Funding Higher Education and Wage Uncertainty:
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Funding Higher Education and Wage Uncertainty: Income Contingent Loan versus Mortgage Loan

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Abstract

In a world where graduate incomes are uncertain and higher education is financed through governmental loans, we build a theoretical model to show whether an income contingent loan (ICL) or a mortgage loan (ML) is preferred for higher levels of uncertainty. Assuming a single lifetime shock on graduate incomes, we compare the individual expected utilities under the two loan schemes, for both risk neutral and risk averse individuals. The theoretical model is calibrated using real data on wage uncertainty and considering the features of the UK Higher Education Reform to observe the implications of the switch from a ML to an ICL and the effect of the top-up fees. Different scenarios are simulated according to individual characteristics and family background. We finally extend the initial model to incorporate stochastic changes of income over time.
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1 Introduction

Investment in education is risky: an individual making schooling decisions is likely to be only imperfectly aware of her abilities, the probability of success, and earnings that may be obtained after completing an education.

Weiss (1974) studies the risk adjusted average rate of return to schooling, which is the subjective discount rate at which the individual would be indifferent between acquiring a certain level of education and having no education at all. Weiss finds that this risk adjusted return sharply decreases as the risk aversion increases. Olson, White and Shefrin (1979) follow the traditional literature focusing on the returns to education but incorporate graduate income uncertainty, and take into account the way higher education is funded. They assume that consumption equals income in each period after schooling and educated individuals get a random stream of income whose mean varies according to the level of education achieved. Olson, White and Shefrin allow borrowing to finance education, and in particular they consider a mortgage loan that is paid back only after the completion of schooling. They find that the estimated real returns of college are large, and the estimated risk adjustments for college are small but positive.

Pistaferri and Padulla (2001) extend the Olson, White and Shefrin's model to consider two types of risk: employment risk and wage uncertainty, within an imperfect credit market framework. Hartog and Serrano (2003) analyze the effects of stochastic post schooling earnings on the optimal schooling length, and show a negative effect of risk on investment.

In this work, starting from the same framework as Hartog and Serrano, we present a theoretical model in which graduates receiving uncertain future income (affected by a single lifetime shock) repay the cost of their education choosing between an income contingent loan and a mortgage loan. We consider only individuals in full-time higher education, with no income and facing the same cost of education, completely financed by the government with loan of equal amount. The government is risk neutral and has no preferences for one system over the other. We assume no default and zero interest rate for both student and government. We do not consider any external effect of education for the whole society. We analyze which loan scheme is more welfare improving in terms of individual lifetime expected utility, for risk neutral and risk averse graduates. Our intuition is that if the graduates expect a high variance in their wages, an income contingent loan that allows to repay the debt only when they have the financial resources to do it provides higher welfare, and it is a better guarantee against high uncertainty.

The interest in the combination wage uncertainty and student loan design is motivated by two empirical facts:

- the observed uncertainty in the real wages of graduates; and
- the reform of the higher education financing system in the UK, that increased the costs of education and introduced an income contingent loan (ICL) to replace a mortgage loan (ML) scheme.

We verify our theoretical intuition calibrating the model on real data on graduate income and its volatility. We consider the features of the UK Higher Education Reform to observe the implications of the switch from a ML to an ICL and the effect of the top-up fees. In particular, we simulate different scenarios where we compare the two loan schemes, observing the changes in the individual welfare. We use the British Cohort Study 1970, that although restricts the sample only on individuals borne in 1970, provides many information on family background. Assuming no selectivity, and following the assumptions of the model, we consider only the wage post graduation and its standard deviation given the individual characteristics (e.g. sex, marital status), the family background (e.g. parental income, parental occupation), and the degree choice (e.g. subjects). The loan system is structured using the past and the current cost of the British higher education.

An important difference between the two loan schemes is the presence of hidden subsidy under an ICL, when the repayment periods are not equal. Assuming risk neutrality, we notice that the implicity subsidy makes an ICL more convenient. Instead when we rule out this possibility the expected costs under an ICL are always higher than the costs under an ML. The effect of the UK Reform is to increase this gap. In terms of utility, a ML provides higher expected utility for risk neutral graduates, and in particular for female graduates, whose income is lower than males' income. Looking at the UK Reform, in general risk neutral graduates do not get benefits from the switch to an ICL system. In case of risk aversion the model offer interesting policy implications. We consider the case without hidden subsidy, and we notice that the effect of the high uncertainty joint with the risk aversion, makes an ICL the preferred system. We find that UK Reform is convenient for people from low educated parents background, males over females, people working in the private sector, because they all receive higher utility from an ICL. A ML instead is the favorite system when the career is quite static, and the income not too high, a typical example is the public sector. Allowing for hidden subsidies, we notice that an ICL becomes more convenient also for low level of uncertainty.

In the second part of our work, the model is extended to incorporate stochastic changes of income over time. The computation of the expected utilities under the two loan schemes requires a numerical solution. The new settings allows us to generate income paths for the entire individual working life, where the uncertainty affects the wage each year. We find again as in the previous part that the higher uncertainty increases the utility of an ICL. However, the results in this case show the importance of the initial income, we notice that the UK Reform is very convenient for low initial wage earners, since for them an ICL is highly preferred.

We now briefly describe the UK Higher Education Reform. The UK higher education financing system has been based on an up-front fee (fixed across universities and courses), together with a ML to finance living expenses. Only students whose family incomes exceeded a given amount used to pay the fee in full, the others were exempted. The Higher Education reform (approved in 2004 and effective from 2006/2007) increases the tuition, enlarges the number of students liable and universities can set their fees up to a maximum £3000 p.a. Fees will be payable up front but will be covered by a system of subsidized loans. The major innovation is the introduction of an income-contingent scheme for repayments. Graduates start to pay back only when their incomes are above $\pounds 15,000$ per year and at a 9 per cent fixed repayment rate. There is a zero real interest rate and repayments are made through the tax system as a payroll deduction. A similar higher education system has been effective in Australia since 1989, the only difference has been in the presence of increasing thresholds of income and increasing repayment rates.

In the next Section we describe the theoretical model, in Section 3 we analyze the case of risk neutrality, in Section 4 we obtain the algebraic form of the expected utilities under risk aversion, while in Section 5 we describe the dataset and show the results of the simulations. In Section 6 we set up the model with the new assumptions on the income and show the results of the simulations. Section 7 concludes.

2 Theoretical Model

This section presents the main assumptions of the theoretical model that hold both a under mortgage loan and under an income contingent loan. We distinguish between schooling period and post graduation period.

2.1 Schooling period

Individuals go to university for s years full time, the education has the same cost for everybody without distinction between courses and subjects. The income during the schooling period is zero, following Olson, White and Shefrin (1979) consumption is always equal to income, therefore during university is also set to zero. The only source of financing allowed is a public loan, again equal for all the students, of fixed size and that covers all the costs of attending university. The real interest rate of the loan is zero[1].

The government finances a constant cost of higher education, through issuing the same amount of debt regardless the repayment scheme (therefore same subsidy for all the students). The debt is paid back only with the graduates' repayments, there is no opting out choice between the 2 loan schemes. The government is risk neutral and does not have any preference over the funding systems. We assume no default[2], this implies that in the long run all the cost of education is equally recovered by both schemes. Since the government could bear different costs of providing the loan according to the scheme, we assume a zero real interest rate on the borrowing[3]. In this way the costs for the government are the same under an ICL and a ML, and the social welfare depends only on the student utility. Moreover, when the repayment periods under the 2 loan schemes are different, the system with longer repayments could provide hidden subsidy to the students. Therefore, we mainly perform the analysis under the condition of no hidden subsidy, and in one case we also consider different repayment periods to verify the differences.

[1]This is not a simplicity assumption, but a real feature of the income contingent loans as implemented by the national governments in the UK and Australia. There is only an adjustment to the inflation. In our model for fair comparison we assume zero real interest also for a mortgage loan

[2]The case of the students' default is analyzed in another work.

[3]The case with positive real interest on the borrowing, is object of analysis in a further extension of this paper.

2.2 Post graduation period

Upon graduation, the individuals start working immediately, and as in Hartog and Serrano (2003) they obtain an uncertain wage because subject to a random shock. For simplicity, the shock has a single lifetime realization, after which the income remains constant. We can imagine an initial random draw that fixes the income at a certain level and remains unchanged for all the working life. Let $y > 0$ be the shock with $E(y) = 1$ and $Var(y) = \sigma^2$.

Individuals cannot insure the wage risk and seek to maximize the expected lifetime utilities. Consumption is equal to income and positive now; utility is defined over the individuals' income stream.

In this model we focus only in the post graduation period, and in all the following analysis it is developed distinguishing between repayment period and after repayment period. Graduates must start repay their educational loan straight after graduation and for T years, then they receive their entire income for all their life, assumed infinite. Considering a general repayment scheme, we define R as the general per-period payment. The expected utility is:

$$V = E\left\{ \int_{s}^{T+s} e^{-\rho t} u(y - R)\, dt + \int_{T+s}^{\infty} e^{-\rho t} u(y)\, dt \right\} \tag{1}$$

where $R < y$, and ρ is the subjective discount rate that measures how much the present is taken in consideration with respect to the future. A loan scheme is described fully by (T, R).

2.3 Mortgage Loan and Income Contingent Loan

We stress that cost of the loan is equal to the total cost of education C, and it is the same under both systems. The way it is repaid produces different individual utilities because of the random income. If we assume no uncertainty and same repayment rates the 2 systems are equal.

2.3.1 Mortgage Loan

The individual takes out a loan equal to C and repays through T equal, fixed and periodical instalments φ, at a zero real interest rate. The repayment period is just

$$T = C/\varphi. \tag{2}$$

2.3.2 Income Contingent Loan

The individuals borrow an amount equal to C, and start to pay back their loan after graduation according to level of their income. Under this scheme if the wage is below a minimum threshold no payment is due. If the wage increases, a greater portion of the debt is repaid and all the loan is paid off in less time. Therefore, compared to a mortgage loan the ICL repayment period, $\tilde{T}$, is *random*. In our model, for simplicity, we assume no initial threshold and the total cost of schooling is given by a fixed percentage (γ) of the random graduate income.

$$C = \gamma \int_{s}^{\tilde{T}+s} y \, dt \tag{3}$$

therefore the repayment period is:

$$\tilde{T} = \frac{C}{\gamma y} \quad . \tag{4}$$

We can now define the following assumption concerning the expected repayments under the two schemes.

Assumption 1. *With a ML,*

$$R < y \quad \Longleftrightarrow \quad \varphi < y$$

and the expected repayment is

$$EP_{ML} = T \times \varphi = C$$

Assumption 2. *Under an ICL the annual repayment is:*
$\gamma \times y$ *until the loan is repaid, and the expected repayment is*

$$EP_{ICL} = T\gamma y = C$$

Proposition 1. *Expected repayment periods under the two systems*

$$\begin{aligned} &(a) \quad if \;\; \gamma = \varphi & E[T_{ICL}] > T_{ML} \\ &(b) \quad if \;\; \gamma = \varphi \times E(\frac{1}{y}) & E[T_{ICL}] = T_{ML} \end{aligned}$$

where $E(\frac{1}{y}) > 1$

Proof. *See Appendix A*

The first proposition highlights one of the main differences between the two schemes: a feature of an ICL is to spread the same cost under a longer repayment period with respect to a ML, assuming the same repayment rates. However, this implies also the presence of hidden subsidy whenever we compare a long-term loan, such as an ICL, with a short-term loan, such as ML. Therefore, in point (b) of Proposition 1, we find the condition that rules out the hidden subsidy and allows a comparison of the two scheme on the same basis. In particular, we notice that to have same repayment periods we require a higher repayment rate under an ICL.

3 Risk Neutrality and Expected Costs

When the individuals are risk neutral $u(y) = y$, and we need consider only the costs to compare the two repayment schemes. We work out the present value of the costs, substituting for each scheme the respective repayment period, T and $\tilde{T}$, and discount to $t = 0$.

Proposition 2. *Assuming risk neutral individuals,*

$$V_{ICL} > V_{ML}$$

when $\gamma = \varphi$, instead

$$V_{ICL} < V_{ML}$$

when $\varphi = \frac{\gamma}{E(1/y)}$

Proof. See Appendix B

In the Appendix B, we prove analytically the Proposition 2 when $\gamma = \varphi$. Then, we argue that for the case $\varphi = \frac{\gamma}{E(1/y)}$ we require a numerical solution using real data from our BCS70 dataset.
The result highlights in terms of expected utilities the differences between the two systems raised in Proposition 1. The presence of implicit subsidies changes completely the preferences and makes an ICL more convenient. Assuming same repayment periods, instead, we need to increase the ICL repayment rate and a ML gives higher benefit.

To give a broad intuition of the result of Proposition 2, we assume a general repayment method R and we compute the present value of the education cost.

$$PVC = \int_0^T R\, e^{-\rho t} dt = \frac{R}{\rho}\left[1 - e^{-\rho T}\right]$$

Taking the derivatives of PVC with respect to T, we can easily observe that this function is concave [4]. Consider now a first loan with a certain repayment period of 10 years, and a second loan with two even probability repayment periods of 5 and 15 years; therefore we have $T_1 = ET_2$, and we rule out the hidden subsidy. The concavity property implies that the expected present value of the cost of a certain repayment period is lower than the present value of the cost of the expected repayment period:

$$E[PVC(T_1)] < PVC[E(T_2)] \quad \Longrightarrow E[PVC(10)] < PVC(10).$$

We now analyze the second result of Proposition 2, through a numerical calibration. We first estimate $E(1/y)$ by its sample analogue, then fix γ and get φ accordingly. The method is explained in detail in the Appendix B, we report in the next section the results of the simulations.

3.1 Equal repayment periods

We compute the expected costs under a ML and under an ICL for all the graduates and then distinguishing between males and females. Referring to the British Higher Education Reform the annual cost of education can be set up to a max of £3000 pounds, while before the Reform the cost was £1150 a year. Assuming a 3-year degree, we fix 2 levels of total cost: £3450 and £9000. We set $\rho = [0.08 \quad 0.15 \quad 0.3]$ and $\gamma = [0.02 \quad 0.09 \quad 0.2]$. Looking at the top of Figure 1, we report on the vertical axes the difference between the expected costs $EC_{ICL} - EC_{ML}$. We refer with y to the income of all the graduates, and with ym and yw to the income of male and female graduates respectively. Setting $\gamma = 9\%$, we observe that the expected costs under an ICL are always higher than the expected costs under a ML, and the effect of the Reform is to increase this gap. This means that when there are no hidden subsidies the benefits of an ICL for risk neutral graduates decrease strongly. Moreover, although the repayment periods are equal between the 2 systems, they differ among the graduates. For all graduates, the repayment period increases from 5.6 to 14.7 years, for male from 5.1 to 13.3 years and for females from 5.7 to 14.9 years. The males are those with the highest income and therefore the shortest repayment period, females are close to the sample mean. These differences among graduate categories are reflected in the expected costs, and we observe that for males the gap between ICL and ML is lower than for females, although they both prefer ML. According

[4] $\frac{\vartheta^2 PVC}{\vartheta T^2} = -R\rho\, e^{-\rho T} < 0$ for all T.

to these results the UK Reform is not convenient for risk neutral individuals.

In the bottom of Figure 1, we keep fixed the cost to £9000, same γ as above, therefore the repayment periods are unchanged, however we let the subjective discount rate increase. We notice an increase of the gap in the expected costs when the individuals discount more their future, that is they take more into account the present. A ML is still preferred.

4 Comparing Mortgage and Income Contingent Loans under Risk Aversion

In this section we consider individuals who are risk averse and work out their expected utility (represented by equation (1)), under a mortgage loan and an income contingent loan system. We consider the assumptions stated in Section 2 and we develop the analysis using constant relative risk aversion (CRRA) utility function[5]. We omit the majority of calculations that are showed in more detail in the Appendices C and D.

4.1 Expected Utility with a Mortgage Loan

Under a ML, the expected utility is obtained by substituting $R = \varphi$ into equation (1):

$$V_{ML} = \int_{s}^{T+s} e^{-\rho t} E\left[u\left(y-\varphi\right)\right] dt + \int_{T+s}^{\infty} e^{-\rho t} E\left[u\left(y\right)\right] dt. \tag{5}$$

To get a closed-form solution for V_{ML}, we use a second order Taylor expansion around the mean $E[y-\varphi] = 1-\varphi$ [6] for the utility during the repayment period, and around $E[y] = 1$ for the utility after the repayment period:

$$E[u(y-\varphi)] \simeq u\left(1-\varphi\right) + \frac{1}{2}u^{''}\left(1-\varphi\right)\sigma^2. \tag{6}$$

$$E[u(y)] \simeq u(1) + \frac{1}{2}u^{''}(1)\sigma^2. \tag{7}$$

We develop our analysis using a CRRA utility function:

$$u(y) = \frac{y^b}{b}.$$

[5]The analysis is developed also using a constant absolute risk aversion (CARA) utility function, but we omit the results that are showed in the working paper version.

[6]See Pistaferri and Padula (2001) and Hartog and Serrano (2003).

where a is the risk aversion parameter and $b = 1 - a$.

After simplifying [7], we get:

$$\begin{aligned} V_{ML_{CRRA}} = & \frac{e^{-\rho s}}{\rho}\{\left(1 - e^{-\frac{\rho C}{\varphi}}\right)\left[\frac{(1-\varphi)^b}{b} + \frac{1}{2}(b-1)(1-\varphi)^{b-2}\sigma^2\right] \\ & + e^{-\frac{\rho C}{\varphi}}\left[\frac{1}{b} + \frac{1}{2}(b-1)\sigma^2\right]\}. \end{aligned} \tag{8}$$

4.2 Expected Utility with the Income Contingent Loan

Under an ICL we do not know how long people take to repay their education debt, therefore in the general equation of the expected utility the random income appears twice: first in the integral's bounds as random repayment period, second as argument of the utility function.

$$V_{ICL} = E\left\{\int_s^{\frac{C}{\gamma y}+s} e^{-\rho t} u\left[y\left(1-\gamma\right)\right] dt + \int_{\frac{C}{\gamma y}+s}^{\infty} e^{-\rho t} u\left(y\right) dt\right\} \tag{9}$$

Solving the integral we get the following equation:

$$V_{ICL} = \frac{e^{-\rho s}}{\rho} E\left\{\left[1 - e^{-\frac{\rho C}{\gamma y}}\right] u\left[y\left(1-\gamma\right)\right] + \left[e^{-\frac{\rho C}{\gamma y}}\right] u\left(y\right)\right\}. \tag{10}$$

To simplify the calculations we define all the expression included in the expected value operator as $g(y)$. This trick allows us to apply a second order Taylor expansion of $E[g(y)]$ around the mean $E\left[y\right] = 1$. Then, the equation (10) becomes:

$$V_{ICL} = \frac{e^{-\rho s}}{\rho}\left[g(1) + g''(1)\frac{\sigma^2}{2}\right]. \tag{11}$$

The remaining procedure (explained in Appendix D) consists of calculating the value of $g(1)$ and $g''(1)$, in general and with a CRRA utility function in particular. Finally, we substitute the expressions found in equation (11), and we obtain the following results. After simplifying, the expected utility

[7] See Appendix C for the proof.

[8] is:

$$V_{ICL_{CRRA}} = \frac{e^{-\left(s+\frac{C}{\gamma}\right)\rho}}{2b\gamma^2\rho}\{e^{\frac{\rho C}{\gamma}}(1-\gamma)^b\gamma^2[2+(b-1)b\sigma^2] - [(1-\gamma)^b - 1]$$
$$\cdot [2\gamma^2 + ((b-1)b\gamma^2 + 2(b-1)C\gamma\rho + C^2\rho^2)\sigma^2]\}. \quad (12)$$

5 Empirical Background and Simulations

In this Section we first illustrate the BCS79 dataset used as basis to calibrate the theoretical model . We describe the graduate income and its standard deviation in four possible environments, in order to get an idea of the wage uncertainty. An important assumption is the absence of selection bias, although we know that it could matter for variance comparisons (Chen, 2004) however we are more interested in observing how the theoretical model works under different potentially real situations[9].

In the second part of this Section we show the results of our calibrations and discuss the implications.

5.1 Data

Statistics are based on the 1970 British Cohort Study (BCS70), that takes as its subjects all 17,000 British births in the week 5-11 April 1970. Subsequently, full sample surveys took place at ages 5, 10, 16, 26 and 30. BCS70 highlights all aspects of the health, educational and social development of its subjects as they passed through childhood and adolescence. In later sweeps, the information collected covers their transitions to adult life, including leaving full-time education, entering the labour market, setting up independent homes, forming partnerships and becoming parents. (Bynner, Butler et al., 2002). For the purposes of our work, we merge the sweeps 1999-2000, 1980 and 1986. The first contains the latest information on the cohort members education, and working situation. The other two sweeps are used because provide information on the family background, that is family income and

[8] The expected utility with an income contingent loan is equal to the expected utility with a mortgage loan if $\varphi = \gamma$ and the variance of the income is zero.

[9] The presence of selection bias is potentially an issue of what we are aware, however there is few evidence in the literature concerning the selection into subjects and into job sectors. We will try to address these questions in a further analysis.

parental education. The initial sample in follow-up 1999-00 consists of 11261 respondents aged 30, then we add the other two sweeps and include observations if: respondents have an NVQ 4 equivalent qualification in 2000[10]; they are in the labour market and earn a positive wage after graduation[11]In particular, we consider those that got a degree from 1987 to 2000 and start working not earlier than the same year of graduation. This implies that the longest working period is 13 years, but we only consider the wages in 1999-2000 and for full time or part time employees[12]. According to these criteria in the final sample there are 1177 respondents.

5.2 Descriptive Statistics

We observe the average annual gross wage and its standard deviation according to the individual characteristics, family background, degree subjects and job sector.
The average income in the sample is around £24000 with a standard deviation of £18300. Male average income is around 40% higher than female wage, but also more than twice volatile (Table 1). Married graduates are around 63% of the sample and their income is slightly higher than single graduates, but quite more uncertain (Table 2). The trend higher-income higher-standard deviation is inverted when we look at the presence of kids. Those who have 1 or more kids (22% of the total) have a lower income but more uncertain (Table 3).

We consider then the family income of the cohort members in 1980, when they are 10 years old (Table 4). Unfortunately, the graduates from poor family are very few (just 2% of the sample), they get the highest income but also with an extremely high standard deviation. The value of this information is probably not too relevant, but we keep this data because

[10]The variable has been generated according to the UK national qualifications framework, NVQ equivalent level 4 includes academic qualifications (Degree and HE Diploma), vocational qualifications (BTEC Higher Certificate/ Diploma, HNC/HND) and occupational qualifications (NVQ level 4, Professional degree level qualifications, Nursing/paramedic, Other teacher training qualification, City & Guilds Part 4, RSA Higher Diploma).

[11]We exclude those working before and during education because this is a specific assumption in the theoretical model.

[12]This restriction allow us to clean from many inconsistencies in the earnings, and it is based on work undertaken by Lorraine Dearden and Alissa Goodman, Institute for Fiscal Studies.

in the theoretical model represent an extreme case of very high uncertainty. The data for medium and high income family look more reasonable and with a relatively low uncertainty compared to the graduate average. Observing the graduates' income given the mother qualifications in 1980, those with a graduate mother get the highest income, but the most uncertain is when the mother hold an O level (secondary school qualification). Controlling for father occupation in 1986 (cohort members are 16 years old), those with a father in a professional occupation have an income higher than graduate average and quite stable. Instead, when the father is unskilled the individuals get the highest income but also the most uncertain. We have to stress also in this case that the number observations is very low (4% of the sample).

Table 7 shows three degree subjects, the incomes are above the average in all the cases, and quite close among them. However, those that took a degree in Sciences (around 25% of the sample) have the lowest standard deviation. Finally, looking at the job sectors (Table 8): 62% of the graduates work in the private sector and earn around 30% more than those in the public sector. However, in the latter the career is quite static and this is reflected by a very low level of uncertainty.

5.3 Simulations under Risk Aversion

We perform the simulations according to the different levels of income and uncertainty described in Section5.2. The method is the same followed in Section 3.1, however in this case we calibrate the equations (12) and (8), both when the repayment periods are equal and when they differ. We divide the simulations in 4 broad categories, using the average wage and standard deviation provided by the BCS79 sample:

1. individual characteristics: sex, marital status, kids;
2. family background: family income 1980, mother's qualifications 1980, father's social class 1986;
3. degree subjects: science, social science, art and humanity.
4. public versus private sector.

In all the computations we set the following parameters:

- risk aversion: $a = [0.25 \quad 0.5 \quad 0.75 \quad 1.5]$, following the literature (Olson, White and Shefrin, Weiss)

- subjective discount factor: $\rho = [0.08 \quad 0.15 \quad 0.3]$, following the literature;
- cost of education: $C = [£3450 \quad £9000]$, before and after the UK Higher Education Reform;
- ICL repayment rate: $\gamma = [0.02 \quad 0.09 \quad 0.2]$, where 0.09 is the current rate fixed by the UK Reform;

In the following analysis when we change a parameter we keep the others constant at these levels: ICL repayment rates 9% (for UK relevance), subjective discount rate 8%, cost £9000 (UK current cost for 3-year degree), risk aversion 0.5.

5.3.1 Individual Characteristics

In the top left graph of Figure 2, we compare the expected utilities for increasing costs of education. We notice two initial important results: when the cost is very low the difference between the 2 systems is small, although females prefer a ML and males an ICL. For increasing costs, uncertainty matters more: compared to females, males income standard deviation is almost double (see Table 1). We observe that the gap in the preferences gets bigger for higher costs. Looking at the UK Reform, the switch to an ICL is more convenient for males than females, and the first have higher benefit for increasing costs. Looking at the graph on the top right of Figure 2, we assume increasing subjective discount rate. For females the preferences are unchanged, instead for males the utility from an ICL reduces. We have to notice that the repayment periods although equal for the two systems, they differ according to the income, therefore females with lower wages have longer repayment periods.

The central left graph of Figure 2 shows that when the incomes are relatively high and close to the average, as it is for single and married (see Table 2), an ICL is preferred and increasingly for higher costs. The gap observed is due to the effect of the uncertainty, since married have bigger income standard deviation they receive higher utility from an ICL. This also means that they have benefits from the UK Reform. The graph on the central right of Figure 2 highlights the effect of risk aversion. For both groups of graduates, higher risk aversion strengthens the preference for the ICL, and the gap remains unchanged.

The bottom left graph of Figure 2 shows that those with kids have higher standard deviation than graduates with no kids, but lower income (see Table

3). The effect of the uncertainty implies a preference for an ICL over a ML for those with kids. However, the higher income of those without kids gives higher utilities within the ICL system. In the graph on the bottom right of Figure 2 we set the highest cost and we observe that when risk aversion is very high for those with kids the utility from an ICL reduces sharply. A possible explanation is the big difference in the repayment periods, for those with no kids it is around 12 years, for graduates with kids around 18 years. Moreover, φ is endogenous and depends on both γ and $E(1/y) = \overline{h}$, when $\overline{h}$ is high, as in the case of graduates with kids, the ML rate decreases. Smaller instalments and high risk aversion makes an ML more advantageous.

5.3.2 Family Background

The top left graph of Figure 3 shows the variation of the expected utilities for increasing costs of education, assuming that the graduates come from family with different incomes in 1980. The effect of low family income is not too relevant because there are few observations, however it can be used to see what happens when the standard deviation is almost the double of the income (see Table 4). An ICL is highly preferred for higher costs, and looking also at the graph to the right, when the risk aversion is very big the ICL expected utility drops sharply. This is due, as mentioned above, to the high $\overline{h}$ and the long repayment period (19 years compared to around 13 under the other level of family income). When graduates come from family with high incomes, they obtain a wage quite stable and this is reflected in a preference for a ML. Incomes from medium family are lower and more uncertain, therefore an ICL provides higher utility. The same trend is confirmed for increasing risk aversion. The UK Reform looks more convenient for graduates from poor and medium income families.

Controlling for mother education (central left graph Figure 3 and Table 5), those with a graduate mother get the highest income and least uncertain, therefore they prefer a ML. Comparing those with mother no qualified and mother with secondary school qualification, income and sd are above the average for both, but for the first the difference (wage minus sd) is higher and they prefer an ICL with more intensity. Same trends confirmed for increasing risk aversion, although when it becomes too high the utility from ICL reduces. The UK Reform seems more beneficial for graduates with low educated parents.

We now consider the father social class in 1986 (bottom left graph Figure 3 and Table 6). For unskilled fathers the observations are too few again, but since the standard deviation is very high there is a strong preference for an

ICL. Instead those with a father in a skilled occupation, get an income above the average and very stable, and they are almost indifferent among the two systems. If the father is professional the values are close to the average, therefore the incomes have a certain degree of uncertainty that makes an ICL preferred. For increasing risk aversion we observe a slightly increasing preference for an ICL.

5.3.3 Degree Subjects and Sectors

The graduates that took the 3 subjects observed (see Table 7) have income above the average and quite similar. The less uncertain is the degree in Sciences and in fact the graduates are almost indifferent among the two systems (Figure 4). Those with a degree in Art and Humanity have the most uncertain incomes, and they strongly prefer an ICL, with higher utility for increasing cost and risk aversion. In general, when graduates discount more the future, their preference for an ICL reduces. If they become more risk averse an ICL is more preferred, and since the repayment periods are very similar there is no drop in the utilities for extremely high risk aversion.

We consider now the effects on the expected utilities for graduates working in the private and public sector (see Table 8). The graphs on the left side show the difference of utilities when the repayment periods are equal, therefore without hidden subsidies. In the graphs on the right we assume different repayment periods, that is according to Proposition 1 we set $\gamma = \varphi$ and we have $E(T_{ICL}) > T_{ML}$[13]. In the top graphs of Figure 5 we compare the two loan schemes for increasing cost. When there are no hidden subsidies, as expected, in the private sector graduates prefer an ICL, since they get a higher and more uncertain income. Instead in the public sector, the income is lower but also less uncertain and a ML is more convenient. The preferences are strengthened for higher costs. Observing the case with different repayment periods, we notice soon how an ICL is much more preferred in the private sector. But the interesting result is that also in public sector the graduates prefer now an ICL. Moreover, according to our settings, for increasing cost the repayment period under a ML rises, but under an ICL it increases more and the latter system is more convenient. Therefore, the effect of the hidden subsidies is very strong, up to change the preferences also when the levels of uncertainty are low. The same behavior is confirmed in the graphs in the middle of Figure 5 where we increase the risk aversion. We finally compare the expected utilities under the same system in

[13] We set a fixed ML instalment to £900 pounds, that means a repayment period with the lowest cost of around 4 years, and with the highest 10 years.

the two different sectors (graph in the bottom left corner). The difference between ICL in the private sector and ICL in the public sector is negative, and the same happens under a ML, with a slight change for increasing costs. This means that the effect of low uncertainty prevails, and under the same scheme a career more stable gives higher utility. Finally, considering the UK Reform, if there are no hidden subsidies the switch to an ICL is more convenient to graduates working in the private sector; the fees top-up augments the utility under an ICL, therefore those working in the private sector get lower benefits.

5.4 Conlusion

The analysis remarks some clear effects, when there is very high uncertainty an ICL is preferred in any case because gives better guarantees. This means that the UK Reform is convenient for graduates with very uncertain wages. Excluding the extreme situations, the preference for an ICL depends strongly on the gap income-standard deviation, compared to the all graduates average. Therefore, if the income is high and the sd below the average a ML gives higher utility (mother degree). In this situation, the switch to an ICL under the UK Reform is not advantageous. If both income and sd are slightly above the sample mean, and very close among them (1 or more kids, or mother with O level), an ICL is preferred but with less intensity. When the costs of education are high and the uncertainty as well, an ICL gives higher utility. Moreover, the effect of risk aversion in general is to increase the preference for an ICL. However, it is particular interesting if combined with high cost and long repayment periods, because for very high risk aversion the utility for an ICL drops drastically. If we consider income careers quite homogenous (degree subjects) and above the sample mean, what matters is the relative level of uncertainty, if low a ML is preferred. The UK Reform is quite convenient for graduates working in the private sector. Finally, looking at the job sectors we notice that when we compare a stable career with another quite dynamic, in the first case a ML gives always higher utility. However, if we don't rule out the hidden subsidies we observe a sharp increase in the utilities under an ICL, up to invert the initial preferences for a ML also for low uncertain incomes.

6 Increasing Income

In this Section we extended our model to incorporate stochastic changes of income over time, we make the model more realistic and verify which conditions still hold with respect to the case of static income. We assume that the graduate income is no longer affected by a single life time shock, but there is a shock each year throughout the working life. To model this assumption we consider an income growth rate following a geometric Brownian motion $W(t)$[14]. This means that $y(t)$ satisfies

$$dy(t)/y(t) = \lambda dt + \sigma dW(t). \tag{13}$$

This expression can be interpreted heuristically as expressing the relative or percentage increment dy/y in y during an instant of time dt. λ is the deterministic growth rate and σ its standard deviation. Solving the stochastic differential equation (13) we obtain the stochastic income:

$$y(t) = y(0)\exp[(\lambda - \frac{1}{2}\sigma^2)t + \sigma W(t)] \tag{14}$$

Equation (14) represents the new income we use to compute the expected utilities under the two loan schemes. Since it is not straightforward to obtain an algebraic solution for the expected utilities under an ICL we adopt a numerical method. We consider a discrete form of equation (14) because it is more related to our problem. The method is explained in detail in the Appendix E; briefly, we generate many incomes paths of the same length (equal to a working life period of 40 years), and we use them to compute the utilities. Each income path produces one level of utility, therefore we average for the number of paths created. At the end we get the average expected utilities under an ICL (AU_{ICL}) and ML (AU_{ML}). What is important to stress under this new approach is that two new parameters enter the model, looking at the equation (14) they are the initial income (y_0) and its deterministic growth rate (λ). Moreover, σ is the volatility of the Brownian motion and represents the maximum variation of the income in the interval t (for us 1 year). As we did in the first part of this work we use real data to calibrate the model and simulate different scenarios.

6.1 Setting the new model

Under the new stochastic framework the only variable that we can set according to the real data is the initial income. In our BCS70 dataset, we generate a new variable for the initial wages, considering the graduates that started the current job within 3 years after graduation[15]. As showed in Ta-

[14] $W(t)$ is Normally distributed with $E(W(t)) = 0$ and $Var(W(t)) = t$.

[15] These incomes are not properly the starting wages because some graduates have been working for more than 5 years, so their wages in 1999-00 has probably increased. However, we consider only those in the same occupation since graduation.

ble 9, there are only 405 observations for the initial wage with an average of almost £24000. We consider also the wages below the 10th percentile, whose mean is £8600 pounds, and those above the 95th percentile (mean £53000). The degree of uncertainty represented by the standard deviation is no longer valid under our new framework. The uncertainty now affects the income each year and concerns a single individual. We choose three levels of sigma (0.02 0.05 0.15) in order to have different intensities of the effect of the stochastic shock on income. For example, a $\sigma = 0.05$ means that the maximum annual variation of the income can be 5%, with respect to the deterministic growth, that is $\sigma = 0$.

In our dataset we do not have information on the growth rate of the wages because we observe a cross section in 1999-00. Merging it with the BCS70 sweep of 1996 we loose a lot observations, since that survey was conducted through mail questionnaires and many people did not answer. Therefore, according to our sample restrictions to generate the initial wages, we would end up with just few observations. We set three values for the deterministic growth rate ($\lambda = 0.5 \quad 1 \quad 1.5$). Assuming no uncertainty, for example $\lambda = 0.5$ corresponds to a total increase of initial income in 40 years of around 40%, that is ,on average, a constant increase of 1% per year. If $\lambda = 1$ the total increase of the income at the end of the working life is 63%, that is 2.4% p.a. Finally, $\lambda = 1.5$ corresponds to a 77% increase of the initial income after 40 years, that is 4% p.a.

To make the model closer to the reality, we consider an initial threshold of £15000 (as in the UK Reform) but only for an ICL. This means that the graduates start paying the loan when their income is above £15000, and the tax rate is levied only on the wage difference[16]. The cost of education is set to £9000 (3-year degree), and under a ML we fix three possible instalments in order to have three repayment periods. When $\varphi = £500$ the repayment period is $T_{ML} = 18$, with $\varphi = £1000 \quad T_{ML} = 9$, with $\varphi = £3000 \quad T_{ML} = 3$. We finally set the other parameters as in the simulation in Section 5.

6.2 Simulations

The results of the simulations are reported in Tables 10-13, where each time we change one parameter keeping the others equal to those in Table 10. We show the results for both low and high income. Looking at Table 10, we notice a first important effect: the preference for one system over the other

[16] For example, if the annual wage is £20000 the tax rate is levied on £5000.

depends strongly on the level of the initial income. For low starting wages an ICL is the favorite system, but for high starting wages the utility under an ICL declines sharply and a ML can be preferred. Another important result is that the level of uncertainty matters less with respect to the case with a static income, although the direction of the effect is the same. We observe in fact that for higher income volatility the utility of an ICL increases but at a very slow pace. When the incomes are high the effect of uncertainty is more evident, in fact for $\rho = 8\%$ the initial preference for a ML is replaced by a preference for an ICL.

For increasing subjective discount rates, we notice a reduction of the gap between the two systems if the income is low. For high income the preference for an ICL is increasing but the two systems give almost equal utility. In Table 11, we first increase and then decrease the fixed ML instalment with respect to the baseline case. When the ML repayment period is very long, an ICL is still preferred for low incomes, but no more for high incomes. Conversely, short ML repayment periods make an ICL always favorite.
The effect of increasing risk aversion is to strengthen the preferences for one system. Low income earners prefer an ICL, and becoming more risk averse increase their utility under this system. Instead for high income earners the effect of risk aversion is correlated to the level of uncertainty. If σ is low, they prefer a ML also for increasing risk aversion. If the level of σ is high the two systems are more or less equal, although for high risk aversion a ML becomes the favorite. Finally, we consider an increase of the deterministic growth rate (Table 13), from 1% to 4% per year, therefore with respect to the baseline case is like having a higher income. And indeed for low income earners the utility from an ICL reduces, and for high income earners the preference for a ML increases.

6.3 Conclusion

The simulations show that the preferences for one system over the other are remarkably driven by the level of the initial income and the size of the ML instalment. Low starting wage earners have strong preference for an ICL and therefore for them the UK Reform is highly convenient. When we control on parameters that affect the income (such as the growth rate) and make it higher, then the utility of an ICL decreases. For increasing uncertainty the utility of an ICL rises, but by very small amounts. Risk aversion increases the utility for the the system already preferred. Finally, the size of the ML instalments can change the preferences, in fact for higher instalment the utility of a ML reduces sharply.

7 Conclusion

In this work we presented a theoretical model to compare two loan schemes for higher education, when graduate incomes are uncertain. The findings of the model have been calibrated using real data on graduate wages, obtained by the British Cohort Study 1970. We also used the features of the UK Higher Education Reform to observe the implications of the switch from a ML to an ICL and the effect of the top-up fees. In the first part of the work we assume that the graduate income is affected by a single lifetime shock, and we compute the individual expected utilities under an ICL and a ML for risk neutral and risk averse people. Our first result, supported by the empirical simulations, is that for risk neutral individual the preferences depend strongly on the presence of hidden subsidies. Assuming different repayment periods between the 2 systems, the expected costs under a ML are higher than the costs under an ICL; otherwise, with the same repayment period the effect is the opposite. This means that the UK Reform gives low benefits to risk neutral graduates. For risk averse individuals we evaluated the effects of our model under different possible scenarios. We used information on graduate wages and relative uncertainty controlling for individual characteristics, family background, degree courses and job sector. The main result is that for high wage uncertainty an ICL is the preferred system. The UK Reform becomes very convenient for people from low educated parents background, males over females, people working in the private sector, because they all prefer an ICL. Instead those with high income and low uncertainty, such as those with mother highly educated, prefer a ML. For increasing costs the graduates that prefer an ICL obtain higher utility, therefore for them the fees top-up is much more convenient compared to increased costs under a ML system. Another important result is that for increasing risk aversion an ICL is more convenient, this confirms the conviction that this system is a better guarantee for uncertain future. Also in this case the Uk Reform is beneficial. Moreover, a static career in the public sector gives an income not too high but also the most stable in terms of uncertainty, and a ML produces higher utility. However, if we don't rule out the hidden subsidies an ICL becomes convenient also when the uncertainty is very low. In general, the UK Reform is more convenient in the private sector than the public sector. In the second part of our work, we changed the assumptions on income, allowing a stochastic growth along the working life. The results of the new framework have in common with the previous the preference for an ICL for increasing uncertainty. However, the factors that affect the choice of one system over the other are different. The size of the starting wage is the main discriminant: when the income is low an ICL is the favorite system. This implies that the UK Reform is convenient for this category of graduates. For high initial income instead a ML gives higher utility. Finally, increasing the size of the ML instalment makes an ICL the preferred system.

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A Appendix: Proof of Proposition 1

We know that $T_{ICL} = \frac{C}{\gamma y}$ and $T_{ML} = \frac{c}{\varphi}$, and given the assumption $E(y) = 1$ we compute the expected value of the repayment period under an ICL.

$$E(T_{ICL}) = E(\frac{C}{\gamma y}) = \frac{C}{\gamma} \times E(\frac{1}{y})$$

by the Jensen's inequality we know that

$$\frac{C}{\gamma} \times E(\frac{1}{y}) > \frac{C}{\gamma E(y)}$$

that implies

$$E(\frac{1}{y}) > 1.$$

Given this result it is straightforward to prove the point (a):
if $\gamma = \varphi$ then

$$\frac{C}{\gamma} \times E(\frac{1}{y}) > \frac{C}{\varphi} \rightarrow E(T_{ICL}) > T_{ML}.$$

Point (b)
We assume $E(T_{ICL}) = T_{ML}$ that means

$$\frac{C}{\gamma} \times E(\frac{1}{y}) = \frac{C}{\varphi}$$

we get γ:

$$\gamma = \varphi \times E(\frac{1}{y}) \quad \Longrightarrow \gamma > \varphi$$

since $E(\frac{1}{y}) > 1$.

B Appendix: Proof of Proposition 2

Under risk neutrality equation (1) becomes

$$V = E(\int_s^\infty e^{-\rho t} y \; dt) - E(\int_s^{T+s} e^{-\rho t} R \; dt) \tag{15}$$

So we can compare only the expected costs. Under ML the present value of the cost of size C is:

$$\begin{aligned} PVC_{ML} &= \int_s^{T+s} \varphi \, e^{-\rho t} \, dt \\ &= e^{-\rho s} \, \frac{\varphi}{\rho} \, [1 - e^{-\rho \frac{C}{\varphi}}] \, . \end{aligned} \tag{16}$$

Under ICL the present value of the cost of size C is:

$$\begin{aligned} PVC_{ICL} &= \int_s^{\tilde{T}+s} y \, \gamma \, e^{-\rho t} \, dt \\ &= e^{-\rho s} \, \frac{\gamma \, y}{\rho} \left[1 - e^{-\rho \frac{C}{\gamma y}} \right] \quad . \end{aligned} \tag{17}$$

Knowing that $E(y) = 1$, we take the expected value of both the equations above.

$$E(PVC_{ML}) = \frac{\varphi}{\rho} \, [1 - e^{-\rho \frac{C}{\varphi E(y)}} \,] e^{-\rho s} \tag{18}$$

$$E(PVC_{ICL}) = E\left[\frac{\gamma\, y}{\rho}\,(1 - e^{-\rho\frac{C}{\gamma y}})e^{-\rho s}\right] \tag{19}$$

Case $\gamma = \varphi$

Under this condition we have $E[T_{ICL}] > T_{ML}$. We can easily observe that the expected values of the costs can be written:

$$E(PVC_{ML}) = f[E(y)]$$

$$E(PVC_{ICL}) = Ef(y)$$

Since $f(y) = \frac{\gamma\, y}{\rho}\left(1 - e^{-\frac{\rho\, C}{\gamma\, y}}\right)e^{-\rho s}$ is a concave function [17]by the Jensen inequality we obtain that the expected costs under ICL are lower than the expected costs under ML: $E(PVC_{ICL}) < E(PVC_{ML})$. According to equation (15) the expected utility under ICL is higher than the expected utility under ML.

Case $E[T_{ICL}] = T_{ML}$

To verify which costs are higher we have to substitute $\varphi = \frac{\gamma}{E(1/y)}$ in equation (18) and then compare with equation(19). As we can see it is not straightforward, therefore we adopt a numerical solution using the real data on graduate income provided in our dataset from BCS70.

In order to be consistent with the assumption $E(y) = 1$, we first standardize the annual gross income of the graduates. We call w_i the wages in the sample and divide each of the them by the sample mean, and we call this new variable z.

$$z_i = \frac{w_i}{\frac{1}{n}\sum_{i=1}^{n} w_i} \quad for \;\; i = 1...n$$

then

$$\overline{z} = \frac{1}{n}\sum_{i=1}^{n} z_i = 1$$

and we use the sample analogue $\overline{z}$ to estimate $E(y) = 1$. Instead, to estimate $E(\frac{1}{y})$ we generate its sample analogue

$$\overline{h} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{z_i} \quad .$$

[17] $f''(y) = -\frac{\rho C^2 e^{-\left(s+\frac{C}{\gamma y}\right)\rho}}{\gamma y^3}$. It is reasonable to assume that γ, ρ and C are all greater or equal than zero. Therefore, the second derivative of $f(y)$ is always negative when the shock on income is positive: $f''(y) < 0, \quad \forall\, y > 0$.

In our sample $\overline{h} > \frac{1}{\bar{z}}$, then the Jensen's inequality holds.

C Appendix: Expected Utility with a Mortgage Loan

The Taylor approximation in equation (6) is the following

$$\begin{aligned} E[u(y-\varphi)] &= E\left\{u\left(1-\varphi\right)+u^{'}\left(1-\varphi\right)\left(y-1\right)+\frac{1}{2}u^{''}\left(1-\varphi\right)\left(y-1\right)^{2}\right\} \\ &= u\left(1-\varphi\right)+u^{'}\left(1-\varphi\right)E(y-1)+\frac{1}{2}u^{''}\left(1-\varphi\right)E(y-1)^{2} \\ &= u\left(1-\varphi\right)+\frac{1}{2}u^{''}\left(1-\varphi\right)\sigma^{2}. \end{aligned} \tag{20}$$

Plugging the equations (6) and (7) in the equation (5), substituting $T = C/\varphi$ and solving the integral, we obtain:

$$\begin{aligned} V_{ML} = &\frac{e^{-\rho s}}{\rho}\left(1-e^{-\frac{\rho C}{\varphi}}\right)\left[u\left(1-\varphi\right)+\frac{1}{2}u^{''}\left(1-\varphi\right)\sigma_s^2\right] \\ &+\frac{e^{-\rho s}}{\rho}e^{-\frac{\rho C}{\varphi}}\left[u\left(1\right)+\frac{1}{2}u^{''}\left(1\right)\sigma_s^2\right]. \end{aligned} \tag{21}$$

Finally, substituting a CRRA utility function in equation(21) and simplifying we get equation (8).

D Appendix: Expected Utility with an Income Contingent Loan

In Section (4.2) we defined a new function $g(y)$ as:

$$g(y) = \left[1-e^{-\frac{\rho C}{\gamma y}}\right]u\left[y\left(1-\gamma\right)\right]+\left[e^{-\frac{\rho C}{\gamma y}}\right]u\left(y\right) \tag{22}$$

We rewrite the equation (10)

$$V_{ICL} = \frac{e^{-\rho s}}{\rho}E[g(y)] \tag{23}$$

and we apply a second order Taylor expansion to $E[g(y)]$, around the mean $E\left[y\right]=1$, then:

$$\begin{aligned} E[g(y)] &= E\left\{ g(1)+g'(1)(y-1)+g''(1)\frac{(y-1)^2}{2} \right\} \\ &= g(1)+g'(1)E(y-1)+\frac{g''(1)}{2}E\left(y-1\right)^2 \\ &= g(1)+g''(1)\frac{\sigma_s^2}{2}. \end{aligned} \tag{24}$$

The equation (23) becomes

$$V_{ICL}=\frac{e^{-\rho s}}{\rho}\left[g(1)+g''(1)\frac{\sigma_s^2}{2}\right] \tag{25}$$

From now on we follow this procedure:

1. we work out the value of g(1), in general and with a CRRA utility function;
2. we work out the first derivative and the second derivative of g(y), both in general and with a CRRA utility function;
3. we calculate $g'(1)$ and $g''(1)$ using a CRRA utility function;
4. we substitute the equations of $g(1)$ and $g''(1)$, using a CRRA utility function, in the equation (25) and we obtain equations (32) and (12).

- **Value of g(1)**

  In general,

$$g(1)=\left[1-e^{-\frac{\rho C}{\gamma}}\right]u\left[(1-\gamma)\right]+\left[e^{-\frac{\rho C}{\gamma}}\right]u\left(1\right) \tag{26}$$

Using a CRRA utility function we have

$$g(1)_{CRRA}=\frac{1}{b}[-e^{-\frac{\rho C}{\gamma}}((1-\gamma)^b-1)+(1-\gamma)^b]. \tag{27}$$

- **Value of g'(y)**

  In general,

$$g'(y) = u'[y(1-\gamma)](1-\gamma)\left[1-e^{-\frac{\rho C}{\gamma y}}\right] + u[y(1-\gamma)]\left[\frac{-\rho C e^{-\frac{\rho C}{\gamma y}}}{\gamma y^2}\right] + u'(y)[e^{-\frac{\rho C}{\gamma y}}] + u(y)\left[\frac{\rho C e^{-\frac{\rho C}{\gamma y}}}{\gamma y^2}\right] \tag{28}$$

  using a CRRA utility function:

$$g'(y)_{CRRA} = (y(1-\gamma))^{b-1}(1-\gamma)\left[1-e^{-\frac{\rho C}{\gamma y}}\right] + (y(1-\gamma))^b\left[\frac{-\rho C e^{-\frac{\rho C}{\gamma y}}}{b\gamma y^2}\right] + y^{b-1}[e^{-\frac{\rho C}{\gamma y}}] + \left[\frac{y^{b-2}\rho C e^{-\frac{\rho C}{\gamma y}}}{b\gamma}\right]. \tag{29}$$

- **Value of g"(y)**

$$\begin{aligned} g''(y) = {} & \frac{e^{-\frac{\rho C}{\gamma y}}\rho C(2\gamma y - \rho C)}{y^4\gamma^2}u[y(1-\gamma)] + \frac{e^{-\frac{\rho C}{\gamma y}}\rho C(-2\gamma y + \rho C)}{y^4\gamma^2}u(y) \\ & - \frac{2e^{-\frac{\rho C}{\gamma y}}\rho C(1-\gamma)}{y^2\gamma}u'[y(1-\gamma)] + \frac{2e^{-\frac{\rho C}{\gamma y}}\rho C}{y^2\gamma}u'(y) \\ & + \left[1-e^{-\frac{\rho C}{\gamma y}}\right](1-\gamma)^2u''[y(1-\gamma)] + \left[e^{-\frac{\rho C}{\gamma y}}\right]u''(y). \end{aligned} \tag{30}$$

Now we work out $g''(y)$ using a a CRRA and evaluating in $y = 1$

$$\begin{aligned} g''(1)_{CRRA} = {} & \frac{1}{b\gamma^2}\{e^{-\frac{\rho C}{\gamma}}[(b-1)b\gamma^2[1+(e^{\frac{\rho C}{\gamma y}}-1)(1-\gamma)^b] \\ & + 2\rho C(b-1)\gamma(1-(1-\gamma)^b) + C^2\rho^2(1-(1-\gamma)^b)]\}. \end{aligned} \tag{31}$$

- **Results**

Substituting $g(1)$ and $g''(1)$ in equation (25) we get the general expected utility under an income contingent loan:

$$\begin{aligned} V_{ICL} = & \left[1 - e^{-\frac{\rho C}{\gamma}}\right] u\left[(1-\gamma)\right] + \left[e^{-\frac{\rho C}{\gamma}}\right] u\left(1\right) \\ & + [\frac{e^{-\frac{\rho C}{\gamma}} \rho C(2\gamma - \rho c)}{\gamma^2} u[1-\gamma] + \frac{e^{-\frac{\rho C}{\gamma}} \rho C(-2\gamma + \rho c)}{\gamma^2} u(1) \\ & - \frac{2e^{-\frac{\rho C}{\gamma}} \rho C(1-\gamma)}{\gamma} u'[1-\gamma] + \frac{2e^{-\frac{\rho C}{\gamma}} \rho C}{\gamma} u'(1) \\ & + \left[1 - e^{-\frac{\rho C}{\gamma}}\right] (1-\gamma)^2 u''[1-\gamma] + \left[e^{-\frac{\rho C}{\gamma}}\right] u''(1)] \frac{\sigma_s^2}{2}. \end{aligned} \tag{32}$$

Substituting in equation (25) the equations for $g(1)$ and $g''(1)$ with a CRRA utility function, we obtain equation (12).

E Appendix: Numerical Method - Brownian Motion

E.1 Numerical Method

1. We generate a path of annual incomes for an individual working life. Since the problem requires a discrete solution, we apply the Euler-Maruyama method that takes the form

$$y_j = y_{j-1} + y_{j-1}\lambda\Delta t + y_{j-1}\sigma(W(\tau_j) - W(\tau_{j-1})). \tag{33}$$

To generate the increments $W(\tau_j) - W(\tau_{j-1})$ we compute discretized Brownian motion paths, where $W(t)$ is specified at discrete t values. As explained in Higham (2001) we first discretize the interval $[0, I]$. We set $dt = I/N$ for some positive integer N, and let W_j denote $W(t_j)$ with $t_j = jdt$. According to the properties of the standard Brownian motion $W(0) = 0$ and

$$W_j = W_{j-1} + dW_j \tag{34}$$

where dW_j is an independent random variable of the form $\sqrt{dt}N(0,1)$. The discretized brownian motion path is a 1-by-N array, where each element is given by the cumulative sum in equation(34). To generate equation(33), we define $\Delta t = I/L$ for some positive integer L, and $\tau_j = \Delta t$. As in Higham (2001) we choose the stepsize Δt for the numerical method to be an integer multiple $R \geq 1$ of the Brownian motion increment dt: $\Delta t = Rdt$. Finally, we get the increment in equation(33) as cumulative sum:

$$W(\tau_j) - W_(\tau_{j-1}) = W(jRdt) - W((j-1)Rdt) = \sum_{h=jR-R+1}^{jR} dW_h. \quad (35)$$

The Brownian motion of equation (34) is produced setting $I = 1$ and $N = 160$ in order to have a small value of dt. Using a random number generator we produce 160 "pseudorandom" numbers from the N(0,1) distribution. The increments of equation (35) are computed setting $R = 4$, in order to have 40 annual incomes.

2. Income contingent loan. We work out the yearly repayments as fixed percentage of the stochastic incomes generated. If the income is higher than £15000 the payments are positive, and the tax rate is levied on the difference between the wage and 15000; if the income is lower than £15000 the payments are zero. We then built a vector whose elements are the cumulative sum of the repayments, in order to see the amount of loan repaid. To obtain the repayment period, we observe the years in which the cumulative sum of the payments is equal[18] to the cost of education. We work out the individual utility as discounted sum of the net incomes during and after the repayment period, up to the end of the working life. We use a CRRA utility function.

3. Mortgage loan. We set the fixed repayment period as the ratio between the cost of education and the annual instalment. The individual utility is given by the discounted sum of the net incomes during and after the repayment period. We use a CRRA utility function. However, it can happen that the annual income is lower than the instalment, in a usual mortgage loan the individual repays in the subsequent years at a higher interest rate. Here to highlight the loss of utility in the case of no repayment in one year, we compute the level of the utility for that year as a negative percentage[19] of the annual income. We repeat the same procedure taking as repayment period that computed above for an ICL, to have the case with no hidden subsidy.

4. From steps (2) and (3) we obtain a single value for the utility for an individual income path generated in point (1). We generalize our

[18] Since it is almost impossible to get a value equal to the cost, when the repayment is greater than it, we infer with certainty that debt has been paid off.

[19] We set this percentage equal to the average-low interest rate for a typical mortgage loan e.g. around 5%.

method generating a high number of income paths (1000) and for each path we compute a level of utility. We then work out the average utility under both financing scheme and the difference of the average in order to compare the two systems.

5. We let the various parameters change and we repeat steps (1) to (4), observing the trend of the difference of the average utility under the two funding schemes.

Table 1: Annual Gross Wages Graduates

	Mean	**Std. Dev.**	**N**	**Percent**
male	27897.81	22576.99	623	52.93
female	19665.85	10406.5	554	47.07
Total	24023.12	18368.97	1177	100.00

Table 2: Marital Status

	Mean	**Std. Dev.**	**N**	**Percent**
single/divorced	23345	15786.7	437	37.13
married/cohabiting	24423.58	19735.23	740	62.87
Total	24023.12	18368.97	1177	100.00

Table 3: Kids 0-16 years old

	Mean	**Std. Dev.**	**N**	**Percent**
no kid/ non answ	24973.56	17884.25	913	77.57
1 or more kids	20736.22	19638.16	264	22.43
Total	24023.12	18368.97	1177	100.00

Table 4: Family income 1980

	Mean	**Std. Dev.**	**N**	**Percent**
not stated	25383.69	20007.07	144	12.23
low	32384.32	56743.81	25	2.12
medium	23052.64	16881.78	759	64.49
high	25355.04	13181.55	249	21.16
Total	24023.12	18368.97	1177	100.00

Table 5: Mother qualifications 1980

	Mean	**Std. Dev.**	**N**	**Percent**
no quals	22306.22	18956.85	354	30.08
O-level	25773.47	22907.52	265	22.51
degree	27149.5	15615.49	72	6.12
other quals	23856.14	15164.33	486	41.29
Total	24023.12	18368.97	1177	100.00

Table 6: Father social class 1986

	Mean	**Std. Dev.**	**N**	**Percent**
missed/not stated	22796.19	15091.37	275	23.36
profes/interm	25969.63	19536.18	491	41.72
skilled occupation	21821.74	12973.58	364	30.93
other occupation	27916.26	42101.93	47	3.99
Total	24023.12	18368.97	1177	100.00

Table 7: Degree subjects

	Mean	**Std. Dev.**	**N**	**Percent**
does not apply	21249.42	14876.29	554	47.07
Sciences	26782.23	16828.23	292	24.81
Social Sciences	25857.94	21385.41	146	12.40
Art and humanities	26526.32	25277.39	185	15.72
Total	24023.12	18368.97	1177	100.00

Table 8: Job sector

	Mean	**Std. Dev.**	**N**	**percent**
other / not answ	18456.66	6864.195	83	7.05
private sector	26434.32	21703.36	736	62.53
public sector	20356.57	9910.831	358	30.42
Total	24023.12	18368.97	1177	100.00

Table 9: Annual gross initial income

	Mean	**Std. Dev.**	**N**
low (10th perc)	8640.983	2652.527	40
high (mean)	53002.737	20505.394	38
total	23987.433	13147.324	405

Table 10: Increasing Income $AU_{ICL} - AU_{ML}$ - ρ and σ changing

	$\varphi = £1000$	$C = £9000$	$T_{ML} = 9$
	$\gamma = 9\%$	$\lambda = 1\%$	$ra = 0.5$
Low Initial Income			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$\rho = 8\%$	67.2721	67.2994	67.4311
$\rho = 15\%$	51.5189	51.5387	51.6331
$\rho = 30\%$	32.7445	32.7559	32.8088
High Initial Income			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$\rho = 8\%$	-0.3704	-0.2421	0.0227
$\rho = 15\%$	0.7800	0.8651	1.1011
$\rho = 30\%$	1.4404	1.4808	1.6262

Table 11: Increasing Income $AU_{ICL} - AU_{ML}$ - φ and σ changing

	$\gamma = 9\%$	$C = £9000$ $\lambda = 1\%$	$\rho = 8\%$ $ra = 0.5$
Low Initial Income			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$\varphi = £500$ $T_{ML} = 18$	48.8080	48.8332	48.9630
$\varphi = £1000$ $T_{ML} = 9$	67.2721	67.2994	67.4311
$\varphi = £3000$ $T_{ML} = 3$	90.6606	90.6905	90.8231
High Initial Income			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$\varphi = £500$ $T_{ML} = 18$	-10.9630	-10.8355	-10.5687
$\varphi = £1000$ $T_{ML} = 9$	-0.3704	-0.2421	0.0227
$\varphi = £3000$ $T_{ML} = 3$	10.9286	11.0558	11.3084

Table 12: Increasing Income $AU_{ICL} - AU_{ML}$ - ra and σ changing

	$\varphi = £1000$ $\gamma = 9\%$	$C = £9000$ $\lambda = 1\%$	$T_{ML} = 9$ $\rho = 8\%$
Low Initial Income			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$ra = 0.25$	6.9826	6.9870	7.0090
$ra = 0.5$	67.2721	67.2994	67.4311
$ra = 1.2$	38271.00	38265.00	38242.00
High Initial Income			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$ra = 0.25$	-0.0224	-0.0121	0.0106
$ra = 0.5$	-0.3704	-0.2421	0.0227
$ra = 1.2$	-743.0753	-594.7867	-340.0252

Table 13: Increasing Income $AU_{ICL} - AU_{ML}$ - λ and σ changing

	$\varphi = £1000$	$C = £9000$	$T_{ML} = 9$
	$\gamma = 9\%$	$\rho = 8\%$	$ra = 0.5$
Low Initial Income			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$\lambda = 1\%$	67.2721	67.2994	67.4311
$\lambda = 2.4\%$	65.3690	65.2274	63.9582
$\lambda = 4\%$	54.9972	55.0309	55.2131
High Initial Income			
	$\sigma = 2\%$	$\sigma = 5\%$	$\sigma = 15\%$
$\lambda = 1\%$	-0.3704	-0.2421	0.0227
$\lambda = 2.4\%$	-2.3783	-2.2142	-1.7014
$\lambda = 4\%$	-2.8460	-2.8486	-3.2024

Figure 1: $EC_{ML} - EC_{ICL}$ - Expected costs and Risk Neutrality

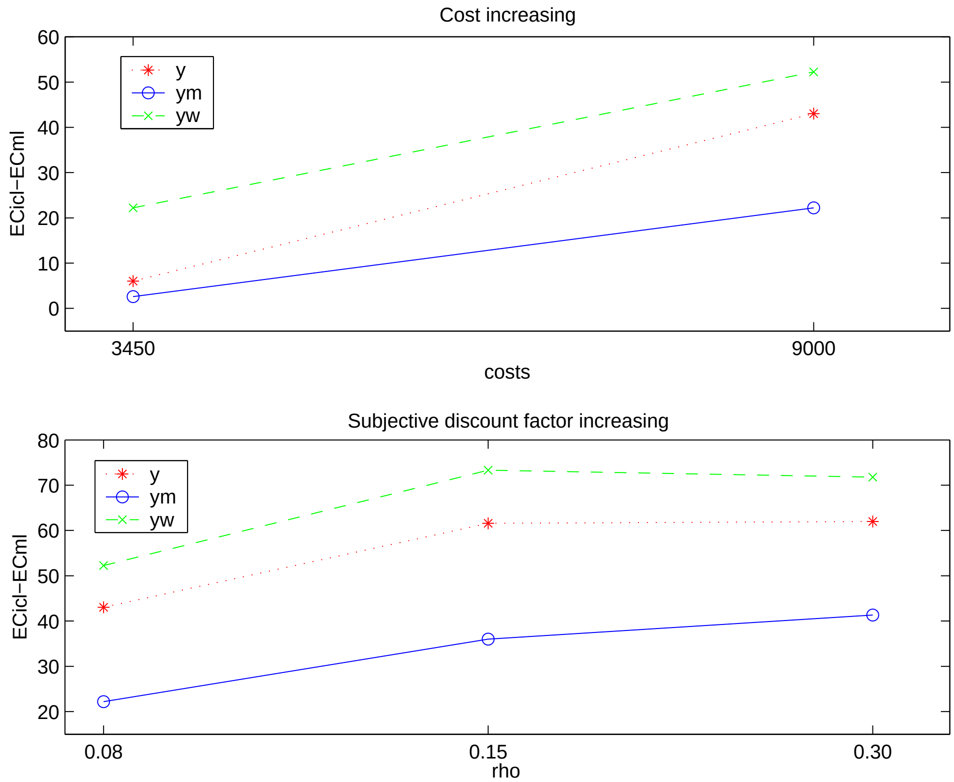

Figure 2: $EU_{ICL} - EU_{ML}$ - Individual Characteristics

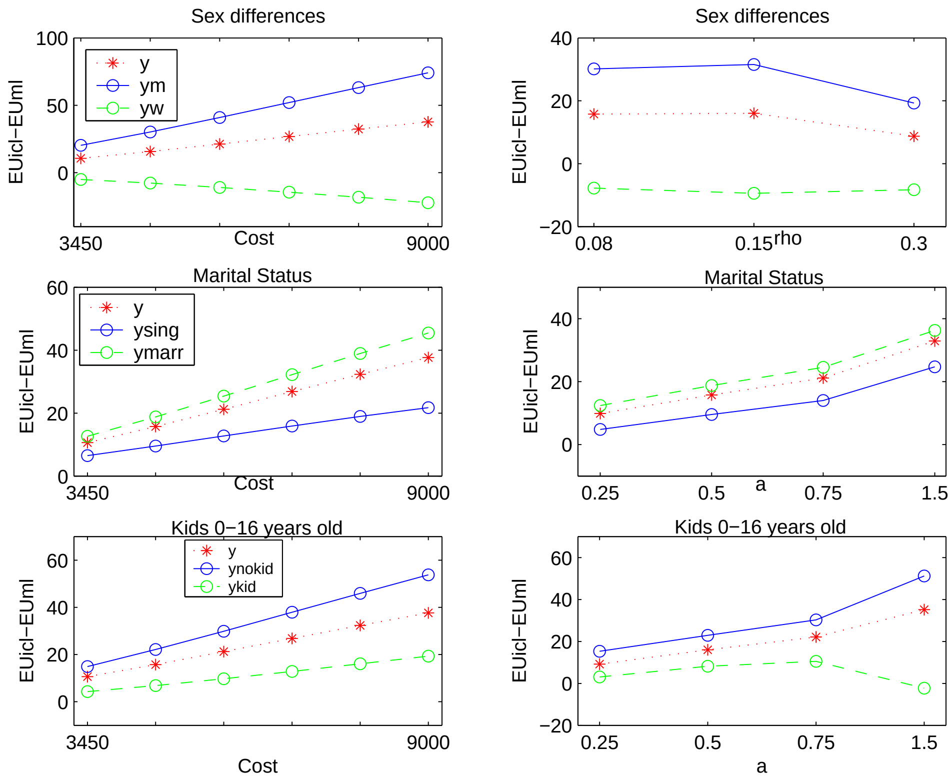

Figure 3: $EU_{ICL} - EU_{ML}$ - Family Background

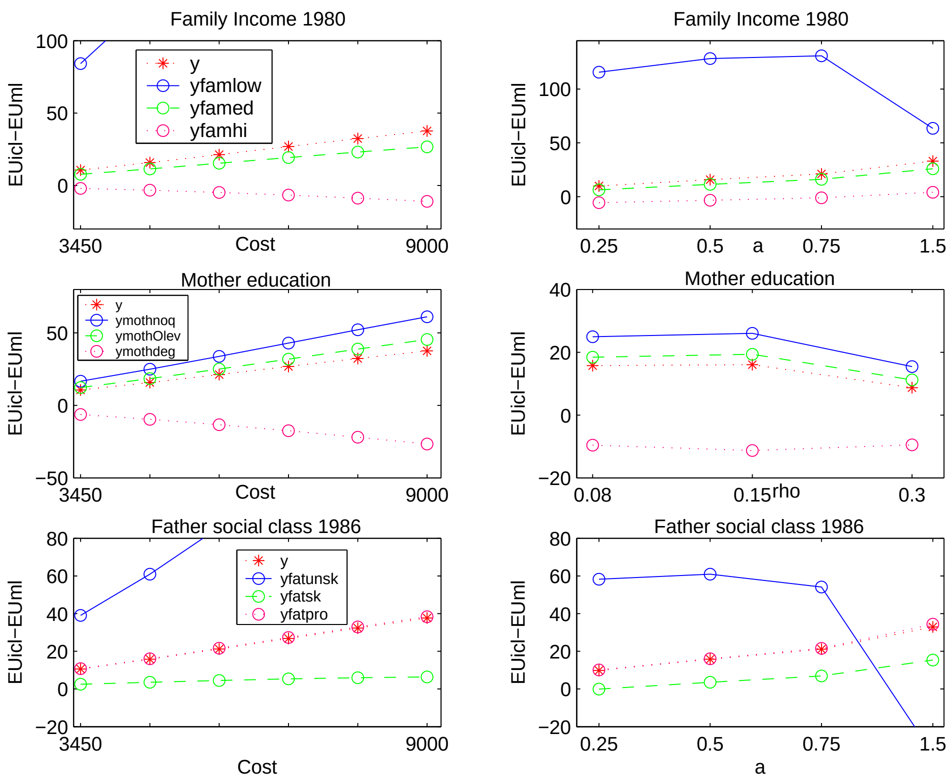

Figure 4: $EU_{ICL} - EU_{ML}$ - Degree Subjects

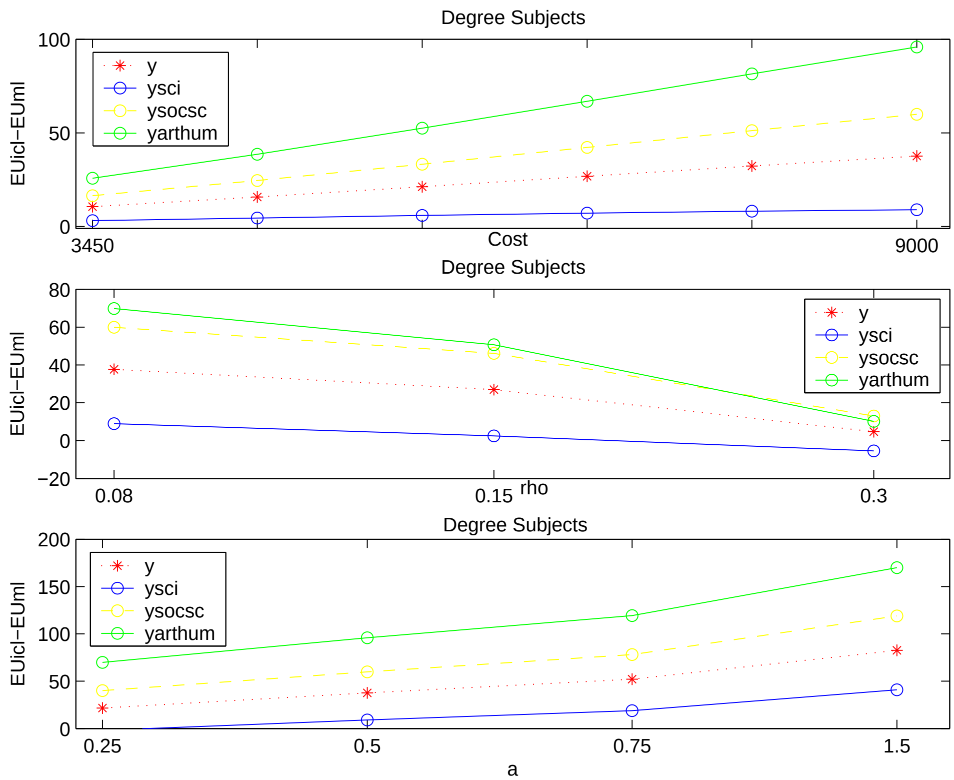

Figure 5: $EU_{ICL} - EU_{ML}$ - Public versus Private sector