}$ such that $\left(U^1 \left(\mathbf{X}^1, N^1 \right), ..., U^H \left(\mathbf{X}^H, N^H \right) \right)' > \left(U^1 \left(\mathbf{X}^1, N^1 \right), ..., U^H \left(\mathbf{X}^H, N^H \right) \right)$ for all $h = 1, ..., H$. For a bargaining problem to exist, there must be at least one point of $\mathbf{U}$ which strictly dominates V, that is which yields a higher utility to each agent than that obtained at the disagreement point. This is necessary since no one would agree to an allocation which left them worse off than if they were not a member of the household.

What are the sources of the gains to household formation? Gains may result from the presence of household shared goods, from economies of scale in the production of household goods and from the benefits of love and companionship. If such potential gains do not exist, there can be no bargaining problem. If there are potential gains, then the agents have an incentive to become or to remain members of the household. The bargaining problem involves choosing a feasible level of household consumption and leisures, and distributing the gains from household membership among household members. Cooperative bargaining models try to predict the compromise the agents will reach. A solution to a bargaining problem is a rule that associates with each bargaining problem, $(\mathbf{U}, V)$, the compromise which will be reached, $f(\mathbf{U}, V)$. Each solution satisfies a set of properties or axioms which represent what is thought to be sensible or fair. Several solutions

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have been explored in the literature on cooperative bargaining. The first axiomatic characterization of a solution is the Nash solution. This solution maximizes the product of utility gains from the disagreement point, $\prod_{h=1}^{H}\left(U^h - V^h\right)$, where $U, V \in \mathbf{U}$, and $U \geq V$. Other solutions include the Kalai-Smorodinsky solution, the egalitarian solution, the dictatorial solution, the utilitarian solution, and others (see Thomson (1994) for a review of cooperative bargaining theory).

The early models of intrahousehold allocations which explicitly adopt a cooperative bargaining framework are those of Manser and Brown (1978, 1980), Brown and Manser (1977, 1978), McElroy and Horney (1981) and McElroy (1990). In those papers, the authors explore extensively the demand functions and comparative static properties of the 2-person, symmetric Nash model. The 2-person dictatorial and Kalai-Smorodinsky models are also briefly considered. They also examine the empirical distinctions between those models and those of the traditional approach outlined above. In later papers (Chiappori 1988, 1992), the author departs from the cooperative bargaining framework and proposes a model in which the intrahousehold allocations are assumed to be Pareto efficient (and this is the only assumption made). However it is shown later on in this paper that this model is in fact identical to an intrahousehold model employing a generalized utilitarian bargaining solution, the only difference being that the disagreement points are set to zero. Chiappori (1992) also proposes another approach to intrahousehold modelling, the sharing rule approach, and suggests that this is equivalent to the efficiency model. However it is also shown later that this model yields predictions which are different from those of the efficiency model.

3.1 Derivation of comparative statics and demand systems

In what follows, person h's preferences are represented by the utility function $U^h\left(\mathbf{X}^h, N^h\right)$ which is twice continuously differentiable, quasi-concave and strictly increasing in all of its arguments. In each model discussed below, the household's objective and constraint functions are taken to be real-valued, twice continuously differentiable, quasiconcave functions defined on $\mathbf{R}^R$, where R is the number of choice variables in the model. The household's problem is to choose

$$\left(X_1^{1*}, ..., X_n^{1*}, N^{1*}, ..., X_1^{H*}, ..., X_n^{H*}, N^{H*}\right) \in \mathbf{R}^R$$

so as to maximize the overall household objective function. The assumptions on the objective and constraint functions ensure that the first-order

conditions derived below are necessary and sufficient for a global maximum.

In order to derive the demand systems and the comparative statics for each model, the fundamental equations of comparative statics need to be obtained. This is done by totally differentiating each first-order condition for a particular household member with respect to each choice variable and exogenous parameter, re-arranging and solving simultaneously for the comparative static equations. It is assumed that the solutions are interior solutions and that the household budget constraint is in effect at

$$\left(X_1^{1*}, ..., X_n^{1*}, N^{1*}, ..., X_1^{H*}, ..., X_n^{H*}, N^{H*} \right).$$

The bordered Hessian matrix is evaluated at given levels of the exogenous parameters.

In matrix notation, the fundamental equations for a particular member h are given by:

$$
\begin{bmatrix}
(n) & \mathbf{X}_{i\mathbf{P}}^h & \mathbf{X}_{iW^h}^h & \mathbf{X}_{i\mathbf{W}^{-h}}^h & \mathbf{X}_{i\mathbf{I}}^h & \mathbf{X}_{i\mathbf{V}}^h & \mathbf{X}_{i\alpha}^h & \mathbf{X}_{i\lambda}^h \\
(1) & N_{\mathbf{P}}^h & N_{W^h}^h & N_{\mathbf{W}^{-h}}^h & N_{\mathbf{I}}^h & N_{\mathbf{V}}^h & N_\alpha^h & N_\lambda^h \\
(r) & \boldsymbol{\gamma}_{\mathbf{P}} & \boldsymbol{\gamma}_{W^h} & \boldsymbol{\gamma}_{\mathbf{W}^{-h}} & \boldsymbol{\gamma}_{\mathbf{I}} & \boldsymbol{\gamma}_{\mathbf{V}} & \boldsymbol{\gamma}_\alpha & \boldsymbol{\gamma}_\lambda \\
 & (n) & (1) & (h-1) & (h) & (h) & (h) & (h)
\end{bmatrix}
$$

$$
= -
\begin{bmatrix}
(n) & \mathbf{L}_{\mathbf{X}_i^h \mathbf{X}_i^h} & \mathbf{L}_{\mathbf{X}_i^h N^h} & \mathbf{L}_{\mathbf{X}_i^h \gamma} \\
(1) & \mathbf{L}_{N^h \mathbf{X}_i^h} & \mathbf{L}_{N^h N^h} & \mathbf{L}_{N^h \gamma} \\
(r) & \mathbf{L}_{\gamma \mathbf{X}_i^h} & \mathbf{L}_{\gamma N^h} & \mathbf{L}_{\gamma \gamma} \\
 & (n) & (1) & (r)
\end{bmatrix}^{-1}
$$

$$
\times
\begin{bmatrix}
(n) & \mathbf{L}_{\mathbf{X}_i^h \mathbf{P}} & \mathbf{L}_{\mathbf{X}_i^h W^h} & \mathbf{L}_{\mathbf{X}_i^h \mathbf{W}^{-h}} & \mathbf{L}_{\mathbf{X}_i^h \mathbf{I}} & \mathbf{L}_{\mathbf{X}_i^h \mathbf{V}} & \mathbf{L}_{\mathbf{X}_i^h \alpha} & \mathbf{L}_{\mathbf{X}_i^h \lambda} \\
(1) & \mathbf{L}_{N^h \mathbf{P}} & \mathbf{L}_{N^h W^h} & \mathbf{L}_{N^h W^{-h}} & \mathbf{L}_{N^h \mathbf{I}} & \mathbf{L}_{N^h \mathbf{V}} & \mathbf{L}_{N^h \alpha} & \mathbf{L}_{N^h \lambda} \\
(r) & \mathbf{L}_{\gamma \mathbf{P}} & \mathbf{L}_{\gamma W^h} & \mathbf{L}_{\gamma \mathbf{W}^{-h}} & \mathbf{L}_{\gamma \mathbf{I}} & \mathbf{L}_{\gamma \mathbf{V}} & \mathbf{L}_{\gamma \alpha} & \mathbf{L}_{\gamma \lambda} \\
 & (n) & (1) & (h-1) & (h) & (h) & (h) & (h)
\end{bmatrix},
$$

where r is the number of constraints in the optimization problem; $\mathbf{X}_{i\mathbf{P}}^h$, for instance, represents the matrix $\left[\frac{\partial X_i^h}{\partial \mathbf{P}} \right]$ in which the number of rows is the number of elements in the vector $\mathbf{X}^h$, and the number of columns is the number of elements in the vector $\mathbf{P}$; the superscript $-h$ to a vector represents that vector with the h-th term omitted; λ represents the h-vector of intrahousehold bargaining weights (described later); L represents the Lagrangian of the problem; $\mathbf{L_{XY}} = \left[\frac{\partial^2 L}{\partial \mathbf{X} \partial \mathbf{Y}} \right]$ is a matrix in which the number of rows is equal to the number of elements in the vector $\mathbf{X}$ and the number of

columns is the number of rows in the vector $\mathbf{Y}$; and $\underset{(1\times h)}{\mathbf{i}'} = (1,1,1,......,1,)$.

The numbers in parentheses indicate the number of rows and columns in each matrix.

The matrix

$$= - \begin{bmatrix} (n) & \mathbf{L}_{\mathbf{X}_i^h \mathbf{X}_i^h} & \mathbf{L}_{\mathbf{X}_i^h N^h} & \mathbf{L}_{\mathbf{X}_i^h \gamma} \\ (1) & \mathbf{L}_{N^h \mathbf{X}_i^h} & \mathbf{L}_{N^h N^h} & \mathbf{L}_{N^h \gamma} \\ (r) & \mathbf{L}_{\gamma \mathbf{X}_i^h} & \mathbf{L}_{\gamma N^h} & \mathbf{L}_{\gamma\gamma} \\ & (n) & (1) & (r) \end{bmatrix}^{-1}$$

is the bordered Hessian matrix evaluated at given levels of the exogenous parameters and can be partitioned as

$$\begin{bmatrix} \underset{((n+1)\times(n+1))}{\mathbf{A}} & \underset{((n+1)\times(r))}{\mathbf{G}} \\ \underset{((r)\times(n+1))}{\mathbf{G}'} & \underset{(r\times r)}{\mathbf{Q}} \end{bmatrix}.$$

It is assumed that:

$$\det \begin{bmatrix} \underset{((n+1)\times(n+1))}{\mathbf{A}} & \underset{((n+1)\times(r))}{\mathbf{G}} \\ \underset{((r)\times(n+1))}{\mathbf{G}'} & \underset{(r\times r)}{\mathbf{Q}} \end{bmatrix} \neq 0,$$

and

$$\text{Rank} \underset{((n+1)\times(r))}{\mathbf{G}} = r.$$

These assumptions along with the existence of a local maximum ensure that the second order sufficient conditions for a maximum hold, namely:

$$\xi' \mathbf{A} \xi < 0$$

for all

$$\xi \neq 0$$

such that

$$\mathbf{G}\xi = 0.$$

From this we have that

$$\begin{bmatrix} \underset{((n+1)\times(n+1))}{\mathbf{A}} & \underset{((n+1)\times(r))}{\mathbf{G}} \\ \underset{((r)\times(n+1))}{\mathbf{G}'} & \underset{(r\times r)}{\mathbf{Q}} \end{bmatrix}^{-1}$$

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exists and is given by

$$
\left[
\begin{array}{ccc}
(n+1) & \mathbf{a} & \mathbf{g} \\
(r) & \mathbf{g}' & q \\
 & (n+1) & (r)
\end{array}
\right],
$$

and by the Caratheodory-Samuelson theorem, $\underset{((n+1)\times(n+1))}{\mathbf{a}}$ is symmetric and negative semi definitie.

The solutions to the fundamental equations are derived in the appendix. In the following sections, the demand functions and some of the key comparative statics are presented.

3.2   A generalized Nash bargaining solution

In this section, the 2-person symmetric Nash model of intrahousehold distribution is extended to a generalized Nash model for H household members. The possibility of an asymmetric distribution of bargaining power is explicitly allowed, and the demand functions and comparative statics of the model are presented. In a generalized Nash model, each person's utility is assigned a bargaining weight $\lambda^h\left(\mathbf{P},\mathbf{W},\mathbf{I},\boldsymbol{\alpha}\right)$, where the sum of these weights add up to one. The weights may be thought of as representing the degree of bargaining power that each member has, and the distribution of bargaining power is taken to be a function of the exogenous prices, wages, incomes and shift parameters. The generalized Nash problem is therefore one of maximizing

$$
\prod_{h=1}^{H}\left[U^h\left(\mathbf{X}^h,N^h\right)-V^h\left(\mathbf{P},I^h,W^h,\alpha^h\right)\right]^{\lambda^h(\mathbf{P},\mathbf{W},\mathbf{I},\boldsymbol{\alpha})}
$$

subject to

$$
\mathbf{P}'\mathbf{X}+\mathbf{W}'\mathbf{N}=\mathbf{I}'\mathbf{i}+\mathbf{W}'\mathbf{T}.
$$

Taking logs, the resulting Lagrangian is

$$
L=\sum_{h=1}^{H}\lambda^h\left(\mathbf{P},\mathbf{W},\mathbf{I},\boldsymbol{\alpha}\right)Log\left[U^h\left(\mathbf{X}^h,N^h\right)-V^h\left(\mathbf{P},I^h,W^h,\alpha^h\right)\right]+\gamma\left[\mathbf{I}'\mathbf{i}+\mathbf{W}'\left(\mathbf{T}-\mathbf{N}\right)-\mathbf{P}'\mathbf{X}\right],
$$

where γ is the Lagrange multiplier, and the first-order conditions are given by:

$$
\begin{aligned}
\frac{\partial L}{\partial X_i^h} &= \lambda^h\left(\mathbf{P},\mathbf{W},\mathbf{I},\boldsymbol{\alpha}\right)\frac{1}{g_h}\frac{\partial U^h}{\partial X_i^h}-\gamma P_i\leq 0, X_i^h\geq 0, \\
h &= 1,......,H
\end{aligned}
$$

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$$
\begin{aligned}
i &= 1, \ldots\ldots, n, \\
\frac{\partial L}{\partial N^h} &= \lambda^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right) \frac{1}{g_h} \frac{\partial U^h}{\partial N^h} - \gamma W_h \leq 0, N^h \geq 0, \\
h &= 1, \ldots\ldots, H \\
\frac{\partial L}{\partial \gamma} &= \mathbf{I}'\mathbf{i} + \mathbf{W}'\left(\mathbf{T} - \mathbf{N}\right) - \mathbf{P}'\mathbf{X} \geq 0, \gamma \leq 0,
\end{aligned}
$$

where

$$
g_h = U^h \left(\mathbf{X}^h, N^h\right) - V^h \left(\mathbf{P}, I^h, W^h, \alpha^h\right).
$$

3.2.1 Demand functions and comparative statics of a generalized Nash model

The generalized Nash commodity and leisure demand functions for person h are:

$$
\begin{aligned}
\mathbf{X}_i^h &= \mathbf{X}_i^h \left[\mathbf{P}, W^h, V^h \left(\mathbf{P}, W^h, I^h, \alpha^h\right), \lambda^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right)\right], \\
N^h &= N^h \left[\mathbf{P}, W^h, V^h \left(\mathbf{P}, W^h, I^h, \alpha^h\right), \lambda^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right)\right], \\
\forall i &= 1, \ldots\ldots, n \\
h &= 1, \ldots\ldots, H
\end{aligned}
$$

where

$$
\lambda^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right) = 1 - \sum_{j=1}^{H} \lambda^j \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right), j \neq h.
$$

The total uncompensated effect of a price change in this model is different from that of the traditional model. It is given by:

$$
\frac{\partial X_i^h}{\partial P_k} = \frac{\partial X_i^h}{\partial P_k}\Big|_{\bar{U}} + \left[\frac{\partial X_i^h}{\partial V^h}\frac{\partial V^h}{\partial P_k} + \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial P_k}\right] + \frac{1}{H}\left[\sum_{j=1}^{H}\left(\frac{\partial X_i^h}{\partial I^j} - \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial I^j}\right) - \frac{\partial X_i^h}{\partial V^h}\frac{\partial V^h}{\partial I_h}\right]\left(-X_k\right).
$$

From this one can see that apart from the usual substituion and income effects of the traditional model, there are two additional effects which have consequences for individual demands. These are (i) the change in the intrahousehold distribution of bargaining power due to the price change, and (ii) the change in person h's disagreement point brought about by the price change.

A change in person j's wage affects h's demands via an income effect and via the redistribution of intrahousehold bargaining power:

$$
\frac{\partial X_i^h}{\partial W_j} = \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial W_j} + \frac{1}{H}\left[\sum_{j=1}^{H}\left(\frac{\partial X_i^h}{\partial I^j} - \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial I^j}\right) - \frac{\partial X_i^h}{\partial V^h}\frac{\partial V^h}{\partial I^h}\right]\left(T - N^j\right).
$$

Non-wage incomes affect individual demands as follows:

$$\frac{\partial X_i^h}{\partial I^h} = \frac{\partial X_i^h}{\partial V^h}\frac{\partial V^h}{\partial I^h} + \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial I^h} - g_i,$$

$$\frac{\partial X_i^h}{\partial I^j} = \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial I^j} - g_i, \forall j = 1, ..., H, j \neq h.$$

In addition to the usual income effect, there is also a change in the intra-household distribution of bargaining power which affects demands. In the case of a change in person h's income this also affects person h's demands via their disagreement point.

The impact of a change in the disagreement points are:

$$\frac{\partial X_i^h}{\partial V^h} \neq 0, \frac{\partial X_i^h}{\partial V^j} = 0, \forall j = 1, ..., H, j \neq h.$$

While person h's disagreement point affects $h's$ demand, a change in j's disagreement point has no such effect.

The effect of the shift parameters on individual h's demands are given by:

$$\frac{\partial X_i^h}{\partial \alpha^h} = \frac{\partial X_i^h}{\partial V^h}\frac{\partial V^h}{\partial \alpha^h} + \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial \alpha^h},$$

$$\frac{\partial X_i^h}{\partial \alpha^j} = \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial \alpha^j}, \forall j = 1, ..., H, j \neq h.$$

Finally, as one would expect, an improvement in h's bargaining power has a positive effect on h's demands while an improvement in j's bargaining power has a negative effect:

$$\frac{\partial X_i^h}{\partial \lambda^h} > 0, \frac{\partial X_i^h}{\partial \lambda^j} < 0, \forall j = 1, ..., H, j \neq h, \frac{\partial \lambda^h}{\partial \lambda^j} = -1.$$

3.3 A generalized utilitarian solution

Before discussing the contributions of Chiappori (1988, 1992), first a generalized utilitarian model is proposed. It will then be shown that the intra-household model suggested by Chiappori based on the assumption of Pareto efficiency is a special case of the generalized utilitarian model in which the disagreement points are set to zero. The generalized utilitarian bargaining solution implies that the household maximizes the weighted sum of individual utility gains:

$$\sum_{h=1}^{H} \lambda_h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right) \left[U^h \left(\mathbf{X}^h, N^h\right) - V^h \left(\mathbf{P}, I^h, W^h, \alpha^h\right)\right]$$

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subject to the household budget

$$\mathbf{P'X} + \mathbf{W'N} = \mathbf{I'i} + \mathbf{W'T}.$$

The resulting Lagrangian is:

$$L = \sum_{h=1}^{H} \lambda^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right) \left[U^h \left(\mathbf{X}^h, N^h\right) - V^h \left(\mathbf{P}, I^h, W^h, \alpha^h\right)\right] + \gamma \left[\mathbf{I'i} + \mathbf{W'} \left(\mathbf{T} - \mathbf{N}\right) - \mathbf{P'X}\right],$$

where γ is the Lagrange multiplier, and the first-order conditions are given by:

$$\frac{\partial L}{\partial X_i^h} = \lambda^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right) \frac{\partial U^h}{\partial X_i^h} - \gamma P_i \leq 0, X_i^h \geq 0,$$
$$h = 1, \dots, H$$
$$i = 1, \dots, n,$$
$$\frac{\partial L}{\partial N^h} = \lambda^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right) \frac{\partial U^h}{\partial N^h} - \gamma W_h \leq 0, N^h \geq 0,$$
$$h = 1, \dots, H$$
$$\frac{\partial L}{\partial \gamma} = \mathbf{I'i} + \mathbf{W'} \left(\mathbf{T} - \mathbf{N}\right) - \mathbf{P'X} \geq 0, \gamma \leq 0.$$

3.3.1 Demand functions and comparative statics for the generalized utilitarian model

The commodity and leisure demand functions of the generalized utilitarian model for individual h are:

$$\mathbf{X}_i^h = \mathbf{X}_i^h \left[\mathbf{P}, W^h, \lambda^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right)\right],$$
$$N^h = N^h \left[\mathbf{P}, W^h, \lambda^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right)\right],$$
$$\forall i = 1, \dots, n$$
$$h = 1, \dots, H$$

where

$$\lambda^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right) = 1 - \sum_{j=1}^{H} \lambda^j \left(\mathbf{P}, \mathbf{W}, \mathbf{I}, \boldsymbol{\alpha}\right), j \neq h.$$

In the generalized utilitarian model, the total uncompensated effect of a price change consists of the usual income and substitution effects and an

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additional effect which reflects the impact of price changes on the intra-household distribution of bargaining power and the effect this has on h's demands. This is given by:

$$\frac{\partial X_i^h}{\partial P_k} = \frac{\partial X_i^h}{\partial P_k}\bigg|_{\bar{U}} + \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial P_k} + \frac{1}{H}\left[\sum_{j=1}^{H}\left(\frac{\partial X_i^h}{\partial I^j} - \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial I^j}\right)\right](-X_k).$$

The effect of individual j's wage on h's demands consists of an income effect and an effect via the distribution of bargaining power:

$$\frac{\partial X_i^h}{\partial W_j} = \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial W_j} + \frac{1}{H}\left[\sum_{j=1}^{H}\left(\frac{\partial X_i^h}{\partial I^j} - \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial I^j}\right)\right]\left(T - N^j\right).$$

The effect of a change in non-wage incomes on individual demands are given by:

$$\frac{\partial X_i^h}{\partial I^j} = \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial I^j} - g_i, \forall j = 1, ..., H.$$

The disagreement points in this model have no impact on demands:

$$\frac{\partial X_i^h}{\partial V^j} = 0, \forall j = 1, ..., H.$$

The shift parameters affect individual h's demands via their impact on the bargaining power distribution:

$$\frac{\partial X_i^h}{\partial \alpha^j} = \frac{\partial X_i^h}{\partial \lambda^h}\frac{\partial \lambda^h}{\partial \alpha^j}, \forall j = 1, ..., H.$$

Finally, as in the Nash model, an improvement in h's bargaining power has a positive effect on h's demands while an improvement in j's bargaining power has a negative effect:

$$\frac{\partial X_i^h}{\partial \lambda^h} > 0, \frac{\partial X_i^h}{\partial \lambda^j} < 0, \forall j = 1, ..., H, j \neq h, \frac{\partial \lambda^h}{\partial \lambda^j} = -1.$$

3.4 Chiappori's model of intrahousehold distribution

3.4.1 The efficiency approach

In Chiappori (1992), a model of collective decision making for a 2-person, 3-good household with egoistic agents is proposed in which the only assumption made is that the chosen intrahousehold allocation is Pareto efficient.

No restriction is imposed a priori on which point of the Pareto frontier the household chooses. The Pareto efficiency assumption is formally expressed as:

Maximize

$$U^h \left(\mathbf{X}^h, N^h \right)$$

subject to

$$U^j \left(\mathbf{X}^j, N^j \right) \geq u^j \left(\mathbf{P}, \mathbf{W}, \mathbf{I} \right), \forall j = 1, ..., H, j \neq h,$$

and

$$\mathbf{P'X} + \mathbf{W'N} = \mathbf{I'i} + \mathbf{W'T},$$

for some given utility level

$$u^j \left(\mathbf{P}, \mathbf{W}, \mathbf{I} \right), \forall j = 1, ..., H, j \neq h.$$

For any given price-income combination, the chosen Pareto efficient outcome varies as $u^j \left(\mathbf{P}, \mathbf{W}, \mathbf{I} \right)$ varies within its domain.

Although Chiappori argues that this characterization of the problem is more general than those which employ a specific cooperative bargaining solution, it is shown in this section that this approach is equivalent to the generalized utilitarian setting with the disagreement points set to zero. The demand functions and comparative statics implied by the efficiency approach are identical to those of the generalized utilitarian approach when the disagreement points are zero. As a result, this efficiency approach is in fact nested within the generalized utilitarian model. It is also shown later that the generalized utilitarian model is in turn nested within the generalized Nash bargaining model.

First note that Chiappori correctly points out that the efficiency model may equivalently be written as:

Maximize

$$U^h \left(\mathbf{X}^h, N^h \right) + \sum_{j=1, j \neq h}^{H} \mu_j \left(\mathbf{P}, \mathbf{W}, \mathbf{I} \right) U^j \left(\mathbf{X}^j, N^j \right)$$

subject to the household budget, where the μ_j's represent the Lagrange multipliers of the first $H - 1$ constraints in the previous specification of the problem. If this new maximand is multiplied through by the constant $\frac{1}{1+\mu}$, where $\mu = \sum_j \mu_j$, then the maximand becomes

$$\frac{1}{1+\mu} U^h \left(\mathbf{X}^h, N^h \right) + \sum_{j=1, j \neq h}^{H} \frac{\mu_j \left(\mathbf{P}, \mathbf{W}, \mathbf{I} \right)}{1+\mu} U^j \left(\mathbf{X}^j, N^j \right),$$

where the utility weights now sum to one. The problem may now equivalently be written as maximize

$$\sum_{h=1}^{H} \lambda_h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}\right) U^h \left(\mathbf{X}^h, N^h\right)$$

subject to

$$\mathbf{P}'\mathbf{X} + \mathbf{W}'\mathbf{N} = \mathbf{I}'\mathbf{i} + \mathbf{W}'\mathbf{T},$$

where

$$\sum_{h=1}^{H} \lambda_h \left(\mathbf{P}, \mathbf{W}, \mathbf{I}\right) = 1.$$

This is exactly the generalized utilitarian bargaining solution, the only difference here being that if the agents fail to reach a compromise, they receive nothing.

Demand functions and comparative statics for the efficiency model
Derivation of the first order conditions for the efficiency model show that they are identical to those of the generalized utilitarian model. Likewise the demand functions and comparative statics of the efficiency model are the same as those of the generalized utilitarian model. The only difference is in the role of the shift parameters which Chiappori did not include in the efficiency model, hence

$$\frac{\partial X_i^h}{\partial \alpha^j} = 0, \forall j = 1, ..., H.$$

Recall that the generalized utilitarian approach implied that

$$\frac{\partial X_i^h}{\partial \alpha^j} = \frac{\partial X_i^h}{\partial \lambda^h} \frac{\partial \lambda^h}{\partial \alpha^j} \neq 0, \forall j = 1, ..., H.$$

3.4.2 The sharing rule approach

Chiappori (1992) points out that an alternative way to think about the efficiency approach is to assume that the decision process is a two-stage budgeting one. Household members first divide the total nonlabour income received by the household among themselves, according to some predetermined sharing rule. Once household income has been allocated in this way, the household members then carry out their individual constrained utility

maximizations. This approach may therefore be characterized as each agent maximizing

$$U^h\left(\mathbf{X}^h, N^h\right)$$

subject to

$$\mathbf{P}'\mathbf{X}^h + W^h N^h = I^h + W^h T + \phi^h\left(\mathbf{P}, \mathbf{W}, \mathbf{I}\right) = \Phi^h\left(\mathbf{P}, \mathbf{W}, \mathbf{I}\right),$$

where

$$\sum_{h=1}^{H} \phi^h\left(\mathbf{P}, \mathbf{W}, \mathbf{I}\right) \equiv 0,$$

$$\sum_{h=1}^{H} \Phi^h\left(\mathbf{P}, \mathbf{W}, \mathbf{I}\right) = \sum_{h=1}^{H}\left(I^h + W^h T + \phi^h\left(\mathbf{P}, \mathbf{W}, \mathbf{I}\right)\right)$$

$$= \sum_{h=1}^{H} I^h + T \sum_{h=1}^{H} W^h.$$

In this problem, the predetrmined sharing rule is represented by $\phi^h\left(\mathbf{P}, \mathbf{W}, \mathbf{I}\right)$ and is a function of the given exogenous parameters. Any solution to these H individul constrained maximization problems is Pareto efficient. As the exogenous prices and incomes vary, the sharing rule varies and a set of Pareto efficient outcomes is generated.

The Lagrangian for any of the H problems is given by:

$$L = U^h\left(\mathbf{X}^h, N^h\right) + \gamma\left[\phi^h\left(\mathbf{P}, \mathbf{W}, \mathbf{I}\right) + I^h + W^h\left(T - N^h\right) - \mathbf{P}'\mathbf{X}^h\right],$$

where γ is the Lagrange multiplier, and the first-order conditions are:

$$\frac{\partial L}{\partial X_i^h} = \frac{\partial U^h}{\partial X_i^h} - \gamma P_i \leq 0, X_i^h \geq 0,$$
$$h = 1, \ldots, H$$
$$i = 1, \ldots, n,$$
$$\frac{\partial L}{\partial N^h} = \frac{\partial U^h}{\partial N^h} - \gamma W_h \leq 0, N^h \geq 0,$$
$$h = 1, \ldots, H$$
$$\frac{\partial L}{\partial \gamma} = \phi^h\left(\mathbf{P}, \mathbf{W}, \mathbf{I}\right) + I^h + W^h\left(T - N^h\right) - \mathbf{P}'\mathbf{X}^h \geq 0, \gamma \leq 0.$$

Demand functions and comparative statics for the sharing rule approach   An examination of the demand functions and comparative statics of the sharing rule approach shows that they are different from those of the efficiency approach (recall that the latter were identical to the generalized utilitarian demands and comparative statics). The sharing rule demand functions are given by:

$$
\begin{aligned}
\mathbf{X}_i^h &= \mathbf{X}_i^h \left[\mathbf{P}, W^h, \phi^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I} \right) \right], \\
N_i^h &= N_i^h \left[\mathbf{P}, W^h, \phi^h \left(\mathbf{P}, \mathbf{W}, \mathbf{I} \right) \right], \\
\forall h &= 1, ..., H, \\
i &= 1, ..., n.
\end{aligned}
$$

The total uncompensated effect of a price change consists of a substitution effect, an income effect, and an effect due to the intrahousehold redistribution of income which occurs:

$$
\begin{aligned}
\frac{\partial X_i^h}{\partial P_k} &= \frac{\partial X_i^h}{\partial P_k}\Big|_{\bar{U}} + \frac{\partial X_i^h}{\partial \phi^h} \left(\frac{\partial \phi^h}{\partial P_k} - X_k^h \right) \\
&= \frac{\partial X_i^h}{\partial P_k}\Big|_{\bar{U}} + \frac{\partial X_i^h}{\partial \phi^h} \frac{\partial \phi^h}{\partial P_k} + \frac{\partial X_i^h}{\partial \phi^h} \left(-X_k^h \right).
\end{aligned}
$$

The effect of individual j's wage on h's demands are given by:

$$
\frac{\partial X_i^h}{\partial W_j} = \frac{\partial X_i^h}{\partial \phi^h} \frac{\partial \phi^h}{\partial W_j}.
$$

As in the efficiency model, j's wage affects h's demands by changing the intrahousehold distribution of bargaining power. However unlike the efficiency model, the income effect of a change in j's wage on h's demands in this model is zero.

The effect of a change in non-wage incomes on individual demands are given by:

$$
\frac{\partial X_i^h}{\partial I^j} = \frac{\partial X_i^h}{\partial \phi^h} \frac{\partial \phi^h}{\partial I^j}, \forall j = 1, ..., H.
$$

Recall that there were no disagreement points nor shift parameters in the sharing rule model, and so the effects of these parameters on individual demands are zero:

$$
\begin{aligned}
\frac{\partial X_i^h}{\partial V^j} &= 0, \forall j = 1, ..., H, \\
\frac{\partial X_i^h}{\partial \alpha^j} &= 0, \forall j = 1, ..., H.
\end{aligned}
$$

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Finally, while there are no bargaining weights in the sharing rule approach, the sharing rule itself may be taken as a measure of the intrahousehold distribution of bargaining power. In this case, the comparative statics for the sharing rules are similar to those for the bargaining weights in the efficiency model. Recall that in the efficiency model we have:

$$\frac{\partial X_i^h}{\partial \lambda_h} > 0, \frac{\partial X_i^h}{\partial \lambda^j} < 0, \forall j = 1, ..., H, j \neq h, \frac{\partial \lambda^h}{\partial \lambda^j} = -1.$$

In the sharing rule approach we have instead:

$$\frac{\partial X_i^h}{\partial \phi^h(.)} > 0, \frac{\partial X_i^h}{\partial \phi^j(.)} < 0, \forall j = 1, ..., H, j \neq h, \frac{\partial \phi^h(.)}{\partial \phi^j(.)} = -1.$$

4 Nesting structure of traditional and cooperative bargaining models

In McElroy (1990) it is shown that the comparative statics of the traditional model were nested within those of the 2-person symmetric Nash model. In the preceeding sections of this paper, the demand functions and comparative statics were derived for the traditional approach as well as for certain cooperative bargaining approaches to intrahousehold resource allocation. The results of McElroy (1990) can now be extended to show that the comparative statics of the traditional model are nested within those of Chiappori's efficiency model which are in turn nested within those of the generalized utilitarian model which are in turn nested within the comparative statics of the generalized Nash model. (To see clearly the nesting structure which is suggested, substitute the proposed restrictions into the fundamental matrix equations of the relevant model in order to obtain the restricted model. See the appendix for the fundamental matrix equations).

More specifically, restriction of the generalized Nash comparative static equations so that:

$$V_{\mathbf{P}}^h = V_{W^h}^h = V_{I^h}^h = V_{\alpha^h}^h = 0, g^h = 1, \text{and } g_{V^h}^h = 0$$

yields the generalized utilitarian comparative static equations. Restriction of the generalized utilitarian comparative static equations so that:

$$\lambda_\alpha^h = 0$$

yields the comparative statics of Chiappori's efficiency model. Finally, restriction of the comparative static equations of Chiappori's efficiency model

so that:

$$\lambda_{\mathbf{P}}^h = \lambda_{\mathbf{W}}^h = \lambda_{\mathbf{I}}^h = 0, \lambda^h = c, \text{const.} \forall h = 1, ..., H, \lambda_{\boldsymbol{\lambda}}^h = 0$$

yields the comparative statics of the traditional model.

The discussion so far shows that the generalized Nash solution is the most general of the solutions discussed so far, and this contradicts Chiappori's (1992) claim that the efficiency model is more general than the Nash model. Chiappori also argued that the efficiency approach to intrahousehold allocations was equivalent to the sharing rule approach. Again the analysis of the preceeding sections would suggest that this was not the case. The comparative statics of both the efficiency and the sharing rule approaches are distinctly different from each other, and neither one is clearly nested within the other. Likewise, the compartive statics of the sharing rule approach are different from those of the generalized utilitarian and the generalized Nash models and are not nested within the latter.

However, when the sharing rule approach is compared to the traditional approach, one can see that imposing

$$\phi_{\mathbf{P}}^h = \mathbf{0}, \phi_{W^h}^h = 0, \phi_{\mathbf{I}}^h = \mathbf{i}', \text{and } \phi_{\mathbf{W}^{-h}}^h = \mathbf{T}' - \mathbf{N}^{-h\prime}$$

on the comparative statics of the sharing rule model yields the comparative statics of the traditional model. Note that

$$\phi_{\mathbf{I}}^h = \mathbf{i}'$$

implies that

$$\phi_{Ij}^h = 1, \forall j = 1, ..., H,$$

and hence

$$\phi^h = \sum_{j=1}^{H} I^j, \forall h = 1, ..., H.$$

From this point of view, the restriction

$$\phi_{\mathbf{W}^{-h}}^h = \mathbf{T}' - \mathbf{N}^{-h\prime}$$

can be interpreted as

$$\frac{\partial \phi^h}{\partial W^j} = \frac{\partial \sum_{j=1}^{H} I^j}{\partial W^j} = T - N^j, \forall j = 1, ..., H, j \neq h,$$

which is what one obtains in the traditional model.

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It is interesting to note that by imposing the restrictions

$$\lambda_\alpha^h = \lambda_{\mathbf{P}}^h = \lambda_{\mathbf{W}}^h = \lambda_{\mathbf{I}}^h = 0, \lambda^h = c, \text{const.} \forall h = 1, ..., H, \lambda_\lambda^h = 0$$

on the generalized utilitarian model, one obtains the same comparative statics as in the traditional model. Hence maximizing

$$c \sum_{h=1}^H \left[U^h \left(\mathbf{X}^h, N^h \right) - V^h \left(\mathbf{P}, I^h, W^h, \alpha^h \right) \right],$$

which is additively separable, subject to the household budget constraint implies the same restrictions on the demand functions as maximizing

$$U^h \left(X^1, ..., X^H, N^1, ..., N^H \right)$$

which is not additively separable. The reason for this is that the first-order conditions of the restricted generalized utilitarian model and of the traditional model are such that in the fundamental matrix equations, while the matrix

$$\begin{bmatrix} (n) & \mathbf{L}_{\mathbf{X}_i^h \mathbf{X}_i^h} & \mathbf{L}_{\mathbf{X}_i^h N^h} & \mathbf{L}_{\mathbf{X}_i^h \gamma} \\ (1) & \mathbf{L}_{N^h \mathbf{X}_i^h} & \mathbf{L}_{N^h N^h} & \mathbf{L}_{N^h \gamma} \\ (r) & \mathbf{L}_{\gamma \mathbf{X}_i^h} & \mathbf{L}_{\gamma N^h} & \mathbf{L}_{\gamma \gamma} \\ & (n) & (1) & (r) \end{bmatrix}$$

differs in each case, the matrix

$$\begin{bmatrix} (n) & \mathbf{L}_{\mathbf{X}_i^h \mathbf{P}} & \mathbf{L}_{\mathbf{X}_i^h W^h} & \mathbf{L}_{\mathbf{X}_i^h W^{-h}} & \mathbf{L}_{\mathbf{X}_i^h \mathbf{I}} & \mathbf{L}_{\mathbf{X}_i^h V} & \mathbf{L}_{\mathbf{X}_i^h \alpha} & \mathbf{L}_{\mathbf{X}_i^h \lambda^h} \\ (1) & \mathbf{L}_{N^h \mathbf{P}} & \mathbf{L}_{N^h W^h} & \mathbf{L}_{N^h W^{-h}} & \mathbf{L}_{N^h \mathbf{I}} & \mathbf{L}_{N^h V} & \mathbf{L}_{N^h \alpha} & \mathbf{L}_{N^h \lambda^h} \\ (r) & \mathbf{L}_{\gamma \mathbf{P}} & \mathbf{L}_{\gamma W^h} & \mathbf{L}_{\gamma W^{-h}} & \mathbf{L}_{\gamma \mathbf{I}} & \mathbf{L}_{\gamma V} & \mathbf{L}_{\gamma \alpha} & \mathbf{L}_{\gamma \lambda^h} \\ & (n) & (1) & (h-1) & (h) & (h) & (h) & (h) \end{bmatrix}$$

turns out to be identical in both cases. When solving for the comparative static equations, the former matrix is inverted and then substituted out entirely so that the fact that it is different for both models does not matter for the solved out comparative static equations.

5 Intrahousehold sharing rules

Chiappori's sharing rule approach explicitly thinks of the household decision as consisting of two stages: first, an intrahousehold distribution of household income takes place, and then second, each household member maximizes his

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or her own utility subject to the income available to them after the sharing rule has been determined. However in this model, the intrahousehold sharing rule is pre-determined and the focus is placed on the second stage, i.e. maximization of each individual utility subject to the sharing rule. It should be noted that this is not the only model which implies an intrahousehold sharing rule. Chiappori's Pareto efficiency model (PE), the generalized utilitarian model (U) and the generalized Nash model (N) also imply the existence of an intrahousehold sharing mechanism. In these cases however, the sharing rule is endogenously determined. This can be seen by recalling that each of these models will lead to an intrahousehold allocation that is Pareto efficient. The structure of the models all imply that any allocation on the Pareto frontier can be achieved for some appropriately chosen distribution of bargaining power; likewise any distribution of bargaining power will lead to the choice of a unique allocation on the Pareto frontier. The intrahousehold sharing rule in PE, U and N can therefore be taken to be represented by the intrahousehold distribution of bargaining weights. The latter are choice variables of the models and hence depend on the exogenous parameters. (Note that the models can be rewritten so that the bargaining weights are equivalently treated as Lagrange multipliers and hence are choice variables of the models). Chiappori's sharing rule model therefore restricts the chosen intrahousehold allocation to that which would be chosen under some pre-specified sharing rule. Chiappori's Pareto efficiency model, the generalized utilitarian model, and the generalized Nash model, on the other hand, allow the intrahousehold sharing rule to be determined within the model.

6 The dictatorial model

In this section, an intrahousehold model is developed in which one household member has the power to determine the distribution of the gains of household membership among the remaining members. Such a model was considered in Manser and Brown (1980) for the 2-person household. In that paper, it is suggested that the Slutsky matrix need not be a symmetric negative semidefinite matrix. In this section, the demand functions and comparative statics are systematically derived and it is shown in the appendix that the Slutsky matrix is indeed symmetric and negative semidefinitie. Clearly the dictatorial solution may appear to be an extreme assumption to adopt, however whether or not it may be appropriate in specific contexts is an empirical question.

According to the dictatorial solution where person h is the dictator, h maximizes her or his gain while constraining the remaining household members to their disagreement points and satisfying the household budget. The problem is therefore to maximize

$$U^h\left(\mathbf{X}^h, N^h\right) - V^h\left(\mathbf{P}, I^h, W^h, \alpha^h\right)$$

subject to

$$U^k\left(\mathbf{X}^k, N^k\right) = V^k\left(\mathbf{P}, I^k, W^k, \alpha^k\right), \forall k = 1, ..., H, k \neq h$$

and

$$\mathbf{P'X} + \mathbf{W'N} = \mathbf{I'i} + \mathbf{W'T}.$$

The Lagrangian and first-order conditions are respectively

$$\begin{aligned}
L &= U^h\left(\mathbf{X}^h, N^h\right) - V^h\left(\mathbf{P}, I^h, W^h, \alpha^h\right) \\
&+ \sum_{k=1, k\neq h}^{H} \rho_k\left[U^k\left(\mathbf{X}^k, N^k\right) - V^k\left(\mathbf{P}, I^k, W^k, \alpha^k\right)\right] \\
&+ \gamma\left[\mathbf{I'i} + \mathbf{W'}\left(\mathbf{T} - \mathbf{N}\right) - \mathbf{P'X}\right],
\end{aligned}$$

where the ρ's and γ represent Lagrange multipliers, and

$$\begin{aligned}
\frac{\partial L}{\partial X_i^h} &= \frac{\partial U^h}{\partial X_i^h} - \gamma P_i \leq 0, X_i^h \geq 0, \\
i &= 1, ..., n, \\
\frac{\partial L}{\partial N^h} &= \frac{\partial U^h}{\partial N^h} - \gamma W_h \leq 0, N^h \geq 0, \\
\frac{\partial L}{\partial \rho_k} &= U^k\left(\mathbf{X}^k, N^k\right) - V^k\left(\mathbf{P}, I^k, W^k, \alpha^k\right) \geq 0, \rho_k \leq 0, \\
k &= 1, ..., H, k \neq h \\
\frac{\partial L}{\partial \gamma} &= \mathbf{I'i} + \mathbf{W'}\left(\mathbf{T} - \mathbf{N}\right) - \mathbf{P'X} \geq 0, \gamma \leq 0.
\end{aligned}$$

6.1 Demand functions and comparative statics of the dictatorial model

In the dictatorial model, the dictator's leisure and commodity demand functions are given by

$$\mathbf{X}_i^h = \mathbf{X}_i^h \left[\mathbf{P}, W^h, \sum_{j=1}^{H} I^j, V^j \left(\mathbf{P}, W^j, I^j, \alpha^j \right), j = 1, ..., H, j \neq h \right],$$

$$N_i^h = N^h \left[\mathbf{P}, W^h, \sum_{j=1}^{H} I^j, V^j \left(\mathbf{P}, W^j, I^j, \alpha^j \right), j = 1, ..., H, j \neq h \right],$$

$$\forall i = 1, ..., n,$$

while those of the other household members are given by: $\bar{\mathrm{U}}$

$$\mathbf{X}_i^j = \mathbf{X}_i^j \left(\mathbf{P}, I^j, W^j, \alpha^j \right), j = 1, ..., H, j \neq h,$$

$$N^j = N^j \left(\mathbf{P}, I^j, W^j, \alpha^j \right), j = 1, ..., H, j \neq h,$$

$$\forall i = 1, ..., n.$$

The total uncompensated effect of a price change on the dictator's demands consists of a substitution effect, an income effect and a further effect which is the impact that a price change has on the disagreement points of each household member and the impact of the latter on the dictator's demands:

$$\frac{\partial X_i^h}{\partial P_k} = \frac{\partial X_i^h}{\partial P_k}\bigg|_{\bar{U}} + \sum_{j=1, j \neq h}^{H} \frac{\partial X_i^h}{\partial V^j} \frac{\partial V^j}{\partial P_k} + \frac{\partial X_i^h}{\partial I^h} \left(-X_k \right).$$

The effect of individual j's wage on the dictator's demands consists of an income effect and also an effect via the change in j's disagreement point:

$$\frac{\partial X_i^h}{\partial W_j} = \frac{\partial X_i^h}{\partial V^j} \frac{\partial V^j}{\partial W_j} + \frac{\partial X_i^h}{\partial I^h} \left(T - N^j \right).$$

The effect of a change in non-wage incomes on dictatorial demands consists of the regular income effect as well as an effect via the impact on j's disagreement point:

$$\frac{\partial X_i^h}{\partial I^h} = -g_i^2,$$

$$\frac{\partial X_i^h}{\partial I^j} = \frac{\partial X_i^h}{\partial V^j} \frac{\partial V^j}{\partial I^j} - g_i^2, \forall j = 1, ..., H, j \neq h,$$

where g_i^2 is the i-th element in the vector $\underset{((n+1)\times 1)}{\mathbf{g}^2}$ (see appendix).

The dictator's own disagreement point has no impact on his or her own demands. However, a change in the disagreement point of the other household members will affect the demands of the dictator:

$$\frac{\partial X_i^h}{\partial V^h} = 0, \frac{\partial X_i^h}{\partial V^j} \neq 0, \forall j = 1, ..., H, j \neq h.$$

The impact of the shift parameters on dictatorial demands is given by:

$$\frac{\partial X_i^h}{\partial \alpha^h} = 0, \frac{\partial X_i^h}{\partial \alpha^j} = \frac{\partial X_i^h}{\partial V^j} \frac{\partial V^j}{\partial \alpha^j}, \forall j = 1, ..., H, j \neq h.$$

Finally, since the dictator possesses all the bargaining power in this model, there is no question of a possible change in the intrahousehold distribution of bargaining power. The only options available to the other household members are either to accept the level of utility consistent with their disagreement points or to leave the household.

7   The egalitarian model

The egalitarian solution has so far not been applied to the analysis of intrahousehold resource allocation. According to the egalitarian solution, the gains to household membership are distributed equally among the members of the household. The problem is therefore to maximize

$$U^h\left(\mathbf{X}^h, N^h\right) - V^h\left(\mathbf{P}, I^h, W^h, \alpha^h\right)$$

subject to

$$U^h\left(\mathbf{X}^h, N^h\right) - V^h\left(\mathbf{P}, I^h, W^h, \alpha^h\right) = U^k\left(\mathbf{X}^k, N^k\right) - V^k\left(\mathbf{P}, I^k, W^k, \alpha^k\right), \forall k = 1, ..., H, k \neq h$$

and

$$\mathbf{P}'\mathbf{X} + \mathbf{W}'\mathbf{N} = \mathbf{I}'\mathbf{i} + \mathbf{W}'\mathbf{T}.$$

The Lagrangian is given by

$$
\begin{aligned}
L = \ & U^h\left(\mathbf{X}^h, N^h\right) - V^h\left(\mathbf{P}, I^h, W^h, \alpha^h\right) \\
& + \sum_{k=1, k \neq h}^{H} \rho_k \left[U^h\left(\mathbf{X}^h, N^h\right) - V^h\left(\mathbf{P}, I^h, W^h, \alpha^h\right) - U^k\left(\mathbf{X}^k, N^k\right) + V^k\left(\mathbf{P}, I^k, W^k, \alpha^k\right) \right] \\
& + \gamma \left[\mathbf{I}'\mathbf{i} + \mathbf{W}'\left(\mathbf{T} - \mathbf{N}\right) - \mathbf{P}'\mathbf{X} \right],
\end{aligned}
$$

where the ρ's and γ represent Lagrange multipliers, and the first-order conditions are

$$\frac{\partial L}{\partial X_i^h} = \frac{\partial U^h}{\partial X_i^h} + \sum_{k=1,k\neq h}^{H} \rho_k \frac{\partial U^h}{\partial X_i^h} - \gamma P_i \leq 0, X_i^h \geq 0,$$

$$i = 1, \ldots\ldots, n,$$

$$\frac{\partial L}{\partial N^h} = \frac{\partial U^h}{\partial N^h} + \sum_{k=1,k\neq h}^{H} \rho_k \frac{\partial U^h}{\partial N^h} - \gamma W_h \leq 0, N^h \geq 0,$$

$$\frac{\partial L}{\partial \rho_k} = U^h\left(\mathbf{X}^h, N^h\right) - V^h\left(\mathbf{P}, I^h, W^h, \alpha^h\right) - U^k\left(\mathbf{X}^k, N^k\right) + V^k\left(\mathbf{P}, I^k, W^k, \alpha^k\right) \geq 0, \rho_k \leq 0$$

$$k = 1, \ldots\ldots, H, k \neq h$$

$$\frac{\partial L}{\partial \gamma} = \mathbf{I}'\mathbf{i} + \mathbf{W}'\left(\mathbf{T} - \mathbf{N}\right) - \mathbf{P}'\mathbf{X} \geq 0, \gamma \leq 0.$$

7.1 Demand functions and comparative statics of the egalitarian solution

The egalitarian demand functions for person h are:

$$\mathbf{X}_i^h = \mathbf{X}_i^h\left[\mathbf{P}, W^h, V^j\left(\mathbf{P}, W^j, I^j, \alpha^j\right), j = 1, ..., H,\right],$$

$$N_i^h = N_i^h\left[\mathbf{P}, W^h, V^j\left(\mathbf{P}, W^j, I^j, \alpha^j\right), j = 1, ..., H,\right],$$

$$\forall i = 1, \ldots\ldots, n.$$

The total uncompensated effect of a price change on person h's demands consists of a substitution effect, an income effect, and an effect caused by the change in the disagreement points of each household member:

$$\frac{\partial X_i^h}{\partial P_k} = \frac{\partial X_i^h}{\partial P_k}\bigg|_{\bar{U}} + \sum_{j=1}^{H} \frac{\partial X_i^h}{\partial V^j}\frac{\partial V^j}{\partial P_k} + \left[\frac{\partial X_i^h}{\partial I^h} - \frac{\partial X_i^h}{\partial V^h}\frac{\partial V^h}{\partial I^h}\right]\left(-X_k\right).$$

As in the dictatorial model, the Slutsky matrix is symmetric and negative semi-definite. The arguments are exactly analogous to those of the dictatorial model.

The effect of individual j's wage on h's demands consists of an income effect and an effect via the change in j's disagreement point:

$$\frac{\partial X_i^h}{\partial W_j} = \frac{\partial X_i^h}{\partial V^j}\frac{\partial V^j}{\partial W_j} + \left[\frac{\partial X_i^h}{\partial I^h} - \frac{\partial X_i^h}{\partial V^h}\frac{\partial V^h}{\partial I^h}\right]\left(T - N^j\right).$$

The effect of a change in non-wage incomes on individual demands is given by:

$$\frac{\partial X_i^h}{\partial I^j} = \frac{\partial X_i^h}{\partial V^j}\frac{\partial V^j}{\partial I^j} - g_i^2, \forall j = 1, ..., H,$$

where g_i^2 is the i-th element in the vector $\underset{((n+1)\times 1)}{\mathbf{g^2}}$ (see appendix).

The impact of the disagreement points on the demands of person h is nonzero:

$$\frac{\partial X_i^h}{\partial V^j} \neq 0, \forall j = 1, ..., H.$$

The impact of shift parameters on individual h's demands is given by:

$$\frac{\partial X_i^h}{\partial \alpha^j} = \frac{\partial X_i^h}{\partial V^j}\frac{\partial V^j}{\partial \alpha^j}, \forall j = 1, ..., H.$$

Finally, in an egalitarian model, as in the dictatorial model, there is no role for bargaining weights since the household members agree to equalize the gains to household membership.

Examination of the fundamental comparative static equations for both the dictatorial and the egalitarian models suggest that there are no set of restrictions which, when imposed on the egalitarian model, allow the egalitarian model to collapse to the dictatorial model. Neither model is nested within the other. Instead, these two models suggest two sets of non-nested hypotheses which need to be tested as such.

References

[Ashenfelter, O. and J. Heckman (1974).] 'The Estimation of Income and Substitution Effects in a Model of Family Labour Supply.' *Econometrica*, Vol. 43, pp. 73-85.

[Barten, A. P. (1977).] 'The Systems of Consumer Demand Functions Approach: A Review.' Econometrica, Vol. 45, pp. 23-51.

[Brown, M. and M. Manser (1977).] 'Estimation of the Demand for Marriage Based on a Bargaining Model.' Discussion Paper, No. 419, State University of New York at Buffalo.

[Brown, M. and M. Manser (1978).] 'Neoclassical and bargaining Approaches to Household decision-Making with Application to the Household Labour Supply Decision.' Discussion Paper, No. 401, State University of New York at Buffalo.

[Chiappori, P-A. (1988).] 'Rational Household Labour Supply.' *Econometrica*, Vol. 56, No. 1, pp. 63-89.

[Chiappori, P-A. (1992).] 'Collective Labour Supply and Welfare.' *Journal of Political Economy*, Vol. 100, No. 3, pp. 437-467.

[Goldberger, A. S. (1967).] 'Functional Form and Utility: A Review of Consumer Demand Theory.' Workshop Paper SFM6703, Social Systems Research Institute, University of Wisconsin.

[Leuthold, J. (1968).]   'An Empirical Study of Formula Income transfers and the Work Decision of the Poor.' *The Journal of Human Resources*, Vol. 3, pp. 312-323.

[Manser, M. and M. Brown (1978).]   'Bargaining Analysis of Household Decisions.' Discussion Paper, No. 423, State University of New York at Buffalo.

[Manser, M. and M. Brown (1980).]   'Marriage and Household Decision-Making: A Bargaining Analysis.' *International Economic Review*, Vol. 21, No. 1, pp. 31-44.

[McElroy, M. (1990).]   'The Empirical Content of Nash-Bargained Household Behaviour.' *The Journal of Human Resources*, Vol. 25, No. 4, pp. 559-583.

[McElroy, M. and M. Horney (1981).]   'Nash-Bargained Household Decisions: Toward a Generalization of the Theory of Demand.' *International Economic Review*, Vol. 22, No. 2, pp. 333-349.

[Thomson, W. (1994).]   'Cooperative Models of Bargaining' in Aumann R. J. and S. Hart (ed.) *Handbook of Game Thoery, Vol. 2*, Elsevier Science B.V., 1994.

[Wales, T. J. and A. D. Woodland (1976).]   'Estimation of Household Utility Functions and Labour Supply Response.' *International Economic Review*, Vol. 17, pp. 397-410.

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APPENDIX: Solution of the fundamental equations of comparative statics

Following the method outlined in section 3.1, the fundamental equations of comparative statics for each of the models discussed in the paper are derived and solved below.

Generalized Nash model

In deriving the comparative statics and demand system predicted by the generalized Nash solution it is noted that:

$$
\begin{bmatrix} \mathbf{X}_{iV^h}^h \\ N_{V^h}^h \end{bmatrix}
= -\mathbf{a} \begin{bmatrix} \mathbf{L}_{\mathbf{X}_i^h V^h} \\ \mathbf{L}_{N^h V^h} \end{bmatrix} - \mathbf{g} \begin{bmatrix} \mathbf{L}_{\gamma V^h} \end{bmatrix}
$$

$$
= -\mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X_i^h}\left(-\frac{1}{g_h^2}\right)\lambda^h(-1) \\ \frac{\partial u^h}{\partial N^h}\left(-\frac{1}{g_h^2}\right)\lambda^h(-1) \end{bmatrix},
$$

and

$$
\begin{bmatrix} \mathbf{X}_{i\lambda^h}^h \\ N_{\lambda^h}^h \end{bmatrix}
= -\mathbf{a} \begin{bmatrix} \mathbf{L}_{\mathbf{X}_i^h \lambda^h} \\ \mathbf{L}_{N^h \lambda^h} \end{bmatrix} - \mathbf{g} \begin{bmatrix} \mathbf{L}_{\gamma \lambda^h} \end{bmatrix}
$$

$$
= -\mathbf{a} \begin{bmatrix} \frac{1}{g_h}\frac{\partial u^h}{\partial X_i^h} \\ \frac{1}{g_h}\frac{\partial u^h}{\partial N^h} \end{bmatrix}.
$$

Also solving for **g** in the Nash model, note that

$$
\begin{bmatrix} \mathbf{X}_{iI^h}^h & \mathbf{X}_{i\mathbf{I}^{-h}}^h \\ N_{I^h}^h & N_{\mathbf{I}^{-h}}^h \end{bmatrix}
$$

$$
= -\mathbf{a} \begin{bmatrix} \mathbf{L}_{\mathbf{X}_i^h I^h} & \mathbf{L}_{\mathbf{X}_i^h \mathbf{I}^{-h}} \\ \mathbf{L}_{N^h I^h} & \mathbf{L}_{N^h \mathbf{I}^{-h}} \end{bmatrix} - \mathbf{g} \begin{bmatrix} \mathbf{L}_{\gamma I^h} & \mathbf{L}_{\gamma \mathbf{I}^{-h}} \end{bmatrix}
$$

$$
= -\mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X_i^h}\left(-\frac{1}{g_h^2}\right)\left[\lambda^h\left(-\frac{\partial V^h}{\partial I^h}\right) - g_h\left(\frac{\partial \lambda^h}{\partial I^h}\right)\right] & \frac{\partial u^h}{\partial X_i^h}\left(\frac{1}{g_h}\right)\left(\frac{\partial \lambda^h}{\partial I^{-h}}\right) \\ \frac{\partial u^h}{\partial N^h}\left(-\frac{1}{g_h^2}\right)\left[\lambda^h\left(-\frac{\partial V^h}{\partial I^h}\right) - g_h\left(\frac{\partial \lambda^h}{\partial I^h}\right)\right] & \frac{\partial u^h}{\partial N^h}\left(\frac{1}{g_h}\right)\left(\frac{\partial \lambda^h}{\partial I^{-h}}\right) \end{bmatrix} - \mathbf{g}\mathbf{i}^{\cdot\prime}
$$

$$
= \begin{bmatrix} \left(\mathbf{X}_{iV^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right) + \left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^{-h}}\right) \\ \left(N_{V^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right) + \left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^{-h}}\right) \end{bmatrix} - \mathbf{g}\mathbf{i}^{\cdot\prime}.
$$

Post-multiplying both sides by $\mathbf{i}$ yields

$$\begin{bmatrix} \mathbf{X}_{iI^h}^h & \mathbf{X}_{i\mathbf{I}^{-h}}^h \\ N_{I^h}^h & N_{\mathbf{I}^{-h}}^h \end{bmatrix}\mathbf{i} = \begin{bmatrix} \left(\mathbf{X}_{iV^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right)+\left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}}\right) \\ \left(N_{V^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right)+\left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}}\right) \end{bmatrix}\mathbf{i} - \mathbf{g}\mathbf{i}'\mathbf{i},$$

$$-\mathbf{g}H = \left[\begin{bmatrix} \mathbf{X}_{iI^h}^h & \mathbf{X}_{i\mathbf{I}^{-h}}^h \\ N_{I^h}^h & N_{\mathbf{I}^{-h}}^h \end{bmatrix} - \begin{bmatrix} \left(\mathbf{X}_{iV^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right)+\left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}}\right) \\ \left(N_{V^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right)+\left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}}\right) \end{bmatrix}\right]\mathbf{i},$$

and

$$\mathbf{g} = -\frac{1}{H}\left[\begin{bmatrix} \mathbf{X}_{iI^h}^h & \mathbf{X}_{i\mathbf{I}^{-h}}^h \\ N_{I^h}^h & N_{\mathbf{I}^{-h}}^h \end{bmatrix} - \begin{bmatrix} \left(\mathbf{X}_{iV^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right)+\left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}}\right) \\ \left(N_{V^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right)+\left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}}\right) \end{bmatrix}\right]\mathbf{i}.$$

It is now possible to solve the comparative static equations. The total price effects are:

$$\begin{bmatrix} \mathbf{X}_{i\mathbf{P}}^h & \mathbf{X}_{iW^h}^h \\ N_{\mathbf{P}}^h & N_{W^h}^h \end{bmatrix}$$

$$= -\mathbf{a}\begin{bmatrix} \mathbf{L}_{\mathbf{X}_i^h\mathbf{P}} & \mathbf{L}_{\mathbf{X}_i^hW^h} \\ \mathbf{L}_{N^h\mathbf{P}} & \mathbf{L}_{N^hW^h} \end{bmatrix} - \mathbf{g}\begin{bmatrix} \mathbf{L}_{\gamma\mathbf{P}} & \mathbf{L}_{\gamma\overline{W}^h} \end{bmatrix}$$

$$= \mathbf{a}\gamma\mathbf{I}$$

$$-\mathbf{a}\begin{bmatrix} \frac{\partial u^h}{\partial X_i^h}\left(-\frac{1}{g_h^2}\right)\left[\lambda^h\left(-\frac{\partial V^h}{\partial \mathbf{P}}\right)-g_h\left(\frac{\partial \lambda^h}{\partial \mathbf{P}}\right)\right] & \frac{\partial u^h}{\partial X_i^h}\left(-\frac{1}{g_h^2}\right)\left[\lambda^h\left(-\frac{\partial V^h}{\partial W^h}\right)-g_h\left(\frac{\partial \lambda^h}{\partial W^h}\right)\right] \\ \frac{\partial u^h}{\partial N^h}\left(-\frac{1}{g_h^2}\right)\left[\lambda^h\left(-\frac{\partial V^h}{\partial \mathbf{P}}\right)-g_h\frac{\partial \lambda^h}{\partial \mathbf{P}}\right] & \frac{\partial u^h}{\partial N^h}\left(-\frac{1}{g_h^2}\right)\left[\lambda^h\left(-\frac{\partial V^h}{\partial W^h}\right)-g_h\left(\frac{\partial \lambda^h}{\partial W^h}\right)\right] \end{bmatrix}$$

$$-\mathbf{g}\begin{bmatrix} -\mathbf{X}' & T-N^h \end{bmatrix}$$

$$= \mathbf{a}\gamma\mathbf{I}$$

$$+\begin{bmatrix} \left(\mathbf{X}_{iV^h}^h\right)\left(\frac{\partial V^h}{\partial \mathbf{P}}\right)+\left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial \mathbf{P}}\right) & \left(\mathbf{X}_{iV^h}^h\right)\left(\frac{\partial V^h}{\partial W^h}\right)+\left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial W^h}\right) \\ \left(N_{V^h}^h\right)\left(\frac{\partial V^h}{\partial \mathbf{P}}\right)+\left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial \mathbf{P}}\right) & \left(N_{V^h}^h\right)\left(\frac{\partial V^h}{\partial W^h}\right)+\left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial W^h}\right) \end{bmatrix}$$

$$+\frac{1}{H}\left[\begin{bmatrix} \mathbf{X}_{iI^h}^h & \mathbf{X}_{i\mathbf{I}^{-h}}^h \\ N_{I^h}^h & N_{\mathbf{I}^{-h}}^h \end{bmatrix} - \begin{bmatrix} \left(\mathbf{X}_{iV^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right)+\left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}}\right) \\ \left(N_{V^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right)+\left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}}\right) \end{bmatrix}\right]\mathbf{i}$$

$$\times\begin{bmatrix} -\mathbf{X}' & T-N^h \end{bmatrix},$$

while the cross-wage effects are given by:

$$\begin{bmatrix} \mathbf{X}^h_{i\mathbf{W}^{-h}} \\ N^h_{\mathbf{W}^{-h}} \end{bmatrix}$$

$$= -\mathbf{a} \begin{bmatrix} \mathbf{L}_{\mathbf{X}^h_i \mathbf{W}^{-h}} \\ \mathbf{L}_{\mathbf{N}^h \mathbf{W}^{-h}} \end{bmatrix} - \mathbf{g} \begin{bmatrix} \mathbf{L}_{\gamma \mathbf{W}^{-h}} \end{bmatrix}$$

$$= -\mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X^h_i} \left(\frac{1}{g_h} \right) \left(\frac{\partial \lambda^h}{\partial \mathbf{W}^{-h}} \right) \\ \frac{\partial u^h}{\partial N^h} \left(\frac{1}{g_h} \right) \left(\frac{\partial \lambda^h}{\partial \mathbf{W}^{-h}} \right) \end{bmatrix} - \mathbf{g} \left(\mathbf{T}^{-h} - \mathbf{N}^{-h} \right)'$$

$$= \begin{bmatrix} \mathbf{X}^h_{i\lambda^h} \\ N^h_{\lambda^h} \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \mathbf{W}^{-h}} \end{bmatrix}$$

$$+ \frac{1}{H} \left[\begin{bmatrix} \mathbf{X}^h_{iI^h} & \mathbf{X}^h_{i\mathbf{I}^{-h}} \\ N^h_{I^h} & N^h_{\mathbf{I}^{-h}} \end{bmatrix} - \begin{bmatrix} \left(\mathbf{X}^h_{iV^h} \right) \left(\frac{\partial V^h}{\partial I^h} \right) + \left(\mathbf{X}^h_{i\lambda^h} \right) \left(\frac{\partial \lambda^h}{\partial I^h} \right) & \left(\mathbf{X}^h_{i\lambda^h} \right) \left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}} \right) \\ \left(N^h_{V^h} \right) \left(\frac{\partial V^h}{\partial I^h} \right) + \left(N^h_{\lambda^h} \right) \left(\frac{\partial \lambda^h}{\partial I^h} \right) & \left(N^h_{\lambda^h} \right) \left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}} \right) \end{bmatrix} \right] \mathbf{i}$$

$$\times \left(\mathbf{T}^{-h} - \mathbf{N}^{-h} \right)',$$

The impact of changes in nonwage incomes are:

$$\begin{bmatrix} \mathbf{X}^h_{iI^h} & \mathbf{X}^h_{i\mathbf{I}^{-h}} \\ N^h_{I^h} & N^h_{\mathbf{I}^{-h}} \end{bmatrix}$$

$$= -\mathbf{a} \begin{bmatrix} \mathbf{L}_{\mathbf{X}^h_i I^h} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{I}^{-h}} \\ \mathbf{L}_{N^h I^h} & \mathbf{L}_{N^h \mathbf{I}^{-h}} \end{bmatrix} - \mathbf{g} \begin{bmatrix} \mathbf{L}_{\gamma I^h} & \mathbf{L}_{\gamma \mathbf{I}^{-h}} \end{bmatrix}$$

$$= -\mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X^h_i} \left(-\frac{1}{g_h^2} \right) \left[\lambda^h \left(-\frac{\partial V^h}{\partial I^h} \right) - g_h \left(\frac{\partial \lambda^h}{\partial I^h} \right) \right] & \frac{\partial u^h}{\partial X^h_i} \left(\frac{1}{g_h} \right) \left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}} \right) \\ \frac{\partial u^h}{\partial N^h} \left(-\frac{1}{g_h^2} \right) \left[\lambda^h \left(-\frac{\partial V^h}{\partial I^h} \right) - g_h \left(\frac{\partial \lambda^h}{\partial I^h} \right) \right] & \frac{\partial u^h}{\partial N^h} \left(\frac{1}{g_h} \right) \left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}} \right) \end{bmatrix} - \mathbf{g} \mathbf{i}'$$

$$= \begin{bmatrix} \left(\mathbf{X}^h_{iV^h} \right) \left(\frac{\partial V^h}{\partial I^h} \right) + \left(\mathbf{X}^h_{i\lambda^h} \right) \left(\frac{\partial \lambda^h}{\partial I^h} \right) & \left(\mathbf{X}^h_{i\lambda^h} \right) \left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}} \right) \\ \left(N^h_{V^h} \right) \left(\frac{\partial V^h}{\partial I^h} \right) + \left(N^h_{\lambda^h} \right) \left(\frac{\partial \lambda^h}{\partial I^h} \right) & \left(N^h_{\lambda^h} \right) \left(\frac{\partial \lambda^h}{\partial \mathbf{I}^{-h}} \right) \end{bmatrix} - \mathbf{g} \mathbf{i}'.$$

Also

$$\begin{bmatrix} \mathbf{X}^h_{iI^h} \\ N^h_{I^h} \end{bmatrix} - \begin{bmatrix} \mathbf{X}^h_{iI^j} \\ N^h_{I^j} \end{bmatrix}, \forall j = 1, ..., H, j \neq h$$

is given by

$$\begin{bmatrix} \mathbf{X}^h_{iI^h} & \mathbf{X}^h_{iI^j} \\ N^h_{I^h} & N^h_{I^j} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

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$$
= \begin{bmatrix} \left(\mathbf{X}_{iV^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right) + \left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^j}\right) \\ \left(N_{V^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right) + \left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) & \left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^j}\right) \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \mathbf{g}\,(1,1)\begin{bmatrix} 1 \\ -1 \end{bmatrix}
$$

$$
= \begin{bmatrix} \left(\mathbf{X}_{iV^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right) + \left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) \\ \left(N_{V^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right) + \left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^h}\right) \end{bmatrix} - \begin{bmatrix} \left(\mathbf{X}_{i\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^j}\right) \\ \left(N_{\lambda^h}^h\right)\left(\frac{\partial \lambda^h}{\partial I^j}\right) \end{bmatrix} \neq 0.
$$

From this it is clear that if

$$
\left(\frac{\partial \lambda^h}{\partial I^h}\right) = \left(\frac{\partial \lambda^h}{\partial I^j}\right),
$$

then

$$
\begin{bmatrix} \mathbf{X}_{iI^h}^h \\ N_{I^h}^h \end{bmatrix} - \begin{bmatrix} \mathbf{X}_{iI^j}^h \\ N_{I^j}^h \end{bmatrix} = \begin{bmatrix} \left(\mathbf{X}_{iV^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right) \\ \left(N_{V^h}^h\right)\left(\frac{\partial V^h}{\partial I^h}\right) \end{bmatrix} \neq 0.
$$

The impact of person h's own disagreement point on his or her demands are:

$$
\begin{bmatrix} \mathbf{X}_{iV^h}^h \\ N_{V^h}^h \end{bmatrix} = -\mathbf{a}\begin{bmatrix} \mathbf{L}_{\mathbf{X}_i^h V^h} \\ \mathbf{L}_{N^h V^h} \end{bmatrix} - \mathbf{g}\begin{bmatrix} \mathbf{L}_{\gamma V^h} \end{bmatrix}
$$

$$
= -\mathbf{a}\begin{bmatrix} \frac{\partial u^h}{\partial X_i^h}\left(-\frac{1}{g_h^2}\right)\lambda^h\,(-1) \\ \frac{\partial u^h}{\partial N^h}\left(-\frac{1}{g_h^2}\right)\lambda^h\,(-1) \end{bmatrix} \neq 0,
$$

while the impact of another member's disagreement point is given by:

$$
\begin{bmatrix} \mathbf{X}_{i\mathbf{V}^{-h}}^h \\ N_{\mathbf{V}^{-h}}^h \end{bmatrix} = -\mathbf{a}\begin{bmatrix} \mathbf{L}_{\mathbf{X}_i^h \mathbf{V}^{-h}} \\ \mathbf{L}_{N^h \mathbf{V}^{-h}} \end{bmatrix} - \mathbf{g}\begin{bmatrix} \mathbf{L}_{\gamma \mathbf{V}^{-h}} \end{bmatrix} = \mathbf{0}.
$$

Person h's shift parameters affect their own demand functions as follows:

$$
\begin{bmatrix} \mathbf{X}_{i\alpha^h}^h \\ N_{\alpha^h}^h \end{bmatrix} = -\mathbf{a}\begin{bmatrix} \mathbf{L}_{\mathbf{X}_i^h \alpha^h} \\ \mathbf{L}_{N^h \alpha^h} \end{bmatrix} - \mathbf{g}\begin{bmatrix} \mathbf{L}_{\gamma \alpha^h} \end{bmatrix}
$$

$$
= -\mathbf{a}\begin{bmatrix} \frac{\partial u^h}{\partial X_i^h}\left(-\frac{1}{g_h^2}\right)\left[\lambda^h\left(-\frac{\partial V^h}{\partial \alpha^h}\right) - g_h\left(\frac{\partial \lambda^h}{\partial \alpha^h}\right)\right] \\ \frac{\partial u^h}{\partial N^h}\left(-\frac{1}{g_h^2}\right)\left[\lambda^h\left(-\frac{\partial V^h}{\partial \alpha^h}\right) - g_h\left(\frac{\partial \lambda^h}{\partial \alpha^h}\right)\right] \end{bmatrix} - \mathbf{g}\mathbf{0}
$$

$$
= \begin{bmatrix} \mathbf{X}_{iV^h}^h\left(\frac{\partial V^h}{\partial \alpha^h}\right) + \mathbf{X}_{i\lambda^h}^h\left(\frac{\partial \lambda^h}{\partial \alpha^h}\right) \\ N_{V^h}^h\left(\frac{\partial V^h}{\partial \alpha^h}\right) + N_{\lambda^h}^h\left(\frac{\partial \lambda^h}{\partial \alpha^h}\right) \end{bmatrix},
$$

while person j's shift parameters affect person h's demands as follows:

$$
\begin{bmatrix} \mathbf{X}^h_{i\alpha^{-h}} \\ N^h_{\alpha^{-h}} \end{bmatrix} = -\mathbf{a} \begin{bmatrix} \mathbf{L}_{\mathbf{X}^h_i \alpha^{-h}} \\ \mathbf{L}_{N^h \alpha^{-h}} \end{bmatrix} - \mathbf{g} \begin{bmatrix} \mathbf{L}_{\gamma \alpha^{-h}} \end{bmatrix}
$$

$$
= -\mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X^h_i}\left(\frac{1}{g_h}\right)\left(\frac{\partial \lambda^h}{\partial \alpha^{-h}}\right) \\ \frac{\partial u^h}{\partial N^h}\left(\frac{1}{g_h}\right)\left(\frac{\partial \lambda^h}{\partial \alpha^{-h}}\right) \end{bmatrix} - \mathbf{g}\mathbf{0}'
$$

$$
= \begin{bmatrix} \mathbf{X}^h_{i\lambda^h}\left(\frac{\partial \lambda^h}{\partial \alpha^{-h}}\right) \\ N^h_{\lambda^h}\left(\frac{\partial \lambda^h}{\partial \alpha^{-h}}\right) \end{bmatrix}.
$$

Finally, the intrahousehold distribution of bargaining power affects person h's demands as follows:

$$
\begin{bmatrix} \mathbf{X}^h_{i\lambda^h} & \mathbf{X}^h_{i\lambda^{-h}} \\ N^h_{\lambda^h} & N^h_{\lambda^{-h}} \end{bmatrix} = -\mathbf{a} \begin{bmatrix} \mathbf{L}_{\mathbf{X}^h_i \lambda^h} & \mathbf{L}_{\mathbf{X}^h_i \lambda^{-h}} \\ \mathbf{L}_{N^h \lambda^h} & \mathbf{L}_{N^h \lambda^{-h}} \end{bmatrix} - \mathbf{g} \begin{bmatrix} \mathbf{L}_{\gamma \lambda^h} & \mathbf{L}_{\gamma \lambda^{-h}} \end{bmatrix}
$$

$$
= -\mathbf{a} \begin{bmatrix} \frac{1}{g_h}\frac{\partial u^h}{\partial X^h_i} & -\frac{1}{g_h}\frac{\partial u^h}{\partial X^h_i} \\ \frac{1}{g_h}\frac{\partial u^h}{\partial N^h} & -\frac{1}{g_h}\frac{\partial u^h}{\partial N^h} \end{bmatrix} \neq \mathbf{0},
$$

since

$$
\sum_{h=1}^{H} \lambda^h = 1.
$$

Also the following must be true:

$$
\begin{bmatrix} \mathbf{X}^h_{i\lambda^h} \\ N^h_{\lambda^h} \end{bmatrix} = \begin{bmatrix} \mathbf{X}^h_{i\lambda^j} \\ N^h_{\lambda^j} \end{bmatrix}\begin{bmatrix} \frac{\partial \lambda^j}{\partial \lambda^h} \end{bmatrix} = -\begin{bmatrix} \mathbf{X}^h_{i\lambda^j} \\ N^h_{\lambda^j} \end{bmatrix} \forall j \neq h.
$$

Generalized utilitarian model
In this model, the matrix

$$
\begin{bmatrix}
(n) & \mathbf{L}_{\mathbf{X}^h_i \mathbf{P}} & \mathbf{L}_{\mathbf{X}^h_i W^h} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{W}^{-h}} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{I}} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{V}} & \mathbf{L}_{\mathbf{X}^h_i \alpha} & \mathbf{L}_{\mathbf{X}^h_i \lambda} \\
(1) & \mathbf{L}_{N^h \mathbf{P}} & \mathbf{L}_{N^h W^h} & \mathbf{L}_{N^h \mathbf{W}^{-h}} & \mathbf{L}_{N^h \mathbf{I}} & \mathbf{L}_{N^h \mathbf{V}} & \mathbf{L}_{N^h \alpha} & \mathbf{L}_{N^h \lambda} \\
(r) & \mathbf{L}_{\gamma \mathbf{P}} & \mathbf{L}_{\gamma W^h} & \mathbf{L}_{\gamma \mathbf{W}^{-h}} & \mathbf{L}_{\gamma \mathbf{I}} & \mathbf{L}_{\gamma \mathbf{V}} & \mathbf{L}_{\gamma \alpha} & \mathbf{L}_{\gamma \lambda} \\
 & (n) & (1) & (h-1) & (h) & (h) & (h) & (h)
\end{bmatrix}
$$

is given by

$$
\times \begin{bmatrix}
(n) & \frac{\partial u^h}{\partial X^h_i}\frac{\partial \lambda^h}{\partial \mathbf{P}} - \gamma \mathbf{I} & \frac{\partial u^h}{\partial X^h_i}\frac{\partial \lambda^h}{\partial W^h} & \frac{\partial u^h}{\partial X^h_i}\frac{\partial \lambda^h}{\partial \mathbf{W}^{-h}} & \frac{\partial u^h}{\partial X^h_i}\frac{\partial \lambda^h}{\partial \mathbf{I}} & \mathbf{0} & \frac{\partial u^h}{\partial X^h_i}\frac{\partial \lambda^h}{\partial \alpha} & \frac{\partial u^h}{\partial X^h_i} & -\frac{\partial u^h}{\partial X^h_i} \\
(1) & \frac{\partial u^h}{\partial N^h}\frac{\partial \lambda^h}{\partial \mathbf{P}} & \frac{\partial u^h}{\partial N^h}\frac{\partial \lambda^h}{\partial W^h} - \gamma & \frac{\partial u^h}{\partial N^h}\frac{\partial \lambda^h}{\partial \mathbf{W}^{-h}} & \frac{\partial u^h}{\partial N^h}\frac{\partial \lambda^h}{\partial \mathbf{I}} & \mathbf{0} & \frac{\partial u^h}{\partial N^h}\frac{\partial \lambda^h}{\partial \alpha} & \frac{\partial u^h}{\partial N^h} & -\frac{\partial u^h}{\partial N^h} \\
(1) & -\mathbf{X}' & T - N^h & \mathbf{T}' - \mathbf{N}^{-h\prime} & \mathbf{i}' & \mathbf{0}' & \mathbf{0}' & \mathbf{0} & \mathbf{0}' \\
 & (n) & (1) & (h-1) & (h) & (h) & (h) & (1) & (h-1)
\end{bmatrix}.
$$

Again, in order to solve the matrix equations, note that:

$$\left[\begin{array}{c} \mathbf{X}_{i\mathbf{I}}^{h} \\ N_{\mathbf{I}}^{h} \end{array}\right] \mathbf{i} \;\; = \;\; -\mathbf{a} \left[\begin{array}{c} \frac{\partial u^{h}}{\partial X_{i}^{h}} \frac{\partial \lambda^{h}}{\partial \mathbf{I}} \\ \frac{\partial u^{h}}{\partial N^{h}} \frac{\partial \lambda^{h}}{\partial \mathbf{I}} \end{array}\right] \mathbf{i} - \mathbf{g}\mathbf{i}'\mathbf{i}$$

$$= \;\; \left[\begin{array}{c} \mathbf{X}_{i\lambda^{h}}^{h} \\ N_{\lambda^{h}}^{h} \end{array}\right] \left[\begin{array}{c} \frac{\partial \lambda^{h}}{\partial \mathbf{I}} \end{array}\right] \mathbf{i} - \mathbf{g}H,$$

and so,

$$\mathbf{g} = -\frac{1}{H} \left[\left[\begin{array}{c} \mathbf{X}_{i\mathbf{I}}^{h} \\ N_{\mathbf{I}}^{h} \end{array}\right] - \left[\begin{array}{c} \mathbf{X}_{i\lambda^{h}}^{h} \\ N_{\lambda^{h}}^{h} \end{array}\right] \left[\begin{array}{c} \frac{\partial \lambda^{h}}{\partial \mathbf{I}} \end{array}\right]\right] \mathbf{i}.$$

Also note that

$$\left[\begin{array}{c} \mathbf{X}_{i\lambda^{h}}^{h} \\ N_{\lambda^{h}}^{h} \end{array}\right] = -\mathbf{a} \left[\begin{array}{c} \frac{\partial u^{h}}{\partial X_{i}^{h}} \frac{\partial \lambda^{h}}{\partial \lambda^{h}} \\ \frac{\partial u^{h}}{\partial N^{h}} \frac{\partial \lambda^{h}}{\partial \lambda^{h}} \end{array}\right] = -\mathbf{a} \left[\begin{array}{c} \frac{\partial u^{h}}{\partial X_{i}^{h}} \\ \frac{\partial u^{h}}{\partial N^{h}} \end{array}\right].$$

The total price effects are given by:

$$\left[\begin{array}{cc} \mathbf{X}_{i\mathbf{P}}^{h} & \mathbf{X}_{iW^{h}}^{h} \\ N_{\mathbf{P}}^{h} & N_{W^{h}}^{h} \end{array}\right] \;\; = \;\; -\mathbf{a} \left[\begin{array}{cc} \frac{\partial u^{h}}{\partial X_{i}^{h}} \frac{\partial \lambda^{h}}{\partial \mathbf{P}} - \gamma\mathbf{I} & \frac{\partial u^{h}}{\partial X_{i}^{h}} \frac{\partial \lambda^{h}}{\partial W^{h}} \\ \frac{\partial u^{h}}{\partial N^{h}} \frac{\partial \lambda^{h}}{\partial \mathbf{P}} & \frac{\partial u^{h}}{\partial N^{h}} \frac{\partial \lambda^{h}}{\partial W^{h}} - \gamma \end{array}\right] - \mathbf{g}\left[\begin{array}{cc} -\mathbf{X}' & T - N^{h} \end{array}\right]$$

$$= \;\; \mathbf{a}\gamma\mathbf{I} - \mathbf{a} \left[\begin{array}{cc} \frac{\partial u^{h}}{\partial X_{i}^{h}} \frac{\partial \lambda^{h}}{\partial \mathbf{P}} & \frac{\partial u^{h}}{\partial X_{i}^{h}} \frac{\partial \lambda^{h}}{\partial W^{h}} \\ \frac{\partial u^{h}}{\partial N^{h}} \frac{\partial \lambda^{h}}{\partial \mathbf{P}} & \frac{\partial u^{h}}{\partial N^{h}} \frac{\partial \lambda^{h}}{\partial W^{h}} \end{array}\right] - \mathbf{g}\left[\begin{array}{cc} -\mathbf{X}' & T - N^{h} \end{array}\right]$$

$$= \;\; \mathbf{a}\gamma\mathbf{I} - \mathbf{a} \left[\begin{array}{c} \frac{\partial u^{h}}{\partial X_{i}^{h}} \\ \frac{\partial u^{h}}{\partial N^{h}} \end{array}\right] \left[\begin{array}{cc} \frac{\partial \lambda^{h}}{\partial \mathbf{P}} & \frac{\partial \lambda^{h}}{\partial W^{h}} \end{array}\right]$$

$$+\frac{1}{H} \left[\left[\begin{array}{c} \mathbf{X}_{i\mathbf{I}}^{h} \\ N_{\mathbf{I}}^{h} \end{array}\right] + \mathbf{a} \left[\begin{array}{c} \frac{\partial u^{h}}{\partial X_{i}^{h}} \\ \frac{\partial u^{h}}{\partial N^{h}} \end{array}\right] \left[\begin{array}{c} \frac{\partial \lambda^{h}}{\partial \mathbf{I}} \end{array}\right]\right] \mathbf{i} \left[\begin{array}{cc} -\mathbf{X}' & T - N^{h} \end{array}\right]$$

$$= \;\; \mathbf{a}\gamma\mathbf{I} + \left[\begin{array}{c} \mathbf{X}_{i\lambda^{h}}^{h} \\ N_{\lambda^{h}}^{h} \end{array}\right] \left[\begin{array}{cc} \frac{\partial \lambda^{h}}{\partial \mathbf{P}} & \frac{\partial \lambda^{h}}{\partial W^{h}} \end{array}\right]$$

$$+\frac{1}{H} \left[\left[\begin{array}{c} \mathbf{X}_{i\mathbf{I}}^{h} \\ N_{\mathbf{I}}^{h} \end{array}\right] - \left[\begin{array}{c} \mathbf{X}_{i\lambda^{h}}^{h} \\ N_{\lambda^{h}}^{h} \end{array}\right] \left[\begin{array}{c} \frac{\partial \lambda^{h}}{\partial \mathbf{I}} \end{array}\right]\right] \mathbf{i} \left[\begin{array}{cc} -\mathbf{X}' & T - N^{h} \end{array}\right],$$

and the cross-wage effects by:

$$\left[\begin{array}{c} \mathbf{X}_{i\mathbf{W}^{-h}}^{h} \\ N_{\mathbf{W}^{-h}}^{h} \end{array}\right] \;\; = \;\; -\mathbf{a} \left[\begin{array}{c} \frac{\partial u^{h}}{\partial X_{i}^{h}} \frac{\partial \lambda^{h}}{\partial \mathbf{W}^{-h}} \\ \frac{\partial u^{h}}{\partial N^{h}} \frac{\partial \lambda^{h}}{\partial \mathbf{W}^{-h}} \end{array}\right] - \mathbf{g}\left(\mathbf{T}' - \mathbf{N}^{-h'}\right)$$

$$= \begin{bmatrix} \mathbf{X}^h_{i\lambda^h} \\ N^h_{\lambda^h} \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \mathbf{W}^{-h}} \end{bmatrix}$$

$$+ \frac{1}{H} \left[\begin{bmatrix} \mathbf{X}^h_{i\mathbf{I}} \\ N^h_{\mathbf{I}} \end{bmatrix} - \begin{bmatrix} \mathbf{X}^h_{i\lambda^h} \\ N^h_{\lambda^h} \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \mathbf{I}} \end{bmatrix} \right] \mathbf{i} \left(\mathbf{T}' - \mathbf{N}^{-h\prime} \right).$$

Non-wage incomes affect person h's demands as follows:

$$\begin{bmatrix} \mathbf{X}^h_{i\mathbf{I}} \\ N^h_{\mathbf{I}} \end{bmatrix} = -\mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X^h_i} \frac{\partial \lambda^h}{\partial \mathbf{I}} \\ \frac{\partial u^h}{\partial N^h} \frac{\partial \lambda^h}{\partial \mathbf{I}} \end{bmatrix} - \mathbf{g} \mathbf{i}'$$

$$= \begin{bmatrix} \mathbf{X}^h_{i\lambda^h} \\ N^h_{\lambda^h} \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \mathbf{I}^h} \end{bmatrix} - \mathbf{g} \mathbf{i}',$$

and person h's income may affect demands differently from person j's income:

$$\begin{bmatrix} \mathbf{X}^h_{iI^h} & \mathbf{X}^h_{iI^j} \\ N^h_{I^h} & N^h_{I^j} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \mathbf{X}^h_{i\lambda^h} \\ N^h_{\lambda^h} \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial I^h} & \frac{\partial \lambda^h}{\partial I^j} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \mathbf{g} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{X}^h_{i\lambda^h} \\ N^h_{\lambda^h} \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial I^h} & \frac{\partial \lambda^h}{\partial I^j} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \neq 0.$$

Disagreement points have no impact in this model:

$$\begin{bmatrix} \mathbf{X}^h_{i\mathbf{V}} \\ N^h_{\mathbf{V}} \end{bmatrix} = \mathbf{0}.$$

Shift parameters affect demands as follows:

$$\begin{bmatrix} \mathbf{X}^h_{i\alpha} \\ N^h_{\alpha} \end{bmatrix} = -\mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X^h_i} \frac{\partial \lambda^h}{\partial \alpha} \\ \frac{\partial u^h}{\partial N^h} \frac{\partial \lambda^h}{\partial \alpha} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{X}^h_{i\lambda^h} \\ N^h_{\lambda^h} \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \alpha} \end{bmatrix}.$$

Finally, the bargaining weights enter individual h's demand functions as follows:

$$\begin{bmatrix} \mathbf{X}^h_{i\lambda^h} & \mathbf{X}^h_{i\lambda^{-h}} \\ N^h_{\lambda^h} & N^h_{\lambda^{-h}} \end{bmatrix} = -\mathbf{a} \begin{bmatrix} \mathbf{L}_{\mathbf{X}^h_i \lambda^h} & \mathbf{L}_{\mathbf{X}^h_i \lambda^{-h}} \\ \mathbf{L}_{N^h \lambda^h} & \mathbf{L}_{N^h \lambda^{-h}} \end{bmatrix}$$

$$= -\mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X^h_i} & -\frac{\partial u^h}{\partial X^h_i} \\ \frac{\partial u^h}{\partial N^h} & -\frac{\partial u^h}{\partial N^h} \end{bmatrix} \neq 0,$$

since
$$\sum_{h=1}^{H} \lambda^h = 1.$$

Also the following must be true:

$$\left[\begin{array}{c} \mathbf{X}_{i\lambda^h}^h \\ N_{\lambda^h}^h \end{array}\right] = \left[\begin{array}{c} \mathbf{X}_{i\lambda^j}^h \\ N_{\lambda^j}^h \end{array}\right] \left[\frac{\partial \lambda^j}{\partial \lambda^h}\right] = -\left[\begin{array}{c} \mathbf{X}_{i\lambda^j}^h \\ N_{\lambda^j}^h \end{array}\right] \forall j \neq h.$$

Chiappori's efficiency model
In Chiappori's Pareto efficiency model, the matrix

$$\left[\begin{array}{cccccccc} (n) & \mathbf{L_{X_i^h P}} & \mathbf{L_{X_i^h W^h}} & \mathbf{L_{X_i^h W^{-h}}} & \mathbf{L_{X_i^h I}} & \mathbf{L_{X_i^h V}} & \mathbf{L_{X_i^h \alpha}} & \mathbf{L_{X_i^h \lambda}} \\ (1) & \mathbf{L_{N^h P}} & \mathbf{L_{N^h W^h}} & \mathbf{L_{N^h W^{-h}}} & \mathbf{L_{N^h I}} & \mathbf{L_{N^h V}} & \mathbf{L_{N^h \alpha}} & \mathbf{L_{N^h \lambda}} \\ (r) & \mathbf{L_{\gamma P}} & \mathbf{L_{\gamma W^h}} & \mathbf{L_{\gamma W^{-h}}} & \mathbf{L_{\gamma I}} & \mathbf{L_{\gamma V}} & \mathbf{L_{\gamma \alpha}} & \mathbf{L_{\gamma \lambda}} \\ & (n) & (1) & (h-1) & (h) & (h) & (h) & (h) \end{array}\right]$$

is given by

$$\times \left[\begin{array}{ccccccccc} (n) & \frac{\partial u^h}{\partial X_i^h}\frac{\partial \lambda^h}{\partial \mathbf{P}} - \gamma \mathbf{I} & \frac{\partial u^h}{\partial X_i^h}\frac{\partial \lambda^h}{\partial W^h} & & \frac{\partial u^h}{\partial X_i^h}\frac{\partial \lambda^h}{\partial \mathbf{W^{-h}}} & \frac{\partial u^h}{\partial X_i^h}\frac{\partial \lambda^h}{\partial \mathbf{I}} & 0 & 0 & \frac{\partial u^h}{\partial X_i^h} & -\frac{\partial u^h}{\partial X_i^h} \\ (1) & \frac{\partial u^h}{\partial N^h}\frac{\partial \lambda^h}{\partial \mathbf{P}} & \frac{\partial u^h}{\partial N^h}\frac{\partial \lambda^h}{\partial W^h} - \gamma & & \frac{\partial u^h}{\partial N^h}\frac{\partial \lambda^h}{\partial \mathbf{W^{-h}}} & \frac{\partial u^h}{\partial N^h}\frac{\partial \lambda^h}{\partial \mathbf{I}} & 0 & 0 & \frac{\partial u^h}{\partial N^h} & -\frac{\partial u^h}{\partial N^h} \\ (1) & -\mathbf{X'} & T - N^h & & \mathbf{T'} - \mathbf{N}^{-h\prime} & \mathbf{i'} & \mathbf{0'} & \mathbf{0'} & 0 & \mathbf{0'} \\ & (n) & (1) & & (h-1) & (h) & (h) & (h) & (1) & (h-1) \end{array}\right].$$

First solving for $\mathbf{g}$ in the matrix equations, note that:

$$\left[\begin{array}{c} \mathbf{X}_{i\mathbf{I}}^h \\ N_{\mathbf{I}}^h \end{array}\right]\mathbf{i} = -\mathbf{a}\left[\begin{array}{c} \frac{\partial u^h}{\partial X_i^h}\frac{\partial \lambda^h}{\partial \mathbf{I}} \\ \frac{\partial u^h}{\partial N^h}\frac{\partial \lambda^h}{\partial \mathbf{I}} \end{array}\right]\mathbf{i} - \mathbf{g}\,\mathbf{i'i}$$

$$= \left[\begin{array}{c} \mathbf{X}_{i\lambda^h}^h \\ N_{\lambda^h}^h \end{array}\right]\left[\begin{array}{c} \frac{\partial \lambda^h}{\partial \mathbf{I}} \end{array}\right]\mathbf{i} - \mathbf{g}H,$$

and so,

$$\mathbf{g} = -\frac{1}{H}\left[\left[\begin{array}{c} \mathbf{X}_{i\mathbf{I}}^h \\ N_{\mathbf{I}}^h \end{array}\right] - \left[\begin{array}{c} \mathbf{X}_{i\lambda^h}^h \\ N_{\lambda^h}^h \end{array}\right]\left[\begin{array}{c} \frac{\partial \lambda^h}{\partial \mathbf{I}} \end{array}\right]\right]\mathbf{i}.$$

Also note that

$$\left[\begin{array}{c} \mathbf{X}_{i\lambda^h}^h \\ N_{\lambda^h}^h \end{array}\right] = -\mathbf{a}\left[\begin{array}{c} \frac{\partial u^h}{\partial X_i^h}\frac{\partial \lambda^h}{\partial \lambda^h} \\ \frac{\partial u^h}{\partial N^h}\frac{\partial \lambda^h}{\partial \lambda^h} \end{array}\right] = -\mathbf{a}\left[\begin{array}{c} \frac{\partial u^h}{\partial X_i^h} \\ \frac{\partial u^h}{\partial N^h} \end{array}\right].$$

The total price effects are given by:

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$$\begin{bmatrix} \mathbf{X}_{i\mathbf{P}}^h & \mathbf{X}_{iW^h}^h \\ N_{\mathbf{P}}^h & N_{W^h}^h \end{bmatrix} = -\mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X_i^h} \frac{\partial \lambda^h}{\partial \mathbf{P}} - \gamma \mathbf{I} & \frac{\partial u^h}{\partial X_i^h} \frac{\partial \lambda^h}{\partial W^h} \\ \frac{\partial u^h}{\partial N^h} \frac{\partial \lambda^h}{\partial \mathbf{P}} & \frac{\partial u^h}{\partial N^h} \frac{\partial \lambda^h}{\partial W^h} - \gamma \end{bmatrix} - \mathbf{g} \begin{bmatrix} -\mathbf{X}' & T - N^h \end{bmatrix}$$

$$= \mathbf{a}\gamma\mathbf{I} - \mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X_i^h} \frac{\partial \lambda^h}{\partial \mathbf{P}} & \frac{\partial u^h}{\partial X_i^h} \frac{\partial \lambda^h}{\partial W^h} \\ \frac{\partial u^h}{\partial N^h} \frac{\partial \lambda^h}{\partial \mathbf{P}} & \frac{\partial u^h}{\partial N^h} \frac{\partial \lambda^h}{\partial W^h} \end{bmatrix} - \mathbf{g} \begin{bmatrix} -\mathbf{X}' & T - N^h \end{bmatrix}$$

$$= \mathbf{a}\gamma\mathbf{I} - \mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X_i^h} \\ \frac{\partial u^h}{\partial N^h} \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \mathbf{P}} & \frac{\partial \lambda^h}{\partial W^h} \end{bmatrix}$$

$$+ \frac{1}{H} \left[\begin{bmatrix} \mathbf{X}_{i\mathbf{I}}^h \\ N_{\mathbf{I}}^h \end{bmatrix} + \mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X_i^h} \\ \frac{\partial u^h}{\partial N^h} \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \mathbf{I}} \end{bmatrix} \right] \mathbf{i} \begin{bmatrix} -\mathbf{X}' & T - N^h \end{bmatrix}$$

$$= \mathbf{a}\gamma\mathbf{I} + \begin{bmatrix} \mathbf{X}_{i\lambda^h}^h \\ N_{\lambda^h}^h \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \mathbf{P}} & \frac{\partial \lambda^h}{\partial W^h} \end{bmatrix}$$

$$+ \frac{1}{H} \left[\begin{bmatrix} \mathbf{X}_{i\mathbf{I}}^h \\ N_{\mathbf{I}}^h \end{bmatrix} - \begin{bmatrix} \mathbf{X}_{i\lambda^h}^h \\ N_{\lambda^h}^h \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \mathbf{I}} \end{bmatrix} \right] \mathbf{i} \begin{bmatrix} -\mathbf{X}' & T - N^h \end{bmatrix},$$

and the cross-wage effects by:

$$\begin{bmatrix} \mathbf{X}_{i\mathbf{W}^{-h}}^h \\ N_{\mathbf{W}^{-h}}^h \end{bmatrix} = -\mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X_i^h} \frac{\partial \lambda^h}{\partial \mathbf{W}^{-h}} \\ \frac{\partial u^h}{\partial N^h} \frac{\partial \lambda^h}{\partial \mathbf{W}^{-h}} \end{bmatrix} - \mathbf{g} \left(\mathbf{T}' - \mathbf{N}^{-h\prime} \right)$$

$$= \begin{bmatrix} \mathbf{X}_{i\lambda^h}^h \\ N_{\lambda^h}^h \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \mathbf{W}^{-h}} \end{bmatrix}$$

$$+ \frac{1}{H} \left[\begin{bmatrix} \mathbf{X}_{i\mathbf{I}}^h \\ N_{\mathbf{I}}^h \end{bmatrix} - \begin{bmatrix} \mathbf{X}_{i\lambda^h}^h \\ N_{\lambda^h}^h \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \mathbf{I}} \end{bmatrix} \right] \mathbf{i} \left(\mathbf{T}' - \mathbf{N}^{-h\prime} \right).$$

The non-wage incomes affect person h's demands as shown below:

$$\begin{bmatrix} \mathbf{X}_{i\mathbf{I}}^h \\ N_{\mathbf{I}}^h \end{bmatrix} = -\mathbf{a} \begin{bmatrix} \frac{\partial u^h}{\partial X_i^h} \frac{\partial \lambda^h}{\partial \mathbf{I}} \\ \frac{\partial u^h}{\partial N^h} \frac{\partial \lambda^h}{\partial \mathbf{I}} \end{bmatrix} - \mathbf{g}\mathbf{i}'$$

$$= \begin{bmatrix} \mathbf{X}_{i\lambda^h}^h \\ N_{\lambda^h}^h \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial \mathbf{I}} \end{bmatrix} - \mathbf{g}\mathbf{i}'.$$

Also person h's income may affect demands differently from person j's income:

$$\begin{bmatrix} \mathbf{X}_{iI^h}^h & \mathbf{X}_{iI^j}^h \\ N_{I^h}^h & N_{I^j}^h \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{i\lambda^h}^h \\ N_{\lambda^h}^h \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^h}{\partial I^h} & \frac{\partial \lambda^h}{\partial I^j} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \mathbf{g} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

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$$= \left[\begin{array}{c} \mathbf{X}^h_{i\lambda^h} \\ N^h_{\lambda^h} \end{array} \right] \left[\begin{array}{cc} \frac{\partial \lambda^h}{\partial I^h} & \frac{\partial \lambda^h}{\partial I^j} \end{array} \right] \left[\begin{array}{c} 1 \\ -1 \end{array} \right] \neq 0$$

unless

$$\frac{\partial \lambda^h}{\partial I^h} = \frac{\partial \lambda^h}{\partial I^j}.$$

Disagreement points and shift parameters do not enter the individual demand functions:

$$\left[\begin{array}{c} \mathbf{X}^h_{i\mathbf{V}} \\ N^h_{\mathbf{V}} \end{array} \right] = \left[\begin{array}{c} \mathbf{X}^h_{i\alpha} \\ N^h_\alpha \end{array} \right] = \mathbf{0}.$$

The distribution of bargaining power affects demands as follows:

$$\left[\begin{array}{cc} \mathbf{X}^h_{i\lambda^h} & \mathbf{X}^h_{i\lambda^{-h}} \\ N^h_{\lambda^h} & N^h_{\lambda^{-h}} \end{array} \right] = -\mathbf{a} \left[\begin{array}{cc} \mathbf{L}_{\mathbf{X}^h_i \lambda^h} & \mathbf{L}_{\mathbf{X}^h_i \lambda^{-h}} \\ \mathbf{L}_{N^h \lambda^h} & \mathbf{L}_{N^h \lambda^{-h}} \end{array} \right]$$

$$= -\mathbf{a} \left[\begin{array}{cc} \frac{\partial u^h}{\partial X^h_i} & -\frac{\partial u^h}{\partial X^h_i} \\ \frac{\partial u^h}{\partial N^h} & -\frac{\partial u^h}{\partial N^h} \end{array} \right] \neq \mathbf{0},$$

since

$$\sum_{h=1}^{H} \lambda^h = 1.$$

Also the following must be true:

$$\left[\begin{array}{c} \mathbf{X}^h_{i\lambda^h} \\ N^h_{\lambda^h} \end{array} \right] = \left[\begin{array}{c} \mathbf{X}^h_{i\lambda^j} \\ N^h_{\lambda^j} \end{array} \right] \left[\frac{\partial \lambda^j}{\partial \lambda^h} \right] = - \left[\begin{array}{c} \mathbf{X}^h_{i\lambda^j} \\ N^h_{\lambda^j} \end{array} \right] \forall j \neq h.$$

Chiappori's sharing rule approach
In this model, the matrix

$$\left[\begin{array}{cccccccc} (n) & \mathbf{L}_{\mathbf{X}^h_i \mathbf{P}} & \mathbf{L}_{\mathbf{X}^h_i W^h} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{W}^{-h}} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{I}} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{V}} & \mathbf{L}_{\mathbf{X}^h_i \alpha} & \mathbf{L}_{\mathbf{X}^h_i \lambda} \\ (1) & \mathbf{L}_{N^h \mathbf{P}} & \mathbf{L}_{N^h W^h} & \mathbf{L}_{N^h \mathbf{W}^{-h}} & \mathbf{L}_{N^h \mathbf{I}} & \mathbf{L}_{N^h \mathbf{V}} & \mathbf{L}_{N^h \alpha} & \mathbf{L}_{N^h \lambda} \\ (r) & \mathbf{L}_{\gamma \mathbf{P}} & \mathbf{L}_{\gamma W^h} & \mathbf{L}_{\gamma \mathbf{W}^{-h}} & \mathbf{L}_{\gamma \mathbf{I}} & \mathbf{L}_{\gamma \mathbf{V}} & \mathbf{L}_{\gamma \alpha} & \mathbf{L}_{\gamma \lambda} \\ & (n) & (1) & (h-1) & (h) & (h) & (h) & (h) \end{array} \right]$$

is given by

$$\times \left[\begin{array}{cccccccc} (n) & -\gamma \mathbf{I} & \mathbf{0} & & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (1) & \mathbf{0} & -\gamma & & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (1) & \frac{\partial \phi^h}{\partial \mathbf{P}} - \mathbf{X}^h & \frac{\partial \phi^h}{\partial W^h} + \left(T - N^h \right) & \frac{\partial \phi^h}{\partial \mathbf{W}^{-h}} & \frac{\partial \phi^h}{\partial \mathbf{I}} & \mathbf{0}' & \mathbf{0}' & \mathbf{0} & \mathbf{0}' \\ & (n) & (1) & & (h-1) & (h) & (h) & (h) & (1) & (h-1) \end{array} \right].$$

First solving for $\mathbf{g}$ in the above matrix equations, note that

$$\begin{bmatrix} \mathbf{X}^h_{i\phi^h} \\ N^h_{\phi^h} \end{bmatrix} = -\mathbf{a}\begin{bmatrix} \mathbf{L}_{\mathbf{X}^h_i\phi^h} \\ \mathbf{L}_{N^h\phi^h} \end{bmatrix} - \mathbf{g}\begin{bmatrix} \mathbf{L}_{\gamma\phi^h} \end{bmatrix}$$
$$= -\mathbf{g}.1 = -\mathbf{g}.$$

The total price effects are

$$\begin{bmatrix} \mathbf{X}^h_{i\mathbf{P}} & \mathbf{X}^h_{iW^h} \\ N^h_{\mathbf{P}} & N^h_{W^h} \end{bmatrix} = \mathbf{a}\gamma\mathbf{I} - \mathbf{g}\left[\begin{bmatrix} \frac{\partial\phi^h}{\partial\mathbf{P}} - \mathbf{X}^h \end{bmatrix} \begin{bmatrix} \frac{\partial\phi^h}{\partial W^h} - N^h \end{bmatrix} \right]$$
$$= \mathbf{a}\gamma\mathbf{I} + \begin{pmatrix} \mathbf{X}^h_{i\phi^h} \\ N^h_{\phi^h} \end{pmatrix}\left[\begin{bmatrix} \frac{\partial\phi^h}{\partial\mathbf{P}} - \mathbf{X}^h \end{bmatrix} \begin{bmatrix} \frac{\partial\phi^h}{\partial W^h} - N^h \end{bmatrix} \right],$$

and the cross-wage effects are

$$\begin{bmatrix} \mathbf{X}^h_{i\mathbf{W}^{-h}} \\ N^h_{\mathbf{W}^{-h}} \end{bmatrix} = -\mathbf{g}\frac{\partial\phi^h}{\partial\mathbf{W}^{-h}}$$
$$= \begin{pmatrix} \mathbf{X}^h_{i\phi^h} \\ N^h_{\phi^h} \end{pmatrix}\frac{\partial\phi^h}{\partial\mathbf{W}^{-h}}.$$

The non-wage incomes affect person h's demands as shown below:

$$\begin{bmatrix} \mathbf{X}^h_{i\mathbf{I}} \\ N^h_{\mathbf{I}} \end{bmatrix} = \begin{pmatrix} \mathbf{X}^h_{i\phi^h} \\ N^h_{\phi^h} \end{pmatrix}\frac{\partial\phi^h}{\partial\mathbf{I}} \neq 0.$$

Also person h's income may affect demands differently from person j's income:

$$\begin{bmatrix} \mathbf{X}^h_{iI^h} & \mathbf{X}^h_{iI^j} \\ N^h_{I^h} & N^h_{I^j} \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{pmatrix} \mathbf{X}^h_{i\phi^h} \\ N^h_{\phi^h} \end{pmatrix}\begin{bmatrix} \frac{\partial\phi^h}{\partial I^h} & \frac{\partial\phi^h}{\partial I^j} \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \neq 0.$$

Disagreement points, shift parameters and bargaining weights do not enter the individual demand functions:

$$\begin{bmatrix} \mathbf{X}^h_{i\mathbf{V}} \\ N^h_{\mathbf{V}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}^h_{i\alpha} \\ N^h_{\alpha} \end{bmatrix} = \begin{bmatrix} \mathbf{X}^h_{i\lambda^h} \\ N^h_{\lambda^h} \end{bmatrix} 0.$$

Dictatorial model
Proof that the Slutsky matrix is symmetric and negative semi-definite:
In the dictatorial model, the fundamental matrix equations are given by

$$
\begin{bmatrix}
(n) & \mathbf{X}^h_{i\mathbf{P}} & \mathbf{X}^h_{iW^h} & \mathbf{X}^h_{i\mathbf{W}^{-h}} & \mathbf{X}^h_{i\mathbf{I}} & \mathbf{X}^h_{i\mathbf{V}} & \mathbf{X}^h_{i\alpha} & \mathbf{X}^h_{i\lambda} \\
(1) & N^h_{\mathbf{P}} & N^h_{W^h} & N^h_{\mathbf{W}^{-h}} & N^h_{\mathbf{I}} & N^h_{\mathbf{V}} & N^h_{\alpha} & N^h_{\lambda} \\
(h-1) & \rho_{\mathbf{P}} & \rho_{W^h} & \rho_{\mathbf{W}^{-h}} & \rho_{\mathbf{I}} & \rho_{\mathbf{V}} & \rho_{\alpha} & \rho_{\lambda} \\
(1) & \gamma_{\mathbf{P}} & \gamma_{W^h} & \gamma_{\mathbf{W}^{-h}} & \gamma_{\mathbf{I}} & \gamma_{\mathbf{V}} & \gamma_{\alpha} & \gamma_{\lambda} \\
& (n) & (1) & (h-1) & (h) & (h) & (h) & (h)
\end{bmatrix}
$$

$$
= -
\begin{bmatrix}
(n) & \mathbf{L}_{\mathbf{X}^h_i \mathbf{X}^h_i} & \mathbf{L}_{\mathbf{X}^h_i N^h} & \mathbf{L}_{\mathbf{X}^h_i \rho} & \mathbf{L}_{\mathbf{X}^h_i \gamma} \\
(1) & \mathbf{L}_{N^h \mathbf{X}^h_i} & \mathbf{L}_{N^h N^h} & \mathbf{L}_{N^h \rho} & \mathbf{L}_{N^h \gamma} \\
(h-1) & \mathbf{L}_{\rho \mathbf{X}^h_i} & \mathbf{L}_{\rho N^h} & \mathbf{L}_{\rho\rho} & \mathbf{L}_{\rho\gamma} \\
(1) & \mathbf{L}_{\gamma \mathbf{X}^h_i} & \mathbf{L}_{\gamma N^h} & \mathbf{L}_{\gamma\rho} & \mathbf{L}_{\gamma\gamma} \\
& (n) & (1) & (h-1) & (1)
\end{bmatrix}^{-1}
$$

$$
\times
\begin{bmatrix}
(n) & \mathbf{L}_{\mathbf{X}^h_i \mathbf{P}} & \mathbf{L}_{\mathbf{X}^h_i W^h} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{W}^{-h}} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{I}} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{V}} & \mathbf{L}_{\mathbf{X}^h_i \alpha} & \mathbf{L}_{\mathbf{X}^h_i \lambda} \\
(1) & \mathbf{L}_{N^h \mathbf{P}} & \mathbf{L}_{N^h W^h} & \mathbf{L}_{\mathbf{N}^h \mathbf{W}^{-h}} & \mathbf{L}_{N^h \mathbf{I}} & \mathbf{L}_{N^h \mathbf{V}} & \mathbf{L}_{N^h \alpha} & \mathbf{L}_{N^h \lambda} \\
(h-1) & \mathbf{L}_{\rho \mathbf{P}} & \mathbf{L}_{\rho W^h} & \mathbf{L}_{\rho \mathbf{W}^{-h}} & \mathbf{L}_{\rho \mathbf{I}} & \mathbf{L}_{\rho \mathbf{V}} & \mathbf{L}_{\rho \alpha} & \mathbf{L}_{\rho \lambda} \\
(1) & \mathbf{L}_{\gamma \mathbf{P}} & \mathbf{L}_{\gamma W^h} & \mathbf{L}_{\gamma \mathbf{W}^{-h}} & \mathbf{L}_{\gamma \mathbf{I}} & \mathbf{L}_{\gamma \mathbf{V}} & \mathbf{L}_{\gamma \alpha} & \mathbf{L}_{\gamma \lambda} \\
& (n) & (1) & (h-1) & (h) & (h) & (h) & (h)
\end{bmatrix} .
$$

The matrix

$$
\begin{bmatrix}
(n) & \mathbf{L}_{\mathbf{X}^h_i \mathbf{X}^h_i} & \mathbf{L}_{\mathbf{X}^h_i N^h} & \mathbf{L}_{\mathbf{X}^h_i \rho} & \mathbf{L}_{\mathbf{X}^h_i \gamma} \\
(1) & \mathbf{L}_{N^h \mathbf{X}^h_i} & \mathbf{L}_{N^h N^h} & \mathbf{L}_{N^h \rho} & \mathbf{L}_{N^h \gamma} \\
(h-1) & \mathbf{L}_{\rho \mathbf{X}^h_i} & \mathbf{L}_{\rho N^h} & \mathbf{L}_{\rho\rho} & \mathbf{L}_{\rho\gamma} \\
(1) & \mathbf{L}_{\gamma \mathbf{X}^h_i} & \mathbf{L}_{\gamma N^h} & \mathbf{L}_{\gamma\rho} & \mathbf{L}_{\gamma\gamma} \\
& (n) & (1) & (h-1) & (1)
\end{bmatrix}
$$

is the symmetric bordered Hessian matrix evaluated at given levels of the exogenous parameters and can be partitioned as

$$
\begin{bmatrix}
\underset{((n+1)\times(n+1))}{\mathbf{A}} & \underset{((n+1)\times(h))}{\mathbf{G}} \\
\underset{((h)\times(n+1))}{\mathbf{G}'} & \underset{(h\times h)}{\mathbf{Q}}
\end{bmatrix} .
$$

It is assumed that:

$$
\det
\begin{bmatrix}
\underset{((n+1)\times(n+1))}{\mathbf{A}} & \underset{((n+1)\times(h))}{\mathbf{G}} \\
\underset{((h)\times(n+1))}{\mathbf{G}'} & \underset{(h\times h)}{\mathbf{Q}}
\end{bmatrix} \neq 0,
$$

and

$$
\text{Rank} \underset{((n+1)\times(h))}{\mathbf{G}} = h.
$$

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These assumptions along with the existence of a local maximum ensure that the second order sufficient conditions for a maximum hold, namely:

$$\xi' \mathbf{A} \xi < 0$$

for all

$$\xi \neq 0$$

such that

$$\mathbf{G}\xi = 0.$$

From this we have that

$$
\begin{bmatrix}
\underset{((n+1)\times(n+1))}{\mathbf{A}} & \underset{((n+1)\times(h))}{\mathbf{G}} \\
\underset{((h)\times(n+1))}{\mathbf{G}'} & \underset{(h\times h)}{\mathbf{Q}}
\end{bmatrix}^{-1}
$$

exists and is given by

$$
\begin{bmatrix}
(n+1) & \mathbf{a} & \mathbf{g} \\
(h) & \mathbf{g}' & q \\
 & (n+1) & (h)
\end{bmatrix},
$$

and by the Caratheodory-Samuelson theorem, $\underset{((n+1)\times(n+1))}{\mathbf{a}}$ is symmetric and negative semi definitie.

Solution of the comparative static equations:

In first solving for the matrix $\mathbf{g}$, note that $\mathbf{g}$ has $(n+1)$ rows and (h) columns which can be partitioned into two matrices $\mathbf{g}^1$ and $\mathbf{g}^2$, where $\mathbf{g}^1$ has $(n+1)$ rows and $(h-1)$ columns, while $\mathbf{g}^2$ has $(n+1)$ rows and (1) column. So

$$
\underset{(n+1)\times h}{\mathbf{g}} =
\begin{bmatrix}
\underset{(n+1)\times(h-1)}{\mathbf{g}^1} & \underset{(n+1)\times 1}{\mathbf{g}^2}
\end{bmatrix}.
$$

Also note that

$$
\begin{bmatrix}
\mathbf{X}^h_{iV^h} & \mathbf{X}^h_{i\mathbf{V}^{-h}} \\
N^h_{V^h} & N^h_{\mathbf{V}^{-h}}
\end{bmatrix}
= -\mathbf{a}
\begin{bmatrix}
\mathbf{L}_{\mathbf{X}^h_i V^h} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{V}^{-h}} \\
\mathbf{L}_{N^h V^h} & \mathbf{L}_{N^h \mathbf{V}^{-h}}
\end{bmatrix}
- \mathbf{g}
\begin{bmatrix}
\mathbf{L}_{\rho V^h} & \mathbf{L}_{\rho \mathbf{V}^{-h}} \\
\mathbf{L}_{\gamma V^h} & \mathbf{L}_{\gamma \mathbf{V}^{-h}}
\end{bmatrix}
$$

$$
= -
\begin{bmatrix}
\underset{(n+1)\times(h-1)}{\mathbf{g}^1} & \underset{(n+1)\times 1}{\mathbf{g}^2}
\end{bmatrix}
\begin{bmatrix}
\mathbf{L}_{\rho V^h} & \mathbf{L}_{\rho \mathbf{V}^{-h}} \\
\mathbf{L}_{\gamma V^h} & \mathbf{L}_{\gamma \mathbf{V}^{-h}}
\end{bmatrix}
$$

$$
=
\begin{bmatrix}
- \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \mathbf{L}_{\rho V^h} - \underset{(n+1)\times 1}{\mathbf{g}^2} \mathbf{L}_{\gamma V^h}
\end{bmatrix}
$$

$$
\left[- \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \mathbf{L}_{\rho \mathbf{V}^{-h}} - \underset{(n+1)\times 1}{\mathbf{g}^2} \mathbf{L}_{\gamma \mathbf{V}^{-h}} \right]
$$

$$
= \left[\; [\mathbf{0}] \quad \left[\underset{(n+1)\times(h-1)}{-\mathbf{g}^1} \cdot \underset{(h-1)\times(h-1)}{-\mathbf{I}} \right] \; \right],
$$

and

$$
\left[\begin{array}{cc} \mathbf{X}^h_{i I^h} & \mathbf{X}^h_{i \mathbf{I}^{-h}} \\ N^h_{I^h} & N^h_{\mathbf{I}^{-h}} \end{array} \right] = -\mathbf{a} \left[\begin{array}{cc} \mathbf{L}_{\mathbf{X}^h_i I^h} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{I}^{-h}} \\ \mathbf{L}_{N^h I^h} & \mathbf{L}_{N^h \mathbf{I}^{-h}} \end{array} \right] - \mathbf{g} \left[\begin{array}{cc} \mathbf{L}_{\rho I^h} & \mathbf{L}_{\rho \mathbf{I}^{-h}} \\ \mathbf{L}_{\gamma I^h} & \mathbf{L}_{\gamma \mathbf{I}^{-h}} \end{array} \right]
$$

$$
= - \left[\underset{(n+1)\times(h-1)}{\mathbf{g}^1} \quad \underset{(n+1)\times 1}{\mathbf{g}^2} \right] \left[\begin{array}{ccc} \underset{(h-1)\times 1}{\mathbf{0}} & \underset{(h-1)\times(h-1)}{-\mathbf{I}} & \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial I^k}} \\ 1 & & \left(\underset{1\times(h-1)}{\mathbf{i}} \right) \end{array} \right]
$$

$$
= \left[\; [-\mathbf{g}^2.1] \quad \left[-\mathbf{g}^1 \; \underset{(h-1)\times(h-1)}{-\mathbf{I}} \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial I^k}} \; -\mathbf{g}^2 \left(\underset{1\times(h-1)}{\mathbf{i}} \right) \right] \; \right]
$$

$$
= \left[\; [-\mathbf{g}^2.1] \quad \left[\left(\begin{array}{c} \mathbf{X}^h_{i \mathbf{V}^{-h}} \\ N^h_{\mathbf{V}^{-h}} \end{array} \right) \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial I^k}} \; -\mathbf{g}^2 \left(\underset{1\times(h-1)}{\mathbf{i}} \right) \right] \; \right],
$$

where $\underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial I^k}}$ is a diagonal matrix with $\frac{\partial V^k}{\partial I^k}, k = 1, ..., H, k \neq h$ on the diagonal and zeros on the off-diagonals.

From this it is clear that

$$
-\mathbf{g}^2.1 = \left(\begin{array}{c} \mathbf{X}^h_{i I^h} \\ N^h_{I^h} \end{array} \right),
$$

and

$$
-\mathbf{g}^2 \left(\underset{1\times(h-1)}{\mathbf{i}} \right) = \left(\begin{array}{c} \mathbf{X}^h_{i \mathbf{I}^{-h}} \\ N^h_{\mathbf{I}^{-h}} \end{array} \right) - \left(\begin{array}{c} \mathbf{X}^h_{i \mathbf{V}^{-h}} \\ N^h_{\mathbf{V}^{-h}} \end{array} \right) \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial I^k}} .
$$

The total price effects are:

$$
\left[\begin{array}{cc} \mathbf{X}^h_{i \mathbf{P}} & \mathbf{X}^h_{i W^h} \\ N^h_{\mathbf{P}} & N^h_{W^h} \end{array} \right]
$$

$$
= -\mathbf{a} \left[\begin{array}{cc} \mathbf{L}_{\mathbf{X}^h_i \mathbf{P}} & \mathbf{L}_{\mathbf{X}^h_i W^h} \\ \mathbf{L}_{N^h \mathbf{P}} & \mathbf{L}_{N^h W^h} \end{array} \right] - \mathbf{g} \left[\begin{array}{cc} \mathbf{L}_{\rho \mathbf{P}} & \mathbf{L}_{\rho W^h} \\ \mathbf{L}_{\gamma \mathbf{P}} & \mathbf{L}_{\gamma W^h} \end{array} \right]
$$

$$
= \mathbf{a}\gamma\mathbf{I}
$$

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$$
-\begin{bmatrix} \mathbf{g}^1 \\ {\scriptstyle (n+1)\times(h-1)} & \mathbf{g}^2 \\ {\scriptstyle (n+1)\times 1} \end{bmatrix} \begin{bmatrix} \begin{pmatrix} -\mathbf{I} & \frac{\partial V_k}{\partial \mathbf{P}} \\ {\scriptstyle (h-1)\times(h-1)} & {\scriptstyle (h-1)\times n} \end{pmatrix} & \mathbf{0} \\ {\scriptstyle (h-1)\times 1} \\ -\mathbf{X}' & T-N^h \end{bmatrix}
$$

$$
= \mathbf{a}\gamma\mathbf{I}
$$

$$
+\left[\left[\begin{pmatrix} -\mathbf{g}^1 & -\mathbf{I} \\ {\scriptstyle (n+1)\times(h-1)} & {\scriptstyle (h-1)\times(h-1)} \end{pmatrix} \frac{\partial V^k}{\partial \mathbf{P}}_{(h-1)\times n} - \mathbf{g}^2_{(n+1)\times 1}(-\mathbf{X}') \right] \quad \left[-\mathbf{g}^2_{(n+1)\times 1}\left(T-N^h\right) \right] \right]
$$

$$
= \mathbf{a}\gamma\mathbf{I}
$$

$$
+\left[\left[\begin{pmatrix} \mathbf{X}^h_{iV^{-h}} \\ N^h_{\mathbf{V}^{-h}} \end{pmatrix} \frac{\partial V^k}{\partial \mathbf{P}}_{(h-1)\times n} + \begin{pmatrix} \mathbf{X}^h_{iI^h} \\ N^h_{I^h} \end{pmatrix}(-\mathbf{X}') \right] \quad \left[\begin{pmatrix} \mathbf{X}^h_{iI^h} \\ N^h_{I^h} \end{pmatrix}\left(T-N^h\right) \right] \right],
$$

and the cross-wage effects are:

$$
\begin{bmatrix} \mathbf{X}^h_{i\mathbf{W}^{-h}} \\ N^h_{\mathbf{W}^{-h}} \end{bmatrix}
$$

$$
= -\mathbf{a}\begin{bmatrix} \mathbf{L}_{\mathbf{X}^h_i \mathbf{W}^{-h}} \\ \mathbf{L}_{\mathbf{N}^h \mathbf{W}^{-h}} \end{bmatrix} - \mathbf{g}\begin{bmatrix} \mathbf{L}_{\rho \mathbf{W}^{-h}} \\ \mathbf{L}_{\gamma \mathbf{W}^{-h}} \end{bmatrix}
$$

$$
= -\begin{bmatrix} \mathbf{g}^1 \\ {\scriptstyle (n+1)\times(h-1)} & \mathbf{g}^2 \\ {\scriptstyle (n+1)\times 1} \end{bmatrix} \begin{bmatrix} -\mathbf{I} & \frac{\partial V^k}{\partial \mathbf{W}^k} \\ {\scriptstyle (h-1)\times(h-1)} & {\scriptstyle (h-1)\times(h-1)} \\ \left(\mathbf{T}^{-h}-\mathbf{N}^{-h}\right)' \end{bmatrix}
$$

$$
= -\mathbf{g}^1_{(n+1)\times(h-1)} \cdot -\mathbf{I}_{(h-1)\times(h-1)} \frac{\partial V^k}{\partial \mathbf{W}^k}_{(h-1)\times(h-1)} - \mathbf{g}^2_{(n+1)\times 1}\left(\mathbf{T}^{-h}-\mathbf{N}^{-h}\right)'
$$

$$
= \begin{pmatrix} \mathbf{X}^h_{iV^{-h}} \\ N^h_{\mathbf{V}^{-h}} \end{pmatrix} \frac{\partial V^k}{\partial \mathbf{W}^k}_{(h-1)\times(h-1)} + \begin{pmatrix} \mathbf{X}^h_{iI^h} \\ N^h_{I^h} \end{pmatrix}\left(\mathbf{T}^{-h}-\mathbf{N}^{-h}\right)',
$$

where $\frac{\partial V^k}{\partial \mathbf{W}^k}_{(h-1)\times(h-1)}$ is a diagonal matrix with $\frac{\partial V^k}{\partial W^k}, k=1,...,H, k\neq h$ on the diagonal and zeros on the off-diagonals.

The non-wage incomes affect person h's demands as shown below:

$$
\begin{bmatrix} \mathbf{X}^h_{iI^h} & \mathbf{X}^h_{i\mathbf{I}^{-h}} \\ N^h_{I^h} & N^h_{\mathbf{I}^{-h}} \end{bmatrix} = -\mathbf{a}\begin{bmatrix} \mathbf{L}_{\mathbf{X}^h_i I^h} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{I}^{-h}} \\ \mathbf{L}_{N^h I^h} & \mathbf{L}_{N^h \mathbf{I}^{-h}} \end{bmatrix} - \mathbf{g}\begin{bmatrix} \mathbf{L}_{\rho I^h} & \mathbf{L}_{\rho \mathbf{I}^{-h}} \\ \mathbf{L}_{\gamma I^h} & \mathbf{L}_{\gamma \mathbf{I}^{-h}} \end{bmatrix}
$$

$$
= -\begin{bmatrix} \mathbf{g}^1 \\ {\scriptstyle (n+1)\times(h-1)} & \mathbf{g}^2 \\ {\scriptstyle (n+1)\times 1} \end{bmatrix} \begin{bmatrix} \mathbf{0} & -\mathbf{I} & \frac{\partial V^k}{\partial \mathbf{I}^k} \\ {\scriptstyle (h-1)\times 1} & {\scriptstyle (h-1)\times(h-1)} & {\scriptstyle (h-1)\times(h-1)} \\ 1 & \begin{pmatrix} \mathbf{i} \\ {\scriptstyle 1\times(h-1)} \end{pmatrix} \end{bmatrix}
$$

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$$= \left[\begin{array}{cc} -\mathbf{g}^2.\mathbf{1} & \left(-\mathbf{g}^1 \underset{(h-1)\times(h-1)}{-\mathbf{I}} \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial I^k}} -\mathbf{g}^2 \left(\underset{1\times(h-1)}{\mathbf{i}} \right) \right) \end{array} \right]$$

$$= \left[\begin{array}{cc} -\mathbf{g}^2.\mathbf{1} & \left(\left(\begin{array}{c} \mathbf{X}^h_{iV^{-h}} \\ N^h_{V^{-h}} \end{array} \right) \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial I^k}} -\mathbf{g}^2 \left(\underset{1\times(h-1)}{\mathbf{i}} \right) \right) \end{array} \right],$$

where $\underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial I^k}}$ is a diagonal matrix with $\frac{\partial V^k}{\partial I^k}, k = 1, ..., H, k \neq h$ on the diagonal and zeros on the off-diagonals.

Also

$$\left[\begin{array}{c} \mathbf{X}^h_{iI^h} \\ N^h_{I^h} \end{array} \right] - \left[\begin{array}{c} \mathbf{X}^h_{iI^j} \\ N^h_{I^j} \end{array} \right], \forall j = 1, ..., H, j \neq h$$

is given by

$$\left[\begin{array}{cc} \mathbf{X}^h_{iI^h} & \mathbf{X}^h_{iI^j} \\ N^h_{I^h} & N^h_{I^j} \end{array} \right] \left[\begin{array}{c} 1 \\ -1 \end{array} \right]$$

$$= \left[\begin{array}{cc} -\mathbf{g}^2.\mathbf{1} & \left[\left(\begin{array}{c} \mathbf{X}^h_{iV^j} \\ N^h_{V^j} \end{array} \right) \underset{(h-1)\times1}{\frac{\partial V^j}{\partial I^j}} -\mathbf{g}^2.\mathbf{1} \right] \end{array} \right] \left[\begin{array}{c} 1 \\ -1 \end{array} \right]$$

$$= \left(\begin{array}{c} \mathbf{X}^h_{iV^j} \\ N^h_{V^j} \end{array} \right) \underset{(h-1)\times1}{\frac{\partial V^j}{\partial I^j}}.$$

The impact of the disagreement points are as follows:

$$\left[\begin{array}{cc} \mathbf{X}^h_{iV^h} & \mathbf{X}^h_{iV^{-h}} \\ N^h_{V^h} & N^h_{V^{-h}} \end{array} \right] = -\mathbf{a} \left[\begin{array}{cc} \mathbf{L}_{\mathbf{X}^h_i V^h} & \mathbf{L}_{\mathbf{X}^h_i V^{-h}} \\ \mathbf{L}_{N^h V^h} & \mathbf{L}_{N^h V^{-h}} \end{array} \right] - \mathbf{g} \left[\begin{array}{cc} \mathbf{L}_{\rho V^h} & \mathbf{L}_{\rho V^{-h}} \\ \mathbf{L}_{\gamma V^h} & \mathbf{L}_{\gamma V^{-h}} \end{array} \right]$$

$$= - \left[\begin{array}{cc} \underset{(n+1)\times(h-1)}{\mathbf{g}^1} & \underset{(n+1)\times1}{\mathbf{g}^2} \end{array} \right] \left[\begin{array}{cc} \mathbf{L}_{\rho V^h} & \mathbf{L}_{\rho V^{-h}} \\ \mathbf{L}_{\gamma V^h} & \mathbf{L}_{\gamma V^{-h}} \end{array} \right]$$

$$= \left[\begin{array}{c} - \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \mathbf{L}_{\rho V^h} - \underset{(n+1)\times1}{\mathbf{g}^2} \mathbf{L}_{\gamma V^h} \end{array} \right]$$

$$\left[\begin{array}{c} - \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \mathbf{L}_{\rho V^{-h}} - \underset{(n+1)\times1}{\mathbf{g}^2} \mathbf{L}_{\gamma V^{-h}} \end{array} \right]$$

$$= \left[\begin{array}{cc} [\mathbf{0}] & \left[\underset{(n+1)\times(h-1)}{-\mathbf{g}^1} \cdot \underset{(h-1)\times(h-1)}{-\mathbf{I}} \right] \end{array} \right].$$

The impact of the shift parameters are given by:

$$\left[\begin{array}{cc} \mathbf{X}^h_{i\alpha^h} & \mathbf{X}^h_{i\alpha^{-h}} \\ N^h_{\alpha^h} & N^h_{\alpha^{-h}} \end{array} \right] = -\mathbf{a} \left[\begin{array}{cc} \mathbf{L}_{\mathbf{X}^h_i \alpha^h} & \mathbf{L}_{\mathbf{X}^h_i \alpha^{-h}} \\ \mathbf{L}_{N^h \alpha^h} & \mathbf{L}_{N^h \alpha^{-h}} \end{array} \right] - \mathbf{g} \left[\begin{array}{cc} \mathbf{L}_{\rho \alpha^h} & \mathbf{L}_{\rho \alpha^{-h}} \\ \mathbf{L}_{\gamma \alpha^h} & \mathbf{L}_{\gamma \alpha^{-h}} \end{array} \right]$$

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$$= -\left[\underset{(n+1)\times(h-1)}{\mathbf{g}^1} \quad \underset{(n+1)\times1}{\mathbf{g}^2} \right] \left[\begin{array}{ccc} 0 & \underset{(h-1)\times(h-1)}{-\mathbf{I}} & \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial \alpha^k}} \\ 0 & 0 & \end{array} \right]$$

$$= \left[0 \quad \left[-\underset{(n+1)\times(h-1)}{\mathbf{g}^1}\underset{(h-1)\times(h-1)}{-\mathbf{I}}\underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial \alpha^k}} \right] \right]$$

$$= \left[0 \quad \left[\left(\begin{array}{c} \mathbf{X}^h_{i\mathbf{V}^{-h}} \\ N^h_{\mathbf{V}^{-h}} \end{array} \right) \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial \alpha^k}} \right] \right],$$

where $\underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial \alpha^k}}$ is a diagonal matrix with $\frac{\partial V^k}{\partial \alpha^k}, k = 1, ..., H, k \neq h$ on the diagonal and zeros on the off-diagonals.

Egalitarian model

Proof that the Slutsky matrix is symmetric and negative semi-definite:

The argument is exactly analogous to that for the dictatorial Slutsky matrix.

Solution of the comparative static equations:

First solving for the matrix $\mathbf{g}$, note that $\mathbf{g}$ has $(n + 1)$ rows and (h) columns which can be partitioned into two matrices $\mathbf{g}^1$ and $\mathbf{g}^2$, where $\mathbf{g}^1$ has $(n + 1)$ rows and $(h - 1)$ columns, while $\mathbf{g}^2$ has $(n + 1)$ rows and (1) column. So

$$\underset{(n+1)\times h}{\mathbf{g}} = \left[\underset{(n+1)\times(h-1)}{\mathbf{g}^1} \quad \underset{(n+1)\times1}{\mathbf{g}^2} \right].$$

Also note that

$$\left[\begin{array}{cc} \mathbf{X}^h_{iV^h} & \mathbf{X}^h_{i\mathbf{V}^{-h}} \\ N^h_{V^h} & N^h_{\mathbf{V}^{-h}} \end{array} \right] = -\mathbf{a} \left[\begin{array}{cc} \mathbf{L}_{\mathbf{X}^h_i V^h} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{V}^{-h}} \\ \mathbf{L}_{N^h V^h} & \mathbf{L}_{N^h \mathbf{V}^{-h}} \end{array} \right] - \mathbf{g} \left[\begin{array}{cc} \mathbf{L}_{\rho V^h} & \mathbf{L}_{\rho \mathbf{V}^{-h}} \\ \mathbf{L}_{\gamma V^h} & \mathbf{L}_{\gamma \mathbf{V}^{-h}} \end{array} \right]$$

$$= -\left[\underset{(n+1)\times(h-1)}{\mathbf{g}^1} \quad \underset{(n+1)\times1}{\mathbf{g}^2} \right] \left[\begin{array}{cc} \mathbf{L}_{\rho V^h} & \mathbf{L}_{\rho \mathbf{V}^{-h}} \\ \mathbf{L}_{\gamma V^h} & \mathbf{L}_{\gamma \mathbf{V}^{-h}} \end{array} \right]$$

$$= \left[-\underset{(n+1)\times(h-1)}{\mathbf{g}^1}\mathbf{L}_{\rho V^h} - \underset{(n+1)\times1}{\mathbf{g}^2}\mathbf{L}_{\gamma V^h} \right]$$

$$\left[-\underset{(n+1)\times(h-1)}{\mathbf{g}^1}\mathbf{L}_{\rho \mathbf{V}^{-h}} - \underset{(n+1)\times1}{\mathbf{g}^2}\mathbf{L}_{\gamma \mathbf{V}^{-h}} \right]$$

$$= \left[-\underset{(n+1)\times(h-1)}{\mathbf{g}^1} \left(\underset{(h-1)\times1}{-\mathbf{i}} \right) \right] \left[-\underset{(n+1)\times(h-1)}{\mathbf{g}^1}\underset{(h-1)\times(h-1)}{\mathbf{I}} \right],$$

and

$$\left[\begin{array}{cc} \mathbf{X}^h_{iI^h} & \mathbf{X}^h_{i\mathbf{I}^{-h}} \\ N^h_{I^h} & N^h_{\mathbf{I}^{-h}} \end{array} \right] = -\mathbf{a} \left[\begin{array}{cc} \mathbf{L}_{\mathbf{X}^h_i I^h} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{I}^{-h}} \\ \mathbf{L}_{N^h I^h} & \mathbf{L}_{N^h \mathbf{I}^{-h}} \end{array} \right] - \mathbf{g} \left[\begin{array}{cc} \mathbf{L}_{\rho I^h} & \mathbf{L}_{\rho \mathbf{I}^{-h}} \\ \mathbf{L}_{\gamma I^h} & \mathbf{L}_{\gamma \mathbf{I}^{-h}} \end{array} \right]$$

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$$
= -\mathbf{g} \left[\begin{array}{cc} \left(\begin{array}{c} -\mathbf{i} \\ {\scriptstyle (h-1)\times 1} \end{array} \right) \dfrac{\partial V^h}{\partial I^h} & \underset{(h-1)\times(h-1)}{\mathbf{I}} \underset{(h-1)\times(h-1)}{\dfrac{\partial V^k}{\partial I^k}} \\[2em] 1 & \left(\begin{array}{c} \mathbf{i} \\ {\scriptstyle 1\times(h-1)} \end{array} \right) \end{array} \right]
$$

$$
= \left[-\mathbf{g}^1 \left(\begin{array}{c} -\mathbf{i} \\ {\scriptstyle (h-1)\times 1} \end{array} \right) \dfrac{\partial V^h}{\partial I^h} - \mathbf{g}^2.1 \right]
$$

$$
\left[-\mathbf{g}^1 \underset{(h-1)\times(h-1)}{\mathbf{I}} \underset{(h-1)\times(h-1)}{\dfrac{\partial V^k}{\partial I^k}} \quad -\mathbf{g}^2 \left(\begin{array}{c} \mathbf{i} \\ {\scriptstyle 1\times(h-1)} \end{array} \right) \right]
$$

$$
= \left[\left[\left(\begin{array}{c} \mathbf{X}^h_{iV^h} \\ N^h_{V^h} \end{array} \right) \dfrac{\partial V^h}{\partial I^h} - \mathbf{g}^2.1 \right] \quad \left[\left(\begin{array}{c} \mathbf{X}^h_{iV^{-h}} \\ N^h_{V^{-h}} \end{array} \right) \underset{(h-1)\times(h-1)}{\dfrac{\partial V^k}{\partial I^k}} - \mathbf{g}^2 \left(\begin{array}{c} \mathbf{i} \\ {\scriptstyle 1\times(h-1)} \end{array} \right) \right] \right]
$$

where $\underset{(h-1)\times(h-1)}{\dfrac{\partial V^k}{\partial I^k}}$ is a diagonal matrix with $\dfrac{\partial V^k}{\partial I^k}, k = 1,...,H, k \neq h$ on the diagonal and zeros on the off-diagonals.

From this it is clear that

$$
-\mathbf{g}^2.1 = \left(\begin{array}{c} \mathbf{X}^h_{iI^h} \\ N^h_{I^h} \end{array} \right) - \left(\begin{array}{c} \mathbf{X}^h_{iV^h} \\ N^h_{V^h} \end{array} \right) \dfrac{\partial V^h}{\partial I^h},
$$

and

$$
-\mathbf{g}^2 \left(\begin{array}{c} \mathbf{i} \\ {\scriptstyle 1\times(h-1)} \end{array} \right) = \left(\begin{array}{c} \mathbf{X}^h_{iI^{-h}} \\ N^h_{\mathbf{I}^{-h}} \end{array} \right) - \left(\begin{array}{c} \mathbf{X}^h_{iV^{-h}} \\ N^h_{\mathbf{V}^{-h}} \end{array} \right) \underset{(h-1)\times(h-1)}{\dfrac{\partial V^k}{\partial I^k}} .
$$

The total price effects are:

$$
\left[\begin{array}{cc} \mathbf{X}^h_{i\mathbf{P}} & \mathbf{X}^h_{iW^h} \\ N^h_{\mathbf{P}} & N^h_{W^h} \end{array} \right]
$$

$$
= -\mathbf{a} \left[\begin{array}{cc} \mathbf{L}_{\mathbf{X}^h_i \mathbf{P}} & \mathbf{L}_{\mathbf{X}^h_i W^h} \\ \mathbf{L}_{N^h \mathbf{P}} & \mathbf{L}_{N^h W^h} \end{array} \right] - \mathbf{g} \left[\begin{array}{cc} \mathbf{L}_{\rho \mathbf{P}} & \mathbf{L}_{\rho W^h} \\ \mathbf{L}_{\gamma \mathbf{P}} & \mathbf{L}_{\gamma W^h} \end{array} \right]
$$

$$
= \mathbf{a}\gamma\mathbf{I}
$$

$$
- \left[\begin{array}{cc} \mathbf{g}^1 & \mathbf{g}^2 \\ {\scriptstyle (n+1)\times(h-1)} & {\scriptstyle (n+1)\times 1} \end{array} \right] \left[\begin{array}{cc} \left[\left(\begin{array}{c} -\mathbf{i} \\ {\scriptstyle (h-1)\times 1} \end{array} \right) \underset{(1\times n)}{\dfrac{\partial V^h}{\partial \mathbf{P}}} + \underset{(h-1)\times(h-1)}{\mathbf{I}} \underset{(h-1)\times n}{\dfrac{\partial V^h}{\partial \mathbf{P}}} \right] & \left(\begin{array}{c} -\mathbf{i} \\ {\scriptstyle (h-1)\times 1} \end{array} \right) \dfrac{\partial V^h}{\partial W_h} \\ -\mathbf{X}' & T - N^h \end{array} \right]
$$

$$
= \mathbf{a}\gamma\mathbf{I}+
$$

$$
\begin{bmatrix}
- \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \left(\left(\underset{(h-1)\times 1}{-\mathbf{i}} \right) \underset{(1\times n)}{\frac{\partial V^h}{\partial \mathbf{P}}} + \underset{(h-1)\times(h-1)}{\mathbf{I}} \underset{(h-1)\times n}{\frac{\partial V^k}{\partial \mathbf{P}}} \right) - \underset{(n+1)\times 1}{\mathbf{g}^2} \left(-\mathbf{X}' \right)
\end{bmatrix}
$$

$$
\begin{bmatrix}
- \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \left(\underset{(h-1)\times 1}{-\mathbf{i}} \right) \frac{\partial V^h}{\partial W_h} - \underset{(n+1)\times 1}{\mathbf{g}^2} \left(T - N^h \right)
\end{bmatrix}
$$

$$
= \mathbf{a}\gamma\mathbf{I} +
$$

$$
\begin{bmatrix}
\begin{pmatrix} \mathbf{X}^h_{iV^h} \\ N^h_{V^h} \end{pmatrix} \underset{(1\times n)}{\frac{\partial V^h}{\partial \mathbf{P}}} + \begin{pmatrix} \mathbf{X}^h_{iV^{-h}} \\ N^h_{V^{-h}} \end{pmatrix} \underset{(h-1)\times n}{\frac{\partial V^k}{\partial \mathbf{P}}} + \left[\begin{pmatrix} \mathbf{X}^h_{iI^h} \\ N^h_{I^h} \end{pmatrix} - \begin{pmatrix} \mathbf{X}^h_{iV^h} \\ N^h_{V^h} \end{pmatrix} \frac{\partial V^h}{\partial I^h} \right] \left(-\mathbf{X}' \right)
\end{bmatrix}
$$

$$
\begin{bmatrix}
\begin{pmatrix} \mathbf{X}^h_{iV^h} \\ N^h_{V^h} \end{pmatrix} \frac{\partial V^h}{\partial W_h} + \left[\begin{pmatrix} \mathbf{X}^h_{iI^h} \\ N^h_{I^h} \end{pmatrix} - \begin{pmatrix} \mathbf{X}^h_{iV^h} \\ N^h_{V^h} \end{pmatrix} \frac{\partial V^h}{\partial I^h} \right] \left(T - N^h \right)
\end{bmatrix},
$$

while the cross-wage effects are given by:

$$
\begin{bmatrix} \mathbf{X}^h_{i\mathbf{W}^{-h}} \\ N^h_{\mathbf{W}^{-h}} \end{bmatrix}
$$

$$
= -\mathbf{a} \begin{bmatrix} \mathbf{L}_{\mathbf{X}^h_i\mathbf{W}^{-h}} \\ \mathbf{L}_{\mathbf{N}^h\mathbf{W}^{-h}} \end{bmatrix} - \mathbf{g} \begin{bmatrix} \mathbf{L}_{\rho\mathbf{W}^{-h}} \\ \mathbf{L}_{\gamma\mathbf{W}^{-h}} \end{bmatrix}
$$

$$
= - \begin{bmatrix} \underset{(n+1)\times(h-1)}{\mathbf{g}^1} & \underset{(n+1)\times 1}{\mathbf{g}^2} \end{bmatrix} \begin{bmatrix} \underset{(h-1)\times(h-1)}{\mathbf{I}} \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial \mathbf{W}^k}} \\ \left(\mathbf{T}^{-h} - \mathbf{N}^{-h} \right)' \end{bmatrix}
$$

$$
= \begin{bmatrix} - \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \underset{(h-1)\times(h-1)}{\mathbf{I}} \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial \mathbf{W}^k}} - \underset{(n+1)\times 1}{\mathbf{g}^2} \left(\mathbf{T}^{-h} - \mathbf{N}^{-h} \right)' \end{bmatrix}
$$

$$
= \begin{bmatrix} \begin{pmatrix} \mathbf{X}^h_{iV^{-h}} \\ N^h_{V^{-h}} \end{pmatrix} \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial \mathbf{W}^k}} + \left[\begin{pmatrix} \mathbf{X}^h_{iI^h} \\ N^h_{I^h} \end{pmatrix} - \begin{pmatrix} \mathbf{X}^h_{iV^h} \\ N^h_{V^h} \end{pmatrix} \frac{\partial V^h}{\partial I^h} \right] \left(\mathbf{T}^{-h} - \mathbf{N}^{-h} \right)' \end{bmatrix},
$$

where $\underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial \mathbf{W}^k}}$ is a diagonal matrix with $\frac{\partial V^k}{\partial W^k}, k = 1, ..., H, k \neq h$ on the diagonal and zeros on the off-diagonals.

The impact of the non-wage incomes are described by:

$$
\begin{bmatrix} \mathbf{X}^h_{iI^h} & \mathbf{X}^h_{i\mathbf{I}^{-h}} \\ N^h_{I^h} & N^h_{\mathbf{I}^{-h}} \end{bmatrix} = -\mathbf{a} \begin{bmatrix} \mathbf{L}_{\mathbf{X}^h_i I^h} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{I}^{-h}} \\ \mathbf{L}_{\mathbf{N}^h I^h} & \mathbf{L}_{\mathbf{N}^h \mathbf{I}^{-h}} \end{bmatrix} - \mathbf{g} \begin{bmatrix} \mathbf{L}_{\rho I^h} & \mathbf{L}_{\rho \mathbf{I}^{-h}} \\ \mathbf{L}_{\gamma I^h} & \mathbf{L}_{\gamma \mathbf{I}^{-h}} \end{bmatrix}
$$

$$= -\begin{bmatrix} \mathbf{g}^1 \\ {\scriptstyle (n+1)\times(h-1)} & \mathbf{g}^2 \\ {\scriptstyle (n+1)\times 1} \end{bmatrix} \begin{bmatrix} \begin{pmatrix} -\mathbf{i} \\ {\scriptstyle (h-1)\times 1} \end{pmatrix} \frac{\partial V^h}{\partial I^h} & \mathbf{I} \\ {\scriptstyle (h-1)\times(h-1)} \frac{\partial V^k}{\partial I^k} \\ {\scriptstyle (h-1)\times(h-1)} \\ 1 & \begin{pmatrix} \mathbf{i} \\ {\scriptstyle 1\times(h-1)} \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -\mathbf{g}^1 \begin{pmatrix} -\mathbf{i} \\ {\scriptstyle (h-1)\times 1} \end{pmatrix} \frac{\partial V^h}{\partial I^h} - \mathbf{g}^2.1 \end{bmatrix}$$

$$\begin{bmatrix} -\mathbf{g}^1 \underset{{\scriptstyle (h-1)\times(h-1)}}{\mathbf{I}} \underset{{\scriptstyle (h-1)\times(h-1)}}{\frac{\partial V^k}{\partial I^k}} & -\mathbf{g}^2 \begin{pmatrix} \mathbf{i} \\ {\scriptstyle 1\times(h-1)} \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} \begin{pmatrix} \mathbf{X}^h_{iV^h} \\ N^h_{V^h} \end{pmatrix} \frac{\partial V^h}{\partial I^h} - \mathbf{g}^2.1 \end{bmatrix} & \begin{bmatrix} \begin{pmatrix} \mathbf{X}^h_{i\mathbf{V}^{-h}} \\ N^h_{\mathbf{V}^{-h}} \end{pmatrix} \underset{{\scriptstyle (h-1)\times(h-1)}}{\frac{\partial V^k}{\partial I^k}} - \mathbf{g}^2 \begin{pmatrix} \mathbf{i} \\ {\scriptstyle 1\times(h-1)} \end{pmatrix} \end{bmatrix} \end{bmatrix}$$

where $\underset{{\scriptstyle (h-1)\times(h-1)}}{\frac{\partial V^k}{\partial I^k}}$ is a diagonal matrix with $\frac{\partial V^k}{\partial I^k}, k = 1, ..., H, k \neq h$ on the diagonal and zeros on the off-diagonals.

Also note that

$$\begin{bmatrix} \mathbf{X}^h_{iI^h} \\ N^h_{I^h} \end{bmatrix} - \begin{bmatrix} \mathbf{X}^h_{iI^j} \\ N^h_{I^j} \end{bmatrix}, \forall j = 1, ..., H, j \neq h$$

is given by

$$\begin{bmatrix} \mathbf{X}^h_{iI^h} & \mathbf{X}^h_{iI^j} \\ N^h_{I^h} & N^h_{I^j} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} \begin{pmatrix} \mathbf{X}^h_{iV^h} \\ N^h_{V^h} \end{pmatrix} \frac{\partial V^h}{\partial I^h} - \mathbf{g}^2.1 & \begin{pmatrix} \mathbf{X}^h_{iV^j} \\ N^h_{V^j} \end{pmatrix} \underset{{\scriptstyle (h-1)\times 1}}{\frac{\partial V^j}{\partial I^j}} - \mathbf{g}^2.1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{pmatrix} \mathbf{X}^h_{iV^h} \\ N^h_{V^h} \end{pmatrix} \frac{\partial V^h}{\partial I^h} - \begin{pmatrix} \mathbf{X}^h_{iV^j} \\ N^h_{V^j} \end{pmatrix} \underset{{\scriptstyle (h-1)\times 1}}{\frac{\partial V^j}{\partial I^j}} .$$

The disagreement points affect h's demands as follows:

$$\begin{bmatrix} \mathbf{X}^h_{iV^h} & \mathbf{X}^h_{i\mathbf{V}^{-h}} \\ N^h_{V^h} & N^h_{\mathbf{V}^{-h}} \end{bmatrix} = -\mathbf{a} \begin{bmatrix} \mathbf{L}_{\mathbf{X}^h_i V^h} & \mathbf{L}_{\mathbf{X}^h_i \mathbf{V}^{-h}} \\ \mathbf{L}_{N^h V^h} & \mathbf{L}_{N^h \mathbf{V}^{-h}} \end{bmatrix} - \mathbf{g} \begin{bmatrix} \mathbf{L}_{\rho V^h} & \mathbf{L}_{\rho \mathbf{V}^{-h}} \\ \mathbf{L}_{\gamma V^h} & \mathbf{L}_{\gamma \mathbf{V}^{-h}} \end{bmatrix}$$

$$= -\begin{bmatrix} \mathbf{g}^1 \\ {\scriptstyle (n+1)\times(h-1)} & \mathbf{g}^2 \\ {\scriptstyle (n+1)\times 1} \end{bmatrix} \begin{bmatrix} \mathbf{L}_{\rho V^h} & \mathbf{L}_{\rho \mathbf{V}^{-h}} \\ \mathbf{L}_{\gamma V^h} & \mathbf{L}_{\gamma \mathbf{V}^{-h}} \end{bmatrix}$$

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$$= \left[- \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \mathbf{L}_{\rho V^h} - \underset{(n+1)\times 1}{\mathbf{g}^2} \mathbf{L}_{\gamma V^h} \right]$$

$$\left[- \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \mathbf{L}_{\rho \mathbf{V}^{-h}} - \underset{(n+1)\times 1}{\mathbf{g}^2} \mathbf{L}_{\gamma \mathbf{V}^{-h}} \right]$$

$$= \left[\left[- \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \underset{(h-1)\times 1}{\left(\begin{array}{c}-\mathbf{i}\end{array}\right)} \right] \quad \left[- \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \underset{(h-1)\times(h-1)}{\mathbf{I}} \right] \right]$$

$$\neq \quad \mathbf{0}.$$

Finally, the shift parameters affect person h's demands as follows:

$$\left[\begin{array}{cc} \mathbf{X}^h_{i\alpha^h} & \mathbf{X}^h_{i\alpha^{-h}} \\ N^h_{\alpha^h} & N^h_{\alpha^{-h}} \end{array} \right] = -\mathbf{a} \left[\begin{array}{cc} \mathbf{L}_{\mathbf{X}^h_i \alpha^h} & \mathbf{L}_{\mathbf{X}^h_i \alpha^{-h}} \\ \mathbf{L}_{N^h \alpha^h} & \mathbf{L}_{N^h \alpha^{-h}} \end{array} \right] - \mathbf{g} \left[\begin{array}{cc} \mathbf{L}_{\rho \alpha^h} & \mathbf{L}_{\rho \alpha^{-h}} \\ \mathbf{L}_{\gamma \alpha^h} & \mathbf{L}_{\gamma \alpha^{-h}} \end{array} \right]$$

$$= - \left[\underset{(n+1)\times(h-1)}{\mathbf{g}^1} \quad \underset{(n+1)\times 1}{\mathbf{g}^2} \right] \left[\begin{array}{cc} \underset{(h-1)\times 1}{\left(\begin{array}{c}-\mathbf{i}\end{array}\right)} \frac{\partial V^h}{\partial \alpha^h} & \underset{(h-1)\times(h-1)}{\mathbf{I}} \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial \alpha^k}} \\ 0 & 0 \end{array} \right]$$

$$= \left[\left[- \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \underset{(h-1)\times 1}{\left(\begin{array}{c}-\mathbf{i}\end{array}\right)} \frac{\partial V^h}{\partial \alpha^h} \right] \quad \left[- \underset{(n+1)\times(h-1)}{\mathbf{g}^1} \underset{(h-1)\times(h-1)}{\mathbf{I}} \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial \alpha^k}} \right] \right]$$

$$= \left[\left[\left(\begin{array}{c} \mathbf{X}^h_{i V^h} \\ N^h_{V^h} \end{array}\right) \frac{\partial V^h}{\partial \alpha^h} \right] \quad \left[\left(\begin{array}{c} \mathbf{X}^h_{i \mathbf{V}^{-h}} \\ N^h_{\mathbf{V}^{-h}} \end{array}\right) \underset{(h-1)\times(h-1)}{\frac{\partial V^k}{\partial \alpha^k}} \right] \right].$$

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