The Law of Demand in Tiebout Economies

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Abstract: We consider a general equilibrium local public goods economy in which agents have two distinguishing characteristics. The first is 'crowding type,' which is publicly observable and provides direct costs or benefits to the jurisdiction (coalition or firm) the agent joins. The second is taste type, which is not publicly observable, has no direct effects on others and is defined over private good, public goods and the crowding profile of the jurisdiction the agent joins. The law of demand suggests that as the quantity of a given crowding type (plumbers, lawyers, smart people, tall people, nonsmokers, for example) increases, the compensation that agents of that type receive should go down. We provide counterexamples, however, that show that some agents of a given crowding type might actually benefit when the proportion of agents with the same crowding type increases. This reversal of the law of demand seems to have to do with an interaction effect between tastes and skills, something difficult to study without making these classes of characteristics distinct. We argue that this reversal seems to relate to the degree of difference between various patterns of tastes. In particular, if tastes are homogeneous, the law of demand holds.

1 Introduction

Tiebout's (1956) central insight was that many types of public goods are subject to crowding and congestion. As a result it would be impractical and inefficient to provide them at the level of national governments. Instead, amenities like education, police and fire protection are services produced by local jurisdictions. In choosing where to live, consumers evaluate the bundles of public goods, taxes and other amenities each jurisdiction offers. In making their locational choices, in effect they reveal their willingness to pay for public goods. Thus, the preference revelation and free riding problem pointed out by Samuelson (1954) for the case of pure public goods disappears in this economic environment.

Tiebout's paper stimulated an enormous theoretical literature. Subsequent authors have shown that, although efficient Tiebout sorting may not occur in completely general circumstances, adding economic restrictions that are natural in the study of clubs or local public goods provide support for Tiebout's hypothesis. Wooders (1978), for example, shows that when there is only one private good , agents crowd each other anonymously (only the numbers of agents sharing the public goods matters), and all gains to coalition-forming are realized in small groups (or coalitions), the core can be decentralized with anonymous prices. On the other hand, Bewley's (1981) early attempt to formalize the Tiebout hypothesis led to largely negative conclusions. Bewley shows that, in some cases, anonymous decentralization of efficient outcomes is not possible and, in other cases, anonymous prices may only serve to decentralize inefficient outcomes. Bewley's formalization, however, has not gained wide acceptance. Key concerns are that in his model the numbers of jurisdictions are some cases is fixed, public goods are not subject to congestion, and most important, small groups are not effective (more on this below).

The local public goods approach to the provision of congestable public goods centers on agents making a locational choice among competing jurisdictions offering distinct public good bundles, and addresses the general equilibrium question of how the entire population of a large economy can be best sorted into non-overlapping and exhaustive coalitions. There are, however, significant classes of congestable goods that are provided by coalitions not connected to a location. For example, agents join country clubs, fitness clubs, private schools churches, professional organizations and so in order to enjoy both the public goods they provide and the company of the other members. Note that agents can belong to one club, many clubs or even no clubs. Thus, unlike the local public goods case, there is no need to require that the outcome be a partition of the agents. Buchanan (1965) is generally credited as being the first to write a formal model of clubs, but the roots can be seen as far back as the early papers on tolls and congested roads by Pigou (1920) and Knight (1924)

Since Buchanan published his seminal paper the club literature has developed in several different directions. In a model with essentially homogeneous agents, Pauly (1967, 1970) explored the issue of optimal club size and the stability of its membership. Tollison (1972), Ng and Tollison (1974), Berglas (1976), and DeSerpa (1977) present clubs in which crowding is nonanonymous. Wooders (1978) considered anonymous crowding and anonymous prices – prices which do not depend on unobservable characteristics of agents. McGuire (1974) and Wooders (1978) address whether club membership will be homogeneous when agents differ in tastes or endowments. Questions of core and equilibrium existence in club economies arose in works such as Pauly (1967,1970) and Wooders (1978,1980). Issues involving the potential costs for excluding unwanted members of an exclusive club are presented by Davis and Whinston (1967), Millward (1970), Nichols, Smolensky and Tiedman (1971), Oakland (1972), and Kamien, Swartz and Roberts (1973). Early explorations of uncertainty in clubs models include DeVany and Saving (1977) and Hillman and Swan (1979). More recent directions of investigation include multiproduct clubs, variable usage, and intergenerational clubs.

In this paper we take on a new question: when will a law of demand hold for skills or other 'crowding characteristics' in coalition economies, and in particular, in economies with local public goods. For example, will the compensation that gregarious people experience from joining social groups decrease if more people become outgoing; will smart college applicants receive less college aid if the population at large gets smarter, will the wage of teachers go down if more teachers are trained, and so on. In labor markets there is a strong intuition that the law of demand should hold. The question is, does it continue to hold in Tiebout economies?

The central issue we encounter in addressing this question was already nicely pointed out by Adam Smith (1776):

"The whole of the advantages and disadvantages of the different employments of labor and stock must, in the same neighborhood, be either perfectly equal or continually tending to equality. If in the same neighborhood there was any employment evidently either more or less advantageous than the rest, so many people would crowd into it in one case, and so many would desert it in the other, that its advantages would soon return to the level of other employments."

As Adam Smith recognized, wage differentials are required to equalize the total monetary and non-monetary advantages and disadvantages amongst alternative employments; a job with favorable conditions can attract labor at relatively low wages while a job with unfavorable conditions must offer a compensating wage premium to attract workers. This well known theory of equalizing differences, is suggested to be 'the fundamental market equilibrium construct in labor economics'[1] and is an example of the central question we will consider in this paper. Clearly, whether this theory holds in Tiebout economies: clubs with attractive memberships and public goods offerings can charge more for admission.

The value of a worker's skills are determined by the market values of the product he is able to generate. How conditions of employment are valued, however, depends on the tastes of individual workers. For example, whether indoor or outdoor work is preferred depends on tastes of workers. If there is an abundance of workers who prefer to stay indoors, then outdoor work may fetch a premium. Thus, when we allow for equalizing differences, the tastes

[1] S. Rosen (1986).

of workers become important determinants of labor market equilibrium. We find that getting the most out of an economy's resources requires matching the appropriate type of worker with the appropriate type of firm: "the labor market must solve a type of marriage problem of slotting workers into their proper 'niche' within and between firms."[2]

It is difficult to address the process of assigning workers to firms in a general equilibrium model since each commodity, including labor, is treated as a homogeneous good which is allocated to productive uses, without reference to the agent who supplied it. In other words, there is a structural de-bundling of the tastes and skills of workers inherent in the model. Under these circumstances, and given diminishing marginal productively of labor, one expects a "law of demand" to hold. That is, as the quantity supplied of a given skill increases the price it receives in equilibrium should go down.

We will therefore explore the law of demand in the context of the *crowding types model* introduced in Conley and Wooders (1996, 1997). The advantage of this model in examining law of demand issues is that it sets up a formal distinction between the tastes and crowding effects of agents. Crowding characteristics are publicly observable and generate direct effects on other

[2] Rosen (1986).

agents. Crowding characteristics include, for example, gender, whether one is a smoker, skills and abilities, personality characteristics, appearance, and languages spoken. Note that some of these characteristics are exogenously attached to agents (gender) and some are endogenously chosen in response to market and other incentives (skills and professional qualifications). See Conley and Wooders (2002) for more discussion of the latter. Tastes, on the other hand, are assumed to be private information and in themselves produce no direct effects on other agents.

The key observation underlying the crowding types approach is that an agent is a bundle of tastes and observable characteristics such as education. These cannot be taken as independent. Thus, it is the joint distribution of tastes and crowding types and not their separate distributions that determines the equilibrium outcome of the economy. Modelling this feature allows us to explore explicitly how the tastes of agents determine the compensating differentials needed to induce agents to joint different jurisdictions/firms/coalitions and in turn to see when a law of demand for skills, for example, will and will not hold in a Tiebout economy.

To do so, we consider a coalitional economy in which small groups are strictly effective. Informally, this means that all per capita gains can be realized in groups that are small relative to the size of the population and that no particular type is scarce (and thus might have monopoly power). In these circumstances the core has the equal treatment property, that is, all agents of a given type must receive the same utility in any core allocation.

To address whether the law of demand holds, we consider two economies that differ only in that the number of one particular crowding type is larger in one than the other. We show that at a core allocation, the law of demand need not hold. We demonstrate this through a pair of examples; some agents of the relatively more abundant crowding type may benefit. In fact,.the average compensation of agents possessing the crowding type that has become more abundant in the population may go either up or down. For example, if there is an increase in the number of plumbers in the world, it might be that plumbers who have a taste for working hot steam tunnels actually benefit from the overall increase. Similarly, while computer programmers in general might oppose the free immigration of programmers from India, it might still be the case that some types of programmers (say game writers) might actually benefit from this migration.

This failure of the law of demand seems to be due to interactions between tastes and crowding characteristics and especially to how they are bundled.

As we will discuss, it is immediate that if all agents have the same tastes, then the law of demand holds.

2 The model

We consider economies in which agents are described by two characteristics, their taste types and crowding types. An agent has one of T different taste types, denoted by $t \in 1, ..., T \equiv \mathcal{T}$ and one of C different crowding types, denoted $c \in 1, \ldots, C \equiv \mathcal{C}$. We assume no correlation between c and t.

A group of agents is described by a vector $m = (m_{11}, \ldots, m_{ct}, \ldots, m_{CT})$ where m_{ct} denotes the number of agents with crowding type c and taste type t in the group. The *crowding profile* of a group m is a vector $(m_1, ..., m_C)$, where $m_c = \sum_t m_{ct}$. A crowding profile simply lists the numbers of agents of each crowding type in the coalition or economy. An economy is determined by the group of agents $N = (N_{11}, \ldots, N_{ct,}, \ldots, N_{CT})$. A *club* $m = (m_{11}, \ldots, m_{ct}, \ldots, m_{CT}) \leq N$ describes a group of agents whose membership collectively produces and consumes a public good The set of all feasible clubs contained in N is denoted by $\mathcal{N}$.

A *partition n of the population* is a set of clubs $\{n^1, ..., n^K\}$ satisfying

$\sum_k n^k = N$. We will write $n^k \in n$ when a club n^k belongs to the partition n. It will sometimes be useful to refer to an individual agent, denoted by $i \in \{1, \ldots, I\} \equiv \mathcal{I}$, where $I = \sum_{c,t} N_{ct}$ is the size of the population. We let $\theta : \mathcal{I} \rightarrow \mathcal{C} \times \mathcal{T}$ be a mapping describing the crowding and taste types of individual agents; thus,

$$|\{i \in \mathcal{I},\ i \in N : \theta(i){=}(c,t)\}| = N_{ct}.$$

We will say an agent i has *type* (c,t) if $\theta(i) = (c,t)$.

With a slight abuse of notation, if agent i is a member of the club described by m, we shall write $i \in m$, and if i belongs to the economy determined by N we write $i \in N$.

An economy has one private good x and club goods $y_1, y_2, ..., y_A$ that are provided by clubs exclusively for their own memberships. A vector $y = (y_1, y_2, ..., y_A) \in \mathbb{R}^A_+$ gives club good production. Each agent belongs to exactly one club. Each agent $i \in \mathcal{I}$ of taste type t is endowed with $\omega_t \in \mathbb{R}_+$ of the private good and has a quasi linear utility function

$$u_t(x, y, m) = x + h_t(y, m)$$

where $i \in m$ and y is the club good production of club m containing agent i. The cost in terms of the private good of producing y club good in club with membership m is given by a production function

$$f(y, m).$$

A particular combination of preferences and endowments for players in the economy N and production possibilities available to clubs is referred to as the *structure* of the economy.

We shall assume preferences satisfy *taste anonymity in consumption* (TAC), and production functions satisfy *taste anonymity in production* (TAP) defined as follows:

TAC: For all $m, \widehat{m} \in \mathcal{N}$, if for all $c \in C$ it holds that $\sum_t m_{ct} = \sum_t \widehat{m}_{ct}$ then for all $x \in \mathbb{R}_+$, all $y \in \mathbb{R}_+^A$, and all $t \in \mathcal{T}$ it holds that $(x, y, m) \sim_t (x, y, \widehat{m})$.

TAP: For all $m, \widehat{m} \in \mathcal{N}$, if for all $c \in C$ it holds that $\sum_t m_{ct} = \sum_t \widehat{m}_{ct}$ then for all $y \in \mathbb{R}_+^A$ it holds that $f(y, m) = f(y, \widehat{m})$.

TAC and TAP capture the idea that agents care only about the crowding types and not the taste types of the agents that are in their respective clubs. These conditions can be seen as defining crowding types rather than imposing restrictions on preferences. To illustrate, the cost of production depends on the skill mix of the people in the jurisdiction, but whether or not skilled workers like warm or cool climates is of no relevance. As for consumption, we might care about the age of other people but are indifferent to whether or not they are danger averse.[3] We will assume throughout that all economic structures satisfy both TAC and TAP.

A *feasible state* of the economy $(X, Y, n) \equiv ((x_1, \ldots, x_I), (y_1, \ldots y_K), (n_1, \ldots n_K))$ consists of a partition n of the population, an allocation $X = (x_1, \ldots, x_I)$ of private goods to agents, and a club goods production plan for each club, $Y = (y_1, \ldots y_K)$, such that

$$\sum_k \sum_{ct} n^k_{ct} \omega_t - \sum_i x_i - \sum_k f(y^k, n^k) \geq 0. \tag{1}$$

We also say that (x, y) is a *feasible allocation for a club m* if

$$\sum_{c,t} m_{ct} \omega_t - \sum_{i \in m} \overline{x}_i - f(\overline{y}, m) \geq 0$$

A club $m \in \mathcal{N}$ producing a feasible allocation $(\overline{x}, \overline{y})$ *can improve upon* a feasible state (X, Y, n) if for all $i \in m$,

$$u_t(\overline{x}_i, \overline{y}, m) > u_t(x_i, y_k, n_k). \tag{2}$$

[3] You may well indirectly care about the tastes of agents you live with through the eventual choice of public good y. However, given y, TAC and TAP imply your welfare does not directly depend on the tastes of other agents.

where i is of taste type t and, in the original state, $i \in n_k$ and $n_k \in n$. A feasible state of the economy (X, Y, n) is a *core state of the economy* or simply a *core state* if it cannot be improved upon by any group m acting as a coalition.[4] This simply says that a feasible state is in the core if it is not possible for a coalition of agents to break away and, using only resources of its members, provide all its members with preferred consumption bundles. A utility vector $\widehat{u} \in \mathbb{R}^I$ is a *core utility* if, for some core state of the economy (X, Y, n), $u_i(x_i, y_k, n_k) = \widehat{u}_i$.

Since we have restricted to economies with quasi-linear preferences, we can also define the core entirely in terms of vectors of utilities. Given a club $m \in \mathcal{N}$ define $v(m)$ as the maximum total utility that can be achieved by the club; that is,

$$v(m) = \max_{(\overline{x}, \overline{y})} \sum_{i \in m} u_t(\overline{x}_i, \overline{y}, m)$$

where the maximum is taken over the set of feasible allocations for the club m. Define $V(N)$ as the maximum total utility that can be achieved by the

[4] Note that we can define the core as the set of feasible states that cannot be improved upon by any club (rather than by a coalition forming perhaps multiple clubs) since there is no benefits to be gained from trade between clubs. This contrasts to work,s such as Wooders (1989), for example., with multiple private goods

entire economy when it can partition into jurisdictions; that is,

$$V(N) = \max \sum_{k} v(n^k)$$

where $n = (n_1, ..., n_K)$ is a partition of N for some K. It is easy to show that a utility vector $\widehat{u} \in \mathbb{R}^I$ is a core utility if and only if

$$\sum_{i \in m} \widehat{u}_i \geq v(m)$$

and

$$\sum_{i \in N} \widehat{u}_i = V(N).$$

This paper will focus solely on economies in which small groups are strictly effective. An economy satisfies *strict small group effectiveness*, SSGE, if there exists a positive integer B such that:

1. For all core states (X, Y, n) and all $n_k \in n$, it holds that $|n_k| < B$.

2. For all $c \in \mathcal{C}$ and all $t \in \mathcal{T}$ it holds that either $N_{ct} > B$ or $N_{ct} = 0$.

SSGE is a relatively strong formalized version of the sixth assumption in Tiebout's paper that there be "an optimal community size" - condition one stating that any coalition with more than B agents can be improved upon while condition two says that this limit of B is small relative to a population which contains at least B agents of each type. As recent literature shows, however, economies satisfying apparently mild conditions can be approximated by ones satisfying SSGE (cf., Kovalenkov and Wooders 2003 and references therein).

2.1 Equal treatment

The first result follows immediately from SSGE and shows that any core state must have the *equal treatment property*, that is any two agents of the same type must be equally well off in any core state.[5]

Theorem 1: Let (X, Y, n) be a core state of an economy satisfying SSGE. For any two individuals $i, \hat{\imath} \in \mathcal{I}$ such that $\theta(i) = \theta(\hat{\imath}) = (c, t)$, if $i \in n^k$ and $\hat{\imath} \in n^{\hat{k}}$ then $u_t(x_i, y, n^k) = u_t(\hat{x}_{\hat{\imath}}, \hat{y}, n^{\hat{k}})$.

Proof. See Conley and Wooders (1997).

One consequence of this result is that with any core state (X, Y, n) we can associate a vector of payoffs $u = (u_{11}, ..., u_{ct}, ..., u_{CT}) \in \mathbb{R}^{CT}$ where u_{ct}

[5] More general versions of this result appear in Wooders (1983, Theorem 3) and in Kovalenkov and Wooders (2001).

is the utility of an agent with crowding type c and taste type t.

Note that Theorem 1 cannot be directly verified by looking at the observable data. Wages received by agents of a given could be widely different, provided the nonobservable nonmonetary compensations of joining a club offset the wage differences. The next result provides a directly observable counterpart to Theorem 1 and is a key feature of the crowding types model.

Theorem 2: Let (X, Y, n) be a core state of an economy satisfying SSGE. Suppose that for some club $n^k \in n$, for some crowding type $c \in \mathcal{C}$, and for two taste types $t, t' \in \mathcal{T}$, both $n^k_{ct} > 0$ and $n^k_{ct'} > 0$. Then for all $i, j \in k$ such that $\theta(i) = (c, t)$ and $\theta(j) = (c, \widehat{t})$, it holds that

$$\omega_t - x_i = \omega_{\widehat{t}} - x_{\widehat{i}} \equiv \rho_c(y^k, n^k).$$

Proof. See Conley and Wooders (1997).

Theorem 2 illustrates that in a core state players of the same crowding type will be offered the same 'price' (which may be positive or negative) to enter clubs. Thus, there is anonymity in the sense that the prices of club membership for two individuals who have the same crowding type do not depend on tastes.

From now on, given a core state, we will interpret $\rho_c(y, m)$ as the *admission price for players of crowding type c to enter the club m producing y of the club good.* For the special case of production, or coalition production, admission prices will generally be negative and are interpreted as the wages paid by firms to workers.

2.2 Core equivalence

Expanding on the above, a *Tiebout price system for crowding type c* associates to each possible club good level and possible club (containing at least one player with crowding type c) an admission price, which applies to all players of crowding type c. Thus, players know the price to join any possible jurisdiction and we also see that prices are anonymous in the sense that they do not depend on the tastes of agents.[6] A *Tiebout price system* is simply a collection of price systems, one for each type, and is denoted by ρ.

We define a *Tiebout equilibrium* as a feasible state $(X, Y, n) \in F$ and a Tiebout price system ρ such that

1. For all $n^k \in n$, all individuals $i \in n^k$ such that $\theta(i) = (c, t)$, all

[6] Formally we also require that for all $m, \widehat{m} \in N$, if for all $c \in C$ it holds that $\sum_t m_{ct} = \sum_t \widehat{m}_{ct}$ then for all y it holds that $\rho(y, m) = \rho(y, \widehat{m})$.

alternative clubs $m \in \mathcal{N}_c$, and for all levels of public good production $y \in \mathbb{R}^A_+$,

$$\omega_t - \rho_c(y^k, n^k) + h_t(y^k, n^k) \geq \omega_t - \rho_c(y, m) + h_t(y, m)$$

2. For all potential clubs $m \in \mathcal{N}$ and all $y \in \mathbb{R}^A_+$,

$$\sum_{c,t} m_{ct}\rho_c(y, m) - f(y, m) \leq 0$$

3. For all $n^k \in n$,

$$\sum_{c,t} n^k_{ct}\rho_c(y^k, n^k) - f(y^k, n^k) = 0$$

It can be seen that a Tiebout equilibrium is a decentralized market equilibrium. Condition 1 states that, given prices to join clubs, every agent is in his preferred club. Condition 2 states that, given the price system, no new club could make positive profits while existing clubs make zero profit.[7]

Under strict small group effectiveness, a strong result can be proven about the relationship between the core and Tiebout equilibrium.

Theorem 3: If an economy satisfies SSGE then the set of states in the core of the economy is equivalent to the set of Tiebout equilibrium states.

Proof. See Conley and Wooders (1997).

[7] From a firm perspective this does not imply that the firm makes zero profit, it means that any profit has been redistributed to the workers and owners of that firm.

Theorem 3 confirms that in the crowding types model efficient allocations can be decentralized through an anonymous price system. Thus, when we consider firm formation, all workers can choose amongst jobs to maximize their utilities and the resulting outcome will be an efficient stable outcome in which workers and firms are optimally matched.[8] Note that, unlike the situation in private goods exchange economies, we have equivalence of core and equilibrium outcomes in economies with a finite number of agents. This is due to our assumption of SSGE. There are no new effective clubs that arise when the economy becomes larger. This is in contrast to situations as in Wooders (1997) where, for some results, forever increasing returns to jurisdiction size are allowed.

The crowding types model allows us to consider firm, jurisdiction or region formation, taking account of both the tastes of workers and their productivity. As such, it gives us a reasonably complete way to model the theory of

[8] We note that a major difference between this result and analogous results for differentiated crowding models as in Wooders (1997), for example, is that prices do not depend on the tastes of agents, only on their crowding types. Analogous results for models with anonymous crowding, as in Wooders (1978) and subsequent papers, are special cases of Theorem 3 since anonymous crowding models are crowding types models but with only one crowding type.

equalizing differences. The rest of the paper uses this model to consider the relevance of the law of demand when crowding types and taste types are taken into account.

3 The law of demand

In this section we formally develop both positive and negative results regarding the law of demand. This is done by way of comparative statics exercises in two economies. These economies have identical technologies and identical populations of all agents except for one particular crowding type c. The second economy has an increased population of crowding type c spread in some arbitrary way across taste types. Thus, for example, the two economies have the same number of plumbers who like football, plumbers who like hockey, plumbers who like baseball, lawyers who like football, lawyers who like baseball, etc. However, the second economy might have twice as many doctors who like football, one additional doctor who likes hockey, and the same number of doctors who like baseball.

Formally, consider two economies S and G with agent sets $S = (S_{11}, \ldots, S_{ct,} \ldots, S_{CT})$ and $G = (G_{11}, \ldots, G_{ct,} \ldots, G_{CT})$, where S_{ct} is interpreted as the total number of agents with crowding type c and taste type t in economy S and where G_{ct} is interpreted as the total number of agents with crowding type c and taste type t in economy G. The most recent and also the most general versions of the following result appear in Kovalenkov and Wooders (2005).[9] Because of our assumption of SSGE, the proof below is particularly simple.

Theorem 4: Let S and G be as above and assume both economies satisfy SSGE. Assume also that there are vectors $u^S = (u^S_{11}, ..., u^S_{ct}, ..., u^S_{CT}) \in \mathbb{R}^{CT}$ and $u^G = (u^G_{11}, ..., u^G_{ct}, ..., u^G_{CT}) \in \mathbb{R}^{CT}$ representing core payoffs in the equal treatment core of economies S and G respectively. Then

$$(u^S - u^G) \cdot (S - G) \leq 0.$$

Proof. From the assumption that u^S is the core of the economy S, $u^S \cdot m \geq v(m)$ for all jurisdictions m with $\|m\| \leq B$. We claim that $u^S \cdot G \geq v(G)$. Let n^G be a partition of G into jurisdictions n^G_k satisfying $\|n^G_k\| \leq B$ for each jurisdiction and supporting the core allocation u^G (for each n^G_K it holds that $u^G \cdot n^G_k = v(n^G_k)$ and $u^G \cdot G = \sum\limits_k v(n^G_k)$). Then $u^S \cdot m \geq v(m)$ for all

[9] Kovalenkov and Wooders (to appear) provides a detailed discussion of related literature.

jurisdictions m implies $u^S \cdot n_k^G \geq v(n_k^G)$ for each k and .

$$u^S \cdot G = \sum_k u^S \cdot n_k^G \geq \sum_k v(n_k^G) = v(G).$$

Similarly, $u^G \cdot S \geq v(S)$. Also, since u^S is a core utility for the economy S and u^G is a core utility for the economy G it holds that $u^S \cdot S = v(S)$ and $u^G \cdot G = v(G)$. We now have

$$0 \geq v(G) - u^S \cdot G + v(S) - u^G \cdot S$$

$$= u^G \cdot G - u^S \cdot G + u^S \cdot S - u^G \cdot S$$

$$= u^G \cdot G - u^S \cdot G + u^S \cdot S - u^G \cdot S$$

$$= (u^S - u^G) \cdot (S - G)$$

The conclusion now follows from some simple algebra.$\square$

One immediate consequence of Theorem 4 is that a certeris paribus increase in the number of players with of particular type (that is, a particular c, t combination) cannot be beneficial to all players of that type.

Corollary 1. Let S and G be as above and assume both economies satisfy SSGE. Assume also that there are vectors $u^S = (u_{11}^S, ..., u_{ct}^S, ..., u_{CT}^S) \in \mathbb{R}^{CT}$ and $u^G = (u_{11}^G, ..., u_{ct}^G, ..., u_{CT}^G) \in \mathbb{R}^{CT}$ representing core payoffs in the equal treatment core of economies S and G respectively. Then, given the core vectors u^S and u^G, if $S_{ct} < G_{ct}$ and $S_{c't'} = G_{c't'}$ for all other types (c', t') then it must hold that

$$u^S_{ct} \geq u^G_{ct}.$$

Corollary 1 states that the law of demand applies on a type-by-type basis. The problem with this is that the taste component of a type is not observable. Thus, the data cannot tell us anything about the relative increases for agents of crowding type c of different taste types.

One particular case in which we can obtain a law of demand is when all agents have the same taste types. This case may be important in application to empirical economics.

Corollary 2. Let S and G be as above and assume both economies satisfy SSGE. Assume also that for all taste types t and t' it holds that $u_t \equiv u_{t'}$. Then if $u^S = (u^S_{11}, ..., u^S_{ct}, ..., u^S_{CT}) \in \mathbb{R}^{CT}$ and $u^G = (u^G_{11}, ..., u^G_{ct}, ..., u^G_{CT}) \in \mathbb{R}^{CT}$ represent core payoffs in the equal treatment core of economies S and G respectively it holds that

1. For each crowding type c it holds that $u^S_{ct} = u^S_{ct'}$ for all t, t' and similarly for u^G.

2. If $S_{ct} < G_{ct}$ and $S_{c't'} = G_{c't'}$ for all other types (c', t') then it must hold that for any t' that

$$u^S_{ct'} \geq u^G_{ct'}.$$

This can easily been seen since the core vectors given in Theorem 4, $u^S = (u^S_{11}, ..., u^S_{ct}, ..., u^S_{CT}) \in \mathbb{R}^{CT}$ and $u^G = (u^G_{11}, ..., u^G_{ct}, ..., u^G_{CT}) \in \mathbb{R}^{CT}$ will be in the spaces $\mathbb{R}^C$ (since $T = 1$).and the relationship $(u^s - u^G) \cdot (S - G) \leq 0$. will hold. Another situation where the law of demand will hold is where all agents have the same genetic type and can acquire any crowding type at the same cost (as in Conley and Wooders 1996). Of course, if there is only one crowding type (that is, if crowding is anonymous) then the law of demand will also hold. While these cases may be important empirically, from a theoretical perspective they are quite narrow.

In view of the observability of crowding types, of particular interest is a ceteris paribus increase in the number of players with a particular crowding type. The following result shows that not all agents of a crowding type can gain from an increase in the numbers of agents with that crowding type.

Corollary 3. If $S_{ct'} \leq G_{ct'}$ for all $t' \in T$ and $S_{c't'} = G_{c't'}$ for all $c' \in \mathcal{C}$, $c' \neq c$, and for all $t' \in T$ then $u^S_{ct} \geq u^G_{ct}$ for at least one type t and, moreover, if $u^S_{ct'} < u^G_{ct'}$ for some type t' then there exists some t such that $u^S_{ct} > u^G_{ct}$.

One interesting aspect of the Corollary is that if the numbers of agents of any crowding type c increases while the numbers of agents of every other crowding type, taste type pair is held constant, if one taste type gains from the increase then another taste type must loose. We cannot, however, say what will happen to average payoffs of the agents with a particular taste type, as the following example illustrates.

Example 1:. Suppose that there is only one crowding type and two tastes types. Agents with taste type 1 like to work agents with taste type 2 gain no utility from work. A pair of agents can work together and produce output worth \$10.00. If a type 1 agent works he experiences a utility increase just from working worth \$7.00 while a type two agent gains 0 utility from working. An equal-treatment core utility vector assigns payoff of \$12.00 worth of utility to a player of type 1 and \$5.00 to each player of type 2. If the numbers of agents of taste type 1 increases then core utilities on average increase while the opposite holds if the numbers of agents of taste type 2 increase. (Note that for this example, the core will be nonempty only if there is an even number of agents but if there are 'many' agents, approximate cores will be nonempty and give most players of type 1 approximately \$12.00 and most of type 2 approximately \$5.00 (Wooders 1994, 2004). •

This is a rather trivial example, and could be made more elaborate without changing the point. What it demonstrates is that given the differences in how agents value different kinds of work situations and so on, the simple proposition that the average payoffs to agents of any given crowding type must go down as the number of that crowding type increases in the population is false.

4 Failures of the law of demand

In this section we provide an that demonstrate that the law of demand need not hold for all agents when the crowding type they posses increases. This example considers that case of crowding in consumption. An example that treats crowding in production is available from the authors upon request.

Example 2: There are 3 taste types - people who like music at work (L), hate music at work (H) and do not mind some music at work (I). There are 3 crowding types - people who sing/whistle at work (W), do not sing

(D) and occasionally sing. (O). People join together to form partnerships and produce a good, say a building service. Note that all agents are equally productive in production of the good. An agent's utility from a partnership depends on his tastes and the crowding profile of the partnership. The utility of belonging to a partnership can be detailed:

$$U_H(W,W) = 0 \quad U_L(W,W) = 4 \quad U_H(O,D) = 3 \quad U_L(O,D) = 1$$

$$U_H(W,O) = 1 \quad U_L(W,O) = 3 \quad U_H(D,D) = 4 \quad U_L(D,D) = 0$$

with all other partnerships giving utility 2. For example, if someone who sings at work but does not like music at work joins with someone who occasionally sings he receives payoff $U_H(W,O) = 3$. If he joins with someone who does not sing he receives payoff $U_H(W,D) = 2$.

We contrast two economies where the number of players of each type is either zero or as given in this table:

type	WH	WI	OH	OL	DI	DL
number of type in economy S	6	4	2	4	4	4
number of type in economy G	6	4	4	6	4	4

Note that the number of players with crowding type O has increased.[10]

Two possible core allocations can be detailed as follows:

[10] As stated the number of players of type OI remains the same at zero. In the two

1. Economy S: 4×(WI, OL), 4×(WH, DL), 2×(OH, DI) and 2×(WH, DI).

2. Economy G: 4×(WI, OL), 4×(WH, DL), 4×(OH, DI) and 2×(WH, OL).

Giving core payoffs:

type	WH	WI	OH	OL	DI	DL
payoff in economy S	2	3	3	2	2	2
payoff in economy G	1.5	2.5	2	2.5	3	2.5

To see that these represent core states we detail the relevant parts of the value function: With the following exceptions, the worth of any pair of agents is 4.

composition	total utility	composition	total utility	composition	total utility
WH, WH	0	WI, OH	3	DL, DL	0
WH, WI	2	DI, OH	5	DI, DL	2
WI, OL	5	DL, OL	2		
WH, OH	2	DI, OL	3		

We observe that agents of type OL receive a higher payoff in economy G despite the increase in agents with crowding type O and type OL. So why,

core states given we could have partnerships (OI, OI) giving core payoffs of 2 to players of type OI. Thus, we could easily consider two economies where the number of players of type OI also increases by 2.

intuitively, are agents of type OL able to gain? Given that agents of type OL like to listen to music they would naturally want to form a partnership with agents who whistle (crowding type W) as opposed to those who do not whistle (D). Conversely, agents of type OH would naturally want to form a partnership with agents who do not whistle (D) as opposed to those who do (W). In economy S it so happens that agents with crowding type W are doing relatively well and agents with crowding type D relatively poorly; this has the side effect (or knock on effect, in British English) that agents of type OL receive a relatively low payoff and agents of type OH a relatively high payoff. In economy G the increased number of agents of type OH sees their 'bargaining position' reduced and consequently their payoffs fall. This feeds through into an increased 'bargaining power' for those agents who do not whistle and a decreased bargaining power for those who whistle. As the 'bargaining power' of whistlers falls agents of type OL are able to increase their payoff. Basically, there are cross-type influences whereby agents of type OL gain more 'bargaining power' by the increased number of players of type OH than they lose by the increased number of players with their own type OL.

Before concluding this section we note that, with more structure on the model, as in Brueckner (1994), it may be possible to avoid examples such as those above. It would be interesting to have further characterizations of economic situations with crowding types where the law of demand continues to hold.

5 Conclusion

In this paper we extended the basic approach pioneered by Tiebout to reexamine the theory of equalizing differences. We do this by drawing a connection between local public goods and the non-wage attributes of jobs. That is, the attributes that necessitate equalizing differences, such as danger, cleanliness, climate and the range of local amenities can all be seen as club goods.

The analogy of local public goods led us to consider the crowding types model of Conley and Wooders. This model has many desirable properties from a public economic sense and we find these qualities equally useful in the context of firm and region formation. Thus, the model allowed us to present a more complete model of equalizing differences in which we can account for the compensating wages between differing taste types while also modeling the markets for different productivity and skill levels. In doing so we make assumption standard to the literature: free mobility, no redistribution between clubs (e.g. no governments) and perfect information on the types of jobs available. We assume that a player's crowding type is observable and that crowding types are independent of taste types. We should acknowledge, that there are some contexts in which these assumptions may not be reasonable. For example, it may not be possible to Fully observe how smart or honest a potential new employee is, and it may be that smokers (a crowding type) like to smoke (a taste). However, there are many other circumstances in which these assumptions can be justified.

Having introduced the model, we turned to an application of particular interest - whether, following a ceteris paribus increase in the supply of a factor of production the per-unit return to that factor can increase. The introduction of compensating differentials means that taste types become important parts of the labor market - if one player prefers the attributes of the firm or region you can afford to pay that person a lower wage. This creates an independence in the money wage that players with the same skills, but different tastes, can earn and as such the arbitrage to equalize wages that we would expect within the standard market paradigm no longer applies.

From the general perspective of modeling equalizing differences there remains one significant area of further study. Compensating differentials apply to a wide variety of attributes, many of which can be modeled as above; the model can be used to look at regional compensations because of climate, local amenities and scenery etc. We have also considered firm and individual specific attributes, which can include cleanliness, vacations, shift work, pension packages, probability of unemployment and danger etc. The results above, however, do not apply to compensating differentials on the basis of human capital. That is, we have not considered the equalizing variations resulting from the cost and time spent learning a trade or skills. To do so would require us to look at the model from a different perspective - we have been comparing the payoff to players with the same crowding type but different taste type, while modeling human capital would require us to consider the payoffs to players with the same tastes but different crowding type. This paper shows the way to do this, however, the issue of human capital neatly fits the model of genetic types introduced in Conley and Wooders (2000). This paper generalizes the crowding types model so that players are endowed with a genetic type and not a crowding type. Players then purchase their crowding types at costs dependent on their genetic type. For example, the genetic type may be the level of intelligence and people purchase their skill level, with players with a higher intelligence finding it cheaper to purchase a high skill level. This question naturally fits the issue of human capital and would allow us to present a very interesting discussion of the role education and training plays in the process of equalizing differences. One further issue we note for future consideration is the possibility of players belong to more than one club. That is, a person joins a firm, then chooses the type of region he wants to live in and finally chooses the type of jurisdiction, meaning that an agent belongs to three distinct coalitions, or alternatively an agent may belong to two firms. This opens up a whole range of issues as to how the model can be extended and what we can learn from doing so.

In conclusion, this paper has presented a new way of considering two very old economic issues. Using the crowding types model we have analyzed the process of compensating differentials in the labor market and applied this to question the law of supply. The crowding types model has previously only been used to model public good economies but clearly it can have a very interesting role to play in modeling firm formation. This paper has merely looked at one possible application but there are a whole range of issues that still remain to be studied.

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