Financing Experimentation

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Abstract

Entrepreneurs must experiment to learn how good they are at a new activity. What happens when the experimentation is financed by a lender? Under common scenarios, i.e., when there is the opportunity to learn by "starting small" or when "no-compete" clauses cannot be enforced ex-post, we show that financing experimentation can become harder precisely when it is more profitable, i.e., for lower values of the known-arm and for more optimistic priors. Endogenous collateral requirements (like those frequently observed in micro-credit schemes) are shown to be part of the optimal contract.

Keywords: Experimentation, Moral Hazard, Adverse Selection, Starting Small, Competition.

JEL Codes: D81, D86, G30.

"Each of us has much more hidden inside us than we have had a chance to explore. Unless we create an environment that enables us to discover the limits of our potential, we will never know what we have inside of us."

Muhammad Yunus, Founder of Grameen Bank

1 Introduction

When people start a new activity, they might not know how profitable it is, or how good they will be doing it. They can only learn by trying it out. In other words, people must experiment to learn about the activity or about themselves. An important example of such a scenario is a person starting a business. This may be a poor woman in a slum in India trying to open a small shop, or an IT-entrepreneur in Silicon Valley hoping to found the next Google. In either case, if initial capital has to be borrowed, the lender – be it a microfinance institution in India or a venture capitalist in the US – finances the experimentation.

What happens when the experimentation is financed by a lender? The lender should take into account that the borrower might misbehave, for example, by shirking or by diverting the loan; also, the borrower might (privately) acquire some information relevant to the continuation of the project. In order to study such a setting, this paper builds a simple model that embeds a two-period experimentation problem into a lending relationship. The central insight of the paper is to show how, in the context of experimentation, projects with higher net present value can be systematically harder to finance.

A standard experimentation setting arises, broadly speaking, when certain activities undertaken today generate valuable information that can be used in future decision making.[1] In its simplest form, standard experimentation involves, in at least two periods, a choice between one activity with known returns (the so-called known arm), and another activity with initially unknown returns (the so-called unknown arm). Experimentation is then a particular form of investment: it involves a trade-off between short-term costs of generating information and long-term benefits of using it. Therefore, the higher the discount factor, the lower the value of the activity with known returns and the more optimistic is the prior belief about the unknown arm, the more the decision maker finds

[1] See Dirk Bergemann & Juuso Välimäki (2008) for a survey.

it attractive to experiment.

The paper studies a two-period model in which in each period an agent can start a project. Initially, both the agent and the lender are uninformed about the effort costs needed to complete the project. Upon starting the project, the agent learns her effort costs. While it is optimal to complete the project regardless of the agent's effort costs since the investment is already sunk, the agent might decide not to exert the effort and to divert the capital for private benefit. In the second period the agent can obtain another loan, depending on the first-period outcome and her communication with the lender.

Equipped with this simple benchmark, we study the resulting financing problem under a number of plausible scenarios. First, we consider the case in which the borrower can experiment by "starting small".[2] We find that obtaining credit to finance the experimentation might become harder precisely when experimenting is more valuable. By experimenting, the borrower privately learns about herself and, therefore, in addition to the standard moral hazard problem associated with borrowing, there is an adverse selection dimension which emerges after the loan has been disbursed. The prospect of a larger second-period project might make the selection of the right type of borrower more difficult since entrepreneurs have high incentives to fake short-term performance in order to enjoy higher rents in the future. For the same reason, a lower payoff of the known arm, i.e., the outside option, makes financing experimentation more difficult. In other words, the future rents which are helpful in solving the moral hazard problem (see, e.g., William P. Rogerson (1985) and Patrick Bolton & David S. Scharfstein (1990)) come at the cost of rendering the adverse selection problem more severe.

Second, we consider the case in which the borrower can leave the relationship with the original lender and seek finance from alternative lenders in the second period. Motivated by empirical evidence, we consider two different scenarios. First, we consider the case in which "non-compete" clauses can be enforced, as in venture capital contracts (see, e.g., Steven N. Kaplan & Per Strömberg (2003)). Under this scenario, we show that the results described above are completely robust to ex post competition. Second, we consider the case in which "non-compete" clauses cannot be enforced, as is likely the case for bank lending to SMEs (see, e.g., Vasso Ioannidou & Steven Ongena (2010)) or in microcredit

[2] A large body of work notes how firms and relationships initially start small and then grow over time (see, e.g., James E. Rauch & Joel Watson (2003) for a theoretical analysis and Jonathan Eaton, Marcela Eslava, Maurice Kugler & James Tybout (2008) and Lucia Foster, John Haltiwanger & Chad Syverson (2012) for empirical evidence on sales patterns in new foreign and domestic markets, respectively).

lending in developing countries (see, e.g., Dean Karlan & Jonathan Morduch (2010)). Under this scenario, we obtain a new result: financing experimentation can become harder when initial priors about the profitability of the unknown arm are sufficiently optimistic. This happens because a higher likelihood of successful experimentation allows an outside lender to offer better contractual terms to the borrower once the initial sunk cost of experimentation has been financed by the inside lender.

Finally, we explore the robustness of these results to the case in which the borrower has access to a saving technology. The insight that experimentation can become harder to finance precisely when it is most valuable is robust to this extension. In addition, the analysis also highlights how access to savings and ex post competition among lenders interact to shape access to finance.

The optimal contract in our model is similar to contracts typically offered in practice. The model highlights how retained earnings can be used to finance payments which induce the bad type of borrower to relinquish the project in the second period. This can be achieved, for example, by using retained earnings to endogenously build up collateral. The optimal contract, therefore, can mimic compulsory saving requirements (CSRs), a common practice observed in microcredit that has, however, received little theoretical attention.[3] Similarly, in venture capital "purchase options" allocate to the investor the right to acquire control over the project at a pre-specified price. When the investor exercises the option she effectively pays an exit fee to the entrepreneur (see, e.g., Kaplan & Strömberg (2003)). Besides rationalizing contractual features that appear to be used in practice, the model yields a number of testable predictions on the relationship between collateral, loan terms and project outcomes that are discussed in detail at the end of the paper.

Related Literature

This paper belongs to a growing literature that combines experimentation and agency problems. We apply our framework to a financing setting, which suggests to focus on a different mix of agency problems and, more importantly, to consider several extensions, e.g., scalability, competition, access to savings, which are usually left unexplored in the

[3] Under CSRs, a share of the repayment from earlier loan cycles is locked in into a saving account until the completion of the final loan cycle. CSRs are a pervasive, yet understudied, feature of microfinance schemes (see, e.g., Jonathan Morduch (1999)). Most of the theoretical work on microfinance has focused on joint liability, a far less common contractual element of those schemes (see surveys in Maitreesh Ghatak & Timothy Guinnane (1999) and Karlan & Morduch (2010)).

literature. As a result we derive a number of novel results, e.g., the non-monotonicity of access to finance with respect to the discount factor, the outside option and the prior.

Dirk Bergemann & Ulrich Hege (2005) consider an agent who can either explore an innovative project or shirk, in which case the project outcome (failure) is not informative.[4] As in other dynamic contracting models without commitment (see, e.g., Jean-Jacques Laffont & Jean Tirole (1987)) they find that a higher discount factor can render financing more difficult when the agent's actions are observable. A key difference with our paper is that we assume the lender has full commitment power. In Gustavo Manso (2011) the agent can experiment, shirk or exploit a known activity. He shows that motivating experimentation requires dramatically different incentives from standard pay-for-performance schemes, e.g., rewards for failure. Our application to financing suggests to consider different agency problems and focus on different comparative statics leading to the central insight that projects with higher net present value can be systematically harder to finance and implement. A contemporaneous paper by Matthieu Bouvard (2012) studies a real-option model where a borrower experiments and the timing of financing is one of the contractual variables. There, the borrower starts being better informed than the investor about the probability of success while the costs of experimentation are exogenous. There are no results about the effects of the discount factor. Moreover, as mentioned above, none of these papers considers ex post competition between lenders nor access to savings by the agent.[5]

In Steven D. Levitt & Christopher M. Snyder (1997) and Roman Inderst & Holger M. Mueller (2010) the principal also faces the combination of the moral hazard and interim adverse selection where the project is terminated (or the agent is fired) following bad news revealed by the agent. However, the mechanism at work there is different from ours. In these two papers, the project outcome is a signal about the agent's effort and is used to elicit the effort. If the project is terminated, the outcome stays unknown and, therefore, acting upon information ex post intervenes with the provision of incentives ex ante. In our model, acting upon information obtained in the first period means deciding about the second-period project which does not depend on the first-period effort. Our mechanism is

[4]See also Dirk Bergemann & Ulrich Hege (1998) which is "a preliminary analysis of the same basic model" (Bergemann & Hege (2005), p. 723).

[5]Other papers related to Bouvard (2012), such as Steven R. Grenadier & Andrey Malenko (2011) and Erwan Morellec & Norman Schürhoff (2011) are also mainly concerned how a better informed firm can signal its private information through the financial contracts it offers to investors.

that the second-period moral hazard rent makes the interim information revelation more costly. While the mechanisms are different, the interaction of moral hazard and adverse selection is crucial in all three papers: each of them becomes trivial if only moral hazard or adverse selection is present.[6]

The paper is also related to the literatures on the role of collateral (see, e.g., Helmut Bester (1985) and David Besanko & Anjan V. Thakor (1987*a*)) and relational lending (see, e.g., Steven A. Sharpe (1990) and Mitchell A. Petersen & Raghuram G. Rajan (1995)) in facilitating access to credit. There are, however, important differences. The literature on collateral has typically focused on the availability of exogenously given amounts of collateral. In contrast, in our setting the value of collateral available in the second period of the relationship to separate borrowers is endogenous. The relational lending literature, instead, focuses on the effects of ex post competition from outside lenders but ignores the role of endogenous savings and collateral. In our setting, ex post competition from outside lenders does affect the ability to finance the project despite the endogenous collateral that can be created through savings.

The rest of the paper is organized as follows. Section 2 presents the model with a unique project size. Section 3 introduces the extension with two project sizes and derives the results on the effects of the discount factor and the outside option. Section 4 studies the effects of competition and shows that a better agent, in the sense of lower expected effort costs, may find financing her project more difficult. Section 5 explores robustness of the results to savings. Section 6 finds a realistic contract that replicates the direct mechanism of Section 3, interprets microfinance contracts in the light of our model and discusses testable implications. Section 7 concludes. The proofs are in the Appendix.

[6] In Jacques Crémer & Fahad Khalil (1992) and Jacques Crémer, Fahad Khalil & Jean-Charles Rochet (1998), the agent may become informed at a cost, and the principal adjusts the contract to provide the agent with optimal incentives for information acquisition. These papers (as well as Levitt & Snyder (1997) and Inderst & Mueller (2010)), however, are essentially static and do not consider the intertemporal trade-offs involved. Other models mixing moral hazard and adverse selection are discussed in, e.g., Jean-Jacques Laffont & David Martimort (2002) (ch. 7) and Patrick Bolton & Mathias Dewatripont (2005) (ch. 6).

2 The Model

2.1 Setup

There is an agent that lives for two periods, $\tau = 1, 2$. In each period the agent has the opportunity to undertake a project that needs an initial capital investment of 1 and yields return r when completed. A project that is not completed fails and yields 0.

The agent has no assets and needs to borrow 1 unit of capital in order to start the project. She is protected by limited liability. The agent and lenders have a common discount factor $\delta \in [0, 1]$ across the two periods. The complete description of the timing of events and the contracts is postponed until Section 2.3.

To complete the project the agent needs to appropriately invest the unit of capital and to exert effort. The agent can divert a share $\psi \leq 1$ of the initial investment for private consumption. If she does so, the project fails. The parameter ψ reflects the difficulty for the lender of monitoring the investment and transaction costs in diverting the investment.

There are two types of agent, good G and bad B, which remain constant over the two periods. The cost of effort for the good agent is $e_G = 0$, and $e_B = e > 0$ for the bad agent.[7] Initially, both the agent and the lenders are uninformed about the type of agent and have a common prior ρ about the probability of the agent being the good type. The agent privately learns her type upon starting the project in period 1 but does not if she doesn't start the project. After having learned her type, she decides whether to exert effort and whether to divert the capital.

Whenever effort is exerted and investment is not diverted, the project succeeds and yields r, which is observable and verifiable. In any period in which the agent does not undertake the project, she takes an outside option $u > 0$.

We make the following parametric assumptions:

Assumption 1 $r - 1 < u + e$.

Assumption 2 $u < \psi$.

Assumption 3 $\max\{1, e\} < r - \psi$.

[7] The model can be also interpreted with the effort cost being a characteristic of the project, rather than of the agent.

The first assumption implies that it is not optimal to invest if the agent is (known to be) bad: the opportunity costs of investment $1 + u$ are higher than revenues r net of effort costs e.

The second assumption implies that the agent always prefers to start the project with borrowed money rather than take her outside option u.

Finally, the third assumption has two implications. First, $r - 1 > \psi$ implies that the project generates enough revenues to solve the moral hazard problem of the good type. Second, $r > \psi + e$ implies that, once the project is started and the initial outlay of 1 unit of capital is sunk, it is optimal to complete the project regardless of the agent's type.

2.2 Optimal Experimentation by a Self-Financed Agent

Let us first consider the benchmark case in which the agent has enough wealth so that she does not need to borrow. In this case the agent is the residual claimant of the project: there are no incentive problems and, therefore, the *first-best allocation* is chosen.

Once she has started the project in period 1, the agent exerts effort and completes the project regardless of her type (Assumption 3). In period 2, she invests and completes the project again if she has learned that she is of the good type, since $r - 1 > u$. If she has learned that she is of the bad type she prefers to take her outside option (Assumption 1). Conditional on having started the project in period 1, this is the first-best allocation.

Investment in period 1 can be thought of as experimentation: its costs are borne in period 1 while the benefits are realized in period 2. After the agent has learned her type, she will be able to make an informed decision. The costs of experimentation are given by the difference between the opportunity cost u and the expected surplus created by the project in period 1, i.e., $r - 1 - (1 - \rho)e$. The benefits of experimentation are due to better decision-making in period 2. With probability ρ, the information gathered through experimentation leads the agent to start a project, instead of taking the outside option. With probability $1 - \rho$, instead, the agent learns she is of the bad type and takes her outside option. In this case, the information gathered through experimentation does not change her decision.[8] The value of information therefore equals $\delta\rho(r - 1 - u)$. Experimentation is optimal if its costs are lower than its benefits.

[8] The agent is considering whether to experiment or not in period 1. If she decides to not experiment in period 1, then she optimally does not experiment in period 2 either.

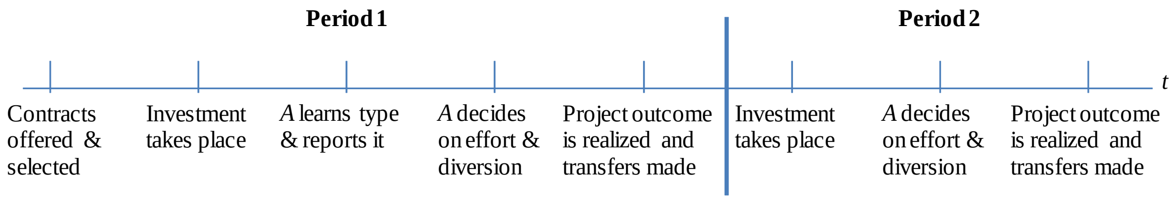

Figure 1: Timing of events

Lemma 1 *If the agent does not need to borrow, experimentation (investment in period* 1*) is optimal if and only if* $\delta \geq \delta_E$*, where*

$$\delta_E \equiv \frac{u + e(1-\rho) - (r-1)}{\rho(r-1-u)}. \tag{1}$$

As in standard experimentation models, starting the project in period 1 becomes profitable if δ is high enough, if the agent is sufficiently confident about being of the good type (high ρ), if the value of the known activity is not too high (low u) and if the project yields high returns (high $r-1$).

2.3 Contracts and Timing of Events

We now describe contracts and the structure of the credit market. Lenders compete in the market and make zero profits in expectation.[9] They have full commitment power and offer two-period contracts. The project is financed in period 1. For simplicity, we initially assume that i) the agent cannot change her lender in period 2 (but she can take her outside option u), ii) the agent cannot save on her own. We relax these two assumptions in Section 4 and Section 5.

The timing of events is the following. Immediately after the agent learns her type, she sends message $m \in \{G, B\}$ to the lender.[10] According to the message, the contract specifies the agent's actions in period 1, a transfer conditional on the project outcome in period 1 and a re-financing policy in period 2. The contract also specifies a transfer in period 2 conditional on project outcomes in periods 1 and 2. The timing of events is summarized in Figure 1.

[9] The main insights of the paper are preserved if the contract maximizes lender's profits subject to the borrower incentive and participation constraints. See the discussion at the end of Section 2.5.

[10] Since lenders have commitment power and contracts are exclusive, the Revelation Principle applies and we can focus on direct revelation mechanisms. We consider an indirect mechanism in Section 6.

We say that an allocation can be financed if there exists a contract that gives appropriate incentives to the agent and satisfies the lender's zero-profit constraint. In the next Section we analyze when a lender can finance the first-best allocation described above. In Section 2.5 we show which allocation is financed if the first best is not possible.

2.4 Financing the First Best

In this Section we study when the first-best allocation, that is, the one chosen by a self-financed agent, is financed. To do so, we proceed in two steps. First, we find the cost-minimizing contract, that is, the contract that finances the first best with the least possible transfers. Second, we find for which parameter values this contract allows the lender to earn non-negative profits.

To find the cost-minimizing contract we need to consider all the relevant incentive compatibility, truth-telling and limited liability constraints for the two types.[11] Remember that in the first-best allocation only the good type is refinanced in period 2 but both types must complete the project in period 1. The following constraints, therefore, need to be satisfied. First, the good type must prefer to complete the project in both period 1 and period 2. Second, the bad type must prefer to complete the project in period 1. Third, both types must have an incentive to reveal their type truthfully. Finally, the contract must satisfy all relevant limited liability constraints.

We first prove the following Lemma.

Lemma 2 *The net present value of the required minimum transfers to the good and bad types to implement the first best is given by*

$$T_G^* = \psi + e + \delta u \text{ and } T_B^* = \psi + e, \tag{2}$$

respectively.

Proof. See Appendix. ■

In period 1, the project should be completed independently of the type of agent since, at that stage, the initial outlay of 1 unit of capital is sunk (Assumption 3). Since the bad type is not given a project in period 2, the contract must give a transfer worth at

[11] To keep exposition simple and avoid too much notation in the main text, we relegate to the Appendix the formal exposition of all relevant constraints.

least $\psi + e$ to compensate for not stealing and for her effort cost. This, however, gives an incentive to the good type to pretend to be the bad type. Hence a minimum transfer of $\psi + e$, with an additional compensation for not taking the project in period 2, must be paid to the good type as well.

Are those transfers sufficient to satisfy the other constraints? It turns out they are. In principle, the good type also needs to be given incentives to complete the project in period 2. The minimum amount of rents necessary to induce the good type to complete the project in period 2 is equal to ψ. However, $\delta \leq 1$ implies that these rents are smaller than those required to induce the bad type to complete the project in period 1. Since rents to the good type can be paid in period 2, a contract that induces the good type to reveal her type truthfully pays sufficient rents to ensure the project in period 2 is completed.[12] Conversely, the bad type does not want to pretend to be the good type and try to get a project in period 2.

The first best can be financed when the project revenues are large enough to pay the cost-minimizing transfers characterized in Lemma 2, i.e., when

$$(r-1)(1+\delta\rho) \geq \rho T_G^* + (1-\rho) T_B^*.$$

This expression can be rewritten as

$$\delta \geq \underline{\delta}^{FB} \equiv \frac{\psi + e - (r-1)}{\rho(r-1-u)}. \tag{3}$$

This leads to the following proposition.

Proposition 1 *The first-best allocation is financed if and only if* $\delta \geq \underline{\delta}^{FB}$.

Threshold $\underline{\delta}^{FB}$ is higher than the one of the self-financing agent, δ_E in (1). Incentive problems create rents that make experimentation more expensive but do not change the nature of the problem. Comparative statics on $\underline{\delta}^{FB}$ are similar to the one on δ_E: experimentation is more likely to be financed for more optimistic priors ρ, higher discount factor δ, higher project profits $r-1$ and for lower values of the outside option u, effort costs e and the share of funds that can be diverted, ψ.

[12] This is similar to the "reusability of punishments" introduced by Dilip Abreu, Paul Milgrom & David Pearce (1991) according to which one punishment can be used to provide incentives for the agent to exert effort over many periods.

2.5 Financing the Second Best

The first-best allocation cannot be financed for every configuration of parameters in which experimentation is profitable. The reason is that inducing the bad type to complete the first-period project requires paying informational rents to the good type as well, and this might be too costly if the bad type is very unlikely. The borrower and the lender may then agree on a contract that let the bad type fail in period 1 and finances the project in period 2 conditional on the successful completion of the project in period 1. In other words, as in standard adverse selection models, the lender may shut down the bad type if its probability is low enough. The contract then only needs to solve the moral hazard problem of the good type. This is the *second-best allocation.*[13]

The good type has to be incentivized to complete the project in period 1, which requires a transfer worth at least $\psi + \delta u$. Since $\delta \leq 1$, these rents are sufficient to also ensure that the good type completes the project in period 2, which requires a transfer worth $\delta\psi$. The bad type, on the other hand, does not require any transfer since she does not complete the project in period 1 and then takes her outside option in period 2.

The second best, therefore, can be financed when the project revenues are larger than the transfers required to induce the good type to repay in both periods, i.e.,

$$\rho r - 1 + \delta\rho\,(r-1) \geq \rho\,(\psi + \delta u)\,.$$

This expression can be rewritten as

$$\delta \geq \underline{\delta}^{SB} \equiv \frac{1-\rho\,(r-\psi)}{\rho\,(r-1-u)}. \tag{4}$$

The comparative statics follows the standard logic: a higher δ, a higher ρ and a lower u expand the region in which the second best can be financed. The next proposition characterizes the region where the second best is financed and Figure 2 illustrates it.

[13] The contract could implement the allocation in which the bad type receives a project in the second period as well. It is easy to show, however, that the first best can be financed whenever this allocation can be financed, and, therefore, inefficient continuation of projects in period 2 does not occur in equilibrium. A previous version of the paper showed that, with more than two types, inefficient continuation can be part of the constrained optimal contract.

Proposition 2 *The second-best allocation is financed if and only if*

$$\underline{\delta}^{SB} \leq \delta < \underline{\delta}^{FB}.$$

Proof. See Appendix. ■

A monopolistic lender which maximizes profits subject to the agent participation and incentive compatibility constraints trades off efficiency and rents. In particular, for ρ and δ such that $\delta = \underline{\delta}^{FB}$ and $\delta > \underline{\delta}^{SB}$ the lender's profits from financing the first-best allocation are zero (by construction) while financing the second-best allocation yields positive profits. A monopolistic lender then chooses the second-best allocation. The region where the first-best allocation is financed shrinks while the one of the second-best allocation expands. However, comparative statics with respect to δ, ρ and u are qualitatively preserved.

3 Starting Small

In many contexts, an agent might decide to experiment by "starting small" and then later to scale up the project if she learns that the activity is profitable. In our context, "starting small" has the additional advantage that it might reduce the informational rents that must be paid to the agent to reveal her type and exert effort. In this Section we show that allowing the agent to "start small" generates novel implications that are qualitatively different from the results obtained in the previous Section: experimentation might become harder to finance when it is more profitable.

We now assume that a small project is also available. The small project is a proportionally scaled down version of the project studied above (that we will call a large project for clarity). Specifically, the small project yields revenues σr, costs an initial investment equal to σ (and so $\sigma\psi$ can be diverted) and requires effort costs σe from the bad type. Starting the small project still perfectly reveals the agent's type.

As a benchmark, consider a self-financed agent.

Lemma 3 *A self-financed agent never implements the small project.*

Proof. See Appendix. ■

In order to avoid a lengthy taxonomy of cases we make an additional assumption:

Assumption 4 $\frac{u}{(\psi+e)} < \sigma < \frac{u}{(r-1)}$.

The assumption $\sigma < \frac{u}{(r-1)}$ implies that a small project is *per se* unprofitable: the only reason to undertake a small project is to learn the type of the agent. The assumption, therefore, rules out cases in which the small project is financed in both periods. The assumption $\sigma > \frac{u}{(\psi+e)}$ ensures the agent's participation in the project.

Assumption 4 implies that we can restrict attention to four allocations. Two allocations we considered above in which a large project is financed in period 1, that is, the first best of Section 2.4 and the second best of Section 2.5. Two new allocations are the ones in which a small project is financed in period 1 and is either completed or not, and the large project is financed in period 2 is the agent is of the good type. For clarity, we refer to those allocations as *first best when starting small* and *second best when starting small.*[14]

Let us find out the conditions under which financing the first best when starting small is possible. As in Section 2.4, we again proceed in two steps. First, we find the contract that finances the allocation with the least possible transfers. This is the next Lemma. Second, we find for which parameter values this contract allows the lender to earn non-negative profits.

Lemma 4 *Define $\delta^* = \sigma\frac{\psi+e}{\psi-u}$. The net present value of the required minimum transfers to the good and bad types to implement the first best when starting small is given by*

$$\begin{cases} T_G^S = \sigma(\psi+e)+\delta u \\ T_B^S = \sigma(\psi+e) \end{cases}, \textit{ if } \delta \leq \delta^* \textit{ and } \begin{cases} T_G^S = \delta\psi \\ T_B^S = \delta(\psi-u) \end{cases}, \textit{ if } \delta > \delta^*. \tag{5}$$

Proof. See Appendix. ■

The small project of period 1 should be completed independently of the type of agent. Since the bad type is not given a project in period 2, the contract must give a transfer worth at least $\sigma(\psi+e)$ to compensate for not stealing and for her effort cost. This gives an incentive to the good type to pretend to be the bad type.

In contrast to the case in which the project has the same size in both periods, however, these transfers may not be sufficient to satisfy other constraints. If $\delta\psi > \sigma(\psi+e)+\delta u$,

[14] Analogously to fn. 13 above, it is easy to show that the bad type is never given a small project in period 2. If this is feasible, then it is also feasible to provide incentives to efficiently terminate the project.

the bad type is tempted to pretend to be the good type, get a project in period 2 and to run with the money. Which constraint binds, therefore, depends on whether $\delta\psi \gtrless \sigma(\psi + e) + \delta u$, i.e., $\delta \gtrless \delta^*$, as described in Lemma 4. Note that this inequality can be rewritten as $\frac{\sigma}{\delta} \gtrless \frac{\psi - u}{\psi + e}$. It is then clear that there is a one-to-one correspondence between the discount factor δ and scale σ, i.e., what matters is the weight, in present value terms, of the first-period rent relative to the second-period rent.

If $\sigma \geq \frac{\psi-u}{\psi+e}$ the first-period rents determine the costs of implementing any given allocation and, therefore, the analysis proceeds as in Section 2 with the first-period project rescaled by factor σ. If, instead, $\sigma < \frac{\psi-u}{\psi+e}$ the analysis might change. In the reminder of the paper, we focus on this case.

Assumption 5 $\sigma < \frac{\psi-u}{\psi+e}$.[15]

The *first best when starting small* can be financed when the project revenues are large enough to pay the cost-minimizing transfers characterized in Lemma 4, that is, $(r-1)(\sigma + \delta\rho) \geq \rho T_G^S + (1-\rho) T_B^S$. If $\delta \leq \delta^*$, this expression can be rewritten as

$$\delta \geq \underline{\delta}_S^{FB} \equiv \sigma \frac{\psi + e - (r-1)}{\rho(r-1-u)}. \tag{6}$$

If $\delta > \delta^*$, this expression can be rewritten as

$$\delta \leq \overline{\delta}_S^{FB} \equiv \begin{cases} \sigma \frac{r-1}{(1-\rho)(\psi-u)-\rho(r-1-\psi)} & \text{if } \rho \leq \frac{(\psi-u)-\sigma(r-1)}{r-1-u} \\ 1 & \text{otherwise} \end{cases}. \tag{7}$$

This leads to the following proposition. Define

$$\rho^* = \left(1 - \frac{r-1}{\psi+e}\right) \frac{\psi - u}{r-1-u}, \tag{8}$$

so the curves $\underline{\delta}_S^{FB}$ and $\overline{\delta}_S^{FB}$ intersect at (ρ^*, δ^*).

Proposition 3 *(i) First best when starting small can be financed if and only if* $\underline{\delta}_S^{FB} \leq \delta \leq \overline{\delta}_S^{FB}$.

(ii) There is a region where the first best when starting small is financed. In particular, it is financed at (ρ^*, δ^*).

[15] As a consequence of this and Assumption 4, $\psi > 2u$.

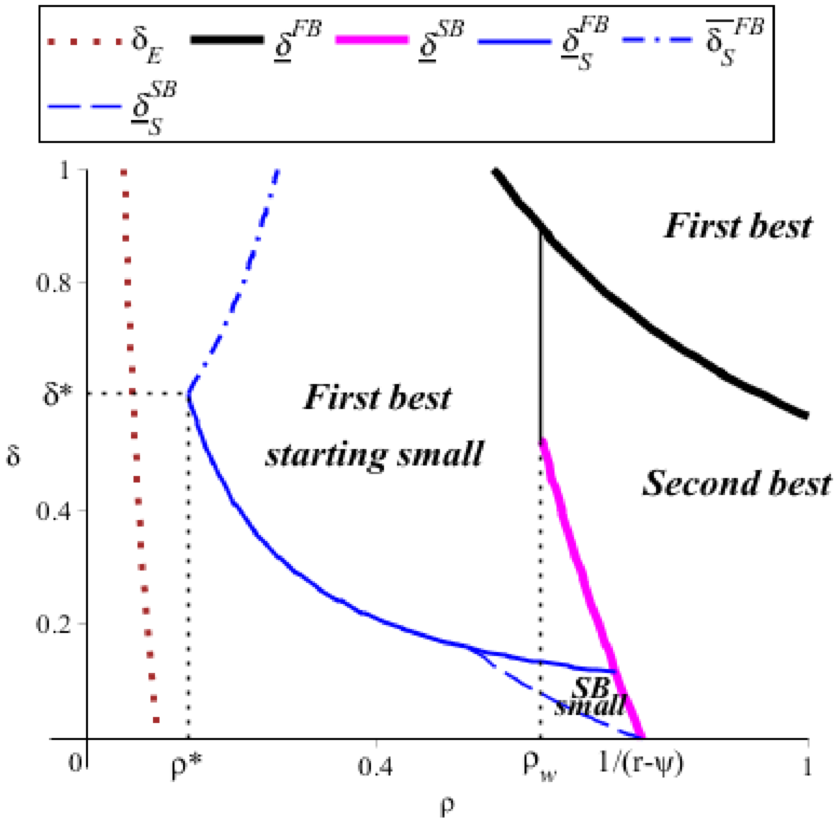

Figure 2: Financed allocations for $r = 1.7, u = 0.15, \psi = 0.4, e = 0.61$ and $\sigma = 0.15$. At $\rho_w = \frac{1+\sigma(r-1-e)}{r-\sigma e}$ the welfare in the second best allocation equals the one in the first best when starting small.

Proof. See Appendix. ■

For the proof of part (ii) we show that at (ρ^*, δ^*) neither the first-best nor the second-best allocations in which the large project is financed in period 1 are possible. In Figure 2 we draw a numerical example showing where each allocation is financed.[16]

Constraint (6) is very similar to (1) and (3): for low enough δ the profits earned in period 2 can be used to finance the agent's rent that must be paid to complete the project. When δ is sufficiently high, however, the bad type is tempted to "take the money and run" in period 2, that is, the truth-telling constraint of the bad type may become binding. In period 2, the lender needs to pay $\psi - u$ to prevent the bad type from obtaining a project. The lender faces a deficit of $(1-\rho)(\psi - u) - \rho(r - 1 - \psi)$ which has to be financed by the first-period profits $\sigma(r-1)$. A higher δ, therefore, reduces the value of period 1 profits relative to the second-period deficit and makes it harder to finance

[16]For completeness, we derive in the Appendix the region where the *second best when starting small* can be financed (see Proposition 8).

experimentation. Thus, the direction of (7), that δ has to be below a certain threshold, is the opposite to the direction of (1), (3) and (6).[17]

While it is generally perceived that future rents associated with a project are helpful to solve moral hazard (see, e.g., Rogerson (1985) and Bolton & Scharfstein (1990)), this paper shows that under initial uncertainty about these rents, they might attract undesirable borrowers and, therefore, lower the ex ante borrowing capacity. Interestingly, these rents are increasing in the net present value of the project, implying that more profitable projects might be harder to finance.

The logic is illustrated by the comparative statics with respect to the discount factor δ, the outside option u and the scale σ. If $\delta \leq \delta^*$, a higher δ expands the interval of values of ρ for which the first best when starting small can be financed. If $\delta > \delta^*$, a higher δ shrinks this interval. Similarly, the comparative statics with respect to the outside option u is non-monotonic. When $\delta \leq \delta^*$, a higher u reduces the costs of being denied access to credit in period 2. This shifts $\underline{\delta}_S^{FB}$ upwards (see (6)) and, hence, shrinks the region in which financing the first best when starting small is possible. When $\delta > \delta^*$, a higher u reduces the rent needed to keep the bad type out in period 2. This shifts $\overline{\delta}_S^{FB}$ upwards (see 7)) and expands the region where financing the first best when starting small is possible. Thus, in contrast to the case of a self-financed agent (1) and the first best (3), a higher outside option makes lending easier. Analogously, a lower σ facilitates financing for $\delta \leq \delta^*$ (see (6)) and hampers it for $\delta > \delta^*$ (see (7)). The latter point implies that the agency problem puts a lower bound on the downsizing of the experimentation round. The remaining comparative statics, however, have the expected sign.

We then summarize the discussion by its corollary.

Corollary 1 *There exists a region in the space (ρ, δ) where the first best when starting small is financed and which shrinks with a lower u and where a higher δ requires a higher ρ. In that region, a higher value of experimentation makes financing it more difficult.*

[17]A useful analogy is dynamic adverse selection models without commitment (see, e.g., Laffont & Tirole (1987) and Jean-Jacques Laffont & Jean Tirole (1988)) in which the principal pays a high rent to the good type which then attracts the bad type (ratchet effect). In contrast, here the lender can commit to a two-period contract and the source of the rent is the possibility of diverting the investment in period 2.

4 Ex Post Competition

In Section 2 we have considered the case in which the borrower cannot seek finance from outside lenders in period 2. This Section relaxes this assumption. The Section has two goals: i) check the robustness of the main result in Section 3 to the presence of ex post competition, and ii) derive additional results on the relationship between competition, value of the project, and financing constraints.

We follow the relational lending literature (see, e.g., Sharpe (1990)) and assume that outside lenders do not observe the communication between the inside lender and the borrower but can observe the first-period outcome of the project. We then consider two different scenarios, depending on whether the original lender can enforce loan contracts that are contingent on whether the borrower takes outside finance from an alternative lender (for simplicity, contingent contract case) or not (noncontingent contract case). Both scenarios are likely to be relevant depending on the context. For example, J.B. Barney, Lowell Busenitz, Jim Fiet & Doug Moesel (1994) and Kaplan & Strömberg (2003) find that venture capital contracts commonly include "non-compete" and "vesting provision" clauses that make it harder for the entrepreneur to hold-up the venture capitalist. In other contexts, however, lenders do not have the ability to condition the terms of their relationship with borrowers on whether borrowers access other sources of finance following the termination of their relationship. An example of such a circumstance is (micro)credit to small, typically informal, microenterprises in developing countries (see, e.g., Craig McIntosh & Bruce Wydick (2005) and Karlan & Morduch (2010) for a discussion). Even in countries with developed financial systems, the type of hold-up we consider prominently features in discussions of bank finance to SMEs (see, e.g., Dietmar Harhoff & Timm Körting (1998), Allen N. Berger & Gregory F. Udell (2006) and Ioannidou & Ongena (2010)).

The exact effects of competition depend on the contracts that the inside lender can offer. With contingent contracts the inside lender counteracts outside lenders' offers successfully and, therefore, competition in period 2 has no effect (Proposition 4) on the results. With noncontingent contracts, instead, competition qualitatively changes the results. In particular, the first-best allocation cannot be financed at all. Moreover, a higher probability of the good type, ρ, may have a negative effect on the possibility to finance experimentation (Proposition 5). Albeit along a different dimension, the result confirms the main finding in Section 3 that financing experimentation might become harder precisely when it is most valuable.

Preliminary Observations and Robustness of the Result in Section 3

In the first best both types complete the project in period 1 and, therefore, outside lenders do not know the type of the agent that applies to them. As outside lenders have one-period relationship with the agent, they cannot finance the small project (Assumption 4) and they have to let the bad type fail (Assumption 2 and 3). Then, they prefer to pay $\psi - u$ to the agent who reports to be of the bad type rather than finance the project that costs one (this can be done, e.g., by giving a small loan). Thus, outside lenders free ride on the information generated by the inside lender.

Competition between outside lenders makes them pay the highest possible rent to the good type driving their profits to zero. The inside lender, however, always structures a contract that gives incentives to the bad type to seek funds from outside lenders as this makes it harder for outside lenders to compete. The highest rent outside lenders can pay to good type while still breaking even in expected terms is therefore given by $r - 1 - \frac{1-\rho}{\rho}(\psi - u)$. This rent has to be above ψ for the agent to complete the project, i.e.,

$$\rho > \rho^{comp} \equiv \frac{\psi - u}{r - 1 - u} \tag{9}$$

is necessary for the outside lenders to be able to offer loans in period 2. For $\rho \leq \rho^{comp}$ outside lenders are unable to attract the good type without making losses. When this is the case, the conditions for implementing all the allocations are the same as in Sections 2 and 3. Since ρ^*, defined in (8), is smaller than ρ^{comp}, there always exists a region where the first-best when starting small is financed and where the comparative statics are as described in Corollary 1.

Corollary 2 *There exists a region in which the comparative statics described in Corollary 1 holds when there is ex-post competition from outside lenders.*

4.1 Competition with Contingent Contracts

We begin by considering the case in which the lender can offer contracts that are contingent on whether the borrower completes, fails, or does not take up a project financed by an outside lender.

Proposition 4 *When the inside lender can write contingent contracts, ex post competition does not bite. The regions in which each allocation can be financed are as characterized in Propositions 1, 2, 3 and 8.*

Proposition 4 can be easily proven by construction. In particular, consider any contract that implements the desired allocation in the absence of ex post competition. To respond to ex post competition, the inside lender has to include in that contract the following "vesting provision": the borrower has the option to purchase the right to continue the project in period 2 at a price F. To exercise the option, the borrower needs to borrow $1+F$ from the outside lender. A price $F > r-1-\psi$ is sufficient to ensure that there does not exist a contract in which i) the borrower obtains sufficient funds, invests and repays the loan, and ii) the outside lender makes non-negative profits (see Philippe Aghion & Patrick Bolton (1987) for a similar logic).

4.2 Competition with Noncontingent Contracts

We now consider the case in which the lender cannot write contracts contingent on what the borrower does upon leaving the relationship in period 2. We focus on the case when $\rho > \rho^{comp}$, i.e., when competition from outside lenders is possible.

Proposition 5 *Under competition from outside lenders with noncontingent contracts:*

(i) the first best cannot be financed,

(ii) the first best when starting small can be financed if $\underline{\delta}^{comp} \equiv \sigma\frac{\psi+e-(r-1)}{(1-\rho)(\psi-u)} \leq \delta \leq \sigma\frac{r-1}{\rho(\psi-u)} \equiv \overline{\delta}^{comp}$*,*

(iii) the region in which the second best can be financed is as in Proposition 2.[18]

Proof. See Appendix. ■

Figure 3 illustrates the Proposition through an example. Proposition 5 contains two main results. First, the first best is impossible to finance (part (i)). This happens because the second-period profits of the inside lender, which are limited by competition to $\delta(1-\rho)(\psi-u)$, are not sufficient to compensate for his first-period loss of $\psi+e-(r-1)$.

More importantly, Proposition 5 shows that a higher ρ is detrimental to the financing of experimentation (part (ii)). Since $\underline{\delta}^{comp}$ increases in ρ while $\overline{\delta}^{comp}$ decreases in ρ, a

[18] For completeness, we also show that the region in which the second best when starting small can be financed shrinks relative to the characterization in Proposition 8 in the Appendix.

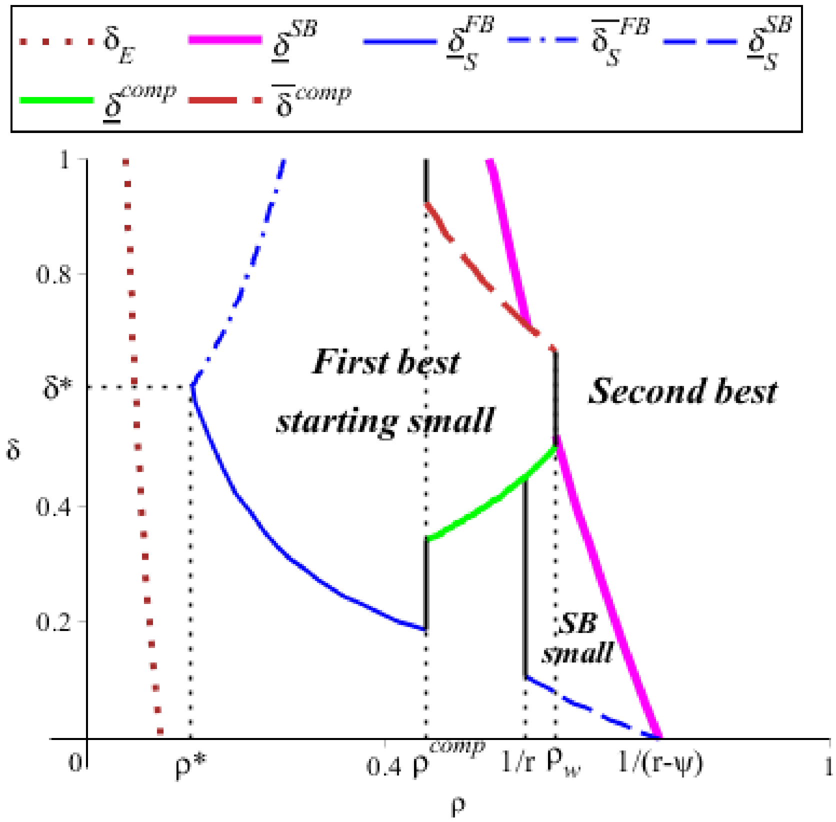

Figure 3: Financed allocations under competition with noncontingent contracts for $r = 1.7, u = 0.15, \psi = 0.4, e = 0.61$ and $\sigma = 0.15$. At $\rho_w = \frac{1+\sigma(r-1-e)}{r-\sigma e}$ the welfare in the second-best allocation equals the one in the first best when starting small.

higher ρ makes financing the first best when starting small more difficult. A higher ρ might make the first best when starting small impossible to finance while no other allocation can be financed either. The intuition for these results is that outside lenders "bite the hand that feeds them". Attracting the good type, they increase the inside lender's costs. A higher ρ allows outside lenders pay a higher rent to the good type to a point that cannot be matched by the inside lender, who also bears the costs of experimentation. But without the information generated by the first period financing, outside lenders cannot survive for some intermediate values of ρ and, therefore, the market completely shuts down.

Finally, if outside lenders believe that only bad types do not complete period 1 projects (second-best allocations), the good type can no longer pretend to be the bad type without loosing access to outside lenders in period 2. The bad type does not get any transfer and all transfers to the good type can be paid upon successful completion of both projects. Whether the region in which a second-best allocation can be financed is affected by competition or not, then, simply depends on whether the rent paid by outside lenders to the good type in period 2, i.e., $\delta(r-1)$, is larger than the rent necessary to have the project completed in both period under no competition. In the second best, it turns out it is not: the inside lender is in any case paying high rents to complete a large project in period 1.

5 Savings

This Section shows that the main results derived in Section 3 and Section 4 are robust if the borrower can (partially) self-finance the period 2 project through endogenous savings acquired in period 1.

The good type, in particular, may prefer to divert ψ in period 1, self-finance the project in period 2 and obtain returns $r-1$. If diverting ψ in period 1 is not enough to self-finance the project in period 2, the agent may apply to outside lenders when they are available. For simplicity, we assume a costless saving technology for the agent. The agent earns an interest rate $1+i=\frac{1}{\delta}$ on her private savings: if the agent saves s in period 1 her savings are worth $s(1+i)=\frac{s}{\delta}$ in period 2.

We first study the case when there are no outside lenders. When a large project is financed in period 1, self-financing is then possible if $\frac{\psi}{\delta}\geq 1$. With the small project financed in period 1, the condition is $\frac{\sigma\psi}{\delta}\geq 1$. The next proposition shows that the possibility to save and self-finance in period 2 does not matter in the first-best allocations but does matter in the second-best allocations.

Proposition 6 *When the agent can save on her own and there are no outside lenders,*

(i) The regions where the first best and the first best when starting small can be financed are as characterized in Section 3,

(ii) The second best cannot be financed for $\delta\leq\psi$ and $\rho<\frac{1}{r-\psi}$; the second best when starting small cannot be financed if $\delta\leq\sigma\psi$ and $\rho<\frac{1}{r-\psi}$.[19]

Proof. See Appendix. ■

When the agent completes the project in period 1 she gets a high rent that makes her prefer to stay with the lender rather than divert the first-period funding and self-finance.

[19] Otherwise, the second best can be financed as characterized in Proposition 2 and the second best when starting small can be financed as characterized in the proof of Proposition 8 in the Appendix.

Consider the first best when starting small. The good type gets at least $\sigma(\psi+e)+\delta u$ when staying with the lender if both projects are successful. Diverting ψ and self-financing she gets $\sigma\psi+\delta(r-1)$. The condition $\delta \leq \sigma\psi$ (which is necessary for self-finance to be possible) then implies that the value of self-finance is always smaller than the rents obtained by completing the project. The same argument applies for the first best (replacing σ by 1).

In the second-best allocations, in contrast, the agent gets a smaller rent since she does not complete the first project. Getting $\psi+\delta(r-1)$ (or $\sigma\psi+\delta(r-1)$) by self-financing is better than the minimum transfer $\psi+\delta u$ (or $\sigma\psi+\delta u$). Thus, the lender has to increase her transfer and the possibility of savings shrinks the region in which the second-best allocations can be financed.[20]

We now turn to the case in which the agent can save and there are outside lenders from whom she might borrow if her savings are not enough to finance the project. We characterize when the first-best allocations can be financed focusing on the case of non-contingent contracts.

Proposition 7 *When the agent can save and borrow from outside lenders under non-contingent contracts,*

(i) the first best cannot be financed,

(ii) the first best when starting small cannot be financed if $\delta \leq \sigma\frac{\psi}{\psi-u}$. For $\delta > \sigma\frac{\psi}{\psi-u}$, there exist thresholds $\underline{\delta}_{sav}^{comp} > \underline{\delta}^{comp}$, $\overline{\delta}_{sav}^{comp} < \overline{\delta}^{comp}$ and $\rho_{sav}^{comp} < \rho^{comp}$ such that: a] if $\rho \leq \rho_{sav}^{comp}$ the characterization in Proposition 3 applies, and b] if $\rho > \rho_{sav}^{comp}$ the first best when starting small can be financed if $\underline{\delta}_{sav}^{comp} \leq \delta \leq \overline{\delta}_{sav}^{comp}$.[21]

Proof. See Appendix. ■

The main message of Proposition 7 is that the borrower's ability to save and borrow from outside lenders interact to make lending even more difficult. The interaction stems from the fact that outside lenders can separate types at a lower cost. In particular, the constraint that the rent of the good type has to be at least ψ can be satisfied more easily since outside lenders invest less into the project and pay less to the bad type.

[20] In our model the possibility to save and self-finance only makes financing experimentation harder. The reason is that financing decision in period 2 is always efficient. In the models built on Bolton & Scharfstein (1990), where inefficient termination is used in the equilibrium, saving and self-financing has an efficiency benefit allowing the agent to continue when the lender would terminate as, for example, in Roman Inderst & Holger M. Mueller (2003).

[21] The results for the second-best allocations are omitted and available from the authors upon request.

If the agent can save more than $\delta(\psi - u)$, outside lenders can separate types at no cost since the bad type does not try to obtain funds from outside lenders and to divert ψ in period 2. Only the good type then applies for the loan and gets all the project rent $r - 1$. If the inside lender matches this rent to keep the good type, he does not make any profit in period 2. When the agent cannot save that much, the bad type also wants to apply for the loan from outside lenders and the analysis is then similar to the one of competition without savings as in Section 4.1. In particular, a higher ρ still makes financing the first best when starting small more difficult (i.e., $\underline{\delta}_{sav}^{comp}$ increases with ρ while $\overline{\delta}_{sav}^{comp}$ decreases with ρ).

We summarize this discussion noting

Corollary 3 *The results in Section 3 and Section 4 are qualitatively robust to the case in which the borrower has access to a saving technology. In particular, there always exist regions in the space (ρ, δ) where 1.] the first best when starting small is financed and which shrinks with a lower u and where a higher δ requires a higher ρ; 2.] a higher ρ makes financing impossible. In these regions, a higher value of experimentation makes financing it more difficult.*

6 Indirect Mechanism

We have investigated so far which allocations, if any, can be financed. It is important, however, to know whether there are realistic contracts that replicate the direct mechanism that implements a given allocation and to derive testable implications. This section answers both questions. We consider only the first best when starting small as in Section 3 to keep the paper at a reasonable length.

Due to the agent's risk-neutrality, the structure of payments in the optimal contract is not uniquely determined. To choose a particular contract, we impose a "minimum consumption spread" refinement. Among all the contracts that implement the first best when starting small, we focus on those that minimize i) the difference in the net present value of consumption across types, and ii) the difference in consumption across periods for each type.[22]

[22]Essentially, we assume that the agent has a utility function which is concave in consumption and separable in effort and consumption, i.e., $U(c) - e$, with $U'(\cdot) > 0$ and $U''(\cdot) < 0$, and then take the limit when $U(\cdot)$ converges pointwise to the linear function $c - e$.

Denote by c_τ^i, $i = G, B$, $\tau = 1, 2$, the consumption of type i in period τ. We proceed in three steps. First, we derive the net present value of consumption given to each type. Second, we derive the consumption allocation to each type in each period taking into account necessary incentive constraints. Finally, we describe a contract that induces the resulting consumption allocation.

Denote by Π the difference between the expected project revenues of the relationship and the minimum transfers necessary to implement the first best when starting small, T_G^S and T_B^S (characterized in Lemma 4):

$$\Pi = (\sigma + \delta\rho)(r - 1) - \left[\rho T_G^S + (1 - \rho) T_B^S\right].$$

The project can be financed if $\Pi \geq 0$. Remember that $T_G^S = T_B^S + \delta u$. We can rewrite

$$\Pi = (\sigma + \delta\rho)(r - 1) + (1 - \rho)\delta u - T_G^S.$$

Since $T_G^S > (1 + \delta) u$, it follows that, if the project can be financed, the lender can design a contract in which the constraint $c_2^B \geq u$ (which would stem from the non-transferability of u) never bites. Transferable revenues, $(\sigma + \delta\rho)(r - 1)$, and non-transferable payoffs, $(1 - \rho)\delta u$, generated by the relationship can be aggregated and competition among lenders ensures that the net present value of consumption for each type is equal to $C(\rho) = (\sigma + \delta\rho)(r - 1) + \delta(1 - \rho) u$.[23]

Contracts satisfying the "minimum consumption spread" refinement implement the following consumption allocation:

1. Perfect consumption smoothing across types, $c_1^B + \delta c_2^B = c_1^G + \delta c_2^G = C(\rho)$;
2. Perfect consumption smoothing across periods for the bad type, $c_1^B = c_2^B = \frac{C(\rho)}{1+\delta} > u$;
3. Perfect consumption smoothing for the good type $c_1^G = c_2^G = \frac{C(\rho)}{1+\delta}$ if $C(\rho) \geq (1+\delta)\psi$. Otherwise, $c_1^G = C(\rho) - \delta\psi < c_2^G = \psi$.

The optimal contract provides full consumption insurance to the borrower against bad realizations of her entrepreneurial talent. The contract also provides perfect consumption smoothing across the two periods for the bad type since, conditional on completing the

[23] We derive testable predictions by considering correlation patterns driven by heterogeneity in ρ across borrowers. For expositional purposes, we omit other parameters entering $C(\cdot)$.

project in period 1, no further constraint must be satisfied. Furthermore, in each period the bad type consumes more than her outside option u. The contract, however, might fail to achieve perfect consumption smoothing for the good type. Indeed, since the good type has to obtain at least ψ in period 2 to complete the project, perfect consumption smoothing is possible only if ψ can also be paid in period 1, i.e., if $C(\rho) \geq (1+\delta)\psi$.

An Optimal Contract: Application to Microfinance

Is there an indirect mechanism that implements the consumption allocation described above and resembles a real world contract? As an example, consider the contract $\mathcal{C} \equiv \{d_1, d_2, s^C, i\}$ defined as follows. The agent borrows σ in the beginning of period 1. If the project yields revenue σr, the agent repays d_1 at the end of period 1. The borrower can apply and obtain funding in period 2 under two conditions: i) she has repaid period 1 loan, and ii) she posts collateral at least equal to s^C. If the borrower seeks and obtains funding in period 2, she borrows one unit of capital and repays d_2 if the project yields revenue r. Otherwise, she defaults and loses the posted collateral. Finally, i is the interest rate paid by the lender on the saving account held by the borrower.

Denote by s^B and s^G the saving chosen by the bad type and good type. Consumption patterns are then defined by

$$\begin{aligned} c_1^G &= \sigma r - d_1 - s^G \text{ and } c_1^B = \sigma r - d_1 - s^B \text{ in period 1, and} \\ c_2^G &= r - d_2 + (1+i)s^G \text{ and } c_2^B = (1+i)s^B + u \text{ in period 2} \end{aligned} \tag{10}$$

As in Section 5, let us set, with no loss of generality, $(1+i) = \frac{1}{\delta}$. The remaining terms of the contract can be computed substituting the appropriate consumption values in (10). In period 2 the bad type consumes more than the income she derives from taking the outside option, $c_2^B = \frac{C(\rho)}{1+\delta} > u$. The consumption in excess of income in period 2 gives a positive saving balance $s^B(\rho) = \delta\left(\frac{C(\rho)}{1+\delta} - u\right)$. Substituting into $c_1^B = \frac{C(\rho)}{1+\delta}$, the amount to be repaid to the lender is equal to $d_1(\rho) = \sigma r - C(\rho) + \delta u$, which is decreasing in ρ.[24] The model implies that better borrowers consume (and save) more and receive better terms on the period 1 loan.

Substituting $d_1(\rho)$ and the appropriate consumption values in (10), we find $s^G(\rho) =$

[24] The described contract is feasible if $d_1(\rho) \geq 0$, i.e., if $\sigma \geq \delta\rho(r-1-u)$. The condition is verified, e.g., at (δ^*, ρ^*) defined in Lemma 4 and (8).

$\max\{\delta(\psi - u), s^B(\rho)\}$. In turn, this implies $d_2 = r - u$. When $C(\rho) < (1+\delta)\psi$, the good type consumes less and saves more than the bad type in period 1.

Note that $s^B(\rho)$ and $s^G(\rho)$ are not part of the contract with the lender. Given $s^B(\rho)$ and a requested collateral s^C, the bad type does not apply for a loan in period 2 (on which she would default) if

$$\psi + \frac{\left(s^B(\rho) - s^C\right)}{\delta} \leq \frac{C(\rho)}{1+\delta},$$

i.e., if $s^C \geq \delta(\psi - u)$. We saw this condition in Section 5 (see discussion after Proposition 7) under which outside lenders can separate the types at no cost.

When $C(\rho) \geq (1+\delta)\psi$, both types optimally save more than s^C but only the good type applies for a loan. When $C(\rho) < (1+\delta)\psi$, however, the bad type saves less than s^C. The good type, instead, is *required* to save $s^C = \delta(\psi - u)$ to obtain the loan. The optimal contract, therefore, requires the borrower to save a larger amount in order to continue borrowing in period 2.

The contract uses retained earnings to endogenously build up collateral and screen out the bad type. One way in which this can be achieved, is through compulsory saving requirements (CSRs). An example of a loan contract with compulsory saving requirements is found in microfinance, broadly defined as the provision of small uncollateralized loans to poor borrowers in developing countries. CSRs are a common feature of microcredit schemes (whenever the regulatory framework allows MFIs to collect deposits). For instance, the three largest microfinance institutions in Bangladesh (Grameen Bank, BRAC and ASA) have been collecting compulsory regular savings from their clients from the very start of their programs (see, e.g., Asif Dowla & Dewan Alamgir (2003)). All of the five major microfinance institutions described by Morduch (1999) use combinations of borrowing and saving. In recent years, many MFIs have also started offering more flexible savings products (see, e.g., Nava Ashraf, Dean Karlan, Nathalie Gons & Wesley Yin (2003)). CSRs are payments that are required for participation in the scheme, are part of loan terms, and are required in place of collateral. The amount, timing, and access to these deposits are determined by the policies of the institution rather than by the clients who are typically allowed to withdraw at the end of the loan term, after a predetermined amount of time, or when they terminate their membership.

When the second-best allocation is financed, CSRs are never needed. Indeed, the bad type reveals herself by defaulting in period 1. Therefore, the model implies that CSRs are more likely to be observed when the contract induces all borrowers to repay their loans. This suggests a connection between extremely high repayment rates and the prevalent use of CSRs observed in microfinance, as informally discussed in Morduch (1999).[25]

Empirical Predictions on the Use of Collateral

Besides rationalizing contractual features used in practice, the model yields a number of testable predictions on the relationship between collateral, loan terms and project outcomes. Many models predict that lower risk borrowers pledge more collateral (see, e.g., Besanko & Thakor (1987*a*), David Besanko & Anjan V. Thakor (1987*b*), Yuk-Shee Chan & Anjan V. Thakor (1987) and Yuk-Shee Chan & George Kanatas (1985)). This observation appears to be at odds with lending practices that associate the use of collateral with riskier borrowers (see, e.g., Kose John, Anthony W. Lynch & Manju Puri (2003)). In the model, the good type obtains the loan in period 2 and is required to post collateral worth $s^C = \delta (\psi - u)$. Since the bad type never obtains a loan in period 2, the model implies *no* relationship between amount of collateral and risk in a cross-section of period 2 borrowers.

Suppose the borrower has some wealth at time zero which can be posted, at some small variable cost, as collateral. This extension of the model *does* imply the observed empirical relationship between collateral and risk. If the borrower is credit constrained (i.e., cannot finance the first-best allocation) she would post the minimum collateral necessary to obtain the loan. Since financing requirements are given while the surplus available is increasing in ρ, borrowers with lower ρ post higher collateral to obtain funds. In other words, in a cross-section of borrowers, the model predicts a positive relationship between collateral posted and likelihood of termination (if a first-best type allocation is implemented) or likelihood of default (if a second-best type of allocation is implemented).[26]

[25] The model can be applied to other contexts besides microcredit contracts. For example, the payment to the (bad type of) borrower to make her relinquish the project can be interpreted as shift of the control from the entrepreneur to the investor in venture capital finance. This can be implemented through a "purchase option": when the investor exercises this option he effectively pays an exit fee to the entrepreneur (see, e.g., Kaplan & Strömberg (2003)).

[26] As in standard models of moral hazard or adverse selection in credit market (see, e.g., Dean Karlan & Jonathan Zinman (2010) for a discussion) the model also predicts that, conditional on the size of the loan, higher interest rates positively correlate with the likelihood of termination and default.

7 Conclusion

Exploration of unknown activities lies at the heart of this model. What happens when such activities are financed by a lender? The paper has shown that introducing agency problems changes the nature of experimentation. In particular, we have shown how, in the context of experimentation, projects with higher net present value can be systematically harder to finance. This might happen for higher discount factors, for lower values of the known arm and, in the presence of ex post competition, when priors are more optimistic about the unknown arm. We have highlighted the role of endogenous saving requirements in mitigating these problems and related the predictions of the model to contractual forms observed in practice and to a number of testable implications. A multi-period version of this model and the case of the lender's imperfect commitment are left for future research.

Appendix

Proof of Lemma 2. In the end of period 1 the agent receives a transfer which is conditional on her report and first-period performance. Denote the first-period transfers as $t_{i,1}^{p_1}$, where i is the reported type, $i = G, B$, and p_1 is the first-period performance taking values s (success) and f (failure).

The second-period transfers are conditional on the entire history of the relationship, that is, the agent's report, her first-period performance and second-period performance if the project is funded in period 2. Denote the second-period transfers as $t_{B,2}^{p}$, when the bad type is reported, and $t_{G,2}^{p}$, when the good type is reported, where p is the performance in the two periods taking values ss (both successes), sf (success in period 1 and failure in period 2) and f (failure in period 1).

A contract consists of four first-period transfers, $t_{G,1}^{s}, t_{G,1}^{f}, t_{B,1}^{s}, t_{B,1}^{f}$, five second-period transfers, $t_{B,2}^{s}, t_{B,2}^{f}, t_{G,2}^{ss}, t_{G,2}^{sf}, t_{G,2}^{f}$ and the rule that the second-period project is financed if and only if the reported type is "good" and there is success in period 1. Limited liability of the agent means that all the transfers have to be non-negative.

The contract has to give incentives to report the truth and to complete the project in period 1 for both types and also to complete the project in period 2 for the good type.

The incentive constraints are

for the good type :

$$\begin{aligned}
& t^s_{G,1} + \delta t^{ss}_{G,2} \geq t^f_{G,1} + \psi + \delta \left(t^f_{G,2} + u \right) && IC_{G,1} \\
& t^s_{G,1} + \delta t^{ss}_{G,2} \geq t^s_{B,1} + \delta \left(t^s_{B,2} + u \right) && TT_G \\
& t^s_{G,1} + \delta t^{ss}_{G,2} \geq t^f_{B,1} + \psi + \delta \left(t^f_{B,2} + u \right) && IC_{G,1} - TT_G \\
& t^{ss}_{G,2} \geq t^{sf}_{G,2} + \psi && IC_{G,2}
\end{aligned}$$

for the bad type :

$$\begin{aligned}
& t^s_{B,1} + \delta \left(t^s_{B,2} + u \right) \geq t^f_{B,1} + \psi + e + \delta \left(t^f_{B,2} + u \right) && IC_{B,1} \\
& t^s_{B,1} + \delta \left(t^s_{B,2} + u \right) \geq t^s_{G,1} + \delta \max\{ t^{ss}_{G,2} - e, t^{sf}_{G,2} + \psi \} && TT_B \\
& t^s_{B,1} + \delta \left(t^s_{B,2} + u \right) \geq t^f_{G,1} + \psi + e + \delta \left(t^f_{G,2} + u \right) && IC_{B,1} - TT_B
\end{aligned}$$

For each type, constraints $IC_{i,1}$ rule out failing the project in period 1 while reporting the type truthfully, constraints TT_i rule out lying while completing the project in period 1 and constraints $IC_{i,1} - TT_i$ rule out the joint deviation of failing the project in period 1 and lying. Finally, constraint $IC_{G,2}$ makes sure that the good type completes the project in period 2. Since financing the bad type in period 2 is off the equilibrium path, the contract may or may not give incentives to complete the project in period 2 for the bad type. That is why there is the term $\max\{t^{ss}_{G,2} - e, t^{sf}_{G,2} + \psi\}$ in the right-hand side of TT_B.

Note that the transfers after the failure in either the first or the second period enter only the right-hand sides of the constraints. Thus, the lender sets $t^f_{G,1} = t^f_{B,1} = t^f_{B,2} = t^{sf}_{G,2} = t^f_{G,2} = 0$. Rewrite the constraints

for the good type :

$$\begin{aligned}
& t^s_{G,1} + \delta t^{ss}_{G,2} \geq \psi + \delta u && IC_{G,1} \\
& t^s_{G,1} + \delta t^{ss}_{G,2} \geq t^s_{B,1} + \delta \left(t^s_{B,2} + u \right) && TT_G \\
& t^s_{G,1} + \delta t^{ss}_{G,2} \geq \psi + \delta u && IC_{G,1} - TT_G \\
& t^{ss}_{G,2} \geq \psi && IC_{G,2}
\end{aligned}$$

for the bad type :

$$\begin{aligned}
& t^s_{B,1} + \delta \left(t^s_{B,2} + u \right) \geq \psi + e + \delta u && IC_{B,1} \\
& t^s_{B,1} + \delta \left(t^s_{B,2} + u \right) \geq t^s_{G,1} + \delta \max\{ t^{ss}_{G,2} - e, \psi \} && TT_B \\
& t^s_{B,1} + \delta \left(t^s_{B,2} + u \right) \geq \psi + e + \delta u && IC_{B,1} - TT_B
\end{aligned}$$

Note that if the project fails in period 1, the agent is not given the project in period 2 and, thus, her report does not matter. So, the constraints $IC_{i,1}$ and $IC_{i,1} - TT_i$ are identical for each type.

To simplify notation, omit superscripts s and ss as this does not create any confusion. Also, denote $T_i = t_{i,1} + \delta t_{i,2}$ the total transfer to each type. Rewrite the constraints

$$\begin{aligned}
&T_G \geq \psi + \delta u && IC_{G,1} \\
&T_G \geq T_B + \delta u && TT_G \\
&t_{G,2} \geq \psi && IC_{G,2} \\
&T_B + \delta u \geq \psi + e + \delta u && IC_{B,1} \\
&T_B + \delta u \geq t_{G,1} + \delta \max\{t_{G,2} - e, \psi\} && TT_B
\end{aligned} \tag{11}$$

From $IC_{B,1}$, $T_B \geq \psi + e$ and, thus, TT_G implies $IC_{G,1}$.

It is easy to check that $T_G = \psi + e + \delta u$ (with $t_{G,2} = \psi > 0$ and $t_{G,1} = \psi + e + \delta u - \delta\psi > 0$) and $T_B = \psi + e$ (split between $t_{B,1} \geq 0$ and $t_{B,2} \geq 0$ in any way) satisfy the constraints TT_G, $IC_{G,2}$, $IC_{B,1}$ and TT_B as equalities and thus cannot be decreased. ■

Proof of Proposition 2. As in the analysis of the first best, we first find the cost-minimizing transfers and then we plug them into the lender's zero-profit condition.

Lemma 5 *The net present value of the required minimum transfers to the good and bad types to implement the second best is given by*

$$T_G^{SB} = \psi + \delta u \text{ and } T_B^{SB} = 0, \tag{12}$$

respectively.

Proof. Since the good type completes the projects in period 1 while the bad type fails it, the project outcome in period 1 reveals the agent's type and the lender does not have to ask for the report. Thus, the first-period transfers are conditional only on the outcome of the first-period project, t_1^s and t_1^f, and the second-period transfers are conditional on the first-period outcome and the second-period one if the second project is financed, t_2^{ss}, t_2^{sf} and t_2^f.

A contract consists of five transfers, $t_1^s, t_1^f, t_2^{ss}, t_2^{sf}$ and t_2^f and the rule that the second-period project is financed if and only if there is success in period 1. Limited liability of the agent means that all the transfers have to be non-negative.

The contract has to give incentives to complete the projects in both periods for the good type and to fail the project in period 1 for the bad type. The incentive constraints are

$$\begin{array}{ll} t_1^s + \delta t_2^{ss} \geq t_1^f + \psi + t_2^f + \delta u & IC_{G,1} \\ t_1^f + \psi + t_2^f + \delta u \geq t_1^s - e + \delta \max\{t_2^{ss} - e, t_2^{sf} + \psi\} & IC_{B,1} \\ t_2^{ss} \geq t_2^{sf} + \psi & IC_2 \end{array} \tag{13}$$

Since t_2^{sf} enters only the right-hand side of the constraints, $t_2^{sf} = 0$.

Denote $T_G = t_1^s + \delta t_2^{ss}$ the total transfer to the good type and $T_B = t_1^f + \delta t_2^f$ the total transfer to the bad type.

It is easy to check that $T_G = \psi + \delta u$ (with $t_2^{ss} = \psi > 0$ and $t_1^s = \psi + \delta u - \delta \psi > 0$) and $T_B = t_1^f = t_2^f = 0$ satisfy all the constraints. T_B cannot be made lower and, since $IC_{G,1}$ is binding, T_G cannot be made lower either. ■

The revenues of the lender are $\rho r - 1 + \delta \rho (r-1)$. Plugging transfers (12) into the zero-profit condition, we obtain condition (4). The second best is financed when 1) it is possible and 2) the first best is not possible, that is, when $\underline{\delta}^{SB} \leq \delta < \underline{\delta}^{FB}$. ■

Proof of Lemma 3. Since the scale of the project does not affect the learning process, the decision between "starting small" or "starting large" entirely depends on the comparison of the expected profits from the two technologies in period 1. If $(r-1) - (1-\rho) e < 0$, i.e., if $\rho < 1 - \frac{r-1}{e}$ the small project is chosen. If, instead, $\rho > 1 - \frac{r-1}{e}$ the large project is chosen. Analogously to the case in Section 2.2, the experimentation with the small project yields a higher payoff than the outside option if $\delta \geq \delta_E^S \equiv \frac{u + \sigma[e(1-\rho) - (r-1)]}{\rho(r-1-u)}$. At $\rho = 1 - \frac{r-1}{e}$, however, $\delta_E^S > 1$.[27] Since δ_E^S is decreasing in ρ, it follows that there does not exist values of ρ and δ for which experimenting with the small project is preferred to both the outside option *and* to experimenting with a large project. ■

Proof of Lemma 4. The proof closely follows the proof of Lemma 2.

A contract consists of four first-period transfers, $t_{G,1}^s, t_{G,1}^f, t_{B,1}^s, t_{B,1}^f$, five second-period transfers, $t_{B,2}^s, t_{B,2}^f, t_{G,2}^{ss}, t_{G,2}^{sf}, t_{G,2}^f$ and the rule that the second-period project is financed if and only if the reported type is "good" and there is success in period 1 (the transfers

[27] At $\rho = 1 - \frac{r-1}{e}$ we have $\delta_E^S = \frac{e}{r-1-u} \frac{u}{e-(r-1)}$. Assumption 3 implies $eu > e(e-(r-1)) > (r-1-u)(e-(r-1))$.

are defined as in the proof of Lemma 2). Limited liability of the agent means that all the transfers have to be non-negative.

The contract has to give incentives to report the truth and to complete the project in period 1 for both types and also to complete the project in period 2 for the good type. The incentive constraints are (see the proof of Lemma 2 for their description)

for the good type :

$$\begin{array}{ll} t^s_{G,1} + \delta t^{ss}_{G,2} \geq t^f_{G,1} + \sigma\psi + \delta\left(t^f_{G,2} + u\right) & IC_{G,1} \\ t^s_{G,1} + \delta t^{ss}_{G,2} \geq t^s_{B,1} + \delta\left(t^s_{B,2} + u\right) & TT_G \\ t^s_{G,1} + \delta t^{ss}_{G,2} \geq t^f_{B,1} + \sigma\psi + \delta\left(t^f_{B,2} + u\right) & IC_{G,1} - TT_G \\ t^{ss}_{G,2} \geq t^{sf}_{G,2} + \sigma\psi & IC_{G,2} \end{array}$$

for the bad type :

$$\begin{array}{ll} t^s_{B,1} + \delta\left(t^s_{B,2} + u\right) \geq t^f_{B,1} + \sigma\psi + \sigma e + \delta\left(t^f_{B,2} + u\right) & IC_{B,1} \\ t^s_{B,1} + \delta\left(t^s_{B,2} + u\right) \geq t^s_{G,1} + \delta\max\{t^{ss}_{G,2} - e, t^{sf}_{G,2} + \psi\} & TT_B \\ t^s_{B,1} + \delta\left(t^s_{B,2} + u\right) \geq t^f_{G,1} + \sigma\psi + \sigma e + \delta\left(t^f_{G,2} + u\right) & IC_{B,1} - TT_B \end{array}$$

Note that the transfers after the failure in either the first or the second period enter only the right-hand sides of the constraints. Thus, the lender sets $t^f_{G,1} = t^f_{B,1} = t^f_{B,2} = t^{sf}_{G,2} = t^f_{G,2} = 0$.

Then, the constraints $IC_{i,1}$ and $IC_{i,1} - TT_i$ are identical for each type since when the project fails in period 1, the agent is not given the project in period 2 and, thus, her report does not matter.

From $IC_{B,1}$, $t^s_{B,1} + \delta t^s_{B,2} \geq \sigma\psi + \sigma e$ and, thus, TT_G implies $IC_{G,1}$.

Denote $T^S_G = t^s_1 + \delta t^{ss}_2$ the total transfer to the good type and $T^S_B = t^f_1 + \delta t^f_2$ the total transfer to the bad type.

The relevant constraints are then written as

$$\begin{array}{ll} T_G \geq T_B + \delta u & TT_G \\ t_{G,2} \geq \psi & IC_{G,2} \\ T_B + \delta u \geq \sigma\psi + \sigma e + \delta u & IC_{B,1} \\ T_B + \delta u \geq t_{G,1} + \delta\max\{t_{G,2} - e, \psi\} & TT_B \end{array} \tag{14}$$

If $\delta \leq \delta^*$, then the transfers (2), adapted to the project being small in period 1, work. Indeed, $T^S_G = \sigma\psi + \sigma e + \delta u$ (with $t_{G,2} = \psi > 0$ and $t_{G,1} = \sigma\psi + \sigma e + \delta u - \delta\psi \geq 0$) and

$T_B^S = \sigma\psi + \sigma e$ (split between $t_{B,1} \geq 0$ and $t_{B,2} \geq 0$ in any way) satisfy the constraints TT_G, $IC_{G,2}$, $IC_{B,1}$ and TT_B as equalities and thus cannot be decreased.

If $\delta > \delta^*$, transfers (2) become unfeasible since $t_{G,1} = \sigma\psi + \sigma e + \delta u - \delta\psi < 0$. The best the lender can do is then to set $t_{G,1} = 0$ which implies $T_G^S = \delta\psi$. From TT_B, he also sets $T_B^S = \delta\psi - \delta u$. Then, TT_G and TT_B are binding while $IC_{B,1}$ is not. ■

Proof of Proposition 3. Part (i). The double inequality $\underline{\delta}_S^{FB} \leq \delta \leq \overline{\delta}_S^{FB}$ is (6) and (7) combined.

Part (ii). Let us show that financing the large project in period 1 is not possible at (ρ^*, δ^*). The first best is not possible since the curve $\underline{\delta}_S^{FB}$ lies below $\underline{\delta}^{FB}$. Indeed, $\underline{\delta}_S^{FB} = \sigma\underline{\delta}^{FB} < \underline{\delta}^{FB}$ defined in (3). Thus, no point belonging to $\underline{\delta}_S^{FB}$ can be financed in the first best.

For the second best, we show that ρ^* lies to the left of $\frac{1}{r-\psi+\sigma\frac{\psi+e}{\psi-u}(r-1-u)}$ which is $\underline{\delta}^{SB}$ defined in (4) at δ^*. By Assumption 3, $r - 1 - u > \psi - u$. Then,

$$\begin{aligned}
(r-\psi+u)(r-1-u) &> (r-\psi)(\psi-u) \Leftrightarrow \\
r(r-1-u) &> (r-\psi+r-1-u)(\psi-u) \Leftrightarrow \\
\frac{1}{r-\psi+r-1-u} &> \frac{1}{r}\frac{\psi-u}{r-1-u} \Leftrightarrow \\
\frac{1}{r-\psi+r-1-u} &> \left(1-\frac{r-1}{r}\right)\frac{\psi-u}{r-1-u} \Rightarrow \\
\frac{1}{r-\psi+\sigma\frac{\psi+e}{\psi-u}(r-1-u)} &> \left(1-\frac{r-1}{\psi+e}\right)\frac{\psi-u}{r-1-u},
\end{aligned}$$

where the last step uses the facts that $\sigma\frac{\psi+e}{\psi-u} < 1$ (Assumption 5) and $r > \psi + e$ (Assumption 3).

Thus, at (ρ^*, δ^*) neither the first best nor the second best are possible to finance and, therefore, the first best when starting small is financed. By continuity, there is a region around (ρ^*, δ^*) and satisfying $\underline{\delta}_S^{FB} \leq \delta \leq \overline{\delta}_S^{FB}$ in which the first best when starting small is financed. ■

Proposition 8 *The second best when starting small can be financed in the following regions:*

- *If* $\underline{\delta}_S^{SB} \equiv \max\left\{\sigma\frac{1-\rho(r-\psi)}{\rho(r-1-u)}, \sigma\frac{1-\rho r}{\rho(r-1-\psi)}\right\} \leq \delta \leq \delta^*$;

- *If* $\delta^* \leq \delta \leq \sigma \frac{\rho r - 1 + (1-\rho)(\psi + e)}{(1-\rho)(\psi - u) - \rho(r - 1 - \psi)}$ *if it is positive and for any* $\delta \geq \delta^*$ *otherwise.*

Proof. As in the analysis of the first best, we first find the cost-minimizing transfers and then we plug them into the lender's zero-profit condition.

Lemma 6 *The net present value of the required minimum transfers to the good and bad types to implement the second best when starting small is given by*

$$\begin{cases} T_G = \sigma\psi + \delta u \\ T_B = 0 \end{cases}, \text{ if } \delta \leq \frac{\sigma\psi}{\psi - u}$$
$$\begin{cases} T_G = \delta\psi \\ T_B = 0 \end{cases}, \text{ if } \delta \in \left(\frac{\sigma\psi}{\psi - u}, \delta^*\right] \tag{15}$$
$$\begin{cases} T_G = \delta\psi \\ T_B = \delta(\psi - u) - \sigma(\psi + e) \end{cases}, \text{ if } \delta > \delta^*$$

Proof. The proof closely follows the proof of Lemma 5. Since the good type completes the projects in period 1 while the bad type fails it, the project outcome in period 1 reveals the agent's type and the lender does not have to ask for the report. Thus, the first-period transfers are conditional only on the outcome of the first-period project, t_1^s and t_1^f, and the second-period transfers are conditional on the first-period outcome and the second-period one if the second project is financed, t_2^{ss}, t_2^{sf} and t_2^f.

A contract consists of five transfers, $t_1^s, t_1^f, t_2^{ss}, t_2^{sf}$ and t_2^f and the rule that the second-period project is financed if and only if there is success in period 1. Limited liability of the agent means that all the transfers have to be non-negative.

The contract has to give incentives to complete the projects in both periods for the good type and to fail the project in period 1 for the bad type. The incentive constraints are

$$\begin{aligned} & t_1^s + \delta t_2^{ss} \geq t_1^f + \sigma\psi + t_2^f + \delta u && IC_{G,1} \\ & t_1^f + \sigma\psi + t_2^f + \delta u \geq t_1^s - \sigma e + \delta \max\{t_2^{ss} - e, t_2^{sf} + \psi\} && IC_{B,1} \\ & t_2^{ss} \geq t_2^{sf} + \psi && IC_2 \end{aligned}$$

Since t_2^{sf} enters only the right-hand side of the constraints, $t_2^{sf} = 0$.

Denote $T_G = t_1^s + \delta t_2^{ss}$ the total transfer to the good type and $T_B = t_1^f + \delta t_2^f$ the total transfer to the bad type and rewrite the constraints as

$$\begin{aligned} & T_G \geq T_B + \sigma\psi + \delta u && IC_{G,1} \\ & T_B + \sigma\psi + \delta u \geq t_1^s - \sigma e + \delta \max\{t_2^{ss} - e, \psi\} && IC_{B,1} \\ & t_2^{ss} \geq \psi && IC_2 \end{aligned} \tag{16}$$

If $\delta \leq \frac{\sigma\psi}{\psi - u}$, $T_G = \sigma\psi + \delta u$ (with $t_2^{ss} = \psi > 0$ and $t_1^s = \sigma\psi + \delta u - \delta\psi \geq 0$) and $T_B = t_1^f = t_2^f = 0$ satisfy all the constraints. T_B cannot be made lower and, since $IC_{G,1}$ is binding, T_G cannot be made lower either.

If $\frac{\sigma\psi}{\psi - u} < \delta \leq \delta^*$, $\sigma\psi + \delta u - \delta\psi < 0$, and so the lender sets $t_1^s = 0$. Thus, $T_G = \delta\psi$ since $t_2^{ss} = \psi$ and $T_B = t_1^f = t_2^f = 0$. $IC_{B,1}$ is satisfied since $\sigma\psi + \delta u \geq \delta\psi - \sigma e$.

If $\delta > \delta^*$, the lender still pays $T_G = \delta\psi$ to the good type. He also pays a positive transfer to the bad type to satisfy $IC_{B,1}$. T_B is then equal to $\delta(\psi - u) - \sigma(\psi + e)$ found from $IC_{B,1}$ satisfied as equality. ■

Plugging transfers (15) into the zero-profit condition $\sigma(\rho r - 1) + \delta\rho(r - 1) \geq \rho T_G + (1 - \rho) T_B$ results in the regions described in the statement of the Proposition (the first two regions are joined to make it more concise). ■

Proof of Proposition 5. Part (i). The first best. Outside lenders offer the rent of $r - 1 - \frac{1-\rho}{\rho}(\psi - u)$ to the good type.[28] The inside lender has to counteract this since otherwise the good type leaves and the first best becomes impossible to finance.

Even though we consider the case of noncontingent transfers, the transfer to the good type in period 2 after two success, $t_{G,2}$, is in fact contingent on the good type staying (and succeeding) with the inside lender.[29] If she leaves the relationship, she then fails (or does not get) the project in period 2 and does not receive $t_{G,2}$. The lender then sets

[28] The transfer to the bad type does not depend on her actions in period 2. Thus, the bad type prefers to take $\psi - u$ from an outside lender. Then, the outside lenders face probability ρ having the good type.

[29] As in the proof of Lemma 2, it can be easily shown that all the transfers after any failure should be set to zero.

$t_{G,1} = 0$ and $t_{G,2} = \frac{T_G}{\delta}$ and the new constraints, in addition (11), are

$$\begin{array}{ll} T_G \geq \delta\left(r - 1 - \frac{1-\rho}{\rho}(\psi - u)\right) & S_G \\ T_G \geq T_B + \delta\left(r - 1 - \frac{1-\rho}{\rho}(\psi - u)\right) & TT_G - S_G \\ T_B + \delta(\psi - u) + \delta u \geq T_B + \delta u & S_B \\ T_B + \delta(\psi - u) + \delta u \geq \delta(\psi - u) + \delta u & TT_B - S_B \end{array}$$

Constraint S_G ensures that the good type does not leave the relationship reporting the truth while $TT_G - S_G$ ensures that she does not leave the relationship lying. Constraint S_B ensures that the bad type leaves the relationship if she reports the truth while $TT_B - S_B$ ensures that she leaves the relationship not lying.[30] Both constraints for the bad type are satisfied for any T_B and $TT_G - S_G$ implies S_G. Then, set $T_B = \psi + e$ to give incentives for the bad type to complete the project in period 1 (see the proof of Lemma 2) and $T_G = \psi + e + \delta\left(r - 1 - \frac{1-\rho}{\rho}(\psi - u)\right)$. Since T_G is higher than it was, we have to check TT_B: $T_B + \delta u \geq t_{G,1} + \delta \max\{t_{G,2} - e, \psi\} = \delta \max\{t_{G,2} - e, \psi\} = T_G - \delta e$. As T_G increases in ρ, check for $\rho = 1$. The constraint becomes

$$\psi + e + \delta u \geq \psi + e + \delta(r - 1 - e)$$

which holds since $e + u > r - 1$ (Assumption 1). It still does not matter how T_B is split between the two periods.

The zero-profit condition of the inside lender is then

$$\begin{aligned} (r-1)(1+\delta\rho) &\geq \rho\left(\psi + e + \delta\left(r - 1 - \frac{1-\rho}{\rho}(\psi - u)\right)\right) + (1-\rho)(\psi + e) \Leftrightarrow \\ r - 1 &\geq \psi + e - \delta(1-\rho)(\psi - u) \end{aligned}$$

This condition is easier to satisfy for a higher δ and a lower ρ. However, it is not satisfied even for $\delta = 1$ and $\rho = 0$ since $r - 1 < e + u$ (Assumption 1). Thus, financing the first best is impossible for $\rho > \rho^{comp}$.

[30] Assume that after the failure of the project in period 1 (which is off the equilibrium path), the outside lenders think that the agent is of the bad type and, therefore, do not deal with her.

Part (ii). The first best when starting small. Let us first find the minimum transfers. In addition to constraints (14), we have the following constraints:

$$\begin{array}{ll} T_G \geq t_{G,1} + \delta\left(r - 1 - \frac{1-\rho}{\rho}(\psi - u)\right) & S_G \\ T_G \geq T_B + \delta\left(r - 1 - \frac{1-\rho}{\rho}(\psi - u)\right) & TT_G - S_G \\ T_B + \delta(\psi - u) + \delta u \geq T_B + \delta u & S_B \\ T_B + \delta(\psi - u) + \delta u \geq \delta(\psi - u) + \delta u & TT_B - S_B \end{array}$$

The inside lender sets $t_{G,1} = 0$. Constraint $TT_G - S_G$ implies then S_G and S_B and $TT_B - S_B$ are always satisfied. Since $r - 1 - \frac{1-\rho}{\rho}(\psi - u) \geq \psi$ for $\rho \geq \rho^{comp}$, $TT_G - S_G$ implies TT_G and $IC_{G,2}$. Thus, we have the following constraints

$$\begin{array}{rl} T_G = T_B + \delta\left(r - 1 - \frac{1-\rho}{\rho}(\psi - u)\right) & TT_G - S_G \\ T_B \geq \sigma(\psi + e) & IC_{B,1} \\ T_B + \delta u \geq \max\{T_G - \delta e, \delta\psi\} & TT_B \end{array}$$

Since T_G is higher than it was, we need to check $TT_B : T_B + \delta u \geq \max\{T_G - \delta e, \delta\psi\}$. If $T_B + \delta\left(r - 1 - \frac{1-\rho}{\rho}(\psi - u)\right) - \delta e \geq \delta\psi$, then $TT_B : T_B + \delta u \geq T_B + \delta\left(r - 1 - \frac{1-\rho}{\rho}(\psi - u)\right) - \delta e$ and it is satisfied since $\delta u \geq \delta(r-1) - \delta e$. If $T_B + \delta\left(r - 1 - \frac{1-\rho}{\rho}(\psi - u)\right) - \delta e \leq \delta\psi$, then $TT_B : T_B + \delta u \geq \delta\psi$ as before. Thus, the minimum transfers are $T_G = T_B + \delta\left(r - 1 - \frac{1-\rho}{\rho}(\psi - u)\right)$ and $T_B = \sigma(\psi + e)$ if $\delta \leq \delta^*$ and $T_B = \delta(\psi - u)$ if $\delta \geq \delta^*$.

Now, plugging these transfers into the zero-profit condition $(r-1)(\sigma + \delta\rho) \geq \rho T_G + (1-\rho) T_B$ yields the regions specified in the proposition.

Part (iii). The second best. Since the outside lenders observe the first-period outcome, only the good type obtains a loan from outside lenders. Since they compete with each other, she gets all the rent, that is, $r - 1$. In the absence of competition, the inside lender pays at least $T_G = \psi + \delta u$ to the good type (see Section 2.5). Pay all this transfer in period 2, that is, after two successes. If the good type switches to an outside lender, she gets $\delta(r-1) < \psi + \delta u$. Thus, the good type does not switch under the original contract and competition from outside lenders does not matter.

Part (iv). The second best when starting small. It can be financed in the following regions:

- If $\sigma\frac{1-\rho(r-\psi)}{\rho(r-1-u)} \leq \delta < \sigma\frac{\psi}{r-1-u}$,
- If $\sigma\frac{\psi}{r-1-u} \leq \delta \leq \delta^*$ and $\rho > \frac{1}{r}$,
- If $\delta^* \leq \delta \leq \sigma\frac{\rho r-1+(1-\rho)(\psi+e)}{(1-\rho)(\psi-u)}$.

Since the outside lenders observe the first-period outcome, only the good type obtains a loan from outside lenders. Since they compete with each other, she gets all the rent, that is, $r-1$. Pay T_G entirely in period 2, that is, after two successes. The agent who succeeded in period 1 but left the relationship cannot succeed in period 2, and thus gets no transfer from the inside lender. Thus, in addition to constraints (16), we have the constraint $S_G : T_G \geq \delta(r-1)$, that is, that the good type does not switch to an outside lender in period 2. Since the bad type is revealed in period 1, she cannot switch to an outside lender.

If $\delta < \sigma\frac{\psi}{r-1-u}$, $T_G = \sigma\psi + \delta u$ and $T_B = 0$ as before since S_G does not bind.

If $\sigma\frac{\psi}{r-1-u} \leq \delta \leq \delta^*$, S_G binds and, therefore, $T_G = \delta(r-1)$. Recall $IC_{B,1}$ constraint:

$$T_B + \sigma\psi + \delta u \geq -\sigma e + \max\{T_G - \delta e, \delta\psi\}$$

Since $\max\{\delta(r-1) - \delta e, \delta\psi\} = \delta\psi$, this constraint does not bind for $\delta \leq \delta^*$.

If $\delta \geq \delta^*$, S_G binds and, therefore, $T_G = \delta(r-1)$. Now, $IC_{B,1}$ also binds and $T_B = \delta(\psi-u) - \sigma(\psi+e)$.

Plugging in these transfers into the zero-profit condition $\sigma(\rho r-1) + \delta\rho(r-1) \geq \rho T_G + (1-\rho)T_B$ yields the regions specified above.

Finally, as compared to the characterization of the second best when starting small in Proposition 8, there is a new constraint $S_G : T_G \geq \delta(r-1)$ which is binding for some parameter values. Thus, the region in which the second best when starting small can be financed under competition with noncontingent contracts shrinks relative to its characterization in Proposition 8. ■

Proof of Proposition 6. Two observations are crucial. First, the bad type never wants to self-finance a project because of Assumption 1. Second, the lender pays everything in the end of period 2 to create higher incentives for the agent to stay in the relationship. Thus, the agent that decides to self-finance diverts ψ if the large project is financed in period 1 and $\sigma\psi$ if it is a small project. In the former case her payoff from self-financing is $\psi + \delta(r-1)$ and in the latter one it is $\sigma\psi + \delta(r-1)$.

Part (i). The first best. Self-financing is possible if and only it $\frac{\psi}{\delta} \geq 1$, that is, $\delta \leq \psi$. If the good type stays with the lender, she gets at least $\psi + e + \delta u$ (Lemma 2) which is higher than $\psi + \delta(r-1)$.

The first best when starting small. Self-financing is possible if and only it $\frac{\sigma\psi}{\delta} \geq 1$, that is, $\delta \leq \sigma\psi$. If the good type stays with the lender, she gets at least $\sigma(\psi + e) + \delta u$ (Lemma 4) which is higher than $\sigma\psi + \delta(r-1)$ for $\delta \leq \sigma\psi$.

Part (ii). The second best. Self-financing is possible if and only it $\frac{\psi}{\delta} \geq 1$, that is, $\delta \leq \psi$. The cost-minimizing transfer to the good type is $\psi + \delta u$ and it is zero to the bad type (see Lemma 5). With self-financing the minimum transfer to the good type becomes $\psi + \delta(r-1)$. Then, the second-best allocation can be financed if

$$\rho r - 1 + \delta\rho(r-1) \geq \rho(\psi + \delta(r-1)),$$

that is, if $\rho \geq \frac{1}{r-\psi}$, or if self-financing is impossible, that is, if $\delta > \psi$.

The second best when starting small. Self-financing is possible if and only it $\frac{\sigma\psi}{\delta} \geq 1$, that is, $\delta \leq \sigma\psi$. For such δ, the cost-minimizing transfer to the good type is $\sigma\psi + \delta u$ and it is zero to the bad type (see Lemma 6). With self-financing the minimum transfer to the good type becomes $\sigma\psi + \delta(r-1)$. Then, the second-best allocation can be financed if

$$\sigma(\rho r - 1) + \delta\rho(r-1) \geq \rho(\sigma\psi + \delta(r-1)),$$

that is, if $\rho \geq \frac{1}{r-\psi}$, or if self-financing is impossible, that is, if $\delta > \sigma\psi$. ■

Proof of Proposition 7. Part (i). The first best cannot be financed since it can be financed even when the agent cannot save (see Proposition 5).

Part (ii). The agent can divert $\sigma\psi$ in period 1 in order to decrease the loan amount she takes from outside lenders in period 2. If the bad type obtains the project in period 2 and diverts the funds, she gets $\delta\psi$. If she takes her outside option, she gets $\sigma\psi + \delta u$. By Assumption 1, the bad type never wants to obtain the project in order to complete it. Thus, if $\delta\psi < \sigma\psi + \delta u$, that is, if $\delta < \sigma\frac{\psi}{\psi - u}$ the bad type does not apply for the loan. Then, any agent applying for the loan is of the good type and, therefore, obtains all the project revenues $\delta(r-1)$. The minimum transfers are then $T_G = T_B + \delta(r-1)$

and $T_B = \sigma(\psi + e)$ if $\delta \leq \delta^*$ (as in the proof of Proposition 5, part (ii), for $\rho = 1$; note that $\sigma\frac{\psi}{\psi-u} < \delta^*$). However, the inside lender cannot break even since he makes a loss in period 1 and no profit in period 2, that is,

$$\sigma(r-1) + \delta\rho(r-1) < \rho(\psi + e + \delta(r-1)) + (1-\rho)(\psi + e).$$

If $\delta\psi \geq \sigma\psi + \delta u$, that is, if $\delta \geq \sigma\frac{\psi}{\psi-u}$ the bad type prefers to apply for the loan from the outside lenders in order to divert it. To keep her out, the outside lenders have to pay her $\psi - u - \frac{\sigma\psi}{\delta}$. They pay the rest to the good type, that is, $r - \left(1 - \frac{\sigma\psi}{\delta}\right) - \frac{1-\rho}{\rho}\left(\psi - u - \frac{\sigma\psi}{\delta}\right)$ as they invested $1 - \frac{\sigma\psi}{\delta}$ of their own money. As this rent has to be higher than ψ, ρ has to be higher than $\rho_{sav}^{comp} \equiv \frac{\psi-u-\frac{\sigma\psi}{\delta}}{r-1-u}$ for competition and savings to have any bite. The overall utility of the good type is then $\sigma\psi + \delta\left(r - 1 - \frac{1-\rho}{\rho}\left(\psi - u - \frac{\sigma\psi}{\delta}\right)\right)$ since she spends $\frac{\sigma\psi}{\delta}$ on co-financing the project in period 2.

Analogously to the proof of Proposition 5, part (ii), the minimum transfers are $T_G = T_B + \delta\left(r - 1 - \frac{1-\rho}{\rho}\left(\psi - u - \frac{\sigma\psi}{\delta}\right)\right)$ and $T_B = \sigma(\psi + e)$ if $\delta \leq \delta^*$ and $T_B = \delta(\psi - u)$ if $\delta \geq \delta^*$. Plugging them into the zero-profit condition $(r-1)(\sigma + \delta\rho) \geq \rho T_G + (1-\rho)T_B$ yields the following condition:

$$\underline{\delta}_{sav}^{comp} \equiv \sigma\frac{\psi(2-\rho) + e - (r-1)}{(1-\rho)(\psi - u)} \leq \delta \leq \sigma\frac{r - 1 - \psi(1-\rho)}{\rho(\psi - u)} \equiv \overline{\delta}_{sav}^{comp}.$$

■

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