An Economic Model of Strategic Electoral Rule Choice Under Uncertainty

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An Economic Model of Strategic Electoral Rule Choice Under Uncertainty[1]

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Abstract

We study electoral rule choice in a multi-party model where parties are office-motivated and uncertainty over the electoral outcome is present. We show that when all dominant parties (parties with positive probability of winning the elections) have sufficiently good chances of winning, then they agree to change the PR with a more majoritarian rule. We identify the exact degree of disproportionality of the new rule and we prove that it is increasing in the expected vote share of the minority parties (parties with zero probability of winning). The necessary and sufficient conditions for such collusion in favour of a majoritarian rule are: a) the high rents from a single-party government, b) sufficient uncertainty over the electoral outcome and c) ideological proximity of the dominant parties.

Keywords: electoral reform, majority premium, single-party government, uncertainty, collusion

JEL classification: D72, H10

1 Introduction

The choice of the electoral rule is an important strategic decision made in all parliamentary democracies around the world. In terms of policy implementation, what is equally important to the outcome of the electoral process itself, is the realized allocation of parliamentary seats in the legislature, according to the applied electoral rule. This is so, because the ability of any government, single-party or coalition one, to implement its policies critically depends on its parliamentary strength. That is, the number of seats allocated to the winning party is a significant determinant of political power (Blais, 1991).

One could go one step further, and claim that it is the allocation of seats that matters the most. Under a *Proportional Representation* rule (hereinafter PR), a party that secures a convincing plurality of the vote share can sometimes fail to capture the absolute majority of parliamentary seats, whereas under some more *majoritarian* electoral rule (such as *First-Past-the-Post*) it could have occupied a larger share of parliamentary seats (Blais, 1991). That is, it would have been easier for an *office-motivated* party to form a single-party government and enjoy the spoils of office alone, or advance its policy agenda facing less opposition in the parliament[1]. It is exactly this feature of non-proportional electoral rules to distort the allocation of parliamentary seats in favour of the largest parties, that gives rise to their strategic manipulation by them. Hence, in this paper, we develop a model of electoral rule change, where the electoral rule is a strategic choice variable, optimally chosen by parliamentary parties within the context of electoral competition.

Of course, it can be argued that non-proportional electoral rules are deemed to be unfair and as such rejected by the political systems. Yet, a closer inspection of the real world reveals that in many countries electoral rules other than pure PR are actually applied in order to transform votes into parliamentary seats (Norris, 1997). With this observation in mind, a set of interesting question arises. Firstly, how strategic considerations and opportunistic incentives affect parties' decisions vis-à-vis electoral rule change. Secondly, which electoral rule do parties choose given their expected vote share in the forthcoming elections. And finally, what are the key determinants of the strategic choice of electoral rules. To put it more simply, in this paper we will try to explain why and how parliamentary polities choose their electoral rules and provide some empirical evidence to that.

This approach constitutes a significant diversification from the traditional body of literature on electoral rules. Rather than arguing in a Duvergerian manner (Duverger, 1954) that it is the electoral rule -possibly determined by some pre-existing constitutional arrangement- which is responsible for shaping the political environment, we turn this assumption upside down (Colomer, 2005). In fact, we explore how electoral uncertainty affects dominant parties' strategic incentives to collude and agree on electoral rule reform. The intuition behind this idea is relatively simple. Office-motivated parties derive utility from parliamentary seats but only indirectly. More seats increase their ability to influence the agenda when in opposition, or occupy more ministerial portfolios and pursue their policies with greater success within a coalition government. Moreover, if their seats exceed the parliamentary

[1] In fact, we do not make a distinction between executive and legislative office since in Parliamentary systems, unlike Presidential ones, if a Party controls or leads the executive it usually commands the majority in the Parliament.

majority threshold they can form a single-party government, thus, maximizing the perks of holding office.

In particular, when there is aggregate *uncertainty* about the electoral outcome, dominant parties[2] might have a joint incentive to collude and agree on an electoral rule reform. This strategic behavior is driven by their desire to form a *single-party* government and enjoy the spoils of the office alone. In other words, we expect that in multi-party systems with two dominant parties and competitive elections, the major players will collude in order to consolidate the *status quo*. As a result, a more majoritarian electoral rule will be chosen. In our context, the term "uncertainty with respect to the electoral outcome", implies that elections are competitive and contested by, at least, two parties that have a positive probability of winning (Andrews & Jackman, 2005). Moreover, we are interested in finding out whether dominant parties can also agree on a unique electoral rule reform proposal, other than PR.

To answer all these questions, we construct a multi-party model of Parliamentary democracy with two dominant parties and uncertainty over the electoral outcome. In such a framework, we derive the optimal electoral rule endogenously, as a strategic choice made by office-motivated parties through a parliamentary voting procedure. Their goal is to win as many parliamentary seats as possible, since, increasing the seat share serves a dual purpose. Firstly, it increases its bargaining power, in case of coalition formation, or its share of legislative office and control over the agenda, if in opposition. Secondly, as its seat share rises above a certain threshold, it secures the majority in the next parliament and eventually, the ability to form a single–party government and enjoy the full spoils of the office, both legislative and executive. That is, we consider that the marginal utility of an extra seat is less when the party's seat share exceeds the required majority. Then, the party has already maximized the spoils and any extra seat gains have a smaller impact.

This formulation of parties' preferences that incorporates their desire to form a single-party government is the first novelty introduced by our model. Fig.1 in the Appendix depicts such a utility function. The discontinuity captures parties' preference for a single-party government, due to higher rents and the concavity captures the decreasing marginal utility of an extra seat, as argued above.

Our second novelty to the literature of electoral rule reform is the introduction of uncertainty over the electoral outcome. Uncertainty plays a key role in our model since it is the driving force that allows dominant parties to collude in changing the electoral rule. The third novelty of our model is that we consider a broader class of electoral rules apart from PR and Plurality. In practice, following Sartori (1979) who claims that the most common distortions to the PR rule are the introduction of majority premie and of exclusion clauses[3], we allow our electoral rule reform proposals to take more generic forms. To capture all the possible degrees of dis-proportionality (from a pure PR to FPTP with multi-member districts) we introduce in the theoretical modeling the, so called, *majority premium system* (Sartori, 1976). This rule allocates a fraction of the seats according to PR and the rest are

[2]The term dominant party refers to a party that has a positive probability of winning the election, whereas the rest are minority parties with zero probability of coming first in the forthcoming election.

[3]Such as the 10% entry barrier in Turkey, the 5% in Germany, the 3% in Greece and many others in various parliamentary democracies.

given to the first party as a premium[4]. Hence, by varying the amount of the premium seats we can simulate electoral rules with different degrees of dis-proportionality.

Nevertheless, the above discussion on electoral rule reform omits one extra dimension that affects the choice of electoral rule, namely ideology. Therefore, in the last part of our paper we extend our model to allow for that parties having preferences defined over ideology. In such a case, strategic incentives to collude over adapting a majoritarian electoral rule conflict with ideology differences among the two dominant parties. Moreover, when the political system is characterized by extreme polarization and ideological divergence it is more likely to expect that the PR rule will be chosen over any majoritarian one. The intuition behind this is that by sticking to the PR, dominant parties insure against the risk of a diametrically opposed party forming a single-party government. This intuition is confirmed by some empirical examples, such as Italy and Greece, where extreme ideological polarization has blocked, until recently, any attempt to substitute the PR with a more majoritarian rule. Hence, the interaction between ideology and strategic choice of electoral rule will also be studied in this framework.

So far, the literature on electoral rule reform has focused solely on office-motivated parties. To allow for ideological preferences, in order to study how polarization and ideological divergence can affect electoral rule choice, is the final novelty introduced by our model. But before entering into the specifics of the model, we first give a brief summary of previous literature on electoral rule reform.

2 Literature Review and Motivation

Recent literature on electoral rule reform suggests that the existing variation in electoral rules across Parliamentary democracies is due to the strategic decisions that ruling parties make, anticipating the coordinating consequences of different electoral rules, in order to maximize their representation in the legislature, or even form a single party government. Boix (1999) asserts that if the electoral competition is less uncertain and the existing electoral rule serves the current ruling parties, then status-quo bias prevails and parties have limited incentives to modify the electoral rule. However, if the degree of uncertainty increases due to new voters, or change in their preferences, the ruling parties will consider changing the current electoral rule depending on two conditions: Firstly, the strength of the other parties and secondly, the coordinating capabilities of the dominant parties. Similarly, if the new entrants parties are expected to be weak, a non-PR rule is maintained regardless of the structure of the old political system (e.g. USA).

In the same spirit, Benoit (2004) studies the endogenous choice of electoral rules by parliamentary parties and develops a theoretical framework that classifies patterns of electoral rule change in various political systems. His theoretical model derives conditions for endogenous electoral rule change by *rational, seat-maximizing* political parties. It predicts that electoral rule change occurs endogenously, when two conditions are met. Firstly, if it exists a coalition of parties willing to agree on electoral rule reform, such that each of these parties is expected to score seat gains under the newly proposed rule. And secondly, if the

[4]Greece currenlty uses this electoral rule introduced in 2003 and used in two consecutive elections since 2007.

parties in this coalition can muster enough votes in the current parliament in order to enact this change.

Those results are in line with the theoretical predictions of our model. Yet, our approach differs in two dimensions. Firstly, we derive explicitly the conditions that allow the two dominant parties to agree on an electoral rule change from PR to a more majoritarian one. Secondly, we identify that their strategic behavior stems from their motivation for office. Rather than arguing that parties are simply rational seat maximizers, we take this approach on step further by claiming that parties' desire for more seats serves another end. It is their desire to form a single-party government and take full advantage of the "spoils of the office" that allows them to collude, in conjunction with electoral uncertainty. The latter allows those incentives to align and hence, leaves enough room for collusion. Our model requires that both parties, participating in the coalition that votes in favour of the electoral rule reform, have expected utility gains. The basic difference is that in our model, expected utility gains do not uniquely imply expected seat gains, but also an increase in the probability of forming a single-party government for all parties favoring the reform. Eventually, given agreement on changing the PR rule, only one of them will form a single-party government. Yet, in the presence of electoral uncertainty and in the absence of ideological polarization, this is utility enhancing in expected terms for both parties.

In parallel, Colomer (2005) presents and tests the hypothesis that in fact, it is the number of parties that can explain the choice of electoral rules, rather than the other way around. He argues that existing political parties tend to choose electoral system that allow them to "...crystallize, consolidate and reinforce..." the current party system instead of changing it dramatically. He also concludes that political systems that are dominated by few parties tend to establish majoritarian electoral rules. Our paper formalizes this idea by explicitly showing how the strategic incentives of the two dominant parties align, in order to consolidate the existing party system and increase their chances of forming a single-party government.

Furthermore, our model extends to show how the strategic choice of electoral rule by the parties interacts with ideology. Our paper builds upon those stylized facts and presents some empirical evidence to support the theoretical predictions of the model on how and why parties choose specific electoral rules, within a certain political environment. That is, we do not explicitly model party entry and its potential effect on electoral rule choice. Although, allowing for new party entry is out of the scope of this paper, our theoretical model, nonetheless, provides some insight on how party entry may affect the choice of electoral rule. Palfrey (1984) considers a two-dominant-party model with new party entry. But Palfrey's results relate to the position that the two dominant parties occupy in the political spectrum. Our analysis is distinct, since we focus on another strategic choice made by the two dominant parties, namely the electoral rule.

Finally, in a paper closely related to ours, Ergun (2010) studies the change of electoral rule from plurality to PR. He finds that for office motivated, rational, seat maximizing parties the following two conditions have to be satisfied for any change to take place. Firstly, the government must be formed by a coalition. And secondly, the larger the number of parties and the more equitable the distribution of the spoils amongst them, the more likely the change to a PR rule is. Starting from the opposite direction, Ergun adopts the same counter-Duvergerian approach. That is, strategic motivations and the party structure determine the choice of electoral rules.

Yet, our approach differs in four ways. Namely, we model electoral rule change under uncertainty, we introduce lexicographic preference for office motivated parties where the marginal utility of an extra seat is less when holding office, we introduce ideology as an additional factor influencing the choice of the rule, and finally, we allow for a broader set of electoral rules to be considered as candidates during the electoral reform process. The remainder of the paper is organized as follows. In Section 3, after introducing the basic structure of the model, we formally state our assumptions and definitions. In Section 4, we present our main results, followed by some discussion and the presentation of some stylized empirical evidence. In Section 5, we introduce an extension of the model to incorporate ideology as a determinant of electoral rule choice. Section 6 concludes the discussion and draws attention to the key points of the paper. Finally, the Appendix contains all the graphs, tables and proofs.

3 The Model

3.1 Basic Set Up and Preferences

Formally, we let $N = \{1, 2, 3\}$ to be the set of parties involved in the electoral competition. All parties in N are assumed to be represented in the parliament. Each party $i \in N$ holds a proportion of seats in the preceding parliament s_i^0 , such that $\sum_{i=1}^{n} s_i^0 = 1$. Moreover, parties have information about each other's expected vote share in the forthcoming elections. Party 3, is assumed to be a "minority" party. That is, its vote share shall never exceed that of Party 1 or Party 2. The expectations on future vote shares are formed by information that is commonly available to all Parties. The vote share of the "minority" Party 3, is assumed to be fixed at a level v_3, whereas, the vote shares of the other two parties are subject to uncertainty[5]. Formally, the vote share of Party 1 in the coming elections will be modeled as a random variable:

$$v_1 \sim \Phi_1,$$

where Φ_1 is a uniform distribution in $[a_1, b_1] \subset [0, 1]$.

Equivalently,

$$v_2 \sim \Phi_2,$$

where Φ_2 is a uniform distribution in $[a_2, b_2] \subset [0, 1]$.

Notice that these expectations need to satisfy $\sum_{i=1}^{n} v_i = 1$ and $v_3 \leq v_i$, $\forall i \in N$. That is, both a_1 and a_2 are bigger than v_3, both b_1 and b_2 are smaller than $1-2v_3$ and $a_2 = 1-b_1-v_3$ and $b_2 = 1-a_1-v_3$. The share of seats of party $i \in N$ in the new parliament will be defined as $s_i^l(v_i)$, where l will be the applied electoral rule. As stated before, we do not limit our

[5]Practically, our model can be viewed as a multi-party model in the following sense: v_3 can be thought of as the sum of the vote shares of various smaller parties. As long as v_3 is less than the vote share of the two dominant parties, our "minority" party can be the sum of all smaller parties.

attention to a single electoral rule. Rather, we want to consider the transition from PR to a wider range of possible electoral rules. In order to capture the big diversity of electoral systems existing in the world we will adopt the following mechanism l that is based on a variation of the PR using a majority premium, in a Sartorian manner (1976). The class of those rules allocates a number of seats as a majority premium to the first party. That is, l is the proportion of parliamentary seats allocated to the winning party as bonus. Obviously, $l \in [0,1]$ and, thereafter:

$$s_i^l(v_i) = v_i(1-l) \text{ if } v_i < \tfrac{1-v_3}{2}$$
and
$$s_i^l(v_i) = v_i(1-l) + l \text{ if } v_i > \tfrac{1-v_3}{2}.$$

Obviously, given our assumptions, the third party will never be entitled to the bonus as a result of never winning the election. The above seat allocation mechanism allows us to capture a wide variety of electoral rules, from pure PR to mixed systems and FPTP[6]. The first part is the proportional allocation of the seats minus the reserved premium, whereas the second part is the bonus given to the winner. The utility of a party $i \in N$ will be defined as:

$$u_i(v_1, v_2, v_3, l) = g s_i^l(v_i) + (1-g),$$
$$\text{where } g = 0 \text{ if } s_i^l(v_i) > 1/2 \text{ and } g = 1 \text{ if } s_i^l(v_i) \leq 1/2.$$

That is, parties in this environment are solely office motivated. They care about the proportion of the seats they hold, but only indirectly. When they lack the necessary parliamentary majority, an increase in their seat share increases their utility via two channels. Firstly it increases their bargaining power in the negotiations for the formation of a coalition government. The larger the number of seats that a party possesses, the larger its bargaining power. Consequently, the more MP's a party has, the greater its role in the future government will be (e.g. more ministerial portfolios to its members). Secondly, even if in opposition, higher seat share increases their control over legislative office (e.g. more committee chairpersons) which allows them to exercise more control over agenda-setting. Yet, when they control the majority, since their objective to form a single-party government and maximize the perks of office is satisfied, extra seats offer them smaller gains. This formulation of preferences, depicted in Fig.1, allows us to capture this very intuitive idea.

Practically, this type of preferences is known in economics literature as lexicographic. One can think of the ability of forming a single-party government as the "good" that the parties want to consume. Hence, the two dominant parties care about forming a single-party government. Only in case they cannot form one, because they lack the necessary parliamentary majority, they do care for extra parliamentary seats in order to increase their control over office, both legislative and executive. To put it more formally, the marginal

[6] To see this, consider that the case of $l = 0$. Then, our electoral rule is pure PR, whereas in the case of $l = 1$, it transforms into an one mutli-member district FPTP system (the most disproportional electoral rule possible). For values of l between those two extremes all the other electoral systems can be simulated. Tables 1 and 2 in the Appendix provide the exact calculations on how our proposed mechanism can replicate the electoral results in Greece (PR with majority premium) and the UK (single-member-district FPTP).

utility of an extra seat is larger when a party lacks the required parliamentary majority and has to negotiate with other parties to form a government or occupy legislative posts than when the party has the ability to form a single-party government. Such a utility function is depicted in Fig.2 and is a special case of the one shown earlier in Fig.1, since the marginal utility of an extra seat when the party already controls the majority is zero[7]. This is done solely for computational simplicity.

Lexicographic preferences for office motivated parties have also been used by Ergun (2010). Yet, our utility function also captures the desire of parties to form a single-party government. This is so, because it allows the marginal utility of an extra seat to be different depending on whether the party is above the parliamentary majority threshold or not. Parties' preferences over single-party governments can be motivated not only by rent-seeking behavior and desire to maximize the perks of the office. Tsebelis (1999) provides evidence that the increase of veto players in parliamentary democracies, and most notably the number of parties participating in government, is associated with legislative delay and lower production of significant laws, due to fierce bargaining among the coalition partners. Hence, the desire for single-party governments can also be motivated by an efficiency-maximizing behavior of the parties. In both cases, nevertheless, it is rational to expect parties preferring single-party governments.

3.2 The Game Structure

After defining the preferences of the players, we proceed by providing the structure of the electoral reform game. Formally, the game has three stages:

(i) The current rule is $l = 0$ (pure PR) and the party with the largest share of seats in the current parliament (either party one or party two) shall bear the role of the "Proposer" of an electoral reform. That is, it will propose $l \in [0, 1]$.

(ii) Parties vote on the "proposal" l. If the votes in favor of the reform surpass a given threshold $W \in [0, 1]$ (defined exogenously by the constitution) then, the electoral reform passes and the forthcoming elections take place according to the new rule l^*. In the opposite case, that is, if the proposal does not gather the necessary parliamentary support W, the electoral reform is cancelled and future elections are conducted according to the proportional rule $l = 0$.

(iii) Elections take place and each party, according to its vote share and the applied electoral rule, l^*or $l = 0$, takes its new seat share in the Parliament and computes its utility.

3.3 Understanding the Proposer's Problem

For simplicity, let us assume without any loss in generality, that the Proposer is always Party 1. That is, we assume that $s_1^0 > s_2^0 > s_3^0$. Since the electoral rule determines the seat allocation for the parties and thus, their utility, Party 1 will propose an electoral rule $l \in [0, 1]$ such that, given the threshold $W \in [0, 1]$, it maximizes its expected utility. That is, if $W \leq s_1^0$ Party 1 will propose $l \in [0, 1]$ such that $l \in \arg\max\{Eu_1(v_1, v_2, v_3, l)\}$.

[7] In this specification the MU of an extra seat when $s_i^l(v_i) \leq 1/2$ is equal to $\frac{\partial u_i}{s_i^l(v_i)} = g = 1$, whereas when $s_i^l(v_i) > 1/2$, we have that $\frac{\partial u_i}{s_i^l(v_i)} = 0$. Clearly, $u_i'|_{s_i^l(v_i) \leq 1/2} > u_i'|_{s_i^l(v_i) > 1/2}$, as required.

In other words, if the current seat share of the Proposer exceeds the required threshold for implementing an electoral reform, the Proposer faces an unconstrained maximization program.

On the contrary, if $W > s_1^0$ the proposer needs the support of at least one of the other two parties in order to implement an electoral rule reform. An obvious, but nonetheless useful Lemma, demonstrates the behavior of the minority Party 3 in such cases. We state it below.

Lemma 1 *The minority party never consents to any electoral reform proposal.*

Proof. Since the minority party expects to receive the premium l with probability zero, it just expects utility losses from any distortion in the proportionality of the electoral rule. Its expected utility from any electoral rule is: $Eu_3(v_1, v_2, v_3, l) = v_3(1-l)$, which is clearly decreasing in l. ■

Given the above observation, Party 1 will have to secure Party 2's support in order to proceed with an electoral rule reform. That is, it has to propose $l \in \arg\max\{Eu_1(v_1, v_2, v_3, l)\}$ s.t. $Eu_2(v_1, v_2, v_3, l) \geq Eu_2(v_1, v_2, v_3, 0)$, facing this time a constrained maximization program (Participation Constraint[8] of Party 2).

In general, since we have assumed that v_3 is fixed and common knowledge and that $v_1 = 1 - v_2 - v_3$, the Proposer faces one source of uncertainty (information about v_1 is equivalent to information about v_2). Thus, the proposer's expected utility is given by expression:

$$Eu_1(v_1, v_2, v_3, l) = \tfrac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l)+l]dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} dv_1]$$

and equivalently, Party's 2 expected utility is:

$$Eu_2(v_1, v_2, v_3, l) = \tfrac{1}{(b_2-a_2)}[\int_{a_2}^{(1-v_3)/2} v_2(1-l)dv_2 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_2(1-l)+l]dv_2 + \int_{\frac{1/2-l}{1-l}}^{b_2} dv_2].$$

Notice that there exist two critical vote shares. The first one, $(1-v_3)/2$, defines the necessary vote share for one of the two parties to be first (and get the premium l). The second, $\frac{1/2-l}{1-l}$, is the vote share that the first party needs in order to get the majority of seats in the parliament, given electoral rule l. Obviously, if $(1-v_3)/2 \geq \frac{1/2-l}{1-l}$ the first party will have the a majority in the parliament as long as it wins and thus, the expected utility of Party 1 will be:

$$Eu_1(v_1, v_2, v_3, l) = \tfrac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} dv_1]$$

and for Party 2:

$$Eu_2(v_1, v_2, v_3, l) = \tfrac{1}{(b_2-a_2)}[\int_{a_2}^{(1-v_3)/2} v_2(1-l)dv_2 + \int_{(1-v_3)/2}^{b_2} dv_2]$$

[8]Hereinafter $Eu_2(v_1, v_2, v_3, l) \geq Eu_2(v_1, v_2, v_3, 0)$ is referred to as the PC.

As $\frac{1/2-l}{1-l}$ is decreasing in l, an increase in the dis-proportionality of the electoral rule (an increase in l) not only affects the potential seat gains for a party that runs first in the elections, but also increases the party's probability of having the majority of seats in the parliament and hence, forming a single-party government. It is this dual impact of the electoral rule on parties' utility that makes the electoral reform process such an important strategic decision.

3.4 Definitions

We will classify the results given the following definitions.

Definition 1 *The electoral reform process is trivial if* $W \leq s_1^0$

When the Proposer (Party 1) has a large enough proportion of seats in the current Parliament so as to choose the electoral rule at will, we shall call the reform process is a trivial one, as it only depends on the preferences of the Proposer.

Definition 2 *The electoral reform is possible if* $W \leq 1 - v_3$

This is a direct implication of Lemma 1. The "minority" party never consents to an electoral reform. Thus, if a reform is to take place, W must be such that Party 3 cannot block it.

Definition 3 *The electoral competition is trivial if either* $a_1 > \frac{1-v_3}{2}$, *or* $b_1 < \frac{1-v_3}{2}$

The above definition just describes the case that the probability of Party 1 running first in the coming elections is either 1 or 0. In such cases the winning party, which will also receive the premium l, is known with certainty. On the other hand, when electoral competition is non-trivial, both Parties 1 and 2 have a positive probability of winning and thus, getting the majority premium l.

Definition 4 *The Proposer (Party 1) is the "leading" party if and only if* $\frac{a_1+b_1}{2} > \frac{1-v_3}{2}$

If a party is expected to run first in the elections, then it shall be called the leading party. Since $E(v_1) = \frac{a_1+b_1}{2}$, and $\frac{1-v_3}{2}$ is the threshold above which Party 1 is the winner, Party 1 is the leading party if and only if $E(v_1) > \frac{1-v_3}{2}$. Alternatively, Party 2 is the leading one.

4 Results

Always assuming that an electoral reform is possible ($W \leq 1-v_3$) we can state the following results.

Proposition 1 *When both the electoral reform process and the electoral competition are trivial and the proposer is: (i)the leading party, then* $l^* \geq \max\{0, \frac{1/2-a_1}{1-a_1}\}$*, (ii) not the leading party, then* $l^* = 0$*.*[9]

This result can be viewed as the simplest case scenario. The idea behind this proposition is very simple. Since the electoral reform process is trivial, the proposer holds enough seats in the current parliament to enact any electoral rule reform, without the need to satisfy the PC of Party 2. Hence, the proposer just chooses l^* in order to maximize expected utility. Given that the electoral competition is trivial, if the Proposer is not the leading party (i.e. sure loser) it proposes that the PR rule is not amended ($l^* = 0$). As a result, the electoral rule does not change. Otherwise, it proposes $l^* \geq \max\{0, \frac{1/2-a_1}{1-a_1}\}$. That is, if Party 1 is the leading party (i.e. sure winner), the proposed proportionality distortion l^* will be such that it will guarantee the formation of a single-party government by Party 1. The solution to this unconstrained maximization problem yields with certainty the majority of the seats in the parliament for every possible realization of a_1. Hence, by choosing this level of l^* the leading party ensures the highest possible level of utility. On the other hand, if it is not the leading party it can never get the premium l^*, making its utility strictly decreasing on l^*. Hence, any distortion to the PR rule is not desirable.

Proposition 2 *When the electoral competition is trivial but the electoral reform process is non-trivial, then* $l^* = 0$*.*

The idea that drives the result is that in this case there is no room for collusion. The strategic incentives of the two dominant parties do not align, because there is no uncertainty over the outcome of the electoral competition. The leading party will always prefer a value of $l^* > 0$ but the other party will always reject this proposal because its utility is strictly decreasing in l. Since the electoral reform is non-trivial and requires the consent of both dominant parties, it is obvious why no electoral rule reform will ever be enacted by this parliament. That is, the status-quo is maintained and PR persists as the electoral rule (i.e. $l^* = 0$).

Proposition 3 *When the electoral competition is non-trivial then* $l^* \in \{0, \frac{v_3}{1+v_3}\}$*.*

This is the main result of our paper. In an environment of uncertainty about the electoral outcome, the proportionality distortion l^* that the proposer might introduce into the electoral rule as a majority premium, will be such that it guarantees to the winning the majority of seats in the parliament. That is, in case Party 1, alone or with the support of Party 2, sponsors an electoral rule reform, it will be such that it consolidates the two party system and the current status quo. This result is a summary of the results in the next two propositions. Hence, we restrict further analysis of this result in the following section to combine Propositions 3, 4 and 5 together.

We now present two Propositions that build on the previous result and state explicitly the necessary and sufficient conditions for an electoral rule change to take place. But, before doing so, for expositional ease we define function $f_i(a_i, b_i, v_3)$ which measures the expected gains (or losses) of accepting an electoral reform l^*.

[9] All Proofs in the Appendix.

Definition 5 *For $i = \{1, 2\}$, define:* $f_i(a_i, b_i, v_3) = (b_i - \frac{1-v_3}{2})(2 - b_i - \frac{1-v_3}{2}) - (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_i)(\frac{1-v_3}{2} - a_i)$*, if* $b_i < \frac{1}{2}$ *and* $f_i(a_i, b_i, v_3) = (\frac{1}{2} - \frac{1-v_3}{2})(\frac{3}{2} - \frac{1-v_3}{2}) - (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_i)(\frac{1-v_3}{2} - a_i)$*, if* $b_i \geq \frac{1}{2}$.

Whenever $f_i(a_i, b_i, v_3) \geq 0$, it means that parties expect gains from an electoral rule change. That is $Eu_i(v_1, v_2, v_3, l^*) \geq Eu_i(v_1, v_2, v_3, 0)$, which in turn implies that the PC is also satisfied. Hence, $f_i(.)$ can be viewed as a re-stated version of the PC. Intuitively, $f_i(.) \geq 0$, implies that Party i has enough chances of winning the elections. Otherwise, the opposite is true.

Proposition 4 *When the electoral competition is non-trivial but the electoral reform process is trivial ($W \leq s_1^0$), then the Parliament departs from PR ($l^* = 0$) and adapts $l^* = \frac{v_3}{1+v_3}$ if and only if $f_1(a_1, b_1, v_3) \geq 0$.*

Proposition 5 *When both the electoral competition and the electoral reform process are non-trivial then in order for the Parliament to depart from PR ($l^* = 0$) and adapt $l^* = \frac{v_3}{1+v_3}$ there are two sets of both necessary and sufficient conditions: EITHER (i) the Proposer is the leading party and $f_2(a_2, b_2, v_3) \geq 0$ is satisfied OR (ii) the Proposer is not the leading party and $f_1(a_1, b_1, v_3) \geq 0$.*

In the remainder, we will provide an idea of the proof, followed by a discussion for each case separately, since this is the main result of the paper. First of all, we note that formally the proof is derived from the maximization program of the proposer. As shown in the Appendix and in Fig. 4, $Eu_i(v_{1,}v_{2,}v_{3,}l)$ for $i = \{1, 2\}$ is convex with respect to l (and strictly convex for some values of l) for $l \in [0, \frac{v_3}{1+v_3}]$ and decreasing for $l \in (\frac{v_3}{1+v_3}, 1]$. Hence, the only two candidates for an optimum are either $l^* = 0$ or $l^* = \frac{v_3}{1+v_3}$. Here, electoral competition is non-trivial. Moreover, when the electoral reform process is also non-trivial the Proposer faces a *constrained* maximization program. That is, it maximizes its expected utility $Eu_1(v_1, v_2, v_3, l)$ by choosing $l^* s.t.\ l^* \in \arg\max Eu_1(v_1, v_2, v_3, l)$, satisfying at the same time the PC of Party 2 ($f_2(.) \geq 0$)[10].

We will first argue why the two candidate equilibrium values of l^* are either $l^* = 0$ or $l^* = \frac{v_3}{1+v_3}$. Technically speaking, those two values of l^* are in fact the corner solutions of the maximization program. That is, in equilibrium the proposer proposes one of those two strategies (notice that in each case the equilibrium is unique). Fig. 3 helps illustrating this point.

Ideally, the proposer would prefer to propose the PR rule (i.e. $l = 0$), which is the top of the three lines, if it knew that it will run second and propose $l = \frac{v_3}{1+v_3}$, which is the bottom line, otherwise. Yet, in our environment there exist uncertainty over the outcome of the election. Hence, the proposer has to compare the *expected gain* from proposing the PR ($l = 0$) and running second (area ABCD in the graph)[11], with the *expected gain* from winning in the election and forming a single party government, having proposed $l = \frac{v_3}{1+v_3}$

[10] Recall that by Lemma 1 the minority party always prefers the PR rule and never agrees to accept any l^* other than $l^* = 0$.

[11] Or equivalently, ABCD is the *expected loss* from proposing $l = \frac{v_3}{1+v_3}$ and running second.

(area DEFH)[12]. This statement, graphically depicted in Fig.3, is mathematically expressed by function $f_i(.)$, which measures the difference between areas (DEFH) and (ABCD). Given that Parties 1 and 2 are symmetric the same analysis applies for the receiver of the proposal in deciding whether to accept it or not.

Now, the intuition behind these results is more clear. In case the electoral reform process is trivial the proposer proposes $l = \frac{v_3}{1+v_3}$ whenever the expected gain exceeds the expected loss, and proposes $l = 0$ otherwise. In case the electoral reform is *non-trivial,* and the proposer is the leading party it always prefers to propose $l = \frac{v_3}{1+v_3}$, since for Party 1 expected benefit always exceeds expected loss[13]. But, when the Proposer faces a constrained maximization program it has to satisfy the PC of Party 2, which is analogous to the previous inequality for Party 1. Whenever the PC of Party 2 is satisfied (i.e. $f_2(.) \geq 0$), Party 1 proposes $l = \frac{v_3}{1+v_3}$, which is always accepted. Otherwise, Party 2 will reject any proposal $l \neq 0$. As result, Party 1 proposes the PR ($l = 0$). The case where the Proposer is not the leading party is, in fact, the other side of the same coin. Party 2 is now the leading party and as a result, its PC is satisfied. Hence, it always accepts a proposal of $l = \frac{v_3}{1+v_3}$. But now, it is Party 1 that will propose $l = \frac{v_3}{1+v_3}$ whenever its expected gain exceeds its expected loss ($f_1(.) \geq 0$), despite not being the leading party. Otherwise it proposes $l = 0$. This completes the intuitive argument.

4.1 Discussion of the Results and Assumptions

In this section, we discuss in greater detail the implications of the main results of our model which are stated in Propositions 3, 4 and 5. The main implication of Proposition 3 is that the two dominant parties strategically choose an electoral rule that crystallizes and consolidates the two-party system. In fact, the optimal level of l ($\frac{v_3}{1+v_3}$) is such that it completely eliminates the effect of the third party on the political competition. Of course, the third party never stood any chance of winning elections, not to mention forming a single-party government. Yet, in the absence of a majoritarian electoral rule, such as the one with the premium, the third party could play a role in the formation of coalition governments. In many instances, it would have had the necessary parliamentary seats in order to influence the coalition government and its pursued policies. But in our model, as Proposition 3 implies, the two dominant parties have aligned strategic incentives to collude in order to eliminate the political impact of the third party. Hence, they consolidate the two-dominant-party environment by endogenously choosing the level of the premium l to be $\frac{v_3}{1+v_3}$. Moreover, in Propositions 4 and 5, we show that this happens under relatively mild assumptions. That is, we only require that there is some uncertainty over the electoral race, which has to be competitive.

Therefore, our model provides a theoretical framework for studying the reverse statement of Duverger's Law. That is, we show how the strategic choice of electoral rule can affect the nature of political competition. Starting upside down, we construct a model of electoral

[12] Or equivalently, DEFH is the expected loss from winning in the election but not being able to form a single-party government because it has proposed the PR ($l = 0$).

[13] Being a "leading party" ($\frac{a_i+b_i}{2} > \frac{1-v_3}{2}$) implies $f_i(.) > 0$. But, also notice that in case of expected ties ($\frac{a_1+b_1}{2} = \frac{a_2+b_2}{2} = \frac{1-v_3}{2}$) we still have $f_i(.) > 0$ for $i = 1, 2$. Hence, by continuity of $f_i(.)$ it is still possible to have $f_i(.) > 0$ even if i is *not* the "leading party".

rule reform where parties endogenously choose the electoral rule, in a way that reinforces the status-quo, in our case the two-dominant-party system. The drivers behind this result are two. Firstly, uncertainty over the electoral outcome and secondly, parties' desire to form single-party governments, once they win elections. The latter, generates the strategic incentive for the dominant parties to distort the PR rule. The former aligns their incentives and allows them to collude by proposing a more majoritarian rule than the PR, in order to eliminate the impact of the third party. Hence, uncertainty over the electoral outcome is a *sine-qua-non* condition for coordination. Otherwise, the room for collusion disappears. It is exactly this combination that allows the incentives to align, under some conditions. Those are the PC's of the two dominant parties. Regardless of being or not the leading party, each of the two will agree on electoral rule change if and only if expected gains from an electoral reform (that introduces a majority premium to the PR) are greater than expected losses. In such a case, we can see that the two parties agree on a unique proposal $l = \frac{v_3}{1+v_3}$. Otherwise, as expected, no electoral rule reform takes place. As a result, we are stuck with the PR rule.

Another equally important implication of Propositions 3, 4 and 5 is the fact that the equilibrium outcome is unique. Once the two dominant parties agree to depart from the PR rule, there is a unique value l that is proposed and accepted. Once expected gains from distorting the PR rule exceed expected losses, the two parties face the strategic decision on how large premium l to propose. The answer is unique and it is $\frac{v_3}{1+v_3}$. In fact, the value of l is such that it is the minimum required majority premium that always guarantees to the winner the ability to form a single-party government. This is so, because Eu_i is decreasing with respect to l on $(\frac{v_3}{1+v_3}, 1]$ but convex on $[0, \frac{v_3}{1+v_3}]$. This is depicted in Fig.4 in the Appendix.

The strategic dilemma that parties face is summarized in the following question: "Which electoral rule guarantees a single-party government to the winner but on the same time minimizes the loss of parliamentary seats in case of defeat, given that the outcome is uncertain?". We already know the answer, and it is $\frac{v_3}{1+v_3}$. This is an important feature of the model. It implies that parties, not only can agree to distort the PR rule in order to consolidate the two-party system, but they can also agree on a *unique* new electoral rule.

This point hints on the idea that, for any given political environment, there exist a unique *optimal* electoral rule. In order to check this claim we try to test the predictions of our model on real electoral outcomes from Greece and the UK. Tables 1 and 2 present the actual electoral results, the vote and seat share allocations, the "actual" majority premium $\tilde{l}$[14] and the optimal one l^*[15]. We also present the observed deviations between the actual and the predicted values. Our aim is to test the fit of the model to real electoral data.

In fact, our model seems to perform quite well in predicting the direction of electoral rule change in Greece. Greece underwent four major electoral rule reforms in the period under consideration, namely in 1988, 1991, 2003 and 2006. As one can observe in Fig.5a and 5b, from 1981-1985 the actual electoral rule is less proportional than the optimum. Yet,

[14] As we have previously said, our model can accomodate a wide range of electoral rules. Hence, the actual majority premium $\tilde{l}$ is computed as $\tilde{l} = \frac{\tilde{s}_1 - \tilde{v}_1}{1-\tilde{v}_1}$ and corresponds to the value of the premium when we allow the actual seat allocation to be replicated by our model. That is, we calibrate our model to the actual results.

[15] The optimal premium is just $l^* = \frac{v_3}{1+v_3}$, as predicted by the model.

during the first electoral reform PR is adapted which results in the rule being now more proportional than the optimum. As a result, another reform follows that makes again the rule more majoritarian. During the third electoral reform, which was first applied in the 2007 elections[16] a 40-seat majority premium is adopted, quite close to the predicted optimum. Finally, the greek parliament amended the electoral rule in 2006[17] and introduced a 50-seat premium, in complete accordance with the theoretical predictions of the model. Moreover, with the exception of the 1989 elections, the observed gap between the actual and optimal majority premium is shrinking over time. On aggregate, we observe that the actual and optimal values are not statistically different from each other. That is, the greek parliament has chosen its electoral rule "optimally", according to our model.

Furthermore, observing the UK Parliamentary Election outcomes from WW II and onwards, in Table 2 and Fig.6a and 6b, we can see that although Britain did not undergo any major electoral rule reform during this period, our model can say something about the intended reform under consideration by the coalition government. It is clear that the FPTP rule, in use in the UK, is more disproportional than required by our theory[18]. In fact, our calculations show that on average, and with two notable exceptions where elections failed to produce a single-party government, the majority premium was larger than required for the winner to form a single-party government. Most notably, with the exception of the 1951 elections[19], our model would yield the same results in terms of outcomes. That is, single-party governments would have been achieved, as Fig.6b shows. Moreover, the magnitude of the "extra" parliamentary majority granted to the winner, according to actual data, is very close to the predicted difference between the actual and the optimal premium. This becomes more relevant when one takes into account the projection of the seat allocation that would have resulted under the AV rule[20]. Hugh-Jones (2011) claims that "electoral changes from AV may be marginal". Results show that a more proportional allocation of seats would have been achieved but "...they also do not support the argument that AV will lead to endless coalitions". In a sense, for the British context, the AV seems to fit our definition of an "optimal" electoral rule.

On a final note, some clarifications are in order. First, with respect to the structure of the bargaining process defined in our game. One might worry that the results presented so far critically depend on the fact the structure of the bargaining process between the two dominant parties does not allow for counter-proposals[21]. Whereas, this statement would have been generically true in any other context, in our particular set up enriching the bargaining process plays absolutely no role in driving the results. The reason for this is the shape of the Eu_i, shown in Fig.4. As stressed in Proposition 3, due to the convexity of Eu_i, there are only two candidate-values of l for an optimum. And it is also true that by assuming a *non-trivial* electoral reform process, one of the two parties can always guarantee its most

[16] By then, the Constitution has changed and demanded that electoral rule reform is implemented with a lag.

[17] Although this rule was first applied in the 2009 Parliamentary Elections.

[18] In almost all cases the optimal premium is less than the actual.

[19] Where Labour won the majority of public votes but Conservatives won the majority in the Commons.

[20] See Sanders, D., H. D. Clarke, M. C. Stewart, and P. Whiteley (2011), "Simulating the Effects of the AV in the 2010 UK General Election" in *Parliamentary Affairs 64(1): 5* and Hugh-Jones D. (2011), "Simulating the Effects of the AV in the 2010 UK General Election: A Note", mimeo.

[21] We adopt a "take-it-or-leave-it" bargaining protocol.

preferred outcome, $l = 0$, in case there is no agreement between them. That is, it has a veto power, since it can block the electoral rule reform. Hence, the existence of an alternative bargaining process, where Parties could engage in consecutive counter-proposals of electoral rules, is equivalent to our set-up and the results obtained under any such formulation would have been identical. Therefore, for simplicity, but without any loss in generality, we refrain from adapting a more complex bargaining protocol.

Secondly, a simple comparative statics analysis on the optimal value of l, $l^*(v_3) = \frac{v_3}{1+v_3}$, with respect to the size of the minority party can yield some interesting insight on new party entry and electoral rule reform. Although, as stressed in the introduction, our model does not aspire to explore party entry in this set up, the minority party can be viewed as the sum of many small parties, as long as they do not stand any chance of outperforming any of the two dominant ones. As such, our model can partially accommodate the entry of a new party, modeled as an increase from v_3 to v_3', as long as the new entrant is not expected to upset the dominance of the other two parties. That is, we still have $v_3' < a_i$, $i \in \{1, 2\}$. Then, one can easily check that $\frac{\partial l^*(v_3)}{\partial v_3} = \frac{1}{(1+v_3)^2} > 0$ implies that the entry of a new party causes the electoral rule to become more majoritarian. In fact, this point is made by Boix (1999) who asserts that whenever the new entrants are weak, a majoritarian electoral rule is maintained (or reinforced) regardless of the structure of the old political system.

5 An Extension of the Model: Electoral Reform and Ideology

So far, our discussion has attempted to shed some light on how and why electoral rule change takes place in parliamentary democracies, as a result of strategic choice by the parties. Yet, we did not provide an account on how ideology might influence this choice. And although we do not aspire to conduct an exhaustive analysis on the role of ideology in electoral rule reform, we will provide some results that yield useful insights on the interaction between ideology and electoral rule choice. As one might have observed, the predictions of our model might fit well the Greek and British general election data. Nevertheless, there are some notable cases (e.g. Italy), where a two-dominant-party system failed to produce a more majoritarian electoral rule than the PR. Practically, from the end of WW II and until the proposed electoral reform of 1993, Italy used the PR rule. Nonetheless, political competition was dominated by two major parties (PCI and CD[22]), especially during the mid-1970s when they reached the peak of their appeal to the voters. Then, one might expect that according to our predictions, PR should have been substituted with a more majoritarian rule. We will show, in this extension of the basic model, that this did not happen due to the extreme polarization and ideological divergence that persisted in the Italian political system during the "Cold War" era.

The intuition behind the "Italian Paradox" is that both parties utilized the PR rule as an implicit insurance mechanism against the risk of their ideological opponent forming a single-party government. That is, strategic incentives to collude were mitigated, or even cancelled off, due to extreme ideological divergence. As a result, the PR rule was sustained in order

[22] PCI is the Italian Communist Party and CD is the Christian-Democrat one.

to insure against the risk of having a single-party government led by a polar-opposite party. Hence, when ideology comes into play, strategic incentives are reversed. In the remainder of this section, we will built into our model parties' preferences over ideology, and explore its effect on electoral rule reform. Nevertheless, we need to stress that the scope of this section is limited into a symmetric case, in order to motivate the "Italian Paradox". Of course, more analysis is warranted on the impact of ideological polarization on electoral rule choice. We leave this task for future research.

5.1 The Set-Up

Formally, we focus on the symmetric case where, in expected terms, the two dominant parties are of equal electoral size and also symmetrically positioned in terms of ideology. That is, we have that $a_1 = a_2$ and $b_1 = b_2$[23]. Hence, both are "leading parties" with equal chances of winning the election. Moreover, we let $[-1, 1]$ be the ideology space. Then, each party $i \in N$ has its own ideology denoted by $x_i \in [-1, 1]$, which will implement if it forms a single-party government. If not, then we assume that a grand-coalition of all parties is formed. As a result, each party influences the ideology of the coalition relative to its own seat share $s_i^l(v_i)$. Symmetry in terms of ideological distance also implies that: $x_1 = -\varepsilon$, $x_2 = \varepsilon$ and $x_3 = 0$, such that $\varepsilon \in [0, 1]$. That is, the Proposer (Party 1) is the leftist party, Party 2 is the rightist one and the minority Party 3 is positioned in the centre, equidistant from the other two. The utility of a party $i \in N$ shall be defined as:

$$u_i(v_1, v_2, v_3, l) = g s_i^l(v_i) + (1 - g) - (x_i - \pi)^2,$$
$$\text{where } g = 0 \text{ if } s_i^l(v_i) > 1/2 \text{ and } g = 1 \text{ if } s_i^l(v_i) \leq 1/2.$$

Clearly, $s_i^l(v_i)$ and l are defined as before. The extra term in the function measures the disutility each party receives when ideology other than its own gets implemented. So, $\pi(l, v_i)$ denotes the implemented ideology and clearly is a function of l and v_i, since it depends on the electoral outcome, the electoral rule and the resulting allocation of seats. Thus, a brief comment with respect to π and the formation of government is in order.

In case we have a single-party government, $\pi = x_i$ iff $s_i^l(v_i) > 1/2$. But in case no party has the necessary parliamentary majority, the grand-coalition chooses its π according to the relative parliamentary strength of each of its members[24]. But given symmetry, it is equally likely that the major partner in the coalition government will be one of the two dominant parties. Hence, in expected terms, the ideology implemented will be that of the median[25], that is $\pi = 0$, which in our particular case happens to coincide with the ideology of the centrist party x_3. Then, the Proposer faces the following maximization program[26]:

[23] This implies that: $b_1 = 1 - a_1 - v_3$, which in turn implies that $\frac{a_1+b_1}{2} = \frac{a_2+b_2}{2} = \frac{1-v_3}{2}$.

[24] One can think of a coalition government distributing the ministerial portfolios to the parties based on their parliamentary strength.

[25] We stress that this statement is not an assumption. It is trivially derived when one considers that the ideology of the median is the Condorcet winner.

[26] In this section we focus only on non-trivial electoral reform processes.

$$\max_{l^*\in[0,1]} \left\{ \begin{array}{c} Eu_1(v_1,v_2,v_3,l) = \frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int\limits_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l)+l]dv_1 + \int\limits_{\frac{1/2-l}{1-l}}^{b_1} dv_1 \\ -\int\limits_{a_1}^{1-v_3-\frac{1/2-l}{1-l}} (-\varepsilon-\varepsilon)^2 dv_1 - \int\limits_{1-v_3-\frac{1/2-l}{1-l}}^{\frac{1/2-l}{1-l}} (-\varepsilon-0)^2 dv_1 - \int\limits_{\frac{1/2-l}{1-l}}^{b_1} (-\varepsilon-(-\varepsilon))^2 dv_1] \end{array} \right\}$$

$$\text{s.t. } Eu_2(v_1,v_2,v_3,l) \geq Eu_2(v_1,v_2,v_3,0).^{27}$$

First of all, we note that the first three terms are identical with the maximization program presented in the previous section. Moreover, we can simplify the second line of the expression that refers to the ideological component of the utility by applying the symmetry conditions. Clearly, the probability of Party 1 forming a single-party government and implementing its ideology is identical with that of Party 2 doing the same. That is, $1 - v_3 - \frac{1/2-l}{1-l} - a_1 = b_1 - \frac{1/2-l}{1-l}$. Then with probability $1 - 2(b_1 - \frac{1/2-l}{1-l})$, no party succeeds in forming a single-party government. Thus, the ideology of the median is implemented. Hence we can rewrite the Proposer's Eu_1 as follows:

$$Eu_1(v_1,v_2,v_3,l) = \frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int\limits_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l)+l]dv_1 + \int\limits_{\frac{1/2-l}{1-l}}^{b_1} dv_1$$
$$-4\varepsilon^2(b_1 - \tfrac{1/2-l}{1-l}) - [1 - 2(b_1 - \tfrac{1/2-l}{1-l})]\varepsilon^2 - 0(b_1 - \tfrac{1/2-l}{1-l})]$$

As before, a critical value for l is $l = \frac{v_3}{1+v_3}$, since for every $l \in (\frac{v_3}{1+v_3}, 1]$ the above expression collapses to[28]: $Eu_1(v_1,v_2,v_3,l) = \frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int\limits_{(1-v_3)/2}^{b_1} dv_1] - 2\varepsilon^2$. Finally, we note that apart from imposing symmetry the rest of the model remains as specified in Section 3. Before proceeding to the main results of this section, we will first present a simple illustrative example.

5.2 The Case of Extreme Ideological Divergence

Here, we assume that $\varepsilon = 1$. That is, the two dominant parties occupy polar opposite positions in the ideology space. Thus, the model exhibits its maximum ideological divergence. It is interesting to compare the results of this special case with the results of the standard version of our model, presented in Section 4. There we have implicitly assumed that $\varepsilon = 0$, since there was no ideological component present. That is, we compare the two extreme cases, with minimal and maximal ideological divergence. Then, we will generalize our findings for all values of ε, still maintaining the symmetry conditions.

Proposition 6 *Assume symmetry and a non-trivial electoral reform process. Then, (i) when $\varepsilon = 0$ the optimal choice of electoral rule is $l^* = \frac{v_3}{1+v_3}$, (ii) when $\varepsilon = 1$, the optimal choice is the PR rule ($l^* = 0$).*

[27] By symmetry Party 2 faces an analogous program.

[28] By symmetry, when $l = l^*$ we have that: $b_1 - \frac{1/2-l}{1-l} = b_1 - \frac{1-v_3}{2} = \frac{b_1-a_1}{2}$.

This simple example demonstrates the role of ideological polarization in the strategic choice of electoral rule. Whereas, under complete ideological convergence the two dominant parties were able to collude and substitute the PR rule with a more majoritarian one, in the presence of extreme ideological divergence the incentives to collude disappear. Moreover, they have an incentive to use the PR rule as an implicit insurance in case they fail to win the elections. And although they still care about forming a single-party government, their risk-aversion over the prospect of their ideological opponent doing the same forces them to stick with the PR rule. This could potentially explain why Italy never departed from the PR, especially during the 70s, when polarization and ideological divergence were at their peak. Moreover, after the collapse of the Soviet Union the subsequent separation of the Italian Communist Party (PCI) resulted in a significant decrease in its electoral power and hence of the ideological polarization, since the moderate centre-left fraction emerged as the dominant one. This was confirmed in the 1992 elections. A proposal to adopt a more majoritarian electoral rule followed in 1993.

5.3 The General Case

So far, we focused on the case of extreme ideological divergence ($\varepsilon = 1$). Yet, there are societies where ideological divergence and polarization might exist but at the same time might not be that extreme. Hence, it is important to explore whether the effect of ideology on the choice of electoral rule is still persistent. Therefore, we present the results for the more general case below. Needless to say, symmetry is assumed throughout this section.

Proposition 7 *Assume the electoral reform process is non-trivial. Let $\varepsilon \in [0,1]$. Then $\exists$ ε', and ε'', satisfying $\varepsilon' > \varepsilon'' > 0$ such that: (i) $\forall \varepsilon \in (\varepsilon', 1]$ we have $l^* \in \{0\}$; (ii) $\forall \varepsilon \in (\varepsilon'', \varepsilon']$ we have $l^* \in \{0, l^{**} | \; l^{**} \leq \frac{v_3}{1+v_3}\}$ and (iii) $\forall \varepsilon \in [0, \varepsilon'']$ we have $l^* \in \{0, \frac{v_3}{1+v_3}\}$.*

The above result demonstrates how ideological divergence affects the strategic choice of electoral rule by the parties. It is clear that as ideological divergence increases we end up with more proportional electoral rules. Moreover, for significantly large values of ε, PR is the unique choice of electoral rule that parties are willing to accept. These are exactly the situations where the PR is used as an insurance against the prospect of facing a single-party government with polar-opposite ideology. For intermediate values of ideological divergence, we observe that dominant parties might still collude in choosing a less proportional rule. Yet, compared to the standard case where ideology is absent ($\varepsilon = 0$), the majoritarian rule chosen is less disproportional from $l^* = \frac{v_3}{1+v_3}$, which used to be the optimal choice. Finally, for smaller values of ε, parties face an identical choice as before. But still, even though we do not explicitly compute them, there are some values of $\varepsilon \in [0, \varepsilon'']$ such that $Eu_i(v_1, v_2, v_3, l = 0) > Eu_{i1}(v_1, v_2, v_3, l = \frac{v_3}{1+v_3})$ for $i = \{1, 2\}$. That is, even in this case the PR rule might be chosen again. Contrast this with the initial case, where under all circumstances $l^* = \frac{v_3}{1+v_3}$ was the optimal choice, and one can get a clear idea on how ideological divergence alters the incentives of the dominant parties to collude.

Going back to the Greek Parliamentary election data, on Table 2 and Fig.4 and 4b, we can see yet another application of the electoral rule reform as an insurance against an ideologically distant opponent. During the late 80s the greek political arena was dominated by corruption allegations and heated political debate that lead to the indictment of the then-Prime-Minister in a special court, once opposition became the new government following the 1990 elections. The governing party early understanding the danger its members faced, if opposition were to win the majority in the parliament, amended the electoral rule to pure PR in 1988. It did so to impede the formation of a single-party government. Clearly, it achieved its objective since consecutive elections failed to produce a single-party government. This is depicted as a jump in the graph. Whereas actual and predicted majority premie co-move throughout the whole sample, during those electoral periods, it is clear that the electoral rule was more proportional than before. We attribute this larger degree of proportionality to the increase in the degree of ideological divergence and polarization that was observed back then. Hence, empirical evidence from both Greece and Italy seem to confirm our theoretical findings and intuition.

On a final note, we need to stress that this attempt to account for ideology is not a complete one. So far, we have focused on the symmetric case, both in terms of electoral strength and of ideological divergence. We also made particular assumptions about the coalition formation process. These assumptions might fit the particular cases of Greece[29] and Italy, however our model does not explore what happens when symmetry is dropped. While acknowledging this limitation, we stress that the model's predictions even in its simplest form are in line with empirical observations and intuition. Moreover, it allows us to isolate the impact of ideology on electoral rule choice. For these reasons, we leave those questions open for further study.

6 Conclusions

In summary, this paper has attempted to shed some light on the strategic choice of the electoral rule made by parliamentary parties under an uncertain political environment. With the aid of a multi-party model of electoral competition, we have shown why and how the dominant parties choose the electoral rule. By formalizing recent literature that turns Duverger upside down, we present a model that, we argue, can explain how dominant parties choose an electoral rule that helps them to consolidate the *status quo*. That is,we have tried to explore how the political environment, the structure of the party system and the nature of political competition affect the optimal choice of electoral rule.

Our model, one of endogenous electoral rule reform, introduces a series of novelties to current literature on electoral reform. Firstly, we introduce a clear preference for single-party governments for office motivated parties. The motivation for this can be derived both from the perks of holding office and the desire to be more efficient (Tsebelis, 1999). Secondly, we expand the choice set of electoral rule reform by allowing for a broader set of electoral rules that can be chosen. We model this through the majority premium mechanism which acts as a majoritarian distortion to the PR rule. Moreover, in the extension of the model we introduce ideological preferences in order to study how ideological divergence and polarization affect the choice of electoral rule. Finally, in what we consider to be the most crucial addition, we introduce uncertainty over the electoral outcome and we explore how

[29] For example in Greece the two dominant parties the socialist left PASOK and the conservative ND are approximately of equal strength and have shared almost equal time in office from 1974 until 2009.

the nature of the political competition affects the endogenous choice of the electoral rule and the strategic incentives of the dominant parties to collude.

The main finding of our paper can be summarized in the aligned incentives of the dominant parties to adapt a more majoritarian than the PR electoral rule, if electoral competition is non-trivial. Furthermore, our model identifies the two driving factors that allow strategic collusion of dominant parties in adapting a more majoritarian electoral rule, in the form of introducing a majority premium to the PR. The first key element is the presence of electoral uncertainty. The second is their desire to form a single-party government. When electoral competition is non-trivial, and both dominant parties have chances of winning the election, they have incentives to cooperate in order to eliminate the impact of the third party for their own benefit, in the same spirit that big firms would like to take smaller competitors out of business. That is, dominant parties have incentives to consolidate the party system since the terms of political competition are obviously favorable to them under the status quo. But for this collusion to take place the two conditions mentioned above are indispensable. The desire for single-party government generates the incentive to distort the proportionality of the electoral rule, whereas electoral uncertainty allows for those incentives to be aligned creating enough room for collusion.

Therefore, it is not a coincidence that in our model the *electoral reform equilibrium* is unique. Once the incentives of the two dominant parties align, there is a unique electoral reform proposal, which is accepted by the two dominant parties. That is, given the strength of the minority party or parties, there is a unique optimal value of the majority premium, $l^* = \frac{v_3}{1+v_3}$. In fact, the value of the premium depends on the relative size of the minority party and is such that it completely eliminates its political impact on government formation. If the PR rule were to remain, then in some instances the minority party might had an important role to play in the formation of coalition government, when neither of the two dominant parties were able to form a single-party government. But the introduction of a distortion to the PR guarantees that this is no longer the case.

But in practice, empirical evidence suggest that, even thought some party systems seem to confirm our intuition, there are also several examples where the two dominant parties do not cooperate to introduce a more majoritarian electoral rule. To address this issue, in the extension of the model, we introduce ideology to the preferences of the parties. We show that, a third key condition was implicitly assumed in order for the dominant parties to collude, namely ideological convergence. That is, when ideological divergence and polarization is increasing, our model predicts that the dominant parties' strategic incentives to agree on a more majoritarian rule are reversed. Their fear of their ideological opponent forming a single-party government dominates their desire to form a single-party government. Hence, they utilize the choice of the electoral rule as an insurance device against the risk of having an ideologically opposing government. In the case of high ideologic divergence, our key result states that both parties are better off by sticking to the PR rule. This can explain, to some extend, the prevalence of PR rule in some bipartisan political systems.

We conclude our paper with a discussion on future extensions of the model that incorporate a more rich set-up with respect to ideology and electoral competition. Yet, the key results of this paper confirm our intuition and are in line with empirical observation. Hence, our model explains how the nature of political competition and the structure of the party system affects the strategic choice of electoral rules by parliamentary parties in order to consolidate and crystallize the current status quo.

7 References

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8 Appendix

8.1 Proofs

Proposition 1. Since the electoral reform process is trivial, Party 1 faces an unconstrained maximization program. Moreover, since the electoral competition is trivial, if it is the leading party, its expected utility is given by: $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{\frac{1/2-l}{1-l}} [v_1(1-l) + l]dv_1 + \int\limits_{\frac{1/2-l}{1-l}}^{b_1} dv_1]$. Then, it proposes l^* such that it secures with certainty the majority of the seats in the new parliament. That is, it sets $a_1(1-l^*) + l^* = 1/2$, which implies that $s_1^{l^*} = 1/2$, $\forall a_1$. Then, solving for l^* yields the result. If it is not the leading party, $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{b_1} v_1(1-l)dv_1]$, which is strictly decreasing in l. Hence, $l^* = 0$. This completes the proof. ■

Proposition 2. Since electoral competition is trivial one party is a sure loser and the other is a sure winner. That is, for $i = 1$ or 2, $Eu_i(v_i, l) = \frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{b_1} v_i(1-l)dv_i]$, which is strictly decreasing in l. Hence, one party always prefers $l^* = 0$ but the other, as shown in Proposition 1 prefers $l^* \geq \max\{0, \frac{1/2-a_1}{1-a_1}\}$. Since the electoral reform process is non-trivial, the two parties have to agree on the electoral reform. So, the only equilibrium is $l^* = 0$. ■

Proposition 3. Given the structure of the game, to show that when the electoral competition is non-trivial then $l^* \in \{0, \frac{v_3}{1+v_3}\}$, is quite easy. In the first part of the proof we shall demonstrate that, for both parties the exact bonus l that maximizes their expected utility is either 0 or $\frac{v_3}{1+v_3}$. Then given this result, we will offer a trivial argument to show that if both parties maximize their expected utility with a bonus $\frac{v_3}{1+v_3}$, this specific electoral reform takes place, and in case at least one maximizes its expected utility with $l = 0$ no electoral reform takes place.

For the first part of the proof we need to prove that the expected utility of party 1 is convex in $l \in [0, \frac{v_3}{1+v_3}]$, strictly convex in a subset of $[0, \frac{v_3}{1+v_3}]$ and decreasing in $(\frac{v_3}{1+v_3}, 1]$. The arguments are equivalent for party 2. Since the electoral competition is non-trivial, we have that $b_1 > \frac{1-v_3}{2}$. If $b_1 < 1/2$ then there exist $\hat{l} \in (0,1)$ s.t. $\frac{1/2-\hat{l}}{1-\hat{l}} = b_1$. For $l \in [0, \hat{l}]$ we have that $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int\limits_{(1-v_3)/2}^{b_1} [v_1(1-l)+l]dv_1]$, for $l \in (\hat{l}, \frac{v_3}{1+v_3}]$ we have that $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int\limits_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l)+l]dv_1 + \int\limits_{\frac{1/2-l}{1-l}}^{b_1} dv_1]$ and for $l \in (\frac{v_3}{1+v_3}, 1]$ we have that $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int\limits_{(1-v_3)/2}^{b_1} dv_1]$. One may observe that $\frac{\partial^2 Eu_1(v_1,v_2,v_3,l)}{\partial l^2} = 0$ for $l \in [0, \hat{l}]$, $\frac{\partial^2 Eu_1(v_1,v_2,v_3,l)}{\partial l^2} > 0$ for $l \in (\hat{l}, \frac{v_3}{1+v_3}]$, and $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} < 0$ for $l \in (\frac{v_3}{1+v_3}, 1]$. Moreover, if $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} \geq 0$ for $l \in [0, \hat{l}]$ then $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} > 0$ for $l \in (\hat{l}, \frac{v_3}{1+v_3}]$. That is, $Eu_1(v_1, v_2, v_3, l)$ is convex in $[0, \frac{v_3}{1+v_3}]$, strictly convex in a subset of $[0, \frac{v_3}{1+v_3}]$ and decreasing in $(\frac{v_3}{1+v_3}, 1]$. The only candidates for maximum are $\{0, \frac{v_3}{1+v_3}\}$.

If $b_1 > 1/2$, for $l \in [0, \frac{v_3}{1+v_3}]$ we have that $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int\limits_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l)+l]dv_1 + \int\limits_{\frac{1/2-l}{1-l}}^{b_1} dv_1]$ and for $l \in (\frac{v_3}{1+v_3}, 1]$ we have that $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int\limits_{(1-v_3)/2}^{b_1} dv_1]$. Just, as before $\frac{\partial^2 Eu_1(v_1,v_2,v_3,l)}{\partial l^2} > 0$ for $l \in [0, \frac{v_3}{1+v_3}]$, and $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} < 0$ for $l \in (\frac{v_3}{1+v_3}, 1]$. That is, $Eu_1(v_1, v_2, v_3, l)$ is strictly convex in $[0, \frac{v_3}{1+v_3}]$ and decreasing in $(\frac{v_3}{1+v_3}, 1]$. The only candidates for maximum are $\{0, \frac{v_3}{1+v_3}\}$. This concludes the first part of the proof.

If both parties maximize their expected utility with $l = \frac{v_3}{1+v_3}$ then party one proposes this electoral reform and party 2 votes for it. If the optimal bonus for the proposing party 1 is $l = 0$ then it does not propose any electoral reform. And when $l = 0$ maximizes the expected utility of party 2 then it always votes against any electoral reform. ■

Proposition 4. We will only prove the result for values of $b_1 < \frac{1}{2}$.First let us note that from Proposition 3 we have that when the electoral competition is non-trivial then there are only two possible candidate values for an optimum, that is either $l^* = 0$ or $l^* = \frac{v_3}{1+v_3}$. When the electoral rule reform is trivial then this implies that the proposer faces an unconstrained maximization problem. Hence for a change of the electoral rule from $l^* = 0$ to $l^* = \frac{v_3}{1+v_3}$ it suffices to show that $Eu_1(v_1, v_2, v_3, l = \frac{v_3}{1+v_3}) > Eu_1(v_1, v_2, v_3, l = 0)$.Assume this condition holds. Then this implies that $\frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{(1-v_3)/2} v_1(1 - \frac{v_3}{1+v_3})dv_1 + \int\limits_{(1-v_3)/2}^{b_1} dv_1] >$ $\frac{1}{(b_1-a_1)}[\int\limits_{a_1}^{(1-v_3)/2} v_1 dv_1 + \int\limits_{(1-v_3)/2}^{b_1} v_1 dv_1]$. After some algebraic manipulation this yields $\int\limits_{(1-v_3)/2}^{b_1} dv_1 -$

$\int_{(1-v_3)/2}^{b_1} v_1 dv_1 > \int_{a_1}^{(1-v_3)/2} v_1 dv_1 - \int_{a_1}^{(1-v_3)/2} v_1(1-\frac{v_3}{1+v_3})dv_1$, which implies $\int_{(1-v_3)/2}^{b_1} (1-v_1)dv_1 > (\frac{v_3}{1+v_3}) \int_{a_1}^{(1-v_3)/2} v_1 dv_1$. It is then easily checked that expanding the integrals yields the desired inequality $(b_1 - \frac{1-v_3}{2})(2 - b_1 - \frac{1-v_3}{2}) \geq (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_1)(\frac{1-v_3}{2} - a_1)$ which in turn implies $f_1(a_1, b_1, v_3) \geq 0$. This completes the argument. Reversing the argument assume that the above inequality holds true but no electoral rule change takes place, that is $l^* = 0$. But then simple algebra yields $Eu_1(v_1, v_2, v_3, l = \frac{v_3}{1+v_3}) > Eu_1(v_1, v_2, v_3, l = 0)$ which implies $l^* = \frac{v_3}{1+v_3}$ a clear contradiction. Then the *only if* part follows. ■

Proposition 5. We will be using an argument analogous to Proposition 4. Let us first note that, in this case, the duality of the conditions is due to the fact that the proposer faces the constrained version of the maximization problem (given that the electoral reform process requires the consent of both parties). That is for $l^* = 0$ to change to $l^* = \frac{v_3}{1+v_3}$ both conditions $Eu_1(v_1, v_2, v_3, l = \frac{v_3}{1+v_3}) > Eu_1(v_1, v_2, v_3, l = 0)$ and $Eu_2(v_1, v_2, v_3, l = \frac{v_3}{1+v_3}) > Eu_2(v_1, v_2, v_3, l = 0)$ need to be satisfied simultaneously. It can be easily checked by an analogous argument (as in Proposition 4), that the condition $(b_1 - \frac{1-v_3}{2})(2 - b_1 - \frac{1-v_3}{2}) \geq (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_1)(\frac{1-v_3}{2} - a_1)$, implying $f_1(a_1, b_1, v_3) \geq 0$, is both necessary and sufficient condition for Party 1 to support $l^* = \frac{v_3}{1+v_3}$. A directly analogous condition is also true for Party 2 by symmetric nature of the problem. Hence we only need to check that the 'leading party condition' implies the above inequality for the other party as well. Note that $\frac{a_1+b_1}{2} > \frac{1-v_3}{2}$ implies $b_1 - \frac{1-v_3}{2} > \frac{1-v_3}{2} - a_1$. In turn this implies that $(b_1 - \frac{1-v_3}{2})(2 - b_1 - \frac{1-v_3}{2}) > (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_1)(\frac{1-v_3}{2} - a_1)$.Thus, $f_1(a_1, b_1, v_3) > 0$. By analogy, $f_2(a_2, b_2, v_3) > 0$ when Party 2 is the leading party and with a similar argument as in the proof of Proposition 4 it can be shown that this condition is both necessary and sufficient. This completes the argument. ■

Proposition 6. Firstly, symmetry implies that the electoral competition is non-trivial as well. Secondly, we note that for every $l \in (\frac{v_3}{1+v_3}, 1]$ the function $Eu_1(v_1, v_2, v_3, l)$ is decreasing with respect to l, since $\frac{\partial Eu_1(v_i,l)}{\partial l} < 0$. Hence, we can restrict our search for an optimum in the interval $[0, \frac{v_3}{1+v_3}]$ as we did before. For case (i) we only need to show that symmetry implies the condition of Proposition 5, since then the same reasoning applies. Furthermore, we need only show that the condition is satisfied only for Party 1, due to symmetry. Check that $\frac{a_1+b_1}{2} = \frac{a_2+b_2}{2} = \frac{1-v_3}{2}$ implies $b_1 - \frac{1-v_3}{2} = \frac{1-v_3}{2} - a_1$. But then the condition becomes: $(2 - b_1 - \frac{1-v_3}{2}) \geq (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_1)$. Since $(\frac{v_3}{1+v_3}) < \frac{1}{4}$ and $2 - b_1 - \frac{1-v_3}{2} \geq \frac{1-v_3}{2} + a_1$ for every a_1, b_1, v_3 we conclude that the condition is always satisfied and hence, $l = l^*$. For case (ii) Eu_1 becomes $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l)+l]dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} dv_1 - 2\varepsilon^2(b_1 - \frac{1/2-l}{1-l}) - \varepsilon^2]$. Then one can compute $\frac{\partial Eu_1(.)}{\partial l} = \frac{1}{(b_1-a_1)}[\frac{l^2/2-l-3/8}{(1-l)^2} - \frac{1-v_3-a_1^2}{2}] < 0$ for every $l \in [0, \frac{v_3}{1+v_3}]$, since $\frac{l^2/2-l-3/8}{(1-l)^2} < 0$. Hence, $Eu_1(v_1, v_2, v_3, l)$ is strictly decreasing for every $l \in [0, 1]$ and as a result, the unique candidate for an optimum is $l = 0$. This completes the proof. ■

Proposition 7. First we note that we restrict attention to values of $l \in [0, \frac{v_3}{1+v_3}]$ since for all other values $Eu_1(v_1, v_2, v_3, l)$ is decreasing with respect to l, as shown in Proposition 6. For part (i) we need to show that $\partial Eu_1(v_1, v_2, v_3, l)/\partial l < 0$, that is decreasing, for all $\varepsilon \in [1, \varepsilon')$. We compute: $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} = \frac{1}{b_1 - a_1}\left[\frac{\frac{l^2}{2}(v_3+a_1^2)-l(v_3+a_1^2)+1/8+(v_3+a_1^2)/2-\varepsilon^2}{(1-l)^2}\right]$. Clearly, $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} < 0$ iff $\frac{l^2}{2}(v_3 + a_1^2) - l(v_3 + a_1^2) + 1/8 + \frac{(v_3+a_1^2)}{2} - \varepsilon^2 < 0$ which implies that for every $l \in [0, \frac{v_3}{1+v_3}]$ the following should be satisfied: $\varepsilon > \sqrt{\frac{1}{8} + \frac{(v_3+a_1^2)}{2}} = \varepsilon'$. Then the result follows, since there is a unique candidate for a maximum, namely $l = 0$, due to the fact that $Eu_1(v_1, v_2, v_3, l)$ is decreasing for every l.

For part (ii) we need to show that for every $\varepsilon > \varepsilon''$, $Eu_1(v_1, v_2, v_3, l)$ is concave in $[0, \frac{v_3}{1+v_3}]$. That is we compute: $\frac{\partial^2 Eu_1(v_1,v_2,v_3,l)}{\partial^2 l} = \frac{1}{b_1-a_1}[\frac{1}{4} - 2\varepsilon^2] < 0$ iff $\varepsilon > \sqrt{1/8} = \varepsilon''$. Hence, by concavity of $Eu_1(v_1, v_2, v_3, l)$ there may exist an interior maximizer $l^{**} \in [0, \frac{v_3}{1+v_3})$ such that $l^{**} < \frac{v_3}{1+v_3}$. Since $Eu_1(.)$ is decreasing for $l \in (\frac{v_3}{1+v_3}, 1]$, and by an argument analogous to that of Proposition 3, we conclude that there are only two candidate values for an optimum, namely $l \in \{0, l^{**}\}$. If an interior maximizer does not exist, then we trivially have $l^{**} = l^*$ and we are in case (iii).

For part (iii), we just note that for any other value of ε, such that $\varepsilon \leq \varepsilon''$, $Eu_1(v_1, v_2, v_3, l)$ is convex in $[0, \frac{v_3}{1+v_3}]$. Therefore, as shown in the proof of Proposition 3, the two candidates for a maximum are the same, namely $l \in \{0, l^*\}$. This completes the proof. ■

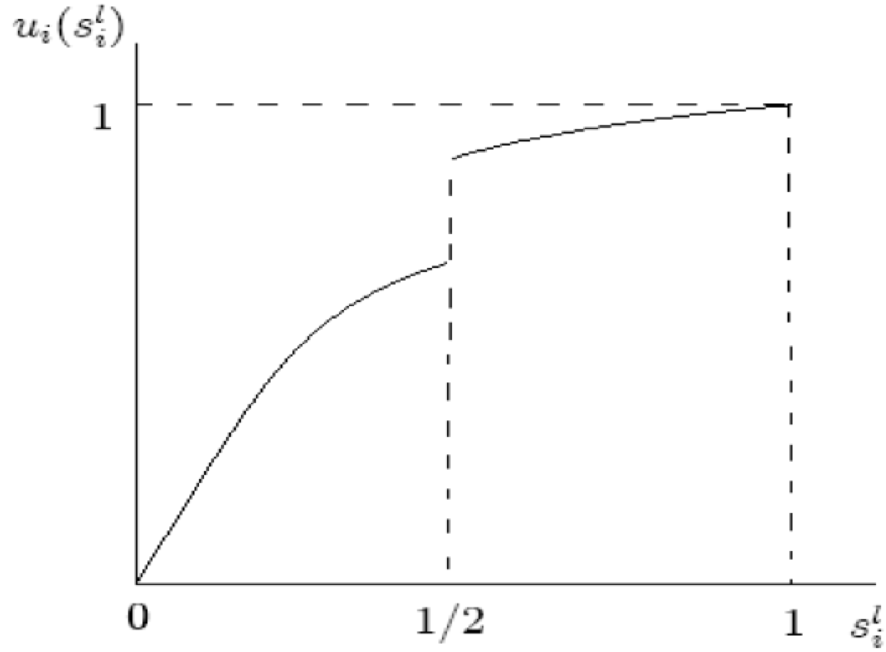

Figure 1: Utility $u_i(s_i^l)$ as a function of seat share s_i^l for $i = 1, 2$.

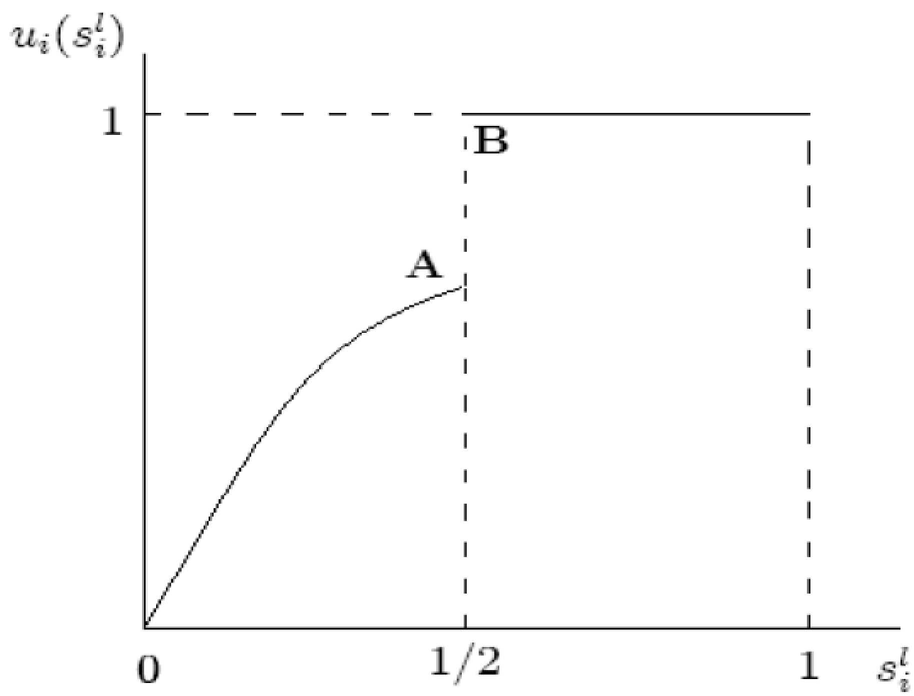

Figure 2: Special case of Utility $u_i(s_i^l)$ as a function of seat share s_i^l

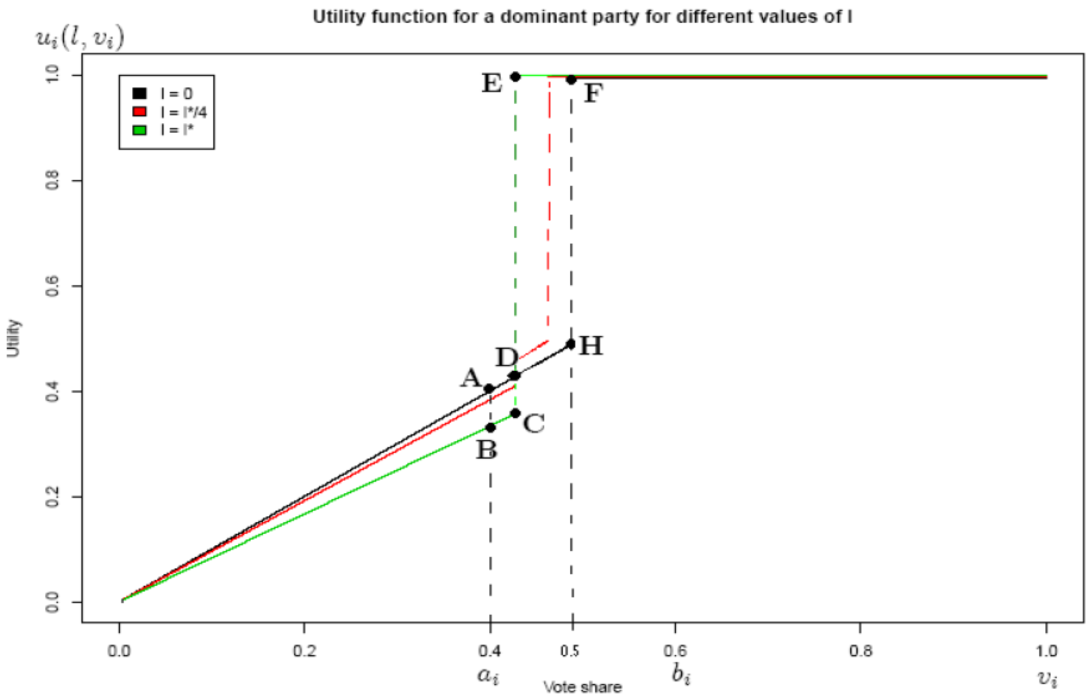

Figure 3: Utility $u_i(v_i, l)$ as a function of vote share for $i = 1, 2$ and for different values of l.

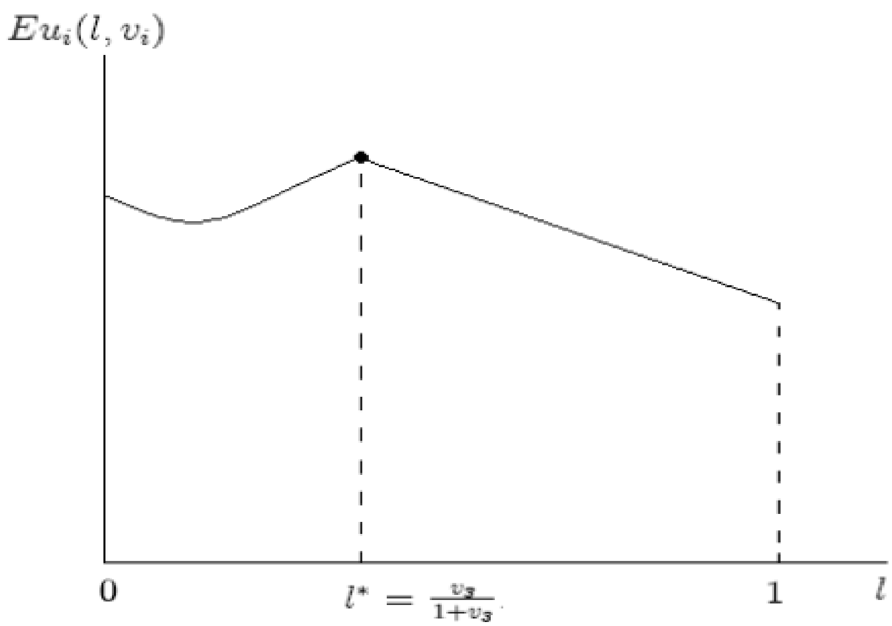

Figure 4: Expected Utility $Eu_i(l, v_i)$ as a function of l for $i = 1, 2$ when $l^* = \frac{v_3}{1+v_3}$ is the optimal choice.

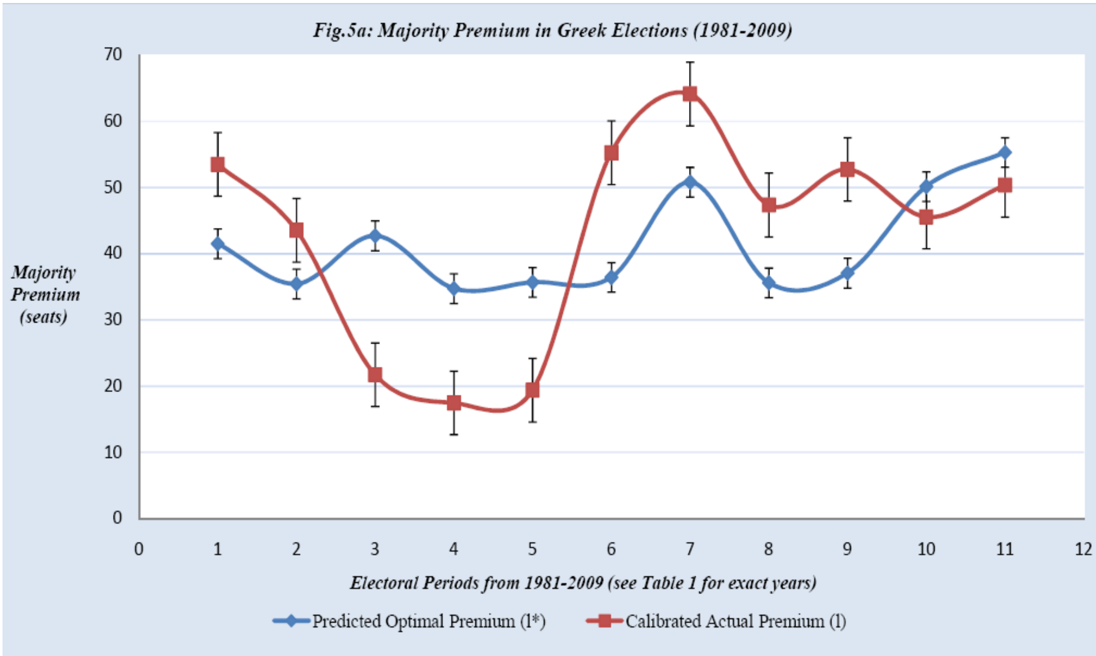
Fig.5a: Majority Premium in Greek Elections (1981-2009)
Majority Premium (seats)
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10
11
12
Electoral Periods from 1981-2009 (see Table 1 for exact years)
Predicted Optimal Premium (l*)
Calibrated Actual Premium (l)

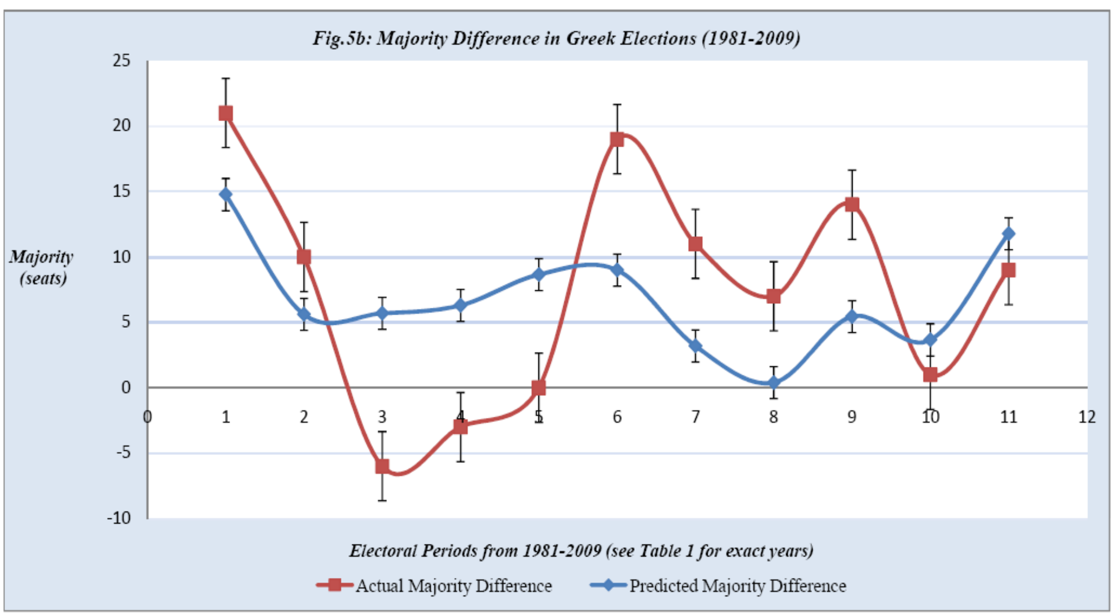
Fig.5b: Majority Difference in Greek Elections (1981-2009)
Majority (seats)
25
20
15
10
5
0
-5
-10
0
1
2
3
4
5
6
7
8
9
10
11
12
Electoral Periods from 1981-2009 (see Table 1 for exact years)
Actual Majority Difference
Predicted Majority Difference

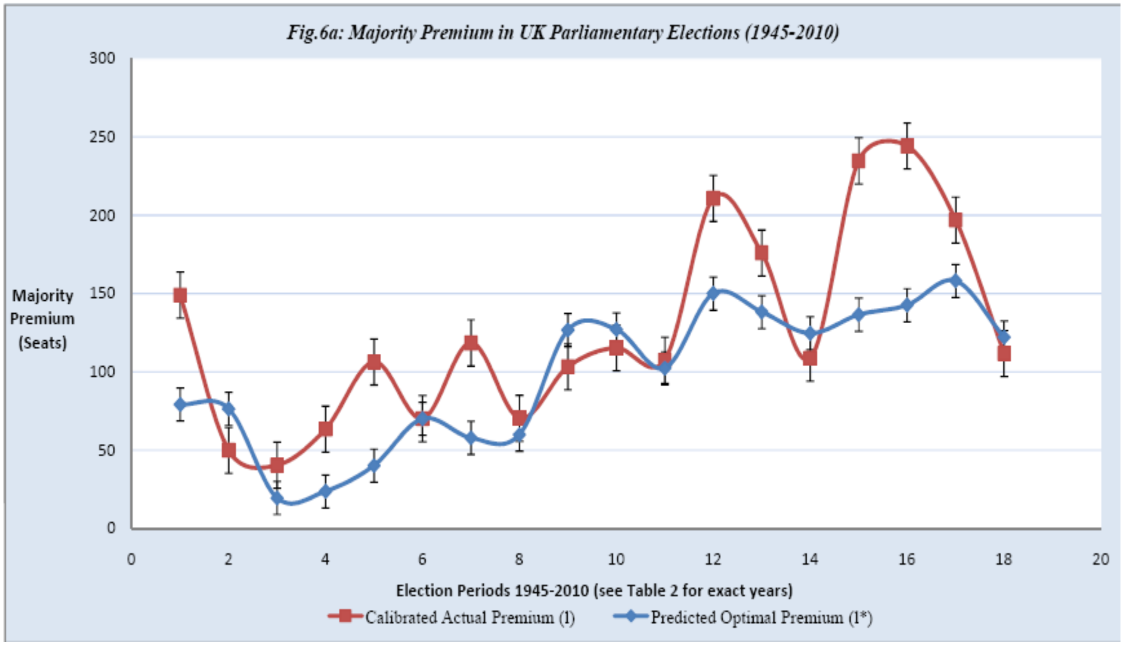
Fig.6a: Majority Premium in UK Parliamentary Elections (1945-2010)
Majority Premium (Seats)
300
250
200
150
100
50
0
0
2
4
6
8
10
12
14
16
18
20
Election Periods 1945-2010 (see Table 2 for exact years)
Calibrated Actual Premium (l)
Predicted Optimal Premium (l*)

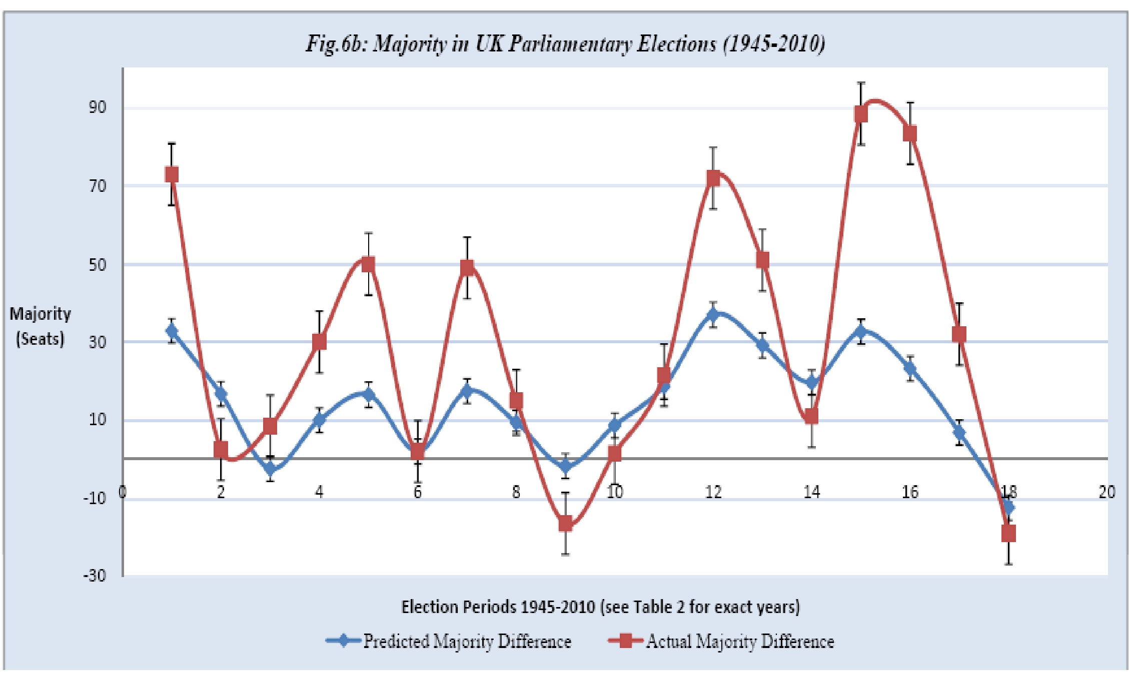
Fig.6b: Majority in UK Parliamentary Elections (1945-2010)
Majority (Seats)
90
70
50
30
10
-10
-30
0
2
4
6
8
10
12
14
16
18
20
Election Periods 1945-2010 (see Table 2 for exact years)
Predicted Majority Difference
Actual Majority Difference

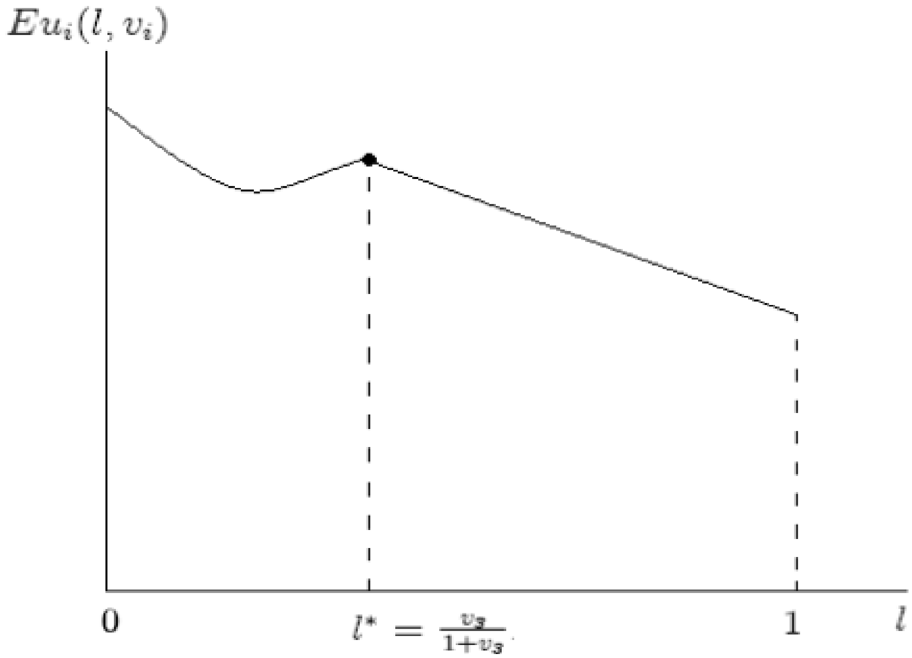

Figure 5: Expected Utility $Eu_i(l, v_i)$ as a function of l for $i = 1, 2$ when $l^* = 0$ is the optimal choice.

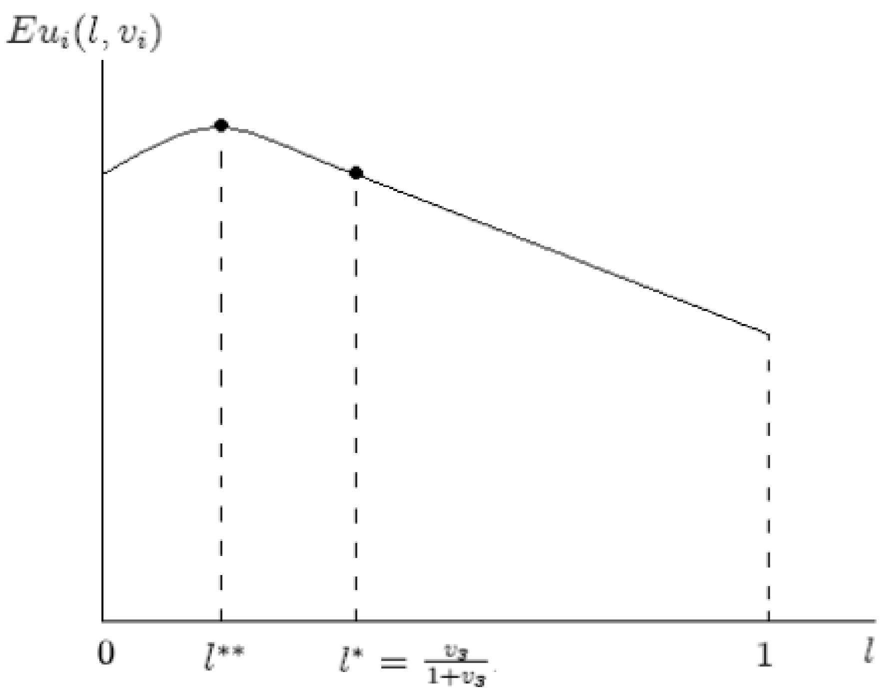

Figure 6: Expected Utility $Eu_i(l, v_i)$ as a function of l for the case (ii) of Proposition 7 when l^{**} is the optimal choice.

Table 1. Greek Legislative Elections and Optimal Majority Premium (1981-2009)

Election Year	Vote Shares V_1	V_2	V_3	Actual Seats (winner)	Seat share S_1	Majority (actual)	Premium l (actual)	Majority (predicted)	Premium l* (predicted)	Premium Gap (l-l*)
1981	0.4807	0.3587	0.1606	172	0.5733333	21	54	15	42	-12
1985	0.458	0.408	0.134	161	0.5366667	10	44	6	35	-8
1989*	0.443	0.391	0.166	145	0.4833333	-6	22	6	43	21
1989*	0.462	0.407	0.131	148	0.4933333	-3	17	6	35	18
1990*	0.469	0.396	0.135	151	0.5033333	0	19	9	36	17
1993	0.4688	0.393	0.1382	170	0.5666667	19	55	9	36	-19
1996	0.4149	0.3812	0.2039	162	0.54	11	64	3	51	-13
2000	0.4379	0.4274	0.1347	158	0.5266667	7	47	0	36	-11
2004*	0.454	0.405	0.141	165	0.55	14	53	5	37	-16
2007*	0.4183	0.381	0.2007	152	0.5066667	1	46	4	50	4
2009	0.4392	0.3348	0.226	160	0.5333333	9	50	12	55	5
Average	0.4496182	0.389373	0.16101	158.5454545	0.5284848	7.545455	42.82880603	6.769463	41.40661118	1.422194849

Table 2. British Legislative Elections and Optimal Majority Premium (1945-2010)

Election Year	Vote Shares V_1	V_2	V_3	Seats actual	Seat share S_1	Majority (actual)	Premium l (actual)	Seats projected	Premium l* (optimal)	Majority predicted	Premium gap (l-l*)
1945*	0.497	0.362	0.141	393	0.6140625	73	149	353	79	33	70
1950*	0.461	0.4	0.139	315	0.504	3	50	329	76	17	-26
1951	0.48	0.488	0.032	321	0.5136	9	40	310	19	-2	21
1955	0.497	0.464	0.039	345	0.54761905	30	63	325	24	10	40
1959	0.494	0.438	0.068	365	0.57936508	50	106	332	40	17	66
1964*	0.441	0.434	0.125	317	0.5031746	2	70	317	70	2	0
1966*	0.48	0.419	0.101	364	0.57777778	49	118	332	58	17	61
1970	0.464	0.431	0.105	330	0.52380952	15	70	324	60	9	10
1974*	0.372	0.379	0.249	301	0.47401575	-17	103	316	127	-2	-23
1974b*	0.392	0.358	0.25	319	0.5023622	2	115	326	127	9	-12
1979	0.439	0.369	0.192	339	0.53385827	22	107	336	102	19	5
1983	0.424	0.276	0.3	397	0.61076923	72	211	362	150	37	61
1987	0.422	0.308	0.27	376	0.57846154	51	176	354	138	29	38
1992	0.419	0.344	0.237	336	0.51612903	11	109	345	125	20	-16
1997*	0.432	0.307	0.261	418	0.63429439	89	235	362	136	33	98
2001*	0.407	0.317	0.276	413	0.62670713	84	244	353	143	23	102
2005*	0.352	0.324	0.324	355	0.5495356	32	197	330	158	7	39
2010	0.361	0.29	0.231	306	0.47076923	-19	112	313	122	-12	-10
Average	0.449	0.40573	0.1345		0.54374106	28	125.683263		98.72272549	14.6806488	27