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ABSTRACT

This paper uses the approach of Im, Pesaran and Shin (2003) to propose seasonal unit root tests for dynamic heterogeneous panels based on the means of the individuals HEGY test statistics. The standardised t-bar and F-bar statistics are simply averages of the HEGY tests across groups. These statistics converge to standard normal variates.

Keywords: Heterogeneous dynamic panels; Monte Carlo; seasonal unit roots;

JEL classification: C12; C15; C22; C23

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1. Introduction

Im, Pesaran and Shin (2003) (IPS) proposed a test for the presence of unit roots in panels, that combines information from the time-series dimension with that from the cross-section dimension, such that fewer time observations are required for the test to have power.

Many economic time series contain important seasonal components and a variety of tests have been proposed to test for seasonal unit roots see Osborn and Ghysels (2001) for a review of these tests. Of these tests the one proposed by Hylleberg, Engle, Granger and Yoo (1990) (HEGY) has proved to be the most popular.

In this paper, we look at using the approach of IPS to investigate the performance of the HEGY test in dynamic heterogeneous panels. Based on Monte Carlo simulations we find that the standardised averaged test statistics from the HEGY auxiliary regression follow a standard normal distribution even for a relatively small number of data points.

The plan of the paper is as follows. Section 2 briefly reviews the IPS approach to unit root testing in panels and sets up the model used to develop the HEGY panel seasonal unit root tests. Section 3 presents the Monte Carlo results.

2. IPS unit root test and basic framework

IPS presented a method to test for the presence of unit roots in dynamic heterogeneous panels. They consider a sample of N cross section units observed over T time periods. The IPS test averages the (Augmented) Dickey-Fuller statistic obtained across the N cross-sectional units of the panel (denoted as $\tilde{t}bar_{NT} = \frac{1}{N}\sum_{i=1}^{N}\tilde{t}_{iT}$, where $\tilde{t}_{iT}$ is the ADF test for the i^{th} cross-sectional unit). and show that a suitable standardisation of the $\tilde{t}bar_{NT}$ statistic, denoted as $Z_{\tilde{t}bar}$, follows a standard normal distribution.

Generalising the HEGY test for seasonal unit roots, to a panel in which there is sample of *N* cross sections (industries, countries) observed over *T* time periods:

$$\varphi_i(L)y_{4it} = \mu_{it} + \pi_{1i}y_{1it-1} + \pi_{2i}y_{2it-1} + \pi_{3i}y_{3it-2} + \pi_{4i}y_{3it-1} + \varepsilon_{it},\ i = 1,\ldots,N,\ t = 1,\ldots,T \quad (1)$$

where $\mu_{it} = \alpha_i + \beta_i t + \sum_{j=1}^{s-1}\gamma_{is}D_{st}$, $D_{st} = \begin{cases} 1 & \text{in season } s \\ 0 & \text{otherwise} \end{cases}$, $\varphi_i(L)$ is a p_i^{th} ordered polynomial in the lag operator, L, $\varepsilon_{it} \sim N(0,\sigma_{\varepsilon_i}^2)$ and $y_{1it} = y_{it} + y_{it-1} + y_{it-2} + y_{it-3}$, $y_{2it} = -y_{it} + y_{it-1} - y_{it-2} + y_{it-3}$, $y_{3it} = -y_{it} + y_{it-2}$ and $y_{4it} = \Delta_4 y_{it} = y_{it} - y_{it-4}$.

HEGY test for the existence of a unit root by testing $H_0 : \pi_1 = 0$ against $H_1 : \pi_1 < 0$, and for the existence of a seasonal unit root by testing $H_0 : \pi_2 = 0$ against $H_1 : \pi_2 < 0$ and simultaneously testing $H_0 : \pi_3 = \pi_4 = 0$ against $H_1 : \pi_3 < 0, \pi_4 \neq 0$. A null hypothesis of a seasonal unit root is only rejected when both the t-test for π_2 and the joint F-test for π_3 and π_4 are rejected. Subsequently, Ghysels *et*. *al*. (1994) suggest using a test of $H_0 : \pi_2 = \pi_3 = \pi_4 = 0$ against $H_1 : \pi_2 < 0, \pi_3 < 0, \pi_4 \neq 0$.

In a panel context, the null hypothesis to test the presence of a unit root, for example, becomes $H_0 : \pi_{1i} = 0 \quad \forall i$ against $H_0 : \pi_{1i} < 0$ for $i = 1, 2\ldots, N_1$, $\pi_{1i} = 0$, for $i = N_1 + 1, N_1 + 2, \ldots, N$. This allows some, but not all, of the individual series to have a unit root, but assumes that a non-zero fraction of the processes are stationary.

3. Monte Carlo simulation results

In this section we undertake Monte Carlo simulation to examine the finite sample properties of the HEGY-IPS test. Simulations are undertaken under the null hypothesis, $\pi_{1i} = \pi_{2i} = \pi_{3i} = \pi_{4i} = 0$ in equation (1):

$$y_{it} - y_{it-4} = \mu_{it} + \sum_{j=1}^{p_i} \varphi_{ji} \Delta_4 y_{it} + \varepsilon_{it}, \quad i = 1, \dots, N, t = 1, \dots, T \tag{2}$$

where $\varepsilon_{it} \sim N(0, \sigma_{\varepsilon i}^2)$ and $\sigma_{\varepsilon i}^2 \sim U[0.5, 1.5]$, and $\sigma_{\varepsilon i}^2$ are generated independently of ε_{it} and are fixed for all replications, where $N = (5, 7, 10, 15, 25, 40)$ and $T = (20, 32, 40, 60, 100)$.

The HEGY statistics from estimating equation (1) for the i^{th} group are given by the t-ratios on $\pi_{ji}, j = 1,2$ and the F-tests of the joint significance of π_{2i}, π_{3i} and $\pi_{2i}, \pi_{3i}, \pi_{4i}$. Denote the estimated t-ratio as $\tilde{t}_{jiT}$,

$$\tilde{t}_{jiT} = \frac{\hat{\pi}_j - 0}{se(\hat{\pi}_j)} \quad j = 1,2$$

and the F-test as $\tilde{F}_{jiT}$,

$$F_{jiT} = (R_j \hat{\pi}_i)' \left[R_j \hat{V}_{\hat{\pi}_i} R_j' \right]^{-1} (R_j \hat{\pi}_i) / j, \quad j = 2,3$$

where $R_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\hat{\pi}_i' = \begin{pmatrix} \hat{\pi}_{1i} & \hat{\pi}_{2i} & \hat{\pi}_{3i} & \hat{\pi}_{4i} \end{pmatrix}'$ and the estimated variance-covariance matrix from equation (1), is written in partitioned form as: $\hat{V}_i = \begin{bmatrix} \hat{V}_{\pi_i} & \hat{C}_{\pi_i \mu_i} & \hat{C}_{\pi_i \varphi_i} \\ \hat{C}_{\pi_i \mu_i} & \hat{V}_{\mu_i} & \hat{C}_{\mu_i \varphi_i} \\ \hat{C}_{\pi_i \varphi_i} & \hat{C}_{\mu_i \varphi_i} & \hat{V}_{\varphi_i} \end{bmatrix}$,

where, for example, $\hat{V}_{\pi_i}$ is the estimated (4×4) variance-covariance matrix for the coefficients on π_i, and $\hat{C}_{\pi_i \mu_i}$ is the estimated variance-covariance matrix between the π_i and μ_i terms.

For a fixed *T* define the average statistics:

$$\tilde{t}_j bar_{NT} = \frac{1}{N}\sum_{i=1}^{N}\tilde{t}_{jiT} \quad j = 1,2$$

and

$$\tilde{F}_j bar_{NT} = \frac{1}{N}\sum_{i=1}^{N}\tilde{F}_{jiT}, \quad j = 2,3.$$

Following IPS, consider the standardised statistics:

$$W_{\tilde{t}_j bar} = \frac{\sqrt{N}\left\{\tilde{t}_j bar_{NT} - \frac{1}{N}\sum_{i=1}^{N} E\left[\tilde{t}_{jiT}(p_i, 0 \mid \pi_i = 0)\right]\right\}}{\sqrt{\frac{1}{N}\sum_{i=1}^{N} Var\left[\tilde{t}_{jiT}(p_i, 0 \mid \pi_i = 0)\right]}} \Rightarrow N(0,1), \quad j = 1,2$$

and

$$W_{\tilde{F}_j bar} = \frac{\sqrt{N}\left\{\tilde{F}_j bar_{NT} - \frac{1}{N}\sum_{i=1}^{N} E\left[\tilde{F}_{jiT}(p_i, 0 \mid \pi_i = 0)\right]\right\}}{\sqrt{\frac{1}{N}\sum_{i=1}^{N} Var\left[\tilde{F}_{jiT}(p_i, 0 \mid \pi_i = 0)\right]}} \Rightarrow N(0,1), \quad j = 2,3$$

where $E\left[\tilde{t}_{jiT}(p_i, 0 \mid \pi_i = 0)\right]$ $\left(E\left[\tilde{F}_{jiT}(p_i, 0 \mid \pi_i = 0)\right]\right)$ and $Var\left[\tilde{t}_{jiT}(p_i, 0 \mid \pi = 0)\right]$ $\left(Var\left[\tilde{F}_{jiT}(p_i, 0 \mid \pi_i = 0)\right]\right)$ are the mean and variance of $\tilde{t}_{jiT}$ ($\tilde{F}_{jiT}$) in the HEGY model, when $\pi_{1i} = \pi_{2i} = \pi_{3i} = \pi_{4i} = 0$.

Table 1 reports the values of $E\left[\tilde{t}_{jiT}(p_i, 0 \mid \pi_i = 0)\right]$ and $Var\left[\tilde{t}_{jT}(p_i, 0 \mid \pi_i = 0)\right]$, $j = 1,2$ and $E\left[\tilde{F}_{jiT}(p_i, 0 \mid \pi_i = 0)\right]$ and $Var\left[\tilde{F}_{jiT}(p_i, 0 \mid \pi_i = 0)\right]$, $j = 2,3$, for different values of *T* and *p*, and for different combinations of deterministic components in the HEGY model. These results are based upon 20,000 replications.

Through simulations it appears that in the HEGY model (when $p_i = 0$), the second moment of t_{jiT} exists only for $T \geq 16$ (when there is a constant and trend) and for $T \geq 20$ (when there are seasonal dummy variables). In addition, for the second moment of F_{jiT} to exist requires at least $T \geq 20$ for all combinations of the deterministic components.

We now consider three Monte Carlo experiments to examine the size and power (at the 5% significance level) of the HEGY-IPS test, using 5,000 replications. Table 2 reports the size of the tests when there is no serial correlation and the model includes a constant and a constant and trend as deterministic components. The tests for both $W_{t_1 bar}$ and $W_{t_2 bar}$ are approximately correctly sized. However, both the $W_{F_2 bar}$ and the $W_{F_3 bar}$ tests are slightly over-sized especially for smaller *N* and *T*. This table also reports the power of the HEGY-IPS test, when the data is generated as $y_{it} = 0.9 y_{it-4} + \varepsilon_{it}, \quad i = 1,\ldots,N, t = 1,\ldots,T$.

In a second set of experiments, we allow for the presence of heterogeneous AR(1) serial correlation in ε_{it}, such that,

$$\varepsilon_{it} = \rho_i \varepsilon_{it-1} + \eta_{it}, \quad i = 1,\ldots N, t = 1,\ldots,T$$

where $\eta_{it} \sim N(0, \sigma^2_{\eta_i})$, $\rho_i \sim U[0.2, 0.4]$ and ρ_i is generated independently of η_{it}. Table 3 reports the size of the HEGY-IPS test for *p*=0,1,2,3,4, when there is only a constant in the HEGY model. The table demonstrates the importance of not underestimating the order of the lag length, with the empirical size for *p*=0, substantially different from the nominal 5%, with $W_{t_1 bar}$ markedly under-sized, but all of the other tests becoming increasingly over-sized as *T* increases. There are little costs in terms of size to over-specifying the lag length, with the empirical size of $W_{F_2 bar}$ and $W_{F_3 bar}$ actually improving. However, the power of all of the tests falls with an over-specified lag length.

In a third set of experiments, we allow for the presence of heterogeneous MA(1) serial correlation in ε_{it}, such that,

$$\varepsilon_{it} = \theta_i \eta_{it-1} + \eta_{it}, \quad i = 1, \ldots N, t = 1, \ldots, T$$

where $\eta_{it} \sim N(0, \sigma^2_{\eta_i})$, $\theta_i \sim U[-0.4, -0.2]$ and θ_i is generated independently of η_{it}. Table 4 reports the size of the HEGY-IPS test for *p*=0,1,2,3,4. In this case there are severe size distortions for *p*=0, with $W_{t_1 bar}$ massively over-sized. The other tests are also over-sized and this becomes increasingly so as both *T* and *N* increase. Increasing *p* improves the size of these tests, but even for *p*=3 there is consistent evidence that $W_{t_1 bar}$, $W_{F_2 bar}$ and $W_{F_3 bar}$ are all still marginally over-sized.

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Table 1: Mean and variance correction for t_1bar and t_2bar

		t_1bar					t_2bar				
p		*T*=20	*T*=32	*T*=40	*T*=60	*T*=100	*T*=20	*T*=32	*T*=40	*T*=60	*T*=100
Constant, seasonal dummies, trend											
0	mean	-1.862	-2.004	-2.042	-2.091	-2.130	-1.320	-1.416	-1.443	-1.474	-1.501
0	variance	0.725	0.622	0.604	0.575	0.567	0.720	0.667	0.676	0.673	0.684
1	mean	-1.721	-1.915	-1.972	-2.044	-2.102	-1.107	-1.277	-1.330	-1.398	-1.454
1	variance	0.751	0.629	0.611	0.578	0.567	0.734	0.683	0.691	0.687	0.693
2	mean	-1.692	-1.903	-1.965	-2.042	-2.101	-1.178	-1.332	-1.376	-1.431	-1.475
2	variance	0.848	0.666	0.641	0.595	0.575	0.793	0.710	0.715	0.700	0.706
3	mean	-1.561	-1.820	-1.898	-1.996	-2.073	-0.994	-1.207	-1.273	-1.360	-1.431
3	variance	0.900	0.685	0.656	0.605	0.580	0.825	0.738	0.735	0.713	0.719
4	mean	-1.517	-1.817	-1.903	-2.007	-2.084	-1.054	-1.264	-1.326	-1.401	-1.459
4	variance	1.002	0.732	0.684	0.623	0.591	0.890	0.766	0.756	0.724	0.724
Constant, seasonal dummies, no trend											
0	mean	-1.314	-1.406	-1.432	-1.470	-1.493	-1.317	-1.407	-1.435	-1.468	-1.496
0	variance	0.737	0.697	0.685	0.682	0.688	0.739	0.687	0.690	0.682	0.690
1	mean	-1.210	-1.339	-1.378	-1.434	-1.471	-1.210	-1.340	-1.380	-1.431	-1.475
1	variance	0.766	0.720	0.705	0.697	0.694	0.764	0.704	0.707	0.697	0.700
2	mean	-1.184	-1.325	-1.369	-1.429	-1.468	-1.183	-1.327	-1.371	-1.426	-1.472
2	variance	0.818	0.750	0.724	0.712	0.703	0.814	0.732	0.732	0.710	0.713
3	mean	-1.093	-1.266	-1.320	-1.395	-1.447	-1.093	-1.267	-1.322	-1.392	-1.451
3	variance	0.862	0.771	0.746	0.727	0.717	0.860	0.758	0.753	0.724	0.726
4	mean	-1.070	-1.262	-1.321	-1.400	-1.452	-1.071	-1.263	-1.323	-1.397	-1.456
4	variance	0.919	0.804	0.768	0.744	0.726	0.922	0.793	0.777	0.736	0.732
Constant, no seasonal dummies, tend											
0	mean	-1.886	-2.007	-2.044	-2.093	-2.134	-0.253	-0.300	-0.319	-0.347	-0.377
0	variance	0.707	0.625	0.613	0.587	0.577	0.897	0.907	0.928	0.945	0.962
1	mean	-1.893	-2.017	-2.051	-2.096	-2.133	-0.191	-0.246	-0.270	-0.308	-0.349
1	variance	0.746	0.657	0.640	0.597	0.582	1.036	0.997	0.995	0.986	0.985
2	mean	-1.885	-2.019	-2.057	-2.102	-2.136	-0.277	-0.314	-0.330	-0.354	-0.380
2	variance	0.826	0.694	0.672	0.617	0.591	0.839	0.886	0.913	0.939	0.964
3	mean	-1.960	-2.050	-2.077	-2.109	-2.138	-0.187	-0.250	-0.275	-0.313	-0.351
3	variance	0.977	0.747	0.706	0.638	0.600	0.969	0.966	0.975	0.978	0.987
4	mean	-1.624	-1.852	-1.926	-2.019	-2.091	-0.189	-0.254	-0.281	-0.321	-0.359
4	variance	0.979	0.758	0.711	0.642	0.603	0.941	0.949	0.958	0.966	0.977
Constant, no seasonal dummies, no trend											
0	mean	-1.322	-1.407	-1.433	-1.472	-1.495	-0.241	-0.290	-0.309	-0.339	-0.371
0	variance	0.761	0.737	0.719	0.711	0.706	0.985	0.962	0.970	0.972	0.978
1	mean	-1.335	-1.412	-1.436	-1.471	-1.493	-0.214	-0.266	-0.288	-0.322	-0.359
1	variance	0.811	0.765	0.744	0.726	0.714	1.044	1.003	1.000	0.990	0.988
2	mean	-1.326	-1.409	-1.435	-1.471	-1.493	-0.262	-0.302	-0.319	-0.346	-0.374
2	variance	0.860	0.797	0.766	0.743	0.723	0.924	0.940	0.955	0.966	0.980
3	mean	-1.354	-1.420	-1.441	-1.472	-1.491	-0.221	-0.274	-0.295	-0.328	-0.362
3	variance	0.934	0.829	0.791	0.758	0.736	0.969	0.970	0.980	0.983	0.991
4	mean	-1.128	-1.284	-1.335	-1.408	-1.457	-0.180	-0.245	-0.272	-0.313	-0.353
4	variance	0.965	0.863	0.818	0.778	0.747	1.030	1.007	1.004	0.994	0.993
No constant, no seasonal dummies, no trend											
0	mean	-0.223	-0.281	-0.299	-0.332	-0.367	-0.226	-0.278	-0.299	-0.331	-0.367
0	variance	1.071	1.034	1.027	1.010	0.995	1.089	1.020	1.014	0.999	0.993
1	mean	-0.240	-0.290	-0.306	-0.336	-0.368	-0.242	-0.287	-0.306	-0.334	-0.367
1	variance	1.030	1.018	1.021	1.007	0.995	1.045	1.008	1.005	0.996	0.994
2	mean	-0.242	-0.293	-0.309	-0.339	-0.370	-0.244	-0.290	-0.309	-0.338	-0.370
2	variance	1.004	1.008	1.011	1.002	0.994	1.024	0.999	0.999	0.993	0.995
3	mean	-0.254	-0.300	-0.315	-0.342	-0.371	-0.256	-0.297	-0.315	-0.341	-0.370
3	variance	0.958	0.986	1.001	0.997	0.994	0.974	0.978	0.988	0.990	0.997
4	mean	-0.169	-0.239	-0.265	-0.308	-0.350	-0.171	-0.237	-0.264	-0.307	-0.349
4	variance	1.124	1.076	1.065	1.033	1.009	1.140	1.069	1.050	1.022	1.008

Table 1 (cont'd): Mean and variance correction for F_2bar and F_3bar

		F_2bar					F_3bar				
p		*T*=20	*T*=32	*T*=40	*T*=60	*T*=100	*T*=20	*T*=32	*T*=40	*T*=60	*T*=100
Constant, seasonal dummies, trend											
0	mean	2.583	2.728	2.784	2.861	2.924	2.745	2.839	2.876	2.926	2.970
0	variance	5.433	4.111	3.982	3.903	3.738	4.822	3.294	3.055	2.815	2.603
1	mean	2.481	2.653	2.723	2.821	2.899	2.521	2.674	2.740	2.832	2.912
1	variance	5.361	3.985	3.870	3.841	3.703	4.342	2.985	2.822	2.682	2.528
2	mean	2.173	2.412	2.518	2.676	2.808	2.529	2.668	2.734	2.827	2.909
2	variance	5.054	3.668	3.621	3.674	3.603	4.771	2.980	2.835	2.684	2.526
3	mean	2.085	2.337	2.457	2.632	2.783	2.247	2.476	2.578	2.723	2.848
3	variance	5.113	3.571	3.531	3.596	3.583	4.428	2.752	2.638	2.549	2.467
4	mean	2.186	2.418	2.533	2.695	2.828	2.329	2.530	2.629	2.765	2.878
4	variance	6.429	3.793	3.666	3.675	3.636	5.879	3.013	2.803	2.649	2.517
Constant, seasonal dummies, no trend											
0	mean	2.643	2.763	2.813	2.881	2.935	2.788	2.861	2.894	2.937	2.976
0	variance	5.287	4.178	4.062	3.983	3.787	4.605	3.309	3.078	2.850	2.624
1	mean	2.513	2.680	2.746	2.838	2.910	2.621	2.750	2.804	2.877	2.941
1	variance	5.179	4.066	3.942	3.907	3.748	4.368	3.136	2.935	2.768	2.579
2	mean	2.299	2.526	2.616	2.747	2.853	2.609	2.741	2.797	2.872	2.938
2	variance	4.967	3.890	3.797	3.807	3.689	4.515	3.095	2.914	2.745	2.565
3	mean	2.222	2.458	2.561	2.706	2.830	2.463	2.636	2.710	2.813	2.904
3	variance	4.983	3.785	3.702	3.738	3.674	4.502	2.981	2.813	2.674	2.541
4	mean	2.243	2.466	2.571	2.719	2.841	2.388	2.571	2.660	2.782	2.887
4	variance	5.663	3.872	3.753	3.759	3.690	5.092	3.053	2.844	2.690	2.542
Constant, no seasonal dummies, trend											
0	mean	0.965	0.957	0.971	0.991	1.017	0.980	0.990	1.006	1.029	1.055
0	variance	1.244	1.034	1.029	1.024	1.037	0.881	0.739	0.730	0.718	0.728
1	mean	0.926	0.945	0.964	0.989	1.016	0.993	1.001	1.014	1.033	1.055
1	variance	1.181	1.022	1.019	1.020	1.032	0.951	0.768	0.746	0.723	0.725
2	mean	1.019	1.013	1.022	1.029	1.039	1.029	1.034	1.046	1.056	1.070
2	variance	1.431	1.161	1.142	1.108	1.068	1.040	0.817	0.796	0.761	0.742
3	mean	1.109	1.049	1.045	1.039	1.041	1.129	1.077	1.073	1.068	1.072
3	variance	1.790	1.259	1.200	1.128	1.074	1.310	0.891	0.840	0.777	0.746
4	mean	1.071	1.012	1.008	1.011	1.022	1.070	1.035	1.038	1.045	1.059
4	variance	1.673	1.158	1.097	1.059	1.033	1.171	0.808	0.772	0.736	0.724
Constant, no seasonal dummies, trend											
0	mean	1.006	0.999	1.008	1.019	1.034	1.035	1.034	1.044	1.055	1.070
0	variance	1.332	1.128	1.111	1.082	1.071	0.969	0.805	0.785	0.754	0.748
1	mean	1.024	1.008	1.015	1.023	1.035	1.065	1.050	1.054	1.060	1.071
1	variance	1.424	1.160	1.131	1.088	1.069	1.061	0.838	0.803	0.758	0.747
2	mean	1.019	1.013	1.022	1.030	1.040	1.056	1.051	1.058	1.064	1.075
2	variance	1.382	1.151	1.137	1.106	1.071	1.049	0.832	0.809	0.769	0.749
3	mean	1.054	1.029	1.032	1.035	1.041	1.093	1.068	1.070	1.070	1.076
3	variance	1.524	1.200	1.166	1.116	1.073	1.156	0.863	0.830	0.778	0.750
4	mean	1.100	1.052	1.045	1.038	1.039	1.118	1.080	1.076	1.071	1.074
4	variance	1.654	1.243	1.180	1.118	1.066	1.197	0.874	0.832	0.773	0.744
No constant, no seasonal dummies, no trend											
0	mean	1.099	1.058	1.056	1.050	1.051	1.130	1.092	1.089	1.084	1.086
0	variance	1.572	1.260	1.214	1.148	1.104	1.149	0.896	0.851	0.794	0.768
1	mean	1.135	1.075	1.067	1.055	1.053	1.146	1.101	1.095	1.087	1.087
1	variance	1.718	1.312	1.246	1.158	1.104	1.199	0.914	0.863	0.797	0.768
2	mean	1.006	1.011	1.021	1.032	1.043	1.079	1.066	1.071	1.073	1.081
2	variance	1.297	1.132	1.129	1.109	1.078	1.053	0.848	0.822	0.780	0.756
3	mean	1.024	1.021	1.029	1.036	1.044	1.078	1.069	1.074	1.076	1.082
3	variance	1.364	1.163	1.150	1.117	1.079	1.065	0.850	0.830	0.784	0.758
4	mean	1.184	1.108	1.091	1.068	1.056	1.212	1.138	1.121	1.099	1.090
4	variance	1.844	1.369	1.287	1.185	1.099	1.349	0.964	0.903	0.815	0.764

Table 2: Size and power of the HEGY-IPS test: No serial correlation

N	T=20				T=32				T=40				T=60				T=100			
	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}
SIZE																				
Constant , no seasonal dummies, no trend																				
5	4.52	4.56	6.88	6.70	4.68	5.02	7.20	6.62	5.00	4.84	6.44	6.54	5.28	4.94	6.64	6.42	4.72	4.96	6.14	5.74
7	4.70	4.34	6.98	7.02	5.18	4.98	6.50	6.44	5.38	5.22	6.76	6.48	4.90	4.94	6.50	6.08	4.60	4.72	6.86	6.04
10	5.04	4.76	6.58	6.70	5.12	4.74	6.88	6.54	4.90	4.90	6.20	5.90	4.68	4.60	6.36	5.80	4.52	4.78	6.18	6.26
15	5.16	4.54	6.58	6.58	5.40	4.72	6.90	6.32	5.24	5.12	6.34	5.82	4.68	4.82	5.88	5.92	4.56	4.44	6.72	6.30
25	5.04	4.68	6.32	5.98	4.90	4.66	6.36	6.00	5.72	4.46	6.06	6.10	4.94	4.40	6.30	6.14	4.66	4.54	6.10	5.88
40	5.08	4.90	5.88	5.80	5.12	4.38	6.02	5.60	5.44	4.74	5.72	5.66	4.86	5.06	6.30	6.02	4.82	4.82	5.82	5.82
Constant, no seasonal dummies, trend																				
5	5.06	4.76	6.86	6.76	5.44	5.18	7.06	6.26	5.34	4.92	6.30	6.52	5.46	5.16	6.68	6.68	5.02	4.92	6.12	5.82
7	5.00	4.34	7.06	7.36	5.30	5.10	6.86	6.56	5.58	5.46	6.52	6.24	5.38	5.10	6.60	6.42	4.38	4.70	6.94	6.18
10	4.76	4.66	6.44	6.52	5.12	4.80	6.66	6.36	5.92	4.88	5.76	5.96	5.14	4.68	6.48	5.96	4.94	4.78	6.16	6.36
15	5.38	4.92	6.68	6.76	5.10	5.22	6.90	6.38	5.66	5.06	6.00	5.66	5.40	4.86	6.04	5.98	4.54	4.48	6.80	6.36
25	5.64	4.52	6.16	6.30	4.76	4.86	6.56	5.94	5.44	4.48	5.96	5.72	5.36	4.36	6.14	6.10	4.62	4.56	5.96	5.88
40	5.08	4.80	5.92	5.88	4.76	4.34	6.12	5.42	5.66	4.68	5.66	5.72	5.12	5.00	6.28	6.06	5.22	4.76	5.92	5.84
POWER																				
Constant , no seasonal dummies, no trend																				
5	6.94	24.02	11.94	14.10	8.00	34.70	20.36	24.44	9.00	40.90	25.94	31.42	11.10	57.16	43.96	53.74	17.44	82.26	76.72	87.34
7	8.40	31.92	12.98	15.66	9.06	46.40	23.88	29.08	11.14	55.34	30.44	37.98	12.66	74.22	53.50	65.78	23.18	94.60	87.86	95.02
10	9.46	42.72	15.34	18.08	10.58	62.74	29.34	35.88	13.30	71.22	38.48	47.88	16.04	89.62	65.66	77.48	31.20	99.20	95.62	99.02
15	10.68	59.96	17.14	22.54	12.82	81.02	36.74	46.12	16.46	89.18	49.06	60.94	21.04	97.60	79.48	89.44	43.00	100.0	99.44	99.96
25	13.54	82.24	21.42	29.44	16.40	96.20	49.90	61.92	21.56	98.66	67.24	79.06	31.00	99.92	94.18	98.46	65.18	100.0	100.0	100.0
40	17.20	95.42	28.20	40.62	22.94	99.66	66.52	79.24	29.50	99.90	84.10	92.80	44.66	100.0	99.26	99.82	84.04	100.0	100.0	100.0
Constant, no seasonal dummies, trend																				
5	7.08	24.72	12.24	15.92	7.28	35.44	22.20	26.62	7.22	41.72	27.70	33.72	7.58	57.50	45.90	56.10	8.86	82.38	77.84	88.18
7	7.14	33.40	13.72	17.34	7.26	47.98	25.48	31.76	8.34	56.26	32.72	40.70	8.04	74.90	56.18	67.80	10.22	94.58	88.80	95.68
10	8.02	44.40	15.68	20.36	8.18	63.82	31.38	39.98	8.44	72.06	40.64	51.14	9.14	89.72	68.10	80.28	12.28	99.18	96.28	99.18
15	9.14	61.26	18.34	25.74	9.18	82.10	40.06	51.12	9.12	89.34	52.16	65.30	10.44	97.54	81.58	91.22	14.60	100.0	99.48	99.98
25	10.44	83.50	22.10	33.32	10.16	96.50	54.10	67.38	10.90	98.70	70.80	83.16	12.08	99.92	95.24	98.88	19.62	100.0	100.0	100.0
40	12.30	95.82	29.82	45.78	11.64	99.72	70.76	83.82	13.12	99.92	86.68	95.10	14.82	100.0	99.52	99.92	27.08	100.0	100.0	100.0

NOTE: For power the DGP is written as $y_{it} = \alpha_i + \phi_i y_{it-1} + \varepsilon_{it}$, where $\phi_i = 0.9$, $\alpha_i = (1-\phi_i)\delta_i$, $\delta_i \sim N(0,1)$, $\varepsilon_{it} \sim N(0,\sigma_i^2)$ and $\sigma_i^2 \sim U[0.5,1.5]$.

Table 3: Size of the HEGY-IPS test: AR(1) errors $\rho_i \sim U[0.2, 0.4]$, constant, no seasonal dummies, no trend

N	p	T=20				T=32				T=40				T=60				T=100			
		W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}
5	0	1.54	7.36	6.24	6.04	1.08	9.98	6.68	7.62	0.92	11.00	6.82	8.32	0.72	14.02	8.46	10.80	1.00	16.40	9.62	14.10
7	0	1.10	8.72	5.96	6.14	0.78	11.20	6.60	7.28	0.68	13.44	6.78	9.02	0.62	16.52	7.96	10.72	0.62	20.58	10.16	15.00
10	0	0.74	10.28	5.70	5.52	0.40	13.62	6.08	6.94	0.32	15.66	6.82	8.64	0.18	19.84	8.26	10.80	0.14	25.32	10.44	16.28
15	0	0.40	11.78	5.72	4.90	0.22	17.12	5.74	6.36	0.22	20.54	6.82	9.02	0.18	25.32	7.98	11.40	0.10	33.34	11.10	17.60
25	0	0.16	15.76	5.04	4.26	0.04	24.34	5.02	6.16	0.04	29.08	6.30	8.84	0.04	37.42	8.64	12.62	0.00	48.00	11.82	22.34
40	0	0.04	21.16	4.48	3.86	0.00	34.54	4.82	6.54	0.00	41.04	5.86	8.90	0.00	53.62	8.10	14.82	0.00	66.04	13.46	27.46
5	1	6.54	4.10	7.54	6.76	5.94	4.58	7.26	6.90	5.02	4.96	7.20	7.00	4.96	5.32	7.16	6.76	5.36	4.46	6.50	6.40
7	1	6.84	4.44	7.08	7.04	5.96	4.36	6.80	6.70	5.30	4.92	6.88	6.70	5.32	4.84	6.76	6.46	5.34	4.12	6.64	6.30
10	1	6.90	4.82	7.18	6.74	6.58	4.44	6.80	6.10	5.40	4.98	7.44	6.70	5.18	4.68	6.84	6.32	5.46	4.84	6.48	5.66
15	1	7.18	4.46	7.32	6.66	6.92	4.40	6.36	5.96	5.48	5.14	6.98	6.64	5.48	4.84	6.90	6.40	5.60	4.28	6.08	5.94
25	1	7.96	4.62	7.52	6.34	6.98	4.56	6.06	5.68	6.18	4.68	7.06	6.58	6.00	4.78	6.48	5.70	6.10	4.50	6.20	5.90
40	1	8.74	4.48	7.94	6.26	7.06	4.58	6.38	5.46	7.36	5.64	6.22	6.28	5.86	4.96	5.86	6.16	6.22	4.18	6.56	5.54
5	2	5.60	4.46	7.52	6.68	5.70	4.60	7.48	7.44	4.82	5.16	6.76	6.86	4.66	5.34	6.78	6.76	5.30	5.08	6.56	6.34
7	2	5.78	4.34	7.32	6.88	5.48	4.50	7.06	7.14	5.26	4.84	7.04	6.68	5.28	4.80	6.72	6.34	5.02	5.08	6.64	5.96
10	2	5.72	4.80	7.18	6.74	5.90	4.50	6.56	6.10	5.28	5.24	6.90	6.88	5.14	4.64	7.04	6.24	5.06	5.03	6.16	5.82
15	2	6.04	4.56	7.34	6.62	5.96	4.28	6.32	5.98	5.06	5.20	6.96	6.20	4.98	4.80	6.22	5.92	5.26	4.95	6.16	5.76
25	2	6.36	4.58	7.60	6.78	5.72	4.58	6.38	5.38	5.42	4.72	6.38	6.52	5.36	4.58	6.34	5.24	5.68	4.94	6.02	5.74
40	2	6.40	4.58	7.52	6.96	6.36	4.44	6.68	5.62	6.64	5.22	6.26	5.80	5.14	4.64	5.80	5.70	5.96	4.88	6.26	5.52
5	3	4.26	4.54	5.74	5.68	4.54	4.84	6.38	6.46	4.12	5.26	6.02	6.36	4.16	5.22	6.44	6.38	5.08	5.09	6.46	5.90
7	3	4.58	4.72	5.62	5.48	4.44	4.52	5.94	5.90	4.52	5.24	5.98	6.30	4.88	4.84	6.26	5.90	4.72	5.17	6.50	5.76
10	3	4.08	5.20	4.82	5.36	4.80	4.66	5.30	5.20	4.32	5.60	5.76	5.96	4.48	4.94	6.48	5.72	4.98	5.09	6.08	5.82
15	3	3.76	5.08	5.20	4.98	4.62	5.00	4.54	4.72	3.72	5.40	5.54	5.22	4.14	5.06	5.60	5.42	5.46	5.06	5.84	5.44
25	3	3.72	5.54	4.50	4.34	3.64	5.56	4.58	4.12	4.22	5.18	4.92	5.22	4.32	5.14	5.44	4.68	5.10	5.06	5.56	5.16
40	3	3.26	5.68	3.48	4.02	3.70	5.56	4.18	3.94	4.40	5.84	4.68	4.74	4.14	5.46	4.62	4.58	5.12	5.07	5.86	5.04
5	4	4.44	4.54	6.08	5.96	4.36	4.76	6.64	5.88	4.00	5.14	6.22	6.70	4.14	5.24	6.56	6.44	5.00	4.68	6.72	6.32
7	4	4.50	4.90	5.56	5.74	4.58	4.32	5.80	5.76	4.34	4.98	5.50	6.24	5.04	4.92	6.10	6.00	4.56	4.34	6.38	5.62
10	4	4.32	5.44	5.34	5.52	4.76	5.00	5.74	5.30	4.44	5.70	5.84	5.74	4.20	4.92	6.28	5.96	5.02	4.80	6.14	5.88
15	4	4.40	4.80	5.42	5.42	4.64	5.06	5.00	4.68	4.20	5.36	5.78	5.48	4.32	5.18	5.66	5.44	5.42	4.02	5.74	5.72
25	4	4.46	5.56	5.00	4.80	4.20	5.22	4.56	4.22	4.64	5.20	5.16	5.14	4.30	5.14	5.54	4.98	5.38	4.78	5.66	5.74
40	4	4.06	5.64	4.24	4.38	4.12	5.64	4.26	4.28	5.18	5.92	4.12	4.50	4.62	5.58	4.92	4.80	5.36	4.54	5.74	5.00

Table 4: Size of the HEGY-IPS test: MA(1) errors $\theta_i \sim U[-0.4,-0.2]$, constant, no seasonal dummies, no trend

N	p	T=20				T=32				T=40				T=60				T=100			
		W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}	W_{t_1bar}	W_{t_2bar}	W_{F_2bar}	W_{F_3bar}
5	0	35.02	4.88	6.76	9.32	44.04	4.22	7.22	9.82	46.68	4.02	6.46	9.08	52.62	3.62	8.72	10.70	57.04	2.34	9.64	10.68
7	0	44.70	4.90	6.08	9.70	54.38	3.98	6.82	10.00	58.82	3.72	6.52	8.96	63.56	3.26	8.24	10.78	68.34	1.84	9.80	11.02
10	0	56.06	3.98	6.08	9.64	67.48	3.34	6.88	10.50	71.08	3.38	6.32	9.66	76.84	2.68	8.88	11.60	81.56	2.04	10.28	12.14
15	0	71.76	3.94	6.20	10.34	81.92	2.90	6.26	11.04	85.50	2.76	6.44	10.34	89.22	2.32	9.20	12.96	92.04	1.34	11.06	12.56
25	0	88.76	3.04	5.88	10.84	94.74	2.38	6.48	12.34	96.56	2.00	6.48	12.08	97.66	1.38	9.68	14.50	98.92	1.10	12.72	15.16
40	0	97.10	2.84	5.08	11.72	99.46	1.86	6.04	13.58	99.62	1.44	6.62	12.84	99.78	0.94	10.26	17.22	99.96	0.42	14.36	17.82
5	1	7.16	5.34	5.88	7.70	9.08	5.82	5.86	9.50	9.74	6.00	6.74	8.72	10.54	5.88	6.80	9.80	12.44	6.12	7.14	9.86
7	1	8.06	5.58	5.82	6.94	10.58	5.94	6.48	9.04	11.56	6.12	6.64	9.06	12.94	6.68	7.14	10.36	14.46	6.26	7.42	10.28
10	1	9.02	5.52	6.14	7.52	11.64	6.28	6.46	9.44	13.34	6.78	6.46	9.42	14.34	6.84	7.04	11.16	17.52	7.14	7.32	10.74
15	1	10.90	6.10	5.22	7.80	14.40	6.26	6.84	9.70	15.86	7.30	6.36	9.62	17.26	7.70	7.50	11.44	21.74	7.20	7.64	11.98
25	1	13.48	5.92	6.18	8.54	18.60	7.08	7.16	11.26	20.82	7.60	7.44	10.72	24.54	9.14	7.34	14.00	29.64	8.36	7.82	13.52
40	1	16.80	7.08	6.26	8.34	25.74	7.82	7.28	11.72	28.26	8.82	7.64	12.80	33.70	9.22	8.30	15.46	39.76	9.50	8.34	15.52
5	2	4.16	5.06	5.76	6.98	5.32	5.26	7.24	7.08	5.14	5.46	6.26	6.50	5.32	4.96	6.62	7.12	6.62	4.95	7.18	6.84
7	2	4.46	4.96	5.62	6.50	5.38	5.06	6.74	6.60	5.74	5.12	5.96	6.50	6.20	4.88	6.72	6.96	6.88	4.97	6.58	6.20
10	2	4.06	5.18	5.86	6.68	5.04	5.08	6.58	7.18	5.24	5.30	5.90	6.40	6.08	4.82	7.24	7.14	6.92	4.84	7.02	6.32
15	2	4.28	5.34	5.76	6.82	5.56	4.70	6.06	6.70	6.30	5.24	5.72	5.90	6.28	5.38	6.72	7.12	7.74	4.74	6.60	6.14
25	2	4.70	4.66	5.30	6.70	5.94	5.16	6.38	6.62	6.04	4.90	5.48	6.16	7.56	5.18	6.82	7.36	9.96	4.58	6.90	6.38
40	2	4.44	4.96	5.20	6.06	6.86	5.40	5.34	6.22	6.68	5.04	5.76	5.92	8.46	4.44	6.66	6.94	10.74	4.56	6.56	6.20
5	3	4.70	5.22	6.60	7.50	5.32	4.82	7.94	7.70	5.08	5.26	6.90	6.42	4.60	5.14	6.64	6.80	5.34	5.03	6.70	6.92
7	3	5.28	5.04	6.62	7.28	5.28	4.96	7.64	7.56	5.74	5.10	6.84	6.74	5.22	5.12	6.84	7.00	5.80	5.17	5.76	6.44
10	3	5.30	5.18	7.50	7.70	5.56	4.74	7.22	7.72	5.62	5.48	6.46	6.94	5.38	4.74	7.04	6.98	5.66	5.05	6.24	6.56
15	3	5.64	5.02	7.66	8.18	5.92	4.52	6.96	7.14	6.02	5.18	6.58	6.38	5.50	5.28	6.56	6.52	5.98	4.99	6.14	6.28
25	3	5.66	4.40	7.76	8.18	6.22	5.02	7.02	7.22	6.14	4.78	6.34	6.58	6.28	5.58	7.08	6.88	7.12	4.94	6.00	6.26
40	3	6.86	4.44	8.34	8.62	7.24	4.94	7.22	7.50	7.06	5.32	7.02	6.56	6.60	5.18	6.74	6.60	7.14	5.02	5.62	6.30
5	4	5.06	5.18	6.42	7.30	5.02	5.22	8.02	7.54	4.80	5.44	6.96	6.84	4.66	5.58	6.76	7.34	5.04	4.46	7.10	6.88
7	4	5.26	5.24	6.50	7.00	4.98	4.92	7.16	7.54	5.32	5.26	6.96	7.10	5.20	5.12	6.94	7.00	5.48	4.52	6.46	6.94
10	4	5.44	5.06	6.98	7.48	5.42	4.82	7.42	7.38	5.12	5.38	6.94	7.18	4.88	4.94	7.30	7.12	5.42	4.96	6.50	6.76
15	4	5.56	5.22	7.66	7.90	6.50	4.82	7.32	7.24	6.28	5.22	6.74	7.08	4.64	5.38	6.66	7.16	5.50	4.76	6.02	6.90
25	4	6.06	4.68	7.64	8.12	6.12	4.88	7.14	7.24	6.04	5.00	6.96	6.82	5.74	5.52	7.50	7.58	6.64	4.38	6.48	7.04
40	4	7.18	4.86	8.28	8.78	7.10	5.22	6.98	7.76	6.22	5.00	7.16	7.00	5.92	4.84	7.34	7.00	6.52	4.44	6.16	6.90