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# Moral hazard, bank runs and contagion

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**Abstract**

We study banking with *ex ante* moral hazard. Resolving the misalignment of the incentives between banks and depositors requires early liquidation with positive probability: efficient risk-sharing between depositors is no longer implementable. In a closed region with a single bank, we show that (i) with costless and perfect monitoring, contracts with bank runs off the equilibrium path of play improve on contracts with transfers, (ii) when the bank's actions are non-contractible, equilibrium bank runs driven by incentives are linked to liquidity provision by banks. With multiple regions linked via an interbank market, with local moral hazard, we show that implementing second-best allocations requires both ex-ante trade in inter-bank markets and contagion after realization of liquidity shocks.

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# 1 Introduction

A key issue in the theoretical literature on banking is the link between illiquid assets, liquid liabilities and bank runs. In the seminal paper by Diamond and Dybvig (1983) (see also Bryant (1980)), efficient risk-sharing between depositors with idiosyncratic, privately observed taste shocks creates a demand for liquidity. Banks invest in illiquid assets but take on liquid liabilities by issuing demand deposit contracts with a sequential service constraint. Although demand deposit contracts support efficient risk-sharing between depositors, the use of such contracts makes banks vulnerable to runs driven by depositor coordination failure. Allen and Gale (2000) extend the analysis of Diamond and Dybvig (1983) to study financial contagion in an optimal contracting scenario.

However, as Diamond and Dybvig point out, when aggregate taste shocks are common knowledge, a demand deposit contract with an appropriately chosen threshold for suspension of convertibility eliminates bank runs while supporting efficient risk-sharing. This point applies to the model of financial contagion developed by Allen and Gale (2000) as well. Taken together, these two remarks raise the following question: without any a priori restrictions on banking contracts, are there scenarios where equilibrium bank runs and equilibrium contagion occur with positive probability in a banking contract?

This paper studies banking with *ex ante* moral hazard but *without* aggregate payoff-relevant uncertainty. We study a model of banking with *ex ante* moral hazard. Resolving the misalignment of the incentives between banks and depositors requires the early asset liquidation with positive probability: efficient risk-sharing between depositors is no longer implementalbe.

Initially, we study banking in a closed region. Although the bank has no investment funds of its own, it has a comparative advantage in operating illiquid assets: no other agent in the economy has the human capital to operate illiquid assets. Consequently, the bank controls any investment made in illiquid assets. The bank has a choice of two illiquid assets to invest in. After depositors endowments have been mobilized, but before the realization of idiosyncratic taste shocks, the bank makes an investment decision. Each illiquid asset generates a stream of "public" and "private" returns. We think of "public" returns as cash flows generated by the asset that the bank cannot access without depositors' consent (for instance, such cash flows are generated by physical capital which can be monitored and seized by depositors). "Private" returns, then, are cash flows generated by the asset which can be accessed by the bank without depositors' consent[1].

We assume that there is a social planner who maximizes the ex-ante utility of a representative depositor[2]. Even with costless, perfect monitoring of the banks actions, we show that using transfers to provide the bank with appropri-

[1] Following Hart and Moore (1998), we think of "public" returns as cash flows generated by the asset that the bank cannot steal because they are publicly verifiable (for instance, they are embedded in physical capital which can be seized by depositors). "Private" returns, then, are cash flows generated by the asset which can be stolen by the bank.

[2] Equivalently, we assume that depositors have all the bargaining power.

ate incentives can result in narrow banking and no liquidity provision. More generally, incentive compatible transfers to the bank will lower consumption for all types of depositors. Nevertheless, our first result shows that it is still possible to implement efficient risk sharing between depositors, without sacrificing consumption, by using a contract which embodies the threat of a bank run off the equilibrium path of play.

When the investment decision of the bank is non-contractible[3], we show that efficient risk-sharing between depositors is no longer implementable. Even with forward looking depositors, the positive probability of an equilibrium bank run is necessary and sufficient to resolve incentive problems in banking. Although the second-best incentive compatible contract improves on autarky, it also generates, endogenously, the risk of a banking crisis.

Next, we extend the model to multiple regions linked by an inter-bank market along the lines of Allen and Gale (2000). In this case, with local moral hazard where only the incentive constraint of the bank in region 1 binds, there is trade in the inter-bank market even allowing for the possibility of bank runs and contagion after the realization of liquidity shocks. Moreover, the second-best allocation is implemented by combination of trade in the inter-bank market with bank runs and contagion induced by the random banking contract. In this sense, global contagion can result with even local moral hazard.

In either case, there is no aggregate uncertainty in preferences and technology: the randomness introduced by banking contracts studied here is uncorrelated with fundamentals and is driven purely by incentives. We believe this is a more primitive explanation for bank runs and contagion. In the formal model studied here, bailouts are equivalent to building in a suspension of convertibility clause in the banking contract. In this sense, the random second-best contracts studied here provides a rationale for the doctrine of "*creative ambiguity*" when the banking regulator makes no ex-ante commitment to a particular bailout policy but instead leaves the banking sector in doubt about its intentions (Goodhart (1999)).

Intervention by central banks or government agents takes place typically after the onset of a crisis (see, for instance, OECD (2002)). Indeed, section 2.5 below for case of closed region, we characterize the structure of second-best intervention in sequential monitoring scenarios where no other agent can replace the bank at $t = 1$. In practice, however, conditional on a bank run, there is also a third option which involves depositor protection via emergency liquidity provision and replacing the existing management of the bank via a takeover by a another bank or nationalization as in the current crisis involving Northern Rock. As such our results justify such an intervention policy the alternatives necessarily require bank runs or contagion.

The rest of the paper is structured as follows. The remainder of the introduction relates the results obtained here with other papers on bank runs. Section 2 studies a simple model of banking with moral hazard and leads up to

[3] In particular, any transfer to the bank cannot be made contingent on the actions chosen by the bank.

the main result of the paper. Section 3 is devoted to contagion issues. The final section concludes.

## 1.1 Related literature

Although to the best of our knowledge, both the model and the results of our paper are new, in what follows, we situate our analysis in the context of related work.

Perhaps the paper closest to the approach we adopt here is Diamond and Rajan (2001) who show that the threat of bank runs off the equilibrium path of play impacts on the bank's ability and incentives to renegotiate loan contracts with borrowers. We obtain a similar result: the threat of bank runs off the equilibrium path of play (when monitoring is both costless and perfect) impacts on the investment decision of the bank. However, they do not obtain equilibrium bank runs as, in their model, whether or not banks renegotiate is observable (though not verifiable and therefore, non-contractible ex-ante).

Calomiris and Kahn (1991) study a model of embezzlement in banking where the bank's temptation to embezzle depends on the realization of an exogenous move of nature and depending on the prevailing state, either the bank will never be tempted to embezzle or will always be tempted to embezzle. Therefore, in Calomiris and Kahn (1991), the positive probability of a bank run relies on the existence of aggregate payoff-relevant uncertainty. Diamond and Rajan (2000), in a framework similar to Diamond and Rajan (2001), also require the additional feature of exogenous uncertainty to obtain equilibrium bank runs. In contrast, in our paper the existence of equilibrium bank runs doesn't rely on aggregate payoff relevant uncertainty. Here bank runs are driven purely by incentives.

Holmström and Tirole ((1997), (1998)), study a model where conditional on the realization of an exogenous liquidity shock, banks incentives have to be aligned with those of the depositors. In their model, ex-ante (before the realization of the exogenous liquidity shock), the threshold (in the space of liquidity shocks) below which the bank is liquidated is set. They show that this threshold will be higher than the first-best threshold when agency costs are taken into account. In this sense, their inefficient termination (relative to the first-best) is driven by exogenous payoff-relevant uncertainty while in our paper inefficient termination doesn't require exogenous payoff-relevant uncertainty.

It is worth remarking that a common feature of Calomiris and Kahn (1991), Holmström and Tirole ((1997), (1998)), and Diamond and Rajan (2001), is their focus on issues of moral hazard that arise conditional on the realization of the liquidity shock. In contrast, here, we study moral hazard issues that arise *ex ante* before the realization of the liquidity shock.

A related branch has focused on the relation between incomplete information about the distribution of taste shocks across depositors and bank runs in banking scenarios with a finite number of depositors. Under the assumption that the social planner can condition allocations on the position a depositor has in the queue of depositors attempting to withdraw their deposits, Green and Lin (2003), building on Wallace ((1998), (1990)), show that it is possible to implement the first-best socially optimal risk-sharing allocation without bank runs. On the other hand, by imposing further restrictions on banking contracts, Peck and Shell (2003) obtain equilibrium bank runs as a feature of the optimal banking contract.

Another branch of the literature has focused on the relation between incomplete information about the future returns of the illiquid asset and bank runs (see, for instance, Gorton (1985), Gorton and Pennacchi (1990), Postlewaite and Vives (1987), Chari and Jaganathan (1988), Jacklin and Bhattacharya (1988), Allen and Gale (1998)). However, in these papers, the variation in the future returns of the illiquid asset is exogenous while here the variation in future returns is a function of the investment decision of the bank and is hence endogenous.

Finally, in our paper, as in Aghion and Bolton (1992), bank runs can be interpreted as a way of allocating control of over banking assets to depositors. However, unlike Aghion and Bolton (1992), the reallocation of control rights isn't triggered by some exogenous event but endogenously via depositor's actions in the second-best banking contract.

# 2 Bank runs with moral hazard

## 2.1 The model

In this section we study a model of banking in a closed region. The model extends Diamond-Dybvig (1983) to allow for moral hazard in banking. There are three time periods, $t = 0, 1, 2$. In each period there is a single perishable good $x_t$. There is a continuum of identical depositors in $[0, 1]$, indexed by $i$, of mass one, each endowed with one unit of the perishable good at time period $t = 0$ and nothing at $t = 1$ and $t = 2$. Each depositor has access to a storage technology that allows him to convert one unit of the consumption good invested at $t = 0$ to 1 unit of the consumption good at $t = 1$ or to 1 unit of the consumption good at $t = 2$.

Depositors preferences over consumption are identical ex-ante, i.e. as of period 0. Each faces a privately observed uninsurable risk of being type 1 or type 2. In period 1, each consumer learns of his type. Type 1 agents care only about consumption in period 1 while for type 2 agents, consumption in period 1 and consumption in period 2 are perfect substitutes. For each agent, only total consumption (and not its period-wise decomposition) is publicly observable. Formally, at $t = 1$, each agent has a state dependent utility function which has the following form:

$$U(x_1, x_2, \theta) = \begin{cases} u(x_1) \; if \; i \; is \; of \; type \; 1 \; in \; state \; \theta \\ u(x_1 + x_2) \; if \; i \; is \; of \; type \; 2 \; in \; state \; \theta \end{cases}$$

In each state of nature, there is a proportion $\lambda$ of the continuum of agents who are of type 1 and conditional on the state of nature, each agent has an equal and independent chance of being type 1. It is assumed that $\lambda$ is commonly known.

In addition, there is a bank, denoted by $b$. The bank's preferences over consumption is represented by the linear utility function $U^b(x_0, x_1, x_2) = x_1 + x_2$[4]. Unlike depositors, the bank has no endowments of the consumption good at $t = 0$. However, the bank is endowed with two different asset technologies, $j = A, B$, that convert inputs of the perishable good at $t = 0$ to outputs of the perishable consumption good at $t = 1$ or $t = 2$. We will assume that the size of the bank is large relative to the size of an individual depositor[5]. As each individual depositor has a (Lebesgue) measure zero, if the bank has the same size as an individual depositor, transfers to the bank can be made without affecting the overall resource constraint. In order to capture the trade-off between making transfers to the bank and efficient risk sharing between depositors, the bank has to be large relative to the depositors.

The output of the perishable consumption good produced by either asset technology has two components: a "private" non-contractible component that only the bank can access and consume and a "public" component which depositors can access and consume as well. Both the "public" and the "private" component of both asset technologies are characterized by constant returns to scale. For each unit of the consumption good invested in $t = 0$, asset technology $j$, $j = A, B$, yields either 1 unit of the "public" component of the consumption good if the project is terminated at $t = 1$ or $R_j > 0$ units of the "public" component of the consumption good at $t = 2$ if the project continues to $t = 2$. In addition, for each unit of the consumption good invested in $t = 0$, asset technology $j$, $j = A, B$, yields 1 unit of the "private" non-contractible component of the consumption good if the project is terminated at $t = 1$, or $R_j^b > 0$ units of the "private" component of the consumption good at $t = 2$ if the project continues to $t = 2$[6]. In addition, at $t = 0$, the bank incurs a direct private utility cost $c_j$ per unit of the consumption good invested in asset $j$ at $t = 0$.

In order to operate either of these two asset technologies, the bank has to mobilize the endowments of the depositors. At $t = 0$, we assume that mobilizing depositors' endowments requires a banking contract which specifies an allocation for each type of depositor and an investment portfolio for the bank.

Any contract used must satisfy the following constraints:

(a) the bank controls any investment that is made into either of these two asset technologies and the operation of both these two asset technologies,

(b) no other agent in the economy has the human capital to operate either of these two technologies,

(c) no other agent can replace the bank to take over the operation of either

[4] The assumption that $u^b(.)$ is linear simplifies the computations and the notation considerably. All the results stated here extend, with appropiately modified computations, to the case where $u^b(.)$ is a strictly increasing in consumption.

[5] Technically, the set of agents is modelled as a mixed measure space where each individual depositor has a Lebesgue measure zero (and therefore is part of an atomless continuum of depositors) while the bank is an atom with measure one. For details on how construct such a measure space see Codognato and Ghosal (2001).

[6] The assumption that within a technology there is no choice as to how much of the investment goes into the public component and how much into the private component is a simplification and nothing essential in our results depends on this analysis.

illiquid asset from the bank at $t = 1$,

(d) at $t = 1$ verifying or observing the investment decision of the bank, made at $t = 0$, is possible only if an appropriate monitoring technology is available,

(e) the public return at $t = 1$ is observed by the depositors and/or an outside agent (a court) only if the asset technology is terminated at $t = 1$ and the public return at $t = 2$ is observed by the depositors and/or the outside agent only at $t = 2$.

The consequence of making these assumptions is that, in the absence of a perfect monitoring technology, the investment decision of the bank at $t = 0$ is non-contractible. The combination of non-contractible actions together with the private non-contractible component to asset payoffs is the source of the moral hazard problem in banking.

In addition, we make some further assumptions on depositor's preferences and the two asset technologies:

$(A1)$ $u(.)$ is strictly increasing, strictly concave, smooth utility function,

$(A2)$ $-\frac{u''(x)x}{u'(x)} > 1$ for all $x > 0$,

$(A3)$ $R_A > 1 > R_B > 0$,

$(A4)$ $R_A + R_A^b > R_B + R_B^b$,

$(A5)$ $1 < R_j^b$, $j = A, B$,

$(A6)$ $c_A < c_B$.

Assumption $(A1)$ implies that each individual type 1 and type 2 depositor is risk-averse. Assumption $(A2)$ implies that whenever there is efficient risk-sharing, the bank has to provide liquidity services: narrow banking is ruled out. Under assumption $(A3)$, it can never be in the depositor's interest for the bank to invest in asset $B$: depositors will prefer to invest their endowments of the consumption good in the storage technology. Assumption $(A4)$ implies that production efficiency requires investment in asset $A$. Assumption $(A5)$ implies that for either asset, the bank prefers the project to continue to $t = 2$. Finally, assumption $(A6)$ implies that the effort cost to the bank of investing in asset $A$ is less than the effort cost of investing in asset $B$.

An allocation is a vector $(\gamma_s, \gamma, x, x^b)$ where $(\gamma_s, \gamma)$ is the asset (equivalently, investment) portfolio (chosen at $t = 0$) and describes the proportion of endowments invested in the storage technology and asset technology $A$ (with proportion $1 - \gamma_s - \gamma$ invested in asset technology $B$), $x = (x_1^1, x_1^2, x_2^1, x_2^2)$ is the consumption allocation of the depositors ($x_t^k$ is the consumption of type $k$ depositor in time period $t$, $k = 1, 2$ and $t = 1, 2$) and describes what each type of depositor consumes in each period and $x^b = \left(x_1^b, x_2^b\right)$ describes the consumption allocation to the bank. A consequence of assumptions $(A4)$ and $(A5)$ is that productive efficiency, and hence social efficiency, requires that $\gamma = 1$.

Throughout the paper we that the social planner maximizes the ex ante utility of a representative depositor. Equivalently, we assume that depositors have all the bargaining power[7]. We first characterize the (constrained) efficient

[7]It can be easily verified (details available on requesti that when banks have all the bargaining power, narrow banking results.

allocation and then, examine the implementation of this allocation using contracts (games). Given the sequential structure of the banking scenario studied here, our notion of implementation requires that agents use dominant actions in every subgame of the banking contract.

## 2.2 Depositor control and contracts without bank runs

Clearly, the ex-ante utility of the representative depositor is

$$g(x) = \lambda u(x_1^1) + (1-\lambda)u(x_1^2 + x_2^2)$$

where $g(x)$ is a weighted sum of type 1 and type 2 depositors preferences where the weights used reflect the aggregate proportions of type 1 and type 2 depositors. When there is no monitoring technology available, the representative depositor cannot condition transfers to the bank at $t = 1$ or $t = 2$, on the investment portfolio chosen by the bank at $t = 0$. In this case, making transfers to the bank will have no impact on the bank's incentives. Without a monitoring technology, in any banking contract written by the representative depositor, no transfers, over and above the private non-contractible payoff the bank receives by operating either asset technology, will be made to the bank.

Consider the case when $R_A^b \geq R_B^b$. By assumption, $c_A < c_B$, and therefore, $R_A^b - c_A \geq R_B^b - c_B$. In this case, we claim that the representative depositor can design a banking contract that implements the efficient risk-sharing without bank runs. The representative depositor solves the following maximization problem (labelled $(P)$ for later reference):

$$\max_{\{\gamma, x, x^b\}} g(x)$$

subject to

$(P1)$ $\quad R_A \geq R_A\left(\lambda x_1^1 + (1-\lambda)x_1^2\right) + \left(\lambda x_2^1 + (1-\lambda)x_2^2\right),$

$(P2)$ $\quad (\hat{A}2) \quad x_t^k \geq 0, k = 1, 2, t = 1, 2,$

$(P3)$ $\quad u\left(x_1^1\right) \geq u\left(x_1^2\right),$

$(P4)$ $\quad u(x_1^2 + x_2^2) \geq u(x_1^1 + x_2^1).$

The solutions to $(P)$ satisfy the equations

(1) $\quad x_1^{2*} = x_2^{1*} = 0,$

(2) $\quad u'(x_1^{1*}) = R_A u'(x_2^{2*}),$

(3) $\quad R_A = \lambda R_A x_1^{1*} + (1-\lambda)x_2^{2*},$

while for the bank

$(4a)$ $\quad \gamma^* = 1,$

$(4b)$ $\quad x_1^{b*} = 0,$

$(4c)$ $\quad x_2^{b*} = R_A^b.$

Allocations characterized by $(1)-(4)$ correspond to the first-best allocations in Diamond and Dybvig (1983). As in their paper, under the assumption $(A1)$, $u''(x) < 0$ while under assumption $(A3)$, $R_A > 1$. Therefore, using (2), it follows that $x_2^{2*} > x_1^{1*}$. This ensures that the truth telling constraints $(P3)$ is satisfied. Under the additional assumption that $-\frac{u''(x)x}{u'(x)} > 1$ it also follows that $x_1^{1*} > 1$

while $x_2^{2*} < R_A$. This implies that whenever there is efficient risk-sharing, the bank has to provide liquidity services: narrow banking is ruled out.

Again, as in Diamond and Dybvig (1983), there is a banking contract $\left(\hat{\gamma}, \hat{r}, \hat{k}\right)$, satisfying a sequential service constraint and with suspension of convertibility[8], that implements $(\gamma^*, x^*)$. Each depositor who withdraws in period 1 obtains a fixed claim $\hat{r}_1 = x_1^{1*}$ per unit deposited at $t = 0$ and convertibility is suspended at $\hat{k} = \lambda$. If banking continues to $t = 2$, each agent who withdraws at $t = 2$, obtains a fixed claim $\hat{r}_2 = x_2^{2*}$ per unit deposited at $t = 0$ and not withdrawn at $t = 1$. Moreover, $\hat{\gamma} = 1$. The argument establishing how such a contract implements first-best risk sharing follows Diamond and Dybvig (1983) and is reported in the appendix.

What happens if $R_A^b < R_B^b$? As long as $R_A^b - c_A \geq R_B^b - c_B$, nothing essential in the preceding argument changes and efficient risk-sharing without bank runs can still be implemented. On the other hand, when $R_A^b - c_A < R_B^b - c_B$[9], if costless and perfect monitoring of the bank's portfolio choice, made at $t = 0$, is possible at $t = 1$, the depositor can write a banking contract that conditions transfers at $t = 1$ on portfolio choices made by the bank at $t = 0$. Whether the representative depositor will actually choose to do so is an issue examined in the next subsection.

## 2.3 Depositor control and bank runs with costless and perfect monitoring

In this subsection, we examine the case where at time $t = 0$, it becomes common knowledge that the representative depositor invested in the monitoring technology and study the case of *costless and perfect* monitoring. With monitoring, we assume that (a) before the bank makes its investment decision, it becomes common knowledge that depositors have invested in the monitoring technology, and (b) the results of monitoring are revealed, publicly, before depositors choose whether or not to withdraw their deposits.

Specifically, we assume that at $t = 0$, it is common knowledge that at the beginning of $t = 1$, the representative depositor observes the investment allocation across assets made by the bank at $t = 0$. We assume that $R_A^b - c_A < R_B^b - c_B$. An obvious additional component in a banking contract is that now the representative depositor can commit to make transfers to the bank at $t = 2$, contingent on

[8] The sequential service constraint implies that (a) withdrawal tenders are served sequentially in random order until the bank runs out of assets and (b) the bank's payoff to any agent can depend only on the agent's place in the line and not on any future information about agents behind him in the line while suspension of convertibility implies that any agent attempting to withdraw at $t = 1$ will receive nothing at $t = 1$ if he attempts to withdraw at $t = 1$ after a fraction $\hat{k}$ of depositors. Note that along the equilibrium path of play, neither the sequential service constraint nor the suspension of convertibility constraint ever binds in any of the banking contracts, whether random or deterministic, studied in this paper.

[9] Taken together, the inequalities $c_A < c_B$ and $R_A^b - c_A < R_B^b - c_B$, imply that from the bank's perspective the project with higher net private utility return at $t = 2$ is also the one with the higher effort cost at $t = 0$. When $R_A^b - c_A < R_B^b - c_B$, as $R_A > R_B$, the long-run interests of the depositors and the bank are no longer aligned.

the actions chosen by the bank at $t = 0$. Note that under our assumptions, the representative depositor cannot make negative transfers to the bank. This is because, by assumption, the payoffs of the bank are private and non-contractible. Therefore, any transfer made to the bank has to be non-negative.

Suppose the representative depositor commits to make a transfer, at $t = 2$, to the bank of $\tau_2^b(\gamma)$, such that $R_A^b + \tau_2^b(1) - c_A = R_B^b - c_B + \varepsilon$, where $\varepsilon > 0$ but infinitesimal, while $\tau_2^b(\gamma) = 0$ for all $\gamma \neq 1$. In this case, the bank will choose $\gamma = 1$ if banking continues to $t = 2$. The resource constraint is

$(P'1)$ $\quad R_A - \tau_2^b(1) \geq R_A\left(\lambda x_1^1 + (1-\lambda)x_1^2\right) + \left(\lambda x_2^1 + (1-\lambda)x_2^2\right)$. Let $\gamma'$, $x'$ denote a solution to the representative depositor's maximization problem with the resource constraint $(P'1)$. Remark that a necessary condition for efficient risk-sharing between type 1 and type 2 depositors is that the equations (1), (2) and the inequality $(P'1)$ be simultaneously satisfied. Remark also that for depositors' participation constraints to be satisfied, any solution to the representative depositor's maximization problem must also satisfy the inequality

(5) $\quad x_1'^1 \geq 1$ *and* $x_1'^2 + x_2'^2 \geq 1$.

The following example demonstrates the (robust) possibility that there is no $x'$ satisfying (1), (2), $(P'1)$ and (5).

**Example 1** *Suppose* $u(x) = \frac{x^{1-\beta}}{1-\beta}$, $\lambda > 0$ *and* $R_A - \tau_2^b(1) < 1 - \varepsilon$. *Suppose to the contrary, there is some* $x'$ *satisfying* (1), (2), $(P'1)$ *and* (5). *Then, any* $x'$ *that satisfies* (1), (2) *must also satisfy the equation* $R_A^{\frac{1}{\beta}} x_1'^1 = x_2'^2$. *Evaluated at* $x_1'^1 = 1$, *the expression on the right hand side of* $(P'1)$ *is* $\lambda R_A + (1-\lambda) R_A^{\frac{1}{\beta}} > 1$ *as* $R_A > 1$ *while the left hand side of* $(P'1)$ *is strictly less than* $1 - \varepsilon$, *a contradiction.*

The above example shows that with transfers, even with costless and perfect monitoring, there is, in general, a trade-off between (a) efficient risk-sharing between type 1 and type 2 depositors and provision of liquidity, and (b) providing the bank with appropriate incentives. In robust banking scenarios, banking contracts with transfers results in no risk-sharing between type 1 and type 2 depositors and consequently, no provision of liquidity i.e. in narrow banking.

In general, however, even if risk-sharing between type 1 and type 2 depositors and providing the bank with appropriate incentives are consistent i.e. if there is a solution to the representative depositor's problem satisfying (1), (2), $(P'1)$ and (5), incentive compatible transfers to the bank will lower consumption for both types of depositors. To make this point, observe that when equations (1) and (2) are satisfied, we have that

$$u'\left(x_1'^1\right) = R_A u'\left(x_2'^2\right)$$

and as $u''(.) < 0$, the preceding equation implicitly defines a function $f(.)$ such that

$$x_2'^2 = f\left(x_1'^1\right)$$

where

$$f\left(x_1'^1\right) = u'^{-1}\left(\frac{u'\left(x_1'^1\right)}{R_A}\right).$$

Consider the inequality

(6) $R_A - \tau_2^b(1) \geq \lambda R_A + (1-\lambda) f(1)$.

By computation, it is easily checked that when (6) holds, an interior solution to the representative depositor's problem is possible. Note that (6) is equivalent to

$$R_A u'\left(\frac{R_A - \tau_2^b(1) - \lambda R_A}{1-\lambda}\right) < u'(1)$$

which implies that as $R_A > 1$ and $u''(.) < 0$, $x_2'^2 > x_1'^1$, $x_2'^2 < f(1)$ and therefore, $x_1'^1 > 1$. But we also have that $x_1'^1 < x_1^{*1}$ and $x_2'^2 < x_2^{*2}$. It follows that if (6) holds, any solution to the representative depositor's problem can be implemented by an appropriately designed banking contract, augmented with transfers and with suspension of convertibility. However, such an contract will inevitably entail a consumption loss for both types of depositors.

As, by assumption, depositors have all the bargaining power, assuming that the action chosen by the bank, at $t = 0$ can be observed costlessly at $t = 1$, can the representative depositor design a banking contract *without transfers* that implements the allocation $x^*$?

The following argument shows that this is indeed possible. The main idea of the argument is that as the representative depositor can observe $\gamma$, and therefore make the terms of the banking contract contingent on $\gamma$ so that if $\gamma = 1$, there is no bank run while if $\gamma < 1$, there is a bank run (equivalently, asset liquidation) with probability one. Such a banking contract would induce the bank to choose $\gamma = 1$ at $t = 0$. Therefore, in the game induced by the banking contract, although bank runs are never observed along the equilibrium path of play, the threat of a bank run off the equilibrium path of play induces the bank to choose $\gamma = 1$ along the equilibrium path of play.

The details are as follows. Let $r'(\gamma)$ be a function defined from $[0,1]$ to $\Re_+^2$ while let $k'(\gamma)$ be a function defined from $[0,1]$ to itself. Consider the banking contract, subject to a sequential service constraint, described by a vector $(\gamma', r', k')$ such that $r_1'(1) = x_1^{1*}$ per unit deposited at $t = 0$, $r_1'(\gamma) = 1$ for $\gamma < 1$, $k'(1) = \lambda$ while $k'(\gamma) = 1$ for $\gamma < 1$, and if banking continues to $t = 2$, $r_2'(1) = x_2^{2*}$ while for $\gamma < 1$, $r_2'(\gamma) = 0$ per unit deposited at $t = 0$ and not withdrawn at $t = 1$. The contract also specifies the bank's asset portfolio where $\gamma' = 1$. It follows that when $\gamma = 1$, it is a dominant action for type one depositors to withdraw and for type two depositors not to withdraw at $t = 1$, while it is a dominant action for all types of depositors to withdraw their deposits at $t = 1$ whenever $\gamma < 1$. Anticipating this behavior by depositors, the bank will choose $\gamma = \gamma' = 1$ as this yields a payoff $R_A^b - c_A > 1 - c_A$ while choosing $\gamma < 1$ yields a payoff $\gamma(1 - c_A) + (1-\gamma)(1 - c_B) < (1 - c_A)$ (since by assumption, $c_A < c_B$ and therefore, $1 - c_A > 1 - c_B$).

The case of imperfect and costly monitoring combines features of the results obtained in this section and section 2.4 below and is omitted.

## 2.4 Depositor control and bank runs with non-contractible actions

What happens if $R_A^b - c_A < R_B^b - c_B$ and there is no available monitoring technology for verifying and observing the investment decision of the bank at $t = 1$? In this case, allowing transfers to the bank will have no impact on the bank's incentives. A banking contract, all of whose Nash equilibria at $t = 1$ involve a zero probability of a bank run, will fail to implement any $\gamma > 0$. As $c_A < c_B$ and $1 - c_A > 1 - c_B$ and if there is enough chance of a bank run (equivalently, asset liquidation)[10] at $t = 1$, so that technology $A$ gets to generate a higher private utility return to the bank than technology $B$, one might get the bank to invest all available resources at $t = 0$ in asset technology $A$. So a run is clearly necessary to implement any allocation with $\gamma > 0$. That it is sufficient is proved below. However requiring $\gamma > 0$ entails a positive probability of a bank run at $t = 1$ and although efficient risk-sharing between type 1 and type 2 depositors is never implemented with probability one, it is achieved with strictly positive probability.

Consider the randomization scheme $(S, \pi)$ where $S = \{s_1, ..., s_M\}$, $M \geq 2$, is some arbitrary but finite set of states of nature and $\pi = \{\pi_1, ..., \pi_M\}$, $\pi_m \geq 0$, $\sum_{m=1}^{m=M} \pi_m = 1$ is a probability distribution over $S$[11]. The randomization scheme works as follows: at $t = 0$, no agent, including the bank, observes $s_m$ while at $t = 1$, before any choices are made, the realized value of $s_m$ is revealed to all agents and as before, each depositor privately observes her own type. A random allocation is a collection $(\tilde{\gamma}, \tilde{x}, \tilde{x}^b)$ where $\tilde{\gamma} \in [0, 1]$, $\tilde{x} : S \to \Re_+^4$ and $\tilde{x}^b : S \to \Re_+^2$. Let $\bar{S} = \left\{ s_m \in S : \tilde{x}_t^k(s_m) \geq 0, \lambda \tilde{x}_1^1(s_m) + (1 - \lambda)\tilde{x}_1^2(s_m) \geq 1 \right\}$, $\bar{M} = \left\{ m : s_m \in \bar{S} \right\}$and let $\bar{\pi} = \sum_{m \in \bar{M}} \pi_m$. The interpretation is that whenever $s_m \in \bar{S}$, the asset needs to be liquidated at $t = 1$ and therefore, $\bar{\pi}$ is the probability of a bank run. Therefore, at $t = 1$, both the bank and the depositors can condition any choices they make on $s_m$.

For $\gamma \in [0, 1]$, let $R_\gamma = \gamma R_A + (1 - \gamma) R_B$. The representative depositor's maximization problem (labelled as $(\tilde{P})$ for later reference) is:

$$\max_{\{S, \pi, \tilde{\gamma}, \tilde{x}, \tilde{x}^b\}} \sum_{s_m \in S} \pi_m g(\tilde{x}(s_m), \tilde{x}^b(s_m))$$

subject to

$\left(\tilde{P}1\right)$ $R_{\tilde{\gamma}}\left(\lambda \tilde{x}_1^1(s_m) + (1 - \lambda)\tilde{x}_1^2(s_m)\right) + \left(\lambda \tilde{x}_2^1(s_m) + (1 - \lambda)\tilde{x}_2^2(s_m)\right) \leq R_{\tilde{\gamma}}, s_m \in S$

$(\tilde{P}2)$ $\tilde{x}_t^k(s_m) \geq 0, k = 1, 2, t = 1, 2, s_m \in S,$

$(\tilde{P}3)$ $u(\tilde{x}_1^1(s_m)) \geq u(\tilde{x}_1^2(s_m)), s_m \in S,$

$(\tilde{P}4)$ $u(\tilde{x}_1^2(s_m) + \tilde{x}_2^2(s_m)) \geq u(\tilde{x}_1^1(s_m) + \tilde{x}_2^1(s_m)), s_m \in S,$

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[10] By assumption, no other agent can replace the bank to take over the operation of either illiquid asset from the bank at $t = 1$ which, in turn, implies that the second-best banking contract studied below is renegotiation proof.

[11] Obviously, there are other ways of introducing randomness in the social planner's problem. We choose the randomization scheme presented here as a matter of convenience.

$(\tilde{P}5)\quad \tilde{\gamma} \in \arg\max_{\gamma \in [0,1]} \left\{ \begin{array}{c} \bar{\pi} + (1-\bar{\pi})\left(\gamma R_A^b + (1-\gamma) R_B^b\right) \\ -\left[\gamma c_A + (1-\gamma) c_B\right] \end{array} \right\}.$

Fix a pair $(S, \pi)$, $M \geq 2$, such that $\bar{S}$ is non-empty. At any socially optimal allocation we must have that $\tilde{\gamma} = 1$. Evaluated at $\tilde{\gamma} = 1$, the payoffs of the bank is given by the expression

$$\bar{\pi} + (1-\bar{\pi}) R_A^b - c_A.$$

For the moral hazard constraint $(\tilde{P}5)$ to be satisfied, we require that

$$\bar{\pi} + (1-\bar{\pi}) R_A^b - c_A \geq \left\{ \begin{array}{c} \bar{\pi} + (1-\bar{\pi})\left(\gamma R_A^b + (1-\gamma) R_B^b\right) \\ -\left[\gamma c_A + (1-\gamma) c_B\right] \end{array} \right\}$$

for all $\gamma \in [0,1]$. When $\bar{\pi} = 0$, as $R_A^b < R_B^b$, $(\tilde{P}5)$ will always be violated for all $\gamma \in [0,1]$. On the other hand when $\bar{\pi} = 1$, as $1 - c_A > 1 - c_B$, $(\tilde{P}5)$ will hold as a strict inequality for all $\gamma \in [0,1]$. Further, both sides of the inequality are continuous in $\pi$ and $R_A^b > 1$, the expression $\bar{\pi} + (1-\bar{\pi}) R_A^b$ is also decreasing in $\bar{\pi}$ at the rate $1 - R_A^b$; moreover, as $R_B^b > 1$, for each $\gamma \in [0,1]$, the expression $\bar{\pi} + (1-\bar{\pi})\left(\gamma R_A^b + (1-\gamma) R_B^b\right)$ is also decreasing in $\bar{\pi}$ at the rate $1 - \left(\gamma R_A^b + (1-\gamma) R_B^b\right)$. It follows that for each $\gamma \in [0,1]$, as $R_B^b > R_A^b > 1$,

$$\begin{array}{cl} & \left|1 - R_A^b\right| \\ = & \left|R_A^b - 1\right| \\ < & \left|\left(\gamma R_A^b + (1-\gamma) R_B^b\right) - 1\right| \\ = & \left|1 - \left(\gamma R_A^b + (1-\gamma) R_B^b\right)\right| \end{array}$$

and therefore, there exists a unique threshold $\tilde{\pi}$, $0 < \tilde{\pi} < 1$, such that for all $\bar{\pi} > \tilde{\pi}$, $\bar{\pi} < 1$, the moral hazard constraint $(\tilde{P}5)$ holds as a strict inequality for all $\gamma \in [0,1]$.

Production efficiency and hence, constrained efficient risk-sharing requires that $\tilde{\gamma} = 1$. Next, note that

$(1')\quad \tilde{x}_1^{2*}(s_m) = 0, s_m \in S,$

$(3')\quad R_A\left(\lambda \tilde{x}_1^{1*}(s_m) + (1-\lambda)\tilde{x}_1^{2*}(s_m)\right) + (1-\lambda)\tilde{x}_2^{2*}(s_m) = R_A, s_m \in S,$

$(4'a)\quad \tilde{x}_1^{b*}(s_m) = r_A^b, s_m \in \bar{S},$

$(4'b)\quad \tilde{x}_1^{b*}(s_m) = 0, s_m \in S \backslash \bar{S},$

$(4'c)\quad \tilde{x}_2^{b*}(s_m) = 0, s_m \in \bar{S},$

$(4'd)\quad \tilde{x}_2^{b*}(s_m) = R_A^b, s_m \in S \backslash \bar{S}.$

By construction,

$(2'a)\quad \tilde{x}_2^{2*}(s_m) = 0, s_m \in \bar{S},$

and as $u'(.) > 0$, using $(\tilde{P}3)$ and $(\tilde{P}4)$, we obtain that

$(2'b)\quad \tilde{x}_1^{1*}(s_m) = \tilde{x}_1^{2*}(s_m), s_m \in \bar{S},$

while using $(3')$,

$(2'c)\quad \tilde{x}_1^{1*}(s_m) = \tilde{x}_1^{2*}(s_m) = 1, s_m \in \bar{S},$

as when there is a bank run, this is the only allocation consistent with the feasibility and participation constraints in $\tilde{P}$. It follows that

$(2'd)\quad \tilde{x}_1^{2*}(s_m) = 0, s_m \in S \backslash \bar{S},$

and
$(2'e)\quad u'(\tilde{x}_1^{1*}(s_m)) = R_A u'(\tilde{x}_2^{2*}(s_m)), s_m \in S\backslash\bar{S},$
and therefore
$(2'f)\quad \tilde{x}_1^{1*}(s_m) = x_1^{1*}, s_m \in S\backslash\bar{S},$
$(2'g)\quad \tilde{x}_2^{2*}(s_m) = x_2^{2*}, s_m \in S\backslash\bar{S}.$

It follows that for a fixed pair $(S, \pi)$, $M \geq 2$, such that $\bar{S}$ is non-empty and $\bar{\pi} \geq \tilde{\pi}$, $\bar{\pi} < 1$, there is a unique random allocation satisfying $(1') - (4')$. For a fixed pair $(S, \pi)$, such that either $\bar{S}$ is empty or $\bar{\pi} < \tilde{\pi}$, we have already established that there is no allocation that satisfies $(1') - (4')$. Finally, for a fixed pair $(S, \pi)$, such that either $S\backslash\bar{S}$ is empty or $\bar{\pi} = 1$, both $\tilde{x}_1^{1*}(s_m) = \tilde{x}_2^{2*}(s_m) = 1$, $\tilde{x}_1^{b*}(s_m) = r_A^b$ and $\tilde{x}_2^{b*}(s_m) = 0$ for all $s_m \in S$. In this case observe that though the moral hazard constraint $(4')$ always holds, there is no state at which there is efficient risk-sharing.

Next, we examine the optimal choice of the pair $(S, \pi)$. First note that at any optimal choice of $(S, \pi)$, generating a unique random allocation satisfying $(1') - (4')$, both $S\backslash\bar{S}$ and $\bar{S}$ will have to be non empty. Fix a pair $(S, \pi)$, $M \geq 2$, generating a unique random allocation satisfying $(1') - (4')$ denoted by $(\tilde{x}, \tilde{x}^b)$. Then, there is a pair $(S', \pi')$, $S' = \{s_1', s_2'\}$ and $\pi' = \{\pi_1', \pi_2'\}$ so that (a) $\tilde{x}(s_m) = \tilde{x}(s_1')$ and $\tilde{x}^b(s_m) = \tilde{x}^b(s_1')$ for all $s_m \in S\backslash\bar{S}$, (b) $\tilde{x}(s_m) = \tilde{x}(s_2')$ and $\tilde{x}^b(s_m) = \tilde{x}^b(s_2')$ for all $s_m \in \bar{S}$, (c) $\pi_1' = (1 - \bar{\pi})$ and $\pi_2' = \bar{\pi}$ and therefore,

$$\sum_{s_m \in S} \pi_m g(\tilde{x}(s_m), \tilde{x}^b(s_m)) = \pi_1' g(\tilde{x}(s_1'), \tilde{x}^b(s_1')) + \pi_2' g(\tilde{x}(s_2'), \tilde{x}^b(s_2'))$$

It follows that without loss of generality, we can restrict attention to $S'$ such that $M = 2$. Finally, as the representative depositor wants to maximize the probability with which efficient risk sharing is implemented, she will choose the lowest value of $\pi_2'$ compatible with $(\tilde{P}5)$ being satisfied as a strict inequality i.e. choose $\pi_2' = \tilde{\pi} + \varepsilon < 1$, where $\varepsilon > 0$ is small but strictly positive number so that $(\tilde{P}5)$ is satisfied as a strict inequality. Setting $\pi_2' = \tilde{\pi}$ will imply that $(\tilde{P}5)$ will be satisfied as an equality in which case the representative depositor will have to rely on the bank choosing a tie-breaking rule in favour of asset technology $A$.

It remains to specify a random banking contract that will implement the random allocation satisfying $(1') - (4')$. A random banking contract[12] is described by the vector $\left(S', \pi', \tilde{\gamma}, \tilde{r}, \tilde{k}\right)$ where the pair $(S', \pi')$ are as in the preceding paragraph, $\tilde{\gamma} = 1$ and $\tilde{r}_1(s_1') = x_1^{1*}$, $\tilde{r}_1(s_2') = 1$, $\tilde{r}_2(s_1') = x_2^{2*}$, $\tilde{r}_2(s_2') = 1$, $\tilde{k}(s_1') = \lambda$, $\tilde{k}(s_2') = 1$. The interpretation is that subject to a sequential service constraint and suspension of convertibility, each depositor who withdraws in period 1 obtains a random claim $\tilde{r}_1(s_m')$, $s_m' \in S'$ per unit deposited at $t = 0$. If banking continues to $t = 2$, each agent who withdraws at $t = 2$, obtains a random claim $\tilde{r}_2(s_m')$, $s_m' \in S'$ per unit deposited at $t = 0$. With such a contract, given $s_m' \in S'$, the payoff to per unit of deposit withdrawn at $t = 1$,

[12] As before we assume that at $t = 0$, no agent, including the bank, observes $s_m$ while at $t = 1$, before any choices are made, the realized value of $s_m$ is revealed to all agents. Thererfore, at $t = 1$, both the bank and the depositers can condition any choices they make on $s_m$.

which depends on the fraction of deposits serviced before agent $j$, $k_j$, is given by the expression

$$\tilde{v}_1(f_j, \tilde{r}_1(s'_m), \tilde{k}(s'_m), s'_m) = \begin{cases} u(\tilde{r}_1(s'_m)), \ if\ f_j \leq \tilde{k}(s'_m) \\ u(0), \ k_j > \tilde{k}(s'_m) \end{cases}$$

while the period 2 payoff per unit deposit withdrawn at $t = 2$, which depends on total fraction of deposits withdrawn in period 1, $k(s'_m)$, is given by the expression

$$\tilde{v}_2(f, \tilde{r}_1(s'_m), s'_m) = \begin{cases} u(\tilde{r}_2(s'_m)), \ if\ 1 > k(s'_m)\,\tilde{r}_1(s'_m) \\ 0, \ otherwise \end{cases}$$

At $t = 1$, for each value of $s'_m \in S'$, the above contract induces a noncooperative game between depositors where each depositor chooses what fraction of their deposits to withdraw. Fix $s'_m \in S'$. Suppose depositor $j$ withdraws a fraction $\mu_j(s'_m)$ Then, a type 1 depositor obtains a payoff $\mu_j(s'_m)\,\tilde{v}_1(f_j, \tilde{r}_1(s'_m), \tilde{k}(s'_m), s'_m)$ while a type 2 depositor obtains a payoff of $\mu_j(s'_m)\,\tilde{v}_1(f_j, \tilde{r}_1(s'_m), \tilde{k}(s'_m), s'_m) + (1 - \mu_j(s'_m))\,\tilde{v}_2(f, \tilde{r}_1(s'_m), s'_m)$. Remark that for a type 1 depositor, $\mu_j(s'_m) = 0$ strictly dominates all other actions. For $s'_1$, as $\tilde{k}(s'_1) = \lambda$, $\tilde{r}_1(s'_1) = x_1^{1*}$ and $\tilde{r}_2(s'_1) = x_2^{2*}$, it follows that $\tilde{v}_2(f, \tilde{r}_1(s'_1), s'_1) > \tilde{v}_1(f_j, \tilde{r}_1(s'_1), \tilde{k}(s'_1), s'_1)$ and for type 2 depositors, $\mu_j(s'_1) = 0$ strictly dominates all other actions. For $s'_2$, as $\tilde{r}_1(s'_2) = 1$ while $\tilde{r}_2(s'_2) = 0$, it follows that for type 2 depositors, $\mu_j(s'_2) = 1$ strictly dominates all other actions. Therefore, (i) for $s'_1$, the unique Nash equilibrium in strictly dominant actions is $\mu_j(s'_1) = 1$ if $j$ is a type 1 depositor while $\mu_j(s'_1) = 0$ if $j$ is a type 2 depositor and (ii) $s'_2$, the unique Nash equilibrium in strictly dominant actions is $\mu_j(s'_1) = 1$ for all $j$. At $t = 0$, the bank's payoffs are:

$$\tilde{v}^b(\gamma) = \pi'_2 + \pi'_2\left(\gamma R_A^b + (1-\gamma) R_B^b\right) - [\gamma c_A + (1-\gamma) c_B]$$

As $(6')$ holds as a strict inequality, it follows that choosing $\gamma = \tilde{\gamma} = 1$ is the strictly dominant choice for the bank.

The above random banking contract implements the allocation satisfying $(1') - (4')$.

We summarize the above discussion with the following proposition:

**Proposition 2** *When $R_A^b - c_A < R_B^b - c_B$, the second-best allocation determined by $(1') - (4')$ is implemented by the random banking contract $\left(S', \pi', \tilde{\gamma}, \tilde{r}, \tilde{k}\right)$.*

The above result makes clear that whenever the moral hazard constraint binds, bank runs are an endemic feature of the banking contract and limit efficient risk-sharing (equivalently, efficient liquidity provision) by banks.

**Remark 3** *In the preceding analysis, what is critical is that the aggregate proportion $\lambda$ of type 1 depositors is commonly observed at $t = 1$. Consider a modification of the problem so that the aggregate proportion $\lambda$ of type 1 depositors can be any one element from a set $\{\lambda_1, ..., \lambda_N\}$ and at $t = 0$ there is a common* *probability distribution over $\{\lambda_1, ..., \lambda_N\}$. However, at $t = 1$, the realized value of $\lambda$ is commonly observed. In such a case, the efficient risk-sharing allocation will be contingent on $\lambda \in \{\lambda_1, ..., \lambda_N\}$ and all the preceding results, after appropriate reformulation, continue to apply. In this sense, our results don't require but can be extended to scenarios with exogenous uncertainty. With exogenous uncertainty, the class of random contracts studied here, introduce noise that is independent of fundamentals in the banking process.*

**Remark 4** *So far we have assumed that the bank's payoffs are non-contractible and cannot be attached or confiscated by an outside agent (a court). Indeed, one consequence of this assumption is that in any banking contract designed by the representative depositor, transfers to the bank have to be non-negative. We drop this assumption here. We examine the case where a proportion $\beta$ of the bank's non-contractible payoffs can be seized directly by the social planner. Assume that a proportion $\beta$, $0 \leq \beta \leq 1$, of the bank's private payoffs can be seized by the social planner. For simplicity, assume that all the other assumptions made in section 2 continue to hold. When $\beta$ is large enough, i.e. when $\beta R_B^b < R_A^b$, it immediately follows that there is a banking contract which will implement the allocation $x^*$. Indeed, consider the banking contract $\left(\hat{\gamma}, \hat{r}, \hat{k}\right)$ augmented by a commitment by the social planner to confiscate $\beta R_B^b$ if banking continues to $t = 2$ and should the planner observe that the public return is consistent with $\gamma < 1$. In such case, it is easily checked that at $t = 0$, the bank will choose $\gamma = 1$ and it will be a dominant action for type one depositors to withdraw at $t = 1$ and type two depositors to withdraw at $t = 2$. Of course, when $\beta R_B^b > R_A^b$, the positive probability of equilibrium bank runs will be required to satisfy the bank's incentives. In this case, appropriately designed random banking contracts Pareto improve on autarchy.*

## 2.5 Monitoring conditional on a bank run

So far all the monitoring scenarios we have studied have the feature that at time $t = 0$, it becomes common knowledge that the representative depositor invested in the monitoring technology. Here, in contrast, we study a monitoring scenario when all monitoring takes place conditional on there being a bank run. The sequence of events involved in such interventions is as follows. Initially, temporary bailout measures are put in place, followed by a discovery phase when the books of the bank are examined and finally, there is a restructuring phase when the a decision is made to either liquidate the bank or leave the bank's status unchanged[13], (see, for instance, Hoggarth and Reidhill (2003)).

As noted in the introduction, there is also a third option which involves depositor protection and either replacing the existing management of the bank

[13] This timing of events is consistent with the sequential service constraint which requires that the return obtained by a depositor depends only on her position in the queue of depositors wishing to withdraw. That a depositor can announce a desire withdraw, then await the signal and decide not to withdraw is equivalent to assuming that she can leave the queue when she changes her mind.

or a takeover (via merger) by a another bank or nationalization. This third scenario isn't studied in this paper as we have assumed that no other agent has the human capital to replace the bank to take over the operation of either illiquid asset at $t = 1$.

In what follows, we characterize the structure of second-best intervention in sequential monitoring scenarios where no other agent can replace the bank at $t = 1$.

In this part of the paper, we will assume that conditional on a bank run it becomes common knowledge that the representative depositor has invested in a monitoring technology with a resource cost $m$. In keeping with the timing of events, we will assume that the resource cost is paid at $t = 1$. By investing in the monitoring technology, conditional on $\gamma$ being chosen by the bank at $t = 0$, the representative depositor observes a signal $\chi$, defined over subsets of $[0, 1]$ so that $\chi = \gamma$ with probability $\hat{q} > 0$ while $\chi = [0, 1]$ with probability $1 - \hat{q}$. Conditional on monitoring at $t = 1$, the resource constraint is

$R_A \geq R_A \left(\lambda x_1^1 + (1 - \lambda) x_1^2 + m\right) + \left(\lambda x_2^1 + (1 - \lambda) x_2^2\right)$.

For simplicity of exposition we focus on the case when there is an allocation, denoted by $x^m$, which is a solution to (1), (2), (5) and the preceding inequality.

Let $S^m = \{s_1^m, s_2^m\}$ and $\pi^m = \{\pi_1^m, \pi_2^m\}$ be a randomization scheme, defined independently, of the randomization scheme $S', \pi'$ studied in section 2.3. Let $\omega \in \{0, 1\}$ where $\omega = 0$ indicates a situation without monitoring and $\omega = 1$ indicates a situation with monitoring. Let $r^m(., ., ., .)$ be a function defined from $S' \times \{0, 1\} \times \quad \times S^m$ to $\Re_+^2$ and let $k^M(., ., .)$ be function defined from $S' \times \{0, 1\} \times \quad \times S^m$ to $[0, 1]$.

Consider the banking contract, subject to a sequential service constraint, described by a vector $(S', \pi', S^m, \pi^m, \gamma^m, r^m, k^m)$ such that:

(i) for all $\omega \in \{0, 1\}$, $\chi \in \quad$ and $s^m \in S^m$,

$$r_1^m(s_1', \{\omega\}, \{\chi\}, s^m) = x_1^{1*}$$

per unit deposited at $t = 0$, and

$$\begin{aligned} k^m(s_1', \{\omega\}, \{\chi\}, s^m) &= \lambda, \\ r_2^m(s_1', \{\omega\}, \{\chi\}, s^m) &= x_2^{2*} \end{aligned}$$

per unit deposited at $t = 0$ and not withdrawn at $t = 1$,

(ii) for all $\chi \in \quad$, $s^m \in S^m$,

$$r_1^m(s_2', \{0\}, \{\chi\}, s^m) = 1$$

per unit deposited at $t = 0$, and

$$\begin{aligned} k^m(s_2', \{0\}, \{\chi\}, s^m) &= 1, \\ r_2^m(s_2', \{0\}, \{\chi\}, s^m) &= 0 \end{aligned}$$

per unit deposited at $t = 0$ and not withdrawn at $t = 1$,

(iii) for all $s^m \in S^m$,

$$r_1^m(s_2', \{1\}, \{1\}, s^m) = x_1^{m,1}$$

per unit deposited at $t = 0$, and

$$\begin{aligned} k^m(s_2', \{1\}, \{1\}, s^m) &= \lambda, \\ r_2^m(s_2', \{1\}, \{1\}, s^m) &= x_1^{m,1} \end{aligned}$$

per unit deposited at $t = 0$ and not withdrawn at $t = 1$,
(iv) when $s' = s_2'$[14],

$$r_1^m(s_2', \{1\}, \{\gamma\}, s^m) = 1 - m$$

per unit deposited at $t = 0$, and

$$\begin{aligned} k^m(s_2', \{1\}, \{\gamma\}, s^m) &= 1, \\ r_2^m(s_2', \{1\}, \{\gamma\}, s^m) &= 0 \end{aligned}$$

per unit deposited at $t = 0$ and not withdrawn at $t = 1$,
(v) when $s' = s_2'$,

$$r_1^m(s_2', \{1\}, \{[0,1]\}, s_1^m) = x_1^{m,1}$$

per unit deposited at $t = 0$,

$$\begin{aligned} k^m(s_2', \{1\}, \{[0,1]\}, s_1^m) &= \lambda, \\ r_2^m(s_2', \{1\}, \{[0,1]\}, s_1^m) &= x_2^{m,2} \end{aligned}$$

per unit deposited at $t = 0$ and not withdrawn at $t = 1$,
(vi) when $s' = s_2'$,

$$r_1^m(s_2', \{1\}, \{[0,1]\}, s_2^m) = 1 - m$$

per unit deposited at $t = 0$,

$$\begin{aligned} k^m(s_2', \{1\}, \{[0,1]\}, s_2^m) &= 1, \\ r_2^m(s_2', \{1\}, \{[0,1]\}, s_2^m) &= 0 \end{aligned}$$

per unit deposited at $t = 0$ and not withdrawn at $t = 1$. The contract also specifies the bank's asset portfolio where $\gamma^m = 1$.

The sequence of events is as follows. At $t = 0$, the representative depositor offers the contract $(S', \pi', S^m, \pi^m, \gamma^m, r^m, k^m)$. Conditional on such a contract being accepted by the bank, depositors' endowments are mobilized by the bank who, then, allocates funds across the two assets. At $t = 1$, a state $s \in S'$ is selected according to the probability distribution $\pi'$ and a proportion $\lambda'$ of

[14] We assume that $0 < m < \hat{m} < 1$ where $\hat{m}$ is sufficiently small so that farsighted depositors always have an incentive to participate in banking.

depositors choose to withdraw their deposits. If $\lambda' \leq \lambda$, there is no monitoring and each depositor $j$ who chooses to withdraw a fraction $\mu_j$ of her deposits obtains a return of $x_1^{1*}$ per unit deposited at $t = 0$ while a depositor $j$ who withdraws a fraction $1 - \mu_j$ of her deposits obtains a return of $x_2^{2*}$ per unit deposited at $t = 0$ and not withdrawn at $t = 1$. If on the other hand, $\lambda' > \lambda$, there is a temporary suspension of convertibility for all depositors, and the representative depositor operates the monitoring technology at a fixed cost $m$. Conditional on monitoring taking place, a state $s^m \in S^m$ is chosen according to the probability distribution $\pi^m$. Conditional on $s \in S'$, the signal $\chi$ and the state of the world $s^m$, the suspension of convertibility threshold is set according to $k^m$. At this point, each depositor who has chosen to withdraw a positive fraction her deposits can leave the queue of depositors. If a depositor $j$ remains in the queue, and continues to choose to withdraw a fraction $\mu_j$ of her deposits, then, subject to a sequential service constraint, she obtains a return of $r_1^m$ per unit deposited at $t = 0$ while a depositor $j$ who leaves the queue obtains a return of $r_2^m$ per unit deposited at $t = 0$ and not withdrawn at $t = 1$.

Using arguments symmetric to the ones used in establishing proposition 2, for each $s \in S'$, it is a dominant action for type one depositors to withdraw, while it is a dominant action for type two depositors not to withdraw at $t = 1$ if the realized state is $s_1'$, and for type two depositors to withdraw at $t = 1$ if the realized state is $s_2'$. Therefore, there is no monitoring at $t = 1$ if the realized state is $s_1'$ while monitoring is triggered if the realized state is $s_2'$. Conditional on monitoring, it is a dominant action for type one depositors to not to leave the queue of depositors. If $\chi = 1$ or $\chi = [0, 1]$ and conditional on monitoring, the realized state is $s_1^m$, then it is a dominant action for type one depositors to not to leave the queue of depositors but if $\chi < 1$ or $\chi = [0, 1]$ and conditional on monitoring, the realized state is $s_2^m$, then it is a dominant action for type two depositors not to leave the queue and continue to want to withdraw their deposits at $t = 1$. Let $\bar{\pi}^m = \pi_2' (1 - \hat{q}) \pi_2^m$. Anticipating this behavior by depositors, the bank will choose $\gamma = \gamma^m = 1$ if and only if

$$\left\{\bar{\pi}^m + (1 - \bar{\pi}^m) R_A^b\right\} \geq \left\{ \begin{array}{c} (\pi_2' \hat{q} + \bar{\pi}^m) \\ + (1 - (\pi_2' \hat{q} + \bar{\pi}^m)) \left(\gamma R_A^b + (1 - \gamma) R_B^b\right) \\ - (\gamma c_A + (1 - \gamma) c_B) \end{array} \right\}$$

It follows, using arguments symmetric to the one used in establishing proposition 2, there exists $\tilde{\pi}^m > 0$, $\tilde{\pi}^m < 1$ such that the bank's incentive compatibility constraint is satisfied iff $\pi^m \geq \tilde{\pi}^m$. Therefore, setting $\pi^m = \tilde{\pi}^m + \varepsilon$, for some $\varepsilon$ strictly positive but close to zero makes it a dominant action for the bank to choose $\gamma = \gamma^m = 1$. In this set-up, conditional on $\gamma = 1$, the probability of termination is $\pi_2' (1 - \hat{q}) \pi_2^m < \tilde{\pi}$: in other words, efficient risk-sharing in implemented with higher probability when there is costly but imperfect monitoring relative to the random contract studied earlier. However, now both bank runs and monitoring occur with positive probability along the equilibrium path of play.

# 3 Local moral hazard and contagion

## 3.1 The model

In this section, we extend the model studied in section 2 to allow for multiple banks and moral hazard in banking along the lines of Allen and Gale (2000). There are three time periods, $t = 0, 1, 2$. In each period there is a single perishable good $x_t$. There are two regions, $r = 1, 2$. In each region, there is a continuum of identical depositors in $[0, 1]$, indexed by $i$, of mass one, each endowed with one unit of the perishable good at time period $t = 0$ and nothing at $t = 1$ and $t = 2$. Each depositor has access to a storage technology that allows him to convert one unit of the consumption good invested at $t = 0$ to 1 unit of the consumption good at $t = 1$ or to 1 unit of the consumption good at $t = 2$.

Depositors preferences over consumption are as before. The main difference is that now in each state of nature, there is a proportion $\lambda_{r,\theta}$ of the continuum of agents in region $r$ who are of type 1 and conditional on the state of nature, each agent has an equal and independent chance of being type 1. For simplicity, it will be assumed that there two states of the world so that $\theta \in \{\theta_1, \theta_2\}$. When $\theta = \theta_1$, in region $r = 1$, $\lambda_1 = \lambda_L$ while in region $r = 2$, $\lambda_2 = \lambda_H$ with $0 < \lambda_L < \lambda_H < 1$. Symmetrically, when $\theta = \theta_2$, in region $r = 2$, $\lambda_2 = \lambda_L$ while in region $r = 1$, $\lambda_1 = \lambda_H$. It is assumed that ex-ante at $t = 0$, there is a prior distribution over $\{\theta_1, \theta_2\}$ given by $\{p, 1 - p\}$.

In addition, in each region $r$, there is a bank, denoted by $b_r$. Bank preferences over consumption is also as before. As before, neither bank has any endowments of the consumption good at $t = 0$ but are endowed with two different asset technologies, $j = A, B$, that convert inputs of the perishable good at $t = 0$ to outputs of the perishable consumption good at $t = 1$ or $t = 2$. As before, we will assume that the size of either bank is large relative to the size of an individual depositor.

The asset technology is similar to the case of a monopoly bank in a closed region. As before, there are two asset technologies. The "public" returns generated by each asset is as in the case of the monopoly bank. In addition, for each unit of the consumption good invested in $t = 0$, asset technology $j$, $j = A, B$, yields 1 unit of the "private" non-contractible component of the consumption good if the project is terminated at $t = 1$, or $R_j^{b_r} > 0$ units of the "private" component of the consumption good at $t = 2$ if the project continues to $t = 2$[15]. In addition, at $t = 0$, the bank incurs a direct private utility cost $c_j$ per unit of the consumption good invested in asset $j$ at $t = 0$.

As before, operating either of these two asset technologies requires each bank to mobilize the endowments of the depositors within its own region: we do not allow for the possibility that the bank in region 1 is able to mobilize the deposits of some depositors in region 2 and vice versa.

At $t = 0$, we assume that mobilizing depositors' endowments requires a

[15] The assumption that within a technology there is no choice as to how much of the investment goes into the public component and how much into the private component is a simplification and nothing essential in our results depends on this analysis.

banking contract in each region that specifies an allocation for each type of depositor and an investment portfolio for the bank within that region.

As before, the investment decision of either bank at $t = 0$ is non-contractible. Further, depositor preferences and asset returns satisfy assumptions $(A1) - (A6)$ above.

An allocation is a vector $(\gamma_{rs}, \gamma_r, x_r, x^{b_r} : r = 1, 2)$ where $(\gamma_{rs}, \gamma_r)$ is the asset (equivalently, investment) portfolio (chosen at $t = 0$) and describes the proportion of endowments invested in the storage technology and asset technology $A$ (with proportion $1 - \gamma_{rs} - \gamma$ invested in asset technology $B$), $x_r = (x_{r1}^1, x_{r1}^2, x_{r2}^1, x_{r2}^2)$ is the consumption allocation of the depositors ($x_{rt}^k$ is the consumption of type $k$ depositor in time period $t$ in region $r$ $k = 1, 2$ and $t = 1, 2$) and describes what each type of depositor consumes in each period and $x^{b_r} = \left(x_1^{b_r}, x_2^{b_r}\right)$ describes the consumption allocation to the bank. A consequence of assumptions $(A4)$ and $(A5)$ is that productive efficiency, and hence social efficiency, requires that $\gamma_r = 1$.

For simplicity, in this section, we assume that depositors have all the bargaining power. In this case, as all depositors are identical ex-ante, a representative depositor, acting on behalf of all other depositors, makes a "take-it-or-leave-it" offer of a banking contract to the bank, which the bank can either accept or reject.

## 3.2 Inter-bank markets and the first-best benchmark

Let $\bar{\lambda} = p\lambda_L + (1 - p)\lambda_H$. Clearly, the objective function of the representative depositor in each region is

$$g(x) = \bar{\lambda} u(x_{r1}^1) + (1 - \bar{\lambda}) u(x_{r1}^2 + x_{r2}^2)$$

where $g(x)$ is the expected utility of type 1 and type 2 depositors preferences. When there is no monitoring technology available, the representative depositor cannot condition transfers to the bank at $t = 1$ or at $t = 2$ on the investment portfolio chosen by the bank at $t = 0$. In this case, making transfers to the bank will have no impact on the bank's incentives. Without a monitoring technology, in any banking contract written by the representative depositor, no transfers, over and above the private non-contractible payoff the bank receives by operating either asset technology, will be made to the bank.

Consider the case when $R_A^{b_r} \geq R_B^{b_r}$. By assumption, $c_A < c_B$, and therefore, $R_A^{b_r} - c_A \geq R_B^{b_r} - c_B$. In this case, we claim that the representative depositor can design a banking contract that implements the ex-ante efficient risk-sharing without bank runs. In each region, the representative depositor solves the following maximization problem (labelled $(R)$ for later reference):

$$\max_{\{\gamma_r, x_r, x^{b_r}\}} g(x)$$

subject to

$(R1)$ $\quad R_A \geq R_A \left(\bar{\lambda} x_{r1}^1 + \left(1 - \bar{\lambda}\right) x_{r1}^2\right) + \left(\bar{\lambda} x_{r2}^1 + \left(1 - \bar{\lambda}\right) x_{r2}^2\right),$

$(R2)\quad x_{rt}^k \geq 0, k = 1,2, t = 1,2,$
$(R3)\quad u\left(x_{r1}^1\right) \geq u\left(x_{r1}^2\right),$
$(R4)\quad u(x_{r1}^2 + x_{r2}^2) \geq u(x_{r1}^1 + x_{r2}^1).$
The solutions to $(R)$ satisfy the equations
$(1'')\quad x_{r1}^{2*} = x_{r2}^{1*} = 0,$
$(2'')\quad u'(x_{r1}^{1*}) = R_A u'(x_{r2}^{2*}),$
$(3'')\quad R_A = \bar{\lambda} R_A x_{r1}^{1*} + \left(1 - \bar{\lambda}\right) x_{r2}^{2*},$
while for the bank
$(4''a)\quad \gamma_r^* = 1,$
$(4''b)\quad x_1^{b_r *} = 0,$
$(4''c)\quad x_2^{b_r *} = R_A^{b_r}.$

Allocations characterized by $(1'') - (4'')$ correspond to the first-best allocations. Clearly $\gamma_r^* = \gamma_{r'}^*, x_r^* = x_{r'}^*, x^{b_r *} = x^{b_{r'} *}$, $r, r' = 1, 2$. Moreover, $\bar{\lambda} x_1^{1*} < 1$. It follows that $x_{r2}^{2*} = x_2^{2*} > x_{r1}^{1*} = x_1^{1*}$ while $x_1^{1*} > 1$ while $x_2^{2*} < R_A$.

In what follows, we assume that $\lambda_H x_1^{1*} > 1$. In this case, note that without a ex-ante interbank market, in the region with the high liquidity shock, there will be inefficiently early liquidation of the long-term asset. It follows that a combination of a regional banking contract (along the lines of Diamond-Dybvig (1983)) with an ex-ante inter-bank market (along the lines of Allen and Gale (2000)) are both required to implement the ex-ante optimal risk-sharing allocation.

Ex-ante, in the interbank market, each bank exchanges claims to half of the deposits mobilized within its own region. Suppose conditional on the realization of the liquidity shock, region 1 faces a high liquidity shock so that proportion of type traders in region is $\lambda_H$. In this case, the bank in region 1 liquidates its claims against the bank in region 2 to meet its own extra liquidity needs which amount to $\left(\lambda_H - \bar{\lambda}\right) x_1^{1*}$. Moreover, by computation, $\left[\lambda_L + \left(\lambda_H - \bar{\lambda}\right)\right] x_1^{1*} = \bar{\lambda} x_1^{1*} < 1$ so that the bank in region 2 doesn't have to liquidate all of its asset either. To prevent bank runs driven by depositor coordination failure, the suspension of convertibility threshold has to be set at $\lambda_H$. At $t = 2$, the bank in region 1 makes a payout of $\left(\bar{\lambda} - \lambda_L\right) x_2^{2*}$ to the bank in region 2. The details of such a contract follows closely the specification of the banking contract in section 2.2 and is omitted. Taken together, the interbank market and suspension of convertibility implements first-best risk sharing between depositors.

The above discussion can be summarized as the following result:

**Proposition 5** *When $R_A^{b_i} - c_A > R_B^{b_i} - c_B$, $i = 1, 2$, the first-best allocation is implemented by combining trade in the inter-bank market with an approprite banking contract embodying suspension of convertibility.*

## 3.3 Bank runs and contagion with local moral hazard

Suppose for some $r$, for concreteness $r = 1$, $R_A^{b_1} < R_B^{b_1}$. As long as $R_A^{b_1} - c_A \geq R_B^{b_1} - c_B$, nothing essential in the preceding argument changes and efficient risk-sharing without bank runs conditional on a liquidity shock can still be implemented. To fix ideas, consider what happens when the two banks seek to implement the first-best risk allocation $(\gamma_r^*, x_r^* : r = 1, 2)$. When $R_A^{b_1} - c_A < R_B^{b_1} - c_B$[16], we argue that the ex-ante optimal allocation can no longer be implemented. Note that implementing the efficient allocation requires that the existence of an inter-bank market where banks exchange claims to each others long term assets. When $R_A^{b_1} - c_A < R_B^{b_1} - c_B$, with a zero probability of a bank run, bank 1 will choose $\gamma_1 = 0$. Anticipating this possibility, bank 2 will be unwilling to hold any of bank $1's$ long-term assets. Thus, the inter-bank market will break down and the efficient allocation can no longer be implemented.

As $c_A < c_B$ and $1 - c_A > 1 - c_B$, provided there is enough chance of a bank run (equivalently, asset liquidation)[17] at $t = 1$, so that technology $A$ gets to generate a higher private utility return to bank 1 than technology $B$, one might get the bank 1 to invest all available resources at $t = 0$ in asset technology $A$. So a run is clearly necessary to implement any allocation with $\gamma > 0$. That it is sufficient and may involve contagion is proved below.

Let $\bar{\pi}_1$ be the ex-ante (before the realization of any liquidity shocks) probability of early liquidation for bank 1. Given $\bar{\pi}_1$, for each $\gamma_1 \in [0, 1]$, bank $1's$ payoff is

$$f_1(\bar{\pi}_1, \gamma_1) = \bar{\pi}_1 + (1 - \bar{\pi}_1)\left(\gamma_1 R_A^{b_1} + (1 - \gamma_1) R_B^{b_1}\right) - [\gamma_1 c_A + (1 - \gamma_1) c_B]$$

We want to ensure that given $\bar{\pi}_1$, $\gamma_1 = 1$ maximizes $f_1(\bar{\pi}_1, \gamma_1)$. This is equivalent to requiring that the following inequality holds for all $\gamma_1 \in [0, 1]$

$$\bar{\pi}_1 + (1 - \bar{\pi}_1) R_A^{b_1} - c_A \geq \left\{ \begin{array}{c} \bar{\pi}_1 + (1 - \bar{\pi}_1)\left(\gamma_1 R_A^{b_1} + (1 - \gamma_1) R_B^{b_1}\right) \\ - [\gamma_1 c_A + (1 - \gamma_1) c_B] \end{array} \right\}$$

When $\bar{\pi}_1 = 0$, as $R_A^{b_1} < R_B^{b_1}$, the preceding inequality will always be violated for all $\gamma_1 \in [0, 1]$. On the other hand when $\bar{\pi}_1 = 1$, as $1 - c_A > 1 - c_B$, the preceding inequality will hold as a strict inequality for all $\gamma_1 \in [0, 1]$. Further, both sides of the inequality are continuous in $\bar{\pi}_1$ and $R_A^{b_1} > 1$, the expression $\bar{\pi}_1 + (1 - \bar{\pi}_1) R_A^{b_1}$ is also decreasing in $\bar{\pi}_1$ at the rate $1 - R_A^{b_1}$; moreover, as $R_B^{b_1} > 1$, for each $\gamma_1 \in [0, 1]$, the expression $\bar{\pi}_1 + (1 - \bar{\pi}_1)\left(\gamma_1 R_A^{b_1} + (1 - \gamma_1) R_B^{b_1}\right)$ is also decreasing in $\bar{\pi}_1$ at the rate $1 - \left(\gamma_1 R_A^{b_1} + (1 - \gamma_1) R_B^{b_1}\right)$. It follows that

[16] Taken together, the inequalities $c_A < c_B$ and $R_A^{b_1} - c_A < R_B^{b_1} - c_B$, imply that in region 1 from the bank 1's perspective the project with higher net private utility return at $t = 2$ is also the one with the higher effort cost at $t = 0$. When $R_A^{b_1} - c_A < R_B^{b_1} - c_B$, as $R_A > R_B$, the long-run interests of the depositors in region 1 and bank 1 are no longer aligned.

[17] By assumption, no other agent can replace the bank to take over the operation of either illiquid asset from the bank at $t = 1$ which, in turn, implies that the second-best banking contract studied below is renegotiation proof.

for each $\gamma_1 \in [0,1]$, as $R_B^{b_1} > R_A^{b_1} > 1$,

$$\begin{aligned} & \left|1 - R_A^{b_1}\right| \\ = & \left|R_A^{b_1} - 1\right| \\ < & \left|\left(\gamma_1 R_A^{b_1} + (1-\gamma_1) R_B^{b_1}\right) - 1\right| \\ = & \left|1 - \left(\gamma_1 R_A^{b_1} + (1-\gamma_1) R_B^{b_1}\right)\right| \end{aligned}$$

and therefore, there exists a unique threshold $\tilde{\pi}_1$, $0 < \tilde{\pi}_1 < 1$, such that for all $\bar{\pi}_1 > \tilde{\pi}_1$, $\bar{\pi}_1 < 1$, the moral hazard constraint for bank 1 holds as a strict inequality for all $\gamma_1 \in [0,1]$. By computation, note that

$$\tilde{\pi}_1 = 1 - \frac{c_B - c_A}{R_B^{b_1} - R_A^{b_1}}.$$

Note that the decision of a depositor to withdraw is made only after she observes her own type. Therefore, necessarily, a bank run on bank 1 can only be implemented after the realization of the liquidity shock. For a bank run on bank 1 not to involve contagion, it must be the case that the bank run occurs conditional on $\theta_2$ when $\lambda_1 = \lambda_H$ and $\lambda_2 = \lambda_L$. However, if $(1-p) < \tilde{\pi}_1$, even if a bank run on bank 1 occurs with probability one conditional on $\theta_2$, the incentive constraint of bank 1 cannot be satisfied. In such cases, there must be a positive probability of a bank run on bank 1 conditional on $\theta_1$ which necessarily implies contagion.

Assume $(1-p) < \tilde{\pi}_1$. Consider a randomization scheme $(S, \pi)$ where $S = \{s_1, s_2\}$ and $\pi(\theta) = \{\pi_1(\theta), \pi_2(\theta)\}$ is a probability distribution over $S$ such that $\pi_1(\theta_1) = \tilde{\pi}_1 - (1-p) + \epsilon$ (where $\epsilon$ is small positive number), $\pi_1(\theta_2) = 1$ and $\pi_2(\theta) = 1 - \pi_1(\theta)$, $\theta \in \{\theta_1, \theta_2\}$. The randomization scheme works as follows: at $t = 0$, no agent, including the bank, observes $s_m$ while at $t = 1$, before any choices are made and after $\theta$ has been observed and each depositor privately observes her own type, the realized value of $s_m$ is revealed to all agents. Ex-ante, in the interbank market, each bank exchanges *contingent* claims to half of the deposits mobilized within its own region where claims are made contingent on $\{s_1, s_2\} \times \{\theta_1, \theta_2\}$. Clearly, claims contingent on $(s_2, \theta_2)$ are not exchanged as the contingency $(s_2, \theta_2)$ has a zero probability. The returns on claims contingent on $(s_1, \theta)$, $\theta \in \{\theta_1, \theta_2\}$, are zero while each unit of a claim contingent on $(s_2, \theta_1)$ yields a $\left(\lambda_H - \bar{\lambda}\right) x_1^{1*}$ at $t = 1$ with a payout of $\left(\bar{\lambda} - \lambda_L\right) x_2^{2*}$ at $t = 2$. The suspension of convertibility threshold is also made contingent on $\{s_1, s_2\} \times \{\theta_1, \theta_2\}$ so that it is set at zero for all contingencies except for $(s_2, \theta_1)$ when its is set at $\lambda_H$. The details of such a contract follows closely the specification of the banking contract in section 2.4 and is omitted. Taken together, the inter-bank market and suspension of convertibility implements first-best risk sharing between depositors.

Therefore, in a second-best contract where the incentive compatibility constraint of bank 1 binds, there is trade in the inter-bank market even allowing for the possibility of bank runs and contagion after the realization of liquidity shocks.

We summarize the above discussion with the following proposition:

**Proposition 6** *When $R_A^{b_1} - c_A < R_B^{b_1} - c_B$, the second-best allocation is implemented by a combination of trade in the inter-bank market alongwith contagion induced by the random banking contract.*

# 4 Conclusion

We interpret the significance of our results in three distinct ways. First, our results show that with moral hazard, bank runs and contagion are necessary elements in second-best banking scenarios and the randomness introduced by banking contracts studied here is uncorrelated with fundamentals driven purely by incentives. In this sense our results provide a theoretical foundation for the doctrine of "creative ambiguity". Second, we show that global contagion can result with even local moral hazard. Third, our result shows that in low asset economies, where productive agents like banks or firms with little or no collateral, appropriately designed random demandable debt contracts Pareto improve on autarky.

Extending our result to examine episodes of twinned bank runs and currency crises is a topic for future research.

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# Appendix

## Proof of proposition 1

Consider the banking contract $\left(\hat{\gamma}, \hat{r}, \hat{k}\right)$ as specified in section 2.2. The payoff to per unit of deposit withdrawn at $t = 1$, which depends on the fraction of deposits serviced before agent $j$, $k_j$, is given by the expression

$$\hat{v}_1(f_j, \hat{r}_1, \hat{k}) = \begin{cases} u\left(\hat{r}_1\right), \ if\ k_j \leq \hat{k}, \\ u\left(0\right), \ k_j > \hat{k} \end{cases}$$

while the period 2 payoff per unit deposit withdrawn at $t = 2$, which depends on total fraction of deposits withdrawn in period 1, $k$, is given by the expression

$$\hat{v}_2(f, \hat{r}_1) = \begin{cases} u\left(\hat{r}_2\right), \ if\ 1 > k\hat{r}_1, \\ 0, \ otherwise \end{cases}$$

At $t = 1$, the above contract induces a noncooperative game between depositors where each depositor chooses what fraction of their deposits to withdraw. Suppose depositor $j$ withdraws a fraction $\mu_j$. Then, a type 1 depositor obtains a payoff $\mu_j \hat{v}_1(k_j, \hat{r}_1, \hat{k})$ while a type 2 depositor obtains a payoff of $\mu_j \hat{v}_1(k_j, \hat{r}_1, \hat{k}) + \left(1 - \mu_j\right) \hat{v}_2(k, \hat{r}_1)$. Remark that for a type 1 depositor, $\mu_j = 1$ strictly dominates all other actions. As $\hat{k} = \lambda$, $\hat{r}_1 = x_1^{1*}$ and $\hat{r}_2 = x_2^{2*}$, it follows that $\hat{v}_2(k, \hat{r}_1) > \hat{v}_1(k_j, \hat{r}_1)$ and for type 2 depositors, $\mu_j = 0$ strictly dominates all other actions and therefore, $k = \hat{k} = \lambda$. The bank's payoffs are

$$\hat{v}^b\left(\gamma\right) = \gamma R_A^b + \left(1 - \gamma\right) R_B^b - \left(\gamma c_A + \left(1 - \gamma\right) c_B\right)$$

There is only one subgame at $t = 1$ (as depositors don't observe the bank's choice of $\gamma$). As $R_A^b \geq R_B^b$, and $c_A < c_B$, at $t = 1$, $1 - \hat{k}\hat{r}_1 > 0$, choosing $\gamma = \hat{\gamma}_A$ is a strictly dominant choice for the bank. ■