**Games of Status and Discriminatory Contracts**

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# Games of Status and Discriminatory Contracts *

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**Abstract**

Following recent empirical evidence which indicates the importance of rank for the determination of workers' wellbeing, this paper introduces status seeking preferences in the form of rank-dependent utility functions into a moral hazard framework with one firm and multiple workers, but no correlation in production. Workers' concern for the rank of their wage in the firm's wage distribution may induce the firm to offer discriminatory wage contracts when its aim is to induce all workers to expend effort.

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'...to understand what makes workers satisfied it is necessary to look at the distribution of wages inside a workplace. We show that rank matters to people. They care about where their remuneration lies within the hierarchy of rewards in their office or factory. They want, in itself, to be high up the pay ordering.'

Brown et al. (2003, p. 30)

# 1 Introduction

There is a wealth of experimental and empirical work that attests to the fact that workers in an organisation care about their position among peers –one recent example is Brown et al. (2003) which provides empirical and experimental evidence on the importance of *rank*.[1] Yet many of the theoretical results in the literature on optimal incentives in organisations assume completely self interested workers. The question of what happens to the nature of optimal contracts when agents can be *other-regarding* has only recently started getting attention.[2]

Itoh (2004) examines moral hazard in incentive contracts when workers have other-regarding preferences, but the analysis is restricted to interdependent and symmetric contracts only. Neilson and Stowe (2004) studies optimal linear contracts for workers with other-regarding preferences. Both sets of authors follow Fehr and Schmidt (1999) in modelling other-regarding preferences. The utility functions allow for both *inequality aversion* and status seeking (e.g Itoh (2004), Neilson and Stowe (2004)).

In contrast, this paper investigates the case of workers who are *status-seeking*, in the sense that they care about their *rank*[3] in the reference group. The focus is on the question of optimal wage contracts in a simple setting of moral hazard with identical status-seeking agents. For simplicity, this work follows the literature in making the assumption that status derives only from one dimension, i.e. the order of realised wages despite the awareness (as pointed out by Shubik (1971)) that status is often multi-dimensional.

There are two distinguishing features in the modelling approach used here. First, in contrast to the above mentioned studies the firm has the possibility to offer asymmetric contracts. A priori if the firm is interested in exploiting incentives from status to reduce its wage cost then not allowing the firm to use asymmetric contracts seems artificially restrictive. Second, differently from the existing literature on other-regarding preferences this work does not follow Fehr and Schmidt (1999) in allowing

[1] Brown et al. (2003) also provide a comprehensive survey of this literature.

[2] There are some early exceptions like e.g. Frank (1984) who argues that the presence of heterogeneous status seeking individuals will lead to wage compression. However he used the framework of perfectly competitive markets and not contract theory.

[3] Rank is based on the order of realised wages only.

wage levels and wage differences to matter to workers. Only ordinal differences are allowed to matter. This is motivated by two considerations: first there is empirical evidence (Brown et al. (2003)) that employees really care about rank and not about the deviation from a certain reference level, and second, these preferences could in principle be generalised to the case of more than two workers[4]. Indeed, making preferences depend on the whole vector of wages (rather than the order) when there are many co-workers in the reference group requires a lot of information on the part of the worker and seems to require strong assumptions about the specific way in which the wages of co-workers enters the utility function.

Shubik (1971) on the other hand had in mind a much simpler notion of status games. In the conversion from a two player game to a *game of status* the set of outcomes reduces to essentially three: Win, Lose or Draw. The natural extension of this to many players suggests that what matters is the number of people below, above or at the same rank. Dubey and Geanakoplos (2004) introduce exactly such a utility function for status seeking students. In other words, rank seems to us to be a more robust way to generalise how wage differences matter in the sense that it is an ordinal measure of status and does not require very precise information on the wage distribution[5]. Motivated both by an interest in exploring the role of *rank* as an indicator of status and the simple and general way in which Dubey and Geanakoplos (2004) allow status to matter through ranks, a modification of their model of status seeking is used.

The main contribution of this work is to show that when agents are conscious about their rank, then under certain conditions the firm finds it worthwhile to use *discriminatory contracts*. This is a surprising result: the firm offers different wage contracts to agents who are ex-ante identical! Such asymmetric contracts can be justified by having, say, higher probabilities of firing workers who benefit from higher expected status[6].

Also, consistent with the literature, it is found that when agents are status-seeking, there is *wage compression*. Finally, it is shown that looking for an optimal contract in this framework involves two steps: designing the game of status that maximises incentives (which then implies a given order of wages between the two workers) and if possible finding the wage levels that satisfy the participation and incentive constraints.

Winter (2004) shows that in an environment of complementarities in production

[4]It is conjectured that the results presented here generalise for more than 2 workers if there is sufficient richness in the distribution of the stochastic shocks.

[5]It might be argued of course that small changes in wage distribution lead to discontinuous changes in rank, but, in principle, some perceptions of changes in rank could be added so that wages would have to change by a significant amount for rank to change. This point is also addressed in the Conclusion.

[6]We are grateful to Debraj Ray for this remark.

and unobservable efforts, optimal mechanisms may be fully *discriminating*, i.e. they require unequal treatment of equals. The driving force in his story is the coordination problems between multiple agents. The more general point of his model is that when peer effects are important more "hierarchy" in organisations should be observed when this hierarchy is not related to different job descriptions. Agents in his model are complementary in production and generate externalities on each other in the sense that the profitability of their own effort level is increasing in the effort of others.

There is no reason why such externalities are generated only when workers are related in production. This paper considers the role of status seeking agents in an environment where effort is not always observable. Status seeking has similar properties as complementarities in production: i.e. the effort that agents put in imposes an externality on other agents. If other agents put in effort then the expected gain from putting in effort for an individual worker increases.

# 2 The Model

In this section a model of moral hazard is built with $n$ agents who are status seeking. The model is standard, apart from the utility functions of workers. The general model with $n$ workers is presented first and then afterwards the discussion is specialised to the main case studied in this paper: that of $n = 2$ workers.

## 2.1 Workers

There are $n$ identical workers who have the choice between two different effort Levels $e_i = \{e^L, e^H\}$ where $e^H > e^L$. A worker's cost of a certain chosen effort level is assumed to be equal to $c_i(e_i)$, which is assumed to be increasing and convex.

Worker $i$'s utility depends on the firm's realised wage distribution $\mathbf{w} = [w_1, w_2]$ and the cost of providing effort level $e_i$:

$$U_i(\mathbf{w}, e_i) = U(\mathbf{w}) - c(e_i).$$

Given $n$ and any realised wage distribution $\mathbf{w}$, *Worker i's rank* is defined as

$$r_i(\mathbf{w}) \equiv \frac{\#j + \gamma\#k - \alpha\#l}{n-1} \tag{1}$$

where $\#j$ (respectively $\#l$) is the number of workers that receive a strictly lower (respectively higher) wage than Worker $i$, and $\#k$ is the number of workers that receive the same wage as Worker $i$, excluding worker $i$. Furthermore $\gamma$ and $\alpha$ are scalars between zero and one, $\gamma, \alpha \in [0;1]$. This is a modification of the status model in Dubey and Geanakoplos (2004): they take rank to be the number of people below minus the number above. Thus $\gamma = 0$ and $\alpha = 1$ in their model[7]. Such preferences will be called *rank dependent*. Note, Worker $i$'s Rank is not differentiable in any of the workers' wages.

Assume that Worker $i$'s utility from a realised wage distribution $\mathbf{w}$ depends both on the magnitude of the wage he receives as a consequence of $\mathbf{w}$ and his rank in the firm's wage distribution $r_i(\mathbf{w})$, i.e.

$$U(\mathbf{w}) = w_i + \beta\rho\left(r_i(\mathbf{w})\right), \qquad \text{with } \rho'(\cdot) > 0,$$

where $\beta \geq 0$.

Given that the number of workers $n$ is fixed, Worker $i$'s direct utility from his rank $r_i$, $\rho(r_i)$, is assumed to be

$$\rho\left(r_i(\mathbf{w})\right) \equiv r_i(n-1)\hat{\rho},$$

where $\hat{\rho} > 0$. This implies that Worker $i$ obtains a direct utility of $\hat{\rho} > 0$ for each worker who receives a realised wage that is strictly smaller than his own wage. Furthermore, for each worker who receives the same wage as Worker $i$, he receives a utility of $\gamma\hat{\rho}$ and for each worker who is above him he receives $-\alpha\hat{\rho}$. This allows for the fact that there is clear empirical evidence that individuals prefer to have a high rank, but there are no clear guidelines yet from the empirical literature for how individuals feel about those who have the same rank as themselves in such a hierarchical framework. For ease of analysis define $\tilde{\rho} \equiv \gamma\hat{\rho}$, and $\rho' \equiv \alpha\hat{\rho}$. Then Worker $i$'s direct utility from his rank $\rho(r_i)$ can be expressed as

$$\rho\left(r_i\right) = (\#j)\hat{\rho} + (\#k)\tilde{\rho} - (\#l)\rho' \tag{2}$$

Having presented the general model for rank dependence for $n$ workers, now the discussion is specialised to the main case discussed in this paper: that of $n = 2$ workers. It is assumed that there are two effort levels and two states of nature. Hence, let $\mathbf{w} = [\mathbf{w}_1, \mathbf{w}_2]$. The rank dependent Utility function is:

$$\begin{aligned} U(\mathbf{w}) &= w_i + \beta\hat{\rho}, \text{ if } w_i > w_j \\ U(\mathbf{w}) &= w_i + \beta\tilde{\rho}, \text{ if } w_i = w_j \\ U(\mathbf{w}) &= w_i - \beta\rho', \text{ if } w_i < w_j \end{aligned} \tag{3}$$

where $\beta \geq 0, \hat{\rho} \geq \tilde{\rho}, \rho' \geq 0$.

[7] Shubik (1971) uses a similar utility function for games of status. He also points out the problem in assigning points for handling ties.

Next, the following definition about workers' preferences with respect to the direct utility derived from their rank is made.

**Definition: Convex preferences on status** are those which satisfy $\hat{\rho}-\tilde{\rho} > \tilde{\rho}+\rho'$, i.e.

$$\hat{\rho} - \rho' > 2\tilde{\rho}.$$

**Concave preferences on status** are those which satisfy $\hat{\rho} - \tilde{\rho} < \tilde{\rho} + \rho'$, i.e.

$$\hat{\rho} - \rho' < 2\tilde{\rho}.$$

Note that if lotteries over contracts are allowed, then convexity (concavity) of preferences over status means that a 50-50 lottery over a contract that puts the agent ahead all the time and a contract that puts the agent behind all the time is (not) preferred to a contract that puts him equal for sure all the time.

Also, $c_i(e_i) = \bar{c}e_i^l$ where $l = L, H$. The workers' choices of effort level is simplified to either exerting effort ($e^H \equiv 1$) or not ($e^L \equiv 0$). It is assumed that a worker who rejects the firm's offered contract and therefore does not enter the employment relationship receives a fixed income of zero. Following Neilson and Stowe (2004, p. 10), the natural assumption is made that if a worker is not in an employment relationship then he does not compare his income to that of other workers, and hence the rank dependent component of the utility function is irrelevant. This implies that Worker $i$'s *reservation utility*, $\bar{U}$, is normalised to zero.

Finally, it will be assumed that the utility from rank is bounded

**Assumption 1.** The degree to which Worker $i$ cares about rank is bounded from above such that

$$\beta < \frac{\bar{c}}{\hat{\rho} + \rho'}.$$

This assumption is made to rule out situations where a worker derives all his utility from status so that he will be ready to work for next to nothing as long as he has status[8].

## 2.2 Technology and Output

There are two different states of nature. Each state of nature is characterised by a different level of output. More specifically the output levels in the two states of

[8]This seems to be the situation in British universities which seem to take full advantage of their status seeking academics to pay them a pittance.

nature $s = H, L$ are such that $y^H > y^L$. Sometimes the adjectives good, and bad are used to refer to the different states of nature. No correlation in agents shocks nor any technological link between them is allowed. These assumptions are made to focus on the case when there are no externalities between agents except those induced by status.

Assume the technology describing the stochastic relationship between effort and output is such that if $e_i = e^H$ then with probability $p^{Hs} > 0$ output is $y_i = y^s$ where $s = L, H$. However, if Worker $i$ chooses effort level $e^L$ instead, then with probability $p^{Ls} > 0$ output is equal to $y_i = y^s$ where $s = L, H$. Thus $y_i$ depends only on the effort level chosen by Worker $i$ and the assumed technology describing the relationship between $e_i$ and $y_i$, but not on the effort level chosen by Worker $j$ where $j, i = 1, 2$ with $j \neq i$. Additionally, for readability define $\Delta p^s \equiv p^{Hs} - p^{Ls}$ for $s = L, H$. Finally, throughout the remainder of this paper the following assumption is made about the technology describing the stochastic relationship between effort and output.

**Assumption 2.** The technology is such that the distribution of output if a worker expends effort *first-order stochastically dominates* the distribution of output if a worker expends no effort such that

$$p^{HL} < p^{LL} \qquad \text{and} \qquad p^{HH} > p^{LH}.$$

## 2.3 The Firm

The firm possesses $n = 2$ identical production facilities $i = 1, 2$, called factories. Each factory employs exactly one worker. It is assumed that Worker 1 is employed at Factory 1 and Worker 2 at Factory 2. This interpretation of the two workers is made to simplify exposition, so that it can be referred to the two factories without confusion. It should not be taken too literally: any two workers in the firm who are not related through production would suffice for the model presented here.

Assuming that the firm is risk-neutral, the firm's choice problem is to choose a profile of wage contracts $\omega$ and a profile of effort levels $\mathbf{e}$ which maximise its expected combined profit $\Pi$ from Factory 1 and 2:

$$\max_{\mathbf{w}(y_1,y_2),\mathbf{e}\in\{(e_1,e_2)|e_i\in\{e^L,e^H\}\ \}} \sum_i \sum_s \left[p_i^{ls}\left(y_i^s - w_i\left(\mathbf{w}\right)\right)\right] \tag{4}$$

where $p_i^{ls}$ denotes the probability of getting the outcome $s$ when effort $e^l, l = L, H$ is exerted.

The constraints for the firm are:

– the workers' *individual rationality constraints*

$$E\left(U_i(\mathbf{w}(y_1, y_2), \mathbf{e})\right) \geq \bar{U} = 0,$$

– the workers' *limited liability constraints*, i.e. for $i = 1, 2$

$$w_i(\mathbf{w}) \geq 0, \tag{5}$$

where $w_i(\mathbf{w})$ is the wage paid to Worker $i$ when the firm observes output $y_1$ and $y_2$ as specified by the wage contract $\mathbf{w}$. There are also *incentive constraints* for the firm which however depend on which effort level is desired for each worker.

The above profit-maximisation problem of the firm is equivalent to the following two-stage problem:

(i) For given effort levels, $e_1$ and $e_2$, the firm minimises its expected wage cost $EWC$.

(ii) The firm maximises expected profit $\Pi$ by comparing the different outcomes of stage (i) with each other.

In this paper, the focus is on the minimisation of the firm's expected wage cost $EWC$. To be able to ignore Stage (ii) throughout the remainder of this paper the following assumption is made.

**Assumption 3.** In terms of the minimisation of its expected wage cost $EWC$ it is optimal for the firm to induce both workers to expend effort.

## 2.4 The Wage Contracts

In principle many different feasible sets of contracts could be distinguished. The most general set is the set of dependent and asymmetric contracts, where wages can be conditioned on the outcome in both factories and in addition on the identity of workers.

For the two worker case there is a convenient representation:

| | H | L |
|---|---|---|
| H | $(w_1^{HH}, w_2^{HH})$ | $(w_1^{HL}, w_2^{HL})$ |
| L | $(w_1^{LH}, w_2^{LH})$ | $(w_1^{LL}, w_2^{LL})$ |

The matrix denotes the four possible events $s_1 \times s_2$ where $s_i$ $(i = 1, 2)$ is the state of nature corresponding to Factory $i$ and worker $i$. Note, whenever this representation is used worker 1 is the column worker and worker 2 is the row worker. A contract is called *dependent* if wages depend on the output levels in both factories. Thus wages are a vector $w_i^m$ where

$$m \in S_D = \{(H,H),(H,L),(L,H),(H,H)\}.$$

A contract is also *dependent and symmetric* if $w_i^{kh} = w_j^{hk}$ where $k$ represents the random shock to worker $i$ and $h$ to worker $j$ .

A contract is *independent* if $w_i^{kh} = w_i^k$, i.e. the wages of $i$ are independent of the shock to worker $j$. Thus wages of worker $i$ are a vector $w_i^m$ where $m \in S_I = \{(H),(L)\}$. Finally a contract is *independent and symmetric* if $w_i^k = w_j^k$.

# 3 Complete Information

As a starting point in accordance with Laffont and Martimort (2002, p. 151) "... assume that the principal and a benevolent court of law can both observe effort. This variable is now verifiable and can thus be included into a contract enforced by the court of law." In the following two cases are distinguished. First, the standard complete information problem with rank-independent preferences ($\beta = 0$) is presented for completeness. Then, secondly, the case with rank-dependent preferences ($\beta > 0$) is studied. The optimal contract when effort is verifiable and hence contractible is described for the case when workers have rank-dependent preferences.

## 3.1 The Benchmark Complete-Information Problem

If Worker $i$'s ($i = 1,2$) preferences are rank-independent, or in other words if $\beta = 0$ then the whole economic problem degenerates to the standard complete information problem. For each worker individually the firm has to find a wage contract which maximises its expected profit or equivalently minimises its expected wage cost, and makes the individual worker expend effort.

If workers are not status seeking and hence have no externalities on each other, the two workers problem is clearly separable in each of them and there is no use in having dependent or asymmetric contracts. hence only worker $i$'s problem is dicussed.

If Worker $i$ expends effort, his *participation or individual rationality constraint* is equal to

$$\sum_m p^{Hm} w^m - \bar{c} \geq 0, \tag{6}$$

with $m \in S_I$. The *complete information optimal contract* $\mathbf{w}^{CI}$ solves the following problem

$$\min_{\mathbf{w}} \sum_m p^{Hm} w^m \qquad \text{subject to} \qquad \sum_m p^{Hm} w^m - \bar{c} \geq 0 \text{ and } w^m \geq 0. \tag{7}$$

This is what the next proposition, a standard result in this field shows.

From (7) it is immediately obvious that there is no unique complete information optimal contract. Instead there is an infinite number of contracts that solve this optimization problem. Any wage contract with $w^m \geq 0$ such that the individual rationality constraint (6) is binding is a complete information optimal contract. The following proposition summarises this standard result of contract theory (see for instance Laffont and Martimort (2002)).

**Proposition 1.** *(Laffont and Martimort (2002)). Let workers' preferences be rank-independent. Given the assumption of complete information, any wage contract* $\mathbf{w}^{CI}$ *with* $w^m \geq 0$ *such that*

$$EWC_i\left(\mathbf{w}^{CI}\right) = \bar{c}$$

*is a complete information optimal contract. The* **first-best cost** *of implementing the high effort level* $e^H = 1$ *is*

$$C_i^{FB} = EWC_i\left(\mathbf{w}^{CI}\right) = \bar{c}.$$

Thus, any wage contract that leads to a minimum wage cost equal to $\bar{c}$ and offers a positive wage in each state of nature is an optimal wage contract. All optimal contracts howver give the same expected wage cost.

This implies for the model with two workers in two different factories that the firm might offer different wage contracts to different workers, but it would not improve the firm's cost structure. The firm's minimum wage cost is always $2\bar{c}$.

## 3.2 Rank-Dependence and Complete Information

What happens to this standard result once rank-dependence is introduced? Now the choice between independent and dependent contracts, or between symmetric and asymmetric contracts is not so trivial.

The most general contract is the asymmetric dependent one. Thus wages are a vector $w_i^m$ with $m \in S_D$. Let $q^m$ denote the joint probability of event $m$ given that both workers expend effort. Thus $q^m = p^{Hs_1} p^{Hs_2}$ were $m = s_1 s_2$.

Given that effort is verifiable, the principal's problem is

$$\min_{\mathbf{w}} EWC = \sum_i \sum_m [q^m w_i^m]$$

subject to the limited liability constraint (5) and the participation constraints

$$\sum_m q^m w_i^m \geq \bar{c} - \beta E\left(\rho(r_i)|\mathbf{w}\right), \tag{8}$$

where $E\left(\rho(r_i)|\mathbf{w}\right)$ is the *expected utility from status*, and it is equal to

$$E\left(\rho(r_i)|\mathbf{w}\right) = q^m \rho(r_i(w_i^m, w_j^m)) \tag{9}$$

First symmetric and independent contracts are examined as they are easy to characterise. It is then shown that no symmetric contract, even if it is dependent can do better. Finally, a characterisation of the optimal asymmetric independent contract is provided and it is shown that no dependent asymmetric contract can do better. It can be concluded that when effort is observable, and agents have convex preferences on status, then asymmetric contracts dominate symmetric contracts.

Let $m \in S_I$. The principal's problem is

$$\min_{\mathbf{w}} EWC = 2\sum_m p^{Hm} w^m$$

subject to the limited liability constraint (5) and the participation constraint

$$\sum_m p^{Hm} w^m \geq \bar{c} - \beta E\left(\rho(r_i)|\mathbf{w}\right), \tag{10}$$

where

$$\begin{aligned} E(\rho(r_i)|\mathbf{w}) &= p^{HH}[p^{HH}\rho(r_i((w_i^H, w_j^H))) + \left(1 - p^{HH}\right)\rho(r_i(w_i^H, w_j^L))] \\ &+ \left(1 - p^{HH}\right)[(p^{HH}(\rho(r_i\left(w_i^L, w_j^H\right))) + \left(1 - p^{HH}\right)(\rho(r_i\left(w_i^L w_j^L\right))))] \end{aligned}$$

Note, given that the firm offers symmetric contracts the two workers have the same participation constraint.

**Lemma 1.** *In the case of independent and symmetric contracts if preferences on status are convex, then the expected utilities from status are maximised when $w^H \neq w^L$. If preferences on status are concave then the expected utilities from status are maximised when $w^H = w^L$.*

**Proof:** Observe that symmetry imposes the condition: $w_1^k = w_2^k$, $k \in \{H, L\}$ Hence only matrices where the following rank payoff entries are already chosen can be considered:

| | H | L |
|---|---|---|
| H | $(\tilde{\rho}, \tilde{\rho})$ | $(X, Y)$ |
| L | $(Y, X)$ | $(\tilde{\rho}, \tilde{\rho})$ |

and $X, Y$ denote the ranks corresponding to the choice of $w^H > (\leq) w^L$. The expected utility from status is equal to

$$E(\rho(r_i)|\mathbf{w}) = \tilde{\rho}\left[[(p^{HH})^2 + \left(1 - p^{HH}\right)^2\right] + p^{HH}(1 - p^{HH})\left(X + Y\right).$$

There are thus two possible choices: the *non-egalitarian* contract where $w_H > w_L$ or vice-versa and in either case the expected utility from status is:

$$E\left(\rho(r_i)|\omega^{\neq}\right) = \tilde{\rho}\left[(p^{HH})^2 + \left(1 - p^{HH}\right)^2\right] + p^{HH}(1 - p^{HH})\left(\hat{\rho} - \rho'\right). \quad (11)$$

If it is an egalitarian contract then $w_H = w_L$ and the expected utility from status is $E\left(\rho(r_i)|\omega^{=}\right) = \tilde{\rho}$.

It is then obvious that whenever preferences on status are convex then the expected utility from status of the workers is higher with the non-egalitarian contract and vice versa when preferences on status are concave.

□

Hence expected minimum costs should be expected to be lower with the non-egalitarian contract when preferences on status are convex. The next proposition shows that this is indeed the case.

**Proposition 2.** *Assume* $0 < p^{HH} < 1$. *(i) If workers have convex preferences on status then with rank-dependent preferences the complete information optimal symmetric and independent contract possesses a non-egalitarian structure, i.e.* $w^H \neq w^L$. *Given rank-dependent preferences the expected wage cost of implementing the high effort level* $e^H = 1$ *is*

$$EWC\left(\mathbf{w}^{CI}|\omega^{\neq}\right) = 2\left\{\bar{c} - \beta E\left(\rho(r_i)|\omega^{\neq}\right)\right\}, \quad (12)$$

*where*

$$E\left(\rho(r_i)|\omega^{\neq}\right) = \tilde{\rho}\left((p^{HH})^2 + (1 - p^{HH})^2\right) + p^{HH}\left(1 - p^{HH}\right)\left(\hat{\rho} - \rho'\right\}.$$

*(ii) If workers have concave preferences on status, then with rank-dependence there is a unique complete information optimal symmetric and independent contract which possesses an egalitarian structure,*

$$\left(w^H, w^L\right) = \left(\bar{c} - \beta\tilde{\rho}, \bar{c} - \beta\tilde{\rho}\right)$$

*The expected wage cost of implementing the high effort level* $e^H = 1$ *is*

$$EWC\left(\mathbf{w}^{CI}|\omega^{=}\right) = 2\left\{\bar{c} - \beta\tilde{\rho}\right\}.$$

**Proof:** Lemma (1) showed that when preferences on status are convex, the non-egalitarian contract gives the maximum expected utility from status and when preferences on status are concave, the egalitarian contract gives maximum expected utility from status. Hence costs are minimised as long as wages $w^H, w^L$ can be found that satisfy the constraints: $w^H \neq w^L$ (non-egalitarian contract) and $w^H = w^L$ (egalitarian contract). Hence, any $w^H \neq w^L$ such that

$$p^{HH}w^H + (1-p^{HH})w^L = \bar{c} - \beta\left\{\tilde{\rho}\left((p^{HH})^2 + (1-p^{HH})^2\right) + p^{HH}\left(1-p^{HH}\right)(\hat{\rho}-\rho')\right\}$$

is a solution to the cost minimising non egalitarian contract. Similarly $w^H = w^L = (\bar{c} - \beta\tilde{\rho})$ is a solution to the cost minimising egalitarian contract.

□

Does the presence of status seeking agents causes *wage compression*? Wage Compression in this setting occurs when in the optimal (symmetric independent) contract, the wage difference $w_i^H - w_i^L$ is lower with rank dependent preferences than in the benchmark case. In the egalitarian contract, $w^H - w^L = 0$ while in the non-egalitarian contract, $w^H - w^L = w^H = \bar{c} - \beta\left\{\tilde{\rho}\left((p^{HH})^2 + (1-p^{HH})^2\right) + p^{HH}\left(1-p^{HH}\right)(\hat{\rho}-\rho')\right\}$, both of which are smaller than $w^H - w^L = \bar{c}$, in the standard case.

Such a result is shown e.g. in Neilson and Stowe (2004) for whom the key driving force that causes wage compression is *behindness aversion* i.e. *changes* in payoff matter more to the worker when he is behind than when he is ahead of co-workers. In the model presented here, behindness aversion is interpreted as the following: Starting from a position of equal wages, $w$, being ahead by an amount $x > 0$, generates a utility of $\hat{\rho} - \tilde{\rho}$, while being behind by an amount $x > 0$ generates a disutility of $\tilde{\rho} + \rho'$. *Thus a worker is behindness averse iff preferences on status are concave.*

Assume that $\max(\tilde{\rho}, \left\{\tilde{\rho}\left((p^{HH})^2 + (1-p^{HH})^2\right) + p^{HH}\left(1-p^{HH}\right)(\hat{\rho}-\rho')\right\}) > 0$. Comparing the optimal contract with rank dependent preferences and the optimal contract in the benchmark case, the setup presented here leads to a wage compression result that holds *independently* of behindness aversion. If preferences on status are concave, then this wage compression result relies on having a positive utility from being equal in rank.

The next question is whether the expected costs of the principal could be lowered even further with dependent and asymmetric contracts. First it is claimed that with dependent *symmetric* contracts the firm can do no better than Proposition (2).

To see this note that symmetry imposes the condition that $w_1^{HH} = w_2^{HH}$,$w_1^{LL} = w_2^{LL}$ and $w_1^{HL} = w_2^{LH}$, $w_1^{LH} = w_2^{HL}$. This leads to the following rank matrix which is the same as the matrix with symmetric and independent contracts.

| | H | L |
|---|---|---|
| H | $(\tilde{\rho}, \tilde{\rho})$ | $(X, Y)$ |
| L | $(Y, X)$ | $(\tilde{\rho}, \tilde{\rho})$ |

Hence, the minimum feasible expected wage cost cannot be lower than that with the symmetric independent contracts. Thus dependence by itself does not buy us any extra degrees of freedom.

Can the firm do better with asymmetric dependent contracts? It is shown that while asymmetry is crucial, dependence does not matter. The firm can do no better with a dependent contract than with an independent asymmetric one. The way this is shown is to consider the optimal asymmetric independent contract and show that this achieves the lowest possible cost, i.e. it is not possible to achieve costs lower than this level. The implication is that no dependent contract can do strictly better. This is shown in the next theorem.

Observe that the game of status is conceptually a strictly competitive game. Hence giving higher expected status to one worker is at the expense of the other worker who must then be compensated suitably for lower expected status. It is therefore not obvious that costs can be lowered by asymmetric contracts. The problem is not trivial because it has been assumed that status only comes from the order of the wages– if there was another dimension of status then, of course costs can always be lowered, since one worker can be given a higher rank but lower wages and vice versa. However it seems highly implausible that high status workers get paid less than low status workers.

The expected wage cost of the firm, $EWC$, depend on both participation constraints:

$$EWC \geq 2\bar{c} - \beta \left( E(\rho(r_1)|\mathbf{w}) + E(\rho(r_2)|\mathbf{w}) \right).$$

**Lemma 2.** *The minimum expected wage cost possible for the firm, $EWC^{min}$ are equal to $2\bar{c} - \beta(\hat{\rho} - \rho')$ if preferences on status are convex and equal to $2\left(\bar{c} - \beta\tilde{\rho}\right)$ otherwise.*

**Proof:** Observe that total costs of the firm are minimised if $E(\rho(r_1)|\mathbf{w}) + E(\rho(r_1)|\mathbf{w})$ is maximised. Now,

$$E(\rho(r_1)|\mathbf{w}) + E(\rho(r_2)|\mathbf{w}) = (p^{HH})^2(x_1) + (p^{HH})(1 - p^{HH})(x_2 + x_3) + (1 - p^{HH})^2(x_4) \tag{13}$$

where $x_i \in \{\hat{\rho} - \rho', 2\tilde{\rho}\}$ for $i = \{1, 2, 3, 4\}$. It is clear that when preferences on status are convex $x_i^* = \hat{\rho} - \rho'$ maximises expected utilities from status while if preferences on status are concave then $x_i^* = 2\tilde{\rho}$ maximises expected utilities from status.

□

It can now be established that the firm can achieve minimum expected wage cost $EWC^{min}$ with independent contracts as long as there exists the possibility that the firm can use asymmetric contracts. Let

$$\bar{p} = \frac{\varphi}{2}\left\{\sqrt{1+\frac{4}{\varphi}}-1\right\},$$

where $\varphi = 1 + \frac{\rho'}{\hat{\rho}+\rho'}$. Note, $1 \leq \varphi \leq 3/2$ given the assumptions made about $\hat{\rho}$ and $\rho'$ and hence $0 < \bar{p} < 1$.

**Theorem 1.** *With complete information for all, there exists a profile of optimal independent wage contracts that guarantees minimum expected wage cost $EWC^{min}$ for the firm.*

*When preferences on status are concave the contracts are symmetric and equal to*

$$w^H = w^L = \bar{c} - \beta\tilde{\rho}.$$

*When preferences on status are convex the contracts are asymmetric and equal to ($i, j = 1, 2$ with $i \neq j$)*

$$\tilde{\mathbf{w}}_i = \left\{\frac{\bar{c} - \beta\left(p^{HH}\hat{\rho} - (1-p^{HH})\rho'\right)}{p^{HH}}\ ,\ 0\right\},$$

*and $\tilde{\mathbf{w}}_j = \{\tilde{w}, \tilde{w}\}$ where*

$$\tilde{w} = \bar{c} - \beta\left(\left(1 - p^{HH}\right)\hat{\rho} - p^{HH}\rho'\right)$$

*whenever $p^{HH} \leq \bar{p}$,*

*and equal to*

$$\breve{\mathbf{w}}_i \equiv \left\{0,\ \frac{\bar{c} - \beta\left((1-p^{HH})\hat{\rho} - p^{HH}\rho'\right)}{(1-p^{HH})}\right\}.$$

*and $\breve{\mathbf{w}}_j = \{\breve{w}, \breve{w}\}$ where*

$$\breve{w} = \bar{c} - \beta\left[p^{HH}\hat{\rho} - (1-p^{HH})\rho'\right]$$

*whenever $p^{HH} \geq 1 - \bar{p}$.*

**Proof:** Suppose preferences on status are concave, then the symmetric independent contract, $w^H = w^L = (\bar{c} - \beta\tilde{\rho})$ implements the $EWC^{min}$. Note that limited liability and participation constraints are satisfied since $\bar{c} > \beta\hat{\rho}$ follows from Assumption 1.

Now suppose preferences on status are convex. Consider firstly the profile of asymmetric and independent wage contracts $\tilde{\omega}$. Observe that given the profile $\tilde{\omega}$, $EWC = EWC^{min}$ if the order of wages satisfies:

$$\begin{aligned} \tilde{w}_i^H &> \tilde{w}_j^H \\ \tilde{w}_j^L &> \tilde{w}_i^L, \end{aligned} \tag{14}$$

where, remember, $\tilde{\mathbf{w}}_j = \{\tilde{w}, \tilde{w}\}$ (since then worker $i$'s expected rank utility is $E\left(\rho(r_i)|\tilde{\omega}\right) = p^{HH}\hat{\rho} - (1-p^{HH})\rho'$ and Worker $j$'s expected rank utility is $E\left(\rho(r_j)|\tilde{\omega}\right) = \left(1 - p^{HH}\right)\hat{\rho} - p^{HH}\rho'$).

It is sufficient to show that the profile $\tilde{\omega}$ is such that both workers participation constraints are satisfied and order (14) is satisfied:

Worker $i$'s participation constraint is

$$p^{HH}\tilde{w}_i^H + \left(1 - p^{HH}\right)\tilde{w}_i^L = \bar{c} - \beta\left[p^{HH}\hat{\rho} - \left(1 - p^{HH}\right)\rho'\right] = E\left(\rho(r_i)|\tilde{\omega}\right),$$

and Worker $j$'s participation constraint is

$$\tilde{w} = \bar{c} - \beta\left[\left(1 - p^{HH}\right)\hat{\rho} - p^{HH}\rho'\right] = E\left(\rho(r_j)|\tilde{\omega}\right).$$

Hence, the participation constraints are clearly satisfied given wage profile $\tilde{\omega}$.

The order of wages and the limited liability constraints imply that it has to be shown that $\tilde{w}_i^H > \tilde{w} > \tilde{w}_i^L = 0$.

Given Assumption 1, $p^{HH} \leq \bar{p}$ is a sufficient condition for the first inequality to be fulfilled:

Note, $\tilde{w}_i^H > \tilde{w}$ iff

$$\bar{c} > \beta\left\{\left[\frac{\left(p^{HH}\right)^2}{1 - p^{HH}}\right]\hat{\rho} + \left[1 - \frac{\left(p^{HH}\right)^2}{1 - p^{HH}}\right](-\rho')\right\}.$$

Assumption 1 implies that $\bar{c} > \beta\left(\hat{\rho} + \rho'\right)$ and hence a sufficient condition for the above (strict) inequality to be fulfilled is

$$\left(\hat{\rho} + \rho'\right) \geq \left\{\left[\frac{\left(p^{HH}\right)^2}{1 - p^{HH}}\right]\hat{\rho} + \left[1 - \frac{\left(p^{HH}\right)^2}{1 - p^{HH}}\right](-\rho')\right\}.$$

However this (weak) inequality is equivalent to

$$\varphi \geq \left[\frac{\left(p^{HH}\right)^2}{1-p^{HH}}\right] \qquad \text{or} \qquad p^{HH} \leq \frac{\varphi}{2}\left\{\sqrt{1+\frac{4}{\varphi}}-1\right\} = \bar{p}.$$

The second inequality follows directly from Assumption 1.

Consider secondly the profile of asymmetric and independent wage contracts $\breve{\omega}$. Observe that given the profile $\breve{\omega}$, $EWC = EWC^{min}$ if the order of wages satisfies:

$$\begin{aligned} \breve{w}_i^L &> \breve{w}_j^L \\ \breve{w}_j^H &> \breve{w}_i^H, \end{aligned} \tag{15}$$

where, remember, $\breve{\mathbf{w}}_j = \{\breve{w}, \breve{w}\}$ (since then Worker $i$'s expected rank utility is $E\left(\rho(r_i)|\mathbf{w}\right) = \left(1-p^{HH}\right)\hat{\rho} - p^{HH}\rho'$. Worker $j$'s expected rank utility is $E\left(\rho(r_j)|\mathbf{w}\right) = p^{HH}\hat{\rho} - \left(1-p^{HH}\right)\rho'$.)

It is sufficient to show that the profile $\breve{\omega}$ is such that both workers participation constraints are satisfied and order (15) is satisfied:

Worker $i$'s participation constraint is

$$p^{HH}\breve{w}_i^H + \left(1-p^{HH}\right)\breve{w}_i^L = \bar{c} - \beta\left[\left(1-p^{HH}\right)\hat{\rho} - p^{HH}\rho'\right] = E\left(\rho(r_i)|\mathbf{w}\right),$$

and Worker $j$'s participation constraint is

$$\breve{w} = \bar{c} - \beta\left[p^{HH}\hat{\rho} - \left(1-p^{HH}\right)\rho'\right] = E\left(\rho(r_j)|\mathbf{w}\right).$$

Hence, the participation constraints are clearly satisfied given wage profile $\breve{\omega}$.

The order of wages and the limited liability constraints imply that it has to be shown that $\breve{w}_i^L > \breve{w} > \breve{w}_i^H = 0$.

Given Assumption 1, $p^{HH} \geq 1-\bar{p}$ is a sufficient condition for the first inequality to be fulfilled:

Note, $\breve{w}_i^L > \breve{w}$ iff

$$\bar{c} > \beta\left\{\left[\frac{\left(1-p^{HH}\right)^2}{p^{HH}}\right]\hat{\rho} + \left[1-\frac{\left(1-p^{HH}\right)^2}{p^{HH}}\right](-\rho')\right\}.$$

Assumption 1 implies that $\bar{c} > \beta\left(\hat{\rho}+\rho'\right)$ and hence a sufficient condition for the above (strict) inequality to be fulfilled is

$$\left(\hat{\rho}+\rho'\right) \geq \left\{\left[\frac{\left(1-p^{HH}\right)^2}{p^{HH}}\right]\hat{\rho} + \left[1-\frac{\left(1-p^{HH}\right)^2}{p^{HH}}\right](-\rho')\right\}.$$

However this (weak) inequality is equivalent to

$$\varphi \geq \left[ \frac{\left(1-p^{HH}\right)^2}{p^{HH}} \right] \quad \text{or} \quad p^{HH} \geq 1 - \frac{\varphi}{2}\left\{ \sqrt{1+\frac{4}{\varphi}} - 1 \right\} = 1 - \bar{p}.$$

The second inequality follows directly from Assumption 1.

□

The following remark follows from the above theorem.

**Remark:** With complete information such that effort is verifiable independent contracts allow the firm to achieve whatever it can achieve with dependent asymmetric contracts as long as it has the ability to choose independent asymmetric contracts.

Notice that, unlike the case of rank independent preferences, it is necessary to condition wages on output and not just effort, in order to achieve minimum cost: the asymmetric contract cannot be replicated by a deterministic one that is conditioned only on effort. However, if it is allowed that contracts are stochastic then the asymmetric contract can be replicated with a symmetric stochastic contract that is conditioned only on effort. In order to provide the same incentives from status, consider a contract that gives worker 1 high wages $w^H > 0$, and worker 2 low wages $w^L = 0$, with probability $p^{HH}$ and vice versa with probability $1 - p^{HH}$ if they both put in high effort. This contract achieves the minimum cost. Thus, in the case of observable effort under some conditions the optimal contract is asymmetric and independent. The intuition behind this result is quite simple − when status and monetary incentives are substitutes, the firm is able to exploit the order of wages to create situations where each worker gets utility from status and hence lowers the expected wage cost. This intuition is easy to see with the stochastic symmetric contract.

This may seem to be unrealistic to assume: after all if firms started paying workers according to random events even when effort is fully observed it might cause a loss of morale. However, the results presented here are consistent with situations where effort is unobserved but is very weakly correlated with output. This suggests that in such situations, linking employee compensation differentially to random events (like changes in the value of the firm's stocks) might occur since it lowers overall costs to the firm by creating artificial hierarchies.

Next the situation when effort is unobservable is studied. What matters here is the expected gain in status when an agent works relative to when he shirks, given that all others are working. The following section addresses the question of whether asymmetric contracts can do better than symmetric contracts, even in the moral hazard setting.

# 4 Moral Hazard with Risk Neutral parties

Assume the firm's aim is to minimise its expected wage cost and to induce both workers to expend effort, $e_1 = e_2 = 1$. If the firm is unable to observe its workers' actions, i.e. their choice of effort, directly, then the firm can offer only a contract that is based on the observable and therefore verifiable output levels of the two factories. However again the firm has the choice between dependent or independent contracts, and symmetric or asymmetric contracts.

The remainder of this section is structured as follows. First, the standard moral-hazard problem is presented. As in the complete information case when there is no rank-dependence in utilities, then the two factories problems are separable and so independent symmetric contracts will do as well as the most general contracts. So in this section only independent and symmetric contracts are considered. Next, the moral-hazard problem in an environment of rank-dependent preferences is set out. In this part independent and dependent contracts are studied. First, the case of independent contracts is presented distinguishing between symmetric and asymmetric independent contracts. It is shown that in some circumstances the firm can improve its situation by offering asymmetric independent contracts, but as long as the firm is restricted to independent contracts it is unable to achieve minimum expected wage cost. Second the case of dependent contracts is presented, and again symmetric and asymmetric contracts are studied. It is shown that the firm cannot improve its situation by using symmetric dependent contracts. However if the firm has the ability to use asymmetric dependent contracts, the most general form of contracts, then it is shown that the firm can achieve minimum expected wage cost. In this way it is established that workers' concern for the rank of their wage in the firm's wage distribution induces the firm to offer discriminatory wage contracts when its aim is to induce all workers to expend effort.

## 4.1 The Benchmark Moral-Hazard Problem

If Worker $i$'s ($i = 1, 2$) utility is rank-independent, or in other words if $\beta = 0$, then the whole economic problem degenerates to the standard moral-hazard problem. For each of its factories, the firm's strategy is to find a wage contract $\omega_i$ which maximises its expected profit or equivalently minimises its expected wage cost, and makes the individual worker expend effort at Factory $i$.[9]

Let $m \in S_I$. With incomplete information, i.e. if Worker $i$'s effort level is not verifiable, the problem of the firm is to find a wage contract $\mathbf{w}$ that minimises

$$\min_{\mathbf{w}} EWC = \sum_m p^{Hm} w^m,$$

[9]Therefore in the remainder of this section the subscript $i$ is omitted.

where $EWC$ represents the expected wage cost at Factory $i$, subject to the limited liability constraint (5), and Worker $i$'s *incentive constraint*[10]:

$$\Delta p^H w^H + \Delta p^L w^L \geq \bar{c}$$

or using the fact that $\Delta p^L = -\Delta p^H$

$$\Delta p^H \left[ w^H - w^L \right] \geq \bar{c}. \tag{16}$$

The following proposition describes the optimal contract for the standard moral-hazard problem. In its main part it repeats Proposition 1 of Itoh (2004).

**Proposition 3.** *For all $m = L, H$ and risk neutral parties the unique optimal contract solving the standard moral-hazard problem is*

$$\mathbf{w}^S = \left\{ w^H, w^L \right\} = \left\{ \frac{\bar{c}}{\Delta p^H}, 0 \right\},$$

*and the expected cost of implementing effort is*

$$EWC_i \left( \mathbf{w}^S \right) = p^{HH} \left( \frac{\bar{c}}{\Delta p^H} \right)$$

*in Factory $i$. The firm's overall expected cost of implementing effort is $2EWC_i \left( \mathbf{w}^S \right)$.*

## 4.2 Rank-Dependence and Moral Hazard

If effort is no longer observable and verifiable, and workers possess rank-dependent preferences then the firm's choice problem changes. Like in the standard moral-hazard problem the firm has to base its contracts on the observable variable output. However, with rank-dependent preferences, when offering a wage contract to one worker the firm has to take into account the likely effects of this contract on the other worker.

The most general type of contract that can be offered is (as before) the Asymmetric Dependent one. Let $E\left(w_i^{s_i}\right) = \sum_{s_j \in \{H,L\}} p^{Hs_j} \; w_i(s_i, s_j)$. Then $E\left(w_i^H\right)$ represents worker $i$'s expected wage when he is in the good state, $s_i = H$, given that $e_j = 1$, and $E\left(w_i^L\right)$ respectively is worker $i$'s expected wage when he is in the bad state, $s_i = L$, given that $e_j = 1$ (note, expectation is taken over the other worker's state of nature). If Contracts are independent then $E\left(w_i^{s_i}\right) = w_i(s_i)$. Similarily, define $E\left(\rho_i^{s_i}\right) = \sum_{s_j \in \{H,L\}} p^{Hs_j} \; \rho_i(r_i(w(s_i, s_j))$. Then $E\left(\rho_i^H\right)$ represents worker $i$'s expected rank payoff when he is in the good state, $s_i = H$, given that $e_j = 1$, and

[10]The individual rationality constraint is implied by the limited liability constraint and the incentive constraint.

$E\left(\rho_i^L\right)$ respectively is worker $i$'s expected rank payoff when he is in the bad state, $s_i = L$, given that $e_j = 1$

The firm's problem is to find contracts that will be conditioned on workers identity and the shocks in both factories to induce both workers to exert effort while minimising its expected wage cost:

$$\min_{\mathbf{w}} EWC = \sum_i \sum_m [q^m w_i^m]$$

subject to the limited liability constraints, which follow from (5),

$$w_i^m \geq 0,$$

the (low effort) participation constraints ($e_i = 0$)

$$p^{LH} E\left(w_i^H\right) + \left(1 - p^{LH}\right) E\left(w_i^L\right) + \beta E\left(\rho(r_i)|e^L\right) \geq 0, \tag{17}$$

where $E\left(\rho_i(r_i)|e^L\right)$ is worker $i$'s expected utility from status given $e_i = 0$ and $e_j = 1$, and the incentive constraints

$$\Delta p^H \left[E\left(w_i^H\right) - E\left(w_i^L\right)\right] \geq \bar{c} - \beta E\left(\Delta\rho_i\right), \tag{18}$$

where $E\left(\Delta\rho_i\right)$ is Worker $i$'s expected gain or loss in expected rank-utility from expending effort. It is equal to

$$E\left(\Delta\rho_i\right) \equiv \Delta p^H [E\left(\rho_i^H\right) - E\left(\rho_i^L\right)].$$

Define worker $i$'s *incentives from status* as

$$I_i = E\left(\rho_i^H\right) - E\left(\rho_i^L\right).$$

Then these incentives from status can be expressed as

$$\begin{aligned} I_i(\mathbf{w}) \quad = \quad & p^{HH}\{\rho(r_i(w(s_i = H, s_j = H))) - \rho(r_i(w(s_i = L, s_j = H)))\} + \\ & (1 - p^{HH})\{\rho(r_i(w(s_i = H, s_j = L))) - \rho(r_i(w(s_i = L, s_j = L)))\}. \end{aligned} \tag{19}$$

Using Equation (18), worker $i$'s incentive constraint can then be written as

$$\left[E\left(w_i^H\right) - E\left(w_i^L\right)\right] \geq \frac{\bar{c}}{\Delta p^H} - \beta I_i(\mathbf{w}) \tag{20}$$

Note, the participation constraints for $e_i = 1$ and $e_j = 1$ can be ignored in this framework, because they are implied by the (low effort) participation constraints and the incentive constraints.

The following lemma provides a straightforward characterisation of the minimum expected wage cost in the case of rank-dependence and moral hazard.

**Lemma 3.** *Let $I_1^* + I_2^*$ be the maximised value of $I_1 + I_2$, the sum of the incentives from status of the two players, across different wage orders. Then, the minimum expected wage cost possible to the firm, $EWC^{min}$, is*

$$EWC^{min} = p^{HH} \left\{ 2\frac{\bar{c}}{\Delta p^H} - \beta \left( I_1^* + I_2^* \right) \right\}.$$

**Proof:** From Equation (20) Worker $i$'s incentive constraint is

$$\left[ E\left(w_i^H\right) - E\left(w_i^L\right) \right] \geq \frac{\bar{c}}{\Delta p^H} - \beta I_i(\mathbf{w})$$

for any wage profile $\omega$. The firm's expected wage cost, $EWC$, can be written as

$$EWC = \sum_i \left\{ p^{HH} \left[ E\left(w_i^H\right) - E\left(w_i^L\right) \right] + E\left(w_i^L\right) \right\}.$$

Using these two information for any wage profile $\omega$ the following inequality has to hold

$$EWC \geq p^{HH} \left\{ 2\frac{\bar{c}}{\Delta p^H} - \beta \left( I_1 + I_2 \right) \right\}.$$

Note, the $RHS$ of the above inequality is minimised iff $I_1 + I_2$ is maximised, hence

$$EWC^{min} = p^{HH} \left\{ 2\frac{\bar{c}}{\Delta p^H} - \beta \left( I_1^* + I_2^* \right). \right\}.$$

□

The minimum feasible cost is achieved when for both workers $E\left(w^L\right) = 0$ and the sum $E\left(w_1^H\right) + E\left(w_2^H\right)$ is just enough to satisfy each of the two incentive constraints. The next lemma characterises the wage orders that maximise the sum of incentives.

**Lemma 4.** *The rank payoff matrix which maximises $I_1 + I_2$ ( the roles of the two workers can be exchanged and the same sum of incentives is obtained) is given by:*

| | $H$ | $L$ |
|---|---|---|
| $H$ | $\hat{\rho}, -\rho'$ | $\hat{\rho}, -\rho'$ |
| $L$ | $-\rho', \hat{\rho}$ | $\tilde{\rho}, \tilde{\rho}$ |

*when preferences on status are convex, and by*

| | $H$ | $L$ |
|---|---|---|
| $H$ | $\tilde{\rho}, \tilde{\rho}$ | $\hat{\rho}, -\rho'$ |
| $L$ | $-\rho', \hat{\rho}$ | $-\rho', \hat{\rho}$ |

*when preferences on status are concave.*

*The maximised value of $I_1 + I_2$ is equal to*

$$I_1^* + I_2^* = p^{HH}(\hat{\rho} + \rho') + 2(1 - p^{HH})(\hat{\rho} - \tilde{\rho})$$

*when preferences on status are convex, and by*

$$I_1^* + I_2^* = 2p^{HH}(\tilde{\rho} + \rho') + (1 - p^{HH})(\hat{\rho} + \rho')$$

*when preferences on status are concave.*

**Proof:**

Step 1: Consider the matrix:

| | H | L |
|---|---|---|
| H | $x_1, y_1$ | $x_2, y_2$ |
| L | $x_3, y_3$ | $x_4, y_4$ |

The problem is to maximize the following expression, by choice of $(x_i, y_i)$ pairs:

$$I_1 + I_2 = p^{HH}\left((x_1 - x_3) + (y_1 - y_2)\right) + \left(1 - p^{HH}\right)\left((x_2 - x_4) + (y_3 - y_4)\right).$$

subject to the constraint that $(x_i, y_i) \in \{(\hat{\rho}, -\rho'); (\tilde{\rho}, \tilde{\rho}); (-\rho', \hat{\rho})\}$. Notice that it has to be $(x_2, y_2) = (\hat{\rho}, -\rho')$ and $(x_3, y_3) = (-\rho', \hat{\rho})$ since $I_1 + I_2$ is increasing in $x_2, y_3$ and decreasing in $x_3, y_2$.

Hence the rank payoff matrix which maximizes $I_1 + I_2$ is of the form:

| | H | L |
|---|---|---|
| H | $x_1, y_1$ | $\hat{\rho}, \rho'$ |
| L | $-\rho', \hat{\rho}$ | $x_4, y_4$ |

**Step 2**: The various possibilities given Step 1 above are the following (upto symmetry between players):

(A) $(x_1, y_1) = (\hat{\rho}, -\rho')$, $(x_4, y_4) = (-\rho', \hat{\rho})$, with $I_1 + I_2 = \hat{\rho} + \rho'$

(B) $(x_1, y_1) = (\hat{\rho}, -\rho')$, $(x_4, y_4) = (\tilde{\rho}, \tilde{\rho})$ with $I_1 + I_2 = p^{HH}(\hat{\rho} + \rho') + 2(1 - p^{HH})(\hat{\rho} - \tilde{\rho})$.

(C) $(x_1, y_1) = (\tilde{\rho}, \tilde{\rho})$, $(x_4, y_4) = (-\rho', \hat{\rho})$, with $I_1 + I_2 = 2p^{HH}(\tilde{\rho} + \rho') + (1 - p^{HH})(\hat{\rho} + \rho')$.

(D) $(x_1, y_1) = (\tilde{\rho}, \tilde{\rho})$ $(x_4, y_4) = (\tilde{\rho}, \tilde{\rho})$ with $I_1 + I_2 = 2p^{HH}(\tilde{\rho} + \rho') + 2(1 - p^{HH})(\hat{\rho} - \tilde{\rho})$.

It is easy to see that the matrix which maximizes $I_1 + I_2$ is (B) when preferences on status are convex and (C) when preferences on status are concave.

□

The above two lemmas give some characterisation of the minimum feasible cost for the firm, $EWC^{min}$. So far it has not been established that there exists a feasible wage profile $\omega$ with a wage order that creates the corresponding incentives from status $I_1^*$ and $I_2^*$. Furthermore it has also not been established that given there exists a wage profile $\omega$ that creates the corresponding incentives from status $I_1^*$ and $I_2^*$ it is such that $EWC = EWC^{min}$.

Like in the case with complete information the most general type of wage contract is an asymmetric dependent one. However, before presenting the case of dependent contracts and especially asymmetric dependent contracts first independent contracts are investigated and two questions are answered: Firstly, which independent contract is minimising the firms expected wage cost, and, secondly, is the firm able to achieve the minimum expected wage cost $EWC^{min}$ with the help of an independent contract.

### 4.2.1 Independent Contracts

Given the restriction on independent contracts, again the firm can choose between symmetric and asymmetric independent contracts. Initially the focus is on symmetric independent contracts. With symmetric contracts each worker receives the same contract, i.e. in the following it is assumed

$$\mathbf{w_1} = \mathbf{w_2} = \left(w^H, w^L\right).$$

The firm's problem is to find symmetric contracts that induces both workers to expend effort and minimises its expected wage cost, i.e.

$$\min_{\omega} EWC = 2\left\{p^{HH}w^H + \left(1 - p^{HH}\right)w^L\right\}$$

subject to worker $i$'s incentive constraint, which follows from (20), and in the case of symmetric wage contracts can be written as

$$\left[w^H - w^L\right] \geq \frac{\bar{c}}{\Delta p^H} - \beta I_i(\mathbf{w}^S), \tag{21}$$

where

$$\begin{aligned} I_i(\mathbf{w}^S) \;\; = \;\; & \Big\{p^{HH}\left(\tilde{\rho} - \rho(r_i(w(s_i = L, s_j = H)))\right) \\ & + \left(1 - p^{HH}\right)\left(\rho(r_i(w(s_i = H, s_j = L))) - \tilde{\rho}\right) \Big\}, \end{aligned}$$

and the limited liability constraints, which follow from (5),

$$w^H, w^L \geq 0.$$

Now the order of wages $w^H, w^L$ has to be found which maximises the incentives of both workers. The rank payoff matrix has less flexibility now as symmetry imposes the conditions that $w_1^k = w_2^k$ for $k = H, L$. The rank payoff matrix is as below with the only choice being whether to choose $w^H \geq w^L$ or vice versa.

| | H | L |
|---|---|---|
| H | $\tilde{\rho}, \tilde{\rho}$ | $X, Y$ |
| L | $Y, X$ | $\tilde{\rho}, \tilde{\rho}$ |

with $X = \hat{\rho}, Y = -\rho'$ if $w^H > w^L$ and $X = -\rho', y = \hat{\rho}$ if $w^H < w^L$. Clearly incentives are maximised if $w^H > w^L$. This is what the next proposition shows.

Let

$$\underline{\rho}'_S = \left( \frac{p^{LH}\left(1 - p^{HH}\right)\hat{\rho} + \left[p^{LH}p^{HH} + \left(1 - p^{LH}\right)\left(1 - p^{HH}\right)\right]\tilde{\rho}}{p^{HH}\left(1 - p^{LH}\right)} \right).$$

The following proposition characterises the optimal symmetric independent contract that minimises the firm's expected wage cost.

**Proposition 4.** *Assume $\rho' \leq \underline{\rho}'_S$. The unique optimal symmetric independent wage contract that minimises the firm's expected wage cost is*

$$\mathbf{w}^{S*} \equiv \left(w^H, w^L\right)^{S*} = \left( \frac{\bar{c}}{\Delta p^H} - \beta\left\{p^{HH}\left(\tilde{\rho} + \rho'\right) + \left(1 - p^{HH}\right)\left(\hat{\rho} - \tilde{\rho}\right)\right\}, 0 \right).$$

*Given the optimal symmetric wage contract, $\mathbf{w}^{S*}$, the firm's expected wage cost is*

$$EWC^S = 2p^{HH}\left\{ \frac{\bar{c}}{\Delta p^H} - \beta\left\{p^{HH}\left(\tilde{\rho} + \rho'\right) + \left(1 - p^{HH}\right)\left(\hat{\rho} - \tilde{\rho}\right)\right\} \right\}, \qquad (22)$$

*where*

$$EWC^S > EWC^{min}.$$

**Proof:** The firm's expected wage cost can be rewritten as

$$EWC = 2\left\{w^L + p^{HH}\left[w^H - w^L\right]\right\}.$$

Using the Workers' limited liability constraints and incentive constraints, in the case of symmetric and independent contracts the following condition must hold:

$$w^L + p^{HH}\left[w^H - w^L\right] \geq p^{HH}\left\{ \frac{\bar{c}}{\Delta p^H} - \beta\left\{p^{HH}\left(\tilde{\rho} - Y\right) + \left(1 - p^{HH}\right)\left(X - \tilde{\rho}\right)\right\} \right\},$$

where $X, Y \in \{\hat{\rho}, \tilde{\rho}, -\rho'\}$ The $RHS$ of this inequality is minimised iff $Y = -\rho'$ and $X = \hat{\rho}$ and hence $w^H > w^L$. The above condition then becomes

$$w^L + p^{HH}\left[w^H - w^L\right] \geq p^{HH}\left\{ \frac{\bar{c}}{\Delta p^H} - \beta\left\{p^{HH}\left(\tilde{\rho} + \rho'\right) + \left(1 - p^{HH}\right)\left(\hat{\rho} - \tilde{\rho}\right)\right\} \right\}.$$

The $LHS$ and hence the firm's expected wage cost are minimised but fulfill the above condition iff $w^L = 0$ and

$$w^H = \frac{\bar{c}}{\Delta p^H} - \beta \left\{ p^{HH} (\tilde{\rho} - 0) + \left(1 - p^{HH}\right) (\hat{\rho} - \tilde{\rho}) \right\}.$$

In other words, the optimal symmetric wage contract is found by choosing the two state-dependent wages such that the workers' incentive constraints are binding and then setting $w^L$ equal to zero. The limited liability constraints are fulfilled given Assumption 1.

Finally, for $\mathbf{w}^{S*}$ to be optimal it has to be checked that the (low) participation constraint of Worker $i$ is fulfilled:

$$p^{LH} w^H + \left(1 - p^{LH}\right) w^L + \beta E\left(\rho(r) | e^L\right) \geq 0$$

where

$$E\left(\rho(r)|e^L\right) = p^{LH}\left(1 - p^{HH}\right)\hat{\rho} + \left[p^{LH} p^{HH} + \left(1 - p^{LH}\right)\left(1 - p^{HH}\right)\right]\tilde{\rho} - p^{HH}\left(1 - p^{LH}\right)\rho'$$

Given the assumption of $\rho'$ being sufficiently small, $\rho' \leq \underline{\rho}'_S$, Worker $i$'s participation constraint is satisfied, because $E\left(\rho(r)|e^L\right) \geq 0$.

From $\mathbf{w}^{S*}$ it follows directly that $EWC^S > EWC^{min}$.

□

As with the complete information case, there is a wage compression in this setting as well: the high wage is smaller than in the benchmark case without rank dependence.

The question now arises: is this the best contract possible or are there as in the complete information case circumstances under which there exists an asymmetric contract that does strictly better than the optimal symmetric independent contract? Next this question is analysed.

After the unique optimal symmetric independent contract has been described above, the aim is now to answer the question whether the firm can improve its outcome by offering asymmetric contracts. In other words is there a profile of asymmetric independent contracts which can make the firm better off than the unique optimal symmetric contract described in Proposition 4.

Define an order on wages $w_1^H, w_1^L, w_2^H, w_2^L$ as a *wage structure*. Observe that by Assumption 1 $w_i^H > w_i^L$. Hence there are only a limited number of such structures that are feasible. Surprisingly, in the following it is shown that all but one of the feasible wage structures that are implied by asymmetric independent contracts lead to higher wage costs for the firm than the symmetric optimal contract. The next

Lemma characterises all such wage structures implied by asymmetric independent contracts. Furthermore it is shown that, as in the case of symmetric independent contracts, it is not possible to obtain the minimum expected wage cost $EWC^{min}$ with asymmetric independent contracts.

**Lemma 5.** *There is no profile of asymmetric independent contracts $\omega^{AS}$ such that the wage structure is either $\mathbf{w_i}^{AS} \gg \mathbf{w_j}^{AS}$ or $\mathbf{w_i}^{AS} \geq \mathbf{w_j}^{AS}$ or $w_i^H > w_j^H > w_j^L > w_i^L$ ($i, j = 1, 2$ with $i \neq j$) which induces both workers to exert effort and makes the firm better off than with the optimal symmetric independent contract, i.e. $EWC\left(\omega^{AS}\right) > EWS^S$ for all $\omega^{AS}$. Furthermore with asymmetric independent contracts it is impossible for the firm to reach $EWC^{min}$.*

**Proof:** This proposition deals with four different cases of asymmetric contracts:

**Case** ($i$) $w_i^H, w_i^L > w_j^H, w_j^L$**:** Worker $i$'s incentive constraint is

$$\left[w_i^H - w_i^L\right] \geq \frac{\bar{c}}{\Delta p^H} - \beta\left\{p^{HH}(\bar{\rho} - \bar{\rho}) + \left(1 - p^{HH}\right)(\bar{\rho} - \bar{\rho})\right\},$$

or simplified

$$\left[w_i^H - w_i^L\right] \geq \frac{\bar{c}}{\Delta p^H}.$$

Similarly Worker $j$'s incentive constraint is

$$\left[w_i^H - w_i^L\right] \geq \frac{\bar{c}}{\Delta p^H}.$$

Note, the firm's expected wage cost is

$$\begin{aligned} EWC &\equiv \left\{p^{HH}w_1^H + \left(1 - p^{HH}\right)w_1^L\right\} + \left\{p^{HH}w_2^H + \left(1 - p^{HH}\right)w_2^L\right\} \\ &= w_1^L + p^{HH}\left[w_1^H - w_1^L\right] + w_2^L + p^{HH}\left[w_2^H - w_2^L\right]. \end{aligned}$$

The expressions for the two workers' incentive constraints imply that the following condition holds for the firm's expected wage cost:

$$EWC > 2p^{HH}\frac{\bar{c}}{\Delta p^H}.$$

But this inequality establishes that deviating from the optimal symmetric independent contract to such a profile of asymmetric independent contracts is not optimal for the firm.

The next two wage structures examined here are the ones which maximise the sum of incentives from status (see Lemma 3 and Lemma 4) and are therefore crucial for the question whether it is possible for the firm to achieve $EWC^{min}$ with asymmetric independent contracts.

**Case (ii)** $w_i^H > w_j^H > w_i^L = w_j^L$**:** Worker $i$'s incentive constraint is

$$\left[w_i^H - w_i^L\right] \geq \frac{\bar{c}}{\Delta p^H} - \beta \left\{p^{HH}\left(\hat{\rho} + \rho'\right) + \left(1 - p^{HH}\right)\left(\hat{\rho} - \tilde{\rho}\right)\right\}.$$

Similarly, Worker $j$'s incentive constraint is

$$\left[w_j^H - w_j^L\right] \geq \frac{\bar{c}}{\Delta p^H} - \beta \left\{\left(1 - p^{HH}\right)\left(\hat{\rho} - \tilde{\rho}\right)\right\}.$$

Given the above described wage structure Worker $i$'s incentive constraint cannot be binding, but Worker $j$'s is. The wage $w_i^H$ is determined by the fact that $w_i^H > w_j^H$. Hence, for the firm's expected wage cost the following inequality holds:

$$EWC > 2p^{HH}\left\{\frac{\bar{c}}{\Delta p^H} - \beta \left\{\left(1 - p^{HH}\right)\left(\hat{\rho} - \tilde{\rho}\right)\right\}\right\}.$$

But this inequality establishes that deviating from the optimal symmetric independent contract to such a profile of asymmetric independent contracts is not optimal for the firm.

**Case (iii)** $w_i^H = w_j^H > w_i^L > w_j^L$**:** Worker $i$'s incentive constraint is

$$\left[w_i^H - w_i^L\right] \geq \frac{\bar{c}}{\Delta p^H} - \beta \left\{p^{HH}\left(\tilde{\rho} + \rho'\right)\right\}.$$

Similarly, Worker $j$'s incentive constraint is

$$\left[w_j^H - w_j^L\right] \geq \frac{\bar{c}}{\Delta p^H} - \beta \left\{p^{HH}\left(\tilde{\rho} + \rho'\right) + \left(1 - p^{HH}\right)\left(\hat{\rho} + \rho'\right)\right\}.$$

Given the above described wage structure Worker $j$'s incentive constraint cannot be binding, but Worker $i$'s is. The wage $w^H$ is determined by the fact that $w_i^L > w_j^L$ and Worker $i$'s binding incentive constraint. Hence, for the firm's expected wage cost the following inequality holds:

$$EWC > 2p^{HH}\left\{\frac{\bar{c}}{\Delta p^H} - \beta \left\{p^{HH}\left(\tilde{\rho} + \rho'\right)\right\}\right\}.$$

But this inequality establishes that deviating from the optimal symmetric independent contract to such a profile of asymmetric independent contracts is not optimal for the firm.

Furthermore this two cases show that even tough it is possible with asymmetric independent contracts to maximise the sum of incentives from status, the wage structures necessary to induce these incentives from status imply that the firm is unable to obtain $EWC^{min}$, because it is not possible given the wage structure and hence the incentives from status to choose the wages in the different states of nature in such a way that all of the incentive constraints are binding.

**Case (iv)** $w_i^H > w_j^H > w_j^L > w_i^L$**:** Worker $i$'s incentive constraint is

$$\left[w_i^H - w_i^L\right] \geq \frac{\bar{c}}{\Delta p^H} - \beta\left\{\hat{\rho} + \rho'\right\}.$$

Similarly, Worker $j$'s incentive constraint is

$$\left[w_j^H - w_j^L\right] \geq \frac{\bar{c}}{\Delta p^H}.$$

Worker $i$'s incentive constraint cannot be binding, but Worker $j$'s is. The wage $w_i^H$ is determined by the fact that $w_j^L > w_i^L = 0$ and Worker $j$'s binding incentive constraint. Hence, for the firm's expected wage cost the following inequality holds:

$$EWC > 2p^{HH}\left\{\frac{\bar{c}}{\Delta p^H}\right\}.$$

But this inequality establishes that deviating from the optimal symmetric independent contract to such a profile of asymmetric independent contracts is not optimal for the firm and hence completes the proof of the above proposition.

□

Given this negative result, there is only one possible wage structure left that could possibly be used to construct an asymmetric independent contract that is better than the optimal symmetric independent contract. This wage structure is $w_i^H > w_j^H > w_i^L > w_j^L$ $(i, j = 1, 2$ with $i \neq j)$.

Let

$$\underline{\rho}'_{AS} = \left(\frac{p^{LH}\left(1 - p^{HH}\right)\hat{\rho}}{1 - p^{LH}\left(1 - p^{HH}\right)}\right).$$

The following proposition describes the minimum expected wage cost with an asymmetric independent contract with the wage structure described above.

**Proposition 5.** *Assume $\rho' \leq \underline{\rho}'_{AS}$. There exists a profile of asymmetric independent wage contracts $\omega^{AS*}$ with a asymmetric implied structure that is strictly increasing such that $(i, j = 1, 2$ with $j \neq i)$*

$$w_i^H > w_j^H > w_i^L > w_j^L,$$

*which induces all workers to expend effort and has expected wages costs $EWC\left(\omega^{AS*}\right)$ given as:*

*If $p^{HH} \leq \frac{1}{2}$:*

$$EWC\left(\omega^{AS*}\right) = 2p^{HH}\left\{\frac{\bar{c}}{\Delta p^{H}} - \beta\frac{(\hat{\rho}+\rho')}{2}\right\} + \epsilon. \tag{23}$$

*If* $p^{HH} > \frac{1}{2}$*:*

$$EWC \geq 2p^{HH}\left\{\frac{\bar{c}}{\Delta p^{H}} - \beta\left\{1-p^{HH}\right\}(\hat{\rho}+\rho')\right\} + \epsilon \tag{24}$$

*where* $\epsilon > 0$ *but equal to the smallest monetary unit.*

**Proof:**

The wage structure implied by $\omega^{AS*}$ implies that Worker $i$'s incentive constraint is

$$\left[w_i^H - w_i^L\right] \geq \frac{\bar{c}}{\Delta p^{H}} - \beta p^{HH}(\hat{\rho}+\rho').$$

Similarly Worker $j$'s incentive constraint is

$$\left[w_j^H - w_j^L\right] \geq \frac{\bar{c}}{\Delta p^{H}} - \beta\left(1-p^{HH}\right)(\hat{\rho}+\rho').$$

From the minimisation of the firm's expected wage cost it follows that $w_j^L = 0$, and taking into account Worker $j$'s incentive constraint gives:

$$w_j^H = \frac{\bar{c}}{\Delta p^{H}} - \beta\left(1-p^{HH}\right)(\hat{\rho}+\rho').$$

Then Worker $i$'s incentive constraint implies that

$$w_i^H \geq \frac{\bar{c}}{\Delta p^{H}} - \beta p^{HH}(\hat{\rho}+\rho') + w_i^L.$$

The condition $w_i^H > w_j^H$ implies that

$$w_i^H > \frac{\bar{c}}{\Delta p^{H}} - \beta\left(1-p^{HH}\right)(\hat{\rho}+\rho').$$

Hence Worker $i$'s incentive constraint is (not) binding iff

$$\beta\left(1-2p^{HH}\right)(\hat{\rho}+\rho') + w_i^L > (<)0,$$

or

$$w_i^L > (<) - \beta\left(1-2p^{HH}\right)(\hat{\rho}+\rho'). \tag{25}$$

Firstly, let $p^{HH} \leq 1/2$, then the limited liability constraint implies that condition (25) is irrelevant because the $RHS$ is negative. Hence, to minimise its expected wage cost the firm sets $w_i^L = \epsilon > 0$ but equal to the smallest monetary unit. From the cost minimisation and Worker $i$'s incentive constraint it follows that

$$w_i^H = \frac{\bar{c}}{\Delta p^H} - \beta p^{HH} (\hat{\rho} + \rho') + \epsilon.$$

This implies the profile of asymmetric wage contracts minimising the firm's expected wage cost $\omega^{AS*} = \left(\left(w_i^H, w_i^L\right), \left(w_j^H, w_j^L\right)\right)$ is

$$\omega^{AS*} = \left(\left(\frac{\bar{c}}{\Delta p^H} - \beta p^{HH} (\hat{\rho} + \rho') + \epsilon, \epsilon\right), \left(\frac{\bar{c}}{\Delta p^H} - \beta \left(1 - p^{HH}\right) (\hat{\rho} + \rho'), 0\right)\right)$$

For $p^{HH} \leq 1/2$, the firm's minimum expected wage cost is

$$EWC\left(\omega^{AS*}\right) = 2p^{HH} \left\{\frac{\bar{c}}{\Delta p^H} - \beta \frac{(\hat{\rho} + \rho')}{2}\right\} + \epsilon.$$

Secondly, let $p^{HH} > 1/2$, then the $RHS$ of condition (25) is strictly positive. Now, it is crucial wether the smallest monetary unit fulfills (25) or not. Suppose not, i.e.

$$\epsilon < -\beta \left(1 - 2p^{HH}\right) (\hat{\rho} + \rho').$$

Then, it follows from the minimisation of the expected wage cost that $w_i^L = \epsilon$ and $w_i^H$ is set such that $w_i^H > w_j^H$. Note, Worker $i$'s incentive constraint is not binding in this case, hence

$$w_i^H = w_j^H + \epsilon = \frac{\bar{c}}{\Delta p^H} - \beta \left(1 - p^{HH}\right) (\hat{\rho} + \rho') + \epsilon.$$

The firm's expected wage cost $EWC$ is then:

$$EWC = 2p^{HH} \left\{\frac{\bar{c}}{\Delta p^H} - \beta \left\{1 - p^{HH}\right\} (\hat{\rho} + \rho')\right\} + \epsilon$$

Suppose next that the smallest monetary unit is such that

$$\epsilon \geq -\beta \left(1 - 2p^{HH}\right) (\hat{\rho} + \rho').$$

Then, it follows from the minimisation of the expected wage cost that $w_i^L = \epsilon$ and $w_i^H$ is set such that Worker $i$'s incentive constraint is binding in this case, hence

$$w_i^H = \frac{\bar{c}}{\Delta p^H} - \beta p^{HH} (\hat{\rho} + \rho') + \epsilon.$$

The firm's expected wage cost $EWC$ is then:

$$EWC\left(\omega^{AS*}\right) = 2p^{HH}\left\{\frac{\bar{c}}{\Delta p^{H}} - \beta\frac{(\hat{\rho}+\rho')}{2}\right\} + \epsilon.$$

Note, with $p^{HH} > 1/2$

$$EWC\left(\omega^{AS*}\right) > EWC = 2p^{HH}\left\{\frac{\bar{c}}{\Delta p^{H}} - \beta\left\{1-p^{HH}\right\}\hat{\rho}\right\} + \epsilon.$$

Note, given Assumption 1 the limited liability constraints are fulfilled. To complete this proof it has to be checked whether the wage contracts specified above by the wage profile $\omega^{AS*}$ fulfill the (low) participation constraints.

Worker $i$'s (low effort) participation constraint is given by

$$p^{LH}w_i^H + \left(1-p^{LH}\right)w_i^L + \beta E\left(\rho(r_i)|e^L\right) \geq 0,$$

where

$$E\left(\rho(r_i)|e^L\right) = \left(p^{LH} + \left(1-p^{LH}\right)\left(1-p^{LH}\right)\right)\hat{\rho} + \left(1-p^{LH}\right)(-\rho').$$

Worker $j$'s (low effort) participation constraint is given by

$$p^{LH}w_j^H + \left(1-p^{LH}\right)w_j^L + \beta E\left(\rho(r_j)|e^L\right) \geq 0,$$

where

$$E\left(\rho(r_j)|e^L\right) = p^{LH}\left(1-p^{HH}\right)\hat{\rho} + \left(p^{LH}p^{HH} + \left(1-p^{LH}\right)\right)(-\rho').$$

Given the assumption of $\rho'$ being sufficiently small, $\rho' \leq \underline{\rho}'_{AS}$, both workers' participation constraints are satisfied, because $E\left(\rho(r_i)|e^L\right) \geq 0$ and $E\left(\rho(r_j)|e^L\right) \geq 0$.

Note, $E\left(\rho(r_i)|e^L\right) \geq 0$ iff

$$\rho' \leq \left(\frac{\left(1-p^{HH}\left(1-p^{LH}\right)\right)\hat{\rho}}{p^{HH}\left(1-p^{LH}\right)}\right),$$

and $E\left(\rho(r_j)|e^L\right) \geq 0$ iff

$$\rho' \leq \left(\frac{p^{LH}\left(1-p^{HH}\right)\hat{\rho}}{1-p^{LH}\left(1-p^{HH}\right)}\right),$$

where

$$\left(\frac{\left(1-p^{HH}\left(1-p^{LH}\right)\right)\hat{\rho}}{p^{HH}\left(1-p^{LH}\right)}\right) > \left(\frac{p^{LH}\left(1-p^{HH}\right)\hat{\rho}}{1-p^{LH}\left(1-p^{HH}\right)}\right) = \underline{\rho}'_{AS}$$

□

Note, $\underline{\rho}'_{AS} < \underline{\rho}'_{S}$. The following theorem establishes the conditions under which a profile of asymmetric independent contracts does strictly better than the optimal symmetric independent contract.

**Theorem 2.** *Assume $\rho' \leq \underline{\rho}'_{AS}$. Let $p^{HH} < 1/2$, then if Worker i's preferences on status are concave, i.e.*

$$\hat{\rho} - \rho' < 2\tilde{\rho},$$

*then there exists an $\bar{\epsilon} \equiv \beta p^{HH}\left[1 - 2p^{HH}\right]\left[2\tilde{\rho} - (\hat{\rho} - \rho')\right]$, such that if $\epsilon < \bar{\epsilon}$, then*

$$EWC\left(\omega^{AS*}\right) < EWC\left(\omega^{S*}\right),$$

*and hence it is optimal for the firm to switch from the profile of symmetric independent contracts $\omega^{S*}$ to the profile of asymmetric independent contracts $\omega^{AS*}$.*

**Proof:** From Proposition (5) the EWC function with asymmetric contracts is known: If $p^{HH} > \frac{1}{2}$, then $EWC$ is described by the condition

$$EWC \geq 2p^{HH}\left\{\frac{\bar{c}}{\Delta p^{H}} - \beta\left\{1 - p^{HH}\right\}(\hat{\rho} + \rho')\right\} + \epsilon > EWC^{S}.$$

Hence with $p^{HH} > 1/2$ it is not profitable for the firm to deviate from its optimal symmetric independent contract to this profile of asymmetric independent wage contracts.

Secondly, let $p^{HH} \leq 1/2$, then

$$EWC\left(\omega^{AS*}\right) = 2p^{HH}\left\{\frac{\bar{c}}{\Delta p^{H}} - \beta\frac{(\hat{\rho} + \rho')}{2}\right\} + \epsilon.$$

$EWC\left(\omega^{AS*}\right) < EWC^{S}$ iff

$$2p^{HH}\left\{\frac{\bar{c}}{\Delta p^{H}} - \beta\frac{(\hat{\rho} + \rho')}{2}\right\} + \epsilon < 2p^{HH}\left\{\frac{\bar{c}}{\Delta p^{H}} - \beta\left[\left(1 - p^{HH}\right)(\hat{\rho} - \tilde{\rho}) + p^{HH}(\tilde{\rho} + \rho')\right]\right\}$$

or simplified

$$\epsilon < \beta p^{HH}\left[1 - 2p^{HH}\right]\left[2\tilde{\rho} - (\hat{\rho} - \rho')\right] = \bar{\epsilon}.$$

If $p^{HH} < 1/2$, then the $RHS$ of this inequality is strictly positive iff

$$2\tilde{\rho} > \hat{\rho} - \rho',$$

completing the proof of the above theorem.

□

The intuition behind this result is as follows. Starting from symmetric independent contracts can the firm do better by switching to an asymmetric independent contract? With the optimal asymmetric independent contract one worker has always higher expected status. For ease of exposition it is assumed this is Worker 1. However what is crucial for the firm is not the expected status of a worker, but the incentives from status. Depending on the parameters it can be either Worker 1 or Worker 2 whose incentives from status increase after the switch to an asymmetric contract. If it is Worker 1 who gains in terms of incentives at the expense of Worker 2 then the firm cannot profit from a switch to an asymmetric contract, because it is the wages of Worker 2 that provide a lower bound for the wages of Worker 1. However note that this lower bound increases due to the switch. Thus, it is only in the case where Worker 2 gains in terms of incentives from status that the firm can benefit from switching to an asymmetric contract. Theorem 2 provides the conditions under which it is indeed Worker 2 who gains in incentives from status while Worker 1 gains in terms of the expected status.

Therefore under some circumstances it might be profitable for the firm to switch from symmetric independent contracts to the asymmetric independent contracts described above. However, if the firm use of contracts is restricted to independent contracts then it is impossible to reach minimum expected wage cost $EWC^{min}$. With asymmetric independent contracts there exists a profile of wage contracts that maximises both workers' incentives from status. However the framework of asymmetric independent contracts is to narrow to obtain $EWC^{min}$, because given the maximised sum of incentives from status the profile of asymmetric independent contracts cannot be chosen in such a way that both incentive constraints are simultaneously binding given $I_1^*$ and $I_2^*$. Next section finally addresses the question whether it is possible for the firm to reach $EWC^{min}$ with the help of dependent contracts.

### 4.2.2 Dependent Contracts

If the firm is not restricted to independent contracts then the firm is able to choose between two different types of dependent contracts: symmetric dependent and asymmetric dependent contracts. The second type of contract is the most general type of contract. The question is whether these more general contracts help the firm to reduce its expected wage cost. In the previous section the minimum expected wage cost given the restriction to independent contracts have been established, but it has been shown, too, that with independent contracts alone the firm is unable to obtain the minimum expected wage cost $EWC^{min}$.

With rank-dependent preferences in a moral-hazard environment symmetric dependent contracts do not help the firm to achieve lower expected wage cost. Recall from the previous section that

$$\underline{\rho}'_S = \left( \frac{p^{LH}\left(1-p^{HH}\right)\hat{\rho} + \left[p^{LH}p^{HH} + \left(1-p^{LH}\right)\left(1-p^{HH}\right)\right]\tilde{\rho}}{p^{HH}\left(1-p^{LH}\right)} \right).$$

The following proposition establishes that the firm can not improve its situation in terms of its expected wage cost with symmetric dependent contracts, because it can achieve the same level of expected wage cost with symmetric independent contracts.

**Proposition 6.** *Let $m \in S_D$ and $i = 1, 2$. Suppose that the cost minimising optimal Symmetric Dependent (SD) contract is given by $w^{SD^*}$, where*

$$w^{SD^*} = (w^*(s_i = H, s_j = H), w^*(s_i = H, s_j = L)), w^*(s_i = L, s_j = H)), w^*(s_i = L, s_j = L))$$

*The principal can achieve the same expected cost by a Symmetric Independent (SI) contract with wages*

$$w_i^H = \{p^{HH} w^*(s_i = H, s_j = H) + (1 - p^{HH}) w^*(s_i = H, s_j = L))\},$$

*and*

$$w_i^L = \{p^{HH} w^*(s_i = L, s_j = H)) + (1 - p^{HH}) w^*(s_i = L, s_j = L))\}.$$

**Proof:**

*Step 1: The rank payoff matrix corresponding to the optimal SD contract is:*

| | H | L |
|---|---|---|
| H | $\tilde{\rho}, \tilde{\rho}$ | $\hat{\rho}, -\rho'$ |
| L | $-\rho', \hat{\rho}$ | $\tilde{\rho}, \tilde{\rho}$ |

To see this note that symmetry imposes the following structure on the rank payoff matrix:

| | H | L |
|---|---|---|
| H | $\tilde{\rho}, \tilde{\rho}$ | $X, Y$ |
| L | $Y, X$ | $\tilde{\rho}, \tilde{\rho}$ |

Correspondingly, with symmetric contracts Worker $i$'s incentives from status are

$$I_i(\mathbf{w}) = \left\{p^{HH}\left(\tilde{\rho} - Y\right) + \left(1 - p^{HH}\right)\left(X - \tilde{\rho}\right)\right\}.$$

Clearly, choosing $X \leq Y$ is suboptimal because *both* $I_1$, and $I_2$ can be increased by choosing $X = \hat{\rho} > Y = -\rho'$. This completes the proof of the Claim. This implies that $w_1^{HL^*} > w_2^{HL^*}$ and $w_2^{LH^*} > w_1^{LH^*}$.

*Step 2: Observe that in a symmetric optimal SD contract* $w_1^{HH^*} = w_2^{HH^*}$, $w_1^{HL^*} = w_2^{LH^*}$ $w_1^{LH^*} = w_2^{HL^*}$ $w_1^{LL^*} = w_2^{LL^*}$. *This implies that* $w_1^H = w_2^H$ *and* $w_1^L = w_2^L$.

*Step 3:* Now it is shown that an SI contract can be found that can achieve the same $EWC$ as the optimal SD contract. Given the construction of the independent contract, it follows from Step 2 that $w_1^H = w_2^H$ and $w_1^L = w_2^L$. Moreover since $w_1^{HL^*} > w_2^{HL^*}$ and $w_2^{LH^*} > w_1^{LH^*}$, it follows that $w_1^H > w_2^L$ and $w_2^H > w_1^L$. Thus the same rank payoff matrix is obtained as the one corresponding to the optimal symmetric SD contract $w^{SD^*}$.

*Step 4:* it has to be shown that the constructed $w_1^H, w_2^H$ and $w_1^L, w_2^L$ satisfy the incentive constraints. Note that the incentive constraints are given by Equation (20). The RHS of the incentive constraints is the same for the SD and the SI contract (since the rank payoffs are the same). But this means that by construction the incentive constraints are satisfied (since the LHS is also the same) and total costs are the same.

It remains to show that the constructed $w_1^H, w_2^H$ and $w_1^L, w_2^L$ satisfy the (low) participation constraints. Note that the participation constraints are given by Equation (17). The LHS of the participation constraint is the same for the SD and the SI contract since the the expected utility from status is the same for both type of contracts and $w_i^H = p^{HH} w_i^{HH^*} + (1 - p^{HH}) w_i^{HL^*}$ and $w_i^L = p^{HH} w_i^{LH^*} + (1 - p^{HH}) w_i^{LL^*}$. For $w_i^{m^*}$ to be the cost minimising optimal symmetric dependent contract $\rho'$ has to be sufficiently small such that the (low) participation constraint is fulfilled. In the case of the SD contract as well as in the case of the SI contract a sufficient condition for the LHS to be positive is $\rho' \leq \underline{\rho}'_S$. However when $w_i^{m^*}$ fulfills the participation constraint the symmetric independent contract $w_i$ fulfills the participation constraint by construction, too.

□

From Proposition 6 together with Proposition 4 it follows immediately that with symmetric dependent contracts it is not possible to achieve minimum expected wage cost $EWC^{min}$. The following remark summerises this finding.

**Remark:** Let $m \in S_D$ and $i = 1, 2$. There exists no symmetric dependent contract $w^{SD}$, where

$$w^{SD} = (w(s_i = H, s_j = H), w(s_i = H, s_j = L)), w(s_i = L, s_j = H)), w(s_i = L, s_j = L))$$

such that $EWC\left(w^{SD}\right) = EWC^{min}$.

The above investigation established that the restrictions imposed by the assumption of symmetry between the contracts is stronger than the freedom obtained from the dependence of the contracts. As long as the firm has to offer both workers the same contracts it does not gain additional degrees of freedom by offering dependent contracts instead of independent contracts.

How does the situation change when the firm has the possibility to over the most general form of contracts, asymmetric dependent contracts? Does the firm gain sufficient degrees of freedom in its choice of wage contracts to achieve minimum expected wage cost $EWC^{min}$? Recall with asymmetric dependent wage contracts the firm's choice problem is

$$\min_{\mathbf{w}} EWC = \sum_i \sum_m [q^m w_i^m]$$

subject to the limited liability constraints

$$w_i^m \geq 0.$$

with $m \in S_D$, the incentive constraints (from Equation (20))

$$\left[E\left(w_i^H\right) - E\left(w_i^L\right)\right] \geq \frac{\bar{c}}{\Delta p^H} - \beta I_i(\mathbf{w})$$

and the (low effort) participation constraints (from Equation (17))

$$p^{LH} E\left(w_i^H\right) + \left(1 - p^{LH}\right) E\left(w_i^L\right) + \beta E\left(\rho(r_i)|e^L\right) \geq 0.$$

Let

$$\underline{\rho}_1' = \left(\frac{\left(1 - p^{HH}\right)\left[p^{LH}\hat{\rho} + (1 - p^{LH})\tilde{\rho}\right]}{p^{HH}}\right)$$

and

$$\underline{\rho}_2' = \left(\frac{p^{LH}\left[(1 - p^{HH})\hat{\rho} + p^{HH}\tilde{\rho}\right]}{1 - p^{LH}}\right)$$

The optimal asymmetric (dependent) contract can now be characterised.

**Theorem 3.** *Assume that*

$$\rho' \leq \min\left[\underline{\rho}_1', \underline{\rho}_2'\right]. \tag{26}$$

*(i) Suppose that preferences on status are convex: then there exists a dependent contract that achieves the minimum expected cost, $EWC^{min}$, with an induced asymmetric wage order:*

$$w_1^{HH} > w_2^{HH} = 0;\; w_2^{LH} > w_1^{HL} > w_2^{HL} = w_1^{LH} = w_1^{LL} = w_2^{LL} = 0.$$

*with*

$$E(w_1^H) = \frac{c}{\Delta p^H} - \beta(p^{HH}(\hat{\rho} + \rho') + (1 - p^{HH})(\hat{\rho} - \tilde{\rho}))$$

*and*

$$E(w_2^H) = \frac{c}{\Delta p^H} - \beta((1-p^{HH})(\hat{\rho}-\tilde{\rho})).$$

*(ii) Suppose preferences on status are concave: then there exists a dependent contract that achieves an expected wage cost that is $\epsilon > 0$ close to the $EWC^{min}$:*

$$EWC = EWC^{min} + \left(1-p^{HH}\right)\epsilon,$$

*where $\epsilon > 0$*[11] *with an induced asymmetric wage order:*

$$w_1^{HH} = w_2^{HH};\ w_2^{LH} > w_1^{LH} = 0;\ w_1^{HL} > w_2^{HL} = 0;\ w_2^{LL} > w_1^{LL} = 0$$

*with*

$$E(w_1^H) = \frac{c}{\Delta p^H} - \beta(p^{HH}(\tilde{\rho}+\rho') + (1-p^{HH})(\hat{\rho}+\rho')),$$

$$E(w_2^L) = \left(1-p^{HH}\right) w_2^{LL} = \left(1-p^{HH}\right)\epsilon$$

*and*

$$E(w_2^H) = \frac{c}{\Delta p^H} - \beta(p^{HH}(\tilde{\rho}+\rho')) + \left(1-p^{HH}\right)\epsilon,$$

**Proof:** Suppose that preferences on status are convex. It is clear that (i) implements the rank payoff matrix (B) (see proof of Lemma 4), since

$$w_1^{HH} > w_2^{HH} = 0; 0 = w_2^{HL} < w_1^{HL}; 0 = w_1^{LH} < w_2^{LH}; w_1^{LL} = w_2^{LL} = 0.$$

This implies $0 = E(w_1^L) = E(w_2^L)$. With the wages given, it has to be shown that the incentive compatibility constraints are satisfied: Indeed, both incentive constraints are binding given that $E(w_i^L) = 0$ and $E(w_i^H) = \frac{c}{\Delta p^H} - \beta I_i$ for $i = 1, 2$. Since $I_1 > I_2$ this implies that $E(w_1^H) < E(w_2^H)$. This is satisfied by choosing $w_2^{LH}$ to satisfy $\left(1-p^{HH}\right) w_2^{LH} = E(w_2^H)$. The limited liability constraints for worker 1 are satisfied if $w_1^{HH}, w_1^{HL} > 0$ and $\frac{c}{\Delta p^H} \geq \beta(p^{HH}(\hat{\rho}+\rho') + (1-p^{HH})(\hat{\rho}-\tilde{\rho}))$, but this is the case since it has been assumed that $\frac{c}{\Delta p^H} > \beta\left(\hat{\rho}+\rho'\right)$. Since $E(w_2^H) > E(w_1^H) > 0$ and given the condition on $w_2^{HH}$ and $w_2^{LH}$ the limited liability constraints for worker 2 are satisfied, too. It remains to check that the participation constraints are satisfied.

The following condition is needed for the participation constraint (low effort) for worker 1 to hold:

$$\begin{aligned} p^{LH}E(w_1^H) + \left(1-p^{LH}\right)E(w_1^L) + \beta\Big[p^{LH}\left(\hat{\rho}\right) \quad &+ \\ \left(1-p^{LH}\right)\left(p^{HH}(-\rho') + (1-p^{HH})(\tilde{\rho})\right)\Big] \quad &\geq \quad 0, \end{aligned} \tag{27}$$

[11] It is again assumed that $\epsilon > 0$ but equal to the smallest monetary unit to be able to fix the optimal contract; otherwise there is always a smaller $\epsilon$.

Since $p^{LH} E(w_1^H) + \left(1 - p^{LH}\right) E(w_1^L) > 0$ it is sufficient to check that

$$p^{LH}\left(\hat{\rho}\right) + \left(1 - p^{LH}\right)\left(p^{HH}(-\rho') + (1 - p^{HH})(\tilde{\rho})\right) \geq 0.$$

Similarly for worker 2 the following condition is needed:

$$p^{LH} E(w_2^H) + \left(1 - p^{LH}\right) E(w_2^L) + \beta\left[p^{HH}\left(-\rho'\right) + \left(1 - p^{HH}\right)\left(p^{LH}(\hat{\rho}) + (1 - p^{LH})(\tilde{\rho})\right)\right] \geq 0, \tag{28}$$

Both of these are satisfied given the assumptions on $\rho'$ being sufficiently small. Note, that if $\rho'$ is sufficiently small such that

$$p^{HH}\left(-\rho'\right) + \left(1 - p^{HH}\right)\left(p^{LH}(\hat{\rho}) + (1 - p^{LH})(\tilde{\rho})\right) \geq 0,$$

then worker 2's and worker 1's participation constraint are both fulfilled. In particular, if $\rho' = 0$ the participation constraints are always satisfied.

(ii)

Suppose that preferences on status are concave. It is clear that (ii) implements the rank payoff matrix (C) (see proof of Lemma 4), since

$$w = w_1^{HH} = w_2^{HH}; 0 = w_2^{HL} < w_1^{HL}; 0 = w_1^{LH} < w_2^{LH}; 0 = w_1^{LL} < w_2^{LL} = \epsilon.$$

This implies that $0 = E(w_1^L) < E(w_2^L) = (1 - p^{HH})\epsilon$. Also, with the wages given, it is necessary to show that the incentive compatibility constraints are satisfied: Note that $E(w_1^L) = 0$, hence worker 1's incentive constraint is binding. Since $E(w_2^L) = (1 - p^{HH})\epsilon$, worker 2's incentive constraint is also binding. Since $I_1 > I_2$ this implies that $E(w_1^H) < E(w_2^H)$. This is satisfied by the choice of $w_2^{LH}$, such that $E(w_2^H) = \left(p^{HH} w + (1 - p^{HH}) w_2^{LH}\right)$ and by the choice of $w_1^{HL}$, such that $E(w_1^H) = \left(p^{HH} w + (1 - p^{HH}) w_1^{HL}\right)$ for any $w$ such that

$$0 \leq w < \frac{E(w_1^H)}{p^{HH}}.$$

The limited liability constraints are satisfied given the above conditions on $w_i^m$ and if $\frac{c}{\Delta p^H} \geq \beta(p^{HH}(\tilde{\rho} + \rho') + (1 - p^{HH})(\hat{\rho} + \rho'))$, but this is the case since it has been assumed that $\frac{c}{\Delta p^H} > \beta\left(\hat{\rho} + \rho'\right)$. Since $E(w_2^H) > E(w_1^H) > 0$ the limited liability constraints for worker 2 are satisfied. It remains to check that the participation constraints are satisfied.

The following condition is needed for the participation constraint of worker 1 to hold:

$$p^{LH} E(w_1^H) + \left(1 - p^{LH}\right) E(w_1^L) + \beta\Big[p^{LH}\left(p^{HH}\tilde{\rho} + (1 - p^{HH})\hat{\rho}\right) + \left(1 - p^{LH}\right)\left(-\rho'\right)\Big] \geq 0, \tag{29}$$

Since $p^{LH}E(w_1^H) + \left(1-p^{LH}\right)E(w_1^L) > 0$ it is sufficient to check that

$$p^{LH}\left(p^{HH}\tilde{\rho} + \left(1-p^{HH}\right)\hat{\rho}\right) + \left(1-p^{LH}\right)(-\rho') \geq 0,$$

Similarly for worker 2 the following condition is needed:

$$\begin{aligned} p^{LH}E(w_2^H) + \left(1-p^{LH}\right)E(w_2^L) + \beta\Big[p^{LH}\left(p^{HH}\tilde{\rho} + \left(1-p^{HH}\right)\hat{\rho}\right) &+ \\ \left(1-p^{LH}\right)\left(p^{HH}(-\rho') + (1-p^{HH})(\hat{\rho})\right)\Big] &\geq 0, \quad (30) \end{aligned}$$

Both of these are satisfied given the assumptions on $\rho'$ being sufficiently small. Note, that if $\rho'$ is sufficiently small such that

$$p^{LH}\left(p^{HH}\tilde{\rho} + \left(1-p^{HH}\right)\hat{\rho}\right) + \left(1-p^{LH}\right)(-\rho') \geq 0,$$

then worker 1's and worker 2's participation constraint are both fulfilled. In particular, if $\rho' = 0$ the participation constraints are always satisfied.

□

Which mechanism is at work in the case of asymmetric dependent contracts that makes it possible to achieve minimum expected wage cost $EWC^{min}$ and which is missing in the case of symmetric dependent contracts? In the setup presented here the main trade off is through the incentive constraints, because they are crucial for the determination of the expected wage cost. Now with asymmetric dependent contracts the incentives from status are higher for worker 1 and lower for worker 2 relative to the situation with the optimal symmetric contract.[12] Denote the asymmetric contract by $AS$ and the symmetric one by $S$, then

$$I_1^{*AS} > I_1^{*S} = I_2^{*S} > I_2^{*AS} \quad \text{and} \quad E(w)_2^{AS} > E(w)_1^S = E(w)_2^S > E(w)_1^{AS}.$$

However, crucial for the firm's decision to use asymmetric dependent contracts instead of symmetric contracts is whether it can achieve lower expected wage cost with asymmetric dependent than with symmetric contracts. To understand the intuition for the results presented here it is useful to look at the status payoff matrix for a symmetric contract:

| | H | L |
|---|---|---|
| H | $\tilde{\rho}, \tilde{\rho}$ | $\hat{\rho}, -\rho'$ |
| L | $-\rho', \hat{\rho}$ | $\tilde{\rho}, \tilde{\rho}$ |

[12]Recall that calling the agent with higher incentives agent 1 is without loss of generality since the roles of the two workers can be exchanged.

By offering an asymmetric contract the firm can take advantage of the fact that the changes in incentives (gain for one worker, loss for the other relative to the symmetric contract) are not symmetric for the two workers. Indeed the gain in incentives (relative to the symmetric contract) and hence the reduction in the expected wage cost for one worker must be larger than the loss in incentives (relative to the symmetric contract) and hence the increase in expected wage cost for the other worker. Starting from the optimal symmetric contract there are two states for which a switch to an asymmetric contract can cause a change in incentives if the firm's intention is to do so: the states when either both factories are in the good state or when both factories are in the bad state. First look at the case when both factories are in the good state. When preferences on status are convex then the firm gains more by increasing worker 1' incentives by increasing his status payoff than it looses by reducing worker 2's incentives by reducing his status payoff in the situation when both factories are in the good state, because worker 1's gain in incentives is $I_1^{*AS} - I_1^{*S} = p^{HH}(\hat{\rho} - \tilde{\rho})$ and worker 2's loss in incentives is $I_2^{*AS} - I_2^{*S} = -p^{HH}(\tilde{\rho} + \rho')$. Second, looking at the case when both factories are in the bad state, then when preferences on status are concave the firm gains more by increasing worker 1' incentives by decreasing his status payoff than it looses by reducing worker 2's incentives by increasing his status payoff in the situation when both factories are in the good state, because worker 1's gain in incentives is $I_1^{*AS} - I_1^{*S} = \left(1 - p^{HH}\right)(\tilde{\rho} + \rho')$ and worker 2's loss in incentives is $I_2^{*AS} - I_2^{*S} = -\left(1 - p^{HH}\right)(\hat{\rho} - \tilde{\rho})$. Furthermore with asymmetric dependent contracts in both cases wage profiles exist which translate these overall gains in incentives into lower expected wage cost. If preferences on status would be linear, i.e. $\hat{\rho} - \tilde{\rho} = \tilde{\rho} + \rho'$, then asymmetry would not help and the optimal contract would be the symmetric one. Also as shown above with asymmetric independent contracts there are no wage profiles which can translate the overall gains in incentives into the minimum expected wage cost $EWC^{min}$.

# 5 Concluding Remarks

Much of contract theory has focused on agents who are self interested. What happens to the theory of optimal contracts when this assumption is relaxed? In particular, what happens when workers care about their *rank*? This question is investigated in a simple model with two status-seeking agents. The model used is the standard moral hazard model used in the literature but with status seeking agents. It is shown that then the problem of finding the optimal contract involves (1) maximising the incentives from status given a particular distribution of the stochastic shocks to the two workers: here essentially a game of status Shubik (1971) is designed to maximise the incentives of both workers. (2) Look for the levels of wages that minimise expected wage costs and satisfy the Incentive Constraints.

The main results are illustrated in a simple two agent model. If agents are not too averse to being behind, it is shown that asymmetric contracts or discriminatory contracts where the two agents are offered different contracts even though they are ex-ante identical, are better than symmetric contracts in the case of observable effort, when preferences on status are convex. In the case of unobservable effort, asymmetric contracts dominate symmetric contracts, as long as the firm is allowed to write down dependent contracts. If the firm is restricted to independent contracts on the other hand, then it is shown that under some conditions, asymmetric independent contracts are also better than symmetric contracts. Moreover, no symmetric contract *even if it is dependent* can replicate the discriminatory one.

Intuitively, the results rely on the fact that there are enough states of nature that any given agent can be both ahead in some states and behind or equal in others[13]. Additionally the result relies on being able to pay one worker just a little bit more than the other (the $\epsilon$). A criticism that might be leveled against this presentation is that people perceive rank differences only when there are noticeable or significant differences in wages (see e.g. Shubik (1971)) . There are two answers to this. First, it is referred to anecdotal evidence from A. Oswald who cites the story of Professor X who refused a job in a top university because he was paid a wage \$ 10 below that of the (then) highest paid professor. In other words, the satisfaction of being top ranked comes from the fact that this hierarchy in wages is common knowledge. This is what is assumed in the model presented here (contracts have to be common knowledge to both workers). Second, what is suggested here (like Winter (2004)) is that there may be benefits to introducing an artificial hierarchy between workers even when the job is ex-ante identical – again it is the common knowledge about status that is important rather than the actual wage differences. Indeed Baron (1988) suggests that reference actors are people who are nor too different from oneself in terms of pay. Pay differences matter more when they are across people in the same job title.

Is it better to have status seeking agents as far as the principal is concerned? Based on this work it might be argued that the extra utility that agents get from rank might cause the total expected wages paid out to be *lower* than in the case of agents who are not status seeking (this is the *wage compression* result that is discussed by many authors in this area (e.g. Frank (1984), Neilson and Stowe (2004)). When symmetric contracts are compared with and without status seeking agents it turns out that the wage compression result holds in the model presented here both with observable effort and with unobservable effort. This is quite intuitive in that when agents get some utility from rank (in the cases when the state is different across workers), they need to be paid less to exert effort.

Finally this paper is concluded with some ideas for extensions of this work. One

[13]In this respect the result presented here parallels the observation in Dubey and Geanakoplos (2004) that to maximize incentives to students from working some randomness must be introduced in the payoffs from rank. However in the presented context the randomness is given as part of the moral hazard set up and it is only possible to play with the ranks.

obvious extension is to investigate the case of status seeking agents who are not identical (adverse selection). Another interesting question is that of information about wage scales. Why can it be observed e.g. that many organizations give broad information about wages (i.e. the bands within which the wages for a given job title lie) but not detailed information (e.g. which employee is getting how much). Is this a case of making the reference group endogenous? How are these bands chosen? It is hoped to tackle these questions in future work.

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