Behavioural Decisions and Welfare

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Abstract

We study decision problems where (a) preference parameters are defined to include psychological/moral considerations and (b) there is a feedback effect from chosen actions to preference parameters. In a standard decision problem the chosen action is required to be optimal when the feedback effect from actions to preference parameters is fully taken into account. In a behavioural decision problem the chosen action is optimal taking preference parameters as given although chosen actions and preference parameters are required to be mutually consistent. Our framework unifies seemingly disconnected papers in the literature. We characterize the conditions under which behavioural and standard decisions problems are indistinguishable: in smooth settings, the two decision problems are generically distinguishable. We show that in general, revealed preferences cannot be used for making welfare judgements and we characterize the conditions under which they can inform welfare analysis. We provide an existence result for the case of incomplete preferences. We suggest novel implications for policy and welfare analysis.

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1 Introduction

This paper studies decision problems where preference parameters are broadly interpreted to include psychological states (e.g. reference points, beliefs, emotions, feelings, self-esteem, will-power, aspirations, etc.) or moral states (e.g. personal commitments, individual values, etc.).

There is considerable evidence from social psychology and experimental economics that such psychological and moral factors affect preferences. High material aspirations may make a person prefer studying business to philosophy; high levels of stress may make her more prone to smoke a cigarette than when she is relax; a high level of self-esteem may guide her to prefer challenging options than safe ones, etc.[1].

In addition, there is also a great deal of evidence which suggests that the relationship between preferences and behavior may also go in the opposite direction: what a person does (or expect to do) may define her psychological and moral states. Lazarus and Folkman (1984) point out that people are able to cope with stress, anger or anxiety, by changing their response to a situation (emotion-focused problem) or by changing the environment (problem-focused coping). In a similar vein, William James (1890/1981) used the term "self-esteem" to refer to the way individuals feel about themselves which in turn depends on the success they have to accomplish those things that they wish to accomplish (in Pajares and Schunk, 2001,2002). Baron (2008, pg. 68) argues that emotions are partly under our control. Individuals can "induce or suppress emotions in themselves almost on cue." Some people may reshape their character, so that their emotional responses change. More generally, there is extensive work in social cognitive theory by Albert Bandura[2], that views human functioning as the product of a dynamic interplay of personal, behavioural, and environmental influence. Bandura points out that the way in which people interpret the results of their own behaviour informs and alters their environments and personal factors which, in turn, inform and alter subsequent behaviour through

[1] Elster (1998) provides a review on how individual preferences are affected by emotions; Sen (1977) discusses how personal values shape preferences; Appadurai (2004) studies the relationship between aspirations and behaviour; Benabou and Tirole (2002) study how self-confidence interact with preferences.

[2] See Bandura (1997, 2001) for a survey.

an "environmental feedback effect." He labelled this concept *reciprocal determinism* (see Bandura 1986)

With this motivation in mind, we study decision problems where preference parameters are potentially endogenous. A decision state is a profile consisting of both actions and preference parameters. A consistent decision state is a profile of actions and preference parameters where the preference parameter is generated by the action profile via the feedback effect. A standard decision problem is one where the chosen action is required to be optimal when the feedback effect from actions to preference parameters is fully taken into account. A behavioural decision problem is one where the chosen actions are optimal taking preference parameters as given although chosen actions and preference parameters are required to be mutually consistent[3].

We begin showing that the decision framework with (potentially) endogenous preference parameters studied here can be obtained as a reduced form representation of seemingly disconnected types of decision making examined in the literature. The papers in question include situations where the reference state corresponds to the decision maker's current state (Tversky and Kahneman, 1991), psychological games with a single active player (Geanakoplos, Pearce and Staccheti, 1989), loss aversion games with a single player (Shalev, 2000), reference dependent consumption and personal equilibrium (Koszegi, 2005; Koszegi and Rabin, 2006, 2007) and aspiration traps (Ray, 2006 and Heifetz and Minelli, 2007). We also show that choice with exogenous frames (Rubinstein and Salant, 2007) or ancillary conditions (Bernheim and Rangel, 2006) are special cases of the class of decision problems studied here[4].

Next, in Section 3, we characterize the necessary and sufficient conditions under which behavioural and standard decision problems are indistinguishable from each other. We also show that in smooth settings, both decision problems are generically

[3] Note that in a standard decision problem the individual fully takes into account all the possible consequences of her actions. However, in a behavioural decision problem, the individual doesn't fully internalize the possible consequences of her actions and as such imposes an externality on herself.

[4] Our framework is also related to dual-self problems proposed by Shefrin and Thaler (1988) and Bernheim and Rangel (2004), Benhabib and Bisin (2004) and Loewenstein and O'Donoghue (2005) among others, to the extent that these models account for the internal interaction between our "emotional" and our "rational" parts.

distinguishable. We then explore some specific outcomes of distinguishable decision problems and we argue that these outcomes have many features in common with two-player games[5].

We explore the welfare and normative implications of our framework. A key requirement in normative economics is that welfare rankings be grounded in individual preferences. To the extent to which individual behaviour reveals the preferences of individuals, it follows that such revealed preferences forms the basis for social welfare. The dilemma raised by behavioural economics is whether such revealed preferences can be used for making welfare judgements. In contrast to Bernheim and Rangel (2006) and Rubinstein and Salant (2007), with endogenous preference parameters, we show that in general, without further restrictions/information about the feedback effect, revealed preferences cannot be used for making welfare judgements.

Motivated by the literature of behavioural economics in which preferences may be neither transitive nor complete or convex, in Section 4 we offer an existence proof for both, standard and behavioural decision problems, which requires neither completeness or transitivity or convexity of preferences and action sets.

Section 5 explores the policy implications of our analysis. Our model provides a general theoretical foundation for policies that directly act on the way a person learns and becomes aware of her own feedback effect and how she eventually internalizes it. These policies include psychotherapy, empowerment, or projects that foster individual introspection or emotional intelligence. Besides suggesting new type of policies for the economics literature, our framework highlights some cases in which existing old policies fail. For example, a policy that provides more information or more opportunities to a decision maker who faces a behavioural decision problem could make her worse-off.

Finally, in an example that we interpret as a model for poverty traps, we work out (a) how extrinsic circumstances of the individual interact with her intrinsic motivation and choices, (b) the nature of policy interventions in such situations.

The remainder of the paper is organized as follows. Section 2 introduces the general model and clarifies the relationships between the framework developed here

[5]This is perhaps not surprising as the feedback effect can be thought of as the reduced form representation of the missing second player. However, we argue that behavioural decision problems in general cannot be obtained as reduced form representations of two-person normal formal games.

and other papers in the literature. Section 3 is devoted to an analysis of indistinguishability and welfare. Section 4 provides existence results for standard and behavioural decision problems with incomplete preferences. Section 5 discusses policy implications. The last section concludes and discusses directions for further research.

2 The model

There are two sets, a set $A \subset \Re^k$ of actions, and a set $P \subset \Re^n$ of preference parameters, where $\Re^k$ and $\Re^n$ are finite dimensional Euclidian spaces. A decision state is a pair of actions and preference parameters (a, p) where $a \in A$ and $p \in P$.

The preferences of the decision-maker are denoted by $\succeq$, a binary relation ranking pairs of decision states in $(A \times P) \times (A \times P)$. The expression $\{(a, p), (a', p')\} \in \succeq$ is written as $(a, p) \succeq (a', p')$ and is to be read as "(a, p) is weakly preferred to (equivalently, weakly welfare dominates) (a', p') by the decision-maker".

There is a map $\pi : A \to P$ modelling the feedback effect from actions to preference parameters and it is assumed that $\pi(a)$ is non-empty for each $a \in A$.

A consistent state is a decision state (a, p) such that $p \in \pi(a)$. Let $\pi(A) = \{p \in P : \exists a \in A \ s.t. \ p \in \pi(a)\}$. Then, $A \times \pi(A)$ is the set of consistent decision states.

There are two types of decision problems studied here:

1. A *standard decision problem* (S) is one where the decision-maker chooses a pair (a, p) within the set of consistent decision states. The outcomes of a standard decision problem are denoted by M where

$$M = \left\{(a, p) \in A \times \pi(A) : (a, p) \succeq \left(a', p'\right) \ for \ all \ \left(a', p'\right) \in A \times \pi(A)\right\}.$$

2. A *behavioural decision problem* (B) is one where the decision maker takes as given the preference parameter p when choosing a. Define a preference relation $\succeq_p$over A as follows:

$$a \succeq_p a' \Leftrightarrow (a, p) \succeq \left(a', p\right) \ for \ p \in P.$$

The outcomes of a behavioural decision problem are denoted by E where

$$E = \left\{(a, p) \in A \times \pi(A) : a \succeq_p a' \ for \ all \ a' \in A\right\}.$$

Suppose $P = A$ and $a \in \pi(a)$. In this case, the decision problems studied here offer a way of modelling situations where "the reference state usually corresponds to the decision maker's current state." (Tversky and Kahneman, 1991, p. 1046). The following examples show that whether the decision-maker correctly anticipates the feedback effect from actions to the reference state or not, will have an impact on the decision outcomes.

Example 1 ($M \subset E$)

Consider a decision problem where $A = P = \{a_1, a_2\}$, $\pi(a_i) = \{a_i\}, i = 1, 2$, and $(a_i, a_i) \succ (a_j, a_i)$, $j \neq i$ and $(a_1, a_1) \succ (a_2, a_2)$. Then, $M = \{(a_1, a_1)\}$ but $E = \{(a_1, a_1), (a_2, a_2)\}$.

Example 2 ($M \neq \phi$, $E \neq \phi$, $M \cap E = \phi$)

Consider a decision problem where $A = P = \{a_1, a_2\}$, $\pi(a_i) = \{a_i\}$, $i = 1, 2$, and $(a_2, a_j) \succ (a_1, a_j)$, $j = 1, 2$, and $(a_1, a_1) \succ (a_2, a_2)$. Then, $M = \{(a_1, a_1)\}$ but $E = \{(a_2, a_2)\}$.

Example 3 ($M \neq \phi$, $E = \phi$)

Consider a decision problem where $A = P = \{a_1, a_2\}$, $\pi(a_i) = \{a_i\}$, $i = 1, 2$, and $(a_j, a_i) \succ (a_i, a_i)$, $i \neq j$, and $(a_1, a_1) \succ (a_2, a_2)$. Then, $M = \{(a_1, a_1)\}$ but E is empty.

2.1 Reduced Form Representation

In this subsection, we show that the model studied here can be obtained as a reduced form representation of several distinctive types of models studied in the literature of behavioural economics.

2.1.1 *Psychological games with a single active player*

Geanakoplos, Pearce and Stacchetti (1989) (hereafter, GPS) study psychological games where the payoffs of each player depend not only on the actions chosen by all other players but also on what other players believe, on what she thinks they believe others believe and so on. Each player takes beliefs and actions of other players as given when choosing her own action. In equilibrium, beliefs are assumed to correspond to actions actually chosen. In the special case where there is a single active player, the payoffs of this single active player can depend on his own actions and the beliefs of other players over his own actions. Consider a two player psychological game. Player 1 is the active player with a set of pure actions S and mixed actions $\Sigma = \Delta(S)$. A belief for player 2 is denoted by $\bar{b} \in \bar{B} = \Sigma$. The payoffs of player 1 over pure actions is given by a utility function $u : A \times \bar{B} \rightarrow \Re$ with $v(\sigma, b) = \sum_{s \in S} \sigma(s) u(s, b)$ being the corresponding payoffs over mixed actions. A psychological equilibrium is a pair $\left(\hat{\sigma}, \hat{b}\right) \in \Sigma \times \bar{B}$ s.t. (i) $\hat{b} = \hat{\sigma}$, (ii) for each $\sigma \in \Sigma$, $u\left(\hat{\sigma}, \hat{b}\right) \geq u\left(\sigma, \hat{b}\right)$. Clearly, by setting $A = P = \Sigma$ and π as the identity map, a behavioural decision problem is a psychological game with one active player. GPS show that there are robust examples where the two sets M and E differ.

2.1.2 *Loss aversion games with a single player*

Shalev (2000) considers a class of games where players have reference dependent utilities and the reference utility depends on the action profile chosen by all players. Shalev defines two notions of equilibrium, a myopic loss aversion equilibrium and a non-myopic loss aversion equilibrium. In either equilibrium notion, each player takes as given the actions of others when choosing her actions. In a myopic loss aversion equilibrium, a player also takes as given the reference utility when choosing her actions (even though changing her actions might change the reference utility). In a non-myopic loss aversion equilibrium, a player takes into account the feedback effect from her actions to the reference utility when choosing her actions. A single player version of Shalev's model has the player choosing a mixed action $\sigma \in \Sigma$ with payoffs $w(\sigma, r) = \sum_{s \in S} \sigma(s) v(u(s), r)$ where

$$v(u(s), r) = \begin{cases} u(s) & \text{if } u(s) \geq r \\ u(s) - \lambda\left(r - u(s)\right) & \text{if } u(s) < r \end{cases},$$

r is the reference utility and $u : S \rightarrow \Re$ is a standard utility function. A consistent reference point r satisfies the equation $r = w(\sigma, r)$. Let $R(\sigma) = \{r \in \Re | r = w(\sigma, r)\}$. Shalev proves that $R(\sigma)$ is single valued and its values are contained in the closed interval $[\underline{r}, \overline{r}]$. Clearly, setting $A = \Sigma$, $P = [\underline{r}, \overline{r}]$ and $\pi(a) = R(\sigma)$, a behavioural decision problem is a myopic loss aversion decision problem while a non-myopic loss aversion decision problem corresponds to a standard decision problem. Shalev shows that in the static version of his model $M = E$ although the two sets differ in dynamic settings.

2.1.3 *Reference dependent consumption and personal equilibrium*

In Kozsegi and Rabin (2006), a person's utility depends not only on her K-dimensional consumption bundle, c, but also on a reference bundle, r. She has an intrinsic "consumption utility" $m(c)$ that corresponds to the standard outcome-based utility. Overall utility is given by $u(c|r) = m(c) + n(c|r)$, where $n(c|r)$ is "gain-loss utility." In their paper, both consumption utility and gain-loss utility are separable across dimensions, so that $m(c) = \sum_k m_k(c_k)$ and $n(c|r) = \sum_k n_k(c_k|r_k)$. They assume that $n_k(c_k|r_k) = \mu\left(m_k(c_k) - m_k(r_k)\right)$, where $\mu(.)$ satisfies the properties of Kahneman and Tversky's [1979] value function. Following Kozsegi (2005) they define a personal equilibrium as a situation where the optimal c computed conditional on forecasts of r coincides with r. Clearly, by setting A and P to be the set of feasible consumption bundles and π to be the identity map, a personal equilibrium can be represented by a behavioural decision problem[6]. However, under the assumptions made in their paper, Koszegi and Rabin (2006) show that in deterministic settings $M = E$ while the two sets differ in stochastic settings

2.1.4 *Aspiration traps*

Appadurai (2004) and Ray (2006) discuss the way an individual can fail to aspire. Based on their insights, Heifetz and Minelli (2007) study a model of aspiration traps where an individual in period $t = 0$ makes a choice $e \in E'$, at a cost $c(e)$. For a given choice e, the decision problem of the individual at $t = 1$ is described by the tuple $G_e = \left(X, u_e, \bar{B}\right)$ where the strategy set of the individual is X, her payoff function is $u_e : X \times \bar{B} \to \Re$, and the utility of the individual depends on her attitude (beliefs, aspirations) $b \in \bar{B}$. When choosing a strategy $x(e, b)$ at $t = 1$ to maximize u_e, the individual takes as given both b and e. However, given e, b is determined by some preference formation mechanism $\beta : E' \to \bar{B}$. At $t = 0$, Heifetz and Minelli consider two modes of choice. When choice is "transparent", the individual would "see through" the preference formation mechanism. At $t = 0$, she would then choose

[6] An analogous statement can be made for Kozsegi and Rabin (2007), since the solution concepts they use (i.e. unacclimating personal equilibrium, UPE and preferred personal equilibrium, PPE) are examples of a "personal equilibrium" defined in Koszegi (2005). The major feature of these solution concepts is that the decision maker does not internalize the effect of her choice on her expectations (or reference point).

e to maximize $u_e(x(e, \beta(e))) - c(e)$. When the individual choice is "self-justifying", her choice of e satisfies a no-regret condition

$$u_e(x(e, \beta(e))) - c(e) \geq u_e\left(x\left(e', \beta(e)\right)\right) - c\left(e'\right) \text{ for all } e' \in E'.$$

By setting $A = E'$, $P = \bar{B}$ and $\pi(a) = \beta(e)$, it is easily checked that a transparent choice problem corresponds to a standard decision problem while a self-justifying choice problem corresponds to a behavioural decision problem. Along the lines of example 1, they show that $M \subset E$.

2.1.5 *Exogenous frames or ancillary conditions*

Bernheim and Rangel (2006) (hereafter BR) and Rubinstein and Salant (2007) (hereafter RS) study decision problems where there is a set of actions A and frames (RS) or ancillary conditions (BR) P which determine choices in A. BR and RS argue that a standard choice situation corresponds to one where an individual chooses between elements in A. Both papers interpret elements in P as additional observable information (like "the order of candidates in a ballot box" (RS) or "the point of time at which a decision is made" (BR) which affects choices in A. Both papers make the point that, in practice, it is difficult to draw a distinction between characteristics of elements in A and variables in P which could also be viewed as characteristics of elements in A. The idea is that, in principle, the decision maker could eventually choose the frame or the ancillary condition. That is, she could eventually choose between generalized decision problems[7]. However, in any actual decision problem studied in their papers, an individual takes the frame or ancillary condition as given when choosing an action. Consistent with this interpretation, it is possible to relate the decision problems studied in BR and RS to those studied in the present paper, by assuming that $\pi(a) = P$ for all $a \in A$. With this interpretation in mind, differently from BR and RS, we argue that a standard choice situation is one where all characteristics of actions are taken into account and therefore, the decision-maker chooses a pair (a, p). Then, the outcomes of a standard decision problem corresponds to one where the objects of choice are any pair $(a, p) \in A \times P$ while a behavioural decision

[7] A generalized decision problem is what RS call an "extended choice function" and BR a "generalized choice function"

problem is one where the objects of choice are $a \in A$ taking as given $p \in P$. With this interpretation, when $\pi(a) = P$ for all $a \in A$, a decision problem with exogenous frames or ancillary conditions corresponds to a behavioural decision problem.

3 Indistinguishability and Welfare

In this section, we first state the conditions under which behavioural and standard decision problems are indistinguishable from each other. Further, we show that in smooth settings, both decision problems are generically distinguishable. We explore some peculiar outcomes of distinguishable decision problems. Finally, we derive necessary and sufficient conditions under which it is appropriate to use revealed preferences for welfare analysis.

3.1 Indistinguishability

A behavioural decision problem is indistinguishable from a standard decision problem if and only if $M = E$. Otherwise, a behavioural decision problem is distinguishable from a standard one.

Note that indistinguishability is, from a normative viewpoint, a compelling property. What matters for welfare purposes is the ranking of consistent decision states. When $M = E$, the outcomes (consistent decision states) of a standard decision problem coincide with that of a behavioural decision problem.

If $\pi(a) = \pi(a')$ for all $a, a' \in A$, a behavioural decision problem is, by construction, indistinguishable from a standard decision problem. So suppose the map π has at least two distinct elements in its range.

Consider the following conditions on preferences:

$(C1)$ for $a, a' \in A$ such that $a \succeq_p a'$ for some $p \in \pi(a)$, $(a, p) \succeq (a', p')$ for each $p \in \pi(a)$ and $p' \in \pi(a')$;

$(C2)$ for $(a, p), (a', p') \in A \times \pi(A)$ such that $(a, p) \succeq (a', p')$, $(a, p) \succeq (a', p)$ for some $p \in \pi(a)$.

Note that the preferences in example 1 violate $(C1)$ but satisfy $(C2)$ while the preferences in example 2 violate both $(C1)$ and $(C2)$. Shalev (2000) shows (in Proposition 1 of his paper) that in the static case his loss averse preferences satisfy both $(C1)$ and $(C2)$. Rabin and Koszegi (2006) show that their reference dependent preferences also satisfy both $(C1)$ and $(C2)$. GPS construct examples where, with one active player, both $(C1)$ and $(C2)$ are violated. Heifetz and Minelli (2007) construct examples where $(C1)$ is violated.

In the choice frameworks developed by BR and RS, when $\pi(a) = P$ for all $a \in A$, $(C1)$ guarantees the existence of revealed preferences namely, a binary relation over A so that for any $p \in P$, and $a, a' \in A$, a is preferred to a' in any choice problem where the only two alternatives are $\{a, a'\}$.

In the following proposition, we state that $(C1)$ and $(C2)$ are the necessary and sufficient conditions for indistinguishability.

PROPOSITION 1. Suppose that both E and M are non-empty. Then, (i) $E \subseteq M$ if and only if $(C1)$ holds. (ii) $M \subseteq E$ if and only if $(C2)$ holds.

Proof: (i) Suppose $(a, p) \in E$. By definition, for all $a' \in A$, $a \succeq_p a'$ for some $p \in \pi(a)$. By $(C1)$, for all $a' \in A$, $(a, p) \succeq (a', p')$ for each $p \in \pi(a)$ and $p' \in \pi(a')$. It follows that $(a, p) \in M$. Next, suppose, by contradiction, $(a, p) \in E \cap M$ but $(C1)$ doesn't hold. As $(a, p) \in E$, for all $a' \in A$, $a \succeq_p a'$ for some $p \in \pi(a)$. As, by assumption, $(C1)$ doesn't hold there exists $a' \in A$ such that $a \succeq_p a'$ for all $p \in \pi(a)$ but $(a, p) \prec (a', p')$ for some $p \in \pi(a)$ and $p' \in \pi(a')$. But, then, $(a, p) \notin M$, a contradiction. (ii) Suppose $(a, p) \in M$. As $(a, p) \succeq (a', p')$ for all $(a', p') \in A \times \pi(A)$, by $(C2)$, $(a, p) \succeq (a', p)$ for some $p \in \pi(a)$. It follows that $(a, p) \in E$. Next, suppose, by contradiction, $(a, p) \in M \cap E$ but $(C2)$ doesn't hold. As $(a, p) \in M$, $(a, p) \succeq (a', p')$ for all $(a', p') \in A \times \pi(A)$. As, by assumption, $(C2)$ doesn't hold, there exists $a' \in A$ such that $a' \succ_p a$ for some $p \in \pi(a)$. But, then, $(a, p) \notin E$, a contradiction.■

3.2 Smooth Decision Problems

To further understand the conditions under which indistinguishability occurs, it is convenient to look at smooth decision problems where decision outcomes are characterized by first-order conditions. We show that for the case of smooth decision problems, behavioural decisions are generically *distinguishable* from standard decisions.

A decision problem is smooth if (a) both A and P are convex, open sets in $\Re^k$

and $\Re^n$ respectively, (b) preferences over $A \times P$ are represented a smooth, concave utility function $u : A \times P \to \Re$ and (c) the feedback map $\pi : A \to P$ is also smooth and concave.

A set of decision problems that satisfies the smoothness assumptions is *diverse* if and only if for each $(a, p) \in A \times P$ it contains the decision problem with utility function and feedback effect defined, in a neighborhood of (a, p), by

$$u + \lambda p$$

and

$$\pi - \mu(a' - a)$$

for each a' in a neighborhood of a and for parameters (λ, μ) in a neighborhood of 0.

A property holds generically if and only if it holds for a set of decision problems of full Lebesgue measure within the set of diverse smooth decision problems.

PROPOSITION 2: For a diverse set of smooth decision problems, a standard decision problem is generically distinguishable from a behavioural decision problem.

Proof. Let $v(a) = u(a, \pi(a))$. The outcome of a standard decision problem $(\hat{a}, \hat{p})$ satisfies the first-order condition

$$\partial_a v(\hat{a}) = \partial_a u(\hat{a}, \pi(\hat{a})) + \partial_p u(\hat{a}, \pi(\hat{a}))\partial_a \pi(\hat{a}) = 0 \qquad (1)$$

while the outcome of a behavioural decision problem (a^*, p^*) satisfies the first-order condition

$$\partial_a u(a^*, p^*) = 0, p^* = \pi(a^*). \qquad (2)$$

For $(a^*, p^*) = (\hat{a}, \hat{p})$, it must be the case that

$$\partial_p u(a^*, p^*)\partial_a \pi(a^*) = 0. \qquad (3)$$

It is easily checked that requiring both $(C1)$ and $(C2)$ to hold is equivalent to requiring that the preceding equation also holds. Consider a decision problem with $(a^*, p^*) = (\hat{a}, \hat{p})$. Perturbations of the utility function and the feedback effect do not affect (2) and hence (a^*, p^*) but they do affect (3) and via (1) affect $(\hat{a}, \hat{p})$. Therefore, $(a^*, p^*) \neq (\hat{a}, \hat{p})$ generically. ■

3.3 Distinguishable Decision Problems

In this part, we present a series of examples illustrating the kinds of behavior possible when a behavioural decision problem is *distinguishable* from a standard decision problem. Clearly examples *1-3* already demonstrate that behavioural decision outcomes have properties normally associated with two-person normal form games. Below we present a few more examples of behavioural decision making with features similar to two-person normal form games. In all these examples, we assume for simplicity that $A = P$ are finite sets and $\pi(a)$ is the identity map. The preferences of the decision maker are represented by an utility function $u : A \times P \to \Re$. We distinguish between pure and random behavioural decisions. Let $\alpha(\hat{a}) = \arg\max_{a\in A} u\left(a, \hat{a}\right)$. A **pure action behavioural equilibrium** is an action profile a^* such that $a^* \in \alpha(a^*)$. Let $\Delta(A)$ denote the set of probability distributions over the set of actions. A random strategy is $\sigma \in \Delta(A)$, where $\sigma(a)$ is the probability attached to action a. A random distribution over the set of parameters is $\mu \in \Delta(A)$, where $\mu(\hat{a})$ is the probability attached to utility parameter $\hat{a}$. A random decision state is a pair (σ, μ). Given a random decision state (σ, μ), the payoff to the decision maker is

$$w(\sigma, \mu) = \sum_{a\in A}\sum_{p\in P} \sigma(a)\mu(\hat{a})u(a, \hat{a})$$

A consistent random decision state is a pair (σ, μ) where $\mu = \sigma$. A **random behavioural equilibrium** is a profile σ^* such that $\sigma^* \in \arg\max_{\sigma\in\Delta(A)} w(\sigma, \sigma^*)$.

In each example, the decision problem is represented by a payoff table where rows are actions and columns are the utility parameters. Under the assumptions made so far, consistent decision states are the diagonal of these payoff tables.

Example 4. *Unique inefficient behavioural equilibrium in dominant actions*
Consider the following payoff table:

	a_1	a_2
a_1	1	-1
a_2	2	0

(Table 1)

Notice that a_2 is dominant action for both values of p. The unique behavioural equilibrium is (a_2, a_2) with a payoff of 0. However, note that there is a consistent decision state (a_1, a_1) with a payoff of 1 and therefore, (a_2, a_2) isn't efficient. Finally, note that the unique inefficient behavioural equilibrium in dominant actions is robust to arbitrary but small perturbations in payoffs.

Example 5. *Unique random equilibrium*

	a_1	a_2
a_1	0	1
a_2	1	0

(Table 2)

Notice that when the utility parameter is a_1, the decision-maker prefers a_2 to a_1 while when the utility parameter is a_2, the decision-maker prefers a_1 to a_2. Therefore, there is no behavioural equilibrium in pure strategies. However, there is a behavioural equilibrium in mixed strategies. It follows that there is a unique random outcome in the payoff table 2, $(\frac{1}{2}a_1 + \frac{1}{2}a_2, \frac{1}{2}a_1 + \frac{1}{2}a_2)$.

Example 6. *Equilibrium in weakly dominated actions and domination by random actions*

	a_1	a_2	a_3
a_1	0	0	0
a_2	0	1	2
a_3	0	2	1

(Table 3)

In this example, there are two behavioural equilibria, one in pure actions, (a_1, a_1) and the other random, $(\frac{1}{2}a_2 + \frac{1}{2}a_3, \frac{1}{2}a_2 + \frac{1}{2}a_3)$. Note that in the pure action equilibrium (a_1, a_1) the decision-maker is choosing a weakly dominated action and at the random equilibrium $(\frac{1}{2}a_2 + \frac{1}{2}a_3, \frac{1}{2}a_2 + \frac{1}{2}a_3)$, the decision-maker is strictly better off than at (a_1, a_1). Note also that there is no pure action that (strictly) dominates a_1 although there are a continuum of random actions $qa_2 + (1 - q)\, a_3$, $0 < q < 1$, that strictly dominates a_1.

Example 7. *Multiple welfare ranked equilibria in undominated actions*

	a_1	a_2
a_1	1	0
a_2	0	2

(Table 4)

In this example, there are two behavioural equilibria in pure undominated actions (a_1, a_1) and (a_2, a_2). Note that the pure action equilibrium (a_1, a_1) is dominated by the pure action equilibrium (a_2, a_2). Note also that there is a random behavioural equilibrium $\left(\frac{2}{3}a_1 + \frac{1}{3}a_2, \frac{2}{3}a_1 + \frac{1}{3}a_2\right)$. However, in addition, sunspots (i.e. payoff-irrelevant events) may play a role in decision-making. Suppose there are two payoff irrelevant states of the world $\{s_1, s_2\}$ with an associated probability distribution $\{\alpha, 1-\alpha\}$. Suppose the decision-maker observes the realization of the sunspot variable before choosing her action. Then, for example, the decision-maker could choose (a_1, a_1) conditional on observing s_1 and (a_2, a_2) conditional on observing s_2 thus obtaining, in expected utility, payoffs in the interval $[1, 2]$. Note that all these features are robust to arbitrary but small perturbations in payoffs.

Example 8. *Larger action sets may make the decision-maker worse-off*

Consider first a situation where the payoff table is

	a_1	a_2
a_1	-1	0
a_2	0	3

(Table 5 (A))

In this case, the decision-maker has a unique efficient undominated action a_2 and there exists a corresponding outcome of the behavioural decision problem (a_2, a_2) with payoff 3. Now, expand the set of choices so that the following payoff table represents the decision problem

	a_1	a_2	a_3
a_1	-1	0	0
a_2	0	3	1
a_3	1	4	2

(Table 5 (B))

In this case, note that a_2 continues to strictly dominate a_1 although now a_3 strictly dominates both a_1 and a_2. The unique behavioural equilibrium is (a_3, a_3) with payoff $2 < 3$. This means that although the action set of the decision-maker has been expanded so that (a) the ranking of existing actions is unaffected and (b) the new action strictly dominates all existing actions, the individual is made worse-off. Note that all these features are robust to arbitrary but small perturbations in payoffs.

Example 9. *More information may make the decision-maker worse-off*

Consider a decision problem with payoff relevant uncertainty, with two states of the world $\{\theta_1, \theta_2\}$ where the payoff tables are

$\theta_1 \rightarrow$

	a_1	a_2	a_3
a_1	-1	0	0
a_2	0	3	$\frac{1}{2}$
a_3	1	4	1

(Table 6 (A))

$\theta_2 \rightarrow$

	a_1	a_2	a_3
a_1	1	4	1
a_2	$\frac{1}{2}$	3	0
a_3	0	0	-1

(Table 6 (B))

Suppose, to begin with, the decision-maker has to choose before uncertainty is resolved. At the time when she makes the decision, the individual attaches a probability $\frac{1}{2}$ to θ_1 and $\frac{1}{2}$ to θ_2. In this case, expected payoff matrix is

	a_1	a_2	a_3
a_1	0	2	$\frac{1}{2}$
a_2	$\frac{1}{4}$	3	$\frac{1}{4}$
a_3	$\frac{1}{2}$	2	0

(Table 6 (C))

It follows that the unique behavioural equilibrium is (a_2, a_2) with expected payoff 3.

Next, suppose that the decision-maker knows with probability one the true state of the world. Then, when the state of the world is θ_1, a_3 strictly dominates all other actions and the unique behavioural equilibrium is (a_3, a_3) with payoff 1 and when the state of the world is θ_2, a_1 strictly dominates all other actions and the unique behavioural equilibrium is (a_1, a_1) with payoff 1. Therefore, the decision-maker is worse-off with more information[8][9].

[8] Note that in this example we are referring only to information that solves the uncertainty about exogenous states of the world. Our statement "the decision-maker is worse-off with more information" would not be right in the case in which additional information helps the decision-maker to be aware of her own feedback effect and to internalize it.

[9] This result is consistent with Carrillo and Mariotti's (2000) results, although they use a dynamic model with time-inconsistent preferences.

Example 10. *Rationality versus bounded rationality*

Consider the payoff table in Table 2. In that example, if the decision maker took into account the feedback effect from actions to the utility parameter and maximized the induced utility function $v(.)$, $v(a_1) = v(a_2) = 0$. Therefore, a fully rational decision-maker who takes into account all the consequences of her actions would obtain a payoff of 0. However when the decision-maker doesn't take this feedback effect into account, we have already seen that there is a unique random outcome of the behavioural decision problem $(\frac{1}{2}a_1 + \frac{1}{2}a_2, \frac{1}{2}a_1 + \frac{1}{2}a_2)$ with an expected payoff of $\frac{1}{2} > 0$. On the face of it, it would seem that a boundedly rational decision-maker will be *better-off* than a fully rational decision-maker. But this interpretation isn't strictly true. In fact, if a fully rational decision maker is also allowed to choose mixed strategies in the payoff matrix in Table 2, she will also randomize $\{a_1, a_2\}$ by choosing the probability distribution $\left\{\frac{1}{2}, \frac{1}{2}\right\}$.

In a behavioural decision problem, the individual imposes an externality on herself that she doesn't fully internalize. Does it follow from this remark that behavioural decision problems are reduced form representations of two person normal formal games? The answer, in general, is no. In all the above examples, as π is the identity map, it follows that the best response of the "missing" second player in pure actions and mixed actions must also be the identity map. However, any best response that attaches positive probability to two or more pure actions can never be single-valued: all pure strategies in the support of mixed strategy must give a player the same (expected) utility and if a mixed strategy is a best-response, any way of randomizing over the support of that mixed strategy must also be a best-response.

3.4 Revealed Preferences and Welfare

The framework studied here suggests that, in general, welfare rankings should take place over consistent decision states $A \times \pi(A)$. In a recent paper, Bernheim and Rangel (2006) define an action a to be a *weak welfare optimum* if and only if for each $a' \in A$ (other than a), a is chosen with a' present (a' may be chosen as well). They also define a *strict welfare optimum* as an action a if and only if for each $a' \in A$ (other than a), either a is chosen and a' is not or it is never the case that a' is chosen and a is not with a present. Their definitions make a clear link between revealed preferences and welfare.

What matters for welfare purposes is the ranking of consistent decision states. The issue is whether revealed preferences over actions can be used to rank consistent decision states as well. The following example shows that this isn't always the case.

Example 11. In examples *2* and *4*, where $\pi(a) = a$ for all $a \in A$, a_2 is always chosen and a_1 is never chosen. Therefore, a_2 is a strict (and hence, weak) welfare optimum as defined by Bernheim and Rangel (2006). However, the decision state (a_2, a_2) is dominated by (a_1, a_1) and so the individual's revealed preferences over actions cannot be used to rank consistent decision states and it is this latter raking what matters for welfare assessments.

As already pointed out, Bernheim and Rangel (2006) work in settings where $\pi(a) = P$ for all $a \in A$. If we look at examples *2* and *4*[10] with Bernheim and Rangel's (2006) lenses, any pair (a', p) is a consistent decision state. Therefore, if a is always chosen over other actions, the individual's revealed preferences over actions can be used to rank consistent decision states.

However, without further restrictions/information about the map π, in general, revealed preferences over actions cannot be used to rank consistent decision states and therefore, cannot be used for making welfare assessments. The following proposition states a necessary and sufficient condition for revealed preferences to rank consistent decision states.

PROPOSITION 3. Let $a \in A$ be a weak welfare optimum. Then, any consistent decision state containing a, weakly welfare dominates any other decision state containing $a' \neq a$, $a' \in A$ if only if $(C1)$ holds.

Proof: Suppose for each $a' \in A$ (other than a), a is chosen with a' present (a' may be chosen as well). By assumption, for all $a' \in A$, $a \succeq_p a'$ for some $p \in \pi(a)$. By $(C1)$, for all $a' \in A$, $(a, p) \succeq (a', p')$ for each $p \in \pi(a)$ and $p' \in \pi(a')$. It follows that any consistent decision state containing a weakly welfare dominates any other decision state containing $a' \neq a$, $a' \in A$. Next, suppose, by contradiction, for each $a' \in A$ (other than a), a is chosen with a' present (a' may be chosen as well), but $(C1)$ doesn't hold. By assumption, for all $a' \in A$, $a \succeq_p a'$ for some $p \in \pi(a)$. As $(C1)$ doesn't hold, there exists $a' \in A$ such that $a \succeq_p a'$ for all $p \in \pi(a)$ but

[10] Recall that, in the examples, $\pi(a) = a$ for all $a \in A$.

$(a,p) \prec (a',p')$ for some $p \in \pi(a)$ and $p' \in \pi(a')$, a contradiction. ■

4 Existence Results

It is not hard to check that as long as both A and P are finite and $\pi(a)$ is single-valued for each $a \in A$, a random equilibrium exists.

Instead, in this section, we study existence in situations where the underlying preferences are not necessarily complete or transitive and underlying action sets are not necessarily convex. Mandler (2005) shows that incomplete preferences and intransitivity is required for "status quo maintenance" (encompassing endowment effects, loss aversion and willingness to pay-willingness to accept diversity) to be outcome rational. Tversky and Kahneman (1979, 1991) argue that reference dependent preferences may not be convex. In this section, we allow preferences to be incomplete, non-convex and acyclic (and not necessarily transitive) and we show existence of a behavioural equilibrium in pure actions extending Ghosal's (2007) result for normal form games to behavioural decision problems[11]. Throughout this section, it will be assumed that $\pi(a)$ is non-empty and closed relative to P for each $a \in A$.

Recall that the preferences of the decision-maker is denoted by $\succeq$ a binary relation ranking pairs of decision states in $(A \times P) \times (A \times P)$. As the focus is on incomplete preferences, in this section, instead of working with $\succeq$, we find convenient to specify two other preference relations, $\succ$ and $\sim$. The expression $\{(a,p),(a',p')\} \in \succ$ is written as $(a,p) \succ (a',p')$ and is to be read as "(a,p) is strictly preferred to (a',p') by the decision-maker". The expression $\{(a,p),(a',p')\} \in \sim$ is written as $(a,p) \sim (a',p')$ and is to be read as "(a,p) is indifferent to (a',p') by the decision-maker". Define

$$(a,p) \succeq (a',p') \Leftrightarrow \textit{either } (a,p) \succ (a',p') \textit{ or } (a,p) \sim (a',p').$$

Once $\succeq$ is defined in this way, the results obtained in the preceding sections continue to apply.

[11] The seminal proof for existence of equilibria with incomplete preferences in Shafer and Sonnenschein (1975) requires convexity both for showing the existence of an optimal choice and using Kakutani's fix-point theorem.

Suppose $\succ$ is (i) acyclic i.e. there is no finite set $\left\{(a^1,p^1),...,(a^T,p^T)\right\}$ such that $(a^{t-1},p^{t-1}) \succ (a^t,p^t)$, $t = 2,...,T$, and $(a^T,p^T) \succ (a^1,p^1)$, and (ii) $\succ^{-1}(a,p) = \{(a',p') \in A \times P : (a,p) \succ (a',p')\}$ is open relative to $A \times P$ i.e. $\succ$ has an open lower section[12]. Suppose both A and P are compact. Then, by Bergstrom (1975), it follows that M is non-empty.

Define

$$a \succ_p a' \Leftrightarrow (a,p) \succ (a',p).$$

The preference relation $\succ_p$is a map, $\succ: P \to A \times A$. If $\succ$ is acyclic, then for $p \in P$, $\succ_p$is also acyclic. If $\succ$ has an open lower section, then $\succ_p^{-1}(a) = \{a' \in A : a \succ a'\}$ is also open relative to A i.e. $\succ_p$ has an open lower section. In what follows, we write $a' \notin \succ_p (a)$ as $a \not\succ_p a'$ and $a' \in \succ_p (a)$ as $a' \succ_p a$.

Define a map $\Psi : P \to A$, where $\Psi(p) = \{a' \in A : \succ_p (a') = \phi\}$: for each $p \in P$, $\Psi(p)$ is the set of maximal elements of the preference relation $\succ_p$.

We make the following additional assumptions:

(A1) A is a compact lattice;

(A2) For each p, and a, a', (i) if $\inf(a,a') \not\succ_p a$, then $a' \not\succ_p \sup(a,a')$ and (ii) if $\sup(a,a') \not\succ_p a$ then $a' \not\succ_p \inf(a,a')$ (quasi-supermodularity);

(A3) For each $a \geq a'$ and $p \geq p'$, (i) if $a' \not\succ_{p'} a$ then $a' \not\succ_p a$ and (ii) if $a \not\succ_p a'$ then $a \not\succ_{p'} a'$ (single-crossing property)[13]

(A4) For each p and $a \geq a'$, (i) if $\succ_p (a') = \phi$ and $a' \not\succ_p a$, then $\succ_p (a) = \phi$ and (ii) if $\succ_p (a) = \phi$ and $a \not\succ_p a'$, $\succ_p (a') = \phi$ (monotone closure).

Assumptions (A2)-(A3) restate, for the case of incomplete preferences, the assumptions of quasi-supermodularity and single-crossing property defined by Milgrom and Shannon (1994). Assumption (A4) is new. The role played by assumption (A4) in obtaining the monotone comparative statics with incomplete preferences is clarified by the examples shown in the Appendix. The following result shows that

[12] The continuity assumption, that $\succ$ has an open lower section, is weaker than the continuity assumption made by Debreu (1959) (who requires that preferences have both open upper and lower sections), which in turn is weaker than the assumption by Shafer and Sonnenschein (1975) (who assume that preferences have open graphs). Note that assuming $\succ$ has an open lower section is consistent with $\succ$ being a lexicographic preference ordering over $A \times P$.

[13] For any two vectors $x, y \in \Re^K$, the ussual component-wise vector ordering is defined as follows: $x \geq y$ if and only if $x_i \geq y_i$ for each $i = 1,..,K$, and $x > y$ if and only if both $x \geq y$ and $x \neq y$, and $x \gg y$ if and only if $x_i > y_i$ for each $i = 1,..,K$.

assumptions (A1)-(A4), taken together, are sufficient to ensure monotone comparative statics with incomplete preferences and ensure the non-emptiness of E.

PROPOSITION 4: Under assumptions (A1)-(A4), each $p \in P$, $\Psi(p)$ is non-empty and a compact sublattice of A where both the maximal and minimal elements, denoted by $\bar{a}(p)$ and $\underline{a}(p)$ respectively, are increasing functions on P. Moreover, $E \neq \phi$.

Proof: See Appendix

A different approach to the existence of equilibrium would be to deduce the existence result for games with incomplete preferences from the standard existence result for games with complete preferences as in Bade (2005). We refer the reader to Ghosal (2007) for details as to why this approach will not work, in general, with the incomplete preferences.

Schofield (1984) shows that if action sets are convex or are smooth manifolds with a special topological property, the (global) convexity assumption made by Shafer and Sonnenschein (1975) can be replaced by a "local" convexity restriction, which, in turn, is equivalent to a local version of acyclicity (and which guarantees the existence of a maximal element). However, here, as action sets are not necessarily convex and are allowed to be a collection of discrete points, Schofield's equivalence does not apply.

5 Policy Implications

5.1 Internalizing the Feedback Effect

A policy intervention for efficiency purposes is only justifiable when both decision problems are distinguishable from each other. One possible policy recommendation for those cases is to directly act on the way the person learns and become aware of her own feedback effect and how to eventually internalize it[14]. In Examples 4 and 6-9, this type of policy would change the behavioural decision problem to a standard decision problem, the "now sophisticated" decision maker would choose among consistent decision states, and as a consequence, she would achieve efficient

[14] An example of such policies could be psychotherapy sessions, projects aiming to foster people's emotional intelligence and empowerment, etc.

outcomes. For instance, we can think of Example 4 as representing addiction: if the individual doesn't take the feedback effect from actions to psychological states into account, she always chooses a_2 *(smoking)* over a_1 *(not smoking)*; however, the reverse would be true, if she took the feedback into account[15].

Likewise, we can interpret Example 7 as an example of self-confidence and aspirations formation. Let a_1=*not going to school* and a_2=*going to school*, with *"low aspirations"* and *"high aspirations"* being the consistent psychological states associated with "*not going to school*" and "*going to school,*" respectively. When decision-maker's aspirations are high, she prefers "*going to school*" $(a_2, a_2) \succ (a_1, a_2)$, while when her aspirations are low, she prefers "*not going to school*" $(a_2, a_1) \succ (a_1, a_1)$. A policy consistent with this type of situations may be an "empowerment" policy, that would help the individual to become aware of her "internal constrains" and thus "gaining control over her own life[16]."

Another added value of our framework is that besides suggesting new type of policies to unsolved economic problems, it also highlights the cases in which existing old policies fail. For instance, as it is illustrated in Example 9, a policy that provides information that reduces the uncertainty of a decision maker facing a behavioural decision problem could make her worse-off. This decision maker will be also worse-off with a policy that increases her opportunity set (as suggested in Example 10). As a corollary, we learn that for a policy to be successful, it should firstly address the type of decision problem faced, and only then, design the appropriate intervention accordingly.

5.2 Poverty Traps

Another type of policy recommendation, this time only consistent with scenarios of multiple behavioural equilibria (Example 7), would be to affect exogenous variables associated with potentially endogenous preference parameters. In many cases, psychological states do not only depend on decision makers' own actions, but also on her extrinsic circumstances (e.g. relative status, social exclusion, poverty, etc.). Appadurai (2004) and Ray (2006), for example, provide an analysis of the nega-

[15] See fo example, Bernheim and Rangel (2004) or Gul and Pesendorfer (2007) for different models and policy discussion on addiction.

[16] See for instance Stern (2004) or World Bank (2002) for a reference on Empowerment.

tive impact of persistent poverty on the "capacity to aspire" and the key role that aspirations play for the poor to alter the conditions of their own poverty. In the remainder of this section, we work out an example of how extrinsic circumstances of the individual interact with her intrinsic motivation and choices[17].

This issue is clearly important for policy purposes: when should policy address the extrinsic circumstances of an individual (like initial wealth social status, health) and when should it address her aspirations? We provide answers for these questions.

Consider an individual whose decision problem involves the following payoff-relevant variables:

(i) a set of actions $A = \{\underline{a}, \overline{a}\}$, $\underline{a} < \overline{a}$, where $\underline{a}$ represents maintaining the existing status quo and $\overline{a}$ represents changing the existing status quo by undertaking higher effort (working harder at school or undertaking additional training), embarking on a new project, etc.;

(ii) a set of extrinsic circumstances Θ where $\theta \in \Theta$ represents the initial wealth or social status or state of health or location or level of nutrition of the individual;

(iii) a set of utility parameters P where $p \in P$ represents the intrinsic motivation or level of confidence or aspirations of the individual.

Assume that both Θ and P are intervals in $\Re$. For concreteness, assume that the extrinsic circumstances of the individual represent her social status. The individual can potentially improve her initial level of social status by choosing effort. Her new social status, $\tilde{\theta}$, is generated by the map $s : A \times \Theta \to \Theta$. Assume that $s(a, \theta)$ is increasing in a and θ, effort is costly and the individual derives benefit from $\tilde{\theta}$. Higher values of p correspond to higher levels of motivation and lowers effort costs. Assume that the preferences of the individual can be represented by a utility function $v(a, p, \tilde{\theta}) = b(\tilde{\theta}) - c(a, p) = b(s(a, \theta)) - c(a, p)$ where $b(\tilde{\theta})$ is the benefit the individual obtains from her new social status and $c(a, p)$, the cost of effort, which is decreasing in p but increasing in a.

As before, assume that there is a feedback effect from actions into intrinsic motivation captured by the map $\pi : \{\underline{a}, \overline{a}\} \times \Theta \to P$ where, now, the intrinsic motivation of the individual depends not only her action but also on her initial social status. Assume that $\pi(a, \theta)$ is increasing in a and θ.

[17] Benabou and Tirole (2003) also study intrinsic motivations but they focus on their interaction with extrinsic incentives.

Let $u(a,p,\theta) = b(s(a,\theta)) - c(a,p)$. For simplicity, assume that $u(\underline{a},p,\theta)$, the individual's utility from preserving the status quo, is normalized to zero for all values of p,θ and for each p,θ, $u(\overline{a},p,\theta)$ is the net gain (or loss) to the individual in deviating from the status quo. Then, under the assumptions made so far, it is easily verified that

(i) for each θ, $u(\overline{a},p',\theta) > u(\overline{a},p,\theta)$, $p' > p$,

(ii) for each p, $u(\overline{a},p,\theta') > u(\overline{a},p,\theta)$, $\theta' > \theta$.

In addition, assume that $u(\underline{a},p,\theta)$ is continuous in p,θ.

Under the assumptions made above, there is a complementarity between actions, the motivation of an individual (with higher values of p representing higher motivational states) and her extrinsic circumstances (with higher values of θ representing more favorable extrinsic circumstances).

For each p,θ, the individual solves the maximization problem

$$\max_{a\in A} u(a,p,\theta)$$

This generates an optimal action correspondence $\alpha(p,\theta)$ and given θ, (a^*,p^*) is a behavioural equilibrium if (i) given θ,p^*, $a^* \in \alpha(p,\theta)$ and (ii) $p^* \in \pi(a^*,\theta)$.

Under our assumptions there is a unique solution, $\hat{p}(\theta)$, to the equation $u(\overline{a},p,\theta) = 0$ with $\hat{p}(\theta)$ decreasing in θ. Given θ, p, the optimal action correspondence of the individual is determined as follows:

(i) whenever $p < \hat{p}(\theta)$, $\underline{a} = \alpha(p,\theta)$;

(ii) whenever $p > \hat{p}(\theta)$, $\overline{a} = \alpha(p,\theta)$;

(iii) whenever $p = \hat{p}(\theta)$, $\{\underline{a},\overline{a}\} = \alpha(p,\theta)$.

Given θ, let $\underline{p}(\theta) = \pi(\underline{a},\theta)$ and $\overline{p}(\theta) = \pi(\overline{a},\theta)$. Note that $\underline{p}(\theta) < \overline{p}(\theta)$.

Let $\underline{\Theta} = \{\theta : \overline{p}(\theta) < \hat{p}(\theta)\}$, $\overline{\Theta} = \{\theta : \hat{p}(\theta) < \underline{p}(\theta)\}$, and $\Theta_M = \{\theta : \underline{p}(\theta) \leq \hat{p}(\theta) \leq \overline{p}(\theta)\}$. Assume that all the three sets $\underline{\Theta}, \overline{\Theta}$ and Θ_M are non-null subsets of Θ. By computation, it follows that

(i) when $\theta \in \underline{\Theta}$, the unique behavioural equilibrium is $(\underline{a},\underline{p}(\theta))$;

(ii) when $\theta \in \overline{\Theta}$, the unique behavioural equilibrium is $(\overline{a},\overline{p}(\theta))$;

(iii) when $\theta \in \Theta_M$, there are two behavioural equilibria, $(\underline{a},\underline{p}(\theta))$ and $(\overline{a},\overline{p}(\theta))$.

Call $(\underline{a},\underline{p}(\theta))$ a type I equilibrium and $(\overline{a},\overline{p}(\theta))$ a type II equilibrium. In a type I equilibrium, there is no change in the status quo while in a type II equilibrium there is a change in the status quo. When a type II equilibrium exists, the individual is always better off at the type II equilibrium decision state relative to the status quo. When both type I and type II equilibria exist, as the type II equilibrium dominates the status quo, a type I equilibrium can be interpreted as an aspirations failure, a low motivation trap for the individual.

The set of equilibria is "weakly increasing" in θ. For an individual of low social status with low θ, the unique equilibrium is type I while for an individual with high social status with high θ the unique equilibrium is type II. For an individual in the middle, with intermediate values of θ, there are multiple welfare ranked equilibria and for such an individual, the theoretical framework developed so far doesn't pin down the equilibrium decision state i.e. the aspirations and choices is indeterminate.

In order get round this difficulty, we develop an equilibrium selection argument that assigns a probability to each behavioural equilibrium as a function of θ[18].

Fix $\theta \in \Theta_M$ and consider the following adaptive dynamics over p:

Step 1: The initial psychological state of an individual is picked at random from $\left[\underline{p}, \overline{p}\right]$ according to some continuous pdf $f(p)$ (with associated cdf $F(p)$)

Step 2: Given the p, the individual chooses $\alpha(p, \theta) \subset \{\underline{a}, \overline{a}\}$ which, in turn, generates a new $p' = \pi(a, \theta)$, for some $a \in \alpha(p, \theta)$.

Note that the above adaptive dynamics will always converge to either a type I or a type II equilibrium. Further, note that the basin of attraction for a type I equilibrium is $[\underline{p}, \hat{p}(\theta))$ while the basin of attraction for a type II equilibrium is $(\hat{p}(\theta), \overline{p}]$. Therefore, the probability that the dynamics will converge to a type I equilibrium is $F(\hat{p}(\theta))$ while the probability that the dynamics will converge to a type II equilibrium is $1 - F(\hat{p}(\theta))$. As $\hat{p}(\theta)$ is decreasing in θ, it follows that there exists a $\hat{\theta}$ such that whenever (a) $\theta < \hat{\theta}$, $F(\hat{p}(\theta)) > \frac{1}{2}$ and a type I equilibrium while will have a higher probability of emerging while (b) $\theta > \hat{\theta}$, $F(\hat{p}(\theta)) < \frac{1}{2}$ and a type II equilibrium while will have a higher probability of emerging.

The preceding discussion can be summarized in the following proposition:

PROPOSITION 5: When multiple welfare ranked behavioural equilibria exist, both aspirations and choices, via equilibrium selection, can be determined as a (stochastic) function of the individual extrinsic circumstances.

[18] In Dalton and Ghosal (2007a) we explore a similar equilibrium selection argument using Dalton and Ghosal's (2007b) n-player version of behavioural decisions.

From our results above we can present some important remarks for policy analysis.

First, the key point in the above equilibrium selection argument is the way the basins of attraction for each of the two equilibria change for different values of θ: the size of the basin of attraction of the type I equilibrium becomes smaller relative to the size of the basin of attraction for a type II equilibrium. There is a critical value of θ below which (respectively, above which) the probability attached to the type I equilibrium is smaller (respectively, larger) than the probability attached to the type II equilibrium. Moreover, the equilibrium selection developed here is non-ergodic i.e. the initial aspiration level determines where the adaptive dynamics ends up. Therefore, the process by which the initial aspiration level is determined, $F(p)$, is of critical importance.

Second, both θ and/or $F(p)$ can interpreted as a characteristic of the individual being studied. For example an individual who has a low social status but has the right motivation could tend to do better than another low status individual with low motivation. From a policy perspective, the relevant instruments will be both θ and/or $F(p)$. For example changes in θ could correspond to things like changes in initial wealth (social status, health, location, nutrition, housing etc.) of an individual while changes in $F(p)$ could correspond to process by which the initial aspirations levels are generated. The formal analysis suggests that direct attempts to change the extrinsic circumstances (by, for example, enhancing the economic status via transfers of wealth) will be welfare enhancing for very poor individuals while for individuals with intermediate wealth levels, policy interventions that directly impact probability with initial aspirations are generated will also be welfare improving. In this sense, the argument presented here distinguishes between absolute and relative deprivation and makes a case for different policy interventions in the two cases.

6 Final Remarks

To summarize, our paper contributes to the literature in distinctive ways. First, we provide a reduced form representation of seemingly disconnected papers in a framework where preference parameters are potentially endogenous. Second, by deriving the conditions under which a standard decision problem is indistinguishable from a behavioral decision problem, we contribute to the small but growing literature on the welfare implications of behavioural economics. In particular, we show that the use of revealed preferences for making welfare judgements is problematic.

Finally, in light of these results, we provide policy prescriptions to bring the outcomes of behavioural decision problems into closer conformity to the normative ones. An insight from our analysis is that a policy intervention for efficiency purposes is only justifiable for the cases in which both decision problems are distinguishable from each other. Moreover, the type of policy recommendation will vary with the class of behavioural decision problem faced by the decision maker.

The results reported here have some empirical caveats. Both, the endogenous preference parameters and the feedback-map are key variables for policy considerations, though they are not directly observable from choice behaviour. One possible approach to identify these "unobservable", may be to use evidence from neuroscience and psychology on the neural processes driving decision making.

Extending the one-person model studied here to $n-$players, dynamic and sequential decision scenarios are topics for future research.

Appendix

The role played by assumption (A4) in obtaining the monotone comparative statics with incomplete preferences is clarified by the following examples. In all these examples, P is single valued and A is the four point lattice in $\Re^2$ $\{(e,e),(f,e),(e,f),(f,f)\}$ where $f > e$.

Example 12. Suppose that $(f,f) \succ (e,e)$ but otherwise no other pair is ranked. Then, Ψ consists of $\{(f,e),(e,f),(f,f)\}$ clearly not a lattice. Note that in this case, $\succ$ satisfies acyclicity (and transitivity) and quasi-supermodularity (and trivially, single-crossing property). However, $\succ$ doesn't satisfy monotone closure: $(f,e) \geq (e,e)$, with $\succ((f,e)) = \phi$ and $(f,e) \nsucc_p (e,e)$, but $\succ((e,e)) \neq \phi$.

Example 13. Suppose that $(f,f) \succ (e,e)$, $(f,e) \succ (e,e)$, $(e,f) \succ (e,e)$ but otherwise no other pair is ranked. Then, Ψ again consists of $\{(f,e),(e,f),(f,f)\}$ clearly not a lattice. Note that in this case, $\succ$ satisfies acyclicity and monotone closure but not quasi-supermodularity.

Example 14. Suppose that $(f,f) \succ (e,e)$, $(f,e) \succ (e,e)$, $(e,f) \succ (e,e)$, $(f,f) \succ (f,e)$, $(f,e) \succ (e,f)$ but the pair $\{(f,f),(e,f)\}$ is not ranked. Note that $\succ$ satisfies acyclicity but not transitivity and also quasi-supermodularity, monotone closure. In this case, again Ψ consists of the singleton $\{(f,f)\}$.

Example 12 demonstrates that with incomplete preferences, quasi-supermodularity on its own, is not sufficient to ensure that the set of maximal elements of $\succ$ is a sublattice of A even when $\succ$ is acyclic (and transitive). Example 12 also demonstrates that $\succ$ can satisfy transitive but not monotone closure. Example 14 demonstrates that $\succ$ can satisfy monotone closure but not transitivity. Therefore, monotone closure and transitivity are two distinct conditions. Example 13 demonstrates that monotone closure without quasi-supermodularity cannot, on its own, ensure that the set of maximal elements of $\succ$ is a sublattice of A.

Proof of PROPOSITION 4:

By assumption, for each p, $\succ_p$ is acyclic, $\succ_p^{-1}(a)$ are open relative to A and A is compact. By Bergstrom (1975), it follows that $\Psi(p)$ is non-empty. As Bergstrom (1975) doesn't contain an explicit proof that $\Psi(p)$ is compact, an explicit proof of this claim follows next. To this end, note that the complement of the set $\Psi(p)$ in A is the set $\Psi^c(p) = \{a' \in A : \succ_p (a') \neq \phi\}$. If $\Psi^c(p) = \phi$, then $\Psi(p) = A$ is necessarily compact. So suppose $\Psi^c(p) \neq \phi$. For each $a' \in \Psi^c(p)$, there is $a'' \in A$ such that $a'' \succ_p a'$. By assumption, $\succ_p^{-1}(a'')$ is open relative to A. By definition of $\Psi(p)$, $\succ_p^{-1}(a'') \subset \Psi^c(p)$. Therefore, $\succ_p^{-1}(a'')$ is a non-empty neighborhood of $a' \in \Psi^c(p)$ and it is clear that $\Psi^c(p)$ is open and therefore, $\Psi(p)$ is closed. As A is compact, $\Psi(p)$ is also compact. Next, I show that for $p \geq p'$ if $a \in \Psi(p)$ and $a' \in \Psi(p')$, then $\sup(a,a') \in \Psi(p)$ and $\inf(a,a') \in \Psi(p')$. Note that as $a' \in \Psi(p')$, $\inf(a,a') \not\succ_{p'} a'$. By part (i) of quasi-supermodularity, it follows that $a \not\succ_{p'} \sup(a,a')$. By part (i) of single-crossing, it follows that $a \not\succ_p \sup(a,a')$. As $a \in \Psi(p)$, $\succ_p (a) \neq \phi$ and therefore, by part (i) of monotone closure, $\succ_p (\sup(a,a')) \neq \phi$ and therefore, $\sup(a,a') \in \Psi(p)$. Next, note that as $a \in \Psi(p)$, $\sup(a,a') \not\succ_p a$. By part (ii) of single-crossing, it follows that $\sup(a,a') \not\succ_{p'} a$ and by part (ii) of quasi-supermodularity, $a' \not\succ_{p'} \inf(a,a')$. As $a' \in \Psi(p')$, $\succ_{p'} (a') \neq \phi$, and by part (ii) of monotone closure, as $a' \not\succ_{p'} \inf(a,a')$, $\succ_{p'} (\inf(a,a')) \neq \phi$ and therefore, $\inf(a,a') \in \Psi(p')$. Therefore, (i) $\Psi(p)$ is ordered, (ii) $\Psi(p)$ is a compact sublattice of A and has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{a}(p)$ and $\underline{a}(p)$, and (iii) both

$\bar{a}(p)$ and $\underline{a}(p)$ are increasing functions from P to A.

Define a map $\Psi : A \times P \to A \times P$, $\Psi(a,p) = (\Psi_1(p), \Psi_2(a))$ as follows: for each (a,p), $\Psi_1(p) = \{a' \in A : \succ_p (a') = \phi\}$ and $\Psi_2(a) = \pi(a)$. By proposition 2, $\Psi_1(p)$ is non-empty and compact and for $p \geq p'$ if $a \in \Psi_1(p)$ and $a' \in \Psi_1(p')$, then $\sup(a,a') \in \Psi_1(p)$ and $\inf(a,a') \in \Psi_1(p')$. It follows that $\Psi_1(p)$ is ordered and hence a compact (and consequently, complete) sublattice of A and has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{a}(p)$ and $\underline{a}(p)$ respectively. By assumption 1, it also follows that for each a, $\pi(a)$ has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{\pi}(a)$ and $\underline{\pi}(a)$ respectively. Therefore, the map $(\bar{a}(p), \bar{\pi}(a))$ is an increasing function from $A \times P$ to itself and as $A \times P$ is a compact (and hence, complete) lattice, by applying Tarski's fix-point theorem, it follows that $(\bar{a}, \bar{p}) = (\bar{a}(\bar{p}), \bar{\pi}(\bar{a}))$ is a fix-point of Ψ and by a symmetric argument, $(\underline{a}(p), \underline{\pi}(a))$ is an increasing function from $A \times P$ to itself and $\left(\underline{a}, \underline{p}\right) = \left(\underline{a}(\underline{p}), \underline{\pi}(\underline{a})\right)$ is also a fix-point of Ψ; moreover, $(\bar{a}, \bar{p})$ and $\left(\underline{a}, \underline{p}\right)$ are respectively the largest and smallest fix-points of Ψ. ■

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