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Abstract

This note shows that there is always a non-empty set of parameter values for which the hybrid equilibrium in the Besley and Smart(2003) model is unstable in the sense of Cho and Kreps. This set may include all the parameter values for which a hybrid equilibrium exists. For these parameter values, it is shown that a fully separating equilibrium always exists, which is Cho-Kreps stable. In this equilibrium, the good incumbent distorts fiscal policy to signal his type. An implication is that equilibrium in their model is not (generically) unique.

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1. Introduction

In an important recent paper, Besley and Smart(2003) present a rich but simple political agency model, which they then use to analyze the impact of tax and yardstick competition on voter welfare. In the model, depending on parameter values, the bad incumbent may either imitate the good incumbent in order to be re-elected (pooling), or take maximum rent in the first period (separating), or randomize between these two alternatives (hybrid). The hybrid equilibrium plays an important role in their analysis of yardstick competition.

This note shows that there is always a non-empty set of parameter values for which the hybrid equilibrium in the Besley and Smart model is unstable in the sense of Cho and Kreps. This set may include all the parameter values for which a hybrid equilibrium exists. For these parameter values, it is shown that a fully separating equilibrium always exists. In this equilibrium, when the cost of public good provision is high, the good type distorts public good provision below the efficient level in order to credibly signal his type. It is also not that not surprising; often, pooling equilibria are unstable, and in the hybrid equilibrium, the bad type pools with positive probability.

An implication is that for parameter values where the hybrid equilibrium is unstable, there are two equilibria in the Besley-Smart model. This result therefore indicates that Lemma 1 in their paper, which claims uniqueness of equilibrium, is incorrect.

2. Analysis

The notation follows Besley and Smart(2003). The hybrid equilibrium in their model occurs when $\hat{s} = (H - L)G_H \geq (1 - \beta)X$, and $q < 0.5$. In that case, the bad incumbent wants to pool, but if he does so, with probability 1, voters will not wish to re-elect him. In the hybrid equilibrium, if the voters observe fiscal policy (G_H, x_H) then: (i) voters re-elect the incumbent with probability $\sigma = \frac{\hat{s}-X}{\beta X}$, and (ii) the bad incumbent chooses to pool i.e. play (G_H, x_H) in cost state L with probability $\lambda = q/(1 - q)$.

Clearly, the hybrid equilibrium is a perfect Bayesian equilibrium. Our claim is that this equilibrium is not stable in the Cho-Kreps sense, under the reasonable assumption that the good type cares about (discounted) future payoffs, as does the bad type. We show this by finding a deviation for the good type which is profitable for the good type, but not profitable for the bad type, given that voters infer that the incumbent is good with probability 1, having observed the deviation.

The key to this profitable deviation is that the good type is not re-elected with probability 1 in the hybrid equilibrium, and this has a second-period cost for him. Specifically, his equilibrium payoff in the hybrid equilibrium, given cost state H, is

$$U^* = v_H(G_H) + \beta\sigma\overline{U} + \beta(1-\sigma)[\pi\overline{U} + (1-\pi)(-X)] \tag{2.1}$$

where $v_\theta(G) = G - C(\theta G)$, $\overline{U} = qv_H(G_H) + (1-q)v_L(G_L)$. This is because (i) with probability σ, the good incumbent is re-elected, in which case we (reasonably) assume that he gets the same expected payoff as the voter i.e. $\overline{U}$, and (ii) with probability $1-\sigma$, the good incumbent is not re-elected, in which case with probability π, he is replaced by a good challenger, and with probability $1-\pi$, with a bad challenger, in which case we again (reasonably) assume that he gets the same expected payoff as the voter i.e. $\pi\overline{U} + (1-\pi)(-X)$.

Now consider a deviation from the hybrid equilibrium where in cost state H, the good incumbent sets $G'' = G' - \varepsilon,\ x'' = HG''$, for some small $\varepsilon > 0$, where

$$(H-L)G' = (1-\beta)X \tag{2.2}$$

Equation (2.2), plus $\varepsilon > 0$, implies that bad incumbent strictly prefers his hybrid equilibrium payoff of X to the payoff from G'', x'', *even if he is re-elected with probability 1,* having chosen G'', x''. This is because

$$(H-L)G'' + \beta X < (H-L)G' + \beta X = X$$

So, in the terminology of Cho and Kreps(1987), G'', x'' is *equilibrium dominated* for the bad type. So, if G'', x'' is observed, the voters infer that the incumbent is good with probability 1, and will always re-elect the incumbent.

What is the good type's payoff from this deviation to G'', x'' in the high-cost state? It is

$$U' = v_H(G'') + \beta\overline{U} \tag{2.3}$$

So, the gain to the good type to deviating is

$$U' - U^* = v_H(G'') - v_H(G_H) + \beta\overline{U}(1-\sigma)(1-\pi)X \tag{2.4}$$

This comprises a first-period loss from distortion of public good supply, plus a second-period gain from a higher probability of re-election.

If $U' - U^* > 0$, then we have found a profitable deviation for the good type that is equilibrium dominated for the bad type, and thus the hybrid equilibrium is unstable. But $U' - U^* > 0$ if

$$\begin{aligned} v_H(G_H) - v_H(G'') &< \beta\overline{U}(1-\sigma)(1-\pi)X \\ &= \overline{U}(1-\pi)((H-L)G_H - (1-\beta)X) \end{aligned} \tag{2.5}$$

where in the second line we have used $\sigma = \frac{(H-L)G_H - X}{\beta X}$.

Generally, (2.5) will hold when G' is close to G_H. To see this, let $G' = G_H - \Delta$, where obviously $\Delta \geq 0$ by the fact that the parameters are such that a hybrid equilibrium exists. Then, using (2.2), (2.5) becomes

$$v_H(G_H) - v_H(G_H - \Delta - \varepsilon) < \overline{U}(1-\pi)(H-L)\Delta$$

where $U(G) = G - C(HG)$, or

$$\frac{v_H(G_H) - v_H(G_H - \Delta - \varepsilon)}{\Delta} < \overline{U}(1-\pi)(H-L) \tag{2.6}$$

But, for Δ, ε small, as G_H maximizes v_H :

$$\frac{v_H(G_H) - v_H(G_H - \Delta - \varepsilon)}{\Delta} \cong v_H'(G_H) = 0$$

So, (2.6) certainly holds for G' close to G_H.

When does it hold for *all* parameter values for which a hybrid equilibrium exists? Assuming that $C(.)$ is strictly convex, $v_H(G)$ is a strictly concave function of G, so for ε small enough,

$$\frac{v_H(G_H) - v_H(G_H - \Delta - \varepsilon)}{\Delta} < v_H'(0) = 1 - C'(0)$$

So, if

$$1 - C'(0) < \overline{U}(1-\pi)(H-L)$$

then (2.6) holds globally. If $C(Z) = Z^2/2$, for example, $G_\theta = \frac{1}{\theta^2}$, $\theta = H, L$, so $v_\theta(G_\theta) = \frac{1}{2\theta^2}$, and this reduces to

$$1 < [\frac{q}{2H^2} + \frac{1-q}{2L^2}](H-L)(1-\pi)$$

which can certainly hold.

The question then arises, if the hybrid equilibrium is unstable in the Kreps-Cho sense, is there a stable equilibrium? If the hybrid equilibrium is unstable, the following separating equilibrium exists and is stable. In the first period:

- the bad type always separates i.e. chooses fiscal policy $G = 0,\ x = X$:

- if the cost state is low, the good type chooses fiscal policy $G_L,\ x_L = LG_L$:

- if the cost state is high, the good type chooses fiscal policy $G',\ x' = LG'$, where G' is defined in (2.2).

- voters have out-of equilibrium beliefs that the type is bad with probability greater than $1-\pi$ (and thus elect the challenger) if they observe any out-of-equilibrium first-period fiscal policy.

This is clearly a perfect Bayesian equilibrium. First, when the cost state is high, the good type does not wish to deviate to any other fiscal policy combination than G', x' because such a deviation gives him a payoff at most

$$U'' = v_H(G_H) + \beta[\pi\overline{U} + (1-\pi)(-X)] \tag{2.7}$$

because he loses the subsequent election. But by inspection of (2.7) and (2.1), U'' is strictly less than U^*, which is in turn less than the equilibrium payoff of U' in (2.3). Second, when the cost state is high, the bad type is indifferent between his equilibrium choice $(0, X)$ and imitating the good type. Finally, the equilibrium is Cho-Kreps stable by construction.

3. References

Besley,T. and M.Smart (2003), "Fiscal Restraints and Voter Welfare", unpublished paper, LSE

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