THE COURNOT-BERTRAND PROFIT DIFFERENTIAL: A REVERSAL RESULT IN A DIFFERENTIATED DUOPOLY WITH WAGE BARGAINING

Mónica Correa López

And

Robin A. Nayor

No 631

WARWICK ECONOMIC RESEARCH PAPERS

THE UNIVERSITY OF WARWICK

DEPARTMENT OF ECONOMICS

The Cournot-Bertrand Profit Differential: a reversal result in a differentiated duopoly with wage bargaining

Mónica Correa López

and

Robin A. Naylor

University of Warwick

October 5, 2001

Abstract

This paper compares Cournot and Bertrand equilibria in a downstream differentiated duopoly in which the input price (wage) paid by each downstream firm is the outcome of a strategic bargain with its upstream supplier (labour union). We show that the standard result that Cournot equilibrium profits exceed those under Bertrand competition - when the differentiated duopoly game is played in imperfect substitutes - is reversible. Whether equilibrium profits are higher under Cournot or Bertrand competition is shown to depend upon the nature of the upstream agents' preferences, on the distribution of bargaining power over the input price and on the degree of product market differentiation. We find that the standard result holds unless unions are both powerful and place considerable weight on the wage argument in their utility function. One implication of this is that if the upstream agents are profit-maximising firms, then the standard result will obtain.

Journal of Economic Literature Classification Numbers: D43, J50, L13.

Key Words: differentiated duopoly; wage bargaining; Cournot; Bertrand.

Corresponding Author:

Robin A. Naylor. Department of Economics, University of Warwick, Coventry CV4 7AL, UK. Tel. No.: +44 2476 523529. Fax No: +44 2476 523032. E-mail address: robin.naylor@warwick.ac.uk.

1. Introduction

A classic result in oligopoly theory is that firms will set quantities rather than prices when goods are imperfect substitutes and *vice versa* when they are imperfect complements. This result was first formalized by Singh and Vives (1984) and has been further refined by Vives (1985), who establishes more general results on the ranking of Cournot and Bertrand outcomes, by Okuguchi (1987) and, in a geometric analysis, by Cheng (1985). The result is a cornerstone of oligopoly theory.

Recently, the early results have attracted renewed interest. Dastidar (1997) shows that in a homogeneous product market the results are sensitive to the sharing rule and are not necessarily valid under asymmetric costs. In the standard model, costs are both symmetric and exogenous. Qiu (1997) develops a model of differentiated duopoly in which there is a two-stage game. In stage 1, each firm chooses a level of cost-reducing research and development (R&D) prior to the standard product market game played in a second stage. Qiu (1997) shows that the relative efficiency of Cournot and Bertrand competition depends upon R&D productivity, the extent of spillovers, and the degree of product market differentiation. Lambertini (1997) extends the standard analysis to the context of a repeated market game in which the firm’s choice of the strategic variable is itself the outcome of a strategic (meta) game. This game is also shown to be characterized by the prisoners’ dilemma. These results do not undermine the key results established by Singh and Vives (1984).

Häckner (2000) has shown that the result concerning the dominance of Cournot over Bertrand profits is sensitive to the duopoly assumption. Häckner (2000) considers an n-firm setting with vertical product differentiation. Our paper, like that of Häckner (2000), can be thought of as testing the robustness of the standard results with respect to alternative market structures. While Häckner (2000) extends the standard model *horizontally* through increasing the number of firms within the product market, our paper extends the analysis *vertically* by examining the consequences of introducing upstream suppliers to the downstream duopolists.

In particular, we address the issue of whether the standard results on the ranking of Cournot and Bertrand equilibrium outcomes under differentiated duopoly are robust to the inclusion of a decentralized wage-bargaining game played between each firm and a firm-specific labour union. There is symmetry across the two union-firm wage bargains. Hence, in equilibrium, we retain the property of symmetric costs, typically assumed in the standard model. As in Qiu (1997) – though for very different reasons – these costs, however, are no longer exogenous in our model. Instead, in the model we develop here, they are the outcome of a strategic game played between each firm and its labour union. This can be interpreted as a particular example of a more general situation of bargaining between an upstream supplier and a downstream retailer in the context of oligopoly in the retail market. The structure of our model is similar to that of Qiu (1997), with wage bargaining rather than R&D choice in the first stage of the game. In stage 2, we consider both Cournot and Bertrand solutions to the non-cooperative product market game.

Our analysis of the Cournot solution is closely related to the model of Horn and Wolinsky (1988), which analyses the incentives to merge among upstream and downstream firms, and how these incentives depend on the degree of product differentiation. The model of Horn and Wolinsky (1988) builds on the concept of strategic substitutes and complements developed by Bulow *et al.* (1985). The analysis of wage determination in unionized oligopolies was first developed by Davidson (1988), who focused on a comparison of local and national bargaining and, like Horn and Wolinsky (1988), adopted the standard Cournot-Nash assumption to describe product market competition. A somewhat more generalized approach to wage setting in the context of imperfect competition in both labour and product markets is described by Dowrick (1988). Similarly, Naylor (1998) and (1999) considers unionized duopoly in the context of international trade and economic integration, and again assumes Cournot behavior in the product market. Grandner (2001) examines the importance of the level of bargaining for consumer prices in vertically connected industries under oligopoly.

The rest of this paper is organized as follows. In Section 2, we present the basic model in which two firms compete in the product market having first bargained independently over wages with a local (firm-specific) labour union. The two firms produce differentiated products. The product market is assumed to be characterized by Cournot competition. We derive sub-game perfect Nash equilibrium values for the key variables of interest. Section 3 presents the corresponding equilibrium values for the case of Bertrand competition in the product market. In Section 4, we compare Cournot and Bertrand equilibrium profits. We show that the standard result that profits are higher under Cournot than under Bertrand competition – in the case of imperfect substitutes – is reversed under certain assumptions regarding the extent of product differentiation, union preferences and bargaining power. In Section 5, we explore the underlying reasons for the reversibility of the standard result. Section 6 closes the paper with conclusions and further remarks.

2. Cournot equilibrium under unionized duopoly

The model of the differentiated product market duopoly follows Singh and Vives (1984) and Qiu (1997). We analyze a non-cooperative two-stage game in which two firms produce differentiated goods. In the first stage (the labour market game), each firm independently bargains over its wage with a local labour union. That is, bargaining is decentralized. The outcome of the labour market game is described by the solution to the two union-firm pairs' sub-game perfect best-reply functions in wages. In the second stage (the product market game), each firm sets its output – given pre-determined wage choices from stage 1 – to maximize profits. In Section 3 of the paper, we consider *price*-competition in the product market.

Preferences of the representative consumer are given by:

$$U(q_i, q_j) = a(q_i + q_j) - (q_i{}^2 + 2cq_i q_j + q_j{}^2)/2 \,,$$

where q_i, q_j denote outputs by firm *i* and *j*, respectively, $a > 0$, and $c \in [0,1]$ denotes the extent of product differentiation with goods assumed to be imperfect substitutes. The derived product market demands are linear and given by:

$$p_i(q_i, q_j) = a - cq_j - q_i \;; \tag{1}$$

$$p_j(q_i, q_j) = a - cq_i - q_j \,. \tag{2}$$

In the standard model, the two firms face the same constant marginal cost, w. Qiu (1997) considers the case in which the firm can influence its marginal cost through R&D expenditure. In the current paper, we assume that the constant marginal cost is the result of a decentralized stage 1 bargain with a local union. We assume that the two firms have the same technology and that the two firm-specific labour unions have the same preferences and the same bargaining power over wages. In symmetric equilibrium, therefore, the two firms will have identical marginal costs: although these will be the outcome of strategic play across the two union-firm pairs.

Profits of firm *i* can be written as:

$$p_i = (p_i - w_i)q_i \, , \tag{3}$$

where w_i denotes the wage paid by firm *i* and is assumed to capture all short-run marginal costs. Under the assumption of a constant marginal product of labour, normalized to unity, q_i represents both output and employment of firm *i*.

Substituting (1) in (3) and differentiating with respect to q_i yields the first-order condition for profit maximization by firm *i*, from which it is straightforward to derive firm *i*'s best-reply function in output space as:

$$q_i(q_j) = \frac{1}{2}(a - cq_j - w_i) \, . \tag{4}$$

Similarly, firm *j*'s best-reply function is given by:

$$q_j(q_i) = \frac{1}{2}(a - cq_i - w_j) \, . \tag{5}$$

As $c > 0$, by assumption, the best-reply functions are downward-sloping: under the Cournot assumption, the product market game is played in strategic substitutes, as is well known.

Eqs. (4) and (5) can be re-written such that each firm's output is a function of the two firms' pre-determined wage levels. Thus,

$$q_i(w_i, w_j) = \frac{1}{4 - c^2}[(2 - c)a - 2w_i + cw_j] \tag{6}$$

and

$$q_j(w_i, w_j) = \frac{1}{4 - c^2}[(2 - c)a - 2w_j + cw_i] \, . \tag{7}$$

where (6) and (7) represent labour demands by firm *i* and *j*, respectively, for given w_i and w_j. These are the derived labour demand functions which will be anticipated by union-firm wage-bargaining pairs in the stage 1 labour market game.

Substituting (1), (6) and (7) in (3) gives firm *i*'s Cournot-Nash equilibrium profits, given w_i, w_j, as:

$$p_i(w_i, w_j) = \frac{1}{(4-c^2)^2}[(2-c)a - 2w_i + cw_j]^2. \qquad (8)$$

We now consider two alternative cases. In Regime 1, wages are exogenously determined and set at the reservation wage level, $\overline{w}$. In Regime 2, wages are set endogenously through decentralized bargaining in the non-cooperative Stage 1 labour market game.

(i) Regime 1: exogenous wages

Assume that, in the absence of labour unions, $w_i = w_j = \overline{w}$. Then symmetric equilibrium Cournot-Nash profits are given by:

$$p^C = \frac{1}{(2+c)^2}(a - \overline{w})^2. \qquad (9)$$

(ii) Regime 2: endogenous wages

Assume that, in Stage 1, firm *i* bargains over the wage, w_i, with a local labour union, union *i*, which has preferences over wages and employment captured by the union utility function

$$u_i(w_i, q_i) = (w_i - \overline{w})^q q_i^{1-q}, \qquad (10)$$

where q denotes the relative strength of the union's preference for wages over employment and $0 \le q < 1$. We rule out the special case of wage-maximization, $q = 1$.[1] This functional form is quite general and encompasses common assumptions such as rent-maximization, arising when $q = 1/2$.

The general asymmetric Nash bargain over wages between union-firm pair *i*, for example, solves

$$w_i = \arg\max\{B_i = u_i{}^b p_i{}^{1-b}\}, \tag{11}$$

where b is the union's Nash bargaining parameter and $0 \le b \le 1$. The union and firm bargain over wages only in the two-stage sequential game: the firm is assumed to have the right-to-manage autonomy over employment.

Substituting (6), (8) and (10) into (11) yields:

$$B_i = \left[\frac{1}{4-c^2}\right]^{2-b(1+q)} \left(w_i - \overline{w}\right)^{bq}[(2-c)a - 2w_i + cw_j]^{2-b(1+q)}, \tag{12}$$

where disagreement payoffs are assumed to be zero. Substituting (12) in (11) and solving, gives a first-order condition which is satisfied when:

$$w_i^C = \overline{w} + \left[\frac{1}{2(2-b)}\right]\left[bq(2-c)(a-\overline{w}) + cbq(w_j - \overline{w})\right], \tag{13}$$

which defines the sub-game perfect best-reply function in wages, ${}^L R_i^C$, of union-firm pair *i* under the assumption of a non-cooperative Cournot-Nash equilibrium in the product market. Differentiating (13) with respect to w_j gives the slope of union-firm pair *i*'s best-reply function in wage-space as:

[1] By ruling out $q = 1$, we avoid the problem of the 'Cheshire cat' monopoly union which sets such a high wage that employment collapses to zero.

$$\frac{\partial w_i^C}{\partial w_j} = \frac{cbq}{2(2-b)} \,. \tag{14}$$

The slope of the best-reply wage function is positive for $c > 0$, $q > 0$, $b > 0$, confirming that the labour market game is played in strategic complements. We also have that the slope is increasing both in q, the relative weight the union attaches to wages in its objective function, and in b, the relative bargaining power of the union in the wage bargain.

In symmetric sub-game perfect equilibrium, $w_i = w_j$ and hence, from (13), equilibrium wages are given by:

$$w_i^C = w_j^C = \overline{w} + \left[\frac{(2-c)bq}{2(2-b)-cbq}\right](a-\overline{w}) = w^C, \tag{15}$$

where w^C signifies that the product market game in stage 2 is described by the non-cooperative Cournot-Nash outcome. Note that $w^C = \overline{w}$ if either $q = 0$ or $b = 0$.

Substituting (15) into the expression for Cournot equilibrium profits, given in (8), we derive sub-game perfect equilibrium profits as

$$p^C = \left[\frac{2}{2(2-b)-cbq}\right]^2 \left(\frac{a-\overline{w}}{2+c}\right)^2 [2-b(1+q)]^2, \tag{16}$$

from which it is readily checked that (16) collapses to (9), replicating the non-union benchmark outcome, for the special case in which $b = 0$.

3. Bertrand equilibrium under unionized duopoly

In this section of the paper, we suppose that the product market game in stage 2 is characterized by price-setting behavior by firms. From (1) and (2), we can write product demand facing firm *i* as:

$$q_i(p_i, p_j) = \frac{1}{1-c^2}[a(1-c) - p_i + cp_j]\,. \tag{17}$$

Similarly, we can re-define profits of firm *i* as a function of prices:

$$\pi_i(p_i, p_j) = \frac{1}{1-c^2}[a(1-c) - p_i + cp_j](p_i - w_i)\,. \tag{18}$$

As in the previous section of the paper, we proceed to solve the 2-stage game by backward induction. In stage 2, firms choose price to maximize profits. From (18), the first-order condition for profit-maximization gives:

$$p_i(p_j) = \frac{1}{2}[a(1-c) + cp_j + w_i]\,, \tag{19}$$

and hence the best-reply function is positive for $c > 0$: the Bertrand product market game is played in strategic substitutes. From (19), and its counterpart for firm *j*, each firm's price can be written as functions of the two wage levels pre-determined in stage 1.

Substituting the resulting prices into (17) yields:

$$q_i(w_i, w_j) = \frac{1}{(4-c^2)(1-c^2)}[(2+c)(1-c)a + cw_j - (2-c^2)w_i]\,, \tag{20}$$

which can be interpreted as the sub-game perfect labour demand function facing union *i* in the stage 1 wage-bargaining game and is the Bertrand equivalent to (6), derived in the foregoing analysis of Cournot competition in the product market. Substitution yields the expression for firm *i*'s profits as:

$$\pi_i(w_i, w_j) = \frac{1}{[4-c^2]^2[1-c^2]}[(2+c)(1-c)a + cw_j - (2-c^2)w_i]^2\,, \tag{21}$$

which is the Bertrand equivalent to (8) for the case of Cournot competition.

As in Section 2, we now distinguish between 2 possible labour market regimes. In the benchmark case, wages are determined exogenously, while in the second case wages are the result of a strategic bargain between each firm and its labour union.

(i) Regime 1: exogenous wages

Assume that, in the absence of labour unions, $w_i = w_j = \overline{w}$. Then symmetric equilibrium Bertrand-Nash profits are given by:

$$p^B = \frac{1-c}{(2-c)^2(1+c)}(a-\overline{w})^2. \tag{22}$$

In the standard model of differentiated duopoly, with marginal costs (wages) determined exogenously, the relation between Cournot and Bertrand profits is based on a comparison of (22) and (9). It is easily demonstrated that in this non-union case, the sign of $(p^C - p^B)$ is equal to the sign of c. Hence, if firms produce imperfect substitutes, $c > 0$, Cournot profits will exceed Bertrand profits in equilibrium. Accordingly, firms would prefer Cournot to Bertrand competition. We now derive the expression for sub-game perfect Bertrand equilibrium profits when wages are subject to bargaining.

(ii) Regime 2: endogenous wages

As in Section 2, we assume that there is an independent wage bargain between each firm and its labour union. Union preferences are given by (10) and the Nash maximand is represented by (11). Substituting (20), (21) and (10) in (11) and solving produces a first-order condition for the Nash maximand that is satisfied when:

$$w_i^B = \overline{w} + \left[\frac{1}{(2-c^2)(2-\boldsymbol{b})}\right][\boldsymbol{bq}(2+c)(1-c)(a-\overline{w}) + c\boldsymbol{bq}(w_j - \overline{w})], \tag{23}$$

which defines the sub-game perfect best-reply function in wages, ${}^L R_i^B$, of union-firm pair i under the assumption of a non-cooperative Bertrand-Nash equilibrium in the product market. Eq. (23) is the Bertrand counterpart of (13) for the case of Cournot competition in the product market. Differentiating (23) with respect to w_j gives the slope of union-firm pair i's best-reply function in wage-space as:

$$\frac{\partial w_i^B}{\partial w_j} = \frac{cbq}{(2-c^2)(2-b)}. \qquad (24)$$

The slope of the best-reply wage function is again positive for $c > 0$, $q > 0$, $b > 0$, confirming that the labour market game is played in strategic complements, independent of the type of product market competition. Again, we note that the slope of the best-reply function in wages is increasing both in q and in b.

In symmetric equilibrium, $w_i = w_j$ and hence, from (24), sub-game perfect equilibrium wages are given by:

$$w_i^B = w_j^B = \overline{w} + \left[\frac{(2+c)(1-c)bq}{(2-c^2)(2-b)-cbq}\right](a-\overline{w}) = w^B, \qquad (25)$$

where w^B signifies that the product market game in stage 2 is described by the non-cooperative Bertrand-Nash outcome. Note that $w^B = \overline{w}$ if either $q = 0$ or $b = 0$.

Substituting (25) into the expression for Bertrand equilibrium profits, given in (21), we derive sub-game perfect equilibrium profits as:

$$p^B = \frac{1-c}{(2-c)^2(1+c)}\left\{\frac{(2-c^2)[2-b(1+q)]}{(2-c^2)(2-b)-cbq}\right\}^2 (a-\overline{w})^2. \qquad (26)$$

We note that (26) replicates (22) for the non-union benchmark case in which $b = 0$. We are now in a position to compare Cournot and Bertrand profits for the case in which wages are determined by decentralized bargaining in stage 1.

4. The Cournot-Bertrand profit differential

Comparison of (16) and (26) yields the expression for the Cournot-Bertrand profit differential, F:

$$F = \left[\frac{1}{(2+c)^2}\left[\frac{2}{2(2-b)-cbq}\right]^2 - \frac{1-c}{(2-c)^2(1+c)}\left[\frac{2-c^2}{(2-c^2)(2-b)-cbq}\right]^2\right]A, \qquad (27)$$

where $A = [2 - b(1+q)]^2 (a - \overline{w})^2 > 0$.

$F = 0$ defines the surface, in $(q, c, b) -$ space, along which the profit differential is zero. For the firms, this can be thought of as an iso-profit or 'indifference surface'. In order to examine the properties of this surface, we first consider cross-sections of the surface in $(b, q) -$ space produced at given values along the c dimension. This yields 'indifference curves' in $(b, q) -$ space, each drawn for given c.

When $F = 0$, it follows that the term in brackets acting on A in (27) must also be equal to zero, as A itself is strictly positive under the assumptions of the model. It is then easily shown that $F = 0$ implies that:

$$q = \frac{2(2-b)}{cb}\hat{A}, \tag{28}$$

where $\hat{A}$ can be defined by:

$$\hat{A} = \frac{y - x}{ey - x},$$

and where

$$y = (2-c)(1+c)^{1/2}; \quad x = (2+c)(1-c)^{1/2}$$

and

$$e = \frac{2}{2-c^2}. \tag{29}$$

The sign of F is given by the sign of $\left[q - \frac{2(2-b)}{cb}\hat{A}\right]$, from (28).

Eq. (29) implies that $e > 1, \ \forall \ c : 0 < c < 1$. From this it follows that for $\hat{A}$ to be strictly positive, $\hat{A} > 0$, it is sufficient that $y - x > 0$. It is easily shown that $y > x \ \ \forall \ c > 0$. Hence, from $e > 1$ and $y > x$ it follows that $0 < \hat{A} < 1, \forall \ c : 0 < c < 1$.

Eq. (28) defines the firms' indifference curve in $(b, q) -$ space, for given c. Differentiation of (28) gives the slope of the indifference curve as:

$$\frac{\partial q}{\partial b} = -\frac{4}{cb^2}\hat{A} < 0 \,. \qquad (30)$$

As $\hat{A} > 0$, it follows from (30) that the indifference curve has a negative slope in $(b, q)-$ space. The key question is whether this indifference curve cuts through the unit-square in $(b, q)-$ space, for $0 < c < 1$. If it does so, then the indifference surface, defined by F in (27) taking the value zero, must cut through the unit-cube in $(q, c, b)-$ space. Our strategy for addressing this question is to show that, for $\forall\, c$ such that $0 < c < 1$, $\exists\; q < 1$ such that $F = 0$ holds for at least some particular value of b satisfying $0 < b \le 1$. For simplicity, we shall consider the case in which $b = 1$.

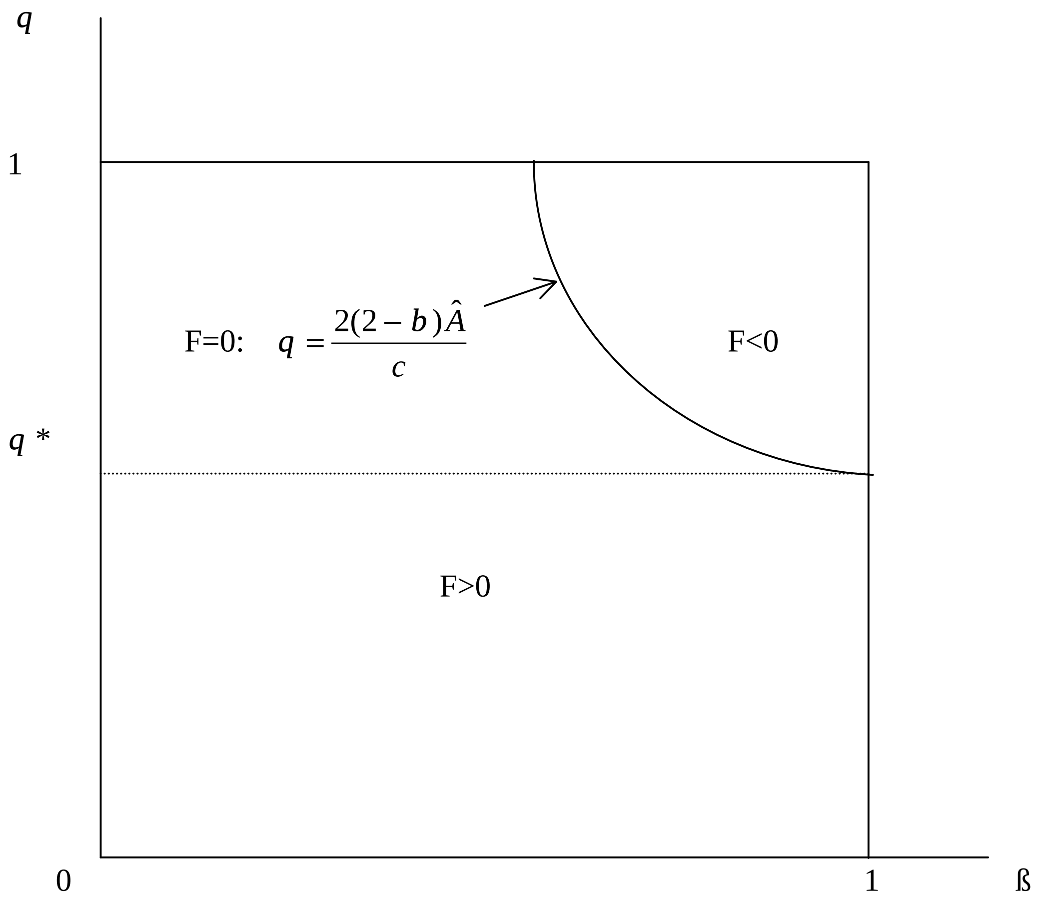

FIGURE 1 The firms' indifference curve in $(b, q)-$ space.

Consider the point on the indifference curve defined by (28) for the particular value, $b = 1$. Fig. 1 depicts an indifference curve in $(b, q)-$ space. When $b = 1$, define q to take the value $q = q^*$. As the indifference curve is downward-sloping, our strategy requires that we show that $0 < q^* < 1$, for $F = 0$ and $0 < c < 1$. From equation (28), this condition becomes, for $0 < c < 1$:

$$0 < \frac{2(2-b)}{cb}\hat{A} < 1 \tag{31}$$

or, substituting in the value of $b = 1$,

$$0 < \frac{2\hat{A}}{c} < 1. \tag{32}$$

As $\hat{A} > 0$, it follows that $2\hat{A}/c > 0$ is satisfied for $c > 0$: the condition for the lower limit in (32) is satisfied. The full condition given by (32) is then satisfied if $c > 2\hat{A}$. This can be shown to be satisfied $\forall\ c$ (see Appendix). Hence it follows that $0 < q^* < 1$, for $F = 0$, $b = 1$ and $0 < c < 1$. Thus, the indifference curve defined by eq. (28) cuts through the unit-square in $(b, q)-$ space. Consequently, the indifference surface, defined by imposing $F = 0$ in (27), cuts through the unit-cube in $(q, c, b)-$ space. The surface is represented in the unit-cube depicted in Fig. 2, drawn from plotting equation (27), for $F = 0$, using the Mathematica program (Wolfram (1999)).

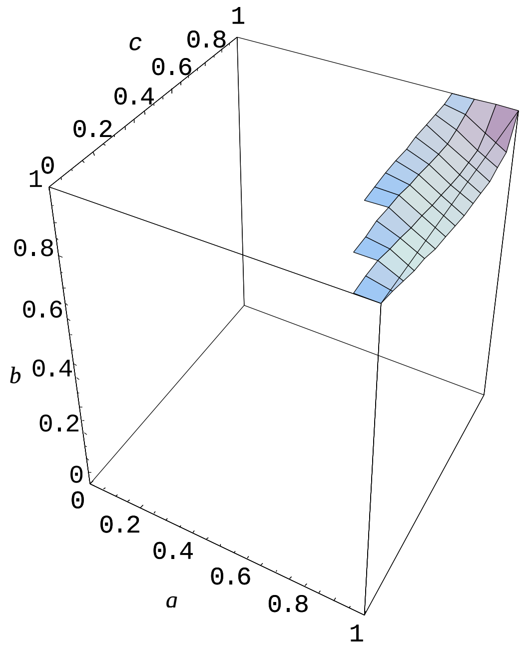

FIGURE 2 The firms' indifference surface, *Surface[* $F = 0$ *]*, in $(q \ , c \ , b \ ) -$ space.

From the numerical solutions represented in Fig. 2 it follows that *Surface[* $F = 0$ *]*, in $(q \ , c \ , b \ ) -$ space, initially falls as c increases and then rises as c increases further. It can be shown numerically that the turning point or critical value in terms of c occurs when *Surface[* $F = 0$ *]* is cross-sectioned at $c = 0.786131$. Numerical analysis carried out in Mathematica (Wolfram (1999)) also confirms the algebraic results that $F > 0$ at any point below *Surface[* $F = 0$ *]* and that F is negative at any point above it. Similarly analysis in Mathematica confirms the slope of the indifference curve, defined in (28) and represented in Fig.1, and confirms that F is positive below and to the left of the indifference curve and is negative above and to the right of the curve. Our results, both analytical and numerical, establish Proposition 1.

PROPOSITION 1. *In sub-game perfect equilibrium, Bertrand profits exceed Cournot profits in the case of imperfect substitutes, for sufficiently high values of* b *and* q. *In other words, the standard result on the ranking of Cournot and Bertrand profits is reversed when upstream suppliers (labour unions) have sufficient bargaining power and place sufficient weight on wages in their utility functions. The threshold levels of* b *and* q *depend on the extent of product differentiation, measured by* c.

As stated in Proposition 1, the standard result concerning the ranking of Cournot and Bertrand profits under duopoly, when products are imperfect substitutes, is reversed only when unions are both relatively powerful in the wage bargain and attach relatively high importance to wages in their objective functions. It follows that under symmetric Nash bargaining, for example, the reversal result does not obtain. Similarly, the standard profit-ranking will not be reversed if unions are simple rent-maximizers, attaching equal weight to wages and employment. A corollary of this is that if upstream agents are profit-maximising firms, then the standard result will obtain: rent-maximising by the union is formally equivalent to profit-maximising by an upstream firm.

The reversal result begs the question as to why the introduction of a stage 1 wage bargaining game into the standard analysis might change the standard ranking of Cournot and Bertrand profits. This issue is the focus of the next section of the paper.

5. Wages under Cournot and Bertrand competition

In the previous section of the paper, we demonstrated the key property of the model that the classic duopoly result regarding the superiority in the level of Cournot over Bertrand profits is overturned when wages are the result of bargaining, for sufficiently high values of b and q, depending upon the value of c. In this Section of the paper, we examine why this reversal result obtains, and why the result depends upon the values of b and q.

Comparison of (9) and (22) demonstrates the superiority of Cournot over Bertrand profits when goods are imperfect substitutes in the absence of unions. One implication of this is that there is more surplus available for potential unions to attempt to capture under Cournot competition. Intuitively, this makes the prospects for rent capture by unions greater in the case of Cournot than Bertrand competition. But this in itself would not explain the profit reversal result. To explain the reversal, we establish two key analytical results. First, we show that SPNE bargained wages are indeed higher under Cournot than under Bertrand product market competition: unions influence wages more aggressively in the case of Cournot competition. In other words, duopolists face higher pre-competition marginal costs under Cournot. We show that the reason that unions bargain higher wages under Cournot competition lies in the fact that product – and hence labour – demands are less sensitive to wages under the Cournot product market regime and hence this induces unions to bargain for correspondingly higher wages. Second, we show that for any given level of wages, Cournot equilibrium profits decrease more steeply in wages than do Bertrand profits. Hence, it follows that if unions are sufficiently powerful and place sufficient weight on wages, then their capacity to bargain higher wages under Cournot than under Bertrand competition can overturn the standard result in differentiated duopoly with exogenous costs.

PROPOSITION 2. *Sub-game perfect Nash equilibrium wages are higher under Cournot than under Bertrand product market competition,* $\forall\ c > 0$.

Proof. From comparison of (16) and (25), it follows that:

$$\Delta^W = w^C - w^B = [(2-c)[(2-c^2)(2-b)-cbq] - (2+c)(1-c)[2(2-b)-cbq]]\tilde{A}, \qquad (34)$$

where $\tilde{A} = \dfrac{bq(a-\overline{w})}{[2(2-b)-cbq][(2-c^2)(2-b)-cbq]} > 0$, $\forall \; w > \overline{w}$. From (34), it follows that sign [Δ^W] = sign [$c^3(2-b(1+q))$]. The latter is positive $\forall \; c > 0$ and hence $w^C > w^B$, which establishes the proposition.

Fig. 3 represents the result in a diagram. Although the best-reply wage functions are steeper under Bertrand competition, the Cournot function for union-firm pair *i* lies vertically above the Bertrand counterpart for all possible equilibrium wage levels.

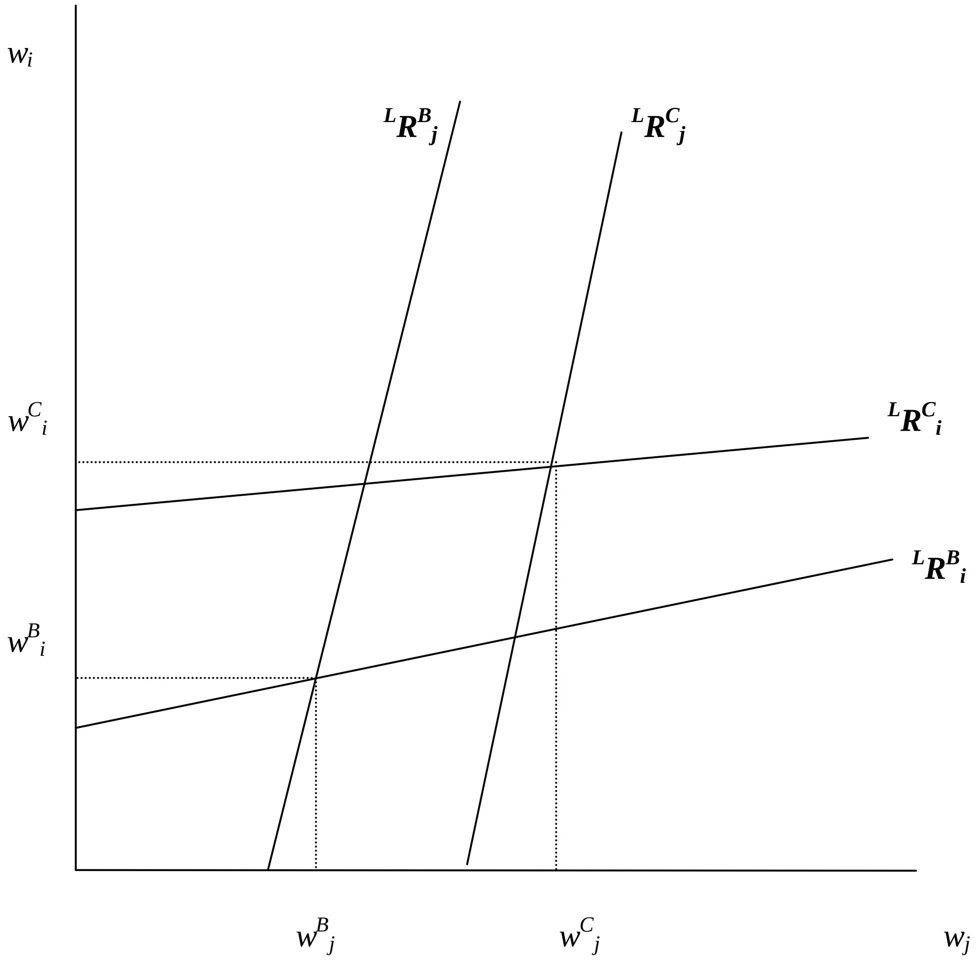

FIGURE 3 SPNE best-reply wage functions for Cournot and Bertrand cases.

The result stated in Proposition 2 is best understood through consideration of the first order conditions solving the wage bargaining problem under the two types of competition. First consider a more general functional form approach. The standard first order condition of the RTM (right-to-manage) model for union-firm pair *i* is given by:

$$\frac{b\ \dfrac{\partial U_i}{\partial w_i}}{U_i} = -\frac{(1-b)\dfrac{\partial \Pi_i}{\partial w_i}}{\Pi_i} \tag{35}$$

where (35) is the solution to (11). Next, we derive the corresponding versions of (35) for both types of product market competition (see Appendix for a complete derivation).

Under Cournot competition, the change in firm *i*'s profitability induced by a wage increase can be decomposed in two effects,

$$\begin{aligned}\frac{\partial \Pi_i^C}{\partial w_i} &= \left(\frac{\partial p_i}{\partial q_j}\frac{\partial q_j}{\partial q_i}\frac{\partial l_i}{\partial w_i}\right) q_i \quad - \quad q_i \\ &\qquad\quad (-) \qquad\qquad\quad - \quad (+) \qquad . \\ &= \quad \textit{strategic effect} \quad - \quad \textit{size effect}\end{aligned} \tag{36}$$

Hence, under Cournot competition the strategic effect on profits induced by a wage increase is strictly negative. The underlying intuition behind the negative strategic effect depends upon the adjustment of the Cournot firm to an exogenous change in marginal costs. In other words, a unit increase in w_i expands firm *j*'s output which in turn induces firm *i* to reduce its price and, hence, total revenue. On the other hand, the size effect captures the negative effect on profits associated with the total costs of producing q_i units after a unit increase in the wage rate.

The Cournot version of (35) can be re-written as follows:

$$\frac{b\,\theta\,w_i}{w_i-\overline{w}} = b\,(1-\theta)\left|\eta_i^C\right| - \frac{(1-b)\left(\dfrac{\partial p_i}{\partial q_j}\dfrac{\partial q_j}{\partial q_i}\dfrac{\partial l_i}{\partial w_i}\right)w_i}{p_i-w_i} + \frac{(1-b)\,w_i}{p_i-w_i} \tag{37}$$

where $\left|h_i^C\right|$ denotes the absolute value of the labour demand wage elasticity when competition in the product market is of the Cournot-type. The LHS of (37) captures the positive effects to the union derived from a wage increase and the RHS represents the negative effects felt by both the union and the firm. Hence, the wage is agreed at the level at which the proportional marginal benefit of a unit increase in the wage rate obtained by the union equals the proportional marginal costs incurred by both parties. Each effect is weighted by the corresponding party's bargaining strength.

Correspondingly, under Bertrand competition firm *i*'s marginal profit from a wage increase can be also decomposed into two effects,

$$\begin{aligned}\frac{\partial \Pi_i^B}{\partial w_i} &= \left(\frac{\partial q_i}{\partial p_j}\frac{\partial p_j}{\partial p_i}\frac{\partial p_i}{\partial w_i}\right)(p_i - w_i) - q_i \\ &\qquad\qquad (+) \qquad\qquad\qquad - \ (+) \\ &= \quad strategic \quad effect \qquad - size \quad effect\end{aligned} \tag{38}$$

Under Bertrand competition the strategic effect is strictly positive. This is due to the way in which firm *i* reacts to the wage increase: a unit increase in w_i leads firm *i* to increase its price, after which, firm *j* follows by increasing its own. The latter is transmitted to an expansion of firm *i*'s output. The increased output multiplied by the price mark-up raises total revenue. This is an important feature of the Bertrand competitor. Thus, the strategic effects are of opposite sign if we compare Cournot and Bertrand perceptions. Qiu (1997) also found an opposite sign in terms of R&D activity. Finally, the size effect again captures the negative effect on profits associated with the total costs of producing q_i units following a unit increase in the wage rate.

The corresponding Bertrand version of (35) can be re-written as follows:

$$\frac{bq\,\dfrac{d\,u(w_i)}{d\,w_i}\,w_i}{u(w_i)-u(\overline{w})}+\frac{(1-b)\left(\dfrac{\partial q_i}{\partial p_j}\dfrac{\partial p_j}{\partial p_i}\dfrac{\partial p_i}{\partial w_i}\right)w_i}{l_i}=b(1-q)\left|h_i^B\right|+\frac{(1-b)w_i}{p_i-w_i} \tag{39}$$

where $\left|h_i^B\right|$ denotes the absolute value of the labour demand wage elasticity when competition in the product market is of the Bertrand-type. The LHS of (39) captures the positive effects to both parties from a wage increase and the RHS captures the negative effects. In other words, the wage is agreed at the level in which the proportional marginal benefit from a unit increase in the wage rate obtained by *both* parties equals the proportional marginal costs incurred by *both* parties: weighted by the corresponding party's bargaining strength. The most important and distinctive feature arising from the comparison of (37) and (39) is that firm *i* perceives a proportional marginal benefit from a wage increase when product market competition is Bertrand whereas it perceives a proportional marginal cost when competition is Cournot. This arises from the fact that the strategic effects are of opposite sign. The above derivation yields Proposition 3.

PROPOSITION 3. *The sub-game perfect labour demand schedule derived under Bertrand competition in the product market is more elastic than the sub-game perfect labour demand schedule derived under Cournot competition.*

Proof: Evaluating $\left|h_i^B\right|$ and $\left|h_i^C\right|$ from (20) and (6) respectively yields the following comparison,

$$\left|h_i^B\right| - \left|h_i^C\right| > 0 \Leftrightarrow$$

$$\frac{(2-c^2)}{(1-c^2)(4-c^2)} \frac{w_i(1-c^2)(4-c^2)}{[(1-c)(2+c)a-(2-c^2)w_i+cw_j]} - \frac{2}{(4-c^2)} \frac{w_i(4-c^2)}{[(2-c)a+cw_j-2w_i]} > 0 \Leftrightarrow$$

$$(2-c^2)[(2-c)a+cw_j-2w_i] > 2[(1-c)(2+c)a-(2-c^2)w_i+cw_j] \Leftrightarrow$$

$$c^3\,(a-w_j) > 0 \quad \forall c > 0.$$

Since $c^3\,(a-w_j)$ is strictly positive $\forall c > 0$, it follows that $\left|h_i^B\right| - \left|h_i^C\right| > 0$.

Proposition 3 implies that a percentage increase in the wage rate will induce a higher percentage reduction in employment under Bertrand than under Cournot competition. The union perceives this difference as a higher proportional marginal cost for a given wage increase when bargaining with a Bertrand-type firm. Thus, union *i* has a stronger incentive to settle for a lower bargained wage rate when facing a Bertrand-type competitor in the product market. This fact explains why the SPNE bargained wage rate is always lower under Bertrand competition than under Cournot competition, despite the fact that the Bertrand firm perceives positive marginal benefits from wage increases, as shown in (38) and (39).

Having established the ranking of SPNE wage rates under both types of competition we now turn to consider the relative sensitivities of Cournot and Bertrand profits to the SPNE bargained wage levels.

PROPOSITION 4. *A unit increase in the wage rate reduces equilibrium profits for both types of product market competition. For any given level of wages, Cournot equilibrium profits decrease more steeply in wages than do Bertrand equilibrium profits.*

Proof: It follows from (9) and (22) that for any given level of wages:

$$p^C = \frac{1}{(2+c)^2}(a-w)^2; \quad p^B = \frac{1-c}{(2-c)^2(1+c)}(a-w)^2.$$

Differentiating with respect to the wage rate yields:

$$\frac{\partial p^C}{\partial w} = -\frac{2}{(2+c)^2}(a-w) \quad < 0 \quad \forall c\ ; \tag{40}$$

$$\frac{\partial p^B}{\partial w} = -\frac{2(1-c)}{(2-c)^2(1+c)}(a-w) \ < 0 \quad \forall c\ . \tag{41}$$

The comparison of (40) and (41) establishes Proposition 4:

$$\frac{\partial p^C}{\partial w} - \frac{\partial p^B}{\partial w} < 0 \Leftrightarrow -\frac{2(a-w)}{(2+c)^2} + \frac{2(1-c)(a-w)}{(2-c)^2(1+c)} < 0 \Leftrightarrow$$

$$\frac{2(a-w)\{-(2-c)^2(1+c)+(1-c)(2+c)^2\}}{(2+c)^2(2-c)^2(1+c)} < 0 \quad \forall c > 0.$$

It is straightforward to demonstrate that $\{-(2-c)^2(1+c)+(1-c)(2+c)^2\} < 0 \ \ \forall c > 0$, from which it follows that $\frac{\partial p^C}{\partial w} - \frac{\partial p^B}{\partial w} < 0$.

Proposition 4 confirms that the negative size effect dominates the positive strategic effect in determining the sign of the marginal profitability from a wage increase of the Bertrand competitor. Moreover, the proposition also demonstrates that Cournot profits are the more adversely affected by wage increases. This is due to the negative strategic effect coupled with the negative size effect induced by a wage increase.

In summary, the essential intuition for the relative profit reversal result is that under Cournot competition unions bargain a higher wage level than under Bertrand competition and that, furthermore, equilibrium Cournot profits are more sensitive to the level of the bargained wage than are Bertrand profits.

6. Conclusions and further remarks

In this paper, we have considered the standard model of differentiated duopoly in which it is well-known that Cournot equilibrium profits are higher than those associated with Bertrand equilibrium when firms produce imperfect substitutes. In the standard model, costs are assumed to be determined exogenously. We have examined the situation in which costs (wages) are determined through a process of decentralized bargaining between each firm and its upstream supplier (labour union). We have found that, under certain conditions, the relative magnitude of Cournot and Bertrand profits is reversed when we allow for bargaining over costs. Specifically, if unions are sufficiently powerful and care enough about wages in their utility function, then Bertrand profits exceed Cournot profits in sub-game perfect Nash equilibrium when goods are (imperfect) substitutes. The key intuition behind this result is that labour demand is less responsive to a change in wages under Cournot competition and this leads unions to bargain for higher wages than when competition in the product market is of the Bertrand type. Furthermore, Cournot profits fall more steeply than do Bertrand profits following any given increase in wages. If unions care sufficiently about wages and are strong enough to influence them substantially, then the fact that unions impact relatively more on wages and profits under Cournot competition overturns the standard result on the ranking of Cournot and Bertrand profits. We note that if the upstream agents are profit-maximising firms, the standard result obtains.

There are a number of obvious directions for further work. First, we have followed standard assumptions in our specification of the basic model. Our results show that the Cournot-Bertrand profit ranking can be reversed, but only when unions are both very powerful and highly geared towards wages in their objective function. It would be interesting to see to how sensitive the results are to alternative or to more general functional forms. Second, we have considered only the case of imperfect substitutes, $c > 0$. Given the symmetry in the standard result concerning the sign of c and the relative magnitude of Cournot and Bertrand profits, it would be interesting to examine whether our results are symmetric. That is, do Cournot profits exceed Bertrand profits when goods are imperfect complements for certain values of the parameters, c, b and q? Third, we have found that if firms can choose cooperatively the strategic variable (price or quantity) with which to play the game, then their choice will depend on the values of b and q, even if $c > 0$. However, we have not considered how b and q influence the outcome of the non-cooperative choice of strategic variable. We leave this for further work.

Appendix

A. *Proof that* $c > 2\hat{A}$ *is satisfied* $\forall\ c$.

From $\hat{A} = \dfrac{y - x}{ey - x}$ the condition can be written as:

$$c > \frac{2y - 2x}{ey - x},$$

or, as $ey - x > 0$,

$$(2 - c)x > (2 - ce)y.$$

Substituting from (29), this becomes:

$$(2 - c)x > \left[2 - \frac{2c}{2 - c^2}\right]y,$$

or

$$(2 - c)(2 - c^2)x > 2(2 + c)(1 - c)y.$$

Substituting $y = (2 - c)(1 + c)^{1/2}$ and $x = (2 + c)(1 - c)^{1/2}$ above and squaring, it simplifies to the condition that:

$$(1 - c)(2 - c^2)^2 > 4(1 + c)(1 - c)^2,$$

After simplification, this reduces to the condition that $c^4 > 0$, which holds $\forall\ c$.

B. *Derivation of the first order condition of Wage Bargaining under Imperfect Competition in the Product Market.*

Define the general profit function of a Cournot competitor with exogenous marginal costs (wages) as $p_i = p_i(q_i(w_i, w_j), q_j(q_i(w_i, w_j))\ q_i(w_i, w_j) - w_i\, q_i(w_i, w_j)$. Assume a short-run production function of the type $q_i = l_i$. Hence, taking the derivative of Cournot profits with respect to the own wage yields:

$$\frac{\partial p_i}{\partial w_i} = \left(\frac{\partial p_i}{\partial q_i} q_i + p_i - w_i \right) \frac{\partial l_i}{\partial w_i} + \left(\frac{\partial p_i}{\partial q_j} \frac{\partial q_j}{\partial q_i} \frac{\partial l_i}{\partial w_i} \right) q_i - q_i \quad ,$$

which can be simplified further since the first term equals zero: it is the first order condition for quantity competition in the product market. Hence, this yields the following expression for marginal profitability:

$$\frac{\partial p_i}{\partial w_i} = \underbrace{\left(\frac{\partial p_i}{\partial q_j} \frac{\partial q_j}{\partial q_i} \frac{\partial l_i}{\partial w_i} \right) q_i}_{\substack{((-)\;(-)\;(-))\;(+) \\ \textit{strategic effect}}} - \underbrace{q_i}_{\substack{(+) \\ \textit{size effec}}} < 0$$

From (1), (5) and (6) it is straightforward to conclude that the strategic effect under Cournot competition is strictly negative.

The F.O.C of the RTM model is obtained by taking the derivative of the bargaining function B_i, as stated in (11), with respect to w_i yielding the following standard condition:

$$\frac{b \dfrac{\partial U_i}{\partial w_i}}{U_i} = -\frac{(1-b)\dfrac{\partial P_i}{\partial w_i}}{P_i} \; .$$

By taking the derivative of (10) with respect to w_i we obtain an expression for the union's marginal utility as:

$$\frac{\partial u_i}{\partial w_i} = [w_i - \overline{w}]^{q-1} [l_i]^{-q} \left[q\, l_i + (1-q)[w_i - \overline{w}] \frac{\partial l_i}{\partial w_i} \right]$$

Introducing (3), (10) and the expressions for marginal profitability and marginal utility in the first order condition of the RTM model and re-arranging yields:

$$\frac{b\,q}{w_i - \overline{w}} = -\frac{b\,(1-q)\dfrac{\partial l_i}{\partial w_i}}{l_i} - \frac{(1-b)\left(\dfrac{\partial p_i}{\partial q_j}\dfrac{\partial q_j}{\partial q_i}\dfrac{\partial l_i}{\partial w_i}\right)}{p_i - w_i} + \frac{(1-b)}{p_i - w_i}$$

Multiplying through by w_i and defining $\left|h_i^C\right|$ as the absolute value of the labour demand wage elasticity under Cournot competition, yields:

$$\frac{b\,q\,w_i}{w_i - \overline{w}} = b\,(1-q)\left|h_i^C\right| - \frac{(1-b)\left(\dfrac{\partial p_i}{\partial q_j}\dfrac{\partial q_j}{\partial q_i}\dfrac{\partial l_i}{\partial w_i}\right)w_i}{p_i - w_i} + \frac{(1-b)\,w_i}{p_i - w_i}\,.$$

Since the derivation for the Bertrand competitor is similar it is omitted here.

References

Bulow, J.I., Geanakoplos, J.D. and Klemperer, P.D., 1985. Multimarket oligopoly: strategic substitutes and complements. Journal of Political Economy 93, 488-511.

Cheng, L., 1985. Comparing Bertrand and Cournot equilibira: a geometric approach. Rand Journal of Economics 16, 146-152.

Dastidar, K.G., 1997. Comparing Cournot and Bertrand in a homogenous product market. Journal of Economic Theory 75, 205-212.

Davidson, C., 1988. Multiunit bargaining in oligopolistic industries. Journal of Labour Economics 6, 397-422.

Dowrick, S.J., 1989. Union-oligopoly bargaining. Economic Journal 99**,** 1123-1142.

Grandner, T., 2001. Unions in oligopolistic, vertically connected industries. European Economic Review 45, 1723-1740.

Häckner, J., 2000. A note on price and quantity competition in differentiated oligopolies. Journal of Economic Theory 93, 233-239.

Horn, H. and Wolinsky, A., 1988. Bilateral monopolies and incentives for merger. Rand Journal of Economics 19, 408-419.

Lambertini, L., 1997. Prisioners' dilemma in duopoly (super) games. Journal of Economic Theory 77, 181-191.

Naylor, R. A., 1998. International trade and economic integration when labour markets are generally unionised. European Economic Review 42, 1251-1267.

Naylor, R. A., 1999. Union wage strategies and international trade, Economic Journal 109, 102-125.

Okuguchi, K., 1987. Equilibrium prices in the Bertrand and Cournot oligopolies. Journal of Economic Theory 42, 128-139.

Qiu, L.D., 1997. On the dynamic efficiency of Bertrand and Cournot equilibria. Journal of Economic Theory 75, 213-229.

Singh, N. and Vives, X., 1984. Price and quantity competition in a differentiated duopoly. Rand Journal of Economics 15, 546-554.

Vives, X., 1985. On the efficiency of Bertrand and Cournot equilibria with product differentiation. Journal of Economic Theory 36, 166-175.

Wolfram, S., 1999. The Mathematica Book, 4th ed., Cambridge University Press, Cambridge.

Legends of Figures:

FIGURE 1 The firms' indifference curve in $(b, q)-$ space.

FIGURE 2 The firms' indifference surface, *Surface[* $F=0$ *]*, in $(q, c, b)-$ space.

FIGURE 3 SPNE best-reply wage functions for Cournot and Bertrand cases.

FIGURES:

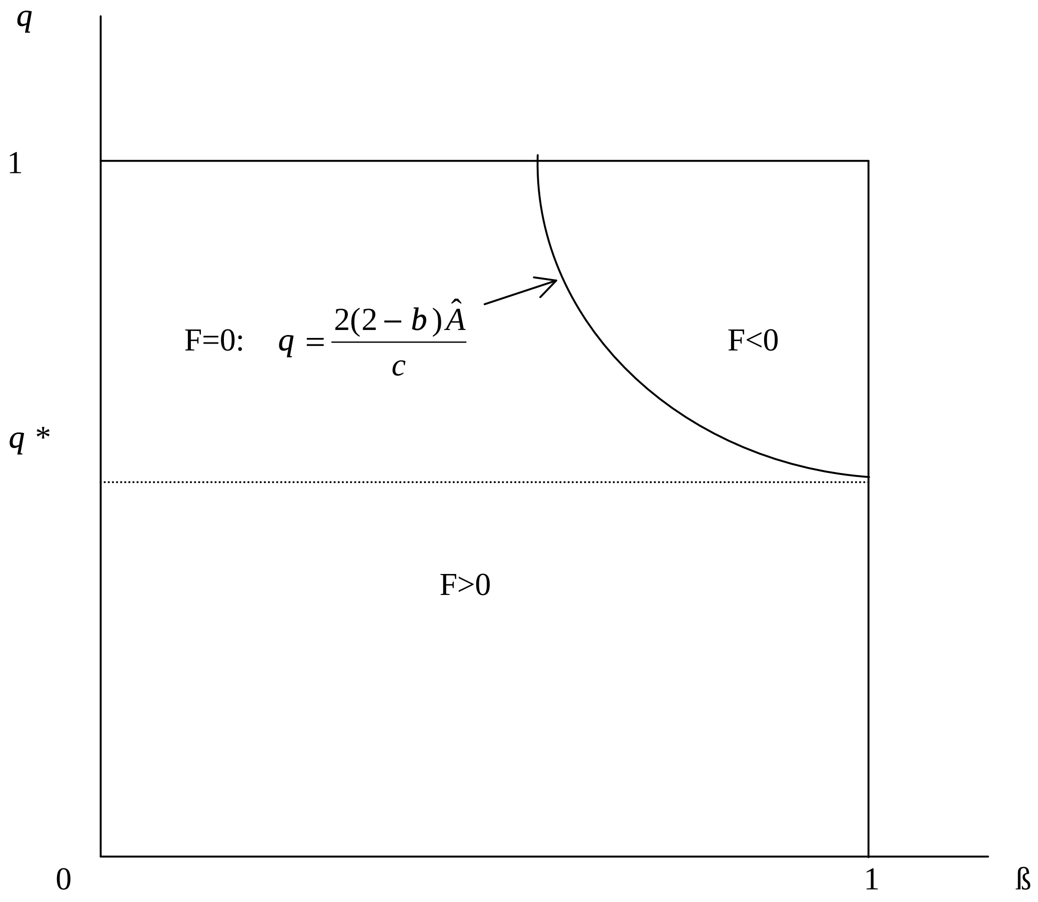

FIGURE 1 The firms' indifference curve in $(b, q)-$ space.

.

FIGURES *(continued)*

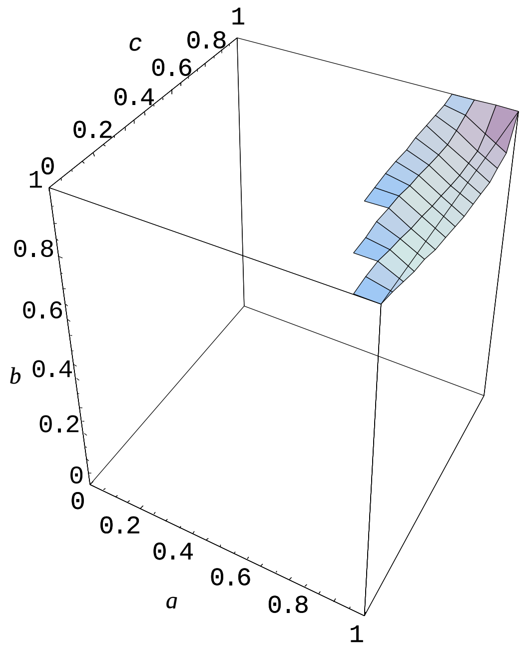

FIGURE 2 The firms' indifference surface, *Surface*[$F = 0$], in $(q, c, b) -$ space.

FIGURES *(continued):*

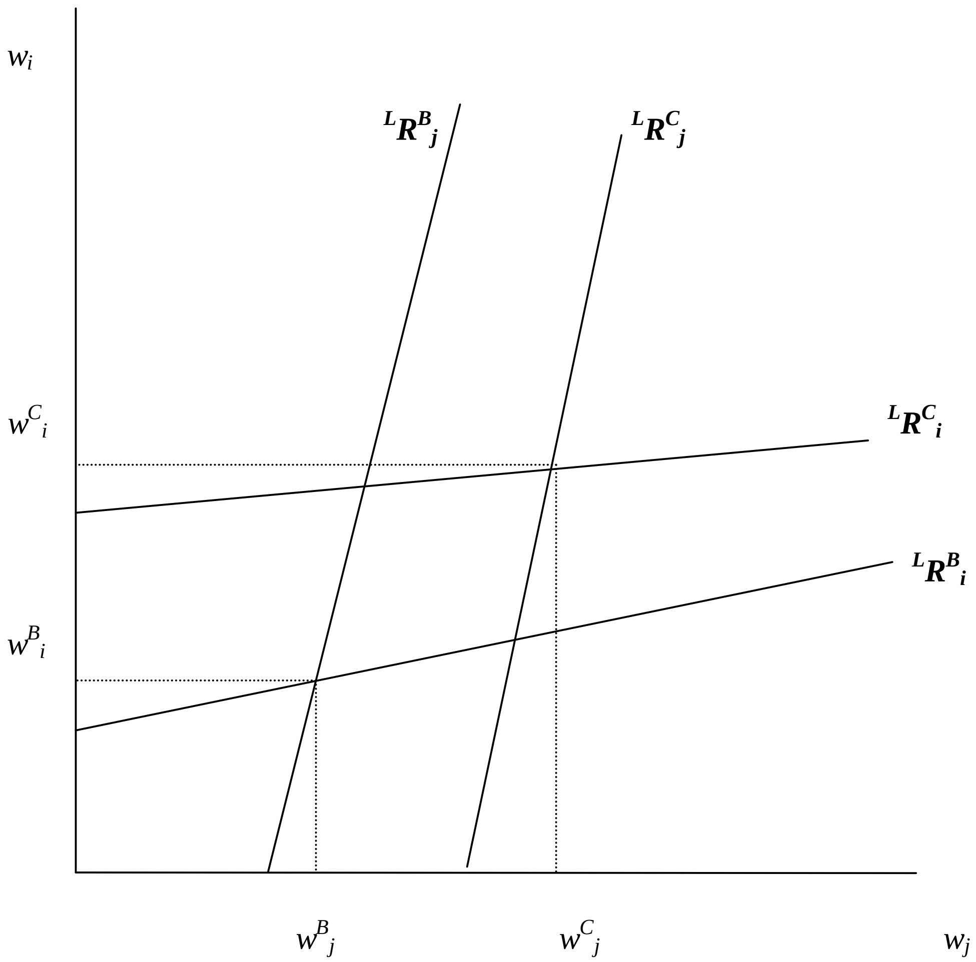

FIGURE 3 SPNE best-reply wage functions for Cournot and Bertrand cases.