

**EXTENDED SENSITIVITY ANALYSIS FOR APPLIED
GENERAL EQUILIBRIUM MODELS.**

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Extended Sensitivity Analysis for Applied General Equilibrium Models*

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ABSTRACT

Previous sensitivity analysis procedures for applied general equilibrium models have focussed on the values of exogenously assigned elasticity parameters, while the *calibrated* parameters - those that are obtained from combining elasticity information with flow or stock data - have been largely ignored. Calibrated parameters are central to a model's specification, and uncertainty surrounding their values influences the credibility of the model's results. This paper introduces and illustrates a calibrated parameter sensitivity analysis (CPSA) which, when combined with previous elasticity sensitivity analysis procedures, allows modelers to undertake sensitivity analysis over the full set of model parameters. The 'extended sensitivity analysis' methodology is illustrated for tax incidence results from an applied general equilibrium model of Côte d'Ivoire.

Key Words: Sensitivity Analysis, Calibration.

JEL classification codes: C63, C68.

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I. Introduction

Previous sensitivity analysis procedures for applied general equilibrium models (Pagan and Shannon, 1985; Pagan and Shannon, 1987; Wigle 1991; Harrison and Vinod 1992; Harrison, Jones, Kimbell and Wigle 1992; DeVuyst and Preckel 1997) have focussed on the values of exogenously assigned elasticity parameters, while the *calibrated* parameters - those that are obtained from combining elasticity information with flow or stock data¹ - have been largely ignored. This omission stems partly from the perception that whereas a model's elasticity values are often obtained through informed guesswork and can therefore be very uncertain, the calibrated parameter values have a more solid empirical foundation in data. However, the considerable uncertainty surrounding the data used for calibration introduces uncertainty into the calibrated parameter values, making them also candidates for sensitivity analysis. This uncertainty arises through measurement error and is augmented by the consistency adjustments made to the observations so that they meet the equilibrium conditions of the model.

The main difficulty for calibrated parameter sensitivity analysis lies in the requirement that the set of calibrated parameters be consistent with an observed 'benchmark' equilibrium. Unlike the exogenously specified elasticities, the set of calibrated parameters in an applied general equilibrium model is jointly determined by the benchmark equilibrium data. A given perturbation to one calibrated parameter would require changes in other parameters to ensure that the system remains a benchmark equilibrium. No such realignment of the remaining parameters is unique, however, and therefore no single change to the model results can be determined from a given perturbation. Thus the approach of perturbing individual parameters to observe the effect on model results which has been adopted in previous elasticity-based sensitivity analysis procedures, is unsuitable for sensitivity analysis with respect to a model's calibrated parameters.

This paper proposes a calibrated parameter sensitivity analysis procedure (CPSA) which circumvents the joint determination problem by conducting sensitivity analysis over sets of calibrated parameters rather than individual parameter values. CPSA is based on the recognition that for a given model, matrix balancing algorithms which provide a unique transformation from an unbalanced 'raw' data matrix into a benchmark equilibrium data set, also yield a fixed mapping from a given set of raw data into a set of jointly determined calibrated parameters. Unlike the

¹ See Mansur and Whalley (1984) for a discussion of calibration.

elements of the benchmark data set, the elements of the raw data matrix are independent of one another and can be individually perturbed.

The CPSA methodology generates a random sample of unadjusted data matrices, each of which is comprised of perturbed elements of the original data matrix. Underlying CPSA is the assumption that the elements of the raw data matrix are observations of stochastically independent random variables for which the modeler can determine *a priori* distributions. Where these random variables are discretely distributed, the support of their joint distribution forms the population of unadjusted matrices from which the modeler samples. If the random variables in the data matrix are continuously distributed, discrete approximations to their distributions are found using the Gaussian quadrature methodology, which specifies discrete approximations that match the lower order moments of the original distributions.² The population from which the modeler then samples, is given by the support of the ensuing approximate discrete joint distribution. Sampling from the support of a joint distribution allows the modeler to attach a probability of being the true data matrix to each unadjusted matrix in the sample.

A fixed algorithm transforms each sample matrix into a balanced benchmark equilibrium data set. The subsequent benchmark data sets map into corresponding sets of calibrated model parameters, which are used to solve the model. Each set of model results is weighted by the probability attached to the unadjusted matrix used in its derivation. From the sample, modelers can determine expectations, standard errors and confidence intervals for the solution values. Thus CPSA translates the modeler's knowledge of uncertainty in the raw data, through the calibrated parameters and into a measure of robustness for the model results. In doing so, it completes the framework for reporting the model's sensitivity to its full numerical specification.

This paper is organized as follows. Section II elaborates on calibration in applied general equilibrium models and on the problems associated with sensitivity analysis for calibrated parameters. In Section III, the CPSA methodology is presented and illustrated using a simple applied general equilibrium model. Section IV proposes and applies an extended sensitivity analysis procedure in which CPSA is combined with elasticity sensitivity analysis. The application examines the sensitivity of personal tax incidence results in a model of Côte d'Ivoire due to Chia, Wahba, and Whalley (1992), to the parameters calibrated from the consumption expenditure matrix and to selected elasticities. Section V concludes with comments on the limitations of the procedure.

² The properties of Gaussian quadrature are discussed in Miller and Rice (1983).

II. The Framework for Calibrated Parameter Sensitivity Analysis

An applied general equilibrium model can be written as a system of m simultaneous equations in which a vector of parameters, α , and a vector of exogenous variables, \mathbf{w} , generate a vector of m endogenous variables, \mathbf{Y} .³ This relationship can be expressed in terms of an implicit mapping, $F: \mathbb{R}^m \rightarrow \mathbb{R}^m$, such that⁴

$$F(\alpha, \mathbf{w}, \mathbf{Y}) = 0. \quad (1)$$

F can be considered to represent the chosen model structure and α to summarize its parameterization. To parameterize a given model, modelers must specify values for the vector α . Ideally, they should be able to draw on econometric estimates with well defined statistical properties to assign values to these parameters,⁵ but in practice the magnitude of the data requirements make such an approach intractable.⁶

³ In a simple applied general equilibrium framework, the vector \mathbf{Y} would include an income for each agent, a price for each commodity and factor, and an activity level for each production sector. Agents' factor and commodity endowments would be included in the vector \mathbf{w} , while elasticities of substitution, input shares and scale parameters in utility and production functions would comprise α . Each value in \mathbf{Y} is associated with an equilibrium condition: equilibrium incomes are values that satisfy budget balance constraints for agents; equilibrium prices satisfy market clearing conditions for commodities and factors; equilibrium activity levels satisfy zero profit conditions in production sectors. These equilibrium conditions also form the basis of the more sophisticated structures discussed in Shoven and Whalley (1992), including models with taxes, joint production, nested functions, intermediate demands, decreasing returns to scale production and intertemporal frameworks.

⁴ A general equilibrium is characterized by a set of complementary slackness conditions where, if equilibrium prices are zero, excess supply can be positive and where, if activity levels are zero, excess profits can be negative. The discussion here is restricted to the case in which prices and activity levels are strictly positive and the equilibrium conditions are satisfied with equality.

⁵ Econometrically derived model parameterizations have been undertaken although the data requirements make such an approach rare. Examples include Clements (1980), Jorgenson, Slesnick and Wilcoxon (1991), and McKittrick (1995).

⁶ Tractability issues surrounding the econometric estimation of applied general equilibrium models are discussed in Mansur and Whalley (1984) and include the following: for most models of sufficient dimensionality to generate insight to policy issues, the time series data requirements for an econometric parameterization are prohibitive; econometric estimates would rely on sufficient excluded exogenous variables to identify the equations in the system, belying the essence of the general equilibrium structure, which is the interdependence of all the modeled variables; the physical units employed in applied general

Instead, parameters are inferred from a set of known values for \mathbf{Y} and \mathbf{w} , $\hat{\mathbf{Y}}$ and $\hat{\mathbf{w}}$ which solve

$$F(\alpha, \hat{\mathbf{w}}, \hat{\mathbf{Y}}) = 0. \quad (2)$$

If the dimensionality of α is greater than m , model parameterization becomes the two stage process discussed in Mansur and Whalley (1984) and Shoven and Whalley (1992). The procedure partitions the vector of parameters α into two subsets: α_1 , a set of parameters which the modeler is free to specify exogenously, and α_2 , the set of 'calibrated' parameters. Calibration yields values for α_2 which ensure that for a given α_1 , $\hat{\alpha}_1$, and $\hat{\mathbf{w}}$, the model produces $\hat{\mathbf{Y}}$ as a solution.⁷ The vector of calibrated parameter values is a function of the exogenously specified parameters and the known solution:

$$\alpha_2 = G(\hat{\alpha}_1, \hat{\mathbf{w}}, \hat{\mathbf{Y}}), \quad (3)$$

where G is an implicit function of F .⁸

Sensitivity analysis provides a means of characterising the robustness of model results to uncertainties in this model parameterization process. Typically, the choice of $\hat{\alpha}_1$ is surrounded by

equilibrium models are defined as the quantity that can be traded for one unit of currency (appropriately adjusted for taxes and subsidies) and such a units convention, which depends on the price level in a base year, presents a challenge to developing a set of units-consistent time series observations.

⁷ Once values for the calibrated parameters have been found, the vector of model parameter values α , and the exogenous variables $\hat{\mathbf{w}}$, can be used in (1) to solve for \mathbf{Y} in a 'replication test.' If the solution values for \mathbf{Y} are the same as $\hat{\mathbf{Y}}$, the calibration procedure has found parameters which are consistent. Policy analysis is undertaken by perturbing some of the model parameters, computing a new equilibrium and comparing the subsequent vector of endogenous variables to the base case vector.

⁸ Calibration can only be undertaken if G satisfies the conditions of an implicit function, that is, if the equations of F are continuously differentiable with respect to \mathbf{Y} , \mathbf{w} , and α and if at $\hat{\mathbf{Y}}$, $\hat{\mathbf{w}}$, and $\hat{\alpha}_1$, the determinant of the Jacobian matrix given by the derivatives of F with respect to α_2 , is non-zero.

a high degree of uncertainty.⁹ In response to this uncertainty, sensitivity analysis procedures which vary the values of α_1 to observe the effect on model results, have been developed.¹⁰

Ideally a sensitivity analysis procedure directed towards the vector of calibrated parameters, α_2 , should also examine the link between the parameter values and the model results directly. This approach, however, is infeasible. Because the calibrated parameters are jointly determined through the requirement that the known values from which they are derived, \hat{w} and \hat{Y} , satisfy the equilibrium conditions of the model, they cannot be individually perturbed; a change to any single calibrated parameter would require other calibrated parameters to change to preserve the equilibrium system. Since many such adjustments are possible, no unique effect on model results can be observed from a specific change to single calibrated parameter.

The joint determination of the calibrated parameters is more evident if the vectors \hat{w} and \hat{Y} are transformed into a square transactions matrix, termed a 'benchmark equilibrium data set' (BED). In the BED, a row, representing receipts, and a column, representing outlays, are assigned to each market, production sector, and agent defined in the model. If the Harberger (1962) convention is adopted whereby units transacted are defined as that quantity which sells for one unit of currency, the equilibrium conditions of the model are reflected in a 'biproportionality' condition for the BED, that is, the BED must satisfy the condition that for a square matrix $[x_{ij}]$,

$$\sum_j x_{ij} = \sum_i x_{ij} \quad \forall i = j. \quad (4)$$

Biproportionality ensures that budget balance holds for agents (incomes equal expenditures), sectors make zero profits (sales equal production costs), and because prices are unity, markets clear, (quantities demanded equal quantities supplied).

⁹ The vector α_1 is comprised largely of elasticities of substitution and transformation. The values for these elasticities are obtained where possible, from literature based econometric estimates, but such estimates are scarce and dated. Modelers occasionally undertake their own estimation for these values. Typically, however, the number of elasticities in an applied general equilibrium model is prohibitively large and insufficient data exists for their estimation. As a result, modelers often derive elasticity values using 'best guesses.'

¹⁰ Elasticities are not the only exogenous parameters for which sensitivity analysis has been undertaken. Rutström (1991) conducts sensitivity analysis over the values of the minimum requirement parameters in a linear expenditure system.

A model's calibrated parameters are functions of ratios of elements of the BED, so that for example, shares of commodity c where $c = 1, \dots, C$ in the consumption of agent q are calculated from the ratio of x_{cq} to $\sum_{c=1}^C x_{cq}$. Perturbing one calibrated parameter is tantamount to changing a ratio or element in the BED. Changing one off-diagonal element¹¹ violates the matrix biproportionality condition, and for calibration, the modeler must rebalance the perturbed matrix into a BED. This rebalancing process, however, is not unique.

Consider for example, a multi-sector model specification where the capital-labor ratio in the agricultural sector is 1. To observe the sensitivity of the model results to a larger value of this ratio, the benchmark data could adjusted in several ways: the overall endowment of factors could be held constant, reducing the capital-labor ratio in one or all of the other production sectors; the endowment of capital in the economy could be increased, increasing income and thus also increasing the share parameter of agricultural sector output in consumption; the labor endowment in the economy could be reduced together with the share parameter of one or all of the remaining sector outputs in consumption; or any of an infinite number of combinations of altering the remaining parameters in the model could be undertaken to rebalance the matrix. Because no unique adjustment exists to rebalance the BED, no unique reconfiguration of the calibrated parameters arises from perturbing a single element of the BED, and sensitivity analysis cannot be undertaken for individual calibrated parameters.

Unlike the elements of the BED, the raw data from which is adjusted to form BED has no consistency restrictions on the values it can assume. The following section describes a calibrated parameter sensitivity analysis which circumvents the problem of joint determination by perturbing the raw data set from which the calibrated parameters are derived. The procedure allows modelers to observe the effect on model results of varying the entire vector of calibrated parameters, rather than of perturbing individual elements of that vector.

¹¹ Changing the ratio of a diagonal element of the BED would preserve the biproportionality condition since rows and columns would be affected equally. Diagonal elements in the BED, however, denote transactions from an agent, a market, or a sector to itself. Since such transactions are devoid of behavioral significance in an applied general equilibrium model, the diagonal elements of the BED are defined to be zero.

III. Calibrated Parameter Sensitivity Analysis

The issue of calibrated sensitivity analysis is addressed by turning to the derivation of the BED from a matrix of initial, unbalanced estimates.¹² At this level, the derivation of the BED falls into the class of matrix balancing problems in which an initial, unbalanced matrix is transformed into a balanced matrix which satisfies a set of linear restrictions and is close to the original matrix under some metric. Many algorithms exist for undertaking such matrix balancing procedures,¹³ and the modeler must choose among these to derive the BED. In the sensitivity analysis which is developed here, the adjustment algorithm remains fixed, but data used as inputs into that algorithm are perturbed. The calibrated parameter sensitivity analysis maps perturbations in the data through the fixed adjustment algorithm, through the resulting BED,¹⁴ through the configuration of calibrated parameters, and ultimately into the model results.

A. Methodology

The general approach for the calibrated parameter sensitivity analysis procedure is one of randomized sampling over alternative values of the initial raw data matrix. Randomized sampling, the approach adopted for elasticity sensitivity analysis in Harrison and Vinod (1992), avoids the prohibitive computational requirements of unconditional systematic sensitivity analysis discussed in Wigle (1991). It has the additional advantage over the Pagan-Shannon sensitivity procedure¹⁵

¹² The derivation of the matrix of unbalanced estimates itself can be a lengthy process. Modelers typically begin with data from disparate sources of varying quality and the data within each source may also vary in its reliability. Modelers are faced with missing values, conflicting data, and with measurement classifications which are inappropriate to the model.

¹³ Günlük-Şenesen and Bates (1988) summarize the general approaches.

¹⁴ The sensitivity of model results to alternative BEDs has been undertaken elsewhere. Roberts (1994) examines the effects the choice of benchmark year for the BED. Adams and Higgs (1990) argue for the use of a synthetic 'typical' BED rather than one derived from a particular 'year of record.' Wiese (1995) derives two BEDs using alternative accounting assumptions for employer contributions to health insurance and traces the effects of these assumptions on model results. These all argue for particular incarnations of the BED rather than providing a systematic analysis of the type proposed here.

¹⁵ This elasticity sensitivity analysis procedure, which relies on a linear approximation of the model results as a function of the parameters, is discussed and applied in Pagan and Shannon (1985), Pagan and Shannon (1987), and Wigle (1991).

of providing global rather than local analysis, which strengthens sensitivity results for non-linear models with large uncertainties in the parameter values. The CPSA procedure employs Gaussian quadrature to find a discrete population of matrices from which to sample, following the methodology used for elasticity parameters in DeVuyst and Preckel (1997). Unlike the methodology employed in Harrison and Vinod, Gaussian quadrature ensures that the moments of the sampling distribution match those of the underlying distribution.¹⁶

CPSA is a procedure in which the data in the initial matrix are considered observations of random variables for which the modeler can determine an *a priori* distribution. Where these distributions are continuous, each is approximated by a set of discrete points and associated probabilities. Together, the distributions are used to form a discrete approximation to the joint distribution for the set of variables comprising the data matrix. A sample of unbalanced data sets is drawn randomly from the joint distribution. Each unbalanced data set in the sample is then balanced using a fixed adjustment algorithm, resulting in a series of BEDs, each of which is used to calibrate and solve the model.

Let \mathbf{Z} be the true BED matrix with elements z_{hi} , where $h = 1, \dots, n$, and $i = 1, \dots, n$. Let row vector \mathbf{X} with elements x_j , $j = 1, \dots, N$, be the vector representation of the set of random variables in \mathbf{Z} and let the x_j be stochastically independently distributed. Under this construction, each x_j represents a unique z_{hi} . The diagonal elements of \mathbf{Z} are zero by definition, so that $N \leq n^2 - n$. Let vector $\bar{\mathbf{A}}$ with elements \bar{a}_j be the best initial estimate of \mathbf{X} . The CPSA methodology is comprised of the following four steps.

1. Specification of an *a priori* distribution for the random matrix elements

The modeler is assumed to be able to specify an *a priori* distribution for each x_j , denoted here by $\{x_j\}$ where $\{x_j\}$ is the probability density function if x_j is a continuous random variable and the probability mass function if x_j is a discrete random variable. To undertake CPSA, the random variable x_j must have finite moments and the support of $\{x_j\}$ must be consistent with the model structure, so that for example, it does not includes negative values for non-negative variables.

¹⁶ See DeVuyst and Preckel (1997) for a comparison of the two methods.

The assumption that the \bar{a}_j are the best initial estimates for x_j implies that in the specified distribution, $E(x_j) = \bar{a}_j$. The variance, $E(x_j - \bar{a}_j)^2$, will be informed by the reliability of the data sources as well as the prior modifications undertaken to form the unadjusted matrix. Because the data in a BED represents transactions in a closed economic system, the modeler's specification of the *a priori* distributions will typically be bounded.

2. Discrete approximation to the continuous probability density functions

If the specification of the distribution is continuous, the sampling methodology in CPSA requires a discrete approximation to $\{x_j\}$, where the discrete approximation is comprised of K pairs of points, \hat{a}_j^k , $k = 1, \dots, K$, and probabilities p_j^k , such that $\sum_k p_j^k = 1$. A discrete approximation is obtained using the Gaussian quadrature approach discussed in Miller and Rice (1983), Preckel and DeVuyst (1992), and DeVuyst and Preckel (1997). For each x_j , Gaussian quadrature chooses K pairs (\hat{a}_j^k, p_j^k) such that

$$\sum_{k=1}^K p_j^k (\hat{a}_j^k)^l = E (x_j - \bar{a}_j)^l, \quad (5)$$

where $l = 0, 1, \dots, 2K-1$. Thus Gaussian quadrature finds a discrete distribution which matches the lower order moments of the original distribution.¹⁷

Two, three, and four point discrete approximations arising from applying Gaussian quadrature to uniform, normal and exponential distributions are given in Miller and Rice (1983), and these provide the discrete approximations employed in the examples of CPSA that follow.¹⁸ As an example, let the raw data vector be

$$\bar{A} = [1 \ 2]$$

¹⁷ The derivation and properties of Gaussian quadrature are given in Miller and Rice (1983).

¹⁸ Other procedures for undertaking Gaussian quadratures are cited in DeVuyst and Preckel (1997).

where element \bar{a}_1 is distributed $N(1, 0.02)$, and element \bar{a}_2 is distributed $N(2, 0.04)$. A three point Gaussian quadrature would approximate the distribution for \bar{a}_1 by the three points and probabilities

$$\begin{array}{ll} \hat{a}_1^1 = 0.755 & p_1^1 = 0.1667 \\ \hat{a}_1^2 = 1.000 & p_1^2 = 0.6666 \\ \hat{a}_1^3 = 1.245 & p_1^3 = 0.1667 \end{array}$$

and \bar{a}_2 by

$$\begin{array}{ll} \hat{a}_2^1 = 1.654 & p_2^1 = 0.1667 \\ \hat{a}_2^2 = 2.000 & p_2^2 = 0.6666 \\ \hat{a}_2^3 = 2.346 & p_2^3 = 0.1667. \end{array}$$

3. Construction of a joint distribution for the initial raw data set

The representation of the joint distribution for the elements of \mathbf{X} , denoted here by $\{\mathbf{X}\}$, is derived from probability mass function representations of the elements in \mathbf{X} . If the *a priori* distributions are discrete, these probability mass functions are simply the $\{x_j\}$, whereas if the *a priori* distributions are continuous, the probability mass functions are given by the Gaussian quadrature discrete approximations to the $\{x_j\}$. Let each x_j have a probability mass function representation with K point and probability pairs. The joint distribution is given by the N^K vectors and probabilities.¹⁹

$$\{\mathbf{X}\} = ([\hat{a}_1^{k_1}, \hat{a}_2^{k_2}, \dots, \hat{a}_N^{k_N}], \prod_{j=1}^N p_j^{k_j}) \quad \forall \quad k_1 = 1, \dots, K, \quad k_2 = 1, \dots, K, \quad \dots, \quad k_N = 1, \dots, K. \quad (6)$$

Where the x_{ij} are discretely distributed, $\{\mathbf{X}\}$ is the true joint distribution. If $\{\mathbf{X}\}$ is formed from Gaussian quadrature approximations to continuous probability density functions for the x_{ij} , the joint distribution also preserves up to and including the $2K-1$ moments of the original, continuous joint distribution. Because this joint distribution is formed under the assumption of stochastic independence of the x_{ij} the covariances and higher order cross moments are zero.²⁰

¹⁹ The derivation of the joint distribution here is taken from Preckel and DeVuyst (1992).

²⁰ The assumption of stochastic independence for such data is supported in applications of the Stone-Byron adjustment algorithm in the social accounting literature, which requires an *a priori* specification of the variance-covariance matrix for a social accounting matrix. For an example, see Crossman (1988). CPSA

The joint distribution for the above example would then consist of the eight vectors and probabilities:

$$\begin{array}{ll}
[\hat{a}_1^1, \hat{a}_2^1] = [0.755, 1.645] & p_1^1 p_2^1 = 0.0278 \\
[\hat{a}_1^1, \hat{a}_2^2] = [0.755, 2.000] & p_1^1 p_2^2 = 0.1111 \\
[\hat{a}_1^1, \hat{a}_2^3] = [0.755, 2.346] & p_1^1 p_2^3 = 0.0278 \\
[\hat{a}_1^2, \hat{a}_2^1] = [1.000, 1.645] & p_1^2 p_2^1 = 0.1111 \\
[\hat{a}_1^2, \hat{a}_2^2] = [1.000, 2.000] & p_1^2 p_2^2 = 0.4444 \\
[\hat{a}_1^2, \hat{a}_2^3] = [1.000, 2.346] & p_1^2 p_2^3 = 0.1111 \\
[\hat{a}_1^3, \hat{a}_2^1] = [1.245, 1.645] & p_1^3 p_2^1 = 0.0278 \\
[\hat{a}_1^3, \hat{a}_2^2] = [1.245, 2.000] & p_1^3 p_2^2 = 0.1111 \\
[\hat{a}_1^3, \hat{a}_2^3] = [1.245, 2.346] & p_1^3 p_2^3 = 0.0278.
\end{array}$$

The derivation of the joint distribution for the elements of the entire BED, denoted here by $\{\mathbf{Z}\}$, is straightforward. The support of $\{\mathbf{Z}\}$ is given by the N^K matrices formed when the set of random variables in \mathbf{Z} take on the values of each vector in the support of $\{\mathbf{X}\}$, and the probability associated with each matrix is that of the vector used in its formation.

4. Sampling

In most applications, the value of N^K is prohibitively large for the modeler to balance the matrix, calibrate and solve the model for each of the matrices which form the points in the support of the joint distribution, as would be the procedure in an unconditional systematic sensitivity analysis.²¹ Instead, the CPSA methodology employs random sampling.

Let $\hat{\mathbf{S}}_1$ be a matrix drawn randomly from $\{\mathbf{Z}\}$, and let P_l be the probability mass of $\hat{\mathbf{S}}_1$. The modeler applies the fixed adjustment algorithm to $\hat{\mathbf{S}}_1$ to generate a microconsistent BED, and through calibration, a set of calibrated parameter values. The model is solved, generating a vector of model results, $\check{\mathbf{R}}_1$.

can, in principle, be extended to the case where elements of \mathbf{Z} are jointly distributed. Preckel and DeVuyst (1992) give a Gaussian quadrature joint distribution for the case in which the x_j are joint normally distributed.

²¹ For example, the BED for the Cote d'Ivoire model of Chia, Wahba and Whalley (1992) has 700 random variables. The support of the joint distribution formed from a 2-point Gaussian quadrature discrete approximation to their distributions would have 2^{700} points!

The process is repeated T times to generate a sample of model results. T is chosen to be sufficiently large that the sample moments are asymptotically consistent estimators of the population moments. To ensure that all matrices in the support of $\{\mathbf{Z}\}$ have the same probability of being sampled, sampling is undertaken with replacement allowing the possibility that the same matrix may be drawn more than once. Each $\check{\mathbf{R}}_t$, $t = 1, \dots, T$, is weighted by P_t to find the expectations and standard deviations for the model results. These moments then allow confidence intervals to be derived from the application of Chebychev's theorem.

While the values in the unbalanced, raw data matrix are the most likely candidates for sensitivity analysis, the CPSA procedure can be generalized to other exogenously specified components of the adjustment process. For example, where the covariance between matrix elements is zero, the adjustment algorithm developed by Stone (1978) and Byron (1978) is a weighted least squares adjustment algorithm in which the weights are given by the variance of the matrix elements. If the modeler is able to specify distributions for the variances of the matrix elements,²² CPSA can be applied to those variances used in the Stone-Byron adjustment algorithm. The illustration of CPSA which follows, examines the sensitivity of model results to uncertainty in the values of data variances.

B. An Illustration of CPSA Using a Simple Tax Model

The model used to illustrate CPSA is the simple 2x2x2 model used in Shoven and Whalley (1984), with two consumers, rich and poor, two factors of production, capital and labor, and two commodities, manufactured and non-manufactured goods. Table 1 summarizes the model structure. The base case version of the model has no taxes. In the counterfactual experiment, a 50 percent tax is levied on the use of capital in the manufacturing sector, resulting in welfare

²² That modelers may be able to specify such a distribution has support in the social accounting literature. For example, in undertaking the Stone-Byron adjustment for Australian data, Crossman (1988) categorizes elements of the National Accounts as being of poor reliability with error margins greater than 10 percent, medium reliability with error margins of 3-10 percent, and good reliability with error margins 0-3 percent. These ranges could reasonably provide the mean and bounds for a uniform or triangular distribution.

Table 1**Structure of the Simple, Illustrative Tax Model**

Production	<ul style="list-style-type: none">• Output is produced using capital and labor combined in proportions implied by CES technology in each sector.• The elasticity of substitution in the production of manufactured goods is 2.0 and in that of non-manufactured goods, 0.5.• Share parameters for the CES function are calibrated from the BED.
Consumption	<ul style="list-style-type: none">• The utility of each consumer is a CES function of manufactured and non-manufactured goods.• The rich consumer's utility function has an elasticity of substitution of 1.5 and the poor consumer's has one of 0.75.• Share parameters for the CES function are calibrated from the BED.
Endowments	<ul style="list-style-type: none">• The rich consumer is endowed with capital and the poor consumer with labor.
Equilibrium	<ul style="list-style-type: none">• Markets clear for all goods and factors.
Conditions	<ul style="list-style-type: none">• Zero profits are made in each sector.• Each consumer's expenditures equals his/her income.
Counterfactual	<ul style="list-style-type: none">• A 50 percent tax is levied on the use of capital in the production of manufactured goods.• The rich consumer receives 40 percent of tax revenues and the poor consumer receives 60 percent.• Welfare changes for each consumer are measured by equivalent variation as a proportion of base income: $EV^i = (U_c^i - U_b^i) / U_b^i$ where U_b^i is the utility of consumer i, $i = \{\text{rich, poor}\}$ in the base case and U_c^i is utility after the imposition of the tax.

changes for both consumers. These welfare changes, measured by the Hicksian equivalent variation as a proportion of base income, provide the basis for the sensitivity analysis.

The initial, unbalanced data for the model is given in Table 2. Under the Stone-Byron algorithm, each element of the unbalanced data set is weighted by its variance. In this example, the variances are assumed to be one percent of the squared value of the initial estimate,²³ and are given in parentheses in Table 2.

Two sets of experiments have been performed to illustrate the CPSA procedure. The first set assumes that the modeler knows with certainty the aggregate incomes and output in the economy as expressed by the row and column totals. These known totals are BED totals for the Shoven and Whalley model and are given in the final row of Table 2. In this case, the constraints in the Stone-Byron adjustment algorithm are that the values of the adjusted matrix sum to the known totals and that the zero elements of the initial matrix are preserved. The second set of experiments assumes that row and column totals are unknown, and the row and column totals are allowed to adjust subject to the constraint that the matrix remains biproportional and zeros are preserved. Both experiments are undertaken assuming a uniform distribution for the variances, where the bounds of the distribution are +/- 20 percent of the central value. These continuous distributions are represented by the three-point discrete Gaussian quadrature approximations given in Table 3, which preserve up to and including the fifth moments of the original distributions.

In each experiment, the support of the variance joint distribution is given by the combinations arising when each of the ten variances assumes one of the three values in the support of its discrete distribution. The result is a set of 10^3 possible configurations, each of which has a probability given by the product of the probabilities of its ten constituent points. A sample of fifty configurations is drawn from this support. Each is used to derive a BED, calibrate and

²³ This assumption for the variances is derived from a time series of annual values for value added in manufacturing for the United States which was found to have a standard deviation of 10.2 percent. The time series was constructed using annual data for 1970 to 1992 taken from the International Bank for Reconstruction and Development (1993) data base. The series "value added in manufacturing" given in current USD was deflated by the ratio of current USD to constant 1985 USD GDP at factor cost to generate a constant series.

Table 2 Transactions Values¹ and Variances For the Illustrative Tax Model (Variances are given in parentheses)						
	Rich	Poor	Manufactures	Non-M'factures	Capital	Labor
Rich					31.3 (11.8)	
Poor						55.0 (36.0)
Manufactures	18.2 (2.6)	16.3 (3.5)				
Non-M'factures	18.1 (3.3)	37.2 (17.0)				
Capital			8.1 (0.7)	30.1 (6.7)		
Labor			22.6 (7.0)	30.9 (11.3)		
Known Total	34.3	60.0	34.9	59.4	34.3	60.0
1. Values were derived as random numbers drawn from a normal distribution with mean equal to the Shoven and Whalley (1984) balanced values and standard deviation equal to 10 percent of the balanced value.						

Table 3 3 Point Gaussian Quadrature Approximations for the Data Matrix Variance Distributions in the Illustrative Model					
	Expected Value and Bounds of the Uniform Distribution	1st Point (probability: 0.278)	2nd Point (probability: 0.444)	3rd Point (probability: 0.278)	
Rich-Capital	11.8, [9.4, 14.1]	9.9	11.8	13.6	
Poor-Labor	36.0, [28.8, 43.2]	30.4	36.0	41.6	
Manufacturing - Capital	0.7, [0.6, 0.9]	0.6	0.7	0.8	
Manufacturing-Labor	7.0, [5.6, 8.4]	5.9	7.0	8.0	
Non-Manufacturing - Capital	6.7, [5.3, 8.0]	5.6	6.7	7.7	
Non-Manufacturing-Labor	11.3, [9.0, 13.5]	9.5	11.3	13.0	
Rich-Manufacturing	2.6, [2.1, 3.1]	2.2	2.6	3.0	
Rich-Non-Manufacturing	3.3, [2.6, 4.0]	2.8	3.3	3.8	
Poor-Manufacturing	3.5, [2.8, 4.2]	3.0	3.5	4.1	
Poor-Non-Manufacturing	17.0, [13.6, 20.4]	14.3	17.0	19.6	

solve the model. Attached to each result is the probability of the configuration used in its derivation. The means, standard deviations and bounds for the results of both experiments are given in Table 4. Where the control totals are known, the standard error in the results is higher than where biproportionality is the underlying adjustment consistency condition. This result is consistent with the additional constraints imposed on the system by control totals.

What emerges from Table 4 is that the central case model results from this experiment are not robust to the range of variances imposed in the CPSA. Although the signs of the welfare changes are preserved, the central case variants lie outside the 95 percent confidence interval. In this case, the sensitivity analysis indicates that the modeler would be ill-advised to present the model results as reliable inputs into the policy process. These sensitivity results are, of course, dependent on the *a priori* specification for the distribution of the variances. If the modeler's *a priori* information about the variances had led to a specification in which the bounds of the uniform distribution for the variances were ± 50 percent of their base values, the central case variances would lie well within the 95 percent confidence interval as is evident in Table 5.

The mechanism by which the variance changes in CPSA affect model results is specific to the model structure. In the illustrative example, the introduction of a tax on the use of capital in the manufacturing sector causes the price of manufactured goods to rise relative to that of non-manufactured goods, and subsequently results in a net decrease in the demand for manufactured goods. The shift in production towards the more labor-intensive, non-manufactured goods forces up the price of labor relative to capital. The rich consumer, who is endowed only with capital and who receives 40 percent of tax revenues, experiences a loss in income. This welfare loss is compounded by the increase in the price of manufactured goods which figure more prominently in the rich consumer's utility function than in the poor consumer's. As a result, in the sensitivity analysis, a higher share of manufactured goods in the constant elasticity of substitution (CES) utility function of the rich consumer and/or a higher share of capital in the production

Table 4**CPSA on the Welfare Effects of Imposing a 50 Percent Tax on the Use of Capital in the Manufacturing Sector****Hicksian Equivalent Variations measured as a proportion of base income**

1. Known Control Totals: Variances with Uniform Distribution

	Central Case ¹	Mean	Standard Error	95% Confidence Interval ²
EV Rich	-0.1258	-0.1292	0.00062	[-0.1321, -0.1265]
EV Poor	0.0630	0.0649	0.00033	[0.0635, 0.0670]

2. Unknown Control Totals: Variances with Uniform Distribution

	Central Case ¹	Mean	Standard Error	95% Confidence Interval ²
EV Rich	-0.1263	-0.1239	0.00053	[-0.1263, -0.1215]
EV Poor	0.0643	0.0707	0.00023	[0.0697, 0.0717]

1. The central case uses the variances given in Table A.2.

2. Confidence intervals are derived using Chebychev's Theorem.

Table 5**Sensitivity of CPSA to the Variance Probability Density Function****Hicksian Equivalent Variations measured as a proportion of base income**

Known Control Totals: Variances with Uniform Distribution, Bounds 50 percent of base value

	Central Case	Mean	Standard Error	95% Confidence Interval
EV Rich	-0.1258	-0.1295	0.00180	[-0.1375, -0.1215]
EV Poor	0.0630	0.0651	0.00096	[0.0608, 0.0694]

function for manufactured goods should result in a greater decrease in the welfare of the rich consumer as the tax on the use of capital in the manufacturing sector is imposed.

This expectation is supported by evidence from a simple experiment. If all but one of the variances are held constant at their central case value and the variance of the poor consumer's consumption of non-manufactured goods, v_{pn} , is allowed to vary,²⁴ the way in which the changes in one variance lead to variations in the model welfare effects can be traced. This path is given in Table 6 for three values of v_{pn} . The first section in Table 6 demonstrates the link between the initial value of v_{pn} and final benchmark consumption values. The lowest value for v_{pn} implies a high reliability for the poor consumer's consumption of non-manufactured goods and yields the adjusted BED value closest to the initial estimate. Increases in the variance cause the adjusted value to move farther from the initial point.

These changes in the adjusted value for the poor consumer's consumption of non-manufactured goods, together with changes in the remainder of the adjusted elements in the BED, have consequences for the values of the calibrated parameters. As v_{pn} increases, the changes in the BED imply that the calibrated share parameter for manufactured goods in the rich consumer's CES utility function increases, while that in the poor consumer's utility function decreases. The greater weight on the rich consumer's share parameter for manufactured goods yields the higher equilibrium consumption values given in the third section of Table 6. These higher share parameters lead to a greater disutility from the increase in the price of the manufactured good and the consequent greater loss in welfare, as given by the rich consumer's equivalent variation in the final section.

The opposite effect is evident for the poor consumer whose share of manufactured goods in utility decreases as v_{pn} increases, and whose subsequent counterfactual demands for manufactured goods decreases with higher values of v_{pn} . Reductions in the credibility of the initial estimate for the poor consumer's consumption of non-manufactured goods thus lead to higher welfare gains for the poor consumer.

²⁴ The adjustment process also assumes that the control totals are known and are the same as those given in Table 2.

Table 6

The Path by which Changes in the Variance of the Poor Consumer's Consumption of Non-Manufactured Goods (v_{pn}) Lead to Changes in the Model Results¹

1. BED Consumption Levels ² (values are in currency units)						
v_{pn}	Rich•Manufactures		Rich•Non- Manufactures	Poor•Manufactures		Poor•Non- Manufactures
1	14.87		19.43	20.03		39.97
17	17.37		16.93	17.53		42.47
25	17.47		16.83	17.43		42.57
↓						
2. Utility Function CES Share Parameters Implied by Alternative BEDs						
v_{pn}	Rich•Manufactures		Rich•Non- Manufactures	Poor•Manufactures		Poor•Non- Manufactures
1	0.308		0.402	0.390		0.778
17	0.358		0.349	0.338		0.818
25	0.360		0.347	0.336		0.819
↓						
3. Counterfactual Demands						
v_{pn}	Rich•Manufactures		Rich•Non- Manufactures	Poor•Manufactures		Poor•Non- Manufactures
1	11.66		18.44	19.97		43.86
17	13.76		16.20	17.47		46.50
25	13.84		16.11	17.34		46.61
↓						
4. Equivalent Variation as a Proportion of Base Income						
v_{pn}	Rich EV	Poor EV	Rich Base Utility	Poor Base Utility	Rich Counterfactual Utility	Poor Counterfactual Utility
1	-0.1249	0.0623	17.25	32.29	15.10	34.30
17	-0.1292	0.0649	17.15	33.65	14.94	35.84
25	-0.1293	0.0650	17.15	33.71	14.93	35.90

1. The variance of the poor consumer's consumption of Non-Manufactured Goods is 17 in the central case. The value 1 is chosen to represent a low value and 25 a high value.

2. The values shown here are those which change as a result of altering v_{pn} . The remaining elements of the BED are the same throughout.

IV. Extended Sensitivity Analysis

While CPSA introduces a means for modelers to undertake sensitivity analysis with respect to the calibrated parameters, the elasticity parameters remain a highly uncertain component of the modeling process. The strength of CPSA, therefore, lies in its contribution to a *comprehensive* sensitivity analysis - one which addresses uncertainty in the full vector of model parameters. The comprehensive procedure proposed here and termed 'extended sensitivity analysis,' combines the CPSA procedures described in Section III with the elasticity sensitivity methodology advocated in DeVuyst and Preckel (1997).

As in Section III, let N be the number of random variables in the BED but let J be the number of exogenously specified parameters, where those parameters are also stochastically independently distributed. In the first step of extended sensitivity analysis, the modeler specifies *a priori* distributions for both the random matrix elements *and* the exogenous parameters. Where these distributions are continuous, have finite moments, and where the model is soluble over their supports, the second step uses Gaussian quadrature to construct discrete approximations. If K is the number of points in the support of each discrete distribution, the joint probability density function found in the third step contains $(N+J)^K$ points. Each point in the support of the joint distribution is comprised of an unadjusted matrix and a set of exogenous parameter values. Its probability mass is given by multiplying the product of the N probabilities of the elements in its unadjusted matrix with the product of the J probabilities in its vector of exogenously specified parameters.

The final step of extended sensitivity analysis samples randomly with replacement from the joint distribution. The unbalanced matrix of each point in the sample is adjusted under a fixed algorithm and the ensuing BED is used together with the exogenous parameter values of the point, to calibrate and solve model. The probability weighted sample of model results generate expectations for the result means, standard deviations and confidence intervals.

This extended sensitivity analysis methodology is illustrated using an existing model developed by Chia, Wahba, and Whalley (1992) for tax incidence analysis in Côte d'Ivoire. The incidence analysis of Chia, Wahba, and Whalley is undertaken for six taxes/subsidies by replacing each with an equal yield, neutral tax on consumption, and finding the associated welfare change for each of seven household types.²⁵ The illustration which follows examines the sensitivity of the personal income tax incidence results to uncertainty in the consumption expenditure data and to the values of consumption and production elasticities of substitution.

A. An Illustration of Extended Sensitivity Analysis Using the Côte d'Ivoire Model

Welfare changes on which tax incidence results are based, derive from household utility functions defined over the consumption of goods and services in the model.²⁶ The CPSA component of the extended sensitivity analysis is undertaken for this consumption expenditure matrix. Changes in utility arise directly from changes in consumption levels, but the extent to which a change in the consumption of a particular good translates into a change in utility is given by the share parameter of that good in the CES utility function. Through calibration, the values

²⁵ The original model is calibrated to a 1986 BED and solved using MPS/GE (Rutherford, 1989), but it has been rewritten in GAMS (see Brooke, Kendrick and Meeraus, 1988) to allow simple incorporation of matrix adjustment, and to facilitate the CPSA component of extended sensitivity analysis.

²⁶ The Côte d'Ivoire model identifies seven socio-economically based household types, each of which receives utility from the consumption of ten goods and services. Incomes derive mainly from capital and labor endowments, as well as interhousehold transfers. Households pay personal income tax and make social security transfers to the government, but also receive income from the government in the form of education and other transfers. The model distinguishes fifteen productive sectors, each of which produces output using value added and intermediate goods. All twelve formal sectors pay production taxes and all formal sectors except the government services sector and the gas, electricity and water sector, also receive subsidies. Eight of the formal productive sectors trade internationally, and since Côte d'Ivoire is modeled as a small, open, price-taking economy, exporters face a perfectly elastic demand function for their output. Traditional exports and exports of primary processed goods are taxed. Imports, used in the production of intermediate goods and in household consumption, are subject to tariffs. The Ivorian price stabilization policy for coffee, cocoa and other exports is captured in the model. In 1986, the benchmark year, the fund experienced a net inflow of revenues and thus the traditional export sector pays into the stabilization fund, while the non-traditional export sector receives only a proportion of those revenues.

in the consumption expenditure matrix²⁷ (together with the elasticity of substitution in consumption), determine the values of these share parameters.

An absence of unadjusted data and information about the construction of the Côte d'Ivoire model BED²⁸ requires the sensitivity analysis to be based on a data set which has been artificially unbalanced from the microconsistent data.²⁹ The unbalanced data is adjusted using two prevalent adjustment algorithms - the Stone-Byron algorithm and RAS.³⁰ Extended sensitivity analysis is undertaken twice - once on the variances used in the Stone-Byron adjustment algorithm and once on the matrix elements which are adjusted using the RAS adjustment algorithm.³¹ To derive the

²⁷ The data used to generate the consumption matrix is assumed to have been obtained from a household survey which reports mean consumption by household type and Chia, Wahba, and Whalley are assumed to have derived an unbalanced estimate of total consumption expenditure by each household type from scaling the survey data by the number of households in each group (given in Table A.2 of Appendix A). In the illustration, the calibrated parameter sensitivity component of the model results is assumed to emanate primarily from uncertainty in the values associated with the household survey data, and it is to values in this matrix that the uncertainty used in CPSA is attached. The household survey data is approximated by the artificially constructed household survey data set given in Table A.1. The elements of this artificial unbalance matrix are randomly drawn from a normal distribution with expected value equal to the known, adjusted value and standard deviation equal to the proportions of the base case given in Table A.1.

²⁸ Chia, Wahba, and Whalley list their primary data sources as the national accounts, the Banque de données financières (from which balance of payments data was obtained), tax data and household budget survey data, but do not state explicitly which elements of the BED derive from which sources.

²⁹ As a result, the sensitivity analysis presented here provides an illustration of the methodology rather informed insight into the Côte d'Ivoire model results. Under current practise modelers do not provide sufficient documentation for outsiders to specify the necessary distributions, and meaningful extended sensitivity analysis can therefore only be undertaken by the modelers who have assembled the data.

³⁰ For an initial matrix with non-negative elements \tilde{a}_{ij} and variances σ_{ij}^2 , known row totals R_i , known column totals S_j , and zero covariances, the Stone-Byron algorithm finds a matrix with elements a_{ij} that minimizes $\sum_{ij} (\tilde{a}_{ij} - a_{ij})^2 \sigma_{ij}^{-2}$ subject to the constraints that $a_{ij} \geq 0$, $\sum_i a_{ij} = S_j$ and $\sum_j a_{ij} = R_i$. The RAS algorithm, attributed to Bacharach (1970) is a scaling algorithm in which each row of the initial matrix is scaled by the ratio of the known control total to the actual total. The columns of the ensuing updated matrix are scaled by the ratio of the known column totals to the updated matrix column totals. This process is applied iteratively until the deviation of the updated matrix totals from the control totals is deemed to be sufficiently close to zero. In the limit, RAS converges to an optimization algorithm that minimizes the logarithmic objective function $\sum_{ij} \tilde{a}_{ij} \ln (\tilde{a}_{ij} a_{ij}^{-1})$.

³¹ The values in the consumption expenditure matrix are assumed to have been derived in a two stage process. In the first stage, aggregate values for total final household consumption of each good consistent with the values for total production, exports, government consumption and intermediate demand would have been found. Similarly, aggregate household consumption expenditure would also have been specified. In the second stage, an adjustment algorithm would have been applied to the initial, unbalanced estimate of the

central case variances used in the Stone-Byron adjustment algorithm, the assumption is made that the reliability of the data differs by good, rather than by household: rice, construction, and financial services are assumed to be the most reliably reported with standard errors of 10 percent of their base value; transportation and non-financial services the least reliably reported with standard errors of 30 percent of their base value; and the data on the remaining goods is assumed to be of intermediate reliability with standard errors of 20 percent of their base value.

Using the variances implied by these standard errors, the Stone-Byron adjustment is applied to the matrix in Table A.1, subject to the consistency constraints that i) the total consumption of each good by each household type, summed across household types is equal to the aggregate final household consumption and ii) the sum across goods of total consumption expenditure by household type is equal to disposable income of each household type, net of interhousehold transfers and savings. These aggregate control totals are given in Table A.3 and are the values used in the original model. Together with the original elasticity values, the resulting balanced matrix is used to calibrate and solve the model to obtain incidence results for the personal income tax. These central case results are given in column (1) of Table 7.

In the first step of the extended sensitivity analysis, a uniform distribution for the variances is assumed. The bounds for the standard errors of the most reliable data in the consumption matrix are assumed to be 5 and 15 percent of the base value, for the data of intermediate reliability, 10 and 30 percent of the base value, and for the least reliable data 10 and 50 percent of the base value. Thus as data becomes less reliable, so too does the modeler's ability to assess the unreliability. As an example, the variance for the first element in the matrix - the consumption of rice by export cropper households - has a value of 7941.36 million CFA³² francs, derived from an average of 3260 CFA francs per export cropper household over 2.436 million households. Its standard deviation assumed to be uniformly distributed and bounded between 5 and 15 percent

consumption matrix under consistency conditions implied by the aggregate values from the first stage.

³² The CFA franc is the currency of the Colonies Françaises d'Afrique of which Côte d'Ivoire is a member.

Table 7

**Extended Sensitivity Analysis Results for Personal Income Tax Incidence in a Model of
Côte d'Ivoire For Alternative Adjustment Algorithms**

Hicksian Equivalent Variations expressed as a percentage of benchmark gross income

	Unbalanced Matrix Adjusted Using the Stone-Byron Algorithm				Unbalanced Matrix Adjusted Using RAS Algorithm			
	Central Case (1)	Expected Value (2)	Standard Error (3)	90% Confidence Interval (4)	Central Case (5)	Expected Value (6)	Standard Error (7)	90% Confidence Interval (8)
Export Croppers	-0.225	-0.197	0.015	(-0.244, -0.150)	-0.224	-0.214	0.013	(-0.282, -0.200)
Savannah Croppers	-0.709	-0.708	0.020	(-0.771, -0.645)	-0.685	-0.690	0.019	(-0.750, -0.630)
Other Food Croppers	-1.632	-1.636	0.023	(-1.709, -1.563)	-1.600	-1.603	0.020	(-1.666, -1.540)
Government Employees	3.516	3.504	0.015	(3.457, 3.551)	3.493	3.490	0.009	(3.462, 3.518)
Formal Households	-0.587	-0.594	0.013	(- 0.635, -0.553)	-0.605	-0.607	0.007	(-0.629,-0.585)
Small Businesses	-1.617	-1.618	0.006	(-1.637, -1.599)	-1.617	-1.614	0.006	(-1.633, -1.595)
Inactive	2.651	2.650	0.018	(2.593, 2.707)	2.666	2.661	0.015	(2.614, 2.708)

of the base value consumption value, implying a variance which is uniformly distributed between 26569 and 239121 with an expected value of 106276.

The extended sensitivity analysis considers the calibrated parameters given by the consumption expenditure matrix together with three sets of elasticities used in CES functions in the model; the elasticity of substitution of consumption goods in preferences,³³ the Armington elasticity of substitution between domestic and imported goods in consumption and the elasticity of substitution between capital and labor in production. As with the variances, the elasticities are assumed to be uniformly distributed. Table A.4 gives their central case values and bounds.

These uniform distributions for the variances and the elasticities are then approximated with three-point discrete approximations obtained from Gaussian quadrature. The support of each approximate distribution has a low, middle and high value. The low value in each approximation is given by the lower bound of the distribution plus 11.27 percent of the range and is associated with a probability of 0.28. The middle value is the lower bound plus 0.5 percent of the range (the central case value) with a probability of 0.44, and the high value, the upper bound minus 11.27 percent of the range, is associated with a probability of 0.28. The three points which comprise the discrete approximation for export croppers' rice consumption variance are 50524 with probability 0.28, 106276 with probability 0.44, and 215166 with probability 0.28.

With 31 elasticities and 70 variances in the consumption expenditure matrix, the support of the discrete joint probability distribution approximation has 3^{101} points. The probability associated with any one of those points is given by the product of the probabilities of its components. Thus the probability of the variance/elasticity configuration in which all values assume the low value in their discrete approximation is 0.28^{101} . A random sample of 500 points is drawn from the joint probability distribution on the assumption that this number is sufficiently large that the sample mean and variance will be consistent estimators of the population mean and variance.

³³ In the central case, these are all 1 implying Cobb-Douglas preferences for households. Sensitivity analysis with respect to this value can therefore also be interpreted as sensitivity over the choice of functional form.

In each case, the variances in the Stone-Byron algorithm are used to adjust the unbalanced matrix. The ensuing BED values are used together with the sample elasticity configuration to calibrate and solve the model, and the expected value for household welfare changes as a result of replacing the personal income tax with a neutral consumption tax is given in column (2) of Table 7. The standard error for the results is given in column (3) and the 90 percent confidence interval in column (4).

The second extended sensitivity analysis is undertaken for the case in which the RAS algorithm is applied to balance the consumption expenditure matrix. The central case results, given in column (5) of Table 7, have the same sign and similar magnitudes to the central case Stone-Byron results. The extended sensitivity analysis procedure using RAS assumes that the elements of Table A.1 are uniformly distributed with the same variances as those used in the central case Stone-Byron algorithm. Thus the example matrix element, export croppers' rice consumption, is distributed uniformly with lower bound 2696 CFA francs, expected value 3260 CFA francs, and upper bound 3825 CFA francs. As in the previous case, a three-point discrete approximation is found for each element in the consumption matrix as well as for each of the elasticities. A joint distribution is derived, and a sample comprised of an unbalanced consumption matrix and an elasticity configuration, is drawn from the support of that joint distribution. The sample unbalanced matrix is adjusted using RAS. The model is calibrated using the ensuing BED and the sample elasticity configuration, and solved. As in the Stone-Byron case, the sensitivity results reported in columns (6), (7), and (8) of Table 7 are built from 500 random samples.

The confidence intervals in Table 7 suggest that under the distributional assumptions made, the model results are robust to uncertainty in the parameters in the sense that the signs of the welfare changes do not change. Furthermore, at the 90 percent confidence interval, the ranking of household groups which benefit from the Ivorian personal income tax remains the same as in the central case. Under both sensitivity experiments, government employees bear most of the burden of the tax with inactive households assuming a secondary burden. Thus if the many assumptions made about the source and nature of uncertainty in the data for the Côte d'Ivoire model hold, the central case model results could be confidently presented as inputs into a debate on tax policy reform.

V. Conclusions

Among the criticisms levelled against applied general equilibrium models is one of empirical weakness - model parameterization relies on point observations which lack the statistical rigour of time series data. One means of addressing this criticism is for modelers to incorporate whatever information they do have about the quality of those single observations into the modeling process via sensitivity analyses. Existing sensitivity analysis methodologies, however, are restricted to the set of exogenously specified parameters. In contrast, the CPSA methodology presented here provides measures of the sensitivity of model results to uncertainty in the data used to derive the benchmark data set, and hence, to uncertainty in the values of the calibrated parameters. When CPSA is combined with existing exogenous parameter sensitivity analysis procedures in an 'extended sensitivity analysis,' modelers can undertake sensitivity analysis for the full set of model parameters.

Extended parameter sensitivity analysis has been described and implemented for the reconciliation of unbalanced matrices into microconsistent data sets using formal matrix adjustment algorithms. In practice, modelers make limited use of such algorithms. Much of the adjustment to the values found in primary data sources occurs in the *ad hoc* procedures used to derive consistent control totals for submatrices, which are then 'fine-tuned' via formal adjustment algorithms. The next challenge would be to capture the sensitivity of model results to these larger adjustments. While the approach of the extended sensitivity analysis procedure is sufficiently general to address such issues, it would require parametric representations of those larger adjustments to do so. Since many of these adjustments are *ad hoc*, records of how and why the data has changed are scarce. Paradoxically, broader sensitivity analyses will require modelers to take greater notice of how they adjust their data, but will dispense with the need to describe that process in detail by summarizing the uncertainties in those adjustments via terse confidence intervals over the model results.

Appendix A

Extended Sensitivity Analysis Specifications for the Côte d'Ivoire Model

Table A.1

**Artificial Household Survey Consumption Expenditure Data
Annual Consumption Expenditure¹ in CFA Francs**

Consumption Good	Household Type						
	Export Croppers	Savannah Croppers	Other Food Croppers	Government Employees	Formal Sector	Small Businesses	Inactive
Rice	3260	4054	1970	10000	18014	8879	10635
Other Subsistence Agr.	35669	52039	47922	35616	40742	35082	27816
Traded Agr. Products	218	0	307	7815	8790	4265	8796
Primary Processed	35651	36759	41063	56167	105131	58828	51389
Manufactures	20209	13847	12817	40251	64624	29694	21507
Electricity, Gas, Water	1817	2530	1659	5401	5965	1951	1884
Construction	2460	1871	1784	6340	5103	3956	2214
Transport	6171	0	9450	34028	30515	12630	34329
Financial Services	576	353	385	4757	2916	1214	1165
Non-Financial Services	6392	7723	6904	18904	34329	1086	13319

1. Unbalanced data were derived as random numbers drawn from a normal distribution with mean equal to the balanced value in the Chia, Wahba, and Whalley model and standard deviation as the following: Rice, Construction and Financial Services, 10% of the balanced value, Other Subsistence Agricultural Products, Traded Agricultural Goods, Primary Processed Goods, Manufactures, and Electricity, Gas, Water, 20% of the balanced value, Transport and Non-Financial Services, 30 % of the balanced value.

Table A.2**Number of Households by Type**

Export Croppers	2 436 000
Savannah Food Croppers	1 320 000
Other Food Croppers	1 524 000
Government Employees	1 416 000
Formal Sector Households	912 000
Small Businesses	2 580 000
Inactive	1 812 000

Table A.3**Control Totals Used for the Consumption Expenditure Matrix in the RAS and Stone-Byron Adjustment Algorithms**

**1. Column Control Totals: Aggregate Consumption Expenditure by Household Type
(million CFA francs)**

Export Croppers	296186
Savannah Food Croppers	157369
Other Food Croppers	185213
Government Employees	375647
Formal Sector Households	285345
Small Businesses	418530
Inactive	334426

**2. Row Control Totals: Aggregate Consumption by Product
(million CFA francs)**

Rice	86484
Non-Rice Subsistence Agricultural Products	516210
Traded Agricultural Products	50164
Goods from Primary Processing	617750
Manufactured Goods	341565
Electricity, Gas, Water	30864
Construction	37600
Transport	201072
Financial Services	17509
Non-Financial Services	153498

Table A.4

Elasticities of Substitution and Bounds Used in the Systematic Sensitivity Analysis

1. Elasticity of Substitution Between Capital and Labor in Production Sectors
(bounds are central case value +/- 0.35)

	Central Value	Lower Bound	Upper Bound
Food	0.4	0.05	0.75
Traditional Exports	0.4	0.05	0.75
Non-Traditional Exports	0.5	0.15	0.85
Formal Sector Primary Processing	0.8	0.45	1.05
Formal Sector Manufacturing	0.8	0.45	1.05
Gas and Electricity	0.8	0.45	1.05
Transportation	0.5	0.15	0.85
Formal Sector Services	0.8	0.45	1.15
Financial Services	0.8	0.45	1.15
Informal Sector Services	0.9	0.55	1.25
Informal Sector Primary Processing	0.9	0.55	1.25
Informal Sector Manufacturing	0.9	0.55	1.25
Informal Sector Construction	0.4	0.05	0.75
Formal Sector Construction	0.4	0.05	0.75

2. Elasticity of Substitution Between Goods in Utility
(bounds are central case +/- 40 percent)

	Central Value	Lower Bound	Upper Bound
All Households	1	0.6	1.4

3. Elasticity of Substitution Between Imports and Domestic Goods in Consumption
(bounds are central case +/- 40 percent)

	Central Value	Lower Bound	Upper Bound
All Goods	2	1.6	2.4

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