Networks and Farsighted Stability

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Abstract

We make two main contributions to the theory of economic and social network formation. First, we introduce the notion of a network formation network or a *supernetwork*. Supernetworks provide a framework in which we can formally define and analyze farsightedness in network formation. Second, we introduce a new notion of equilibrium corresponding to farsightedness. In particular, we introduce the notion of a *farsightedly basic network*, as well as the notion of a *farsighted basis*, and we show that all supernetworks possess a farsighted basis. A farsightedly basic network contained in the farsighted basis of a given supernetwork represents a possible final resting point (or absorbing state) of a network formation process in which agents behave farsightedly. Given the supernetwork representation of the rules governing network formation and the preferences of the individuals, a farsighted basis contains networks which are likely to emerge and persist if individuals behave farsightedly.

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1 Introduction

Overview

Since the seminal paper by Jackson and Wolinsky (1996) there has been a rapidly growing literature on social and economic networks and their stability and efficiency properties (e.g., see Jackson (2001) and Jackson and van den Nouweland (2001)). As noted by Jackson (2001), an important issue that has not yet been addressed in the literature on networks and network formation is the issue of *farsighted stability* (see Jackson (2001), p.21 and p.35)). This issue is the focus of our paper. We make two main contributions to the theory of economic and social network formation. First, we introduce the notion of a network formation network. We call such a network a *supernetwork*. Supernetworks provide a framework in which we can define a farsighted dominance relation over networks, and thus a framework in which we can formally analyze farsightedness in network formation. Second, we introduce a new notion of equilibrium corresponding to our supernetwork formalization of farsightedness. In particular, we introduce the notion of a *farsightedly basic network*, as well as the notion of a *farsighted basis*, and we show that all supernetworks possess a farsighted basis. A farsightedly basic network contained in the farsighted basis of a given supernetwork represents a possible final resting point (or absorbing state) of a network formation process in which agents behave farsightedly. Thus, given the supernetwork representation of the rules governing network formation and the preferences of the individuals, a farsighted basis contains networks which are likely to emerge and persist if individuals behave farsightedly.

A key ingredient in establishing that all supernetworks possess a farsighted basis is the notion of an *inductive supernetwork*. A supernetwork is said to be inductive if given any farsighted domination path through the supernetwork there exists a network (i.e., a node in the supernetwork) which is reachable via a farsighted domination path from every network on the given path.[1] We show that all supernetworks are inductive. Then, by a straightforward application of a classical result due to Berge (1958), we are able to conclude that all supernetworks possess a farsighted basis. Formally, a set of networks is said to be a farsighted basis for the supernetwork, if given any two distinct networks in the basis there exists no finite farsighted domination path connecting the two networks and if for any network not contained in the basis, there exists a finite farsighted domination path from this network to some network contained in the basis. Thus, a farsighted basis is simply a von Neumann-Morgenstern stable set with respect to the relation defined via the farsighted domination paths on the set networks composing the nodes of the supernetwork.[2]

We also show that given any supernetwork, any farsighted basis is a subset of the largest consistent set of networks. Consistency with respect to farsighted dominance and the notion of a largest consistent set were introduced by Chwe (1994) in an abstract game setting. Here we extend the notion of farsighted consistency and the

[1] A farsighted domination path is a sequence of networks, $\{G_k\}_k$, forming a path through the supernetwork such for all k, network G_k farsightedly dominates network G_{k-1}.

[2] Put differently, a farsighted basis is a von Neumann-Morgenstern stable set with respect to the transitive closure of the farsighted dominance relation.

largest consistent set to network formation. Given agent preferences and the rules governing network formation, as represented via the supernetwork, a network is said to be farsightedly consistent if no agent or coalition of agents is willing to alter the network (via the addition, subtraction, or replacement of arcs) for fear that such an alteration might induce further network alterations by other agents or coalitions that in the end leave the initially deviating agent or coalition no better off - and possibly worse off. Our notion of a farsighted basis represents a refinement of our network rendition of the largest consistent set. Moreover, our result establishing that all supernetworks possess a farsighted basis, together with our result showing that any farsighted basis is a subset of the largest consistent set, imply that for any supernetwork the largest consistent set of networks is nonempty.

In order to illustrate the utility of our framework, we apply our supernetwork framework to a three-agent version of the co-author model of Jackson and Wolinsky (1996) and we compute the largest consistent set for two different supernetworks corresponding to the collection of directed co-author networks.[3] Jackson and Wolinsky show that, in general, a pairwise stable co-author network can be partitioned into fully intraconnected components, each of which has a different number of members (see Proposition 4, part (ii) in Jackson and Wolinsky (1996)).[4] In our example, a similar conclusion can be drawn: each farsightedly consistent network in each of the two co-author supernetworks can be partitioned into fully intraconnected components, each of which has a different number of members. Also, in both of our examples, each farsightedly consistent co-author network is Nash as well as Pareto optimal.

Directed Networks vs Linking Networks

We focus on directed networks, and in fact, we extend the definition of directed networks found in the literature. In a directed network, each arc possesses an orientation or direction: arc j connecting nodes i and i' must either go from node i to node i' or must go from node i' to node i.[5] In an undirected (or linking) network, arc j would have no orientation and would simply indicate a connection or link between nodes i and i'. Under our extended definition of directed networks, nodes are allowed to be connected by multiple arcs. For example, nodes i and i' might be connected by arcs j and j', with arc j running from node i to i' and arc j' running in the opposite direction (i.e., from node i' to node i).[6] Thus, if node i represents a seller and node i' a buyer, then arc j might represent a contract offer by the seller to the buyer, while arc j' might represent the acceptance or rejection of that contract offer. Also, under our extended definition loops are allowed and arcs are allowed to be used multiple

[3] It would have been preferable to have included a larger number of agents in our example. However, moving beyond three agents, the number of co-author networks in the supernetwork increases dramatically, making the computation very difficult. The computation of a farsighted basis remains an open question.

[4] A network is pairwise stable if for each pair of agents directly connected via an arc in the network both agents weakly prefer to remain directly connected, and if for each pair of agents not directly connected, a direct connection preferred by one of the agents makes the other agent strictly worse off (i.e., if one agent prefers to be directly connected, the other does not).

[5] We denote arc j going from node i to node i' via the ordered pair $(j,(i,i'))$, where (i,i') is also an ordered pair. Alternatively, if arc j goes from node i' to node i, we write $(j,(i',i))$.

[6] Under our extended definition, arc j' might also run in the same direction as arc j.

times in a given network.[7] For example, arc j might be used to connect nodes i and i' as well as nodes i' and i''. However, we do not allow arc j to go from node i to node i' multiple times in the same direction. By allowing arcs to possess direction and be used multiple times and by allowing nodes to be connected by multiple arcs, our extended definition makes possible the application of networks to a richer set of economic environments. Until now, most of the economic literature on networks has focused on linking networks (see for example, Jackson and Wolinsky (1996) and Dutta and Mutuswami (1997)).

Given a particular directed network, an agent or a coalition of agents can change the network to another network by simply adding, subtracting, or replacing arcs from the existing network in accordance with certain rules represented via the supernetwork.[8] For example, if the nodes in a network represent agents, then the rule for adding an arc j from node i to node i' might require that both agents i and i' agree to add arc j. Whereas the rule for subtracting arc j, from node i to node i', might require that only agent i or agent i' agree to dissolve arc j. This particular set of rules has been used, for example, by Jackson and Wolinsky (1996). Other rules are possible. For example, the addition of an arc might require that a simple majority of the agents agree to the addition, while the removal an arc might require that a two-thirds majority agree to the removal. Given the flexibility of the supernetwork framework, any set rules governing network formation can be represented.

Other Approaches and Related Literature

While the literature on stability in networks is well established and growing (e.g., see Dutta and Mutuswami (1997), Jackson (2001), and Jackson and van den Nouweland (2001)), the literature on farsighted stability in network formation is in its infancy. As far as we know, the work by Page, Wooders, and Kamat (2001) is the first to formally addresses the issue of farsighted stability in network formation. Since Page, Wooders, and Kamat (2001), other papers have appeared focusing on non-myopic behavior in network formation. Most notable are the papers by Watts (2002), Deroian (2003), and Dutta, Ghosal, and Ray (2003). These papers differ from our paper in at least two respects: (1) all three papers analyze the non-myopic formation of linking networks (rather than directed networks) and assume that network formation takes place one link at a time in accordance with a given set of rules usually requiring link addition to be bilateral while allowing link subtraction to be unilateral (thus, in all three of these papers a particular set of network formation rules is assumed); (2) all three papers utilize a notion of farsightedness and farsighted dominance that differs from the notion we use. For example, Dutta, Ghosal, and Ray (2002) offer a preliminary model of dynamic network formation which attempts to capture farsightedness by being dynamic and forward looking in the sense of dynamic programming. Their work is closely related to the work of Konishi and Ray (2002) on dynamic coalition formation. Our approach, while not explicitly dynamic, can be thought of as a dynamic network formation thought experiment, carried out at a particular point in

[7] A loop is an arc going *from* a given node *to* that same node. For example, given arc j and node i, the ordered pair $(j, (i, i))$ is a loop.

[8] Put differently, agents can change one network to another network by adding, subtracting, or replacing ordered pairs, $(j, (i, i'))$, in accordance with certain rules.

time, in which the future network consequences of individual or coalitional defections from the status quo network are fully taken into account.[9] Unlike Watts (2002), Deroian (2003), and Dutta, Ghosal, and Ray (2003), in our network formation model we use Chwe's (1994) notion of farsighted dominance (i.e., in our thought experiment, a move from network G to network G' might occur if network G' farsightedly dominates network G in the sense of Chwe). In a given supernetwork, network G' (a node in the supernetwork) farsightedly dominates network G (another node in the supernetwork) if there exists a finite path through the supernetwork going from G to G' such that each consecutive move along the path (from one network to another) can be brought about by some coalition and for each such coalition the network G' eventually reached by the path is preferred to the intermediate network altered by that coalition. Thus, farsighted dominance allows for coalitional defections from a given network which are not necessarily immediately preferred, but which eventually lead to a network which is preferred (see Li (1992, 1993) for an alternative definition of farsighted dominance requiring that each network along the path be preferred to the previous network by the agent or coalition responsible for altering the previous network). Chwe's notion of farsighted dominance has two progenitors: the notion of effective preferences due to Guilbaud (1949) and the notion of indirect dominance due to Harsanyi (1974).

Other approaches to farsightedness in network formation are suggested by the work of Xue (1998, 2000), Luo (2001), Mariotti and Xue (2002), Bhattacharya and Ziad (2003), and Mauleon and Vannetelbosch (2003) on farsightedness in games and coalition formation. With the exception of Mauleon and Vannetelbosch (2003), all of these papers take as their starting point Greenberg's *Theory of Social Situations* (Greenberg (1990)). Here, we shall follow the approach introduced in Page, Wooders, and Kamat (2001).

[9] Other important works on dynamic network formation which do not explicitly address the issue of farsightedness are, for example, Watts (2001) and Jackson and Watts (2001).

2 Directed Networks

We begin by giving a formal definition of the class of directed networks we shall consider. Let N be a finite set of nodes, with typical element denoted by i, and let A be a finite set of arcs, with typical element denoted by j. Arcs represent potential connections between nodes, and depending on the application, nodes can represent economic agents or economic objects such as markets or firms.[10]

Definition 1 *(Directed Networks)*
Given node set N and arc set A, a directed network, G, is a subset of $A \times (N \times N)$. We shall denote by $\mathbb{N}(N, A)$ the collection of all directed networks given N and A.

A directed network $G \in \mathbb{N}(N, A)$ specifies how the nodes in N are connected via the arcs in A. Note that in a directed network order matters. In particular, if $(j, (i, i')) \in G$, this means that arc j *goes from* node i *to* node i'. Also, note that under our definition of a directed network, loops are allowed - that is, we allow an arc to go from a given node back to that given node. Finally, note that under our definition an arc can be used multiple times in a given network and multiple arcs can go from one node to another. However, our definition does not allow an arc j to go from a node i to a node i' multiple times.

The following notation is useful in describing networks. Given directed network $G \subseteq A \times (N \times N)$, let

$$\left.\begin{array}{c} G(j) := \left\{(i, i') \in N \times N : (j, (i, i')) \in G\right\}, \\ \\ G(i) := \left\{j \in A : (j, (i, i')) \in G \text{ or } (j, (i', i)) \in G\ \right\} \\ \\ G(i, i') := \left\{j \in A : (j, (i, i')) \in G\right\}, \\ \\ G(j, i) := \left\{i' \in N : (j, (i, i')) \in G\right\}. \end{array}\right\} \quad (1)$$

Thus,

$G(j)$ is the *set of node pairs* connected by arc j in network G,
$G(i)$ is the *set of arcs* going from node i or coming to node i in network G,
$G(i, i')$ is the *set of arcs* going from node i to node i' in network G,
and
$G(j, i)$ is the *set of nodes* which can be reached by arc j from node i in network G.

Note that if for some arc $j \in A$, $G(j)$ is empty, then arc j is not used in network G. Moreover, if for some node $i \in N$, $G(i)$ is empty then node i is not used in network G, and node i is said to be isolated relative to network G.

[10] Of course in a supernetwork, nodes represent networks.

If in our definition of a directed network, we had required that $G(j)$ be single-valued and nonempty for all arcs $j \in A$, then our definition would have been the same as that given by Rockafellar (1984).

Suppose that the node set N is given by $N = \{i_1, i_2, \ldots, i_5\}$, while the arc set A is given by $A = \{j_1, j_2, \ldots, j_5, j_6, j_7\}$. Consider the network, G, depicted in Figure 1.

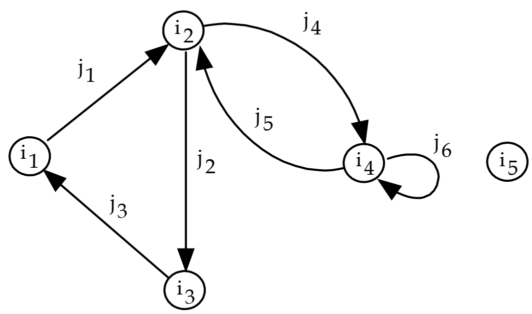


Figure 1: Network G

In network G, $G(j_6) = \{(i_4, i_4)\}$. Thus, $(j_6, (i_4, i_4)) \in G$ is a loop. Also, in network G, arc j_7 is not used. Thus, $G(j_7) = \emptyset$.[11] Finally, note that $G(i_4) = \{j_4, j_5, j_6\}$, while $G(i_5) = \emptyset$. Thus, node i_5 is *isolated* relative to G.[12]

Consider the new network, $G' \in \mathbb{N}(N, A)$ depicted in Figure 2.

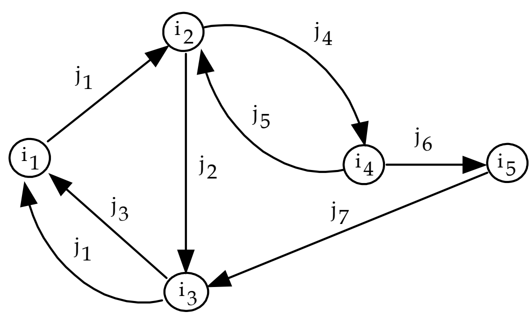


Figure 2: Network G'

In network G', $G'(j_1) = \{(i_1, i_2), (i_3, i_1)\}$. Thus, $(j_1, (i_1, i_2)) \in G'$ and $(j_1, (i_3, i_1)) \in G'$. Note that in network G', node i_5 is no longer isolated. In particular, $G'(i_5) = \{j_6, j_7\}$. Also, note that nodes i_2 and i_4 are connected by two different arcs pointed in

[11] The fact that arc j_7 is not used in network G can also be denoted by writing

$$j_7 \notin proj_A G,$$

where $proj_A G$ denotes the projection onto A of the subset

$$G \subseteq A \times (N \times N)$$

representing the network.

[12] If the loop $(j_7, (i_5, i_5))$ were part of network G in Figure 1, then node i_5 would no longer be considered isolated under our definition. Moreover, we would have $G(i_5) = \{j_7\}$.

opposite directions. Under our definition of a directed network it would be possible to create a new network from network G' by replacing arc j_5 from i_4 to i_2 with arc j_4 *from* i_4 *to* i_2. However, it would *not* be possible under our definition to create a new network by replacing arc j_5 from i_4 to i_2 with arc j_4 from i_2 to i_4 - because our definition does not allow j_4 to go *from* i_2 *to* i_4 multiple times. Finally, note that nodes i_1 and i_3 are also connected by two different arcs, but arcs pointed in the same direction. In particular, $G(i_3, i_1) = \{j_1, j_3\}$.

Remark:

Under our extended definition of a directed network, a directed graph or digraph, as it is sometimes called in the graph theory literature, can be viewed as a special case of a directed network. A directed graph consists of a pair, (N, E), where N is a nonempty set of nodes or vertices and E is a nonempty set of ordered pairs of nodes. Given node set N, arc set A, and directed network $G \in \mathbb{N}(N, A)$, for each arc $j \in A$, $(N, G(j))$ is a directed graph where, recall from expression (1) above, $G(j)$ is the set of ordered pairs of nodes connected by arc j, given by

$$G(j) := \left\{ (i, i^{'}) \in N \times N : (j, (i, i^{'})) \in G \right\}.$$

Thus, a directed network is a collection of directed graphs where each directed graph is labelled by a particular arc.

3 Supernetworks

Let D denote a nonempty set of agents (or economic decision making units) with typical element denoted by d, and let $\Gamma(D)$ denote the collection of all *nonempty* subsets (or coalitions) of D with typical element denoted by S.

Given collection of directed networks $\mathbb{G} \subseteq \mathbb{N}(N, A)$, we shall assume that each agent's preferences over networks in $\mathbb{G}$ are specified via a network payoff function,

$$v_d(\cdot) : \mathbb{G} \to \mathbb{R}.$$

For each agent $d \in D$ and each directed network $G \in \mathbb{G}$, $v_d(G)$ is the payoff to agent d in network G. Agent d then prefers network G' to network G if and only if

$$v_d(G') > v_d(G).$$

Moreover, coalition $S' \in \Gamma(D)$ prefers network G' to network G if and only if

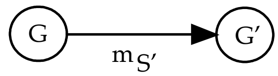
$$v_d(G') > v_d(G) \text{ for all } d \in S'.$$

Note that the payoff function of an agent depends on the entire network. Thus, the agent may be affected by directed links between other agents even when he himself has no direct or indirect connection with those agents. Intuitively, 'widespread' network externalities are allowed.

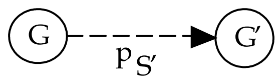
By viewing each network G in a given collection of directed networks $\mathbb{G} \subseteq \mathbb{N}(N, A)$ as a node in a larger network, we can give a precise network representation of the rules governing network formation as well as agents' preferences. To begin, let

$$\mathbb{M} := \{m_S : S \in \Gamma(D)\} \text{ denote the set of move arcs (or } m\text{-arcs for short),}$$
$$\mathbb{P} := \{p_S : S \in \Gamma(D)\} \text{ denote the set of preference arcs (or } p\text{-arcs for short),}$$
$$\text{and}$$
$$\mathbb{A} := \mathbb{M} \cup \mathbb{P}.$$

Given networks G and G' in $\mathbb{G}$, we shall denote by



(i.e., by an m-arc, belonging to coalition S', going from node G to node G') the fact that coalition $S' \in \Gamma(D)$ can change network G to network G' by adding, subtracting, or replacing arcs in network G. Moreover, we shall denote by



(i.e., by a p-arc, belonging to coalition S', going from node G to node G') the fact that each agent in coalition $S' \in \Gamma(D)$ prefers network G' to network G.

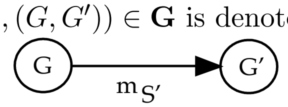
Definition 2 *(Supernetworks)*

Given directed networks $\mathbb{G} \subseteq \mathbb{N}(N, A)$, agent payoff functions $\{v_d(\cdot) : d \in D\}$, and arc set $\mathbb{A} := \mathbb{M} \cup \mathbb{P}$, a supernetwork, $\mathbf{G}$, is a subset of $\mathbb{A} \times (\mathbb{G} \times \mathbb{G})$ such that for all networks G and G' in $\mathbb{G}$ and for every coalition $S' \in \Gamma(D)$,

$$(m_{S'}, (G, G')) \in \mathbf{G} \text{ if and only if coalition } S' \text{ can change network } G \text{ to network } G',$$
$$G' \neq G, \text{ by adding, subtracting, or replacing arcs in network } G,$$
$$\text{and}$$
$$(p_{S'}, (G, G')) \in \mathbf{G} \text{ if and only if } v_d(G') > v_d(G) \text{ for all } d \in S'.$$

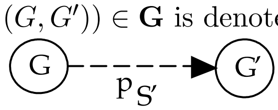
Thus, a supernetwork $\mathbf{G}$ specifies how the networks in $\mathbb{G}$ are connected via coalitional moves and coalitional preferences - and thus provides a *network representation* of agent preferences and the rules governing network formation. Given coalition $S' \in \Gamma(D)$, m-arc $m_{S'} \in \mathbb{M}$, and p-arc $p_{S'} \in \mathbb{P}$

$(m_{S'}, (G, G')) \in \mathbf{G}$ is denoted by



while

$(p_{S'}, (G, G')) \in \mathbf{G}$ is denoted by



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Remarks:

(1) Under our definition of a supernetwork, m-arc loops and p-arc loops are ruled out. Thus, for any network G and coalition S',

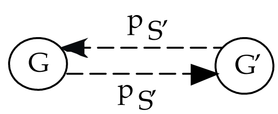
$$(m_{S'}, (G, G)) \notin \mathbf{G} \text{ and } (p_{S'}, (G, G)) \notin \mathbf{G}.$$

While m-arc loops are ruled out by definition, the absence of p-arc loops in supernetworks is due to the fact that each agent's preferences over networks are irreflexive. In particular, for each agent $d \in D$ and each network $G \in \mathbb{G}$, $v_d(G) > v_d(G)$ is not possible. Thus, $(p_{\{d\}}, (G, G)) \notin \mathbf{G}$.

(2) The definition of agent preferences via the network payoff functions,

$$\{v_d(\cdot) : d \in D\},$$

also rules out the following types of p-arc connections:



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Thus, for all coalitions $S' \in \Gamma(D)$ and networks G and G' contained in $\mathbb{G}$,

$$\text{if } (p_{S'}, (G, G')) \in \mathbf{G}, \text{ then } (p_{S'}, (G', G)) \notin \mathbf{G}.$$

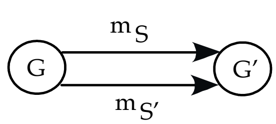
(3) For all coalitions $S' \in \Gamma(D)$ and networks G and G' contained in $\mathbb{G}$, if

$$(p_{S'}, (G, G')) \in \mathbf{G}, \text{ then}$$

$$(p_S, (G, G')) \in \mathbf{G} \text{ for all subcoalitions } S \text{ of } S'.$$

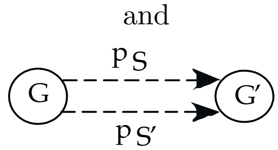
(4) Under our definition of a supernetwork, multiple m-arcs, as well as multiple p-arcs, connecting networks G and G' in supernetwork $\mathbf{G}$ are allowed. Thus, in supernetwork $\mathbf{G}$ the following types of m-arc and p-arc connections are possible:

For coalitions S and S', with $S \neq S'$



,

and



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However, multiple m-arcs, or multiple p-arcs, from network $G \in \mathbf{G}$ to network $G' \in \mathbf{G}$ belonging to the *same* coalition are not allowed - and moreover, are unnecessary. Allowing multiple arcs can be very useful in many applications. For example, multiple

m-arcs (not belonging to the same coalition) connecting networks G and G' in a given supernetwork $\mathbf{G}$ denote the fact that in supernetwork $\mathbf{G}$ there is more than one way to get from network G to network G' - or put differently, there is more than one way to change network G to network G'.

(5) In many economic applications, the set of nodes, N, used in defining the networks in the collection $\mathbb{G}$, and the set of economic agents D are one and the same (i.e., in many applications $N = D$).

4 Farsightedly Basic Networks

4.1 Farsighted Dominance

Given supernetwork $\mathbf{G} \subset \mathbb{A} \times (\mathbb{G} \times \mathbb{G})$, we say that network $G' \in \mathbb{G}$ farsightedly dominates network $G \in \mathbb{G}$ if there is a finite sequence of networks,

$$G_0, G_1, \ldots, G_h,$$

with $G = G_0$, $G' = G_h$, and $G_k \in \mathbb{G}$ for $k = 0, 1, \ldots, h$, and a corresponding sequence of coalitions,

$$S_1, S_2, \ldots, S_h,$$

such that for $k = 1, 2, \ldots, h$

$$(m_{S_k}, (G_{k-1}, G_k)) \in \mathbf{G},$$
and
$$(p_{S_k}, (G_{k-1}, G_h)) \in \mathbf{G}.$$

We shall denote by $G \lhd\lhd G'$ the fact that network $G' \in \mathbb{G}$ farsightedly dominates network $G \in \mathbb{G}$.

Figure 3 below provides a network representation of the farsighted dominance relation in terms of m-arcs and p-arcs. In Figure 3, network G_3 farsightedly dominates network G_0.

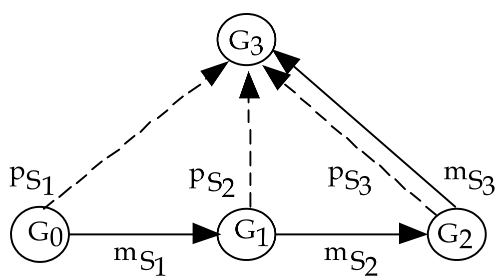

Figure 3: G_3farsightedly dominates G_0

Note that what matters to the initially deviating coalition S_1, as well as coalitions S_2 and S_3, is the ultimate network outcome G_3. Thus, the initially deviating coalition S_1 will not be deterred even if

$$(p_{S_1}, (G_0, G_1)) \notin \mathbf{G}$$

as long as the ultimate network outcome G_3 is preferred to G_0, that is, as long as G_3 is such that

$$(p_{S_1}, (G_0, G_3)) \in \mathbf{G}.$$

4.2 Farsighted Domination Paths and Inductive Supernetworks

Given supernetwork $\mathbf{G} \subset \mathbb{A} \times (\mathbb{G} \times \mathbb{G})$, we say that a sequence of networks $\{G_k\}_k$ in $\mathbb{G}$ is a *farsighted domination path*, written $\triangleleft\triangleleft$-path, *through supernetwork* $\mathbf{G}$ if for any two consecutive networks G_{k-1} and G_k, G_k farsightedly dominates G_{k-1}, that is, if

$$G_{k-1} \triangleleft\triangleleft G_k.$$

We can think of the farsighted dominance relation $G_{k-1} \triangleleft\triangleleft G_k$ between networks G_k and G_{k-1} as defining a $\triangleleft\triangleleft$-arc *from* network G_{k-1} *to* network G_k. Given $\triangleleft\triangleleft$-path $\{G_h\}_h$ through $\mathbf{G}$, the *length* of this path is defined to be the number of $\triangleleft\triangleleft$-arcs in the path. We say that network $G_1 \in \mathbb{G}$ is $\triangleleft\triangleleft$-*reachable* from network $G_0 \in \mathbb{G}$ in $\mathbf{G}$ if there exists a finite $\triangleleft\triangleleft$-path in $\mathbf{G}$ from G_0 to G_1 (i.e., a $\triangleleft\triangleleft$-path in $\mathbf{G}$ from G_0 to G_1 of finite length). If network $G_0 \in \mathbb{G}$ is $\triangleleft\triangleleft$-reachable from network G_0 in $\mathbf{G}$, then we say that supernetwork $\mathbf{G}$ contains a $\triangleleft\triangleleft$-*circuit*. Thus, a $\triangleleft\triangleleft$-circuit in $\mathbf{G}$ starting at network $G_0 \in \mathbb{G}$ is a finite $\triangleleft\triangleleft$-path from G_0 to G_0. A $\triangleleft\triangleleft$-circuit of length 1 is called a $\triangleleft\triangleleft$-loop. Note that because preferences are irreflexive, $\triangleleft\triangleleft$-loops are in fact ruled out. However, because the farsighted dominance relation, $\triangleleft\triangleleft$, is *not transitive*, it is possible to have $\triangleleft\triangleleft$-circuits of length greater than 1.

Given supernetwork $\mathbf{G} \subset \mathbb{A} \times (\mathbb{G} \times \mathbb{G})$, we can use the notion of $\triangleleft\triangleleft$-reachability to define a new relation on the nodes of the supernetwork. We shall write

$$G_1 \trianglerighteq G_0 \text{ if and only if } \begin{cases} G_1 \text{ is } \triangleleft\triangleleft\text{-reachable from } G_0, \text{ or} \\ G_1 = G_0. \end{cases} \tag{2}$$

The relation $\trianglerighteq$ is a weak ordering on the networks composing the nodes of the supernetwork $\mathbf{G}$. In particular, $\trianglerighteq$ is reflexive ($G \trianglerighteq G$) and $\trianglerighteq$ is transitive ($G_2 \trianglerighteq G_1$ and $G_1 \trianglerighteq G_0$ implies that $G_2 \trianglerighteq G_0$). The relation $\trianglerighteq$ is sometimes referred to as the transitive closure of the farsighted dominance relation, $\triangleleft\triangleleft$.

Definition 3 *($\triangleleft\triangleleft$-inductive Supernetworks)*
Let $\mathbb{G} \subseteq \mathbb{N}(N, A)$ be a collection of directed networks and let $\mathbf{G} \subset \mathbb{A} \times (\mathbb{G} \times \mathbb{G})$ be a supernetwork.

Supernetwork $\mathbf{G}$ is said to be $\triangleleft\triangleleft$-Inductive if given any $\triangleleft\triangleleft$-path, $\{G_k\}_k$, through $\mathbf{G}$ there exists a network $G^* \in \mathbb{G}$ such that

$$G^* \trianglerighteq G_k \text{ for all } k.$$

Network G^* above is referred to as the majorant of the sequence.

Due to the finiteness of the node and arc sets, N and A, the collection of all directed networks, $\mathbb{N}(N,A)$, is a finite set. The finiteness of $\mathbb{N}(N,A)$ implies that all supernetworks defined over $\mathbb{N}(N,A)$ have a finite number of nodes, and this in turn implies that all supernetworks are $\lhd\lhd$-Inductive. To see this, let $\{G_k\}_k$ be any $\lhd\lhd$-path through $\mathbf{G}$. If $\{G_k\}_k$ is finite, then the last network in the sequence is a majorant. If $\{G_k\}_k$ is infinite, then because the supernetwork in finite, the sequence contains at least one network which is repeated an infinite number of times, and this infinitely repeated network is a majorant. We summarize these observation in the following Theorem.

Theorem 1 *(All supernetworks are $\lhd\lhd$-inductive)*
Let $\mathbb{G} \subseteq \mathbb{N}(N,A)$ be a collection of directed networks and let $\mathbf{G} \subset \mathbb{A} \times (\mathbb{G} \times \mathbb{G})$ be any supernetwork. Then $\mathbf{G}$ is $\lhd\lhd$-inductive.

4.3 Farsighted Bases

We begin with the definition.

Definition 4 *(Farsighted Bases)*
Let $\mathbb{G} \subseteq \mathbb{N}(N,A)$ be a collection of directed networks and let $\mathbf{G} \subset \mathbb{A} \times (\mathbb{G} \times \mathbb{G})$ be a supernetwork.

A subset $\mathbb{B}$ of directed networks in $\mathbb{G}$ is said to be a farsighted basis for supernetwork $\mathbf{G}$ if

(a) (internal $\trianglerighteq$ -stability) whenever G_0 and G_1 are in $\mathbb{B}$, with $G_0 \neq G_1$, then neither $G_1 \trianglerighteq G_0$ nor $G_0 \trianglerighteq G_1$ hold, and

(b) (external $\trianglerighteq$ -stability) for any $G_0 \notin \mathbb{B}$ there exists $G_1 \in \mathbb{B}$ such that $G_1 \trianglerighteq G_0$.

In other words, a nonempty subset of networks $\mathbb{B}$ is a farsighted basis for supernetwork $\mathbf{G}$ if G_0 and G_1 are in $\mathbb{B}$, with $G_0 \neq G_1$, then G_1 is not reachable from G_0, nor is G_0 reachable from G_1, and if $G_0 \notin \mathbb{B}$, then there exists $G_1 \in \mathbb{B}$ reachable from G_0. Thus, a farsighted basis $\mathbb{B}$ for supernetwork $\mathbf{G}$ is simply a von Neumann-Morgenstern stable set with respect to the relation $\trianglerighteq$ determined by the supernetwork. Note that if a supernetwork possesses a farsighted basis, then by definition it is nonempty.

Theorem 2 *(Existence of a Farsighted Basis for any Inductive Supernetwork, Berge (1958))*

Let $\mathbb{G} \subseteq \mathbb{N}(N,A)$ be a collection of directed networks and let $\mathbf{G} \subset \mathbb{A} \times (\mathbb{G} \times \mathbb{G})$ be any supernetwork. Then $\mathbf{G}$ possesses a farsighted basis.

Theorem 2 is an immediate consequence of Theorem 1 above and a classical result due to Berge (1958) which, in our terminology, states that every inductive supernetwork possesses a farsighted basis.

5 Farsightedly Consistent Networks

In this section, we show that given any supernetwork, any farsighted basis is a subset of the largest consistent set of networks.

5.1 The Largest Consistent Set of Networks

A subset of directed networks $\mathbb{F}$ is said to be farsightedly consistent if given any network $G_0 \in \mathbb{F}$ and any m_{S_1}-deviation to network $G_1 \in \mathbb{G}$ by coalition S_1 (via adding, subtracting, or replacing arcs in accordance with $\mathbf{G}$), there exists further deviations leading to some network $G_2 \in \mathbb{F}$ where the initially deviating coalition S_1 is not better off - and possibly worse off. A network $G \in \mathbb{G}$ is said to be farsightedly consistent if $G \in \mathbb{F}$ where $\mathbb{F}$ is a farsightedly consistent set corresponding to supernetwork $\mathbf{G}$. Formally, we have the following definition.

Definition 5 *(Farsightedly Consistent Sets)*
Let $\mathbb{G} \subseteq \mathbb{N}(N, A)$ be a collection of directed networks and let $\mathbf{G} \subset \mathbb{A} \times (\mathbb{G} \times \mathbb{G})$ be a supernetwork.

A subset $\mathbb{F}$ of directed networks in $\mathbb{G}$ is said to be farsightedly consistent in supernetwork $\mathbf{G}$ if

$$\begin{gathered}\text{for all } G_0 \in \mathbb{F},\\ (m_{S_1}, (G_0, G_1)) \in \mathbf{G} \text{ for some } G_1 \in \mathbb{G} \text{ and some coalition } S_1, \text{ implies that}\\ \text{there exists } G_2 \in \mathbb{F}\\ \text{with } G_2 = G_1 \text{ or } G_2 \rhd\rhd G_1 \text{ such that,}\\ (p_{S_1}, (G_0, G_2)) \notin \mathbf{G}.\end{gathered}$$

There can be many farsightedly consistent sets. We shall denote by $\mathbb{F}^*$ is largest farsightedly consistent set (or simply, the *largest consistent set*). Thus, if $\mathbb{F}$ is a farsightedly consistent set, then $\mathbb{F} \subseteq \mathbb{F}^*$.

Two questions arise in connection with the largest consistent set: (i) given supernetwork $\mathbf{G}$, does there exist a farsightedly consistent set of networks in $\mathbf{G}$, and (ii) is it nonempty? To establish existence, consider the mapping

$$\Lambda_{\mathbf{G}}(\cdot) : 2^{\mathbb{G}} \rightarrow 2^{\mathbb{G}},$$

where $2^{\mathbb{G}}$ denote the collection of *all* subsets of $\mathbb{G}$ (including the empty set). The mapping $\Lambda_{\mathbf{G}}(\cdot)$ is defined as follows:

$$\begin{gathered}\text{for subcollection of networks } \mathbb{H} \in 2^{\mathbb{G}} \text{ and network } G_0 \in \mathbb{G}\ ,\\ G_0 \text{ is contained in } \Lambda_{\mathbf{G}}(\mathbb{H})\\ \text{if and only if}\\ \forall\ G_1 \in \mathbb{G} \text{ such that } (m_S, (G_0, G_1)) \in \mathbf{G} \text{ for some coalition } S \in \Gamma(D)\\ \exists \text{ a network } G_2 \in \mathbb{H} \text{ such that}\\ \text{(i) } G_2 = G_1 \text{ or } G_2 \rhd\rhd G_1, \text{ and}\\ \text{(ii) } (p_S, (G_0, G_2)) \notin \mathbf{G}, \text{ that is, } v_d(G_2) \leq v_d(G_0) \text{ for some } d \in S.\end{gathered} \tag{3}$$

First, note that a set $\mathbb{F}$ is a farsightedly consistent set if and only if $\mathbb{F}$ is a fixed point of the mapping $\Lambda_{\mathbf{G}}(\cdot)$. Second, note that $\Lambda_{\mathbf{G}}(\cdot)$ is monotone increasing; that is, for any subcollections $\mathbb{H}$ and $\mathbb{E}$ of $\mathbb{G}$,

$$\mathbb{H} \subseteq \mathbb{E} \text{ implie that } \Lambda_{\mathbf{G}}(\mathbb{H}) \subseteq \Lambda_{\mathbf{G}}(\mathbb{E}).$$

To see that $\Lambda_{\mathbf{G}}(\cdot)$ has a unique, largest fixed point consider the following. Let

$$\Sigma = \left\{ \mathbb{H} \in 2^{\mathbb{G}} : \mathbb{H} \subseteq \Lambda_{\mathbf{G}}(\mathbb{H}) \right\}.$$

Note that Σ is nonempty since the empty subcollection $\emptyset \in 2^{\mathbb{G}}$ is contained in Σ; that is, $\emptyset \subseteq \Lambda_{\mathbf{G}}(\emptyset)$. Let $\mathbb{F}^* = \bigcup_{\mathbb{H} \in \Sigma} \mathbb{H}$. By the monotonicity of $\Lambda_{\mathbf{G}}(\cdot)$ and the definition of Σ,

$$\mathbb{H} \subseteq \Lambda_{\mathbf{G}}(\mathbb{H}) \subseteq \Lambda_{\mathbf{G}}(\mathbb{F}^*) \text{ for all } \mathbb{H} \in \Sigma.$$

Hence

$$\mathbb{F}^* = \bigcup_{\mathbb{H} \in \Sigma} \mathbb{H} \subseteq \bigcup_{\mathbb{H} \in \Sigma} \Lambda_{\mathbf{G}}(\mathbb{H}) \subseteq \Lambda_{\mathbf{G}}(\mathbb{F}^*).$$

Thus,

$$\mathbb{F}^* \subseteq \Lambda_{\mathbf{G}}(\mathbb{F}^*).$$

Moreover, by the monotonicity of $\Lambda_{\mathbf{G}}(\cdot)$,

$$\Lambda_{\mathbf{G}}(\mathbb{F}^*) \subseteq \Lambda_{\mathbf{G}}\left(\Lambda_{\mathbf{G}}(\mathbb{F}^*)\right).$$

Thus we have, $\Lambda_{\mathbf{G}}(\mathbb{F}^*) \in \Sigma$ and $\Lambda_{\mathbf{G}}(\mathbb{F}^*) \subseteq \mathbb{F}^*$, and we can conclude that

$$\mathbb{F}^* = \Lambda_{\mathbf{G}}(\mathbb{F}^*).$$

The set $\mathbb{F}^*$ is the largest farsightedly consistent set because if $\mathbb{F}'$ is any other fixed point (i.e., if $\mathbb{F}' = \Lambda_{\mathbf{G}}(\mathbb{F}')$), then, $\mathbb{F}' \in \Sigma$ and hence $\mathbb{F}' \subseteq \mathbb{F}^*$.

The method of proving existence above is based on Chwe (1994) and is similar to the method introduced by Roth (1975, 1977). The important point to take away from the proof is that in order to show that the largest consistent set is nonempty, it suffices to show that for some nonempty subset of networks $\mathbb{H} \in 2^{\mathbb{G}}$,

$$\mathbb{H} \subseteq \Lambda_{\mathbf{G}}(\mathbb{H}).$$

5.2 Farsighted Bases and Nonemptiness of the Largest Consistent Set

We begin by defining a new relation on the collection of directed networks $\mathbb{G} \subseteq \mathbb{N}(N, A)$. Let $\mathbf{G} \subset \mathbb{A} \times (\mathbb{G} \times \mathbb{G})$ be a supernetwork and let $\mathbb{B}$ be a farsighted basis for $\mathbf{G}$. Define the relation $\rhd_{\mathbb{B}}$ on the $\mathbb{G}$ as follows: for G^1 and G^0 in $\mathbb{G}$,

$$G_1 \rhd_{\mathbb{B}} G_0 \text{ if and only if } \begin{cases} \text{(a) } G_1 \in P_{\rhd\rhd}(G_0), \text{ and} \\ \text{(b) } \mathbb{B} \cap P_{\rhd\rhd}(G_1) \subseteq P_{\rhd\rhd}(G_0), \end{cases} \quad (4)$$

where

$$P_{\rhd\rhd}(G_0) := \{G \in \mathbb{G} : G \rhd\rhd G_0\}.$$

The new relation $\rhd_{\mathbb{B}}$ is quasi-transitive, that is, if $G_2 \rhd_{\mathbb{B}} G_1$ and $G_1 \rhd_{\mathbb{B}} G_0$, then $G_2 \in \mathbb{B}$ implies that $G_2 \rhd_{\mathbb{B}} G_0$. To see this, first, note that $G_2 \in \mathbb{B} \cap P_{\rhd\rhd}(G_1)$, and because $G_1 \rhd_{\mathbb{B}} G_0$, $\mathbb{B} \cap P_{\rhd\rhd}(G_1) \subseteq P_{\rhd\rhd}(G_0)$. Thus, $G_2 \in P_{\rhd\rhd}(G_0)$. Finally, note that because $G_2 \rhd_{\mathbb{B}} G_1$,

$$\mathbb{B} \cap P_{\rhd\rhd}(G_2) \subseteq P_{\rhd\rhd}(G_1),$$

and because $G_1 \rhd_{\mathbb{B}} G_0$,

$$\mathbb{B} \cap P_{\rhd\rhd}(G_1) \subseteq P_{\rhd\rhd}(G_0).$$

Thus, $\mathbb{B} \cap P_{\rhd\rhd}(G_2) \subseteq \mathbb{B} \cap P_{\rhd\rhd}(G_1) \subseteq P_{\rhd\rhd}(G_0)$ and thus $G_2 \rhd_{\mathbb{B}} G_0$.

Now let $\mathbb{B}^*$ denote the basis with respect to the new relation $\rhd_{\mathbb{B}}$.[13] Such a basis exists, by Berge (1958), because the supernetwork $\mathbf{G}$ is $\rhd_{\mathbb{B}}$-inductive. In particular, given the finiteness of $\mathbf{G}$ it is easy to show that given any $\rhd_{\mathbb{B}}$-path, $\{G_k\}_k$, through $\mathbf{G}$ there exists a network $G^* \in \mathbb{G}$ such that

$$G^* \trianglerighteq_{\mathbb{B}} G_k \text{ for all } k,$$

where

$$G_1 \trianglerighteq_{\mathbb{B}} G_0 \text{ if and only if } \begin{cases} G_1 \text{ is } \rhd_{\mathbb{B}}\text{-reachable from } G_0, \text{ or} \\ G_1 = G_0. \end{cases} \tag{5}$$

But now note that because the relation $\rhd_{\mathbb{B}}$ is quasi-transitive, $\mathbb{B}^*$ is in fact a von Neumann-Morgenstern stable set with respect to the relation $\rhd_{\mathbb{B}}$ determined by supernetwork $\mathbf{G}$. Thus,

(a) $\mathbb{B}^*$ is internally $\rhd_{\mathbb{B}}$-stable; that is, $G \in \mathbb{B}^*$ implies that $\mathbb{B}^* \cap P_{\rhd_{\mathbb{B}}}(G) = \emptyset$, and
(b) $\mathbb{B}^*$ is externally $\rhd_{\mathbb{B}}$-stable; that is, $G \notin \mathbb{E}$ implie that $\mathbb{B}^* \cap P_{\rhd_{\mathbb{B}}}(G) \neq \emptyset$,

where

$$P_{\rhd_{\mathbb{B}}}(G) := \left\{G' \in \mathbb{G} : G' \rhd_{\mathbb{B}} G\right\}.$$

Moreover, by Theorem 4 in Berge (1958), $\mathbb{B}^*$ is the unique $\rhd_{\mathbb{B}}$-stable set.

We can now state our main result on the relationship between the farsighted basis $\mathbb{B}$ (i.e., the basis with respect to $\rhd\rhd$), the basis $\mathbb{B}^*$ with respect to $\rhd_{\mathbb{B}}$, and the largest consistent set $\mathbb{F}^*$.

Theorem 3 *(All Farsightedly Basic Networks Are Farsightedly Consistent)*
Let $\mathbb{G} \subseteq \mathbb{N}(N, A)$ be a collection of directed networks and let $\mathbf{G} \subset \mathbb{A} \times (\mathbb{G} \times \mathbb{G})$ be a supernetwork. Let $\mathbb{B}$ be a farsighted basis for $\mathbf{G}$ and $\mathbb{B}^*$ the corresponding unique $\rhd_{\mathbb{B}}$-stable set. The following statements are true:

1. $\mathbb{B} \subseteq \mathbb{B}^*$.

[13] The farsighted basis $\mathbb{B}$ can be referred to as the basis with respect to the farsighted dominance relation $\rhd\rhd$.

2. *The largest consistent set, $\mathbb{F}^*$, is nonempty and $\mathbb{B}^* \subseteq \mathbb{F}^*$.*

3. *The largest consistent set, $\mathbb{F}^*$, is externally stable with respect to farsighted dominance, that is, if network G is a node in supernetwork $\mathbf{G}$ not contained $\mathbb{F}^*$, then there exists a network G', a node in supernetwork $\mathbf{G}$, contained in $\mathbb{F}^*$ that farsightedly dominants G (i.e., $G' \rhd\rhd G$).*

Proof. (1) $\mathbb{B} \subseteq \mathbb{B}^*$: Suppose network G_0 is contained in $\mathbb{B}$ but not contained in $\mathbb{B}^*$. Since $\mathbb{B}^*$ is $\rhd_{\mathbb{B}}$-stable, $G_0 \notin \mathbb{B}^*$ implies that there exists

$$G_1 \in \mathbb{B}^* \cap P_{\rhd_{\mathbb{B}}}(G_0).$$

$G_1 \in P_{\rhd_{\mathbb{B}}}(G_0)$ implies that $G_1 \in P_{\rhd\rhd}(G_0)$ and $\mathbb{B}\cap\, P_{\rhd\rhd}(G_1) \subseteq P_{\rhd\rhd}(G_0)$. If $G_1 \in \mathbb{B}$ we have a contradiction of the internal $\trianglerighteq$-stability of $\mathbb{B}$. If $G_1 \notin \mathbb{B}$, then there exists a $\lhd\lhd$-path through the supernetwork from G_1 to some $G_2 \in \mathbb{B}$. But then $G_1 \in P_{\rhd\rhd}(G_0)$ together with this fact implies that there exists a $\lhd\lhd$-path through the supernetwork from $G_0 \in \mathbb{B}$ to $G_2 \in \mathbb{B}$, also a contradiction of the internal $\trianglerighteq$-stability of $\mathbb{B}$.

(2) To show that the largest consistent set, $\mathbb{F}^*$, is nonempty and that $\mathbb{B}^* \subseteq \mathbb{F}^*$, it suffices to show that $\mathbb{B}^* \subseteq \Lambda_{\mathbf{G}}(\mathbb{B}^*)$: Suppose network G_0 is contained in $\mathbb{B}^*$ but not contained in $\Lambda_{\mathbf{G}}(\mathbb{B}^*)$. $G_0 \notin \Lambda_{\mathbf{G}}(\mathbb{B}^*)$ implies that there exists a network G_1 with

$$(m_{S_1}, (G_0, G_1)) \in \mathbf{G} \text{ for some coalition } S_1,$$

such that for all networks $G_2 \in \mathbb{B}^*$ with $G_2 = G_1$ or $G_2 \in P_{\rhd\rhd}(G_1)$, $G_2 \in P_{\rhd\rhd}(G_0)$. It suffices to consider two cases:

Case 1: $G_1 \in \mathbb{B}^*$. If $G_1 \in \mathbb{B}^*$, then by the implications of $G_0 \notin \Lambda_{\mathbf{G}}(\mathbb{B}^*)$ stated above, $G_1 \in P_{\rhd\rhd}(G_0)$ and $\mathbb{B}^*\cap P_{\rhd\rhd}(G_1) \subseteq P_{\rhd\rhd}(G_0)$. But $\mathbb{B} \subseteq \mathbb{B}^*$ and $\mathbb{B}^*\cap P_{\rhd\rhd}(G_1) \subseteq P_{\rhd\rhd}(G_0)$ imply that $\mathbb{B} \cap P_{\rhd\rhd}(G_1) \subseteq P_{\rhd\rhd}(G_0)$. Thus, $G_1 \in \mathbb{B}^*$, $G_1 \in P_{\rhd_{\mathbb{B}}}(G_0)$, and $G_0 \in \mathbb{B}^*$, a contradiction of the internal $\rhd_{\mathbb{B}}$-stability of $\mathbb{B}^*$.

Case 2: $G_1 \notin \mathbb{B}^*$. If $G_1 \notin \mathbb{B}^*$, then by the external $\rhd_{\mathbb{B}}$-stability of $\mathbb{B}^*$ there exists

$$G_2 \in \mathbb{B}^* \cap P_{\rhd_{\mathbb{B}}}(G_1)$$

Thus, there exists $G_2 \in \mathbb{B}^*$ with $G_2 \in P_{\rhd\rhd}(G_1)$ and $\mathbb{B} \cap P_{\rhd\rhd}(G_2) \subseteq P_{\rhd\rhd}(G_1)$. Because $\mathbb{B} \subseteq \mathbb{B}^*$

$$\mathbb{B} \cap P_{\rhd\rhd}(G_2) = \mathbb{B}^* \cap \mathbb{B} \cap P_{\rhd\rhd}(G_2),$$

and because $\mathbb{B} \cap P_{\rhd\rhd}(G_2) \subseteq P_{\rhd\rhd}(G_1)$

$$\mathbb{B}^* \cap \mathbb{B} \cap P_{\rhd\rhd}(G_2) \subseteq \mathbb{B}^* \cap P_{\rhd\rhd}(G_1).$$

Therefore,

$$\mathbb{B} \cap P_{\rhd\rhd}(G_2) \subseteq \mathbb{B}^* \cap P_{\rhd\rhd}(G_1).$$

Now recall that $G_0 \notin \Lambda_{\mathbf{G}}(\mathbb{B}^*)$ implies that

$$\mathbb{B}^* \cap P_{\rhd\rhd}(G_1) \subseteq P_{\rhd\rhd}(G_0).$$

Thus, we have $G_2 \in \mathbb{B}^* \cap P_{\triangleright\triangleright}(G_1) \subseteq P_{\triangleright\triangleright}(G_0)$, and thus,

$$G_2 \in P_{\triangleright\triangleright}(G_0) \text{ and } \mathbb{B} \cap P_{\triangleright\triangleright}(G_2) \subseteq P_{\triangleright\triangleright}(G_0),$$

implying that

$$G_2 \in \mathbb{B}^* \cap P_{\triangleright_{\mathbb{B}}}(G_0).$$

This fact together with $G_0 \in \mathbb{B}^*$ contradict the internal $\triangleright_{\mathbb{B}}$-stability of $\mathbb{B}^*$.

(3) Since $\mathbb{B}^* \subseteq \mathbb{F}^*$, $G \notin \mathbb{F}^*$ implies that $G \notin \mathbb{B}^*$. Since $\mathbb{B}^*$ is $\triangleright_{\mathbb{B}}$-stable, there exists a network $G' \in \mathbb{B}^*$, and hence a network $G' \in \mathbb{F}^*$ such that $G' \triangleright_{\mathbb{B}} G$. Given the definition of the relation $\triangleright_{\mathbb{B}}$, $G' \triangleright\triangleright G$. ■

By parts (1) and (2) of Theorem 3, we have

$$\mathbb{B} \subseteq \mathbb{B}^* \subseteq \mathbb{F}^*.$$

Thus, any farsightedly basic network is farsightedly consistent.

6 An Example: The Jackson-Wolinsky Co-author Model

In this section, we compute the largest consistent sets and the Nash equilibria for two supernetwork models of the Jackson and Wolinsky co-author problem. All of our computations are carried out using a *Mathematica* package developed by Kamat and Page (2001).

6.1 Co-author Networks

Consider a situation in which each of three researchers, i_1, i_2, and i_3, is attempting to decide with which other researchers to collaborate. Figure 4 is a directed network representation of one possible configuration of research proposals and researcher collaborations.

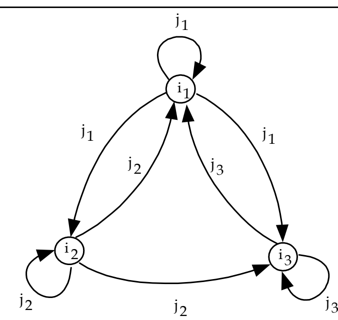

Figure 4: A Directed Co-author Network, G_{32} in Table 1

In Figure 4, the loops, $(j_1, (i_1, i_1))$, $(j_2, (i_2, i_2))$, and $(j_3, (i_3, i_3))$, at each of the nodes indicates that each researcher has undertaken a project on his own. The j_2 arc from node i_2 to i_3 indicates that researcher i_2 has *proposed* a research collaboration to researcher i_3, but since there is no reciprocating arc j_3 from researcher i_3 to researcher i_2 no research collaboration is undertaken. Note that researchers i_1 and i_2

are connected by reciprocal arcs j_1 and j_2 indicating that a research collaboration has been undertaken. Thus, the reciprocating arcs indicate that i_1 and i_2 are co-authors. Note that i_1 and i_3 are also co-authors. Letting n_i denote the number of arcs from researcher i to any researcher i' (including $i' = i$) - and therefore letting n_i denote the total number of research projects *and* research proposals undertaken by researcher i - we shall assume, as do Jackson and Wolinsky (1996), that the payoff to researcher i in any given co-author network is given by

$$\left[\frac{1}{n_i} + \sum_{i'=co-author} \left(\frac{1}{n_i} + \frac{1}{n_{i'}} + \frac{1}{n_i n_{i'}}\right)\right] - c_i(n_i - 1),$$

where $n_i - 1$ is the total number of research proposals and research collaborations and $c_i(n_i - 1)$ is the direct costs of these proposals and collaborations.

Table 1 below, consisting of four panels, lists all possible co-author networks - assuming each researcher undertakes a project on his own. Denote the collection of all possible co-author networks by $\mathbb{G}_{ca}$.

$(j_3,(i_3,i_3))$				
	$(j_2,(i_2,i_2))$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_1))\end{pmatrix}$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_3))\end{pmatrix}$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_1)),\\(j_1,(i_2,i_3))\end{pmatrix}$
$(j_1,(i_1,i_1))$	G_1	G_2	G_3	G_4
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_2))\end{pmatrix}$	G_5	G_6	G_7	G_8
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_3))\end{pmatrix}$	G_9	G_{10}	G_{11}	G_{12}
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_2)),\\(j_1,(i_1,i_3))\end{pmatrix}$	G_{13}	G_{14}	G_{15}	G_{16}

$\begin{pmatrix}(j_3,(i_3,i_3)),\\(j_3,(i_3,i_1))\end{pmatrix}$				
	$(j_2,(i_2,i_2))$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_1))\end{pmatrix}$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_3))\end{pmatrix}$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_1)),\\(j_1,(i_2,i_3))\end{pmatrix}$
$(j_1,(i_1,i_1))$	G_{17}	G_{18}	G_{19}	G_{20}
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_2))\end{pmatrix}$	G_{21}	G_{22}	G_{23}	G_{24}
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_3))\end{pmatrix}$	G_{25}	G_{26}	G_{27}	G_{28}
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_2)),\\(j_1,(i_1,i_3))\end{pmatrix}$	G_{29}	G_{30}	G_{31}	G_{32}

$\begin{pmatrix}(j_3,(i_3,i_3)),\\(j_3,(i_3,i_2))\end{pmatrix}$				
	$(j_2,(i_2,i_2))$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_1))\end{pmatrix}$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_3))\end{pmatrix}$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_1)),\\(j_1,(i_2,i_3))\end{pmatrix}$
$(j_1,(i_1,i_1))$	G_{33}	G_{34}	G_{35}	G_{36}
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_2))\end{pmatrix}$	G_{37}	G_{38}	G_{39}	G_{40}
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_3))\end{pmatrix}$	G_{41}	G_{42}	G_{43}	G_{44}
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_2)),\\(j_1,(i_1,i_3))\end{pmatrix}$	G_{45}	G_{46}	G_{47}	G_{48}

$\begin{pmatrix}(j_3,(i_3,i_3)),\\(j_3,(i_3,i_1)),\\(j_3,(i_3,i_2))\end{pmatrix}$				
	$(j_2,(i_2,i_2))$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_1))\end{pmatrix}$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_3))\end{pmatrix}$	$\begin{pmatrix}(j_2,(i_2,i_2)),\\(j_2,(i_2,i_1)),\\(j_1,(i_2,i_3))\end{pmatrix}$
$(j_1,(i_1,i_1))$	G_{49}	G_{50}	G_{51}	G_{52}
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_2))\end{pmatrix}$	G_{53}	G_{54}	G_{55}	G_{56}
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_3))\end{pmatrix}$	G_{57}	G_{58}	G_{59}	G_{60}
$\begin{pmatrix}(j_1,(i_1,i_1)),\\(j_1,(i_1,i_2)),\\(j_1,(i_1,i_3))\end{pmatrix}$	G_{61}	G_{62}	G_{63}	G_{64}

Table 1: All Possible Co-author Networks, $\mathbb{G}_{ca}$

6.2 Co-author Network Payoffs and Preferences

We construct two possible supernetworks, $\mathbf{G}_{ca1}$and $\mathbf{G}_{ca2}$, over the collection $\mathbb{G}_{ca}$ by varying the payoffs to each researcher of proposing to or collaborating with other researchers.[14] In supernetwork $\mathbf{G}_{ca1}$ we shall assume that the payoffs to each researcher in any given co-author network G contained in the collection $\mathbb{G}_{ca}$ are given by

$$v_{i_k}(G) = \left[\frac{1}{n_{i_k}} + \sum_{i_{k'}=\text{co-author in G}} \left(\frac{1}{n_{i_k}} + \frac{1}{n_{i_{k'}}} + \frac{1}{n_{i_k} n_{i_{k'}}}\right)\right] - .3(n_{i_k} - 1), \tag{6}$$

where $k = 1, 2, 3,\ k' \in \{1, 2, 3\},\ k \neq k'$. Table 2 below, consisting of four panels, summarizes the payoff possibilities in supernetwork $\mathbf{G}_{ca1}$. Each payoff vector - a

[14] By varying the payoffs, we change the configuration of the preference arcs and therefore change the supernetwork.

3-tuple whose components give the payoffs to each researcher - are subscripted by the identification number of the network generating the payoff vector (see Table 1 for networks and identification numbers).

$(1, 1, 1)_1$	$(1, .20, 1)_2$	$(1, .20, 1)_3$	$(1, -.27, 1)_4$
$(.20, 1, 1)_5$	$(1.45, 1.45, 1)_6$	$(.20, .20, 1)_7$	$(1.20, .73, 1)_8$
$(.20, 1, 1)_9$	$(.20, .20, 1)_{10}$	$(.20, .20, 1)_{11}$	$(.20, -.27, 1)_{12}$
$(-.27, 1, 1)_{13}$	$(.73, 1.20, 1)_{14}$	$(-.27, .20, 1)_{15}$	$(.51, .51, 1)_{16}$
$(1, 1, .20)_{17}$	$(1, .20, .20)_{18}$	$(1, .20, .20)_{19}$	$(1, -.27, .20)_{20}$
$(.20, 1, .20)_{21}$	$(1.45, 1.45, .20)_{22}$	$(.20, .20, .20)_{23}$	$(1.20, .73, .20)_{24}$
$(1.45, 1, 1.45)_{25}$	$(1.45, .20, 1.45)_{26}$	$(1.45, .20, 1.45)_{27}$	$(1.45, -.27, 1.45)_{28}$
$(.73, 1, 1.20)_{29}$	$(1.73, 1.20, 1.20)_{30}$	$(.73, .20, 1.20)_{31}$	$(1.51, .51, 1.20)_{32}$
$(1, 1, .20)_{33}$	$(1, .20, .20)_{34}$	$(1, 1.45, 1.45)_{35}$	$(1, .73, 1.20)_{36}$
$(.20, 1, .20)_{37}$	$(1.45, 1.45, .20)_{38}$	$(.20, 1.45, .20)_{39}$	$(1.20, 1.73, 1.20)_{40}$
$(.20, 1, .20)_{41}$	$(.20, .20, .20)_{42}$	$(.20, 1.45, 1.45)_{43}$	$(.20, .73, 1.20)_{44}$
$(-.27, 1, .20)_{45}$	$(.73, 1.20, .20)_{46}$	$(-.27, 1.45, 1.45)_{47}$	$(.51, 1.51, 1.20)_{48}$
$(1, 1, -.27)_{49}$	$(1, .20, -.27)_{50}$	$(1, 1.20, .73)_{51}$	$(1, .51, .51)_{52}$
$(.20, 1, -.27)_{53}$	$(1.45, 1.45, -.27)_{54}$	$(.20, 1.20, .73)_{55}$	$(1.20, 1.51, .51)_{56}$
$(.20, 1, .73)_{57}$	$(1.20, .20, .73)_{58}$	$(1.20, 1.20, 1.73)_{59}$	$(1.20, .51, 1.57)_{60}$
$(.51, 1, .51)_{61}$	$(1.51, 1.20, .51)_{62}$	$(.51, 1.20, 1.51)_{63}$	$(.51, .51, .51)_{64}$

Table 2: Network Payoffs in supernetwork G_{ca1}

For example, in co-author network G_{32} in supernetwork $\mathbf{G}_{ca1}$, the payoffs to researchers are given by

$$(v_{i_1}(G_{32}), v_{i_2}(G_{32}), v_{i_3}(G_{32})) = (1.51, .51, 1.20).$$

In network G_{32}, researcher i_2 is involved in his own project, is involved in one research collaboration with researcher i_1 (hence i_2 and i_1 are co-authors), and proposes a collaboration to researcher i_3 - but because the proposal is not reciprocated, no collaboration between i_2 and i_3 occurs. Thus, $n_{i_2} = 3$ (2 projects and 1 proposal). Researcher i_2's co-author, i_1, is involved in three research projects: one on his own, one with researcher i_1, and one with researcher i_3 (hence $n_{i_1} = 3$). Given payoff function (6), researcher i_2's payoff is computed as follows:

$$v_{i_2}(G_{32}) = \frac{1}{3} + \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{9}\right) - .6 \doteq .51.$$

Here, $.6 = 2 \times .3$ is the direct cost to agent i_2 of his research activities. Thus, his own project generates no direct cost, but does generate an indirect cost expressed as reduction in the average amount of time that can be spent on any one project.

Table 3 below, also consisting of three panels, summarizes the payoff possibilities in supernetwork $\mathbf{G}_{ca2}$. In supernetwork $\mathbf{G}_{ca2}$, we assume that payoffs are given by

$$v_{i_k}(G) = \begin{cases} \left[\frac{1}{n_{i_k}} + \sum_{i_{k'}=\text{co-author in G}} \left(\frac{1}{n_{i_k}} + \frac{1}{n_{i_{k'}}} + \frac{1}{n_{i_k} n_{i_{k'}}}\right)\right] - 1.2, & \text{if } n_{i_k} - 1 = 2 \\ \left[\frac{1}{n_{i_k}} + \sum_{i_{k'}=\text{co-author in G}} \left(\frac{1}{n_{i_k}} + \frac{1}{n_{i_{k'}}} + \frac{1}{n_{i_k} n_{i_{k'}}}\right)\right] - .3, & \text{if } n_{i_k} - 1 = 1. \end{cases} \quad (7)$$

Thus, in supernetwork $\mathbf{G}_{ca2}$ the direct costs of research activities (proposals and collaborations) are increasing.

$(1,1,1)_1$	$(1,.20,1)_2$	$(1,.20,1)_3$	$(1,-.87,1)_4$
$(.20,1,1)_5$	$(1.45,1.45,1)_6$	$(.20,.20,1)_7$	$(1.20,.13,1)_8$
$(.20,1,1)_9$	$(.20,.20,1)_{10}$	$(.20,.20,1)_{11}$	$(.20,-.87,1)_{12}$
$(-.20,1,1)_{13}$	$(.13,1.20,1)_{14}$	$(-.87,.20,1)_{15}$	$(-.09,-.09,1)_{16}$
$(1,1,.20)_{17}$	$(1,.20,.20)_{18}$	$(1,.20,.20)_{19}$	$(1,-.87,.20)_{20}$
$(.20,1,.20)_{21}$	$(1.45,1.45,.20)_{22}$	$(.20,.20,.20)_{23}$	$(1.20,.13,.20)_{24}$
$(1.45,1,1.45)_{25}$	$(1.45,.20,1.45)_{26}$	$(1.45,.20,1.45)_{27}$	$(1.45,-.87,1.45)_{28}$
$(.13,1,1.20)_{29}$	$(1.13,1.20,1.20)_{30}$	$(.13,.20,1.20)_{31}$	$(.91,-.09,1.20)_{32}$
$(1,1,.20)_{33}$	$(1,.20,.20)_{34}$	$(1,1.45,1.45)_{35}$	$(1,.13,1.20)_{36}$
$(.20,1,.20)_{37}$	$(1.45,1.45,.20)_{38}$	$(.20,1.45,.20)_{39}$	$(1.20,1.13,1.20)_{40}$
$(.20,1,.20)_{41}$	$(.20,.20,.20)_{42}$	$(.20,1.45,1.45)_{43}$	$(.20,.13,1.20)_{44}$
$(-.87,1,.20)_{45}$	$(.13,1.20,.20)_{46}$	$(-.87,1.45,1.45)_{47}$	$(-.09,.91,1.20)_{48}$
$(1,1,-.87)_{49}$	$(1,.20,-.87)_{50}$	$(1,1.20,.13)_{51}$	$(1,-.09,-.09)_{52}$
$(.20,1,-.87)_{53}$	$(1.45,1.45,-.87)_{54}$	$(.20,1.20,.13)_{55}$	$(1.20,.91,-.09)_{56}$
$(.20,1,.13)_{57}$	$(1.20,.20,.13)_{58}$	$(1.20,1.20,1.13)_{59}$	$(1.20,-.09,.91)_{60}$
$(-.09,1,-.09)_{61}$	$(.91,1.20,-.09)_{62}$	$(-.09,1.20,.91)_{63}$	$(.69,.69,.69)_{64}$

Table 3: Network Payoffs in supernetwork G_{ca2}

6.3 Computational Results

In Table 4 below we list the largest consistent sets corresponding to supernetworks $\mathbf{G}_{ca1}$ and $\mathbf{G}_{ca2}$. We also list the Nash networks and the Pareto efficient networks for each of our supernetworks.

	$\mathbf{G}_{ca1}$	$\mathbf{G}_{ca2}$
Largest Consistent Set $\mathbb{F}^*$	$\{G_6, G_{25}, G_{35}, G_{30}, G_{40}, G_{59}\}$	$\{G_6, G_{25}, G_{35}\}$
Nash Networks	$\{G_1, G_6, G_{25}, G_{35}, G_{30}, G_{40}, G_{59}\}$	$\{G_1, G_6, G_{25}, G_{35}\}$
Pareto Efficient Networks	$\{G_6, G_{25}, G_{35}, G_{30}, G_{40}, G_{59}\}$	$\{G_1, G_6, G_{25}, G_{35}\}$

Table 4: Summary of Computational Results

Figure 5 depicts the all co-author networks contained in Table 4.

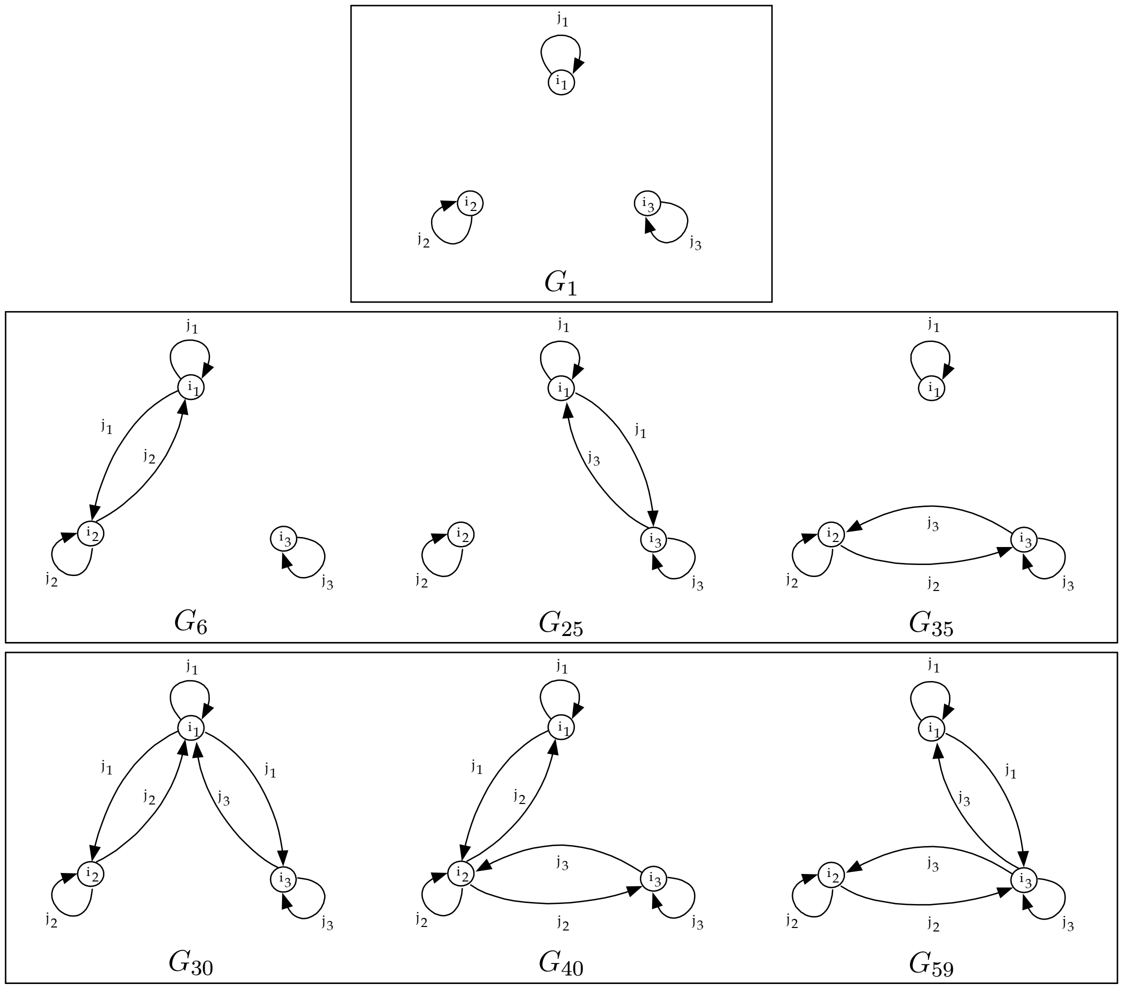

Figure 5: Farsightedly Consistent Networks Relative to Information Sharing Supernetwork G_I

Note that in each supernetwork, the largest consistent set is a subset of the Nash set. Also, note that in each supernetwork, the farsightedly consistent networks as well as the Nash networks are such that all research proposals are reciprocated - thus, in all the farsightedly consistent networks and Nash networks researchers either collaborate or work alone. Finally, note that in supernetwork $\mathbf{G}_{ca2}$ where the costs of research activities are increasing, the farsightedly consistent networks as well as the Nash networks are such that no researcher is involved in more than 1 collaboration - it is simply too costly.

We conclude the co-author example by considering the issue of Pareto efficiency. Define the set of Pareto efficient networks relative to the collection $\mathbb{G}_{ca}$ as follows:

$$\left\{G \in \mathbb{G}_{ca} : \text{there does not exist } G' \text{ such that } v_d(G') > v_d(G) \text{ for all } d \in D\right\}.$$

By examination of the payoffs in Tables 2 and 3, we conclude that in both supernetworks $\mathbf{G}_{ca1}$ and $\mathbf{G}_{ca2}$, the set of farsightedly consistent networks is contained in the set of Pareto efficient networks. In fact, in supernetwork $\mathbf{G}_{ca1}$, the largest consistent set equals the Pareto efficient set. In supernetwork $\mathbf{G}_{ca2}$, only Pareto efficient, Nash network G_1 is not contained in the largest consistent set. But note that G_1 is weakly Pareto dominated by each of the farsightedly consistent networks, G_6, G_{25}, and G_{35}.

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