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Dynamic Price Adjustment Under Imperfect Competition*

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Abstract

We study dynamic price adjustment under imperfect competition when consumers have non-time-separable preferences. In our model an intertemporal link arises in the consumers' maximization problems because current consumption decisions affect the utility of future consumption. Thus future demand depends on the current price and firms must take this into account when making their decisions. The main result is that equilibrium prices follow a dynamic stochastic process in which the current price depends on past prices and on random disturbances. The convergence of prices to the 'long run expected price' is monotonic if current and future consumption are substitutes and oscillatory if they are complements.

JEL classification: C73, D21, D43

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1 Introduction

A number of authors have proposed both competitive and non-competitive theories of dynamic price adjustment. Examples of perfectly competitive theories include Carlton (1978, 1979, 1983) and Deaton and Laroque (1992, 1996) who examine price dynamics using variations of the standard competitive setting. There is also a significant strand of literature studying price dynamics under imperfect competition. For example, Green and Porter (1984) and Rotemberg and Saloner (1986) have considered price movements under collusive supergame equilibria. Blanchard and Kiyotaki (1987), Rotemberg (1982) and Taylor (1980), on the other hand, introduce market imperfections such as menu costs, adjustment costs or price staggering. We use an alternative to these imperfectly competitive approaches by examining a non-collusive equilibrium of a model where the only additional 'imperfection' is that consumers have *non-time-separable* preferences.

When utility functions are not time-separable, current consumption affects the utility a consumer gets from future consumption. Optimal consumption choices will depend on past consumption choices and on expected future consumption. Thus, there is a link between time periods for the consumers' utility maximization problems—this yields dynamic demand functions where market output depends on past as well as current prices. With dynamic demand functions, an intertemporal link is introduced into the firms' profit maximization problem. Given these intertemporal linkages, past prices will affect current demand and the current price will affect future demand. Thus firms must take into account the effect of their current choice on expectations regarding future prices.

Within the context of non-time-separable preferences, there are two strands of related literature: durable goods monopolies and habit persistence. It is clear how durability introduces non-time-separability into the utility function—if one buys a refrigerator today, one's utility from another refrigerator tomorrow is very low. When consumers have habit persistence, their utility functions also are not time-separable. With habit persistence, the more one consumes of a good today, the more of that good one likes to consume in the future. The literature on durable goods (Bulow (1982), Kahn (1986) and Stokey (1981)) focuses on the case when there is a fixed set of consumers for the purpose of evaluating the Coase conjecture. The literature on habit persistence and imperfect competition (Becker, Grossman and Murphy (1990) and Fethke and Jagannathan (1996)) is non-stochastic and thus is limited in its ability to fully examine the dynamics of price adjustment.

The main result is that equilibrium prices follow a dynamic stochastic process in which the current price depends both on past prices and on random disturbances. In particular, temporary shocks can have long lasting effects. In the absence of stochastic shocks, the price would converge to a long run equilibrium (steady state) price. The nature of this convergence depends on whether current and future consumption are substitutes or complements. With substitutes, current price depends positively on past prices and thus convergence is monotonic. If current and future consumption are complements then the current price depends negatively on the past price so convergence is cyclical. In either case, shocks have a long lasting effect on prices. Finally, price and output behave countercyclically in response to cost shocks and procyclically in response to demand shocks.

2 The Model

Suppose there are N infinitely lived firms that sell a good to overlapping generations of representative consumers. Assume that each generation of consumers lives for two discrete time periods. In each period, given market demand, each firm chooses output to maximize the discounted expected value of profits. Given the market price, consumers make consumption decisions to maximize intertemporal utility.

Each generation has a single representative consumer who is born with an endowment of wealth $\bar{w}$ which can be divided between consumption when young, consumption when old[1] and a numeraire that is perfectly substitutable between young and old age. Assume that a consumer born in period t who consumes X_t^y when young, X_t^o when old, and w_t of the numeraire good earns the following utility

$$U(X_t^y, X_t^o, w_t) = a(X_t^y + X_t^o) - \frac{b}{2}(X_t^{y2} + X_t^{o2}) - dX_t^y X_t^o + w_t \tag{1}$$

where $a, b > 0$ and $|d| < b$. The numeraire good can be interpreted as money spent on other goods and its inclusion yields linear demand functions, making the model tractable. The parameter b is an indicator of the elasticity of demand while the parameter d indicates the degree of substitutability or complementarity between current and future consumption. Large values of

[1] A model where consumers live for $M > 2$ periods quickly becomes intractable. Even with $M = 3$, an equilibrium, if it were solvable, would have prices that depended on every price from prior periods as well as the random shock.

$d > 0$ imply greater degrees of substitutability with current and future consumption becoming perfectly substitutable as $d \to b$. Similarly, large negative values of d indicate a greater degree of complementarity.

Suppose that $p_t(X_t; p_{t-1})$ is the market inverse demand function at time t where X_t is total industry output and p_{t-1} is last period's price. In each period, every firm has an identical marginal production cost of c_t. Given firm i's output, x_t^i, the output of other firms, $X_t^{-i} = \sum_{j \neq i} x_t^j$, and last period's, price, p_{t-1}, its t period profit function is given by

$$\pi^i(x_t^i, X_t^{-i}, p_{t-1}, c_t) = (p_t(x_t^i + X_t^{-i}, p_{t-1}) - c_t)x_t^i. \tag{2}$$

Marginal costs are independently and identically distributed over time with $c_t = \bar{c} + \varepsilon_t$ and $E_t \varepsilon_{t+1} = 0$. Firms have discount factor β and in each period, t, choose output to maximize discounted expected profits:

$$\Pi_t^i = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \pi^i(x_\tau^i, X_\tau^{-i}, p_{\tau-1}, c_\tau) \tag{3}$$

where π^i is as in (2).

In addition to considering price responses to changes in costs, this formulation can also be used to consider the problem of firms facing unpredictable tariffs or an unpredictable regulator. I.e., let $c_t = \bar{c} + T_t$ where $\bar{c}$ is marginal cost and T_t is the stochastic import tariff or regulatory tax faced by firms.

We consider other extensions to the basic model in Section 4.

3 Dynamic Oligopoly

We look for Markov perfect equilibria where the state variable is last period's price and the equilibrium price functions are linear. As is common with dynamic games, there may in principle be Markov perfect equilibria with non-linear equilibrium price functions. We concentrate on linear equilibria because they can easily be characterized. Since we are looking for equilibria with linear pricing, it must be the case that demand functions and hence the expected price functions must be linear.

The strategy we use to solve the model is as follows. First we derive the demand functions

by solving the young and old consumers' problems, anticipating that the expected future price will be a linear function of the current price. Next we solve each firm's profit maximization problem and confirm that the expected future price is indeed linear. Finally, we show existence and uniqueness of an equilibrium in linear strategies and where the solutions to the consumers' and the firms' problems are consistent with one another.

3.1 Consumer Demand

First consider an old consumer's utility maximization problem. Old consumers know the price and their level of consumption when they were young. They also know the current price. Since the numeraire good is perfectly substitutable between periods, consumption of the numeraire can be determined in the second period of life. Hence, in period t, an old consumer, born in period $t-1$ chooses X^o_{t-1} and w_{t-1} to maximize utility, given p_{t-1}, X^y_{t-1} and p_t.

$$\max_{X^o_{t-1}, w_{t-1}} U(X^y_{t-1}, X^o_{t-1}, w_{t-1}) = a(X^y_{t-1} + X^o_{t-1}) - \frac{b}{2}({X^y_{t-1}}^2 + {X^o_{t-1}}^2) - dX^y_{t-1}X^o_{t-1} + w_{t-1}$$

$$\text{subject to: } p_{t-1}X^y_{t-1} + p_tX^o_{t-1} + w_{t-1} \leq \bar{w}$$

Provided that $\bar{w}$ is sufficiently large to ensure positive consumption of the numeraire, this is a straight forward maximization problem which yields old consumer demand as a linear function of consumption from last period and the current price.

$$X^o_{t-1} = \frac{a}{b} - \frac{d}{b}X^y_{t-1} - \frac{1}{b}p_t \tag{4}$$

Consumption of the numeraire is given by the remainder of the endowment which was not spent on consumption (i.e., $w_{t-1} = \bar{w} - p_{t-1}X^y_{t-1} - p_tX^o_{t-1}$).

Now consider the young consumer's problem. The young consumer knows current price p_t, has expectations over the future price p_{t+1} and future consumption. In equilibrium, expectations over the future price must be consistent with the firm's profit maximization problem and expectations over X^o_t and w_t must be consistent with the old consumer's utility maximization problem. Since we are only considering equilibria with linear pricing strategies, assume that in equilibrium, the expected price is a linear function of last period's price. In particular, let $E_tp_{t+1} = (1-\lambda)\bar{p} + \lambda p_t$, where λ and $\bar{p}$ are, for the moment, constants where $|\lambda| < 1$. These will turn out to be the coefficients of the equilibrium price function which will be determined

once we solve the firms' optimization problem.

The young consumer's problem is:

$$\max_{X_t^y} \; E_t U(X_t^y, X_t^o, w_t) = E_t\{a(X_t^y + X_t^o) - \frac{b}{2}(X_t^{y2} + X_t^{o2}) - dX_t^y X_t^o + w_t\}$$

$$\text{subject to: } \; p_t X_t^y + E_t\{p_{t+1} X_t^o\} + E_t w_t \leq \bar{w}$$

where expectations over p_{t+1} are as assumed above and expectations over X_t^o are consistent with (4). Solving this problem yields young consumer demand:

$$X_t^y = \frac{a}{b+d} + \frac{d(1-\lambda)\bar{p}}{b^2 - d^2} - \frac{b - d\lambda}{b^2 - d^2} p_t. \tag{5}$$

Given last period's price, p_{t-1} and assuming that old consumers behaved optimally when they were young, we can substitute (5) into (4) to get old demand as a function of p_t and p_{t-1}.

$$X_{t-1}^o = \frac{a}{b+d} - \frac{d^2(1-\lambda)\bar{p}}{b(b^2 - d^2)} + \frac{d(b - d\lambda)}{b(b^2 - d^2)} p_{t-1} - \frac{1}{b} p_t. \tag{6}$$

Finally, summing young and old consumer demand yields aggregate consumer demand.

$$X_t = X_{t-1}^o + X_t^y = \hat{a} - \hat{b} p_t + \hat{d} p_{t-1} \tag{7}$$

where,

$$\hat{a} = \frac{2a}{b+d} + \frac{d(1-\lambda)\bar{p}}{b(b+d)}, \; \hat{b} = \frac{b(b - d\lambda) + (b^2 - d^2)}{b(b^2 - d^2)}, \; \hat{d} = \frac{d(b - d\lambda)}{b(b^2 - d^2)}. \tag{8}$$

As long as $\bar{p} \geq 0$ and given that $a, b > 0$ and $|d| < b$ then $\hat{a}, \hat{b} > 0$, $|\hat{d}| < \hat{b}$ and $\text{sign}(\hat{d}) = \text{sign}(d)$. For the present we will concentrate on the case where current and future consumption are substitutes and defer to Section 3.4, a brief discussion of intertemporal complementarities in consumption.

3.2 The Firms' Problems

Since each period's demand depends only on the current and last period's price, p_{t-1} summarizes each firm's relevant information in a Markov perfect equilibrium. In each period, given last period's price, p_{t-1}, and the realized cost, c_t, each firm chooses output to maximize discounted

expected profits (3).

To solve the problem, we first reformulate it as follows. First, note that given the total output of the other firms, X_t^{-i}, firm i's output choice uniquely determines the price and hence given X_t^{-i}, firm i can instead choose price to maximize profits. Since in equilibrium, each firm's output will be a linear function of last period's price and this period's cost let $X_t^{-i} = f + gp_{t-1} - hc_t$. Using (7), firm i's residual demand can now be written as follows:

$$\begin{aligned} x_t^i &= \hat{a} - \hat{b}p_t + \hat{d}p_{t-1} - X_t^{-i} \\ &= (\hat{a} - f) - \hat{b}p_t + (\hat{d} - g)p_{t-1} + hc_t \end{aligned} \tag{9}$$

The single period profit function can thus be rewritten as a function of last period's price, the current price and marginal cost.

$$\pi^i(p_t, p_{t-1}, c_t) = (p_t - c_t)x_t^i \tag{10}$$

where x_t^i is given by (9).

The first derivatives of the single period profit function with respect to the current and previous price are

$$\frac{\partial \pi^i(p_t, p_{t-1}, c_t)}{\partial p_t} = (\hat{a} - f) - 2\hat{b}p_t + (\hat{d} - g)p_{t-1} + (\hat{b} + h)c_t$$

$$\frac{\partial \pi^i(p_t, p_{t-1}, c_t)}{\partial p_{t-1}} = (\hat{d} - g)(p_t - c_t).$$

The Euler equation is thus

$$\begin{aligned} &\frac{\partial \pi^i(p_t, p_{t-1}, c_t)}{\partial p_t} + \beta E_t \left\{ \frac{\partial \pi^i(p_{t+1}, p_t, c_{t+1})}{\partial p_t} \right\} = \\ &\qquad (\hat{a} - f) - 2\hat{b}p_t + (\hat{d} - g)p_{t-1} + (\hat{b} + h)c_t + \beta(\hat{d} - g)E_t(p_{t+1} - c_{t+1}) = 0. \end{aligned} \tag{11}$$

Since X_t^{-i} has been assumed to represent the equilibrium outputs of all firms but i, (11) determines the behavior of prices in equilibrium. This is a stochastic second-order linear difference

equation with a solution of the following form:

$$p_{t+1} - p_t = (1-\lambda)(\bar{p} - p_t) - \lambda \frac{\hat{b}+h}{\hat{d}-g}(\bar{c} - c_{t+1}) \tag{12}$$

where $\bar{p}$ turns out to be the long run expected price and λ is the smaller of the roots found by factorizing the difference equation. If we take expectations of (12), we see that $E_t p_{t+1}$ is in fact a linear function of p_t with the form assumed in Section 3.1.

The equilibrium value of λ must satisfy:

$$\beta\lambda^2 - 2\frac{\hat{b}}{\hat{d}-g}\lambda + 1 = 0 \tag{13}$$

Since we are looking for $\lambda \in (0,1)$, solving this yields:

$$\lambda = \frac{\hat{b}/(\hat{d}-g) - \sqrt{(\hat{b}/(\hat{d}-g))^2 - \beta}}{\beta}. \tag{14}$$

It is easy to see that for $b > d > 0$, if $\lambda \in (0,1)$, the right hand side of (14) is in $(0,1)$. Solving for the long run expected price yields,

$$\bar{p} = \frac{\hat{a}-f}{2\hat{b} - (1+\beta)(\hat{d}-g)} + \frac{(\hat{b}+h) - \beta(\hat{d}-g)}{2\hat{b} - (1+\beta)(\hat{d}-g)}\bar{c}. \tag{15}$$

Thus λ and $\bar{p}$ are functions of model parameters and $\hat{a}, \hat{b}, \hat{d}, f, g$ and h which are in turn functions of model parameters and λ and $\bar{p}$—in the remainder of this section, we find and characterize their solutions.

Using (12) and summing (9) over all $j \neq i$, yields equations for f, g and h.

$$f = (N-1)(\hat{a}-f) - (N-1)\hat{b}(1-\lambda)\bar{p} + (N-1)\frac{\lambda\hat{b}(\hat{b}+h)}{\hat{d}-g}\bar{c} \tag{16}$$

$$g = -(N-1)\lambda\hat{b} + (N-1)(\hat{d}-g) \tag{17}$$

$$h = (N-1)\frac{\lambda\hat{b}(\hat{b}+h)}{\hat{d}-g} - (N-1)h \tag{18}$$

We can then solve (17) and (18) for g and h.

$$g = \frac{N-1}{N}(\hat{d} - \lambda\hat{b}) \tag{19}$$

$$h = \frac{(N-1)\lambda\hat{b}^2}{\hat{d}} \tag{20}$$

Substituting (19) and (20) into $\lambda(\hat{b}+h)/(\hat{d}-g)$ and then simplifying yields:

$$\lambda\frac{\hat{b}+h}{\hat{d}-g} = \frac{N\lambda\hat{b}}{\hat{d}}. \tag{21}$$

Thus (12) becomes:

$$p_{t+1} = (1-\lambda)\bar{p} + \lambda p_t + \frac{N\lambda\hat{b}}{\hat{d}}\varepsilon_{t+1}. \tag{22}$$

Note from (14) and (19), λ depends on $\hat{b}$ and $\hat{d}$ which depends on λ. As a result, we need to prove the existence of such a λ which solves these equations. In addition, we would like the solution to be unique and to prove some additional properties of the result.

Proposition 1 *For all admissible parameters the Markov perfect equilibrium in linear pricing strategies exists, is unique, and has $\bar{p} > \bar{c}$ and $N\lambda\hat{b}/\hat{d} < 1$.*

Proof: First we define some additional notation. For each N let:

$$Q = \frac{N\lambda s}{1+(1-1/N)N\lambda s}, \quad s = \frac{\hat{b}}{\hat{d}}.$$

Using the results above and this definition we can rewrite (13) as:

$$\beta\lambda^2 - 2Q + 1 = 0$$

Solving this for λ then yields:

$$\lambda = \sqrt{\frac{2Q-1}{\beta}}.$$

Note that, if we find a $\lambda \in (0,1)$ that satisfies (13) the above implies that $1/2 < Q < (1+\beta)/2$.

Now, define the following functions over $1/2 \leq q \leq (1+\beta)/2$:

$$\Lambda(q) = \sqrt{\frac{2q-1}{\beta}} \in [0,1]$$

$$s(q) = \frac{2e^2 - \Lambda(q)e - 1}{e - \Lambda(q)} \in (1,\infty), \; e = \frac{b}{d} > 1$$

$$\eta(q) = \frac{Ns(q)}{\sqrt{\beta}} > N/\sqrt{\beta} > N \tag{23}$$

$$y(q) = \eta(q)\sqrt{2q-1} = N\Lambda(q)s(q)$$

$$\gamma(q) = \frac{y(q)}{1+(1-1/N)y(q)} = \frac{N\Lambda(q)s(q)}{1+(1-1/N)N\Lambda(q)s(q)}$$

$$k(q) = q - \gamma(q).$$

Suppose we knew an equilibrium value of λ. This would imply a value for Q so that $\lambda = \Lambda(Q)$. Then, by construction, $\hat{b}/\hat{d} = s(Q)$, $N\lambda\hat{b}/\hat{d} = y(Q)$ and $Q = \gamma(Q)$. That is, the Q corresponding to an equilibrium will be a fixed point of $\gamma(\cdot)$. Also, if q^* is a fixed point of $\gamma(\cdot)$ it is easily verified that $\Lambda(q^*)$ satisfies the equilibrium conditions. So equilibria will correspond one-to-one with fixed points of $\gamma(\cdot)$ or, equivalently, to points at which $k(q^*) = 0$. Note that all the above functions are continuous over the range of q and all are increasing in q except $k(\cdot)$. With this notation, we can now turn to the proof.

The strategy of the proof is to show that there is a unique $q^* \in [1/2, (1+\beta)/2]$ such that $k(q^*) = 0$ and $y(q^*) < 1$. Now, we can write:

$$k(q) = \frac{q - \eta(q)\sqrt{2q-1}[1-(1-1/N)q]}{1+(1-1/N)\eta(q)\sqrt{2q-1}}$$

and this will be zero if and only if the numerator is zero. Define, for $1/2 \leq q \leq (1+\beta)/2$ and $\eta > N/\sqrt{\beta}$ (since $\eta(q) > N/\sqrt{\beta}$):

$$\mu(q,\eta) = q - \eta\sqrt{2q-1}[1-(1-1/N)q].$$

The following properties of μ are important: μ is continuous, strictly convex in q, and strictly decreasing in η. By construction, an equilibrium corresponds to a point q^* such that $\mu(q^*, \eta(q^*)) = 0$. Note also that the composite function $\mu(q, \eta(q))$ is continuous.

We next show an equilibrium exists. First:

$$\mu(1/2, \eta(1/2)) = 1/2 > 0.$$

Also:

$$\begin{aligned}\mu\left(\frac{1+\beta}{2}, \eta\right) &= \frac{1+\beta}{2} - \eta\sqrt{\beta}\left[1 - \left(1 - \frac{1}{N}\right)\frac{1+\beta}{2}\right] \\ &< \frac{1+\beta}{2} - N\left[1 - \left(1 - \frac{1}{N}\right)\frac{1+\beta}{2}\right], \ \forall \eta > \frac{N}{\sqrt{\beta}}\end{aligned}$$

so

$$\mu\left(\frac{1+\beta}{2}, \eta\left(\frac{1+\beta}{2}\right)\right) < \frac{1+\beta}{2}(1 + N - 1) - N = N\left(\frac{1+\beta}{2} - 1\right) < 0$$

since $\beta < 1$. By continuity, there exists $q^* \in (1/2, (1+\beta)/2)$ such that $\mu(q^*, \eta(q^*)) = 0$.

We now show q^* is unique. Take arbitrary $q \in (1/2, q^*)$. Since $q^* < (1+\beta)/2$, take $\alpha \in (0, 1)$ such that $q^* = \alpha q + (1-\alpha)(1+\beta)/2$. By convexity:

$$\mu(q^*, \eta(q^*)) = 0 < \alpha\mu(q, \eta(q^*)) + (1-\alpha)\mu\left(\frac{1+\beta}{2}, \eta(q^*)\right).$$

The second term on the right-hand side is negative (since the above argument showed this was true for arbitrary η), so the first must be positive. Therefore $\mu(q, \eta(q^*)) > 0$. But $\eta(q)$ is increasing in q so $\eta(q^*) > \eta(q)$. Since μ is decreasing in η, this implies $\mu(q, \eta(q)) > \mu(q, \eta(q^*)) > 0$ so $\mu(q, \eta(q)) > 0$ for all $q \in [1/2, q^*)$. That $\mu(q, \eta(q))$ is not zero for $q \in (q^*, (1+\beta)/2]$ is now obvious since otherwise the same argument would imply $\mu(q^*, \eta(q^*)) > 0$.

We now show that $N\lambda\hat{b}/\hat{d} < 1$. Note that $N\lambda\hat{b}/\hat{d} = y(q^*)$ and if $q^* = \gamma(q^*) < N/(2N-1)$ it must be that $y(q^*) < 1$. So it suffices to show $q^* < N/(2N-1)$. If $N/(2N-1) > (1+\beta)/2$

we are done. Otherwise:

$$\begin{aligned}
\mu\left(\frac{N}{2N-1}, \eta\left(\frac{N}{2N-1}\right)\right) &= \frac{N}{2N-1} - \eta\left(\frac{N}{2N-1}\right)\frac{1}{\sqrt{N-1}}\left[1 - \frac{N-1}{N}\cdot\frac{N}{2N-1}\right] \\
&= \frac{N}{2N-1}\frac{\sqrt{2N-1} - \eta\left(\frac{N}{2N-1}\right)}{\sqrt{2N-1}} \\
&< \frac{N}{2N-1}\frac{\sqrt{2N-1} - N}{\sqrt{2N-1}} \\
&\leq 0
\end{aligned}$$

where the inequalities hold since $\eta > N$ and $N/(2N-1) \leq 1$, $N \geq \sqrt{2N-1}$.[2]

Finally, solving (16) for f and using (18) yields:

$$f = \frac{N-1}{N}\hat{a} - \frac{N-1}{N}\hat{b}(1-\lambda)\bar{p} + h\bar{c}$$

Substituting this and $\hat{a}$ into $\bar{p}$ and solving yields:

$$\bar{p} = \frac{\frac{1}{N}\frac{2a}{b+d} - h\bar{c} + (\hat{b}+h)\bar{c} - \beta(\hat{d}-g)\bar{c}}{2\hat{b} - (1+\beta)(\hat{d}-g) - \frac{1}{N}\frac{d(1-\lambda)}{b(b+d)} - \frac{N-1}{N}\hat{b}(1-\lambda)}. \tag{24}$$

It is easy to show that

$$\hat{b} - \beta(\hat{d}-g) > 2\hat{b} - (1+\beta)(\hat{d}-g) - \frac{1}{N}\frac{d(1-\lambda)}{b(b+d)} - \frac{N-1}{N}\hat{b}(1-\lambda)$$

and therefore $\bar{p} > \bar{c}$. ■

That is, prices follow a first-order autocorrelation process.[3] Thus the equilibrium price has properties that match stylized facts of price fluctuations. Shocks to costs are not fully reflected as changes in price (i.e., $N\lambda\hat{b}/\hat{d} < 1$)[4] and price changes persist into the future. Thus our results look remarkably similar to models of adjustment costs and price staggering (e.g., Rotemberg

[2]This can easily be derived from the fact that $(N-1)^2 \geq 0$.

[3]With more complicated shocks and additional information and behavioral assumptions to ensure solvability, prices can be shown to follow more complicated stochastic processes. We examine one such extension in Section 4.1.

[4]This incomplete passthrough result may admittedly be dependent on the linearity of the model. Indeed, with a static oligopoly model, if demand is linear and marginal costs are constant, there is also incomplete passthrough of cost changes—a constant fraction of cost changes are passed on as price changes. Once linearity is eliminated, prices can change by more than costs, both in absolute and relative terms (see Carlton (1989)). However, due to the difficulty involved in finding an analytic solution to most non-linear dynamic problems, it is unclear whether or not this static argument extends to our dynamic problem.

(1982) and Taylor (1980)) without requiring such restrictions.

In order to examine the cyclical behavior of output, substitute the equilibrium price into aggregate demand.

$$X_t = (\hat{a} - \hat{b}(1-\lambda)\bar{p}) + (\hat{d} - \lambda\hat{b})p_{t-1} - \frac{N\lambda\hat{b}^2}{\hat{d}}\varepsilon_t \tag{25}$$

This reveals that equilibrium output depends positively on past price but negatively on the cost shock. Since price is increasing and output is decreasing in cost shocks, price and output behave countercyclically.

We now demonstrate some of the convergence properties of the equilibrium. In particular,

Corollary 1 *As N tends to infinity, λ tends to zero, $N\lambda\hat{b}/\hat{d}$ tends to one and $\bar{p}$ tends to $\bar{c}$.*

Proof: Since $1/2 < Q < N/(2N-1)$, Q must tend to $1/2$ as N tends to infinity. Since $\lambda = \Lambda(Q)$ and this function is continuous, λ tends to $\Lambda(1/2) = 0$. Note Q is a one-to-one, continuous function of $y = N\lambda s$, so the fact that Q converges implies y converges to some limit point y'. But then it must be that $1/2 = y'/(1+y')$. Solving this yields $y' = 1$.

Note that as $N \to \infty$, $h \to \hat{b}$ and $g \to \hat{d}$. Thus from equation (24),

$$\lim_{N\to\infty} \bar{p} = \frac{0 - \hat{b}\bar{c} + 2\hat{b}\bar{c} - 0}{2\hat{b} - 0 - 0 - \hat{b}} = \bar{c}.$$

■

Hence as the number of firms becomes large, the equilibrium price approaches marginal cost and persistence becomes negligible. This concurs with the standard intuition that as the number of firms grow large, the market approaches perfect competition.

3.3 Comparative Statics

We can also use a subset of the system which was constructed to prove Proposition 1 to demonstrate some comparative static properties of the equilibrium. Write λ, s and q from (23) as

follows:

$$\lambda = m\sqrt{2q-1},\ m = 1/\sqrt{\beta}$$

$$s = \frac{2e^2 - \lambda e - 1}{e - \lambda}$$

$$q = \frac{N\lambda s}{1 + (N-1)s\lambda}$$

This three equation system determines the three endogenous variables λ, s, and q, given the three exogenous variables e, m, and N. While none of these equations have any structural economic interpretation, we can exploit the fact that this system's solution corresponds one-to-one with our economic model's solution. It is, however, important to bear in mind that the above is an artificial system of equations. For example, application of stability conditions to establish properties of the solution (i.e., the correspondence principle in macroeconomics) would be a mistake since stability in the artificial system is meaningless.

To sign partial derivatives of these endogenous variables with respect to exogenous variables, linearize the equations in a neighborhood of an equilibrium by taking the following total differentials:

$$d\lambda = \theta_1 dq + \theta_2 dm$$

$$ds = \theta_3 d\lambda + \theta_4 de$$

$$dq = \theta_5 d\lambda + \theta_6 ds + \theta_7 dN.$$

It can be shown that all the θ_i's are positive. Writing this system in matrix form:

$$\begin{bmatrix} 1 & 0 & -\theta_1 \\ -\theta_3 & 1 & 0 \\ -\theta_5 & -\theta_6 & 1 \end{bmatrix} \begin{bmatrix} d\lambda \\ ds \\ dq \end{bmatrix} = \begin{bmatrix} \theta_2 & 0 & 0 \\ 0 & \theta_4 & 0 \\ 0 & 0 & \theta_7 \end{bmatrix} \begin{bmatrix} dm \\ de \\ dN \end{bmatrix}.$$

The determinant of the matrix on the left is $D = 1 - \theta_1(\theta_3\theta_6 + \theta_5)$. Now:

$$\theta_1 = \frac{m}{\sqrt{2q-1}} > \sqrt{2N-1}$$

since $m > 1$ and $q < N/(2N-1)$.

$$\theta_5 = \frac{Ns}{[1+(1-1/N)N\lambda s]^2} > \frac{N}{[(2N-1)/N]^2} = \frac{N^3}{(2N-1)^2}$$

since $s > 1$ and $N\lambda s < 1$. Therefore $\theta_1\theta_5 > N^3/(2N-1)^{3/2}$. It is possible to show that the last expression equals one when $N = 1$ and is greater than one for $N > 1$. This guarantees that D is negative.

We can now sign the partial derivatives of λ using Cramer's rule:

$$\frac{d\lambda}{dN} = \frac{1}{D}\begin{vmatrix} 0 & 0 & -\theta_1 \\ 0 & 1 & 0 \\ \theta_7 & -\theta_6 & 1 \end{vmatrix} = \frac{\theta_1\theta_7}{D} < 0$$

so λ is decreasing in N. To see why, consider the optimal price path, from the point of view of the firms. Since single period profits are concave in prices, optimality requires some price smoothing. When there is more than one firm, each firm's output decision exerts an externality on other firms by reducing price smoothing. That is, in addition to the static externality one firm's decision imposes on other firms, there is a dynamic externality. As a result of this externality, the degree of price smoothing falls as the number of firms rises.

$$\frac{d\lambda}{dm} = \frac{1}{D}\begin{vmatrix} \theta_2 & 0 & -\theta_1 \\ 0 & 1 & 0 \\ 0 & -\theta_6 & 1 \end{vmatrix} = \frac{\theta_2}{D} < 0$$

so λ is decreasing in m, which implies it is increasing in β, the discount factor. That is, the more patient the firms, the greater the degree of persistence in prices. This is because optimality requires price smoothing. The more patient firms are, the more important are dynamics and thus price smoothing. Therefore, as the discount factor increases, last period's price will have a greater effect on the current price.

$$\frac{d\lambda}{de} = \frac{1}{D}\begin{vmatrix} 0 & 0 & -\theta_1 \\ \theta_4 & 1 & 0 \\ 0 & -\theta_6 & 1 \end{vmatrix} = \frac{\theta_1\theta_4\theta_6}{D} < 0$$

so that λ is decreasing in e. Note that $e = b/d$, so the larger e is, the less important is the term d, which is a measure of the strength of substitutability. This makes perfect sense—it is substitutability between consumption when young and when old that drives the persistence in prices. That is, the less important is intertemporal substitutability, the less the persistence in prices.

In order to get some idea as to the behavior of the term $N\lambda\hat{b}/\hat{d}$, we will need to get similar comparative static results on s. These are:

$$\frac{ds}{dm} = \frac{1}{D}\begin{vmatrix} 1 & \theta_2 & -\theta_1 \\ -\theta_3 & 0 & 0 \\ \theta_5 & 0 & 1 \end{vmatrix} = \frac{\theta_2\theta_3}{D} < 0$$

$$\frac{ds}{de} = \frac{1}{D}\begin{vmatrix} 1 & 0 & -\theta_1 \\ -\theta_3 & \theta_4 & 0 \\ \theta_5 & 0 & 1 \end{vmatrix} = \frac{\theta_4(1+\theta_1\theta_5)}{D} < 0$$

$$\frac{ds}{dN} = \frac{1}{D}\begin{vmatrix} 1 & 0 & -\theta_1 \\ -\theta_3 & 0 & 0 \\ \theta_5 & \theta_7 & 1 \end{vmatrix} = \frac{\theta_1\theta_3\theta_7}{D} < 0$$

Now, differentiating $y = N\lambda s$ with respect to m, e and N yields:

$$\frac{dy}{dm} = Ns\frac{d\lambda}{dm} + N\lambda\frac{ds}{dm} < 0$$
$$\frac{dy}{de} = Ns\frac{d\lambda}{de} + N\lambda\frac{ds}{de} < 0$$
$$\frac{dy}{dN} = \lambda s + Ns\frac{d\lambda}{dN} + N\lambda\frac{ds}{dN}$$

Thus prices are more responsive to shocks if firms are more patient or if intertemporal substitutability of consumption becomes more important. A clear-cut result for the final comparative static is not trivial. It is possible to show that it is strictly positive for $N \geq 3$ (the following proposition), however, for $N < 3$ we can only say that based on extensive simulation exercises, $N\lambda\hat{b}/\hat{d}$ appears to increase as N changes from 1 to 2 to 3. In any case, $N\lambda\hat{b}/\hat{d}$ is strictly in-

creasing in $N \geq 3$. This is consistent with the Carlton (1986) evidence which shows that price sluggishness is correlated with industry concentration.

Proposition 2 *$dy/dN > 0$ for $N \geq 3$.*

Proof: First:

$$\begin{aligned}\frac{dy}{dN} &= \lambda s + N\lambda\frac{ds}{dN} + Ns\frac{d\lambda}{dN} \\ &= \lambda s - N\lambda\frac{\theta_1\theta_3\theta_7}{|D|} - Ns\frac{\theta_1\theta_7}{|D|}\end{aligned}$$

where $|D| = \theta_1(\theta_3\theta_6 + \theta_5) - 1 > 0$. Factoring out $|D|$, we get:

$$\frac{dy}{dN} = \frac{1}{|D|}(\theta_1[\theta_3(\theta_6\lambda s - \theta_7 N\lambda) + \theta_5\lambda s - \theta_7 Ns] - \lambda s)$$

Using the definition of the θs:

$$\frac{dy}{dN} = \frac{1}{|D|}\left(\theta_1\left[\theta_3\left(\frac{N\lambda^2 s}{H^2} - \frac{N\lambda^2 s(1-\lambda s)}{H^2}\right) + \frac{N\lambda s^2}{H^2} - \frac{N\lambda s^2(1-\lambda s)}{H^2}\right] - \lambda s\right)$$

where $H = 1 + (N-1)\lambda s$. So,

$$\begin{aligned}\frac{dy}{dN} &= \frac{1}{|D|}\left(\theta_1\left[\theta_3\frac{N\lambda^3 s^2}{H^2} + \frac{N\lambda^2 s^3}{H^2}\right] - \lambda s\right) \\ &= \frac{1}{|D|}\left(\theta_3\frac{N\lambda^2 s^2}{\beta H^2} + \frac{N\lambda s^3}{\beta H^2} - \lambda s\right)\end{aligned}$$

since $\theta_1 = 1/\beta\lambda$. Factoring out $\lambda s/H^2$ yields:

$$\frac{dy}{dN} = \frac{\lambda s}{|D|H^2}\left(\frac{\theta_3 N\lambda s}{\beta} + \frac{Ns^2}{\beta} - H^2\right)$$

A sufficient condition for this to be positive is that:

$$\frac{Ns^2}{\beta} > H^2$$

or

$$\frac{s}{\sqrt{\beta}}\sqrt{N} > H$$

Now:

$$\begin{aligned}
\frac{s}{\sqrt{\beta}}\sqrt{N} - H &> \sqrt{N} - H \text{ (since } \frac{s}{\sqrt{\beta}} > 1) \\
&= \sqrt{N} - 1 - \frac{N-1}{N} N\lambda s \\
&= \sqrt{N} - 1 - \frac{N-1}{N} y \\
&> \sqrt{N} - 1 - \frac{N-1}{N} \text{ (since } y < 1) \\
&= \frac{N\sqrt{N} - 2N + 1}{N}
\end{aligned}$$

The term in the numerator is negative for N between one and two. However, it is positive for $N = 3$ and is strictly increasing in N for $N \geq 3$, so dy/dN is positive for $N \geq 3$. ■

3.4 Intertemporal Complementarity of Consumption

When $d < 0$, rather than being substitutes, current and future consumption are complements. When there is complementarity,

$$\lambda = \frac{\hat{b}/(\hat{d} - g) + \sqrt{(\hat{b}/(\hat{d} - g))^2 - \beta}}{\beta}. \tag{26}$$

It is easy to see that $-1 < \lambda < 0$. Now, alter a few of the definitions in (23) as follows:

$$\Lambda(q) = -\sqrt{\frac{2q-1}{\beta}} \in [0, 1]$$

$$\eta(q) = -\frac{Ns(q)}{\sqrt{\beta}} > \frac{N}{\sqrt{\beta}} > N$$

In addition, it can be seen that $s(q) < -1$ and $e < -1$. It is easy to see that the proofs of Proposition 1 and Corollary 1 corresponding to the case when $d < 0$ are identical. Since $\lambda < 0$, in the absence of cost shocks, price will converge to while oscillating around the long run steady state price. Since $\hat{d} < 0$ the sign of $N\lambda\hat{b}/\hat{d}$ and $N\lambda\hat{b}^2/\hat{d}$ are positive and so (22) and (25) show that price and output again behave countercyclically.

Similarly, employing the same method for comparative statics, it can be shown that θ_1, θ_2, θ_4, θ_5, $\theta_6 < 0$, θ_3, $\theta_7 > 0$, $\theta_1\theta_5 > N^3/(2N-1)^{3/2}$ and $D < 0$. Thus, $d\lambda/dm$, $d\lambda/de$, $d\lambda/dN$,

ds/dm, ds/de, ds/dN, dy/dm, $dy/de > 0$. Similar to the case when $d > 0$, although the sign of dy/dN is ambiguous, it converges to 1 from below and thus as N grows, it must have a tendency to be increasing in N.

4 Extensions and Applications

4.1 Correlated Shocks

To this point we have considered shocks which are i.i.d. over time. We now briefly consider the case in which there is correlation in the cost shocks. As a simple case, suppose there is first-order autocorrelation in the cost-shock series. In this case, the same method of solving for a Markov-perfect equilibrium with linear pricing strategies does not work. If young consumers form expectations of the next period's price as a linear function of the current price, the firms' problem would be the same as in Section 3.2 and equation (22) would still give the equilibrium response of firms to such a strategy by consumers. However, the last term in equation (22) involves the term ε_{t+1} which will not have expectation of zero unless the current cost shock is zero. Thus, if consumers were to form expectations of the future price assuming the price sequence is first-order autocorrelated, the price sequence firms would choose would be second-order autocorrelated.

To avoid this problem, we instead assume the young consumers observe only the current price and not the cost shock or the history of prices that occurred before they were born. Even with this simple information set, the expectation of the next-period's price will not generally be linear (this depends on the distribution that generates the shocks), so we assume consumers use a least-squares projection to form forecasts of the future price.

Given that consumers use linear forecasts, the firms' problem remains the same and has a solution of the form given in equation (22). Letting z_t denote the deviation in the price at time t from the long-run expected price. That is, $z_{t+1} \equiv p_{t+1} - \bar{p} = \lambda(p_t - \bar{p}) + \lambda(N\hat{b}/\hat{d})\varepsilon_{t+1}$. This can be rewritten in the form:

$$z_{t+1} = \lambda z_t + \lambda e_{t+1}$$

where e_t is proportional to the cost shock in period t. By assumption the cost shock follows a

first-order autocorrelation process, so that:

$$e_{t+1} = \rho e_t + u_{t+1}$$

where the scalar ρ is less than one in absolute value and u_t is white noise. Let $P(z_{t+1}|z_t)$ be the projection of z_{t+1} given z_t. Since z_t is known by consumers born at date t, in order to compute $P(z_{t+1}|z_t)$, we need to find the projection of e_{t+1} given z_t, $P(e_{t+1}|z_t)$. This takes the form:

$$P(e_{t+1}|z_t) = \theta z_t$$

where

$$\theta = \frac{COV(z_t, e_{t+1})}{VAR(z_t)}$$

It can be shown that

$$\theta = \frac{(1-\lambda^2)\rho}{\lambda(1+\rho\lambda)}$$

so that the projection of z_{t+1} on z_t is:

$$P(z_{t+1}|z_t) = \lambda(1+\theta)z_t = \frac{\lambda+\rho}{1+\lambda\rho}z_t \equiv \zeta z_t$$

It is easy to show that ζ as defined above is in the interval $[-1,1]$ whenever $\rho \in [-1,1]$ and $\lambda \in [0,1]$. To show existence of an equilibrium when $d > 0$, define the function $G : [-1,1] \times [0,1] \to [-1,1] \times [0,1]$ as follows. For any given ρ, let:

$$G_1(\zeta, \lambda) = \frac{\lambda+\rho}{1+\lambda\rho}$$

and let G_2 be the right-hand side of (14) where for the solution to the consumers utility maximization problem, we have replaced the λ's appearing in (8) with ζ's.[5] Here the subscripts index the two arguments of G. It is straightforward to show that G is continuous and maps the compact, convex set $[-1,1] \times [0,1]$ back into itself. Therefore G has a fixed point. By construction, fixed points of G correspond to Markov perfect equilibria, so an equilibrium exists. The

[5]For $d < 0$, similarly define G_2 as the right-had side of (26) so that $G : [-1,1] \times [-1,0] \to [-1,1] \times [-1,0]$.

primary difference is that prices now follow a second-order autocorrelation process.

Using similar informational and behavioral assumptions, other stochastic processes can lead to similar results. For example, if shocks are instead assumed to follow a first-order moving average process, it can be shown that prices will then follow an ARMA(1,1) process. Of course more complicated stochastic shocks lead to more complicated price processes.

4.2 Demand Shocks

Suppose that there is no cost uncertainty but that demand is subject to observable, additive stochastic demand shocks. That is, let $X_t = \hat{a} - \hat{b}p_t + \hat{d}p_{t-1} + a_t$ where $a_t = \bar{a} + \xi_t$ and $E_{t-1}\xi_t = 0$. For ease of notation, let $\bar{a} = 0$ and $\bar{c} = 0$. If we now define the equilibrium output of rival firms as $X_t^{-i} = f + gp_{t-1} + k\xi_t$ then firm i's residual demand can be written as

$$x_t^i = (\hat{a} - f) - \hat{b}p_t + (\hat{d} - g)p_{t-1} + (1 - k)\xi_t. \tag{27}$$

The Euler equation is

$$(\hat{a} - f) - 2\hat{b}p_t + (\hat{d} - g)p_{t-1} + (1 - k)\xi_t + \beta(\hat{d} - g)E_t p_{t+1} = 0$$

and has solution

$$p_{t+1} - p_t = (1 - \lambda)(\bar{p} - p_t) + \frac{\lambda(1 - k)}{\hat{d} - g}\xi_{t+1}. \tag{28}$$

Using (28) and summing (27) over all $j \neq i$, we solve for k (the solutions for f and g remain unchanged):

$$k = -(N - 1)\frac{\lambda\hat{b}(1 - k)}{\hat{d} - g} + (N - 1)(1 - k).$$

Solving this for k yields:

$$k = \frac{1 - \dfrac{\lambda\hat{b}}{\hat{d} - g}}{\dfrac{N}{N - 1} - \dfrac{\lambda\hat{b}}{\hat{d} - g}}$$

We know from the earlier discussion in Section 3.2 that $\lambda\hat{b}/(\hat{d} - g) < 1$ and so $k > 0$. Furthermore,

by examination it is easy to see that $k < 1$ and that as N tends towards infinity, k tends towards 1. Using the system of equations constructed in the proof of Proposition 1 (23), the shock coefficient of the equilibrium price process, $\lambda(1-k)/(\hat{d}-g)$, can be rewritten as $q(1-k)/\hat{b}$. Since q and $\hat{b}$ are bounded, as N tends to infinity, this tends to zero. Since the limiting behavior of λ and $\bar{p}$ are the same as before, prices approach marginal cost.

Again substituting the equilibrium price into aggregate demand,

$$X_t = (\hat{a} - \hat{b}(1-\lambda)\bar{p}) + (\hat{d} - \lambda\hat{b})p_{t-1} + \left(1 - \frac{\lambda\hat{b}(1-k)}{\hat{d}-g}\right)\xi_t.$$

Since $\lambda\hat{b}/(\hat{d}-g) < 1$ and $0 < k < 1$, $1-\lambda\hat{b}(1-k)/(\hat{d}-g) > 0$ and positive demand shocks result in increases in output. Thus in contrast to a model with cost shocks, output is increasing in the demand shock and thus price and output behave *procyclically* both when current and future consumption are substitutes ($d > 0$) and when they are complements ($d < 0$). Further, in the limit, as N tends to infinity, the shock coefficient of the output process, $1 - \lambda\hat{b}(1-k)/(\hat{d}-g)$, tends to 1.

Because of the similarity of Bils (1989) to the current model when $N = 1$ and when current and future consumption are complementary,[6] the procyclical behavior of prices and output in response to demand shocks is somewhat puzzling. Bils shows with his model of customer markets that prices and output behave countercyclically. This is due to the fact that with Bils' model, the old generation's demand is invariant to small increases in the price. As a result the monopolist recognizes that its current price fully determines its sales to the soon to be older generation. Thus in periods of high demand, the monopolist charges a low current price, sacrificing profits from the relatively smaller customer base in order to capture a larger customer base which can be exploited in the future. With the current model, there is also an incentive to increase sales in response to an increase in demand. However, the monopolist recognizes that she must still price low enough in the next period in order to sell to the older generation and thus price and output behave procyclically.

[6] In Bils (1989), aggregate quantity demanded, D_t falls with increases in either the current price or the future price while with the current paper, the same is true for $d < 0$.

4.3 Infinite Horizons and Inventories

There is a large body of evidence which suggests that most goods are habit goods (see for example, Blanciforti and Green (1988), Browning (1991) and Pollak and Wales (1969)). On the other hand, there is also evidence which suggests that prices are positively autocorrelated, even for goods that most would agree are habit goods (coffee and tea).[7] This evidence contradicts our result that there is negative autocorrelation in prices when consumption is intertemporally complementary. One hypothesis is that prices may be determined in the commodity market rather than the final consumption market. To examine this possibility, we construct a model with consumers who get utility (or profits) from consumption (or an intermediate good) and can hold inventories.

Suppose we have an infinitely lived, representative agent who gets utility from consumption and from holding inventories. For example, this agent might be thought of as a firm that purchases an intermediate good to be used in its production process.[8] Since this firm may have long term commitments which need to be fulfilled, it must hold inventories to ensure a supply of the intermediate good. To be more specific, suppose that the total period t revenue from 'consuming' y_t and holding i_t inventories is given by: $a_i i_t + a_y y_t - (b_i/2)i_t^2 - (b_y/2)y_t^2 + d i_t y_t$. We assume that $d > 0$ under the following interpretation: one would expect the marginal product of inventories to increase as the rate of consumption increases. Single period profits are therefore total revenue, less the cost of purchases, $p_t(i_{t+1} - i_t + y_t)$. Therefore, the representative agent maximizes:

$$U = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left(a_i i_t + a_y y_t - \frac{b_i}{2} i_t^2 - \frac{b_y}{2} y_t^2 + d i_t y_t - p_t(i_{t+1} - i_t + y_t)\right)$$

where r is the per-period rate of interest and p_t is the price of the good in period t.

In each period t the agent chooses y_t and i_{t+1}. The first-order conditions are:

$$a_y - b_y y_t + d i_t - p_t = 0 \tag{29}$$

[7]Most notably Deaton and Laroque (1992, 1996) however the now accepted empirical regularity that prices behave sluggishly implies positive correlation over time.

[8]This production process may be nothing more than a wholesale stage of the product's distribution. A more in depth analysis where each of the various stages of a product's distribution are modelled is beyond the scope of the current analysis.

$$-p_t + \frac{1}{1+r}(a_i - b_i i_{t+1} + dE_t y_{t+1} + E_t p_{t+1}) = 0 \tag{30}$$

These first-order conditions can be taken without regard to expectations over future choices because of the envelope theorem. Now, solving (29) yields:

$$y_t = \frac{a_y}{b_y} + \frac{d}{b_y} i_t - \frac{1}{b_y} p_t \tag{31}$$

Shifting (31) forward one period and substituting into (30) yields:

$$-(1+r)p_t + a_i - b_i i_{t+1} + d\left(\frac{a_y}{b_y} + \frac{d}{b_y} i_{t+1} - \frac{1}{b_y} E_t p_{t+1}\right) + E_t p_{t+1} = 0 \tag{32}$$

As before, assume $E_t p_{t+1} = (1-\lambda)\bar{p} + \lambda p_t$. Using this to solve (32) yields:

$$i_{t+1} = \frac{a_i + da_y + (b_y - d)(1-\lambda)\bar{p}}{b_i b_y - d^2} - \frac{b_y(1+r) - \lambda(b_y - d)}{b_i b_y - d^2} p_t \tag{33}$$

From (33) and (31):

$$y_t = B_0 - \frac{1}{b_y} p_t - \frac{d(b_y(1+r) - \lambda(b_y - d))}{b_y(b_i b_y - d^2)} p_{t-1}$$

where B_0 is a constant. Total demand for the good, X_t, is therefore given by:

$$\begin{aligned} X_t &= i_{t+1} - i_t + y_t \\ &= \hat{a} - \hat{b} p_t + \hat{d} p_{t-1} \end{aligned} \tag{34}$$

where

$$\hat{b} = \Omega + \frac{1}{b_y}$$
$$\hat{d} = \Omega \frac{b_y - d}{b_y}$$
$$\Omega = \frac{b_y(1+r) - \lambda(b_y - d)}{b_i b_y - d^2}$$

It can then be shown that:

$$\frac{\hat{b}}{\hat{d}} = \frac{b_y}{b_y - d} + \frac{1}{\Omega(b_y - d)}$$

On the assumption that the revenue function is concave in y_t and i_{t+1}, Ω is positive (i.e., differentiate (32)). We cannot sign $b_y - d$ from concavity. However, it would seem this should be positive. To see this, note that the change in y_t with respect to i_t is d/b_y (see (31)). If this were bigger than one it would mean that an exogenous increase in inventories would lead to a larger increase in consumption of the intermediate good. This would seem implausible, so we assume $b_y - d > 0$. Note this implies that $\hat{b}/\hat{d} > 1$ and it is increasing in λ. Thus, this problem yields dynamic demand that has the same properties as the overlapping generations problem. Thus, nearly all of those results go through[9] when this infinite horizon consumer replaces the overlapping generations of consumers.

Note also that the autocorrelation in prices is positive here, even though inventories and consumption are complements. The reason is as follows: if the current price is low, the firm increases its inventories. Other things being equal (as long as the increase in y in the next period is smaller than the increase in i) this lowers purchases in the next period. Thus, current and future demand for the intermediate good are intertemporal substitutes.

4.4 Exchange Rate Passthrough

Suppose that firms sell the product in a foreign market at foreign price p_t and receive $p_t\eta_t$ in domestic currency. Because exchange rates enter the per period profit function multiplicatively, solving a model where they behave as a stochastic process becomes extremely difficult. However, one can model exchange rates as being more or less permanent which are occasionally subject to unanticipated changes. For example, although exchange rates are in most cases 'flexible,' they are often kept within a narrow, fixed band. This band might be occasionally be moved due to exogenous pressures. Thus assume that $\eta_t = \eta$ so that there is no uncertainty.

Solving the problem yields price and output dynamics, identical to those in Sections 3 and 4.2 with the exception that there are no error terms and the long run equilibrium price is given by

$$\bar{p} = \frac{\hat{a} - f}{2\hat{b} - (1+\beta)(\hat{d} - g)} + \frac{\hat{b} - \beta(\hat{d} - g)}{\eta(2\hat{b} - (1+\beta)(\hat{d} - g))}\bar{c}.$$

Now suppose that there is an unanticipated and permanent depreciation of the domestic

[9]The sole exception is the comparative static results with respect to $e = b/d$ as the b_y, b_i and d parameters are not strictly comparable to b and d.

currency (i.e., an increase from η to η'). This will result in a fall in the long run price at which firms are able to sell their product. To see the degree of pass-through, compute the elasticity of the long run price with respect to the exchange rate:

$$\frac{\partial \bar{p}}{\partial \eta}\frac{\eta}{\bar{p}} = \frac{-\dfrac{\hat{b}-\beta(\hat{d}-g)}{\eta(2\hat{b}-(1+\beta)(\hat{d}-g))}\bar{c}}{\dfrac{\hat{a}-f}{2\hat{b}-(1+\beta)(\hat{d}-g)} + \dfrac{\hat{b}-\beta(\hat{d}-g)}{\eta(2\hat{b}-(1+\beta)(\hat{d}-g))}\bar{c}} \tag{35}$$

This is less than one, in absolute value, and therefore we have the result that there is incomplete passthrough of exchange rate changes. In addition, we know from the dynamic process examined earlier that an unanticipated and permanent depreciation will be followed by a slow process of adjustment where the price falls from $\bar{p}$ to $\bar{p}'$. In particular, when current and future consumption are substitutes, this adjustment will be monotonic. When they are complements, this adjustment process will be one of overshooting so that after an unanticipated depreciation, price converges to $\bar{p}'$, with price alternately falling below $\bar{p}'$ and rising above $\bar{p}'$. Not only is there incomplete long run exchange rate passthrough but this passthrough is spread over many periods.

We can also consider the effect of an unanticipated temporary depreciation of the domestic currency. That is, suppose that in the tth period, the exchange rate rises from η to η' but returns to η forever after. By examining the period t first order condition and using the equilibrium price adjustment process ($p_{t+1} = (1-\lambda)\bar{p} + \lambda p_t$), it is easy to confirm the natural intuition that the period t price falls from $\bar{p}$ but not by as much as it would under a permanent depreciation. This difference in the effect of a permanent vs a temporary exchange rate shift is similar to that found in Froot and Klemperer (1989) but in addition, our model is fully dynamic and as a result, we show that even temporary shocks can have long lasting effects.

Under either permanent or temporary changes, we get incomplete exchange rate passthrough. With a permanent shift, the adjustment process for this incomplete passthrough is spread through time. With a temporary shift, there is a small immediate change in the price which then slowly returns to its steady state.

5 Related Literature

We examined a Markov perfect equilibrium of a simple dynamic model of oligopoly where consumer utility functions are not time-separable. That is, there are intertemporal linkages in the

utility that consumers get from consumption. With such linkages and i.i.d. shocks, equilibrium prices follow a first-order autoregressive stochastic process so that temporary shocks can have long lasting effects on prices. This generalizes to more complicated stochastic processes. Furthermore, price and output behave countercyclically in response to cost shocks and procyclically in response to demand shocks.

Models of 'customer markets' (Bils (1989)) or 'consumer switching costs' (Beggs and Klemperer (1992) and To (1996)) have some similarity to the current paper when there are intertemporal complementarities in consumption. With customer markets or consumer switching costs, once a consumer has purchased from a particular producer, they prefer to continue purchasing from the same producer. This brings about an intertemporal link where the higher is a firm's current sales, the greater is its future demand (i.e., there is intertemporal complementarity of consumption for the product of a particular firm). With the model of the current paper, different firms produce homogeneous products for which there is intertemporal complementarity in the consumption of any firm's product. However, since Bils (1989) assumes the industry is monopolistic, his model has a similar interpretation to the current one. In this regard, his countercyclical price and output result is dependent on the fact that the demand of the older generation is invariant to small increases in price. When the demand of the older generation is not perfectly inelastic, this result is reversed.

The evidence from a number of empirical studies show that prices tend to be slow to change (Blinder (1994), Carlton (1986), Cecchetti (1986), and Lach and Tsiddon (1992)). With intertemporal substitutability, the price process is similar to those generated by a number of models with imperfect price adjustment (e.g., adjustment costs, menu costs and price staggering). However, these models have a number of empirical problems. For example, like the other studies, Kashyap (1995) also finds evidence of sluggish price movements but in addition, argues that various theoretical explanations of sluggish price movements are inconsistent with his evidence from retail catalogs. Menu costs can explain sluggish price movements only if menu costs vary over time—both relatively small and relatively large price changes are frequently observed, implying either a small or a large menu cost. The fact that his data also shows prices can change with different magnitudes and with a large change quickly followed by another smaller change is also inconsistent with these theories. Furthermore, in his data, price changes did not occur at fixed intervals and as a result, staggered price models are ruled out.

Furthermore, models with staggered price adjustment, as in Taylor (1980), have two further criticisms. First, when firms prices are fixed for several periods, if firms can choose when to set their prices, the only stable Nash equilibrium occurs when they all choose to set prices at the same time (Fethke and Policano (1986)). Second, in price-setting periods, if firms can predetermine prices for T periods,[10] the effects of temporary shocks will only have price effects for exactly T periods (Fischer (1977)).

Finally, there is a significant body of empirical literature estimating dynamic demand equations where many goods are found to exhibit habit persistence in consumption (e.g., Blanciforti and Green (1988), Browning (1991) and Pollak and Wales (1969)). There is also evidence which suggests that the prices of many goods, including habit goods such as coffee and tea, exhibit positive autocorrelation (Deaton and Laroque (1992, 1996)). This would seemingly contradict our theory which predicts that for habit goods, prices should follow a negative autocorrelation process. What we would suggest is that perhaps prices and consumption of certain products are determined through separate processes. In particular, suppose that for these products there is a commodity market where commodity traders determine the prices at which commodities trade. Given these prices, consumers make their purchases. The fact that commodity traders (who are consumers of a sort) can store their purchases (possibly at some cost) for later resale (as in Deaton and Laroque (1992, 1996)) introduces intertemporal substitutability into the decision making process, resulting in positive autocorrelation in prices. Viewed in this way, the evidence that commodity prices are positively autocorrelated is not inconsistent with our theory of price formation. To this end, we consider an infinite horizon consumer who can hold inventories (Section 4.3) and show that our results under intertemporal substitutability extend to this case.

As with a number of the models discussed, ours is partial equilibrium and thus we can only look at the process of real price adjustment. Nevertheless, our approach has the advantage that we have a dynamic model of imperfect competition where all agents have rational expectations. We fully characterize the unique linear Markov perfect equilibrium. This equilibrium has the property that temporary shocks have long lasting effects.

[10] That is, when firms choose prices, they choose them for T periods, allowing prices to be different in different periods.

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