Control Rights in Complex Partnerships

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ABSTRACT. This paper develops a theory of the allocation of authority between two players who are in a "complex" partnership, that is, a partnership which produces impure public goods. We show that the optimal allocation depends on technological factors, the parties' valuations of the goods produced, and the degree of impurity of these goods. When the degree of impurity is large, control rights should be given to the main investor, irrespective of preference considerations. There are some situations in which this allocation is optimal even if the degree of impurity is very low as long as one party's investment is more important than the other party's. If the parties' investments are of similar importance and the degree of impurity is large, shared authority is optimal with a greater share going to the low-valuation party. If the importance of the parties' investments is similar but the degree of impurity is neither large nor small, the low-valuation party should receive sole authority. We analyze an extension in which side payments are infeasible. We check for robustness of our results in several dimensions, such as allowing for multiple parties or for joint authority, apply our results to interpret a number of complex partnerships, including those involving schools and child custody.
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1. INTRODUCTION

1.1. **Background.** Since Simon's (1951) contribution, authority — that is, the legitimate power to direct the action of others (Weber, 1968) — has become a central concept in many economic formulations of the theory of the firm. As pointed out by Grossman and Hart (1986) and Hart and Moore (1990) (henceforth, GHM), authority can be conferred by the ownership of an asset, which gives the owner the right to make decisions over the use of this asset. Using this notion to analyze the allocation of authority within and between firms involved in the production of *pure* private goods in an environment where contracts are incomplete, GHM show that the main investor should have full control of the asset.

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Although much progress has been accomplished in the case of pure private goods,[1] relatively little has been done to understand the division of responsibilities between the state and the private sector for the provision of public goods. A notable exception is the study by Besley and Ghatak (2001) (henceforth, BG). They apply the GHM notion of incomplete contracting to examine the allocation of authority in public-private partnerships producing *pure* public goods, whose benefits are nonrival and nonexcludable. Contrary to GHM, BG prove that sole authority should be given to the party that values the benefits generated by the goods relatively more irrespective of the relative importance of the investments.[2]

In this paper, we too use this notion of authority when contracts are incomplete to study the allocation of control rights between players who are engaged in a "complex" partnership, that is, a partnership which produces goods that are neither purely private nor purely public.[3] This is important for at least three reasons. First, many public goods — such as highways, airports, courts, and possibly national defense and police services — are subject to congestion. These goods therefore are rival, but nonexcludable to varying degrees (Barro and Sala-I-Martin, 1992). Other public goods — such as schools, universities, television, waterways, parks, zoos, museums, and transportation facilities — are excludable, in the sense that they are public goods for which exclusion by means of price or constraints is costless (Brito and Oakland, 1980; Fang and Norman, 2006). Consumers have access to such goods if they are willing to pay a fee or a license for the services that such goods provide. Otherwise, access can only be achieved if the restrictions imposed (sometimes accidentally) by individual agents and institutions are removed. Second, the considerable expansion of public-private partnerships in many countries in the last twenty years (BG; World Bank, 2002) has produced a variety of *impure* public goods (see also the discussion in the next subsection).[4] Our analysis therefore is important for its implications for policy. Third, by considering impure public goods, our model

[1]See for example Hart (1995), Aghion and Tirole (1997), and Aghion et al. (2004).

[2]Different departures from the GHM's result have been presented in other models with private goods (e.g., De Meza and Lockwood, 1998; Rajan and Zingales, 1998).

[3]Complexity means that our model deals with decision-making rights over a large set of decisions. Only a subset of such decisions will concern asset usage, and, as implied by the GHM-based literature, asset ownership is one of the mechanisms that grant control rights over asset use. Most of the other relevant decision-making rights, which do not have to rely on asset utilization, may be committed to either through the project's governance structure or contractually (Aghion and Tirole, 1997; Hart and Holmström, 2002; Bester, 2005). In what follows, therefore, we employ the terms authority, control rights, and decision-making rights interchangeably.

[4]This expansion has been recently accompanied by a growing economic literature on the properties of different forms of public procurement, including public-private partnerships. Most of these studies, however, are generally cast in a more complete contracting environment than in the GHM-based world used in our paper. See, among others, Martimont and Pouyet (2006).

allows us to assess the robustness of the GHM's and BG's results when there are perturbations away from the pure private and pure public world respectively.

Not only do GHM and BG focus on the two extreme cases of goods (pure private and pure public), but they also restrict attention to two polar cases of authority allocation, those in which one or the other party is allocated full control rights. Clearly, this contrasts with what we observe within firms (as confirmed, for example, by the analysis of Aghion and Tirole (1997) and Aghion et al. (2004)). It is also not consistent with most of the authority arrangements that have emerged between governments and private firms engaged in the provision of impure public goods around the world (see the discussion in Section 7), where authority is often shared. Our analysis shows that there are circumstances in which the two sole authority allocations are dominated by a shared authority allocation in which each party has *some* authority.

1.2. **Examples.** We provide some examples of impure public goods, and draw attention to issues related to their provision and authority allocation. We emphasize where the sources of "impurity" may come from and how authority interacts with investments.

1.2.1. *Public-Private Projects.* The provision of public goods and services through public-private partnerships has increasingly become more common in many industrialized and developing countries.[5] Such partnerships comprise a wide range of collaborations between public and private sector partners, with the involvement of the private sector varying considerably: from designing schools, hospitals, roads, waterways and sanitation services, to undertaking their financing, construction, operation, maintenance, management and, crucially, ownership. BG illustrate their model by considering the case in which a government and a nongovernmental organization (NGO) can invest in improving the quality of a school. It is crucial that the investment levels of the two parties are noncontractible, and that the value created by the investments is a pure public good (i.e., nonrival and nonexcludable). When this is the case, BG show that the party with the highest valuation on the benefits generated by the investment in the school should be the sole owner.

[5]The United Kingdom, Australia, Canada and the United States stand out as world leaders in the number and scale of such projects. For example, in the UK between 1992 and 2003, over 570 public-private projects have been funded for a combined capital value of about £36 billion. Current projects have committed the UK government to a stream of revenue payments to private sector contractors between 2004 and 2029 of about £110 billion (Allen, 2003). In developing countries, 20 percent of infrastructure investments (or about $580 billion) were funded by the private sector over the 1990s (World Development Report, 2002, chapter 8).

Improving the quality of a school or building and operating a new school are valuable public investments, regardless of whether the school is owned by the state or by a private organization. But issues of excludability arise if children of specific groups are excluded from accessing the school, perhaps unintentionally and even if fees are not charged. This may happen for instance when children come from families that are too poor and live too far away from the school, or when they come from religious or ethnic minorities which are unwelcome in the school environment (World Development Report, 2004). Even when, in line with BG, the school is not owned by the state because the NGO cares more about it, the government may impose regulations (e.g., academic curricula and admission rules) which could effectively dilute the value of the project to the NGO. In all these circumstances, as excludability increases, the school services lose part of their public nature, and investment and technology considerations are expected to become more relevant, as in GHM.

1.2.2. *State Funding of Basic Research.* Basic scientific research is typically considered a public good. This is perhaps the reason why most governments around the world provide for its funding. In the United States, since the passage of the 1980 Patent and Trademark Amendments, universities have the right to retain the exclusive property rights associated with inventions deriving from federally funded research. Before 1980, instead, it was the government to have the right to claim all royalties and other income from patents resulting from federally funded research (Henderson et al., 1998). This shift in ownership of patents and intellectual property rights is in line with BG's arguments, as long as universities value the benefits generated by their inventions more than the main investor (the government).

Elements of excludability however arise when inventors (either universities or individual scientists) obtain license agreements with private sector firms (Jensen and Thursby, 2001), or patent through external channels (e.g., setting up new independent firms), or manage to extract large shares of royalties (Lach and Schankerman, 2004). In these circumstances, the government may have little incentive to invest unless it receives (some) ownership of the inventions it funded. In fact, as in the GHM's framework, when exclusion is complete, we may expect the government — as the sole investor — to retain exclusive control rights irrespective of the relative valuations about the benefits of research.[6]

[6]Similar considerations apply in the case of other publicly funded activities, such as fine arts and classical music. Here excludability arise when a piece of art can only be displayed in museums or performed in opera houses at prices that could disproportionately exclude certain groups of citizens, e.g., poor or less educated people (Fenn et al., 2004).

1.2.3. *Child Custody After Divorce.* Children are generally viewed as household pure public goods when parents are married (Becker, 1991). If they retain their (local) pure public nature even after their parents divorce, and if the mother has the highest valuation, then — in line with BG's model — she should receive custody regardless of whether or not she is the key investor.[7] Custody will go to the father instead, if he values the benefits generated by the child relatively more. However, when parents are divorced, children can be seen as impure public goods to the extent that the non-custodial parent is excluded (or limited) to access them by the custodial parent (Weiss and Willis, 1985). An important implication of this exclusion is the very low compliance with court orders on child support payments (Del Boca and Flinn, 1995). In the extreme case of full excludability, whereby the non-custodial parent cannot enjoy the value of the investments in the child and the child is a private good to the custodial parent, custody should be allocated solely on the basis of investment considerations, as in GHM.

1.3. **Our Contribution.** This paper develops a theory of authority allocation between parties that produce impure public goods. In a world with contractual incompleteness, the ex-post allocation of control rights matters, as it does in the standard GHM-based literature. We contribute to this literature in two fundamental ways. First, we focus on *impure* public goods, that is, public goods that, to differing degrees, can be excludable. This adds to the scant knowledge on the pure public goods ownership allocation, for which BG provide the only thorough analysis available to date.[8] Second, we allow parties to *share* authority, that is, each party has control rights over a subset of decisions.[9] As the previous subsection has illustrated, impure public goods often embed complex bundles of goods and services, the provision of which may require parties to exercise rights over, or have differential access to, different critical resources (Rajan and Zingales, 1998). Therefore, the notion of shared authority seems to fit many situations with impure public goods quite naturally. A primary example of this is given by shared child custody after divorce.

[7]BG reach this conclusion provided that parents' investments are complements (p. 1366).

[8]Hart et al. (1997) also consider ownership allocations of pure public goods between the state and a private firm. In their framework, however, ownership is solely driven by technological factors. This is because the private firm does not directly care about the project, unlike in BG and our models. Likewise, BG also briefly touch on the case in which the benefit from the investments has a public as well as a private good component, where the latter can be appropriated by the owner in the event of disagreement. We provide a broader framework which generalizes such a case.

[9]The notion of shared authority is distinct from that of *joint* authority, according to which each party has veto rights over all decisions (as in BG). We shall return to the possibility of joint authority in Section 6.2.

Our baseline model involves two parties, such as a government and an NGO (or a government and a university, or two parents), investing in a common project. The investment will increase the value of the project's service and this is an impure public good to the two parties.[10] Because contracts are incomplete and thus investments are subject to holdup, we have a theory of authority allocation that tells us how control rights over the project's service should be distributed between the two parties to maximize the net surplus generated by their investments. We show that, in a broad range of cases, the optimal allocation of authority depends on the technology structure (as in GHM), the parties' relative valuations of the goods produced (as in BG), and the degree of impurity. When the degree of impurity is very small (and, therefore, we are in a world á la BG), authority should be given to the high-valuation party, irrespective of investment considerations. This is consistent with BG. When the degree of impurity instead is large, control rights should be entirely given to the main investor, irrespective of preference considerations. This is consistent with GHM. In fact, there is a wide range of situations in which this allocation is optimal even if the degree of impurity is low provided that one party's investment is more important than the other party's.

On the other hand, if the parties' investments are of similar importance and the degree of impurity is large, shared authority is optimal, and a relatively greater share should go to the low-valuation party. But the low-valuation party will get sole authority when both parties investments are of similar importance and the degree of impurity is neither large nor small. The two last allocations emerge because the party with the highest valuation would invest anyway (indeed, we are in a world in which both parties invest and their investments are of similar importance), while the low-valuation party would be endowed with greater bargaining power. This specific "balancing out" of bargaining chips is a distinctive feature of a world with impure public goods.

The remainder of the paper is organized as follows. Section 2 lays out our basic model, presents some preliminary results, and discusses our main assumptions. Sections 3 and 4 consider the optimal allocations of authority when only one party invests and when both parties invest, respectively. Section 5 elaborates on an extension to our basic model that deals with equilibrium authority allocations when side payments are not feasible. Section 6 presents a number of additional extensions (e.g., the case in which there are more than two parties, the presence of ex-post uncertainty, the possibility of joint authority, and the case in which the parties' investments are perfect substitutes). Section 7 reviews some applications, especially

[10]Our analysis also holds for any type of organizations (i.e., for-profit firms too) as long as they care about the project (Glaeser and Shleifer, 2001).

to the provision of schools services by public-private partnerships, and to child custody. Section 8 concludes.

2. The Model

Consider a situation in which a government and an NGO discuss whether or not and how to collaborate in the management and running of a project (e.g., a local primary school). The first main issue is to allocate decision-making rights between the two parties. After this is done, the parties undertake project-specific investments. These investments are too costly to be verified by "third" parties (such as the courts), and hence they cannot be contracted upon. Each party will undertake whatever level of investment it wishes to, and, once undertaken, investments are observable by both parties.

Given an allocation of authority and a pair of investment levels, the project's benefits are higher if the parties make decisions cooperatively rather than via the allocated control rights. This means that there exists a surplus, and the parties will negotiate over its partition. Each party's marginal returns to investment are influenced by the outcome of this ex-post bargaining, in which the ex-ante allocated control rights determine the parties' default payoffs from not reaching agreement. Hence, each party's investment incentives indirectly depend on the allocated control rights. A central objective of our analysis is to characterize the optimal allocation of authority, one that maximizes the parties' ex-ante joint payoffs from partnership.

2.1. **Formal Structure.** Two players, government g and NGO n, are to be involved in a joint project. There are three critical dates at which they will interact.

Date 0: Authority — The players jointly select an allocation of authority (control rights) between them. We formalize this choice in a reduced-form manner: a share π (where $\pi \in [0,1]$) of such authority is allocated to g, and the remaining share $1-\pi$ is allocated to n. If $\pi = 1$ ($\pi = 0$), then the government (NGO) is allocated control rights over *all* matters on which decisions need to be taken. This can be interpreted as the government (NGO) having sole authority. But if $\pi \in (0,1)$, and thus each player has *some* power, authority is shared.

Date 1: Investments — At least one of the players has an opportunity to undertake an investment that increases the benefits generated by the project. Let y_i denote the investment level of i ($i = g, n$). The cost of investing y_i, incurred by i at this date, is $C_i(y_i)$. This function satisfies

Assumption 1. *C_i is strictly increasing, convex and twice continuously differentiable, with $C_i(0) = 0$.*

After the investments are sunk, the two parties face the following bargaining situation. If decisions are taken via the allocated control rights, π, the project's benefits are given by $B(y, \pi)$, where $y \equiv (y_g, y_n)$. But if decisions are taken cooperatively, the project's benefits are $b(y)$, where $b(y) > B(y, \pi)$. Both players then can mutually benefit from making decisions cooperatively. We assume that B is linear in π, so that:

$$B(y, \pi) = \pi B^g(y) + (1 - \pi)B^n(y),$$

where $B^i(y)$ denotes the project's benefits when i has sole authority.
Date 2: Bargaining — The players negotiate over whether or not to cooperate in decision-making *and* over the level of a monetary transfer from n to g or from g to n. If agreement is reached, the payoffs to g and n are respectively

$$u_g(y) = \theta_g b(y) + t \quad \text{and} \tag{1}$$

$$u_n(y) = \theta_n b(y) - t, \tag{2}$$

where $\theta_i > 0$ is i's valuation parameter of the project's benefits, and t is a monetary payment from n to g which can be positive or negative. But if they fail to reach agreement, the project operates via the allocated control rights and the default payoffs are

$$\overline{u}_g(y, \pi) = \theta_g \left[\pi B^g(y) + (1 - \alpha)(1 - \pi)B^n(y) \right] \quad \text{and} \tag{3}$$

$$\overline{u}_n(y, \pi) = \theta_n \left[(1 - \alpha)\pi B^g(y) + (1 - \pi)B^n(y) \right], \tag{4}$$

where $\alpha \in [0, 1]$ is a parameter that captures the degree of impurity of the goods generated by the project.[11,12] BG analyze this framework but with the implicit assumption that $\alpha = 0$ (and $\pi \in \{0, 1\}$); they are concerned with pure public goods and do not consider shared authority allocations. Our more general setup allows us to study the full spectrum of goods, from the extreme case of pure private goods

[11] The discussion in the Introduction refers to public goods that can be impure because of rivalry or excludability reasons. But as (3) and (4) show, we model impurity in such a way that it lowers the default payoff of the non-owner; that is, the party who does not have control rights can be excluded from the public good to a certain extent. Rivalry (congestion), however, is likely to affect both owner and non-owner equally. Therefore, our setup can be thought of as being more suitable to analyze excludable public good rather than rival public goods.

[12] Under shared authority, each party will have control rights over a subset of decisions when they fail to reach an agreement. In some contexts, when the degree of impurity is high, this could be interpreted as shared ownership, as in the case of shared custody of children after divorce. In other circumstances, however, this can mean that the two parties are not integrated, and yet they are still engaged in their joint project. For example, when the degree of impurity is small, production can be divided with one party producing inputs and the other party buying such inputs and converting them into final goods.

($\alpha = 1$), which is the focus of GHM, to the other extreme case of pure public goods ($\alpha = 0$).

Since $b(y) > B(y, \pi)$, it follows that $u_g(y) + u_n(y) > \overline{u}_g(y, \pi) + \overline{u}_n(y, \pi)$. Hence, it is mutually beneficial (efficient) for the players to negotiate an agreement and make decisions cooperatively at date 2. To describe the outcome of such negotiations we adopt the Nash bargaining solution, in which the threat (or disagreement) point is defined by the players' default payoffs (3) and (4). We place the following restrictions on the benefit functions:

Assumption 2.
(i) b is a strictly increasing, strictly concave and twice continuously differentiable function satisfying the Inada endpoint conditions, with $b(0, 0) > 0$*.*
(ii) For each $i = g, n$*,* B^i *is a non-decreasing, concave and twice continuously differentiable function, with* $B^i(0, 0) \geq 0$*.*
(iii) For any y, $b_1(y) \geq B_1^g(y) > B_1^n(y)$ *and* $b_2(y) \geq B_2^n(y) > B_2^g(y)$*.*
(iv) For any y, $b_{12}(y) \geq B_{12}^g(y), B_{12}^n(y) \geq 0$*.*

Assumption 2(iii) implies that the marginal return to each player's investment is highest when decisions are made cooperatively, second highest when that player has sole authority, and lowest when the other player has sole authority. Assumption 2(iv) says that investments are weak complements.[13]

2.2. **Preliminary Results.** For any π and y, the Nash-bargained payoff to i gross of the investment cost incurred at date 1 ($i = g, n$) is

$$V^i(y, \pi) = \frac{1}{2}(\theta_g + \theta_n)b(y) + \frac{1}{2}\left[\overline{u}_i(y, \pi) - \overline{u}_j(y, \pi)\right] \quad (j \neq i).$$

That is, V^i equals one-half of the gross surplus plus a factor (the second term) that captures the difference in the players' default payoffs. After substituting for the

[13]In most of the examples discussed in the Introduction and the applications reviewed in Section 7, certain types of investments have a clear-cut weak complement nature. There are other types of investments that are instead substitutable. Therefore, our results may not provide a complete picture of the optimal arrangement in complex partnerships. Incidentally, this is a limitation shared by the related literature (including BG).

default payoffs, using (3) and (4), re-arranging terms and simplifying, we obtain

(5)

$$V^g(y,\pi) = \frac{1}{2}(\theta_g + \theta_n)b(y) + \frac{1}{2}(\theta_g - \theta_n)B(y,\pi) + \frac{\alpha}{2}\left[\theta_n \pi B^g(y) - \theta_g(1-\pi)B^n(y)\right],$$

(6)

$$V^n(y,\pi) = \frac{1}{2}(\theta_g + \theta_n)b(y) - \frac{1}{2}(\theta_g - \theta_n)B(y,\pi) - \frac{\alpha}{2}\left[\theta_n \pi B^g(y) - \theta_g(1-\pi)B^n(y)\right].$$

The first-best investment levels maximize the difference between the gross surplus, $(\theta_g + \theta_n)b(y)$, and the total cost of investments, $C_g(y_g) + C_n(y_n)$. In contrast, the investment levels that are actually chosen are a pure-strategy Nash equilibrium of the date 1 simultaneous-move game in which each player maximizes the difference between its Nash-bargained payoff and its cost of investment.

Lemma A.1 establishes that this investment game has a unique Nash equilibrium, $y^e(\pi) \equiv (y^e_g(\pi), y^e_n(\pi))$, and that it possesses some important properties. (For ease of exposition, the lemma and its proof are detailed in the appendix.) We use the results of this lemma to characterize the optimal value of π. This maximizes the players' date 0 equilibrium net surplus:[14]

$$\max_{0\le\pi\le1} S(\pi) \equiv V^g(y^e(\pi),\pi) + V^n(y^e(\pi),\pi) - C_g(y^e_g(\pi)) - C_n(y^e_n(\pi)). \tag{7}$$

After making use of the first-order conditions which deliver the Nash equilibrium $y^e(\pi)$ (see the appendix) and noting that $V^g_3 + V^n_3 = 0$, the derivative of S with respect to π is

$$S'(\pi) = V^g_2(y^e(\pi),\pi)\left[\frac{\partial y^e_n}{\partial \pi}\right] + V^n_1(y^e(\pi),\pi)\left[\frac{\partial y^e_g}{\partial \pi}\right]. \tag{8}$$

Lemma A.1 shows that the effect of a marginal change in π on the players' respective equilibrium investment levels depends on the signs of V^g_{13} and V^n_{23}. These two cross-partial derivatives capture the effects of a marginal change in π on the players'

[14] Player i's Nash bargained-payoff V^i depends on $y \equiv (y_g, y_n)$ and π, and hence we write it as $V^i(y,\pi)$. For each $i = g, n$, we denote by $V^i_k(y,\pi)$ (or simply V^i_k) the first-order partial derivative of V^i with respect to its k-th argument ($k = 1,2,3$), where the first argument is y_g, the second y_n and the third π. The second-order partial derivatives are denoted by $V^i_{kl}(y,\pi)$ (or simply V^i_{kl}), where $k, l = 1,2,3$.

respective marginal returns on investments. For any π and y,

$$V_{13}^{g} \equiv \frac{\partial}{\partial \pi}\left[\frac{\partial V_g}{\partial y_g}\right] = \frac{1}{2}\left[\underbrace{(\theta_g - \theta_n)[B_1^g - B_1^n]}_{\text{BG effect}} + \underbrace{\alpha[\theta_g B_1^n + \theta_n B_1^g]}_{\text{GHM effect}}\right], \tag{9}$$

$$V_{23}^{n} \equiv \frac{\partial}{\partial \pi}\left[\frac{\partial V_n}{\partial y_n}\right] = \frac{1}{2}\left[\underbrace{(\theta_g - \theta_n)[B_2^n - B_2^g]}_{\text{BG effect}} - \underbrace{\alpha[\theta_g B_2^n + \theta_n B_2^g]}_{\text{GHM effect}}\right]. \tag{10}$$

The right-hand sides of each of these two expressions depend on y, but not on π. Given Claim A.1(i) and (iii) stated in the appendix and Assumption 1, it follows that if, for all y, both V_{13}^g and V_{23}^n are non-negative (non-positive) with at least one of them being strictly positive (strictly negative), then both players' equilibrium investments are strictly increasing (strictly decreasing) in π over its domain. Since $V_2^g > 0$ and $V_1^n > 0$, from (8) we have:

Claim 1. *If, for all y, expressions (9) and (10) are non-negative (non-positive) with at least one of them being strictly positive (strictly negative), then it is optimal to set $\pi = 1$ ($\pi = 0$).*

The right-hand side of (9) decomposes the effect of a marginal change in π on g's investment incentives into two terms. The first term, which we call BG effect, can be positive or negative depending on whether g values the project's benefits more or less than n. The second term, which we label GHM effect, is strictly positive when there exists some degree of impurity ($\alpha > 0$), and zero in the degenerate case of no impurity ($\alpha = 0$). This decomposition shows that g's investment incentives are driven by two potentially opposing forces: preferences and technology.

To gain some intuition, consider the case in which n places a relatively higher value on the project. Then, in the expression for V_{13}^g, the BG effect is negative while the GHM effect is positive. With $\theta_n > \theta_g$ and $B_1^g > B_1^n$, the BG effect arises because n's marginal return to investment is higher when g has sole authority than when n has sole authority. This is the key reason why g's relative bargaining power is higher when n has sole authority than when g has sole authority. In contrast, the GHM effect comes about from the intuition that allocating more authority to an investor increases its relative bargaining power. Consequently, with impure public goods ($0 < \alpha < 1$), g's aggregate relative bargaining power is the sum of these two opposing effects.[15] The trade-off between these two effects will play a central role in the analysis of Sections 3 and 4.

[15] An analogous interpretation applies to (10) in relation to the effect of a marginal change in π on n's investment incentives.

The BG effect entails allocating all of the control rights (sole authority) to the player who values the project's benefits the most. This effect works in the same direction for both parties, in the sense that there is no conflict between g and n. This is not true for the GHM effect, which entails that an investor should be allocated sole authority. As can be seen from (9) (alternatively (10)), the GHM effect is positive (negative), and hence g's (n's) marginal returns are increasing (decreasing) in π. Moreover, when both parties invest, the optimal allocation may require a compromise in the provision of investment incentives to the two players. In some of such cases shared authority will arise at the optimum (see Section 4).

2.3. **Discussion of the Basic Ingredients of the Model.** We underline six features of our model. First, authority is conceptualized in a reduced form fashion. One may think of this formulation along the following line of argument. There are many (formally a continuum of) issues on which decisions have to be taken, all of which are equally important to the project's benefits. An allocation of the large number of control rights is then payoff-equivalent to an allocation of shares.[16] The development of a micro-founded formulation of authority is an important extension which, however, goes beyond the scope of the paper.

Second, as in the GHM-related literature, our model is based on the presumption that if it is optimal for the two players to collaborate and agree to some authority allocation, then they will do so. The focus here is on the analysis of the optimal authority allocation. A sufficient condition for this presumption to hold is that Coase theorem applies: at date 0, the parties bargain in the absence of any friction and, if necessary, can make lump-sum transfers (the extent of which depends on the parties' date 0 outside options and the nature of the optimal authority allocation). In Section 5, however, this condition is relaxed as we examine the way in which authority is allocated in an environment where bargaining is costly and parties cannot make optimal side payments.

Third, in relation to the non-verifiable investment decisions, one feature merits attention. And that is that investments are perfectly observable to the two parties after they are undertaken. While this assumption is standard in the literature, this might be unrealistic. Relaxing the perfect observability assumption means that bargaining over the date 2 surplus takes place under conditions of asymmetric information, and this may imply that with positive probability the players fail to strike an agreement

[16]The fact that there are many decisions to be taken in the context of non-trivial organizations is perhaps unarguable. But there is some loss of generality, of course, in the presumption that these decisions are all equally important. Decisions on some matters (e.g., whether English or Hindi is the main language of instruction in Indian schools) are far more consequential than decisions on other matters (e.g., whether the morning school break is to be at one time or another).

to make decisions cooperatively. This, in turn, may alter some of the main insights on the ex-ante optimal authority allocation.

Fourth, there exists an ex-post surplus. Section 5.3 relaxes this assumption by considering the case in which with a small but positive probability, the surplus does not exist. In this case, with a small probability it is ex-post efficient for the players not to reach an agreement, but to operate under the allocated control rights.

Fifth, the way in which we apply the Nash bargaining solution can be justified by assuming that the players bargain strategically, with the default payoffs being identified as the players' inside option payoffs (Muthoo, 1999). This means that during any significant delay in reaching agreement, the players would operate the project under the default allocated control rights (which is consistent with many real-life public-private partnerships). Of course, in equilibrium, no delay occurs, but these out-of-equilibrium payoffs shape the nature of the equilibrium division of the surplus.[17]

Sixth, the structure of the model is common knowledge. In many circumstances, however, this may not be the case because, for example, a party's valuation would be its private information. This extension is left for future research.

3. Optimal Authority with Sole Investor

The case of a sole investor may be interpreted as a limiting case in which the other player's investment has a negligible impact on the project's benefits. This will provide an explicit understanding of some of the forces at work. It may also be of general interest because in some situations only one party invests. We consider the case in which the sole investor is the government. The analysis therefore is restricted to equation (9), as the issue of the investment incentives for n (and thus (10)) is no longer relevant.

We begin by examining the two extreme cases already studied in the literature (pure public goods and pure private goods), and by considering the optimal allocation when the parties have identical valuations.

Lemma 1 (Benchmark Cases)**.** *Assume that g is the sole investor.*
(a) (*Pure Public Good*) *If $\alpha = 0$, then it is optimal to allocate sole authority to the player who has the relatively higher valuation.*

[17] An alternative way is to treat the default payoffs as the players' outside option payoffs (Muthoo, 1999). This means that during any significant delay in reaching agreement, the project comes to a halt: each player has the option to stop the negotiations unilaterally and get the project going under the allocated control rights, without any further negotiation to reach agreement. The outside-option bargaining approach would alter some, but not all, of the results obtained under the inside-option bargaining approach.

(b) (Pure Private Good) If $\alpha = 1$, then it is optimal to allocate sole authority to the sole investor, g.
(c) (Identical Valuations) In the degenerate case when the parties have the same valuation, any authority allocation is optimal if $\alpha = 0$; but if $\alpha > 0$, then it is optimal to allocate sole authority to the sole investor, g.

Proof. The results of this lemma can be easily derived from Claim 1. Parts (a), (b) and (c) follow immediately from examining the sign of the right-hand side of (9) after substituting for $\alpha = 0$, $\alpha = 1$ and $\theta_g = \theta_n$, respectively. □

As in BG, Lemma 1(a) shows that in the case of a pure public good, sole authority should go to the player who cares most about it, irrespective of the investor and the importance of its investment. Conversely, in the GHM case of a pure private good, Lemma 1(b) shows that control rights should entirely go the sole investor, irrespective of how important its investment is and whether the non-investor has a higher or a lower valuation. Lemma 1(c) confirms BG's result that in the case of a pure public good, authority does not matter when the parties have identical valuations; but for any positive degree of impurity, authority does matter and should be fully given to the sole investor, regardless of how important its investment is.

We now move beyond these benchmark cases. Examining the right-hand side of (9), we see that if g, the sole investor, is the player with the relatively higher valuation, then both BG and GHM effects are in the same direction, and it is optimal to allocate sole authority to g. But if n values the project's benefits more than g does, then the GHM effect is in the opposite direction of the BG effect. In this case the BG effect is negative while the GHM effect is strictly positive, provided there is some degree of impurity, otherwise Lemma 1(a) applies. Suppose $\theta_g < \theta_n$. Equation (9) can be rewritten as

$$V_{13}^g \equiv \frac{\partial}{\partial \pi}\left[\frac{\partial V_g}{\partial y_g}\right] = \frac{1}{2}\left[\left[\theta_g - (1-\alpha)\theta_n\right]B_1^g + \left[\theta_n - (1-\alpha)\theta_g\right]B_1^n\right]. \tag{11}$$

While the first term on the right-hand side of (11) (the term involving B_1^g) can be positive or negative, the second term is strictly positive, since $\theta_g < \theta_n$. The first term is non-negative if and only if $\alpha \geq (\theta_n - \theta_g)/\theta_n$. Hence, under such parametric restrictions the right-hand side of (11) is strictly positive, and hence g should be optimally allocated sole authority. We summarize these results in the following:

Proposition 1. *If g is the sole investor and $\alpha \geq (\theta_n - \theta_g)/\theta_n$, then it is optimal to allocate sole authority to g.*

This means that if the degree of impurity is sufficiently large, control rights should be entirely given to the sole investor. The key insight of GHM is thus robust to small perturbations along the private-public good dimension.

Consider now the remaining set of parameter values for which $\theta_g < \theta_n$ and $\alpha < (\theta_n - \theta_g)/\theta_n$. In this case, the right-hand side of (11) may not keep the same sign for any y (since the first term is negative while the second is positive). Thus, we have to impose some additional structure on the relationship between the marginal returns when g has sole authority and when n has sole authority. Our next result is derived under the assumption that the ratio $B_1^n(y)/B_1^g(y)$ is constant and independent of y. This assumption is borrowed from BG (p. 1355).

Assume that, for any y, $B_1^n(y) = \beta_g B_1^g(y)$, where $\beta_g \in (0,1)$. Following BG and Hart et al. (1997), one may interpret $1 - \beta_g$ as the proportion of the returns on g's investment that cannot be realized without g's continued cooperation. After substituting for $B_1^n(y)$ in (11), simplifying, and rearranging terms, it follows that

$$V_{13}^g \equiv \frac{\partial}{\partial \pi}\left[\frac{\partial V_g}{\partial y_g}\right] = \frac{1}{2}\left[\underbrace{(1-\widehat{\theta})(1-\beta_g)}_{\text{BG effect}} + \underbrace{\alpha(\widehat{\theta}+\beta_g)}_{\text{GHM effect}}\right] B_1^g \gtreqless 0 \Longleftrightarrow \alpha \gtreqless \alpha_g^*, \tag{12}$$

where $\alpha_g^* = [(\widehat{\theta}-1)(1-\beta_g)]/(\widehat{\theta}+\beta_g)$ and $\widehat{\theta} = \theta_n/\theta_g$. We thus obtain:

Proposition 2. *Assume that g is the sole investor and that $B_1^n(y) = \beta_g B_1^g(y)$ (for any y, with $\beta_g \in (0,1)$). If $\alpha > \alpha_g^*$ ($\alpha < \alpha_g^*$), then it is optimal to allocate sole authority to g (n).*

Figure 1 illustrates Proposition 2 in the $(\widehat{\theta}, \alpha)$ space. In the non-shaded area, it is optimal to allocate sole authority to the investor, g, while in the shaded area control rights should be entirely given to the non-investor, n. The optimal allocation does not depend on the importance of the investment (i.e., on the investment's marginal benefits), but depends on the other three key parameters: α (the degree of impurity), $\widehat{\theta}$ (the parties' relative valuation), and β_g (the degree to which the sole investor is dispensable).

For all the $(\widehat{\theta}, \alpha)$ combinations such that $\widehat{\theta} < 1$, allocating sole authority to g regardless of the degree of impurity of the public good is consistent with both BG and GHM. In that region, in fact, the sole investor happens to have also a higher valuation for the project.

But if the non-investor has a relatively higher value, $\widehat{\theta} > 1$, the BG and GHM effects go in opposite directions. On one hand, the GHM result (sole authority to the sole investor) arises whenever there is a sufficiently high degree of impurity, $\alpha > 1 - \beta_g$, which is independent of the parties' relative valuation. On the other hand, the BG result (sole authority to the non-investor) emerges whenever the degree of

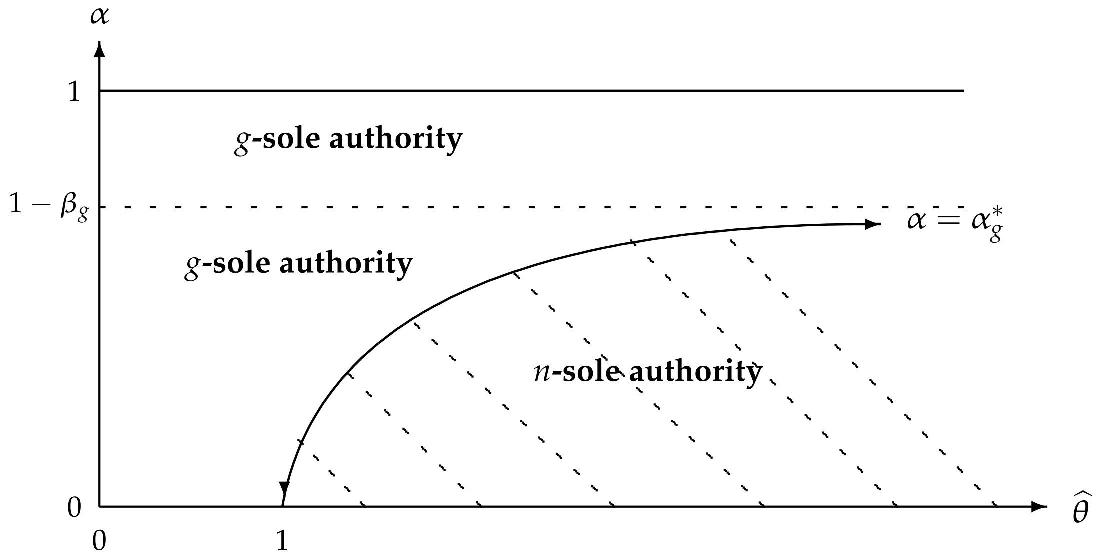

FIGURE 1. An illustration of Proposition 2.

impurity is sufficiently low, $\alpha < \alpha_g^*$, which does depend on $\widehat{\theta}$. Thus, both effects are robust to perturbations along the private-public good dimension.

Notice, however, that there is a non-negligible region in Figure 1 in which sole authority is given to the investor even if the non-investor cares more about the project *and* the degree of impurity is relatively small. This allocation is clearly inconsistent with BG's general insight. It arises because when $\widehat{\theta}(> 1)$ decreases, the size of the BG effect declines at a faster rate than the size of the GHM effect (see the two terms in brackets in (12)) as long as there is a positive (albeit small) degree of impurity, i.e., $\alpha < 1 - \beta_g$. The presence of some excludability over the project's benefit, therefore, makes BG's results less compelling.

Now consider the situation in which the sole investor's dispensability β_g increases. In this case, there is a set of $(\widehat{\theta}, \alpha)$ combinations for which the equilibrium shifts from sole n-authority to sole g-authority. The intuition for this result stems from the fact that as $1 - \beta_g$ declines, the BG effect is weakened relative to the GHM effect.[18] Hence, g should receive sole authority precisely because this maintains its investment incentives.

[18]In the limit (as $\beta_g \to 1$), the BG effect is eliminated. This observation holds more generally. As the difference $B_1^g - B_1^n$ decreases the BG effect shrinks, and as this difference goes to zero, the BG effect disappears.

To conclude the analysis of this section, we emphasize three results. First, both BG and GHM's insights are generally robust to small perturbations along the private-public good dimension. Second, there are however situations in which this result does not hold, with the BG effect being dominated by the GHM effect. This is driven by the fact that we look at impure public goods. Third, shared authority is never optimal. With only one investor, sole authority (either to the investor or to the non-investor) always generates a higher surplus than any form of sharing of that authority.

4. Optimal Authority When Both Invest

We now turn to the general case in which both players undertake investments at date 1. Our first set of results shows the extent to which the results of Proposition 2 apply to this case. Let $\alpha_n^* = [(1-\widehat{\theta})(1-\beta_n)]/(1+\widehat{\theta}\beta_n)$, where both $\widehat{\theta}$ and α_g^* are defined just before Proposition 2.

Proposition 3. *Assume that both parties can invest at date 1,* $B_1^n(y) = \beta_g B_1^g(y)$ *and* $B_2^g(y) = \beta_n B_2^n(y)$ *(for any* y*, with* $\beta_g, \beta_n \in (0,1)$*).*
(a) If $\widehat{\theta} > 1$ *and* $\alpha \leq \alpha_g^*$*, then it is optimal to allocate sole authority to* n *(i.e.,* $\pi^* = 0$*).*
(b) If $\widehat{\theta} < 1$ *and* $\alpha \leq \alpha_n^*$*, then it is optimal to allocate sole authority to* g *(i.e.,* $\pi^* = 1$*).*

Proof. These results are easily derived from Claim 1. Using the hypotheses of this proposition, it is straightforward to verify that

$$V_{13}^g \gtreqless 0 \Longleftrightarrow \alpha \gtreqless \alpha_g^* \quad \text{and} \quad V_{23}^n \gtreqless 0 \Longleftrightarrow \alpha_n^* \gtreqless \alpha.$$

Note that if $\widehat{\theta} > 1$ then $\alpha_g^* > 0 > \alpha_n^*$, and if $\widehat{\theta} < 1$ then $\alpha_n^* > 0 > \alpha_g^*$. □

Figure 2 illustrates these results in the $(\widehat{\theta}, \alpha)$ space. In the two shaded areas the principle under which control rights are allocated is consistent with BG's main insight: sole authority should be given to the high-valuation party. Clearly, technological conditions embedded in β_g and β_n also matter, to the extent that they affect the shape of these two regions. There are, however, some $(\widehat{\theta}, \alpha)$ combinations for which this principle cannot be applied even for small perturbations from the pure public good case. These combinations lie in the non-shaded area of Figure 2 — i.e., for combinations of $(\widehat{\theta}, \alpha)$ such that $\alpha > \max\{\alpha_g^*, \alpha_n^*\}$. In this region the optimal value of π cannot be determined with Claim 1 since $V_{13}^g > 0$ and $V_{23}^n < 0$. To derive the optimal authority allocation for parameter values in this region, we thus impose some more structure on the benefit and cost functions.

Following BG (p. 1355), let $b(y) = a_g\mu(y_g) + a_n\mu(y_n)$, $B^g(y) = a_g\mu(y_g) + \beta_n a_n\mu(y_n)$ and $B^n(y) = \beta_g a_g\mu(y_g) + a_n\mu(y_n)$, where μ is a strictly increasing, strictly concave, and twice differentiable function satisfying the Inada endpoint conditions,

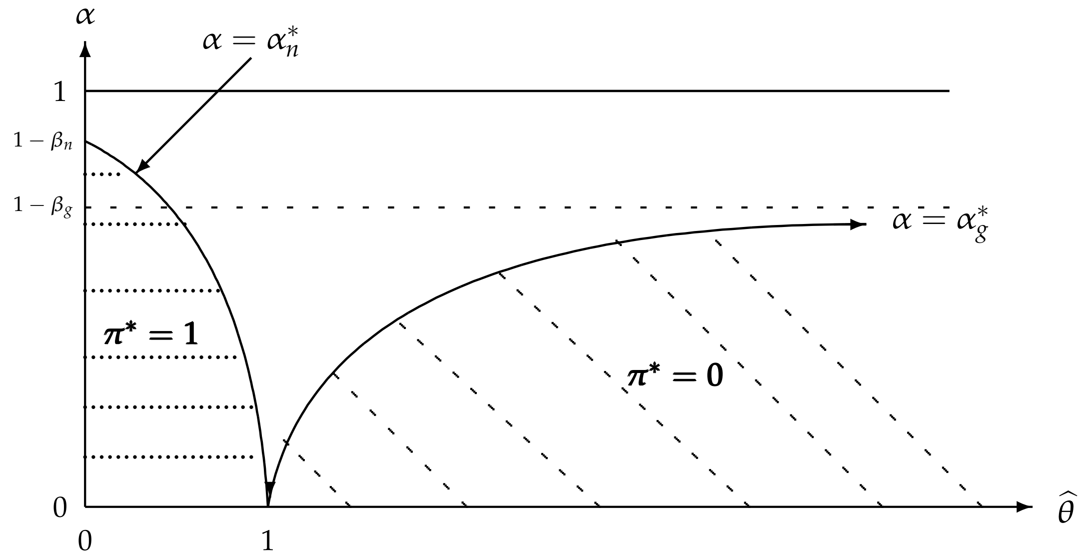

FIGURE 2. An illustration of Proposition 3.

with $\mu(0) > 0$. The strictly positive parameters a_g and a_n denote the importance of the investments, and β_g and β_n are defined in Proposition 3, with $1 - \beta_i$ capturing the proportion of the returns on i's investment that cannot be realized if j has sole authority (i.e., without i's continued cooperation), where $i, j = g, n$ and $j \neq i$. To simplify the algebra in the derivation of the results stated in Propositions 4–6, we assume that $C_i(y_i) = y_i$, and, as in BG, that $\mu(y_i) = 2\sqrt{a_i y_i} + A$, where A is a positive constant.

Our next result considers the optimal authority allocation for the perfectly symmetric scenario, in which the parties' valuations are identical, their investments are of equal importance, and they are equally dispensable:

Proposition 4. *Assume that $\theta_g = \theta_n$, $a_n = a_g$, and $\beta_g = \beta_n$. If $\alpha = 0$ then any $\pi \in [0, 1]$ is optimal, but for any $\alpha > 0$, the optimal authority allocation is $\pi^* = 1/2$ (i.e., equal sharing of authority).*

Proof. In the appendix. □

In this fully symmetric situation, optimality requires allocating an equal amount of authority across the two parties, as long as there is some positive degree of impurity. But, perhaps surprisingly, there is discontinuity at $\alpha = 0$ (the case of a pure public good): in this case, the allocation of authority does not matter.

Moving away from this perfectly symmetric case, we examine two opposite situations. The first is one in which one party's investment is sufficiently more important than the other party's investment (i.e., a_n/a_g is either sufficiently large or sufficiently small). In the second case, we analyze situations in which the importance of both parties' investments is relatively similar.

In the first case, sole authority is preferred to shared authority, and it should be allocated to the party whose investment is relatively more important. This conclusion is valid irrespective of relative valuations, as long as the $(\widehat{\theta}, \alpha)$ combinations lie in the non-shaded region of Figure 2.[19] The intuition of this result is simple: when the investment of one party is more consequential for the success of the project, the GHM effect dominates, whereby control rights must be given exclusively to that party. Formally:

Proposition 5. *Fix any parameter values in the non-shaded region of Figure 2. If one party's investment is sufficiently more important than the other party's investment, then sole authority is preferred to shared authority, and it should be allocated to the party whose investment is relatively more important.*

Proof. In the appendix. □

For the second class of situations, those in which the importance of the parties' investments is relatively similar, our result is given in the following:[20]

Proposition 6. *Assume that the importance of the parties' investments is similar.*
(a) If the degree of impurity is sufficiently small, then sole authority should be allocated to the high-valuation party.
(b) If the degree of impurity is sufficiently large, then shared authority is the optimal allocation, with the low-valuation party receiving a relatively larger share.
(c) If the degree of impurity is neither sufficiently small nor sufficiently large, then sole authority should optimally be allocated to the low-valuation party.

Proof. In the appendix. □

[19]It is worthwhile pointing out that, as β_g and β_n tend to one (i.e., both parties become fully dispensable), the non-shaded area of Figure 2 covers the entire $(\widehat{\theta}, \alpha)$ space, with the exclusion of the $\alpha = 0$ line. Thus, the relevance of Proposition 5 is general in this extreme case. At the limit, when $\alpha = 0$, Proposition 3 shows that control rights should be entirely given to the party with the higher valuation (as in BG).

[20]The appendix contains a more formal characterization of this proposition.

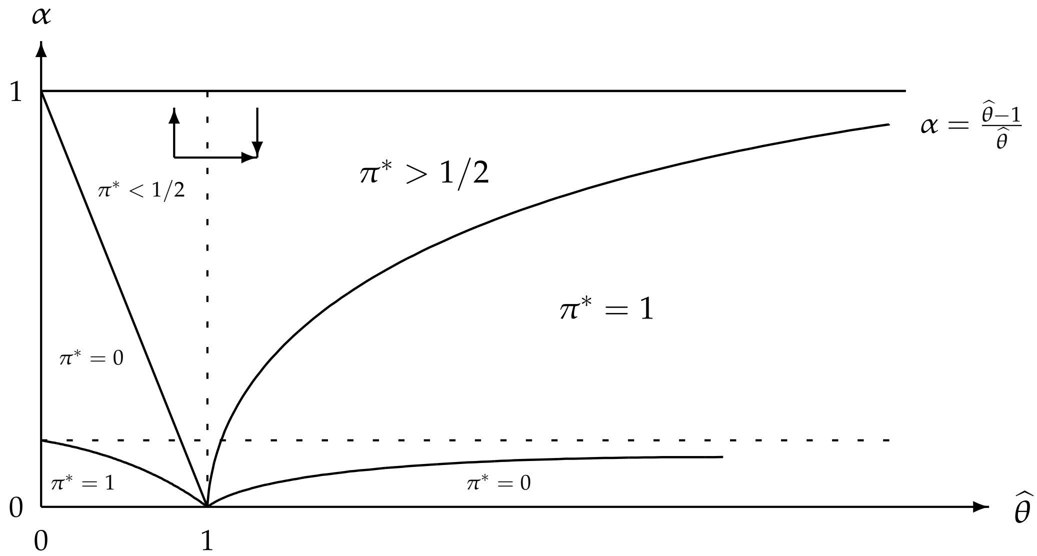

FIGURE 3. An illustration of Proposition 6.

Figure 3 illustrates this result in the $(\widehat{\theta}, \alpha)$ space.[21] We emphasize three points. First, there is a wide range of parameter combinations in which the two parties share authority at the optimum. In such situations, n will have a greater share of control rights if g cares more about the project, and vice versa g will have a greater share if n has a higher valuation. This result goes against BG's main intuition. But it reflects the way in which the GHM effect operates: by providing goods that have some degree of excludability, the two parties must spread their control rights in order to equalize their ex-post bargaining powers. This equalization is a covenant whereby the two parties' investment incentives are balanced out, given that both investments are relatively important.

Second, as the degree of impurity gets smaller, we move from shared authority to sole authority. The way in which control rights are allocated echoes the GHM principle just discussed: sole authority goes to the party who cares relatively *less* about the project, precisely because this restores equal bargaining powers between parties whose investments are similarly relevant. Third, when the extent of excludability is

[21]In the figure we account for additional formal properties of the optimum stated in the appendix. The arrows at the top of the figure indicate the direction in which π^* is strictly increasing (over the region in which π^* lies between zero and one): (i) π^* is strictly increasing in $\widehat{\theta}$; and (ii) π^* is strictly increasing (strictly decreasing) in α when $\widehat{\theta} < 1$ (> 1).

very small, sole authority is again optimal and, consistent with BG, should be given to the party who cares more for the project.

We close this section pointing out three general results. First, as in the case with only one investor, there are circumstances in which both BG and GHM's insights are robust to small perturbations along the private-public good dimension. Second, as in the previous section, there are many situations in which this is not straightforwardly true. In these situations, the notion of equalizing bargaining powers operates in a way such that control rights are optimally spread across parties in order to keep their investment incentives undiminished. Third, in stark contrast with the results of the previous section, there are cases in which shared authority is the preferred allocation, especially when both parties' investments are of comparable importance. This can provide an economic foundation to a number of actual shared authority allocations, such as joint custody of children after divorce (see Section 7).

5. Equilibrium Authority

In this section we develop an extension to our analysis. We analyze the allocation of authority which the parties would agree on at date 0 in a bargaining equilibrium, and discuss the extent to which this equilibrium authority allocation differs from the optimal authority allocation derived in the two previous sections.

In line with the incomplete contract approach, we have so far assumed that the two parties jointly determine the allocation of authority at date 0, by maximizing the joint surplus generated by their project. This is what we denoted with π^*. But will this optimal allocation be realized in equilibrium? And under what conditions would the equilibrium authority allocation π^e be the same as π^*?

Suppose that, at date 0, the two parties, g and n, negotiate over the allocation of authority, i.e., over the choice of $\pi \in [0, 1]$, including side payments (if feasible). Such negotiations are conducted in the shadow of a *status quo* or default authority allocation, denoted by $\pi^d \in [0, 1]$. For simplicity, we do not specify how this default has come about and treat it as a given parameter.[22] To deal with a non-trivial problem, we assume that this default allocation is not the optimal allocation (i.e., $\pi^d \neq \pi^*$). Given this, we now explore whether there exist circumstances under which the bargaining equilibrium π^e coincides with the optimal allocation π^*.

5.1. **Coasian Negotiations.** If the sufficient conditions of the Coase theorem (Coase 1960) hold, then g and n would negotiate and implement the optimal allocation of authority, which implies $\pi^e = \pi^*$. Such conditions require transferable utility, no

[22]For example, π^d could be determined by tradition and be established on the basis of other similar projects which the same two (or other) parties were involved with in the past.

wealth effects, and costless bargaining, so that all information is common knowledge and side payments are feasible. Each party therefore must have enough resources to make either an upfront side payment or a binding commitment over such a payment later. Side payments are essential here since, in the switch from π^d to π^*, one of the parties is likely to worsen its position in relation to the other party's.

As before, for any authority allocation π determined at date 0 and any pair of investment levels $y = (y_g, y_n)$ chosen at date 1, the payoffs at date 2 are stated in (5) and (6) for g and n respectively, with the Nash equilibrium investment levels $y^e(\pi)$ depending on π. After substituting $y^e(\pi)$ into (5) and (6), the date 0 payoff to party i will be $W^i(\pi) \equiv V^i(y^e(\pi), \pi) - C_i(y_i^e(\pi))$, for $i = g, n$. Given this, i's default payoff is $W_i^d \equiv W^i(\pi^d)$, while the payoff of i if π^* is chosen is $W_i^* \equiv W^i(\pi^*)$. The assumption that the default allocation is inefficient implies that $W_g^* + W_n^* > W_g^d + W_n^d$. Using the Nash bargaining solution, both parties will agree to set $\pi^e = \pi^*$ with a side payment from g to n equal to:

$$T^e = \frac{[W_g^* - W_g^d] - [W_n^* - W_n^d]}{2}.$$

If, however, $T^e < 0$, then it is a payment (of $-T^e$) from n to g. Even if both g and n gain by selecting π^*, in the Nash bargained context a party will still need to make a side payment if its net gain becomes relatively larger. It is worth emphasizing that, although the equilibrium allocation reached through Coasian bargaining coincides with π^* regardless of π^d, the party who makes the side payment as well as the magnitude of T^e are sensitive to the default allocation, and hence π^d has distributional consequences.

5.2. **Infeasibility of Side Payments.** There are cases in which bargaining is costly (Anderlini and Felli, 2001) or either g or n (or both) are wealth constrained, so that the parties may not be able to make (or commit to) the equilibrium side payment T^e. To illustrate this scenario very starkly, we assume that no side payment can be made. Our aim is to characterize the conditions under which g and n would agree on the optimal allocation π^* rather than the default allocation π^d, even in the absence of side payments. Clearly, this can happen only when each party prefers the former allocation to the latter, i.e., $W_i^* > W_i^d$ for both $i = g, n$. This means that the date 0 payoff pair associated with the optimal allocation Pareto dominates the corresponding payoff pair associated with the default allocation.

As in some of the analysis of Section 4, we consider the case in which both parties can invest in the project and the importance of the parties' investments is relatively similar.[23] In this context, the optimal allocation of authority is pinned down by the results given in Proposition 6. Our main findings are summarized in the following:

Proposition 7. *Assume that both parties can invest, the importance of the parties' investments is similar, and no side payment between parties can be made. Then, each party is more likely to prefer the optimal allocation π^* over the default allocation π^d as:*
(a) the difference between π^d and π^* increases; or
(b) the absolute difference between the parties' valuations, θ_n and θ_g, increases.

Proof. In the Appendix. □

The intuition behind the results in Proposition 7 comes from trading off the aggregate size of the efficiency gains by moving from the default allocation to the optimal allocation against the loss to a party from doing so. Put differently, the party that is disadvantaged by the switch must compute whether the smaller share of a larger cake is greater than the larger share of a smaller cake. Proposition 7 reveals that the optimal allocation π^* is reached when the difference between the default allocation and the optimal allocation is large (part (a)), because the efficiency gains are increasing in the distance between π^d and π^*. Part (b) of the proposition shows that π^* can also emerge when the parties' valuations are very different, since again the efficiency gains are increasing in the distance between θ_n and θ_g.

In sum, when bargaining is costly and parties are unable (or cannot commit) to make side payments, the authority allocation emerging from a bargaining equilibrium coincides with the optimal allocation as in the world with Coasian negotiations, provided either that the default allocation differs substantially from the optimal allocation or that parties' valuations differ substantially from each other. If neither of these conditions is met, then the default allocation remains in place. Of course, these results apply to the case when side payments are not feasible and parties' investments are of comparable importance. The authority allocations that could be achieved if limited side payments were possible and in other environments (e.g., when one party's investment is sufficiently more important than the other's or when only one party can invest) have not been explored here and are left for future research.

[23]For the sake of brevity, we cannot develop all the other cases developed in Sections 3 and 4. This, however, provides an interesting benchmark example.

6. OTHER EXTENSIONS

In this section we discuss the robustness of our main results to some alternative specifications. First, we consider three or more parties (e.g., a government and multiple NGOs). Second, we discuss whether there are circumstances under which joint authority would dominate the optimal shared authority allocation. Third, we introduce ex-post uncertainty which may lead the parties to operate the project at date 2 under the date 0 allocated control rights. Fourth, we examine the case in which investments are perfect substitutes.

6.1. **Multiple NGOs.** In many public-private projects, the government is involved with more than one NGO.[24] We now study such situations, but only consider the case in which the government is the sole investor. This is equivalent to assuming that all parties invest but the government's investment is significantly more important than the investment of any NGO.

There are N ($N \geq 1$) NGOs, which are denoted by the integers $1, 2, \ldots, N$, and a government, which, for notational convenience, is denoted by $N+1$. Let $\theta_i > 0$ ($i = 1, 2, \ldots, N, N+1$) be player i's valuation of the project's benefits, and π_i its share of authority, where $\pi_i \in [0,1]$ and $\sum_{i=1}^{N+1} \pi_i = 1$. The vector of shares is given by $\pi = (\pi_1, \pi_2, \ldots, \pi_N, \pi_{N+1})$.

If all players reach an agreement to make decisions cooperatively at date 2, then player i's payoff is $u_i(y) = \theta_i b(y) + t_i$, where y denotes the government's date 1 investment, the transfer t_i can be positive or negative, and sum of the transfers equals zero. If, however, the players fail to reach such an agreement, the project operates under the control rights allocated at date 0, and player i's default payoff is $\overline{u}_i(y, \pi) = \theta_i[\alpha \pi_i B_i(y) + (1-\alpha) B(y, \pi)]$, where $B(y, \pi) = \sum_{k=1}^{N+1} \pi_k B_k(y)$ and $B_k(y)$ is the project's benefits when player k has sole authority.

As before, we assume that for any y, $b'(y) > B'_{N+1}(y) > B'_j(y)$ for all $j = 1, 2, \ldots, N$: that is, the marginal returns to the government's investment are highest when the players make decisions cooperatively, and second highest when the government has sole authority. For simplicity and without loss of generality, we rewrite Assumption 2(iii) as follows:

Assumption 3. *For any y, $b'(y) > B'_{N+1}(y) > B'_N(y) > \ldots > B'_2(y) > B'_1(y)$.*

This ranks the marginal returns to government's investment over the set of all possible sole authority regimes (there are $N+1$ such regimes). It implies that the marginal returns are *lowest* when NGO 1 receives sole authority.

[24] One of the few applications with multiple parties is in Hart and Moore (1990).

By definition, the Nash bargained payoff to player i at date 2 is[25]

$$V^i(y,\pi) = \overline{u}_i(y,\pi) + \frac{1}{N+1}\left[(N+1)\overline{\theta}b(y) - \sum_{k=1}^{N+1}\overline{u}_k(y,\pi)\right], \quad \text{where} \quad (N+1)\overline{\theta} = \sum_{k=1}^{N+1}\theta_k.$$

After substituting for the default payoffs, simplifying and collecting terms, we obtain

$$V^i(y,\pi) = \overline{\theta}b(y) + (1-\alpha)(\theta_i - \overline{\theta})B(y,\pi) + \alpha\left[\theta_i\pi_i B_i(y) - \frac{1}{N+1}\sum_{k=1}^{N+1}\theta_k\pi_k B_k(y)\right].$$

Proposition 8. *Suppose there is a finite but arbitrary number of NGOs and a government, who is the sole investor.*
(a) If the degree of impurity is sufficiently large, then it is optimal to allocate sole authority to the government.
(b) If the degree of impurity is sufficiently small, then the optimal authority allocation depends on whether the government's valuation is greater or smaller than the average valuation of the NGOs. When the government's valuation is greater, it is optimal to allocate sole authority to the government. When the government's valuation is lower, it is optimal to allocate sole authority to NGO 1 (under which the marginal returns to government investment are lowest; cf. Assumption 3).

Proof. In the appendix. □

When public-private projects deliver public goods with a high degree of excludability, part (a) of Proposition 8 confirms the basic insight of Propositions 1 and 2: a GHM-type effect dominates, and optimal authority allocations are primarily driven by technology (rather than preferences). If instead the degree of impurity is small and we are close to the BG world, part (b) shows that our previous results are not entirely robust. If, as before and as in BG, the government cares most about the project, then full authority goes to the government. But if the government cares less than the average NGO, then authority should not go to the NGO with the highest valuation but to the NGO under which the marginal returns to the government's investment are the lowest.

[25]In applying the $(N+1)$–player Nash bargaining solution, we assume that the players have a choice between full cooperation and no cooperation. Thus, we rule out the possibility of partial cooperation, whereby a subset (coalition) of players may cooperate while the remaining players make decisions on matters over which they have received allocated control rights. Allowing for partial cooperation is interesting, but it would take us beyond the scope of this paper. Hart and Moore (1990) provide the only study in this literature that considers more than two players with partial cooperation (albeit in the context of pure private goods). They use the Shapley value to determine the outcome of the date 2 bargaining. This means that, by definition, full cooperation is reached in equilibrium, although a player's payoff is in general influenced by the outcomes associated with partial cooperation.

We look at this last result by considering the limiting case of a pure public good ($\alpha = 0$). As in Section 3, the optimal allocation with only one investor is determined by assessing how its investment incentives (or the marginal returns to its investment) vary as the amount of authority allocated to each player varies. If $\alpha = 0$ and the government (player $N+1$) is the sole investor, these incentives are given by

$$\frac{\partial}{\partial \pi_i}\left[\frac{\partial V^{N+1}}{\partial y}\right] = [\theta_{N+1} - \overline{\theta}]B_i'(y),$$

where $i = 1, 2, \ldots, N, N+1$. If $\theta_{N+1} < \overline{\theta}$, the government's incentives are strictly decreasing in each player's share of authority. Because of this, full control rights should be given to the party for which $B_i'(y)$ is minimal. Assumption 3 guarantees that this party is NGO 1. Of course if there were only one NGO (as in previous sections and in BG), this allocation would have been equivalent to the allocation based on the most caring party principle. This discussion however shows that such a principle is not robust to the presence of multiple NGOs.

6.2. **Joint Authority.** We now go back to the basic model with two players and consider what happens when they can opt for joint authority (BG). Under joint authority, each party has veto rights over all decisions. Thus, if at date 0 the parties agree to operate the project under joint authority, they will need to cooperate at date 2. But if the parties fail to agree, the project cannot go ahead, and their disagreement payoffs are zero. Given that at date 2 they bargain over a surplus of size $b(y)$, the Nash-bargained payoff to each player (gross of its investment cost) is $Z(y) = \frac{1}{2}(\theta_g + \theta_n)b(y)$. A direct implication is

Proposition 9. *Suppose that the players can operate the project under joint authority. All the results of Sections 3 and 4 hold except when the investment of the low-valuation party is sufficiently more important than that of the high-valuation party and the degree of impurity is small. Under these circumstances, joint authority is optimal rather than sole authority to the high-valuation party.*

Proof. In the appendix. □

This proposition alters the results from Lemma 1(a) and from Propositions 2, 3, and 6(a). The intuition behind this new result is simple. The low-valuation party whose investment is more important has greater investment incentives under joint authority than when the high-valuation party has full control rights (which is the best allocation among the set of all shared allocations if the degree of impurity is sufficiently small). This is because under joint authority the high-valuation party has no bargaining advantage as it would have had under sole authority. Proposition

9 therefore extends what BG found for $\alpha = 0$ (their Proposition 2) to cases in which there are sufficiently small degrees of impurity.

6.3. **Ex-Post Uncertainty.** So far, we considered equilibrium allocations in which the two parties cooperate at date 2. Suppose instead that there is a positive probability that the project operates under the initially specified control rights (Rasul, 2006). This could arise for several reasons. For example, although at date 2 all parties know that it is mutually beneficial to make decisions cooperatively, they still may fail to strike an agreement.[26] In addition, there might be some adverse circumstances under which the gains from cooperation at date 2 do not exist.

We consider two cases in which there is a *small* probability that at date 2 the parties operate the project under the control rights allocated at date 0. These two cases differ in the way uncertainty occurs. In what follows, we provide intuitive arguments to show that our results are robust to both types of ex-post uncertainty.

First, suppose there exists a fixed probability, ω, with which the parties fail to strike a deal at date 2. Hence, with probability ω the project is operated under the date 0 allocated control rights. At the beginning of date 2, the equilibrium gross expected payoff to i ($i = g, n$) is $(1-\omega)V^i(y,\pi) + \omega\overline{u}_i(y,\pi)$. Clearly, when $\omega = 0$ we are back to the basic model of the two previous sections. Given Assumptions 1 and 2, the equilibrium investments and the optimal shared authority allocation are continuous in ω. Thus, all the results of the basic model will apply for sufficiently small values of ω.

Second, suppose there is a random variable, ξ, that positively affects the project benefits when the parties cooperate. Let $b(y,\xi)$ denote these benefits, with b being strictly increasing and strictly concave in ξ. Let F be the distribution function of ξ, which has finite support over the interval $[0,\overline{\xi}]$. Given these assumptions, for any π and y, there exists a cut-off value of ξ, $\xi^*(\pi,y)$, such that mutually beneficial gains from cooperation exist if and only if the realized value of ξ is greater than or equal to this cut-off value. Hence, with probability $F(\xi^*(\pi,y))$ the project is operated under the control rights π. At the beginning of date 2 and before the realization of ξ, the equilibrium gross expected payoff to i ($i = g, n$) is

$$\overline{u}_i(y,\pi)F(\xi^*(\pi,y)) + \int_{\xi^*(\pi,y)}^{\overline{\xi}} V^i(y,\pi,\xi)dF(\xi).$$

If $\overline{\xi} = 0$, we return to the basic model. Since equilibrium investments and optimal authority allocations are continuous in $\overline{\xi}$, any small perturbation of $\overline{\xi}$ close to zero will deliver our basic model's results.

[26]As in the bargaining theory literature, this failure may arise from either specific procedural features (Muthoo, 1999) or the presence of bargaining costs (Anderlini and Felli, 2001).

6.4. **Perfect Substitutes.** The analysis of the model in Sections 2-4 is based on the assumption that the parties' investments are weak complements. We now briefly consider the case in which investments are perfect substitutes. As in BG (p. 1361), assume that $b(y) = \eta(y_g + y_n)$, $B^g(y) = \eta(y_g + \lambda y_n)$ and $B^n(y) = \eta(\lambda y_g + y_n)$, where $\eta(.)$ is a well-behaved, increasing, concave function satisfying the Inada conditions, and $0 < \lambda < 1$. For simplicity, assume $C_i(y_i) = y_i$, $i = g, n$. Letting $Y = y_g + y_n$ be the aggregate level of investment, the first-best solution maximizes $\eta(Y) - Y$. This is pinned down by Y^*, which solves $\eta'(Y) = 1$, while the precise distribution of total investment between y_g and y_n is irrelevant for joint surplus.

Within this setup, we are able to characterize the optimal authority allocation when the degree of impurity is sufficiently small. If instead the degree of impurity is not small, π^* cannot be characterized without imposing additional structure on the η function.

Proposition 10. *Assume the parties' investments are perfect substitutes.*
(a) Suppose $\widehat{\theta} > 1$. *If* $\alpha < 1 - (1/\widehat{\theta})$, *it is optimal to allocate sole authority to* n *(i.e.,* $\pi^* = 0$*).*
(b) Suppose $\widehat{\theta} < 1$. *If* $\alpha < 1 - \widehat{\theta}$, *it is optimal to allocate sole authority to* g *(i.e.,* $\pi^* = 1$*).*
(c) For values of $(\widehat{\theta}, \alpha)$ *that lie outside of the regions described in cases (a) and (b) above, the optimal authority allocation is ambiguous, unless further assumptions on* η *are introduced.*

Proof. In the Appendix. □

The intuition of this result is simple. Consider case (b), in which the public aspect of the project is sufficiently high (that is, the degree of impurity is small enough with $\alpha < 1 - \widehat{\theta}$). In this case, not only does the government have the highest valuation ($\widehat{\theta} < 1$) but also its investment incentives always dominate n's. Because investments are perfect substitutes, then only g invests (and n never does) regardless of the allocation of authority. Hence, sole authority should be allocated to g (i.e., $\pi = 1$). This result, therefore, confirms the basic insights of Proposition 3 (as well as BG's) even in an environment in which parties' investments are perfect substitutes. The development of case (c), which refers to an environment with a greater degree of impurity, is left for future research.

7. Applications

7.1. **The Provision of School Services: Government or NGOs?** BG provide a compelling argument to address the question of how the responsibilities of the state and the voluntary sector should be optimally allocated in financing (and generally providing inputs to) public projects, such as schools, hospitals, and sanitation services. Their analysis shows that ownership should reside with the party that cares most about the project. For the cases in which the project delivers a pure public good, Sections 3 and 4 confirm this result. However, as pointed out in the Introduction, most public services that are provided through public-private partnerships are characterized by some degree of impurity. In general, this comes in the form of excludability, so that specific groups of consumers can be prevented to enjoy the projects' benefits either by means of price or through the imposition of institutional or nonmarket restrictions.

Moreover, the line separating the state as education policymaker (i.e., setting objectives, curricula, pedagogical methods, and the rules of the game) and as major provider (i.e., providing teacher training, school construction and new information technologies, and more generally running the school system) is often blurred (World Bank, 2004; Woessmann, 2006). In these cases, the standard allocation of authority, according to which the state has full control rights over the school system, is difficult to reconcile with BG's results unless the state is always the more caring party. There are however several recent examples — especially in developing countries — that emphasize the importance of the voluntary sector in providing specific school services (e.g., textbooks, personnel training, and infrastructure maintenance) with the state retaining overall responsibility of the school system (Narayan et al., 2000; World Development Report, 2004). This practice, which also BG acknowledge (see BG, pp. 1363-65), is more in line with the idea of shared authority as presented in Section 4.

There are many situations in which some specific responsibilities are shared between the state or local educational authorities on one hand and the voluntary sector (in the form of either NGOs or community-based organizations) on the other. For instance, many decentralization programs have shifted the responsibility from the government to local schools and parent-run school committees to purchase textbooks and provide teacher incentives (Kremer, 2003, for Kenya), introduce catch-up classes for underperforming pupils in primary schools (Banerjee et al., 2003, for India), and improve school quality through changes in teaching and learning practices (World Bank, 2004, for Cambodia). In all such cases, however, the government keeps the overall governance of the school system by overseeing the design of *all* basic educational policies. This allocation of control rights could be justified without bringing into play arguments based on the government's greater valuation. In fact, sole authority to the government can occur: (a) if the government is the only investor and the degree of excludability from the benefits of school services is high (Proposition 1), or (b) if the government's investment is substantially more important than the parent-run school committee's (Proposition 5).

There are other situations of greater school autonomy. For example, in Chile the introduction of a voucher-type subsidy system and the entry of private organizations in the market have led subsidized private schools to grow considerably in the last 20 years and provide free educational services to more than one-third of all students (Mizala and Romaguera, 2000). Similarly, after introducing a reform that involved parents in key governance issues (such as hiring and dismissing teachers), El Salvador experienced a substantial increase in school participation, with reformed schools comprising more than 40 percent of all students enrolled in rural primary schools and 60 percent of all children in preschools (World Bank, 2004). In these circumstances, NGOs or parents share some of the authority over schools with the government. The prediction of Proposition 6(b) — whereby control rights must be split between parties whose investments in excludable public goods have similar importance — seems to fit well such circumstances.

Similar arguments apply also to other situations in which there is joint provision of impure public goods by the government and NGOs or other concerned parties. Examples range from the provision of social services, agricultural projects, and microlending in developing countries (Eversole, 2003) to the provision of water sanitation services, hospitals and transportation infrastructure within public-private partnerships in industrialized economies (e.g., U.S. Department of Transportation, 2004). How control rights are split between NGOs and the government in these circumstances will depend on parties' preferences (relative valuations) and technologies (investments' importance and dispensability) as well as on the good's degree of impurity. But in general, greater investment incentives (and, hence, greater efficiency gains) can be guaranteed by authority allocations that equalize both parties' bargaining powers.

7.2. **Child Custody.** The norms regulating child custody after divorce generally reflect balances of power between husbands and wives as well as concerns for the rights and needs of both children and parents (Mnookin and Kornhauser, 1979). Our model can be used to interpret such norms. For example, up to the Guardianship of Infants Act of 1925, the British legislation was dominated by an absolute paternal preference rule, whereby the father had unconditional rights in all family matters. A court could not give custody to the mother, even if an abusive father might lose his legal rights to child custody (Maidment, 1984). Such a rule would be hard to justify with the argument that the father values the benefits generated by children relatively more than the mother, that is, on the basis of BG's argument. It could however be interpreted along the lines of Propositions 1 and 2, in which the father may care less for the child than the mother does but he is the sole investor and the degree of excludability is very high (by law).

Instead, BG's main result would suit the widely observed practice of custody rights allocated to the mother, which was the explicit dominant norm in the United States over the course of the twentieth century until the early 1970s (Mnookin, 1975), as long as the mother values child's well-being the most and parents' investments are complements. But an explicit maternal presumption is difficult to legitimize when the limited empirical evidence on parental preferences is mixed, with some studies finding divorced mothers to be less altruistic than divorced fathers and other studies finding the opposite (Flinn, 2000; Del Boca and Ribero, 2001). It is perhaps even harder to uphold in the many countries that, since the beginning of the 1970s, moved away from strict rules (e.g., paternal preference or maternal preference) towards a more discretionary principle based on the best interest of the child (Elster, 1989).

The introduction of this principle has been accompanied by a marked increase in shared custody.[27] This can be interpreted through the insights of Proposition 6(b): whenever one parent can positively prevent the other parent to access their children, some form of shared authority is the preferred allocation since it guarantees maximal investment incentives from both parents. Of course, the exact distribution of custody rights is determined by parental preferences and technologies on the basis of the notion of equalizing bargaining powers. If, for example, both parents equally care for their children and provide investments that are equally important, custody rights should be equally split (Proposition 4).

7.3. **Other Applications.** There are many other situations in which impure public goods are produced by different parties and, thus, the allocation of control rights over such goods and services may be an issue. Here, we briefly discuss a few examples.

Our analysis can be applied to government-sponsored research in universities that leads to patented inventions. Suppose a patent is produced as a result of the investments of both the state and a university. Suppose these investments are not fully contractible, are of comparable importance, and are both equally dispensable. This may occur when the state directly provides the funding and the university provides already existing physical assets and human capital. Our framework suggests that both parties should receive roughly similar control rights over the services (and income) generated by the patent (Proposition 4). This allocation is optimal even when

[27] For instance, in Britain and the United States at the beginning of the twenty-first century, shared custody arrangements account for nearly 30 percent of post-divorce living arrangements for children. Less than 10 percent of cases are awarded to the father, and the remaining three-fifths are awarded solely to the mother. Thirty years earlier, shared custody was nonexistent, and at least 90 percent of cases were awarded to the mother.

the patented goods and services are highly excludable (Proposition 6(b)). But if their degree of impurity is small, full control rights should go to the university provided that the university is the more caring party (Proposition 6(a)).

Another application of the model is to public service broadcasting (PSB). PSB programs and channels are typically free to air, and are funded either by a license fee levied on all television viewers, or through the sale of advertising airtime, or both. If PSB has a low degree of excludability and the state has the highest valuation of such a service, then sole authority should be given to the state regardless of technology considerations. Indeed, this is what we observe in many countries (Djankov et al., 2003), and it is consistent with BG and our Lemma 1(a) and Proposition 3(b). If the state, however, is not the party with the highest valuation,[28] control rights should go to private investors. Significant noncontractible investments by both the state and the private sector will provide arguments in support of other alternative allocations, including shared authority. Of course, if PSB programs are almost entirely excludable, so that television viewing becomes closer to other activities in the "market for ideas" such as books and newspapers (Coase, 1974), authority will have to be entirely allocated on the basis of the relative importance of the parties' investments: in line with GHM, if a commercial broadcaster is the most efficient investor, then this should also possess residual control rights.

Our framework can also be applied to scientific collaborations, where partners invest in the project and directly care about its success. An example of such collaborations is given by coauthorship, in which all authors share costs and benefits of their joint noncontractible investments. Typically, coauthorships are equally shared (whether the authors appear in alphabetical order or otherwise). According to our model (Proposition 4), this arises because all authors have roughly equal valuations, and their contributions — as perceived by (unbiased) readers — are equally important and equally indispensable. Interestingly, coauthorships among academic economists have increased markedly in the last 30 years (Goyal et al., 2006).

Our analysis goes through also if we consider situations in which impure public goods and services are jointly produced by different government units (e.g., local versus state), or by the state, for-profit and not-for-profit firms. An example of the latter situations is given by the provision and management of medical care services. When the degree of impurity of such services is large,[29] and private investors are

[28]Besley and Prat (2004) and Prat and Strömberg (2005), among others, discuss a number of reasons why this may be the case.

[29]This can arise if privately owned hospitals are more responsive than centrally funded hospitals to government financial incentives by cream-skimming the type of patients they serve. For evidence in favor of this possibility, see Duggan (2000).

more efficient, then sole authority to the private sector may be desirable (Proposition 5). This can provide an additional argument for decentralization. But if the importance of the public and private sectors' medical care service investments is comparable, some form of shared division of control rights will be optimal (Proposition 6(b)). Indeed, this fits well the recent experience of many European countries' hospital market, with greater decision-making rights over public facilities given to the private sector and increased encouragement of public-private partnerships.

8. Conclusions

This paper has developed a framework for analyzing the distribution of decision-making power in complex partnerships. Because such partnerships deliver public goods and services that have some element of excludability, we have a theory of allocation of authority over *impure* public goods, which comprise purely private goods and purely public goods as special cases. We highlight six results, which allow us to stress that the optimal allocation is inextricably linked to the degree of impurity of these goods as well as the parties' technologies and preferences.

First, when the degree of impurity is very low, authority should be given to the high-valuation party (in line with BG). Second, when the degree of impurity is large, control rights should be entirely given to the main investor, irrespective of preference considerations (in line with GHM). Third, there are some situations in which this allocation is optimal even if the degree of impurity is low as long as one party's investment is more important than the other party's. Fourth, if the parties' investments are of similar importance and the degree of impurity is large, shared authority is optimal, and a relatively greater share should go to the low-valuation party. Fifth, if the importance of the parties' investments is similar and the degree of impurity is neither large nor small, sole authority should go to the low-valuation party. Sixth, all our main results hold even when side payments cannot be made, provided either that the default allocation differs substantially from the optimal allocation or that parties' valuations differ substantially from each other.

The last four results are new and, at varying degrees, in contrast to the main findings reported in GHM or BG. Each of them should deserve further attention and development in future research efforts. Public-private projects are inevitably "complex" and, as such, may require more sophisticated divisions of authority than those based solely on either investment or preference considerations. Furthermore, our notion of authority is broader than that of asset ownership, which has been generally used in other incomplete contracting models of the firm. It encompasses other sources of power, such as restricted access to critical assets (Rajan and Zingales, 1998), effective control over decisions (Aghion and Tirole, 1997), and transfer of control rights (Aghion et al., 2004) within and between the organizations involved in the provision of impure public goods and services. In a world in which the private sector increasingly interacts with the state to deliver such goods and services, our model and its results are likely to be relevant for understanding how these two parties can be involved in their provision.

Finally, our analysis can be applied to a variety of other situations in which (private) partners jointly produce impure public goods. Two of the examples we have mentioned are parents' investments in children that affect the design of child custody rules and scientific collaborations. Again, our framework is useful to the extent that it provides a basis for thinking about (authority allocation in) these processes, which have grown in the real world far more quickly than our ability to understand them.

APPENDIX

LEMMA A.1 AND ITS PROOF

To establish Lemma A.1, in addition to Assumptions 1 and 2, we need to impose two other technical restrictions: (R1) For each $i = g, n$, $k = 1, 2$, and for any y, $|b_{kk}(y)| \geq |B^i_{kk}(y)|$; and (R2) For each $i = g, n$, $k, l = 1, 2$ with $k \neq l$, and for any y, $|b_{kk}(y)| - |b_{kl}(y)| \geq |B^i_{kk}(y)| - |B^i_{kl}(y)| \geq 0$, where $|x|$ denotes the absolute value of x.[30] These restrictions ensure that the players' Nash bargained payoffs satisfy standard regularity conditions.

LEMMA A.1 (Equilibrium Investments). *For any allocation of authority π, there exists a unique Nash equilibrium of the date 1 investment game. This Nash equilibrium, $y^e(\pi) \equiv (y^e_g(\pi), y^e_n(\pi))$, is the unique solution to the following first-order conditions:*

$$V^g_1(y, \pi) = C'_g(y_g)$$
$$V^n_2(y, \pi) = C'_n(y_n).$$

In the unique Nash equilibrium, each player under-invests relative to his unique first-best investment level. Furthermore,

$$\frac{\partial y^e_g}{\partial \pi} = \frac{1}{\Sigma}\left[V^n_{23}V^g_{12} - V^g_{13}(V^n_{22} - C''_n)\right] \quad \text{and} \quad \frac{\partial y^e_n}{\partial \pi} = \frac{1}{\Sigma}\left[V^g_{13}V^n_{12} - V^n_{23}(V^g_{11} - C''_g)\right],$$

where $\Sigma \equiv (V^g_{11} - C''_g)(V^n_{22} - C''_n) - V^g_{12}V^n_{12} > 0$, with all these second-order partial derivatives evaluated at the Nash equilibrium investment levels.

Proof of Lemma A.1. We first establish the following claim which states some properties of the players' Nash-bargained payoff functions:

[30] The non-negativity conditions in (R2) are the conditions for the Hessian of each benefit function (b, B^g and B^n) to be a dominant diagonal matrix. Furthermore, the first inequality in (R2) holds if and only if the Hessian of the net benefit function $b - B^i$ is a dominant diagonal matrix.

Claim A.1. *Fix an arbitrary $\pi \in [0,1]$. For each $i = g, n$, player i's Nash-bargained payoff function, $V^i(y,\pi)$, satisfies the following properties:*
(i) V^i is strictly increasing and twice continuously differentiable in y, strictly concave in y_i, $\lim_{y_g \to 0} V_1^g(y,\pi) = \lim_{y_n \to 0} V_2^n(y,\pi) = \infty$, $\lim_{y_g \to \infty} V_1^g(y,\pi) = \lim_{y_n \to \infty} V_2^n(y,\pi) = 0$, and $V^i(0,0,\pi) > 0$.
(ii) For any y, the first-order derivative of $V^i(y,\pi)$ with respect to y_i is strictly less than $(\theta_g + \theta_n)b_k(y)$ with $k = 1$ if $i = g$ and $k = 2$ if $i = n$.
(iii) For any y, $V_{12}^i(y,\pi) \geq 0$.
(iv) The Hessian of V^i is a dominant diagonal matrix (i.e., for any y, the absolute values of both V_{11}^i and V_{22}^i are greater than or equal to V_{12}^i).

Proof of Claim A.1. Fix $\pi \in [0,1]$. After rearranging terms and simplifying, V^g and V^n can respectively be usefully rewritten as follows (for expositional convenience, we suppress the arguments of the functions):

$$2V^g = \theta_g\Big[\pi(b+B^g) + (1-\pi)[b+(1-\alpha)B^n]\Big] + \theta_n\Big[(1-\pi)(b-B^n) + \pi[b-(1-\alpha)B^g]\Big]$$
$$2V^n = \theta_g\Big[\pi(b-B^g) + (1-\pi)[b-(1-\alpha)B^n]\Big] + \theta_n\Big[(1-\pi)(b+B^n) + \pi[b+(1-\alpha)B^g]\Big].$$

It is now straightforward to establish Claim A.1 by examining the appropriate derivatives of these two functions. It is easy, first, to verify that the results stated in parts (i) and (ii) of Claim A.1 follow given Assumptions 2(i)–(iii) and technical restriction (R1). Claim A.1(iii) follows given Assumption 2 (iv). Claim A.1(iv) follows given technical restriction (R2).

We can now proceed to prove Lemma A.1. Given the properties of V^i established in Claims A.1(i) and given Assumption 1, i's payoff function, $V^i - C_i$, is continuous in y and strictly concave in y_i. As each player's strategy set is compact, existence of a pure-strategy Nash equilibrium follows from the Debreu-Glicksberg-Fan existence results. Furthermore, a pair y is a Nash equilibrium investment pair if and only if y is a solution to the first-order conditions stated in Lemma A.1. Using these first-order conditions and Claim 1(iv), it is easy to verify that the two best-reply functions are contraction mappings. This implies that there exists at most a unique Nash equilibrium. The under-investment conclusion follows from Claim A.1(ii).

Differentiating the first-order conditions with respect to π, given that investments are set at the Nash equilibrium levels $y^e(\pi)$, and then solving for the derivatives of y_g^e and y_n^e with respect to π, we obtain the expressions stated in Lemma A.1, where (given Claims A.1(i) and A.1(iv), and Assumption 1) $\Sigma > 0$.

PROOFS OF PROPOSITIONS 4–6

To establish Propositions 4 to 6, we first provide a general characterization of the optimal value of π. To simplify the algebra, we assume that $C_i(y_i) = y_i$, and, as in BG, that $\mu(y_i) = 2\sqrt{a_i y_i} + A$, where A is a positive constant. Using these functional forms into (5) and (6), simplifying and collecting terms, it is straightforward to verify that g's and n's Nash-bargained marginal returns to investment are respectively $V_1^g(y) = \lambda_g \mu'(y_g)$ and

$V_2^n(y) = \lambda_n \mu'(y_n)$, where

$$\lambda_g = \frac{a_g \theta_g}{2}\left(\left[(\widehat{\theta}+\beta_g)(\alpha-\alpha_g^*)\right]\pi + \left[1+(1-\alpha)\beta_g + \widehat{\theta}(1-\beta_g)\right]\right) \quad \text{and}$$

$$\lambda_n = \frac{a_n \theta_g}{2}\left(\left[(1+\widehat{\theta}\beta_n)(\alpha_n^*-\alpha)\right]\pi + \left[\alpha + 2\widehat{\theta}\right]\right).$$

The term λ_i $(i = g, n)$, which is always strictly positive, measures i's bargaining power. We now prove a characterization of the optimal authority allocation for the parameter values in the non-shaded region of Figure 2, the value of π that maximizes the equilibrium net surplus S (defined in (7)). We use this to establish Propositions 4–6.

Claim A.2 *Assume that the parameters are such that $(\widehat{\theta}, \alpha)$ lie in the non-shaded region of Figure 2 (i.e., such that $\alpha > \max\{\alpha_g^*, \alpha_n^*\}$). Define*

$$\Gamma_g = \left[\frac{\alpha-\alpha_g^*}{\alpha-\alpha_n^*}\right]\left[\frac{\widehat{\theta}(2-\alpha)(\widehat{\theta}+\beta_g)}{(1+\widehat{\theta}\beta_n)[1+\beta_n+\widehat{\theta}(1-[1-\alpha]\beta_n)]}\right] \quad \textit{and}$$

$$\Gamma_n = \left[\frac{\alpha-\alpha_g^*}{\alpha-\alpha_n^*}\right]\left[\frac{(\widehat{\theta}+\beta_g)[1-\beta_g+\widehat{\theta}(1+\beta_g)+\alpha\beta_g]}{(2-\alpha)(1+\widehat{\theta}\beta_n)}\right].$$

(a) If $a_n \geq a_g\Gamma_n$, then it is optimal to allocate sole authority to n (i.e, π^*=0).
(b) If $a_g\Gamma_g < a_n < a_g\Gamma_n$, then it is optimal to allocate some authority to g and some authority to n, where $\pi^* \in (0,1)$ and it is the unique solution to the following first-order condition:

$$(\theta_g+\theta_n)\left[\frac{\partial\lambda_g}{\partial\pi}+\frac{\partial\lambda_n}{\partial\pi}\right] = \frac{\lambda_g}{a_g}\left[\frac{\partial\lambda_g}{\partial\pi}\right] + \frac{\lambda_n}{a_n}\left[\frac{\partial\lambda_n}{\partial\pi}\right].$$

(c) If $a_n \leq a_g\Gamma_g$, then it is optimal to allocate sole authority to the government (i.e., $\pi^* = 1$). Furthermore, $\Gamma_n > \Gamma_g$; and for all $\widehat{\theta} \neq 0$ and $\widehat{\theta}$ is finite, $\Gamma_i > 0$ ($i = g, n$). If $\widehat{\theta} = 0$ then $\Gamma_g = 0$ and $\Gamma_n > 0$; and in the limit as $\widehat{\theta} \to \infty$, $\Gamma_n \to \infty$ and Γ_g converges to a strictly positive and finite number.

Proof of Claim A.2. Using the adopted benefit and cost functions, it follows from an application of Lemma A.1 that the unique Nash equilibrium investment levels, for any π, are

$$y_g^e(\pi) = \frac{(\lambda_g)^2}{a_g} \quad \text{and} \quad y_n^e(\pi) = \frac{(\lambda_n)^2}{a_n},$$

where λ_g and λ_n are defined above. It is straightforward to verify that for any y,

$$V_2^g(y) = [a_n(\theta_g+\theta_n)-\lambda_n]\mu'(y_n) \quad \text{and} \quad V_1^n(y) = [a_g(\theta_g+\theta_n)-\lambda_g]\mu'(y_g).$$

After making the appropriate substitutions, simplifying and collecting terms, $S'(\pi)$ defined in (8) becomes

$$\frac{S'(\pi)}{2} = (\theta_g+\theta_n)\left[\frac{\partial\lambda_g}{\partial\pi}+\frac{\partial\lambda_n}{\partial\pi}\right] - \frac{\lambda_g}{a_g}\left[\frac{\partial\lambda_g}{\partial\pi}\right] - \frac{\lambda_n}{a_n}\left[\frac{\partial\lambda_n}{\partial\pi}\right].$$

It follows that $S''(\pi) < 0$ for all π, and hence S is strictly concave in π. This implies that: (i) if $S'(0) \leq 0$ then $\pi^* = 0$; (ii) if $S'(1) \geq 0$ then $\pi^* = 1$; and (iii) if $S'(0) > 0 > S'(1)$ then $\pi^* \in (0, 1)$ and it is the unique solution to $S'(\pi) = 0$.

Substituting for $\pi = 0$ and $\pi = 1$ respectively, we obtain

$$S'(0) = \left[\frac{(\theta_g)^2(1+\widehat{\theta}\beta_n)(2-\alpha)(\alpha-\alpha_n^*)}{2}\right]\left[a_g\Gamma_n - a_n\right].$$

$$S'(1) = \left[\frac{(\theta_g)^2(1+\widehat{\theta}\beta_n)[1+\beta_n+\widehat{\theta}(1-[1-\alpha]\beta_n)](\alpha-\alpha_n^*)}{2}\right]\left[a_g\Gamma_g - a_n\right].$$

Claim A.2 now follows from applying the results established here about the equilibrium net surplus function, S. Note that $\Gamma_n > \Gamma_g$ since (due to S being strictly concave) $S'(0) \leq 0$ implies $S'(1) < 0$.

Proof of Proposition 4. After the appropriate substitutions (given the hypothesis that $\theta_g = \theta_n$, $a_g = a_n$ and $\beta_g = \beta_n$), we obtain that $\alpha_g^* = \alpha_n^* = 0$ and $0 < \Gamma_g < 1 < \Gamma_n$. The proof follows because, when $\alpha > 0$, Claim A.2(b) applies, and the first-order condition collapses to $\lambda_g = \lambda_n$. Proposition 3 gives the result when $\alpha = 0$.

Proof of Proposition 5. This proposition follows immediately from a straightforward application of Claim A.2.

Proposition 6 (stated formally). *Assume that $a_n = a_g$, $\widehat{\theta} \neq 1$ and $1-\beta_g = 1-\beta_n = O(\epsilon)$, where $O(\epsilon)$ is a term of order ϵ with ϵ being infinitesimal.*
(a) If $\widehat{\theta} < 1$, then

$$\pi^* = \begin{cases} 1 & \text{if } 0 \leq \alpha \leq O(\epsilon), \\ 0 & \text{if } O(\epsilon) < \alpha \leq 1-\widehat{\theta}+O(\epsilon), \\ \dfrac{\alpha+\widehat{\theta}-1}{\alpha(1+\widehat{\theta})} & \text{if } 1-\widehat{\theta}+O(\epsilon) < \alpha \leq 1. \end{cases}$$

(a) If $\widehat{\theta} > 1$, then

$$\pi^* = \begin{cases} 0 & \text{if } 0 \leq \alpha \leq O(\epsilon), \\ 1 & \text{if } O(\epsilon) < \alpha \leq \dfrac{\widehat{\theta}-1}{\widehat{\theta}}+O(\epsilon), \\ \dfrac{\alpha+\widehat{\theta}-1}{\alpha(1+\widehat{\theta})} & \text{if } \dfrac{\widehat{\theta}-1}{\widehat{\theta}}+O(\epsilon) < \alpha \leq 1. \end{cases}$$

Proof of Proposition 6. The hypothesis of this proposition imply that

$$\Gamma_g = \left[\frac{(2-\alpha)\widehat{\theta}}{2+\alpha\widehat{\theta}}\right] + O(\epsilon) \quad \text{and} \quad \Gamma_n = \left[\frac{2\widehat{\theta}+\alpha}{2-\alpha}\right] + O(\epsilon).$$

Hence, the results of Proposition 3 and an application of Claim A.2 lead to the desired conclusions about the boundaries of the various regions. As for the interior solution, this is obtained by substituting for the various terms in the first-order condition stated in Claim A.2, simplifying and collecting terms: the interior solution is the value of π at which $\lambda_g = \lambda_n$.

PROOF OF PROPOSITION 7

Define $Z_i(\pi^d) = W^i(\pi^*) - W^i(\pi^d)$, $i = g, n$. In what follows we use the functional forms used in Propositions 4–6, and we also restrict attention to the case in which the parties' investments are of similar importance. Thus, π^* is defined in the proof of Proposition 6.

After substituting for π^* in the above expression for $Z_i(.)$ and simplifying, the following properties of the function $Z_g(\pi^d)$ can be established. First, it takes value zero at two values: $\pi^d = \pi^*$ and $\pi^d = \widehat{\pi}^g$, where $\widehat{\pi}^g > \pi^*$. Second, $Z_g'(\pi^*) < 0$ and $Z_g'(\widehat{\pi}^g) > 0$. Putting these properties together, it follows that for any $\pi^d \in [0, \pi^*] \bigcup [\widehat{\pi}^g, 1]$ player g prefers the optimal allocation π^* over the default allocation π^d.

Turning to $Z_n(\pi^d)$, we can establish the following properties. First, it takes value zero at two values: $\pi^d = \pi^*$ and $\pi^d = \widehat{\pi}^n$, where $\widehat{\pi}^n < \pi^*$. Second, $Z_n'(\pi^*) > 0$ and $Z_n'(\widehat{\pi}^n) < 0$. Putting these properties together, it follows that for any $\pi^d \in [0, \widehat{\pi}^n] \bigcup [\pi^*, 1]$ player n prefers the optimal allocation π^* over the default allocation π^d.

Combining these results, it follows that both players prefer the optimal allocation over the default allocation if and only if $\pi^d \in [0, \widehat{\pi}^n) \bigcup (\widehat{\pi}^g, 1]$. Proposition 7 follows from this result.

PROOF OF PROPOSITION 8

Let $y^e(\pi)$ denote the government's date 1 investment level, which is the unique solution to the first-order condition $V_1^{N+1}(y, \pi) = 1$. For simplicity, we assume a constant marginal cost of investment, set equal to unity. At date 0, the net surplus for an arbitrary vector of shares, $\pi = (\pi_1, \pi_2, \ldots, \pi_N, \pi_{N+1})$, is

$$S(\pi) = \sum_{k=1}^{N+1} V^k(y^e(\pi), \pi) - y^e(\pi).$$

Our objective is to find the authority allocation which maximizes $S(\pi)$.

For expositional convenience, we use the following notation. The first-order derivative of $y^e(\pi)$ with respect to π_i is denoted by $y_i^e(\pi)$ (where $i = 1, 2, \ldots, N, N+1$). For each $k = 1, 2, \ldots, N, N+1$, the first-order derivatives of $V^k(y, \pi)$ with respect to y and π_i respectively (where $i = 1, 2, \ldots, N, N+1$) are denoted by V_1^k and V_{i+1}^k. The second order, mixed derivative of $V^k(y, \pi)$ with respect to y and π_i is denoted by $V_{1,i+1}^k$. Finally, the second-order derivative of $V^k(y, \pi)$ with respect to y is V_{11}^k.

In the expression for $S(\pi)$, we first substitute for π_{N+1} by setting $\pi_{N+1} = 1 - \sum_{k=1}^N \pi_k$. Now differentiate S with respect to π_j, where $j = 1, 2, \ldots, N-1, N$. Using the first-order condition, simplifying and collecting terms, for each $j = 1, 2, \ldots, N$, we obtain

$$\frac{\partial S}{\partial \pi_j} = \left[y_j^e(\pi) - y_{N+1}^e(\pi)\right] \sum_{k=1}^{N} V_1^k + \sum_{k=1}^{N+1} [V_{j+1}^k - V_{N+2}^k].$$

It is straightforward to verify that $\sum_{k=1}^{N+1} V_{j+1}^k = 0$ (for each $j = 1, 2, \ldots, N, N+1$). Using the first-order condition, we obtain

$$y_j^e(\pi) - y_{N+1}^e(\pi) = \left[\frac{-1}{V_{11}^{N+1}}\right]\left[V_{1,j+1}^{N+1} - V_{1,N+2}^{N+1}\right].$$

Straightforward computations establish that

$$V_{1,j+1}^{N+1} - V_{1,N+2}^{N+1} = (1-\alpha)[\theta_{N+1} - \overline{\theta}]\left[B_j' - B_{N+1}'\right] - \frac{\alpha}{N+1}\left[\theta_j B_j' + N\theta_{N+1}B_{N+1}'\right].$$

Hence, we have shown that for each $j = 1, 2, \ldots, N$:

$$\frac{\partial S}{\partial \pi_j} = \left[\frac{-\sum_{k=1}^N V_1^k}{V_{11}^{N+1}}\right]\left[(1-\alpha)[\theta_{N+1} - \overline{\theta}]\left[B_j' - B_{N+1}'\right] - \frac{\alpha}{N+1}\left[\theta_j B_j' + N\theta_{N+1}B_{N+1}'\right]\right].$$

The term in the first big brackets is strictly positive and independent of j. So we focus attention on the term in the second big brackets. If α is sufficiently large, it follows that the right hand side of this expression is strictly negative. Hence, $\pi_j^* = 0$. Consequently $\pi_{N+1}^* = 1$ (i.e., it is optimal to allocate sole authority to the government). This then establishes part (a) of the proposition.

Now suppose that α is sufficiently small. If $\theta_{N+1} > \overline{\theta}$, then (given Assumption 3) the right hand side is again strictly negative, and hence $\pi_{N+1}^* = 1$. If, on the other hand, $\theta_{N+1} < \overline{\theta}$, then the right hand side of the expression is strictly positive. Hence, for any $j = 1, 2, \ldots, N$, the derivative of S with respect to π_j is strictly positive. Letting, for convenience, S_j and S_k respectively denote the first-order derivatives of S with respect to π_j and π_k, note that $S_j > S_k$ if and only if $B_{N+1}' - B_j' > B_{N+1}' - B_k'$. Hence, Assumption 3 implies that $\pi_1^* = 1$. This then establishes part (b) of the proposition.

PROOF OF PROPOSITION 9

Consider first the case in which g is the sole investor. If g's investment incentives are higher under the optimal shared authority allocation ($\pi^* \in [0,1]$) than under joint authority — i.e., for any y_g, $V_1^g(y, \pi^*) \geq Z_1(y)$ — then the date 0 equilibrium net surplus under the optimal shared authority allocation is higher than under joint authority. (Similar arguments hold for the opposite case in which g's investment incentives are lower.) Using this observation, we now establish the following set of results:

(i) If $\alpha = 0$, then it is optimal to allocate sole authority to the sole investor provided it is the more caring party. Otherwise (when $\theta_g < \theta_n$), it is optimal to operate the project under joint authority.

(ii) Lemmas 1(b) and 1(c), and Proposition 1.

(iii) If $\alpha > \widehat{\alpha}_g$, where $\widehat{\alpha}_g = (\widehat{\theta} - 1)/\widehat{\theta}$ (which is strictly greater than α_g^* when $\widehat{\theta} > 1$), then it is optimal to allocate sole authority to g. Otherwise (when $\alpha < \widehat{\alpha}_g$) it is optimal to operate the project under joint authority.

It is straightforward to verify that for any y_g,

$$V_1^g(y, \pi^*) \gtreqless Z_1(y) \iff \Upsilon_g \equiv \left[\theta_g - (1-\alpha)\theta_n\right]\pi^* B_1^g(y) + \left[(1-\alpha)\theta_g - \theta_n\right](1-\pi^*)B_1^n(y) \gtreqless 0.$$

After substituting for $\alpha = 0$, it follows that $\Upsilon_g \gtreqless 0 \Leftrightarrow \theta_g \gtreqless \theta_n$. Result (i) follows immediately, and the first part of Lemma 1(c) carries over. After substituting for $\alpha = 1$ and $\pi^* = 1$, it follows that $\Upsilon_g > 0$, and hence Lemma 1(b) carries over. Substituting for $\theta_g = \theta_n$, $\alpha > 0$ and $\pi^* = 1$, it follows that $\Upsilon_g > 0$, and hence the second part of Lemma 1(c) carries over. Given the hypothesis of Proposition 1, it follows after substituting for $\pi^* = 1$ that $\Upsilon_g > 0$, and hence Proposition 1 carries over. As for Proposition 2, assume that $\alpha > \alpha_g^*$. After substituting for $\pi^* = 1$, it follows that $\Upsilon_g > 0$ if and only if $\alpha > \widehat{\alpha}_g$. Now assume that $\alpha < \alpha_g^*$. After substituting for $\pi^* = 0$, it follows that $\Upsilon_g < 0$. Result (iii) follows immediately.

We now turn to the case when both can invest. It is straightforward to verify that for any y,

$$V_2^n(y, \pi^*) \gtreqless Z_2(y) \iff \Upsilon_n \equiv \Big[(1-\alpha)\theta_n - \theta_g\Big]\pi^* B_2^g(y) + \Big[\theta_n - (1-\alpha)\theta_g\Big](1-\pi^*)B_2^n(y) \gtreqless 0.$$

Under the hypotheses of Proposition 4, it follows that both Υ_g and Υ_n are strictly positive. This implies that the conclusion of Proposition 4 carries over. Now we show that Propositions 6(b) and 6(c) carry over. From the proof of Proposition 6 we know that at the optimal shared authority allocation, $\lambda_g = \lambda_n = \lambda^*$. Hence under the optimal shared authority allocation, the parties' marginal investment returns are identical. Furthermore, since (under the hypothesis of Proposition 6) $a_n = a_g = a$, it follows that under joint authority, the parties' marginal investment returns are identical. Thus, the desired conclusion follows once we show these identical returns are lower than the identical returns under the optimal shared authority allocation. To do so, we need to show that $\lambda^* > a(\theta_g + \theta_n)/2$. After substituting the optimal value of π (namely, $\pi^* = (\alpha + \widehat{\theta} - 1)/\alpha(1+\widehat{\theta})$ since $\alpha > \max\{1-\widehat{\theta}, (\widehat{\theta}-1)/\widehat{\theta}\}$) into λ_n, simplifying and collecting terms, it follows that the desired conclusion holds provided that $[\alpha - (1-\widehat{\theta})][\alpha - (\widehat{\theta}-1)/\widehat{\theta}] > 0$, which holds. Proposition 5, which applies when the degree of impurity is sufficiently large, carries over. This is because the payoffs are additive, and given the first part of Result (iii) above. Proposition 6(a) is an application of Proposition 3, and this result alters as we now explain, just like Proposition 2 did for the sole investor case. Suppose g's investment is sufficiently more important than n's, who however is the high valuation party. This means that greater weight in the additive net surplus function is attached to g's investment, and that the optimal shared authority allocation is n-sole authority. The desired conclusion follows since g's investment incentives are even greater under joint authority.

PROOF OF PROPOSITION 10

After substituting for the functional forms and simplifying, it can be shown that

$$\frac{\partial V^g}{\partial y_g} - \frac{\partial V^n}{\partial y_n} = \eta'(y_g^* + \lambda y_n^*)[\theta_g - (1-\alpha)\theta_n](1+\lambda)\pi + \eta'(\lambda y_g^* + y_n^*)[(1-\alpha)\theta_g - \theta_n](1+\lambda)(1-\pi).$$

First consider the parameter values such that $(1-\alpha)\theta_g > \theta_n$. In this case it follows from the the expression above that g's marginal returns are always strictly greater than those of n's.

Hence, $y_g^* > 0$ and $y_n^* = 0$. And since g's marginal returns are strictly increasing in π, it is thus optimal to allocate sole authority to g; i.e., $\pi^* = 1$.

Now consider the case when $(1-\alpha)\theta_n > \theta_g$. In this case the reverse is true: n's marginal returns are always strictly greater than those of g's. Hence, $y_n^* > 0$ and $y_g^* = 0$. And since n's marginal returns are strictly decreasing in π, it is thus optimal to allocate sole authority to n; i.e., $\pi^* = 0$.

It is straightforward to verify that for parameter values that do not satisfy one of these two cases, further restrictions are needed to pin down the optimal authority allocation.

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