

Productivity and the Welfare of Nations

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# Productivity and the Welfare of Nations\*

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## Abstract

We show that the welfare of a country's infinitely-lived representative consumer is summarized, to a first order, by total factor productivity (TFP) and by the capital stock per capita. These variables suffice to calculate welfare changes within a country, as well as welfare differences across countries. The result holds regardless of the type of production technology and the degree of product market competition. It applies to open economies as well, if TFP is constructed using domestic absorption, instead of gross domestic product, as the measure of output. Welfare relevant TFP needs to be constructed with prices and quantities as perceived by consumers, not firms. Thus, factor shares need to be calculated using after-tax wages and rental rates, and will typically sum to less than one. These results are used to calculate welfare gaps and growth rates in a sample of advanced countries with high-quality data on output, hours worked, and capital. We also present evidence for a broader sample that includes both advanced and developing countries.

**JEL:** D24, D90, E20, O47

**Keywords:** Productivity, Welfare, TFP, Solow Residual.

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# 1 Introduction

Standard models in many fields of economics posit the existence of a representative household in either a static or a dynamic setting, and then seek to relate the household's welfare to observable aggregate data. A separate large literature examines the productivity residual defined by Solow (1957), and interprets it as a measure of technical change or policy effectiveness. Yet a third literature, often termed "development accounting," studies productivity differences across countries, and interprets them as measures of technology gaps or institutional quality. To our knowledge, no one has suggested that these three literatures are intimately related. We show that they are. Starting from the standard framework of a representative household that maximizes intertemporal welfare over an infinite horizon, we derive methods for comparing economic well-being over time and across countries. Our results show that under a wide range of assumptions, welfare can be measured using just two variables, productivity and capital accumulation. We take our framework to the data, and measure welfare change within countries and welfare differences across countries.

In the simplest case of a closed economy with no distortionary taxes we show that to a first-order approximation the welfare change of a representative household can be fully characterized by three objects: the expected present discounted value of total factor productivity (TFP) growth as defined by Solow, the change in expectations of the level of TFP, and the growth in the stock of capital per person. The result sounds similar to one that is often proven in the context of a competitive optimal growth model, which might lead one to ask what assumptions on technology and product market competition are required to obtain this result. The answer is, None. The result holds for all types of technology and market behavior, as long as consumers take prices as given and are not constrained in the amount they can buy or sell at those prices. Thus, for example, the same result holds whether the TFP growth is generated by exogenous technological change, as in the Ramsey-Cass-Koopmans model or is the result of spillovers or profit-maximizing investment in R&D as in endogenous growth models. Aggregate TFP can also change without any change in production technology in models with heterogeneous distortions, and our results imply that an increase in aggregate TFP due to reallocation would be as much of a welfare gain for the representative consumer as a change in technology with the same magnitude and persistence.

Our findings suggest a very different interpretation of TFP from the usual one. Usually one argues that TFP growth is interesting because it provides information on the change or diffusion of technology, or measures improvement in institutional quality, the returns to scale in the production function, or the markup of price over marginal cost. We show that whether all or none of these things is true, TFP is interesting for a very different reason. Using only the first-order conditions for optimization of the representative household, we can show that TFP is key to measuring welfare changes within a country and welfare differences across countries. We interpret TFP purely from the household side, producing what one might call "the household-centric Solow residual."<sup>1</sup> Here we follow the intuition of Basu and Fernald (2002), and supply a general proof of their basic insight

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<sup>1</sup>The term is due to Miles Kimball.

that TFP, calculated from the point of view of the consumer, is relevant for welfare.

The intuition for our result comes from noting that TFP growth is output growth minus share-weighted input growth. The representative household receives all output, which *ceteris paribus* increases its welfare. But it also supplies some inputs: labor input, which reduces leisure, and capital input, which involves deferring consumption (and perhaps losing some capital to depreciation). The household measures the cost of the inputs supplied relative to the output gained by real factor prices—the real wage and the real rental rate of capital. TFP also subtracts inputs supplied from output gained, and uses exactly the same prices to construct the input shares. The welfare result holds in a very general setting because relative prices measure the consumer’s marginal rate of substitution even in many situations when they do not measure the economy’s marginal rate of transformation—e.g., if there are externalities, increasing returns or imperfect competition.

This intuition suggests that in cases where prices faced by households differ from those facing firms, it is the former that matter for welfare. We show that this intuition is correct, and here our household-centric Solow residual substantially differs from Solow’s original measure, which uses the prices faced by firms. Proportional taxes are an important source of price wedges in actual economies. We show that the shares in the household-centric Solow residual need to be constructed using the factor prices faced by households. Since marginal income tax rates and rates of value-added taxation can be substantial, especially in rich countries, this modification is quantitatively important, as we show in empirical implementations of our results. We also extend our framework to allow for the provision of public goods and services.

We then move to showing analogous results for open economies. Our previous results need to be modified substantially if we construct TFP using the standard output measure, real GDP. To the three terms discussed above we need to add the present discounted value of expected changes in the terms of trade, the present discounted value of expected changes in the rate of return on foreign assets, and the growth rate of net foreign asset holdings. Intuitively, the first two of these additional terms affect the consumer’s ability to obtain welfare-relevant consumption and investment for a given level of factor supply. Holdings of net foreign assets are analogous to domestic physical capital in that both can be transformed into consumption at a future date.

While these results connect to and extend the existing literature, as we discuss below, they are difficult to take to the data. It is very hard to get good measures of changes in asset holdings by country for a large sample of countries.<sup>2</sup> Furthermore, measuring asset returns in a comparable way across countries would require us to adjust for differences in the risk of country portfolios, which is a formidable undertaking. Fortunately, these difficulties disappear if we use real absorption rather than GDP as the measure of output.<sup>3</sup> In this case, exactly the same three terms that summarize welfare in the closed economy are also sufficient statistics in the open economy. Thus, our approach using the household-centric productivity residual can be applied empirically to measure welfare

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<sup>2</sup>The important work of Lane and Milesi-Ferretti (2001, 2007) has shed much light on this subject, but the measurement errors that are inevitable in constructing national asset stocks lead to very noisy estimates of net asset growth rates.

<sup>3</sup>We are indebted to Mikhail Dmitriev for pointing out this result.

change regardless of the degree of openness of the economy.

Finally, we extend our model to allow for human capital accumulation using the stylized framework of Lucas (1988). This allows us to take into account changes in human capital over time and across countries in the measurement of welfare in a simple yet informative way.

These results pertain to the evolution of welfare in individual economies over time. The indexes we obtain are not comparable across countries. Thus it is natural to ask whether our methods shed any light on a pressing and long-standing question, the measurement of relative welfare across countries using a method firmly grounded in economic theory. It turns out that they do. Perhaps our most striking finding is the result that we can use data on cross-country differences in TFP and capital intensity, long the staples of discussion in the development and growth literatures, to measure differences in welfare across countries.

To understand this result, it helps to deepen the intuition offered above. Our analysis is based on a dynamic application of the envelope theorem, and it shows that the welfare of a representative agent depends to a first order on the expected time paths of the variables that the agent takes as exogenous. In a dynamic growth context, these variables are the prices for factors the household supplies (labor and capital), the prices for the goods it purchases (consumption and investment), and beginning-of-period household assets, which are predetermined state variables and equal to the capital stock in a closed economy. Apart from this last term, the household’s welfare depends on the time paths of *prices*, which are exogenous to the household. Thus, the TFP that is directly relevant for household welfare is actually the *dual* Solow residual, which we transform into the familiar primal Solow residual using the national income identity.

Our cross-country welfare result comes from using the link between welfare and exogenous prices implied by economic theory to ask how much a household’s welfare would differ if it faced the sequence of prices, not of its own country, but of some other country. More precisely, we perform the thought experiment of having a US household optimizing while facing the expected time paths of all goods and factor prices in, say, France, and owning the initial stock of French assets rather than US assets. The difference between the resulting level of welfare and the welfare of remaining in the US measures the gain or loss to a US household of being moved to France. Note that this is a counterfactual thought experiment—the US household in France will typically choose different time paths for consumption, leisure and saving than the French household. Yet we show that the welfare comparisons can be based on the productivity residuals constructed using just the observed data of both countries, without the need to construct any counterfactual quantities.

Importantly, our approach allows performing cross-country welfare comparisons without the need to know individuals’ preference parameters (other than the discount rate) and without having to assume them equal across countries. Note that our welfare comparisons are from a definite point of view—in this example, from the view of a US household. In principle, the result could be different if the USA-France comparison is made by a French household, with different preferences over consumption and leisure. Fortunately, our empirical results are not greatly affected by the choice of the “reference country” used for these welfare comparisons.

Our theoretical results are derived using a first-order approximation, and its accuracy might be a potential concern. To address it, we solve numerically a set of calibrated workhorse models and compare the resulting welfare levels with those obtained from the first-order approximation. Reassuringly, we find that the approximation is highly accurate for assessing welfare differences, both over time and across countries.

We illustrate our methods using data for several industrialized countries for which high-quality data are available: Canada, France, Italy, Japan, Spain, the United Kingdom and the United States. We show the importance of fiscal considerations in constructing measures of welfare change over time. For example, if we assume that government spending is wasteful and taxes are lump-sum, the UK has the largest welfare gain among our group of countries over our sample period, 1985-2005, while Spain lags far behind due to its low TFP growth rate. Indeed, the US, a much richer country, has faster welfare growth than Spain under these assumptions. Allowing for distortionary taxation and assuming that government expenditure is not wasteful, Spain has the highest welfare growth among all countries, with the UK a shade behind, and the US much further back.

However these welfare growth rates are country-specific indexes, and cannot be used to compare welfare across countries. We next apply our methodology to cross country-comparisons for the same seven advanced countries and show how relative welfare levels evolve over time. Although the available data are not as good (for instance, information on hours worked are not available), in addition to the countries listed above we also provide welfare comparisons for a larger set of countries that includes both advanced and developing countries. In our benchmark case of optimal government spending and distortionary taxation, the US is the welfare leader among large countries throughout our sample period (in our larger data set, only Luxembourg enjoys higher welfare). In our smaller sample with high-quality data, with one exception, the US pulls away from other advanced countries in terms of welfare. The exception is the UK, which converges steadily to US levels of welfare over time. In both data sets, the welfare differences among countries are driven to a much greater extent by TFP gaps than by differences in capital intensity. This finding echoes the conclusion of the “development accounting” literature, but for welfare differences rather than GDP gaps.

The paper is structured as follows. The next section presents our analytical framework, and derives the results on the measurement of welfare within single economies and on welfare comparisons across countries. (Full derivations are presented in the appendix.) Section 3 extends the basic framework to allow for multiple goods, distortionary taxes, government expenditure and an open economy, and summarizes our results in their more general form. Section 4 reports simulations of standard growth models to evaluate the accuracy of our approximation. We take our framework to the data in Section 5. Section 6 relates our work to several distinct literatures. Lastly, we summarize our findings and suggest avenues for future research.

## 2 The Productivity Residual and Welfare

Both intuition and formal empirical work link TFP growth to increases in the standard of living, at least as measured by GDP per-capita.<sup>4</sup> The usual justification for studying the Solow productivity residual is that, under perfect competition and constant returns to scale, it measures technological change, which contributes to GDP growth, one major determinant of welfare. Thus, the usual connection between Solow's residual and welfare is a roundabout one. Furthermore, this intuition suggests that we should not care about the Solow residual in an economy with non-competitive output markets, non-constant returns to scale, and possibly other distortions where the Solow residual is no longer a good measure of technological progress. We show that the link between Solow's residual and welfare is immediate and solid, even when the residual does not measure technical change. Here we build on the intuition of Basu and Fernald (2002) and derive rigorously the relationship between a modified version of the productivity residual and the intertemporal utility of the representative household. The fundamental result we obtain is that, to a first-order approximation, utility reflects the present discounted value of productivity residuals (plus the initial stock of capital).

Our results are complementary to those in Solow's classic (1957) paper. Solow established that, if there was an aggregate production function with constant returns to scale and all markets were competitive, then his index measured its rate of change. We now show that under a very different set of assumptions, which are disjoint from Solow's, the familiar TFP index is also the key component of an intertemporal welfare measure. The results are parallel to one another. Solow did not need to assume anything about the consumer side of the economy to give a technical interpretation to his index, but he had to make assumptions about technology and firm behavior. We do not need to assume anything about the firm side (which includes technology and market structure) in order to give a welfare interpretation, but we do need to assume the existence of a representative consumer.<sup>5</sup> Which result is more useful depends on the application, and the trade-off that one is willing to make between having a result that is very general on the consumer side but requires very precise assumptions on technology and firm behavior, and a result that is just the opposite.

### 2.1 Measuring welfare changes over time

We begin by assuming the familiar objective function for a representative household that maximizes intertemporal utility. In a growth context one often neglects the dependence of welfare on leisure, but the work of Nordhaus and Tobin (1972) suggests that this omission is not innocuous (see the discussion in Section 6). Thus, we assume the household derives utility from both consumption and leisure:

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<sup>4</sup>For a review of the literature linking TFP to GDP per worker, in both levels and growth rates, see Weil (2008).

<sup>5</sup>At a technical level, both results assume the existence of a potential function (Hulten, 1973), and show that TFP is the rate of change of that function.

$$W_t = E_t \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} \frac{1}{1-\sigma} C_{t+s}^{1-\sigma} \nu(\bar{L} - L_{t+s}) \quad (1)$$

where  $W_t$  denotes the total welfare of the household,  $C_t$  is the per-capita consumption at time  $t$ ,  $L_t$  are per-capita hours of work and  $\bar{L}$  is the per-capita time endowment.  $N_t$  is population and  $H$  is the number of households, assumed to be fixed and normalized to one from now on. Population grows at a constant rate  $n$  and aggregate per capita variables at rate  $g$ . To ensure the existence of a well-defined steady-state in which hours of work are constant while consumption and the real wage share a common trend, we assume that the utility function has the King, Plosser and Rebelo (1988) form with  $\sigma > 0$  and  $\nu(\cdot) > 0$ .<sup>6</sup> The budget constraint facing the representative consumer and the capital accumulation equation are respectively:

$$P_t^I K_t N_t + B_t N_t = (1-\delta) P_t^I K_{t-1} N_{t-1} + (1+i_t^B) B_{t-1} N_{t-1} + P_t^L L_t N_t + P_t^K K_{t-1} N_{t-1} + \Pi_t N_t - P_t^C C_t N_t \quad (2)$$

and

$$K_t N_t = (1-\delta) K_{t-1} N_{t-1} + I_t N_t \quad (3)$$

$K_t$ ,  $B_t$  and  $I_t$  denote per-capita capital, bonds and investment;  $P_t^K$ ,  $P_t^L$ ,  $P_t^C$  and  $P_t^I$  denote, respectively, the user cost of capital, the hourly wage, the price of consumption goods and of new capital goods;  $(1+i_t^B)$  is the nominal interest rate and  $\Pi_t$  denotes per-capita profits, which are paid lump-sum from firms to consumers. Assume for now that the economy is closed and there is no government, which implies that in equilibrium  $B_t = 0$ . We derive analogous results for the open economy with capital mobility and unbalanced trade ( $B_t \neq 0$ ) in Section 3.4, and extend the results in this section to allow for government expenditure, distortionary or lump-sum taxes, and government bond issuance in Sections 3.1-3.2.

Define ‘‘equivalent consumption’’ per person, denoted by  $C_t^*$ , as the level of consumption per-capita at time  $t$  that, if growing at the steady-state rate  $g$  from  $t$  onward, with leisure set at its steady-state level, delivers the same per-capita intertemporal utility as the actual stream of consumption and leisure. More precisely,  $C_t^*$  satisfies:

$$\begin{aligned} \frac{W_t}{N_t} &= V_t = \sum_{s=0}^{\infty} \frac{(1+n)^s (C_t^* (1+g)^s)^{1-\sigma}}{(1+\rho)^s (1-\sigma)} \nu(\bar{L} - L) \\ &= \frac{1}{(1-\sigma)(1-\beta)} C_t^{*1-\sigma} \nu(\bar{L} - L) \end{aligned} \quad (4)$$

where  $V_t$  denotes per-capita utility and  $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$  is the discount rate in the problem reformulated in terms of stationary variables to allow for steady-state growth. We will measure welfare changes over time in terms of equivalent consumption per-capita and relate them to observable

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<sup>6</sup>If  $\sigma = 1$ , then the utility function must be  $U(C, \bar{L} - L) = \log(C) + \nu(\bar{L} - L)$ . See King, Plosser and Rebelo (1988).



economic variables.

First we define a few of the key variables used in our analysis. Consider a modified definition of the Solow productivity residual:

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} \quad (5)$$

where  $\Delta \log Y_t = s_C \Delta \log C_t + s_I \Delta \log I_t$ .  $\Delta \log Y_t$  is a Divisia index of per-capita GDP growth, where demand components are aggregated using constant steady-state shares.  $s_C$  and  $s_I$  denote the steady-state values of  $s_{C,t} = \frac{P_t^C C_t}{P_t^Y Y_t}$  and  $s_{I,t} = \frac{P_t^I I_t}{P_t^Y Y_t}$  respectively, and  $P_t^Y Y_t$  represents per-capita GDP in current prices.<sup>7</sup> Distributional shares are also defined as the steady-state values,  $s_L$  and  $s_K$ , of  $s_{L,t} \equiv \frac{P_t^L L_t}{P_t^Y Y_t}$  and  $s_{K,t} \equiv \frac{P_t^K K_{t-1} N_{t-1}}{P_t^Y Y_t N_t}$  (note that the household receives remuneration on the capital stock held at the end of the last period). We use the word “modified” in describing the productivity residual for three reasons. First, we do not assume that the distributional shares of capital and labor add to one, as they would if there were zero economic profits and no distortionary taxes.<sup>8</sup> Second, all shares are calculated at their steady-state values and, hence, are not time varying, which is sometimes assumed when calculating the residual.<sup>9</sup> Third, the residual is stated in terms of per-capita rather than aggregate variables, although it should be noted that Solow himself defined the residual on a per-capita basis (1957, equation 2a). Correspondingly, define the log level productivity residual as:

$$\log PR_{t+s} = s_C \log C_{t+s} + s_I \log I_{t+s} - s_L \log L_{t+s} - s_K \log K_{t+s-1} \quad (6)$$

The prices in the budget constraint, equation (2), are defined in nominal terms. It will often be easier to work with relative prices, and disregard complications that arise from price inflation. Taking the purchase price of new capital goods,  $P_t^I$ , as numeraire, define the following relative prices:  $p_t^K = \frac{P_t^K}{P_t^I}$ ,  $p_t^L = \frac{P_t^L}{P_t^I}$  and  $p_t^C = \frac{P_t^C}{P_t^I}$ . Real per-capita profits are defined as  $\pi_t = \frac{\Pi_t}{P_t^I}$ . Our approximations are taken around a steady-state path where the first three relative prices are constant and the wage  $p^L$  grows at rate  $g$ , as in standard one-sector models of economic growth. We also assume that all per-capita quantity variables other than labor hours (for example  $Y_t$ ,  $C_t$ ,  $I_t$ , etc.) grow at a common rate  $g$  in the steady-state. Note these assumptions imply that all of the shares we have defined above are constant in the steady-state and so is the capital output ratio, whose nominal steady-state value will be denoted by  $\frac{P_t^I K}{P_t^Y Y}$ .<sup>10</sup>

<sup>7</sup>For now we set government expenditure to zero, and introduce it in our extension in Section 3.2. Our definition of GDP departs slightly from convention, as value added is usually calculated using time-varying shares. The two definitions coincide to a first-order approximation.

<sup>8</sup>Zero profits are guaranteed in the benchmark case with perfect competition and constant returns to scale, but can also arise with imperfect competition and increasing returns to scale—as long as there is free entry—as in the standard Chamberlinian model of imperfect competition.

<sup>9</sup>Rotemberg and Woodford (1991) argue that in a consistent first-order log-linearization of the production function the shares of capital and labor should be taken to be constant, and Solow’s (1957) use of time-varying shares amounts to keeping some second-order terms while ignoring others.

<sup>10</sup>We conjecture that all our results could be proved in the household environment corresponding to a two-sector growth model as laid out, for example, in Whelan (2003)—assuming that the steady-state shares are also constant,

Under these assumptions we can show that welfare changes, as measured by equivalent consumption,  $C_t^*$ , are, to a first-order approximation, a linear function of the expectation of present and future total factor productivity growth (and its revision), and of the initial capital stock. This first key result is summarized in:

**Proposition 1** *Assume that the representative household in a closed economy with no government maximizes (1) subject to (2), taking prices, profits and interest rates as exogenously given. Assume also that population grows at a constant rate  $n$ , and the wage and all per-capita quantities other than hours worked grow at rate  $g$  in the steady-state. To a first-order approximation, the growth rate of equivalent consumption can be written as:*

$$\Delta \log C_t^* = \frac{(1 - \beta)}{s_c} \left[ E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s-1} + \frac{1}{\beta} \left( \frac{P^I K}{P^Y Y} \right) \Delta \log K_{t-1} \right] \quad (7)$$

**Proof.** Proofs of all propositions and extensions are collected in the Appendix. ■

Proposition 1 implies that the expected present discounted value of current and future Solow productivity residuals, together with the change in the initial stock of capital per-capita, is a sufficient statistic for the welfare of a representative consumer (where we measure welfare as the log change in equivalent consumption). The term  $\Delta E_t \log PR_{t+s} = E_t \log PR_{t+s} - E_{t-1} \log PR_{t+s}$  represents the revision in expectations of the log level of the productivity residual, based on the new information received between  $t - 1$  and  $t$ . Note that the expectation revision terms in the second summation will reduce to a linear combination of the innovations in the stochastic shocks affecting the economy at time  $t$ . Moreover, if we assume that the modified log level productivity residual follows a univariate autoregressive process, then only the innovation of such a process matters for the expectation revision, and the first summation is simply a function of current and past values of productivity.

Since we have not made any assumptions about production technology and market structure, the productivity terms may or may not measure technical change. For example, Solow's residual does not measure technical change in economies where firms have market power, or produce with increasing returns to scale, or where there are Marshallian externalities. Even in these cases, Proposition 1 shows that productivity and the capital stock are jointly a sufficient statistic for welfare. Finally, as we show in Section 3, this basic result can be proved in much more general environments—for example, in an open economy, with government expenditure and debt, distortionary taxes, multiple consumption goods, and human capital.

While the proof of the proposition requires somewhat complex notation and algebra, in the remainder of this sub-section we shall try to convey the economic reasoning for the result by considering the much simpler case of an economy with a zero steady-state growth rate ( $g = 0$ ). (Of course, the formal proof of Proposition 1 allows for  $g > 0$ .) We begin by taking a first-order approximation to the level of utility of the household (normalized by population) around the steady

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as in Whelan's setup.

state.<sup>11</sup> We then use the household's first-order conditions for optimality to obtain:

$$\frac{(V_t - V)}{\lambda p^Y Y} = E_t \sum_{s=0}^{\infty} \beta^s \left[ s_L \widehat{p}_{t+s}^L + s_K \widehat{p}_{t+s}^K + s_\pi \widehat{\pi}_t - s_C \widehat{p}_{t+s}^C \right] + \frac{1}{\beta} \left( \frac{P^I K}{P^Y Y} \right) \widehat{K}_{t-1} \quad (8)$$

Hatted variables denote log deviations from the steady-state ( $\widehat{x}_t = \log x_t - \log x$ ). Variables without time subscripts denote steady-state values. Since  $g = 0$ ,  $\beta = \frac{1+n}{1+\rho}$ .  $\lambda$  is the Lagrange multiplier associated with the budget constraint expressed, like utility, in per-capita terms. Equation (8) follows almost directly from the Envelope Theorem. An atomistic household maximizes taking as given the sequences of current and expected future prices, lump-sum transfers ( $\Pi_t$ ), and predetermined variables (in our environment, just  $K_{t-1}$ ). Thus, only fluctuations in these objects affect welfare to a first order. The Envelope Theorem plus a bit of algebra shows that each change in exogenous prices or profits needs to be multiplied by its corresponding share to derive its effect on welfare—for example, the larger is  $s_C$  the more the consumer suffers from a rise in the relative price of consumption goods. (It may appear that the investment price is missing, but since we normalized the relative price of investment goods to 1 it never changes.) The terms within the summation can be thought of as the dual version of the productivity residual, as we will show shortly.

Two comments are in order. First, the left hand side of the equation has an interesting interpretation: it is the money value of the deviation of per-person utility from its steady-state level, expressed as a fraction of steady-state GDP per person. To understand this interpretation, consider the units. The numerator is in “utils,” which we divide by  $\lambda$ , which has units of utils/investment good (since investment goods are our numeraire). The division gives us the deviation of utility from its steady-state value measured in units of investment goods in the numerator. We then scale the result by the real value of per-capita GDP, also stated in terms of investment goods (recall that  $p^Y$  is a relative price). Second, note that we can express welfare change without knowing the parameters of the utility function, other than the discount factor,  $\beta$ . The right-hand side only contains expectations of the Solow residual (and its revision) and the initial capital stock per capita. Essentially, the parameters of the utility function are embedded in observed choices for consumption, labor and capital, in the expenditure shares, in the distributional shares and in the capital output ratios. Our basic idea is to use a revealed-preference approach, akin to the logic behind the economic theory of index numbers. This approach uses observed choices to infer welfare parameters (as, for example, expenditure shares reveal the relative importance of different prices to the household).

However, we find it more convenient and intuitive to express the left hand side of (8) in terms of equivalent consumption. Using the definition in (4) and taking a first order approximation of  $V_t - V$  in terms of  $\log C_t^*$  we obtain:

$$\frac{(V_t - V)}{\lambda p^Y Y} = \frac{s_C}{(1 - \beta)} (\log C_t^* - \log C) \quad (9)$$

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<sup>11</sup>We approximate the level of  $V$  rather than its log because  $V < 0$  if  $\sigma > 1$ .

where we have used the fact that in the steady state  $C^* = C$  and  $U_C = \lambda p^c$ .

The right hand side of (8) is written as a function of the log deviation from the steady-state of prices, profits and the initial capital stock. Our results can also be presented using the familiar (primal) productivity residual rather than stating them in terms of prices and transfers, as in equation (8). However, if one uses a consistent data set, there is literally no difference between the two. One can show, using the per-capita version of the household budget constraint (2) and the capital accumulation equation (3), that the following relationship must hold at all points in time:

$$s_L \widehat{p}_{t+s}^L + s_K \widehat{p}_{t+s}^K + s_\pi \widehat{\pi}_t - s_C \widehat{p}_{t+s}^C = s_C \widehat{C}_{t+s} + s_I \widehat{I}_{t+s} - s_L \widehat{L}_{t+s} - s_K \widehat{K}_{t+s-1}. \quad (10)$$

Since the budget constraint of the representative household is just the national income accounts identity in per-capita terms, equation (10) says that in any data set where national income accounting conventions are enforced, the primal productivity residual identically equals the dual productivity residual.<sup>12</sup> Thus we can express our results in either form, but using the dual result would require us to provide an empirical measure of lump-sum transfers, which is not needed for results based on the primal residual. Mostly for this reason, we work with the primal.

Using (9) and (10) in (8), we can write:

$$(\log C_t^* - \log C) = \frac{(1 - \beta)}{s_C} \left[ E_t \sum_{s=0}^{\infty} \beta^s \widehat{P} R_{t+s} + \frac{1}{\beta} \left( \frac{P^I K}{P^Y Y} \right) \widehat{K}_{t-1} \right] \quad (11)$$

where now the log deviation of consumption is expressed as a function of the log deviation from steady-state of the productivity residual, defined in equation (6). Taking differences of equation (11) and using the definition of the Solow residual in (5) gives the key equation of Proposition 1, whose proof we have just sketched for the case of  $g = 0$ . When using either equivalent consumption or the money value of utility as a metric for welfare comparisons one does not require knowledge of the parameters of the utility function (except the discount factor  $\beta$ ). The parameters of the utility function are embedded in observed choices for consumption, labor and capital, in the expenditure shares, in the distributional shares and in the capital-output ratio.

## 2.2 Implications for Cross Country Analysis

Proposition 1 pertains to the evolution of welfare in individual economies over time. The indexes we obtain are not comparable across countries. However, in this sub-section we show that similar methods can be used to do a rigorous welfare comparison across countries. More precisely, we show that productivity and the capital stock suffice to calculate differences in welfare across countries, with both variables computed as log level deviations from a reference country.

Welfare comparisons across countries have been investigated recently by Jones and Klenow (2010), who focus on a point-in-time comparison of single-period flow utility. By comparison, we focus on intertemporal (lifetime) utility, and show how out-of-steady-state dynamics are related to

<sup>12</sup>See, for example, Barro and Sala-i-Martin (2004, section 10.2).

capital accumulation and productivity.<sup>13</sup>

A comparison of welfare across countries requires either assuming that their respective representative agents possess the same utility function, or making the comparison from the perspective of the representative agent in a reference country. We favor the second option, and consider the thought experiment of a household from a reference country  $j$  facing the prices, per-capita profits and initial capital stock of country  $i$  instead of those in country  $j$ . We then study the difference in the utility of a representative member of the household and, as in the within-country case, we conduct the comparison by using the concept of equivalent consumption. In this context for the representative agent of the reference country  $j$  living in country  $i$ , equivalent per-capita consumption,  $\tilde{C}_t^{*,i}$  satisfies:

$$\tilde{V}_t^i = \frac{1}{(1 - \sigma^j)(1 - \beta^j)} \left( \tilde{C}_t^{*,i} \right)^{1 - \sigma^j} \nu(\bar{L} - L^j) \quad (12)$$

where  $\tilde{V}_t^i$  denotes per-capita utility of the individual from country  $j$ , facing country  $i$ 's relative prices, per-capita profits and per-capita initial capital stock (we use  $\sim$  to denote these counterfactual quantities). Note that  $\tilde{C}_t^{*,i}$  is defined for a constant level of leisure fixed at country  $j$ 's steady-state level. We will use  $V_t^j$  and  $C_t^{*,j}$  to denote per-capita utility and equivalent consumption of the individual of country  $j$  living in country  $j$ . We take first-order approximations of  $\tilde{V}_t^i$ ,  $V_t^j$ , and the budget constraints around the steady state of country  $j$ .

**Proposition 2** *Assume that in a reference country, country  $j$ , the representative household maximizes (1) subject to (2), under the assumptions of Proposition 1. Assume now that the household of country  $j$  is confronted with the sequence of prices, per-capita profits and initial capital stock of country  $i$ . In a closed economy with no government, to a first order approximation, the difference in equivalent consumption between living in a generic country  $i$  versus country  $j$  can be written as:*

$$\log \tilde{C}_t^{*,i} - \log C_t^{*,j} = \frac{(1 - \beta^j)}{s_C^j} \left[ E_t \sum_{s=0}^{\infty} (\beta^j)^s \left( \log \overline{PR}_{t+s}^i - \log PR_{t+s}^j \right) + \frac{1}{\beta^j} \left( \frac{P^{L,j} K^j}{P^{Y,j} Y^j} \right) \left( \log K_{t-1}^i - \log K_{t-1}^j \right) \right] \quad (13)$$

where the productivity terms are constructed using country  $j$ 's shares in the following fashion:

$$\log \overline{PR}_{t+s}^i = \left( s_C^j \log C_{t+s}^i + s_I^j \log I_{t+s}^i \right) - s_L^j \log L_{t+s}^i - s_K^j \log K_{t+s-1}^i \quad (14)$$

and:

$$\log PR_{t+s}^j = \left( s_C^j \log C_{t+s}^j + s_I^j \log I_{t+s}^j \right) - s_L^j \log L_{t+s}^j - s_K^j \log K_{t+s-1}^j \quad (15)$$

**Proof.** See the Appendix. ■

Welfare differences across countries are therefore summarized by two components. The first component is related to the well-known log difference between TFP levels, which accounts em-

<sup>13</sup>We do not, however, allow for cross country differences in life expectancy or in inequality as in Jones and Klenow (2010).

pirically for most of the difference in per-capita income across countries (Hall and Jones (1999)), although it is the present value of the gap that matters for welfare. In the development accounting literature, this gap is interpreted as a measure of technological or institutional differences between countries. This interpretation, however, is valid under restrictive assumptions on market structure and technology (perfect competition, constant returns to scale, no externalities, etc.). We provide a welfare interpretation of cross-country differences in TFP that applies even when these assumptions do not hold. The second component of the welfare difference reflects the difference in capital intensity between the two countries; *ceteris paribus*, a country with more capital per person can afford more consumption or higher leisure. The development accounting literature also uses capital intensity as the second variable explaining cross-country differences in per-capita income.

Our result holds for any kind of technology and market structure, as long as a representative consumer exists, takes prices as given and is not constrained in the amount he can buy and sell at those prices. Notice however, that our measure of per-capita TFP is modified with respect to the traditional growth accounting measure in two ways. First, measuring welfare differences requires comparing not only current log differences in TFP but the present discounted value of future ones as well. Second, the distributional and expenditure shares used to compute the log differences in TFP between countries need to be calculated at their steady-state values in the reference country.<sup>14</sup>

As in the case of Proposition 1, we shall try to convey the economic reasoning for the result by considering the simple case of an economy with a zero steady-state growth rate ( $g = 0$ ). Assume we confront the household from country  $j$  with the prices, profits and the initial per-capita capital stock of country  $i$ . If we expand the utility of a representative member of the household, denoted by  $\tilde{V}_t^i$ , around the steady state of his own country, we obtain:

$$\begin{aligned} \frac{(\tilde{V}_t^i - V^j)}{\lambda^j p^{Y,j} Y^j} &= E_t \sum_{s=0}^{\infty} (\beta^j)^s [s_L^j (\log p_{t+s}^{L,i} - \log p_{t+s}^{L,j}) + s_K^j (\log p_{t+s}^{K,i} - \log p_{t+s}^{K,j})] \\ &\quad + s_{\Pi}^j (\log \pi_{t+s}^i - \log \pi^j) - s_C^j (\log p_{t+s}^{C,i} - \log p_{t+s}^{C,j}) \\ &\quad + \frac{1}{\beta^j} \left( \frac{P^{L,j} K^j}{P^{Y,j} Y^j} \right) (\log K_{t-1}^i - \log K^j) \end{aligned} \quad (16)$$

Now expand per-capita utility for country  $j$ 's household around its own steady-state and subtract from (16). This yields:

$$\begin{aligned} \frac{(\tilde{V}_t^i - V_t^j)}{\lambda^j p^{Y,j} Y^j} &= E_t \sum_{s=0}^{\infty} (\beta^j)^s [s_L^j (\log p_{t+s}^{L,i} - \log p_{t+s}^{L,j}) + s_K^j (\log p_{t+s}^{K,i} - \log p_{t+s}^{K,j})] \\ &\quad + s_{\Pi}^j (\log \pi_{t+s}^i - \log \pi_{t+s}^j) - s_C^j (\log p_{t+s}^{C,i} - \log p_{t+s}^{C,j}) \\ &\quad + \frac{1}{\beta^j} \left( \frac{P^{L,j} K^j}{P^{Y,j} Y^j} \right) (\log K_{t-1}^i - \log K_{t-1}^j) \end{aligned} \quad (17)$$

Differences in welfare across countries are, therefore, due to differences in their relative prices,

<sup>14</sup>It is standard in the development accounting literature to assume that all countries have the same capital and labor shares in income (often one-third and two-thirds), but to use country-specific shares in expenditure.

per-capita profits and capital intensities.

Use (12) to express differences in utility across countries in term of log differences in equivalent consumption on the left hand side of (17). Now linearize two budget constraints around country  $j$ 's steady state: first, the budget constraint for the household from country  $j$  if moved to country  $i$  and, second, its budget constraint when living in its own country. Subtracting one from the other allows us to write the right hand side of (17) in terms of productivity differences and differences in the initial capital stock. This yields equations (13), (14), and (15) in Proposition 2.<sup>15</sup>

Notice that in stating Proposition 2, we have not needed to assume that either the population growth rate  $n$  or the per-capita growth rate  $g$  is common across countries.<sup>16</sup> Most importantly, we do not need to know the parameters of the utility function (other than  $\beta$ ) to perform cross country welfare comparisons, nor do we need to assume that any of those parameters (including  $\beta$ ) is common across countries. This is because we are making the comparisons in terms of differences in equivalent consumption and from the point of view of the representative individual in the reference country, who is faced with the exogenous (to the household) prices, lump-sum transfers and initial conditions of country  $i$ .

Notice that this thought experiment does not simply assign to the household from the reference country the consumption and leisure choices made by the household from country  $i$ . Rather, our approach allows the reference-country household to re-optimize when facing the conditions of country  $i$ . Even if faced with the same exogenous variables, the choices of the two households will generally differ, unless their preferences are identical. However, to a first order approximation, the algebraic sum of the terms in prices, profits, and initial conditions for the household from country  $j$  facing country  $i$ 's prices, profits, and endowments equals the algebraic sum of the terms in consumption, labor supply and capital chosen by the individual from country  $i$ . We exploit this fact to eliminate the need to construct counterfactual quantities in calculating the welfare change for a household of country  $j$  moving to country  $i$ .<sup>17</sup>

### 3 Extensions

We now show that our method of using TFP to measure welfare can be extended to allow for the presence of taxes and government expenditure, multiple types of consumption and investment goods, and an open economy setting. We also extend the problem of the household to allow for human capital accumulation. These extensions require modifications in the formulas given above for welfare comparisons over time and across countries, and we state the changes to the basic framework that are needed in each case; detailed derivations are given in the appendix. These

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<sup>15</sup>We can show that, in the special case in which all countries are in the steady state, share a common growth rate, and consumers do not derive utility from leisure, cross country welfare comparisons reduce to comparing Net National Product. This result is in the spirit of Weitzman (1976, 2003). We thank Chad Jones for this observation.

<sup>16</sup>Both introspection and the results of Kremer, Onatski and Stock (2001) suggest that it is implausible to assume that countries will diverge perpetually in per-capita terms. Thus, even though we do not need to assume a common  $g$ , we would not view it as a restrictive assumption.

<sup>17</sup>See the equations leading up to (A.28) in the Appendix.

results prove that the basic idea of using TFP to measure welfare holds in a variety of economic environments, but they also demonstrate the advantage of deriving the welfare measure from an explicit dynamic model of the household.

In what follows, we discuss the generalization of our measure of welfare changes over time. Analogous results apply to the measurement of cross-country welfare differences.

### 3.1 Taxes

Consider first an environment with distortionary and/or lump-sum taxes. Since the prices in the budget constraint (2) are those faced by the consumer, in the presence of taxes all prices should be interpreted as after-tax prices. At the same time, the variable that we have been calling “profits,”  $\Pi_t$ , can be viewed as comprising any transfer of income that the consumer takes as exogenous. Thus, it can be interpreted to include lump-sum taxes or rebates. Finally one should think of  $B_t$  as including both government and private bonds (assumed to be perfect substitutes, purely for ease of notation).

More precisely, in order to modify (2) to allow for taxes, let  $\tau_t^K$  be the tax rate on capital income,  $\tau_t^R$  be the tax rate on revenues from bonds,  $\tau_t^L$  be the tax rate on labor income,  $\tau_t^C$  be the *ad valorem* tax on consumption goods, and  $\tau_t^I$  be the corresponding tax on investment goods.<sup>18</sup> Also, let  $P_t^{C'}$  and  $P_t^{I'}$  respectively denote the pre-tax prices of consumption and capital goods, so that the tax-inclusive prices faced by the consumer are  $P_t^{C'}(1 + \tau_t^C)$  and  $P_t^{I'}(1 + \tau_t^I)$ . We assume for the time being that the revenue so raised is distributed back to individuals using lump-sum transfers; we consider government expenditures in the next sub-section. The representative household’s budget constraint now is:

$$P_t^{I'}(1 + \tau_t^I)K_tN_t + B_tN_t = (1 - \delta)P_t^{I'}(1 + \tau_t^I)K_{t-1}N_{t-1} + (1 + i_t^B(1 - \tau_t^R))B_{t-1}N_{t-1} \\ + P_t^L(1 - \tau_t^L)L_tN_t + P_t^K(1 - \tau_t^K)K_{t-1}N_{t-1} + \Pi_tN_t - P_t^{C'}(1 + \tau_t^C)C_tN_t \quad (18)$$

Thus, differently than in the benchmark case, the exogenous variables in the household’s maximization are not only the prices and the initial stocks of capital and bonds, but also the tax rates on labor and capital income, consumption and investment. However, it can be easily shown that the basic results (7) and (13) continue to hold. The only modification is that in defining the Solow productivity residual we need to take account of the fact that the national accounts measure factor payments as perceived by firms – that is, before income taxes – while nominal expenditure is measured using prices as perceived from the demand side, thus inclusive of indirect taxes (subsidies) on consumption and investment. Hence, letting  $P_t^C = P_t^{C'}(1 + \tau_t^C)$  and  $P_t^I = P_t^{I'}(1 + \tau_t^I)$  denote the tax-inclusive prices of consumption and investment goods, the expenditure shares  $s_C$  and  $s_I$  defined earlier are fully consistent with those obtained from national accounts data, but the factor shares

<sup>18</sup>For simplicity, we are assuming no capital gains taxes and no expensing for depreciation. These could obviously be added at the cost of extra notation.



$s_L$  and  $s_K$  defined above refer to the gross income of labor and capital rather than their respective after-tax income. Thus, to be consistent with the data, in the presence of taxes the welfare residual needs to be redefined in terms of the shares of after-tax returns on labor and capital. Specifically, equation (5) can be re-written as:

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - (1 - \tau^L) s_L \Delta \log L_{t+s} - (1 - \tau^K) s_K \Delta \log K_{t+s-1} \quad (19)$$

and an analogous modification applies to (14) and (15).  $\tau^L$  and  $\tau^K$  are the steady-state values of  $\tau_t^L$  and  $\tau_t^K$ . With these modifications, our results generalize to a setting with distortionary time-varying taxes on consumption and investment goods and on the household's income coming from labor, capital or financial assets.

### 3.2 Government Expenditure

With some minor modification, our framework can be likewise extended to allow for the provision of public goods and services (see the Appendix for details). We illustrate this under the assumption that government activity is financed with lump-sum taxes. Using the results from the previous subsection, it is straightforward to extend the argument to the case of distortionary taxes.

Assume that government spending takes the form of public consumption valued by consumers. We rewrite instantaneous utility as:

$$U(C_{t+s}, C_{G,t+s}, L_{t+s}) = \frac{1}{1 - \sigma} \Omega(C_{t+s}; C_{G,t+s})^{1 - \sigma} \nu(\bar{L} - L_{t+s}) \quad (20)$$

where  $C_G$  denotes per-capita public consumption and  $\Omega(\cdot)$  is homogenous of degree one in its arguments. Total GDP now includes public consumption: that is,  $P_t^Y Y_t = P_t^C C_t + P_t^G C_{Gt} + P_t^I I_t$ , where  $P^G$  is the public consumption deflator.

In this setting, our earlier results need to be modified to take account of the fact that public consumption may not be set by the government at the level that consumers would choose. Intuitively, in such circumstances the value that consumers attach to public consumption may not coincide with its observed value as included in GDP, and therefore in the productivity residual as conventionally defined.

Formally, let  $s_{c_{Gt}} = \frac{P_t^G C_{Gt}}{P_t^Y Y_t}$  denote the GDP share of public consumption, and let  $s_{c_{Gt}}^*$  denote the share that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household, rather than using its deflator  $P_t^G$ .<sup>19</sup> The welfare-relevant modified Solow residual (5) now is:

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} + (s_{c_G}^* - s_{c_G}) \Delta \log C_{G,t+s} \quad (21)$$

and an analogous modification applies to (14) and (15). Hence in the presence of public consumption the Solow residual needs to be adjusted up or down depending on whether public consumption

<sup>19</sup>It is easy to verify that  $s_{c_{Gt}}^* = \frac{U_{C_{Gt}} P_t^C C_{Gt}}{U_{C_t} P_t^Y Y_t}$ .

is under- or over-provided (i.e.,  $s_{c_G}^* > s_{c_G}$  or  $s_{c_G}^* < s_{c_G}$  respectively). If the government sets public consumption exactly at the level the utility-maximizing household would have chosen if confronted with a price of  $P_t^G$ , then  $s_{c_G}^* = s_{c_G}$  and no correction is necessary. In turn, in the standard neo-classical case in which public consumption is pure waste  $s_{c_G}^* = 0$ , the welfare residual should be computed on the basis of private final demand – i.e., GDP minus government purchases. With the residual redefined in this way, the growth rate of equivalent consumption now is:<sup>20</sup>

$$\Delta \log (C_t)^* = \frac{(1 - \beta)}{(s_C + s_{c_G}^*)} \left[ E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s-1} + \frac{1}{\beta} \left( \frac{P^I K}{P^Y Y} \right) \Delta \log K_{t-1} \right] \quad (22)$$

### 3.3 Multiple Types of Consumption and Investment Goods

The extension to the case of multiple types of consumption and investment goods is immediate. The instantaneous utility function can be written as:

$$U(C_{1,t+s}, \dots, C_{Z_C,t+s}, L_{t+s}) = \frac{1}{1 - \sigma} C(C_{1,t+s}, \dots, C_{Z_C,t+s})^{1 - \sigma} \nu [\bar{L} - L_{t+s}] \quad (23)$$

where  $C(\cdot)$  is a homogenous functions of degree one and  $Z$  is the number of consumption goods. Denote by  $P_t^{C_z}$  the price of a unit of  $C_{z,t}$ . Similarly, assume that consumers can purchase  $Z_I$  different types of investment goods  $I_{z,t}$  at prices  $P_t^{I_z}$ , and combine them into capital according to a constant-returns aggregate investment index. Thus, investment (in per-capita terms) can be expressed  $I_t = I(I_{1,t}, \dots, I_{Z_I,t})$ , and the trajectory of the capital stock is still described by equation (3). Further, we can define (exact) deflators for consumption and investment  $P_{t+s}^C$  and  $P_{t+s}^I$ , each of which is a linear homogenous function of the prices of the underlying individual types of goods, such that  $P_{t+s}^C C_{t+s} = \sum_{z=1}^{Z_C} P_{t+s}^{C_z} C_{z,t+s}$  and  $P_{t+s}^I I_{t+s} = \sum_{z=1}^{Z_I} P_{t+s}^{I_z} I_{z,t+s}$ .<sup>21</sup> Insert these two definitions in equation (2) to obtain the new budget constraint.

In this framework, our earlier results continue to hold without modification: the applicable expression for the modified Solow residual still is (5), and welfare changes over time and differences across countries continue to be characterized by (7) and (13) respectively. The only new feature is that the GDP shares of consumption and investment can also be expressed as the sums of the shares of their respective disaggregated components evaluated at the steady state; e.g.,  $s_{C_t} =$

<sup>20</sup>Government purchases might also yield productive services to private agents. For example, the government could provide education or health services, or public infrastructure, which – aside from being directly valued by consumers – may raise private-sector productivity. In such case, the results in the text remain valid, but it is important to note that the contribution of public expenditure to welfare would not be fully captured by the last term in the modified Solow residual as written in the text. To this term we would need to add a measure of the productivity of public services, which is implicitly included in the other terms in the expression.

<sup>21</sup>The existence of these perfect price indices under the assumptions made in the text was established by the classic literature on two-stage consumption budgeting; see Lloyd (1977). The extension to investment is discussed by Servén (1995).

$\frac{P_t^C C_t}{P_t^Y Y_t} = \sum_{h=1}^{Z_C} s_{C_{z,t}}$ , where  $s_{C_{z,t}} = \frac{P_t^{C_z} C_{z,t}}{P_t^Y Y_t}$ . Moreover,  $\Delta \log C_t$ , and  $\Delta \log I_t$  are Divisia indexes (with fixed weights) of individual consumption and investment goods.

### 3.4 Open Economy

Our results also apply to an open economy if we just replace GDP with domestic absorption in the definition of the Solow residual, and thus rewrite (5) as:

$$\Delta \log PR_{t+s} = \Delta \log A_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} \quad (24)$$

where  $\Delta \log A_t = s_C \Delta \log C_t + s_I \Delta \log I_t$  is a Divisia index of domestic absorption growth (in real per-capita terms), and  $s_L$ ,  $s_K$ ,  $s_C$  and  $s_I$  are shares out of domestic absorption.  $\Delta \log C_t$  and  $\Delta \log I_t$  are, in turn, Divisia indices of domestically produced and imported goods aggregated with fixed weights, as discussed in the previous subsection. In addition, the steady-state capital-output ratio in (7) and ensuing expressions in the preceding section should also be replaced with the steady-state capital-absorption ratio. It is true that now the initial stock of net foreign assets, and the return on those assets, would appear – along with the initial capital stock, profits, and prices – as a determinant of utility in equation (8). However they would also appear in the budget constraint (10) and would cancel out in the primal version of the productivity residual, provided the latter is defined in terms of absorption (see the Appendix for details). As a consequence, all the results we have stated in terms of the primal productivity residual continue to hold.<sup>22</sup>

Alternatively, one may want to use a standard measure of output, real GDP, defined as consumption, plus investment, plus net exports. Then the welfare-relevant residual can be written as the sum of a conventionally-defined productivity residual plus additional components that capture terms of trade and capital gains effects. Moreover, the initial conditions then should include the initial value of the net foreign asset stock.

To show this, assume that the domestic economy buys imports  $IM_{t+s}N_{t+s}$  at a price  $P_{t+s}^{IM}$  and sells domestic goods abroad  $EX_{t+s}N_{t+s}$  at a price  $P_{t+s}^X$ . The current account balance can be written:

$$B_t N_t - B_{t-1} N_{t-1} = i_t B_{t-1} N_{t-1} + P_t^{EX} EX_t N_t - P_t^{IM} IM_t N_t \quad (25)$$

where  $B_t$  is taken to denote the per-capita foreign asset stock. The applicable Divisia index of per-capita GDP growth now is  $\Delta \log Y_t = s_C \Delta \log C_t + s_I \Delta \log I_t + s_X \Delta \log EX_t - s_M \Delta \log IM_t$ , where  $s_C$ ,  $s_I$ ,  $s_X$  and  $s_M$  are respectively the steady-state shares of consumption, investment, exports and imports out of total value added.

Using these definitions, welfare changes can be related to a productivity residual corrected for

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<sup>22</sup>We could also allow for bond financing of government expenditure. The existence of domestic government bonds, in addition to foreign assets, does not change our results when they are expressed in terms of the primal version of the productivity residual.

terms of trade changes and changes in the rate of return on foreign assets:

$$\begin{aligned} \Delta \log PRTT_{t+s} &= \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} \\ &\quad + s_X \Delta \log p_{t+s}^{EX} - s_M \Delta \log p_{t+s}^{IM} + \left( \frac{Br}{PY\bar{Y}} \right) \Delta \log r_{t+s} \end{aligned} \quad (26)$$

where  $s_L$  and  $s_K$  are also shares out of total value added,  $r$  is the real rate of return on foreign assets, and  $\left( \frac{Br}{PY\bar{Y}} \right)$  is the steady-state ratio of foreign asset income to GDP. Changes in welfare can be summarized by an expression similar to (7), but based on (26) rather than the conventional Solow residual:

$$\begin{aligned} \Delta \log C_t^* &= \frac{(1-\beta)}{s_c} \left[ E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PRTT_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PRTT_{t+s-1} \right. \\ &\quad \left. + \frac{1}{\beta} \left( \frac{P^I K}{PY\bar{Y}} \right) \Delta \log K_{t-1} + \frac{1}{\beta} \left( \frac{B}{PY\bar{Y}} \right) \Delta \log \frac{B_{t-1}}{P_{t-1}^I} \right] \end{aligned} \quad (27)$$

Conceptually, it makes very good sense that all these extra terms come into play when taking the GDP route to the measurement of welfare in the open economy. The effects of an improvement in the terms of trade, as captured by  $s_X \Delta \log p_t^{EX} - s_M \Delta \log p_t^{IM}$  in (26), are analogous to those of an increase in TFP - both give the consumer higher consumption for the same input of capital and labor (and therefore higher welfare); see Kohli (2004) for a static version of this result. In turn, the term in  $\Delta \log r_{t+s}$  captures present and expected future changes in the rate of return on foreign assets, including capital gains and losses on net foreign assets due either to exchange rate movements or to changes in the foreign currency prices of the assets. Finally, the initial conditions now include not only the (domestic) capital stock, but also the net stock of foreign assets.

Measuring these extra terms empirically poses major challenges, however. One needs reliable measures of changes in foreign asset holdings for a large sample of countries. Asset returns would have to be measured in risk-adjusted terms to make them comparable across countries. In addition, forecasts of future asset returns and the terms of trade would be required as well. In contrast, all these problems disappear if the measurement of welfare is based on real absorption rather than GDP as the measure of output, in which case the same terms that summarize welfare in the closed economy suffice to measure it in the open economy. The implication is that we can measure welfare empirically in ways that are invariant to the degree of openness of the economy.

### 3.5 Summing up

We can now go back to the two propositions stated earlier. They were formulated for the special case of a closed economy with no government. In light of the discussion in this section, we can now restate them in a generalized form for an open economy with multiple goods and a government sector, which is more appropriate for empirical implementation.

**Proposition 1'** *Assume an open economy in which the government engages in public consumption, and levies taxes on labor and capital income at rates  $\tau_{t+s}^L$  and  $\tau_{t+s}^K$ , as well as taxes on consumption and investment expenditure at rates  $\tau_{t+s}^C$  and  $\tau_{t+s}^I$ . Assume also that the representative household maximizes intertemporal utility, taking prices, profits, interest rates, tax rates and public consumption as exogenously given. Lastly, assume that population grows at a constant rate  $n$ , and the wage and all per-capita quantities other than hours worked grow at rate  $g$  in the steady state. To a first-order approximation, the growth rate of equivalent consumption can be written as:*

$$\Delta \log (C_t)^* = \frac{(1 - \beta)}{(s_C + s_{C_G}^*)} \left[ E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s-1} + \frac{1}{\beta} \left( \frac{P^I K}{P^A A} \right) \Delta \log K_{t-1} \right] \quad (28)$$

where productivity growth is defined as:

$$\Delta \log PR_{t+s} = s_C \Delta \log C_{t+s} + s_I \Delta \log I_{t+s} + s_{C_G}^* \Delta \log C_{G,t+s} - (1 - \tau^L) s_L \Delta \log L_{t+s} - (1 - \tau^K) s_K \Delta \log K_{t+s-1} \quad (29)$$

All shares are defined relative to total absorption,  $A_t$ .  $\log C_t$  and  $\log I_t$  are share weighted aggregates (with fixed weights) of individual types of domestically produced and imported consumption and investment goods. Shares and tax rates are evaluated at their steady-state values, and  $s_{C_G}^*$  denotes the steady-state share of public consumption in total absorption that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household.

**Proof.** See the Appendix. ■

By stating (28), as well as all the expenditure and income shares entering the productivity residual, in terms of absorption, the proposition applies to both open and closed economies. Further, as explained earlier, the value of  $s_{C_G}^*$  depends on the assumptions made about government consumption: if the latter is set at the level the representative household would have chosen if she were facing the price  $P^G$ , then  $s_{C_G}^* = s_{C_G} = \frac{P^G C_G}{P^C C + P^I I + P^G C_G}$ . Alternatively, if government consumption is pure waste (i.e., if it does not enter the consumption aggregate  $C$  in (20)),  $s_{C_G}^* = 0$ . This implies that the proposition can encompass a variety of cases with respect to taxation and government spending: 1) wasteful government spending with lump sum taxes (in which case distortionary taxes are set to zero in the productivity equation); 2) optimal government spending with lump sum taxes; 3) wasteful government spending with distortionary taxes; 4) optimal government spending with distortionary taxes.

Our main result regarding welfare differences across countries can be restated in a similar way:

**Proposition 2'** *Assume that in a reference country  $j$ , the representative household maximizes intertemporal utility under the assumptions of Proposition 1'. Assume now that the household of country  $j$ , is confronted with the sequence of prices, tax rates, per-capita profits, other lump sum transfers, public consumption, and endowment of country  $i$ . In an open economy with distortionary taxation and government spending, the difference in equivalent consumption between living in coun-*

try  $i$  versus country  $j$  can be written, to a first order approximation, as:

$$\ln \tilde{C}_t^{*,i} - \ln C_t^{*,j} = \frac{(1 - \beta^j)}{(s_C^j + s_{C_G}^{*j})} \left[ E_t \sum_{s=0}^{\infty} (\beta^j)^s \left( \log \overline{PR}_{t+s}^i - \log PR_{t+s}^j \right) + \frac{1}{\beta^j} \left( \frac{P^{I,j} K^j}{P^{A,j} A^j} \right) \left( \log K_{t-1}^i - \log K_{t-1}^j \right) \right] \quad (30)$$

where  $A^j$  denotes absorption and  $s_{C_G}^{*j}$  the steady-state share of public consumption in total absorption that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household. The two productivity terms are constructed with all shares (in terms of absorption) and tax rates evaluated at the reference country's steady-state values:

$$\log PR_t^j = s_C^j \log C_t^j + s_I^j \log I_t^j + s_{C_G}^{*j} \log C_{G,t}^j - (1 - \tau^{L,j}) s_L^j \log L_t^j - (1 - \tau^{K,j}) s_K^j \log K_{t-1}^j \quad (31)$$

$$\log \overline{PR}_t^i = s_C^j \log C_t^i + s_I^j \log I_t^i + s_{C_G}^{*j} \log C_{G,t}^i - (1 - \tau^{L,j}) s_L^j \log L_t^i - (1 - \tau^{K,j}) s_K^j \log K_{t-1}^i \quad (32)$$

where  $\log C$  and  $\log I$  are share weighted aggregates (with the fixed weights of the reference country) of individual types of domestically produced and imported consumption and investment goods.

**Proof.** The proof follows from the proof of Proposition 2, together with the cross country analogues of the extensions contained in Proposition 1'. ■

Proposition 2' shows that our method for comparing welfare across countries applies in a much more general setting than the one we used previously. We can compare two economies with any degree of openness to trade or capital flows, and with differing levels of distortionary taxation or government expenditure. The derivation shows a result that would be hard to intuit *ex ante*, which is that to a first-order approximation only the tax rates of the reference country enter the welfare comparison.<sup>23</sup> This asymmetry implies that welfare rankings may depend on the choice of reference country. In our empirical application in Section 5.3 below we take the US as our reference country, but check the robustness of the results by using France instead.<sup>24</sup>

### 3.6 An extension with human capital

We will use the results summarized in Propositions 1' and 2' to evaluate welfare over time within each country and across countries. In doing so it may be useful, particularly in the cross country comparisons, to account for differences in human capital. It goes beyond the scope of this paper to provide an exhaustive analysis of the implications of human capital accumulation for welfare measurement. However, we will develop in this subsection an extension of our model in the spirit of Lucas (1988). As in Lucas, we will assume that non-leisure time can be used either to work or to accumulate human capital and that the accumulation of human capital is linear in the stock of

<sup>23</sup>Of course, the tax rates of the comparison country will generally change output and input levels in that country through general-equilibrium effects, which will influence the welfare gap between the two countries. However, the tax rates of the comparison country do not enter the formula directly.

<sup>24</sup>We conjecture that the asymmetry may be eliminated by moving to second-order approximations, where instead of using the tax-adjusted shares of the reference country only, one might take an arithmetic average of the shares of the reference and comparison countries. We are investigating this possibility in current research.

human capital.<sup>25</sup> More specifically assume that the per-period utility function of the representative individual is now  $U(C_t; \bar{L} - L_t - E_t)$ , where  $E_t$  denotes the amount of time devoted to human capital accumulation. We assume that the labor income per person can be written as  $P_t^L H_{t-1} L_t$ , where  $L_t$  continues to denote hours worked,  $H_{t-1}$  the initial level of human capital, and  $P_t^L$  the hourly price of one unit of human capital. The human capital accumulation equation is assumed to be:

$$(H_t - H_{t-1}) + \delta_H H_{t-1} = F(E_t) H_{t-1} \quad (33)$$

where  $F'(E_t) > 0$  and  $F(0) = 0$ . It is possible to show that equivalent consumption now also depends upon the change in the initial level of human capital. Moreover, labor input must be adjusted for human capital growth in the definition of productivity growth. Thus, equations (28) and (29) now become:

$$\begin{aligned} \Delta \log(C_t)^* &= \frac{(1-\beta)}{(s_C + s_{C_G}^*)} \left[ E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s-1} \right. \\ &\quad \left. + \frac{1}{\beta} \left( \frac{P^I K}{P^A A} \right) \Delta \log K_{t-1} + \left( \frac{1}{1-\beta} \right) (1-\tau^L) s_L \Delta \log H_{t-1} \right] \end{aligned} \quad (34)$$

and

$$\begin{aligned} \Delta \log PR_{t+s} &= s_C \Delta \log C_{t+s} + s_I \Delta \log I_{t+s} + s_{C_G}^* \Delta \log C_{G,t+s} \\ &\quad - (1-\tau^L) s_L (\Delta \log L_{t+s} + \Delta \log H_{t+s-1}) - (1-\tau^K) s_K \Delta \log K_{t+s-1} \end{aligned} \quad (35)$$

Summarizing, in the presence of human capital, the definition of productivity must account for the effect on total labor input of both hours and human capital changes. Note that human capital investment does not show up as part of GDP because in the Lucas formulation it is only a subtraction from leisure and does not require any other physical input. Moreover, human capital must now be included among the initial conditions, alongside physical capital. In the empirical section, we will present results accounting for human capital in the measurement of welfare differences across countries and over time.

### 3.7 Further Discussion: Non-Price-Taking Households

So far we have assumed that the household is a price-taker in goods and factor markets, and that it faces no constraints other than the intertemporal budget constraint. We have exploited the insight

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<sup>25</sup>See the Appendix for details. Lucas showed that this formulation of the capital accumulation equation generates a positive steady-state growth rate, even if there is no exogenous technological progress. If the production function of goods is constant returns in physical and human capital, then all the relevant quantities grow at the same rate. These results extend to the case in which the leisure choice is endogenized, although in this case there are some parameters configurations for which multiple steady-state balanced growth paths exist (Ladron de Guevara, Ortigueira and Santos (1999)).

that under these conditions relative prices measure the representative consumer’s marginal rate of substitution between goods, even when relative prices do not measure the economy’s marginal rate of transformation. We now discuss whether our conclusions need to be modified in environments where the household does not behave as a price taker or faces quantity constraints.

In our working paper, Basu, Pascali, Schiantarelli and Serven (2012), we present two examples where households are not price takers; we refer the reader to that paper for details. Our examples focus on the labor market, since the price-taking assumption seems most questionable in that setting. Indeed several literatures (on labor search, union wage setting, and efficiency wages, to name three) begin by assuming that households are not price takers in the labor market. Our first example is in the spirit of the dual labor markets literature, where wages are above their market-clearing level in some sectors but not in other. We do not model why wages are higher in the primary sector, but this can be due to the presence of unions or government mandates in formal but not in informal employment, or efficiency wage considerations in some sectors but not in others. Wages in the secondary market are set competitively. The second example is a model of the labor market, where firms pay different wages but workers do not know *ex ante* which firms pay high wages. Knowing the distribution of wages, household need to decide how many workers should participate in the labor market and how many should stay home to enjoy leisure. Households would prefer to supply all their labor to the sector or firm that pays the highest wage, but are unable to do so since all decisions must be made *ex-ante*. In this sense, both examples feature a type of labor market rationing. (In both cases the different wages are paid to identical workers, and are not due to differences in human capital characteristics.)

We show that when marginal and average wages differ, the change in work hours need to be weighted by the marginal wage. In the dual labor markets example, this would be the wage in the secondary sector. Or, in models where workers have market power (as in standard New Keynesian models), one needs to divide the observed wage by the wage markup to create a competitive “shadow wage” that accurately reflects the disutility of work. Even in these cases, though the details of our implementation change, our conclusion that welfare can be summarized by a forward-looking TFP measure and capital intensity remain robust. However, the weight on labor input may differ from the observed labor share.

The second example has households face a whole distribution of wages without knowing which wage they will receive after finding a job. Thus, workers equate the disutility of work to the *ex ante* expected wage, which is also the *ex post* average wage. Even though the law of one price does not hold in this environment—identical workers are paid different wages—we show that the observed labor share is exactly the right weight to apply to changes in work hours. Thus, even in this richer environment, our basic welfare measures summarized in Section 3.5 are accurate and need no modification.

From these examples, it is clear that dropping the assumption that all consumers face the same price for each good or service can—but need not—change the precise nature of the proxies we develop for welfare. While the exact nature of the proxy will necessarily be model-dependent, we



believe that our basic insight applies under fairly general conditions.

## 4 The quality of our approximation: Some examples

A potential concern with our main results, as stated in Proposition 1' and 2', is that they are proved using first-order approximations. This approach may seem especially problematic for cross-country comparisons, where gaps in living standards are often large. We now use simple general-equilibrium models to investigate the quantitative error introduced by our use of approximations. We consider a set of workhorse models that are standard in the macroeconomic literature, solve them, and then compare the calculated welfare values to our approximated measures.<sup>26</sup> The details of these models and their calibration, which are both standard, can be found in the Appendix.

First, we discuss the quality of the approximation in a within-country analysis. Figure (1) reports the impulse response function of our measure of approximated welfare and compares it to welfare measures based on third-order approximations, in four standard macro models subject to different types of shocks. Panel (a) reports the impulse response of both equivalent consumption and our approximated measure of it following a one-standard deviation technology shock in a standard Ramsey growth model. The two lines are practically indistinguishable. On impact, equivalent consumption increases by 19.35% while its approximated value increases by 19.32%.<sup>27</sup> In the following periods, the approximated value converges monotonically to the exact one. The non-linearity of the utility function does not have a large effect on the quality of the approximation. In panel (b), we perform the same experiment as in panel (a) but using an extremely concave utility function: we raise the coefficient of relative risk aversion from a common business-cycle value of 1.1 to 10. Following the technology shock, on impact equivalent consumption increases by 19.19% while its approximated value again increases by 19.32%.<sup>28</sup> As we would expect, the approximation error is larger when the utility function is more concave, but the magnitude of the difference is still quite small and converges to zero quickly.

To ensure that the quality of the approximation is not a peculiarity of the Ramsey model, in the following panels, we perform the same exercise in different theoretical frameworks. Panel (c) considers a technological shock in a Real Business Cycle (RBC) model with standard calibration. Panel (d) considers a tax shock in a RBC model with distortionary income taxes and wasteful public

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<sup>26</sup>We compare welfare across countries assuming that they are at their steady states. These calculations are exact solutions of the non-linear models. For the comparisons over time in within-country, dynamic settings, we solve the models using third-order approximations and compare our results based on first-order approximations to the third-order solutions. To check the third-order approximations, we also solved the models using fourth-order approximations and solved the simpler models using global methods. In both cases the results were barely distinguishable numerically from the third-order solutions, so we think these are a good baseline for the purposes of checking the first-order approximations.

<sup>27</sup>Here and in the rest of the paper, we use percent (%) change to refer to differences in natural logs multiplied by 100.

<sup>28</sup>It is not a coincidence that the approximated first-period welfare change is the same in the two models. Since both models are neoclassical, the time path of TFP is just the exogenous shocks to technology, which is the same in the two cases. Moreover, since we shock both models starting at their steady states, the period  $t - 1$  change in the capital stock is also the same in the two models—zero.

expenditure. Panel (e) considers a public expenditure shock in a RBC model with lump-sum taxes, wasteful public expenditure and production externalities. Note that in these last two cases, our welfare-relevant TFP differs from exogenous technology—in the first case due to taxes, and in the second case because of the externality. In all three cases, however, the first-order approximation gives results that are close to the calculated value for welfare.

We next evaluate the approximation in a cross-country setting and compare steady-state welfare differences between countries. Comparing steady states allows us to solve for the exact values of welfare in the two countries, and compare the gap to our approximated result. In our theoretical results, which we take to the data later in the paper, we allow for both steady-state and transitory welfare gaps between countries. To evaluate the approximation error for this type of comparison, one can take the approximation error for transitory shocks which we just discussed and add it to the approximation error for the steady-state differences.

We use a standard RBC model with distortionary income taxes to analyze welfare gaps between countries. Both countries are assumed to be in their respective steady state. We compute the change in equivalent consumption of a representative agent in a reference country who moves permanently to a different country characterized by different exogenous technology parameters or tax rates. We then compute the approximated change in equivalent consumption and see how it compares to the exact value. We conduct three different experiments, with the results in the three panels of Figure 2. First, in panel (a), we consider an increase in the capital elasticity parameter from 0.28 to 0.39 (thus moving from British to Canadian capital shares, the two extremes in our sample): it produces a steady-state increase in equivalent consumption of 73.77%, while the result from our approximation is 72.78%. Second, in panel (b), we consider an increase in the income tax rate from 30% to 40% (thus moving from the average US tax rate over 1985-2005 to the French average over the same period of time): it produces a reduction in equivalent consumption of 16.58 percent while the approximated change is 13.85 percent. Finally, in panel (c), we consider differences in technology. The figure illustrates the exact and the approximated change in equivalent consumption when the level of technology drop to a fraction  $x$  of its original level. Moving to a country with a level of productivity that is 50% (10%) that of the reference country implies a reduction in equivalent consumption of 69.31 (230.26%), while the approximated value is 65.90% (218.90%). It is interesting to note that the approximation error is largest for differences in tax rates. However, all the changes we consider are large ones. Relative to the large size of the welfare gaps we are considering, we believe the approximation errors are modest and quite acceptable.

## 5 Empirical results

### 5.1 Data and Measurement

We illustrate the potential of our methodology by computing welfare indexes over the period 1985-2005 for a set of large, developed countries for which high quality time series data are available: US, UK, Japan, Canada, France, Italy and Spain. We use two different data sets to compare

welfare within a country and across countries. Given the interest in welfare comparisons for a more heterogeneous group of countries, we use a third data set for cross country comparisons for 63 developed and less developed countries. However, we do not use this sample for our baseline results because consistent data on hours of work (as opposed to employment) are not available for most countries, particularly outside the OECD.

To analyze welfare changes over time for our sample of advanced countries, we combine data coming from the OECD Statistical Database with the EU-KLEMS dataset.<sup>29</sup> Our index of absorption is constructed from the OECD dataset as the weighted growth of household final consumption, gross capital formation and government consumption (where appropriate) at constant national prices, using as weights their respective nominal shares of absorption. Since our theory requires steady-state shares, we use the averages of the observed shares across the twenty years in our sample.

The growth rate of our modified productivity residual is constructed as the log-change in real absorption minus the log changes in capital and labor, each weighted by its income share out of absorption. Data on aggregate production inputs are provided by EU-KLEMS. The capital stock is constructed by applying the perpetual inventory method to investment data. Labor input is the total amount of hours worked by persons engaged. To obtain per-capita quantities, we divide absorption, capital, and labor by total population. We assume that economic profits are zero in the steady-state so that we can recover the gross (tax unadjusted) share of capital as one minus the labor share.

In order to compare welfare across countries, we combine data from the Penn World Tables with hours data from EU-KLEMS dataset. Specifically, our basic measure of real absorption is constructed from the Penn World Tables as the weighted average of PPP-converted log private consumption, log gross investment and log government consumption, using as weights their respective shares of absorption in the reference country; as in the within case, we use shares that are averaged across the twenty years in our sample.

To construct the modified log productivity residual for each country, we subtract share weighted log capital and labor from log real absorption. The shares are the compensation of each input out of absorption in the reference country, also in this case kept constant at their average values. The stock of capital in the economy is constructed using the perpetual-inventory method on the PPP-converted investment time series from the Penn World Tables. Labour input is total hours worked, from EU-KLEMS.

For the comparative welfare calculations in the 63 country sample, absorption, capital, and the factor shares are constructed exactly as before. Since consistent data on hours of work are not available for most countries, we use, as an imperfect proxy for total labor input, aggregate employment from the ILO's Key Indicators database.<sup>30</sup> In this broader sample, we included all

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<sup>29</sup>The EU-KLEMS data are extensively documented by O'Mahony and Timmer (2009). We are unable to include Germany in the sample, since official data for unified Germany are available only since 1995 in EU-KLEMS.

<sup>30</sup>The ILO provides two different measures of total employment. The first gathered at the national level by statistical services and ministries. Data coverage starts in 1980 but differences in the definition of employment make

countries for which sufficient data were available to reconstruct TFP for at least 20 years (1985-2005).

For the empirical exercises including human capital, we construct per-capita human capital stocks as in Caselli (2005, p. 685-686), using the Barro and Lee (2010) data on average years of schooling of the population over 25 years of age. The source reports data at 5-year intervals; we use log-linear interpolation to obtain annual data.

Finally, to take into account of distortionary taxation, we use data on average tax rates on capital and labor provided by Boscá et al. (2005). The tax rates are computed by combining realized tax revenues, from the OECD Revenue Statistics, with estimates of the associated tax bases derived from the OECD National Accounts. These data update the tax rates constructed by Mendoza et al (1994) and introduce some methodological improvements in their calculation, most of which are described in Carey and Tchilinguirian (2000). In essence, they involve some adjustments to the definition of the various tax bases.

## 5.2 Within Results

We construct country specific indexes of welfare change over time for our seven benchmark countries. Since the change in welfare over time depends on the expected present discounted value of TFP growth and its revision, as shown by equation (28), we need to construct forecasts of future TFP. In order to keep our empirical illustration simple and uniform across countries, we estimate univariate time-series models using annual data. The extension to a multivariate forecasting framework is something worth exploring in future work. Our sample period runs from 1985 to 2005 for all countries except Canada, where the EU-KLEMS data end in 2004.

We use the various aggregate TFP measures suggested by our theory (in log levels), and estimate simple AR processes for each country. The persistence and trend of TFP growth are key statistics, since they determine the present discounted value of TFP. For illustration, we report in Table 1 the estimated forecasting equations for two of the four different definitions of TFP that we use throughout the paper.

The first concept is TFP in the case where we assume that government purchases are wasteful, and taxes are lump-sum. For this case, as discussed above, we construct output by aggregating consumption and investment only, but using shares that sum to  $(1 - s_{cG})$ , and we do not correct the capital and labor shares for the effects of distortionary taxes. In this case, the capital and labor shares sum to one. The second case is the one where we assume government spending is optimally chosen, but needs to be financed with distortionary taxes. In this case, the output concept is the share-weighted sum (in logs) of consumption, investment and government purchases, and the capital and labor shares are corrected for both income taxes and indirect taxes. After-tax shares

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this measure not comparable across countries. The second one, instead, provides a standardized measure across countries but is only available only starting from 1991. In constructing the modified productivity residual we use both measures. We use the first one when constructing the TFP measure used to estimate country-specific time-series model (gaps for some countries are filled with log-linear interpolations). We use the second one when calculating welfare differences across countries for 2005.

are now smaller and no longer sum to one. Note that in both cases the output concept measures absorption rather than GDP (unless the economy is closed, in which case the two concepts coincide). Thus, following our discussion in Section 3.4, all the TFP measures that we use in our analysis are appropriate for measuring welfare in open economies. In all cases, we assume that pure economic profits are zero in the steady state.

For all countries, the log level of TFP is well described by either an AR(1) or AR(2) stationary process around a linear trend. In Table 1 we report the estimation results obtained using the two definitions of TFP stated above, together with the Lagrange Multiplier test for residual first order serial correlation (shown in the last line of each panel in the table), confirming that we cannot reject the null of no serial correlation for the preferred specification for each country. For all countries, the order of the estimated AR process is invariant to the TFP measure used. In all cases, we can comfortably reject the null of a unit root in the log TFP process (after allowing for a time trend). We use the estimated AR processes to form expectations of future levels or differences of TFP, which are required to construct our welfare indexes.

We use equation (28) to express the average welfare change per year in each country in terms of changes in equivalent consumption. Given the time-series processes for TFP in each country, we can readily construct the first two terms in equation (28), the present value of expected TFP growth and the change in expectations of that quantity. The third term, which depends on the change in the capital stock can also be constructed using data from EU-KLEMS. We assume that the composite discount rate,  $\beta$ , is common across countries and we set it equal to 0.95.<sup>31</sup>

The results are in Table 2. We see that assumptions about fiscal policy affect the results in significant ways. We first illustrate our methods by discussing the results for the US, which are given in the last row. We then broaden our discussion to draw more general lessons from the full set of advanced countries.

In the first column of Table 2, we construct the output data and the capital and labor shares under the assumption that government expenditure is wasteful and taxes are lump-sum. In this column, "utility-relevant output" comprises just consumption and investment, aggregated using weights that sum to less than one. In this case, the average annual growth rate of welfare in the US is equivalent to a permanent annual increase in consumption of about 2.5 percent.

Now we study the case of optimal government spending, still under the assumption that taxes are not distortionary. Thus, at the margin the consumer is indifferent between an additional unit of private consumption and an additional unit of government expenditures. In this case, output consists of consumption, investment and government purchases, aggregated using nominal expenditure shares that sum to one. In a closed economy this concept of TFP corresponds to the standard Solow residual.

Welfare growth for the US is only slightly higher when we assume that expenditures are optimal: 2.6 versus 2.5 percent for the lump-sum tax cases. (We will see that this result is not universal within our sample of countries.) On the whole, the differing assumptions about the value of

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<sup>31</sup>We construct our measure of  $\beta$  following the method of Cooley and Prescott (1995), who find  $\beta = 0.947$ .

government expenditure do not change the calculated US growth rate of welfare significantly. Note, however, that this result does *not* mean that the US consumer is indifferent between wasteful and optimal government spending. The level of welfare is surely much lower in the case where the government wastes 20 percent of GDP. However, our results show that the difference in welfare in the two cases is almost entirely a *level* difference rather than a *growth rate* difference.

We repeat our welfare calculations under the assumption that the government raises revenue via distortionary taxes. The results are in columns 3 and 4 of Table 2. As shown above, if taxes are distortionary we need to construct the factor shares in the Solow residual using the after-tax wage and capital rental rate perceived by the household. We construct the new shares using the tax rates described in the previous section. The quantitative effect of this change is significant. In both of the cases we consider (wasteful spending and optimal spending), per-capita welfare growth expressed in terms of consumption growth rates is *higher* by nearly half a percentage point per year. Intuitively, if taxes are distortionary then steady-state output is too low; thus, any increase in output, even with unchanged technology, is a welfare improvement. It is quantitatively important to allow for the fact that taxes are distortionary and not lump-sum. For the US, it matters more for the growth rate of welfare than whether we assume that government spending is wasteful or optimal. We take as our benchmark the case shown in the last column, where spending is optimally chosen (from the point of view of the household) and taxes are distortionary. In this case, average US welfare growth is equivalent to a growth rate of per-capita consumption of 3 percent per year.

Assumptions about fiscal policy naturally matter more in countries with a high rate of growth of government purchases per-capita and with a high growth rate of factor inputs. For example, both facts are true for Spain over our sample period. The growth rate of welfare in Spain nearly doubles from the first column, where its 2.1 percent annual welfare growth rate is literally middling, to the last, where its 4 percent growth rate is the highest among all the countries in our sample. Assumptions about fiscal policy also matter significantly for Canada and Japan, and change the welfare growth rates of these countries by a full percentage point or more. In percentage terms, the change is particularly dramatic for Canada. Under three of the four scenarios, the UK leads our sample of countries in welfare growth rates; in the last case, it is basically tied with Spain.

Finally, we show the full time series of the welfare indexes for each country graphically, for our two benchmark cases of wasteful spending with lump-sum taxes and optimal spending with distortionary taxes. In Figures 3 and 4 we report the evolution over time of our welfare indexes for each country, in log deviations from their values in 1985. In Figure 3, the UK is the clear growth leader, with France and the US nearly tied in a second group, and Canada trailing badly. In Figure 4, by contrast, there are three clear groups: the UK and Spain lead, by a considerable margin; the US, France, and Japan comprise the middle group; Italy and Canada have the lowest welfare growth rates. Two countries show significant declines in growth rates, both starting in the early 1990s. The first is Japan, which in the first few years of our sample grew in line with the leading economies, Spain and the UK, and then slowly drifted down in growth rate to end the sample in the middle group, with France and the US. Similarly, Italy used to grow at the pace

of the middle group, but then experienced a slowdown which, by the end of the sample, caused it to leave the middle group and form a low-growth group with Canada. Thus, our results are consistent with the general impression that Italy and Japan experienced considerable declines in economic performance over the last two decades relative to the performance in the earlier postwar period.

In Table 3, we investigate which of the two components of welfare–TFP growth or capital accumulation–contributed more to the growth rate of welfare in our sample of countries. For the purpose of this decomposition, we treat the expectation-revision term as a contribution to TFP. In order to keep the table uncluttered, we drop the case where government spending is optimal and taxes are lump sum. The first column, in which government spending is wasteful and taxes are treated as lump-sum, shows that four of the seven countries have achieved two-thirds or more of their welfare gains mostly via TFP growth. The exceptions are the three countries that are known to have had low TFP growth over our sample period: Japan, Canada, and Spain. Moving to the case of distortionary taxes raises the TFP contribution (by reducing the factor shares), as does changing the treatment of public spending as optimal rather than wasteful (which raises the growth rate of output, and thus TFP). In the case of optimal spending with distortionary taxes, all countries get a majority of their welfare growth from TFP. Only in Japan and Canada is the contribution of TFP to welfare less than 70 percent, and in most cases it is 75 percent or more.

We next check the robustness of the previous results to the inclusion of human capital. As discussed in section 3.7, this implies that labor input growth must be adjusted for the growth of human capital in calculating productivity growth. In addition, equivalent consumption growth must also take into account the growth of the initial human capital stock per capita. The results, for the case of optimal spending with distortionary taxes, are shown in Figure 5. Qualitatively, there is little change relative to the results without human capital. The UK and Spain are still bunched at the top, followed by France, the U.S. and Japan, with the latter two countries virtually on par at the end of the sample period. As before, Italy and Canada lag behind the rest of countries. Overall, the results look very similar to our baseline case.

Finally, we compare the results we have just obtained using our theory-based welfare metric to those implied by standard proxies for welfare change. In Table 4, we present the average growth rates of GDP and consumption per-capita for our group of countries over our sample period, as well as the average growth rate of our baseline welfare measure, which assumes optimal government spending and distortionary taxes. The differences in magnitude are substantial, with welfare growing faster than conventional measures like consumption per capita. We have already remarked that allowing for public spending in the utility function and for distortionary taxes produces larger growth rates of welfare (compare Table 2, columns one and four), especially for countries with high rates of growth of factor inputs and government purchases per capita. Indeed, there are smaller gaps (in absolute value) between our welfare index and the conventional ones when we assume that government spending is wasteful and taxes are lump sum.

There are two more factors that account for the difference, namely the forward-looking nature of

our calculations and the presence of leisure in the utility function. To highlight the role of former, we report in column four of Table 4 our baseline measure calculated assuming that productivity in each country is forecasted to grow at the US rate in the future. We see that Canada, Italy and Japan, countries where productivity was growing more slowly than in the US over the sample period, improve in terms of welfare growth, while the UK, where productivity was growing faster, worsens. Finally, in column five of Table 4 we redo our calculations assuming that leisure does not enter the utility function.<sup>32</sup> Note that average labor hours per capita tend to change modestly during our period, implying that our thought experiment leaves the index of welfare growth unchanged in most cases. The exception is a small improvement in the welfare index for Spain and Canada, where average hours worked experienced a more sizeable increase, and a worsening of the index for Japan, where average labor hours contracted. We shall see that the treatment of leisure in utility is of greater importance in the cross-country comparisons, where it is its level relative to the US that matters.

### 5.3 Cross Country Results

We now turn to measuring welfare differences across the countries in our sample. For each country and time period, we calculate the welfare gap between that country and the US, as defined in equation (30). Recall that this gap is the loss in welfare of a representative US household that is moved permanently to country  $i$  starting at time  $t$ , expressed as the log gap between equivalent consumption at home and abroad. In this hypothetical move, the household loses the per-capita capital stock of the US, but gains the equivalent capital stock of country  $i$ . From time  $t$  on, the household faces the same product and factor prices and tax rates, and receives the same lump-sum transfers and government expenditure benefits as all the other households in country  $i$ . In a slight abuse of language, we often refer to the incremental equivalent consumption as “the welfare difference” or “the welfare gap.”

Note that these gaps are all from the point of view of a US household. Hence, all the shares in (30), even those used to construct output and TFP growth in country  $i$ , are the *US* shares. This naturally raises the question whether our results would be quite different if we took a different country as our baseline. We return to this issue after presenting our basic set of results.<sup>33</sup>

We present numerical results in Table 5. Since the size of the gap varies over time, we present the gap at the beginning of our sample, at the end of our sample, and averaged over the sample period. We present results for three cases: wasteful spending, with lump-sum and distortionary taxes, and optimal spending with distortionary taxes. These numerical magnitudes are useful references in the discussion that follows.

However, the results are easiest to understand in graphical form. We plot the welfare gap for

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<sup>32</sup>We continue to use the country-specific dynamics estimated in the case where leisure is valued, in order to avoid changing both the treatment of leisure and the dynamics at the same time.

<sup>33</sup>We conjecture that if we took a second-order approximation to the welfare gap, then the shares in our computation would be averages of the shares in the two countries, and hence bilateral comparisons would be invariant to the choice of a reference country. We leave the investigation of this hypothesis to future research.



the countries and time periods in our sample in Figures 6 and 7. Note that by definition the gap is zero for the US, since the US household neither gains nor loses by moving to the US at any point in time. The vertical axis shows, therefore, the gain to the US household of moving to any of the other countries at any point in the sample period, expressed in log points of equivalent consumption. Figure 6 shows the results for the case of wasteful spending and lump-sum taxes. Figure 7 shows the results for our benchmark case, where we allow for distortionary taxes and assume that government expenditure is optimally chosen. Since both figures show qualitatively similar results, for brevity we discuss only the benchmark case.

It is instructive to begin by focusing on the beginning and end of the sample. At the beginning of the sample, expected lifetime welfare in both France and the UK was about 20 percent lower than in the US (gaps of 16 and 19 percent, respectively). This relatively small gap reflects both the long-run European advantage in leisure and the fact that in the mid-1980s the US was still struggling with its productivity slowdown, while TFP in the leading European economies was growing faster than in the US. Capital accumulation was also proceeding briskly in those countries. By the end of the sample, the continental European economies, Canada and Japan are generally falling behind the US, because they had not matched the pickup in TFP growth and investment experienced in the US after 1995. Italy experiences the greatest relative “reversal of fortune,” ending up with a welfare gap of nearly 50 percent relative to the US. The results for France are qualitatively similar, but far less extreme. France starts with a welfare gap of 16 percent, and slowly slips further behind, ending with a gap of 21 percent. In continental Europe, only Spain shows convergence to the US in terms of welfare: it starts with a gap of 41 percent, and ends with a gap of 36 percent. However, after 1995 Spain holds steady relative to the US, but does not gain further.

The only economy in our sample that exhibits convergence to the US throughout our sample is the UK. Indeed, as Figure 6 shows, under the assumption of wasteful spending and lump-sum taxes, the UK overtakes the US by the end of our sample period. Table 5 and Figure 7 show that in more realistic cases where taxes are assumed to be distortionary the welfare level of the UK is always below that of the US, but the UK shows strong convergence, slicing two-thirds off the welfare gap in two decades. This result is interesting, because the UK experienced much the same lack of TFP growth in the late 1990s and early 2000s as the major continental European economies. However, the UK had very rapid productivity growth from 1985 to 1995. The other “Anglo-Saxon” country in our sample, Canada, had a welfare level about 30 percent below that of the US in 1985, but the welfare gap had grown by an additional 50 percent by the end of the sample. This result is due primarily to the differential productivity performance of the two countries: TFP in Canada actually fell during the 1990s, and rose only slowly in the early 2000s.

Perhaps the most striking comparison is between the US and Japan. Even in 1985, when its economic performance was the envy of much of the world, Japan was the least attractive country in our sample to a US household contemplating emigration; such a household would give up nearly 50 percent of consumption permanently in order to stay in the US instead of moving to Japan. However, like the UK and Spain, Japan was closing the gap with the US until the start of its ‘lost

decade' in 1991. The relative performance of the three countries changes dramatically from that point: unlike the UK, which continues to catch up, and Spain, which holds steady, Japan begins to fall behind the US, first slowly and then more rapidly. Having closed to within 43 percent of the US welfare level in 1991, Japan ends our sample 53 percent behind.

As we did for the within-country results, we investigate whether the cross-country welfare gaps are driven mostly by the TFP gap or by differences in capital per worker. The results are shown in Table 6. We focus on the last column of Panel C, which gives results averaged over the full sample period for our baseline case of optimal spending with distortionary taxes. We find that for five of the six countries, TFP is responsible for the vast majority of the welfare gap relative to the US. Indeed, for Japan TFP accounts for more than 100 percent of the gap (meaning that Japan has generally had a higher level of capital per person than the US). Thus we arrive at much the same conclusion as Hall and Jones (1999), although our definition of TFP is quite different from the one they used, and we do not focus only on steady-state differences. The exception to this rule is the UK. The average welfare gap between the US and the UK is driven about equally by TFP and by capital. Panel B shows that by the end of the sample, the UK had surpassed the US in "welfare-relevant TFP," and more than 100 percent of the gap was driven by the difference in per-capita capital between the two economies.

We now check the robustness of the preceding results along three dimensions. First, as noted above, we wish to see whether our welfare ranking is sensitive to the choice of the reference country. We thus redo the preceding exercises taking France as the baseline country. France is the largest and most successful continental European economy in our sample, and by revealed preference French households place much higher weight on leisure than do US ones. We summarize the results for our baseline case of optimal spending with distortionary taxes in Figure 8. For ease of comparison with the preceding cross-country figures, we still normalize the US welfare level to zero throughout, even though the comparison is done from the perspective of the French household and is based on French shares. Reassuringly, we see that the qualitative results are unchanged. France and the UK start closest to the US in 1985, but even they are well behind the US level of welfare. The UK converges towards the US welfare level and so, from a much lower starting point, does Spain. All the other economies, including France, fall steadily farther behind the US over time. Interestingly, from the French point of view almost all the other countries are shifted down vis-a-vis the US relative to the rankings from the US point of view.

In our second robustness check, we bring human capital into the analysis. From the derivation in section 3.6, this may change our welfare results for two reasons. First, countries may differ in their initial human capital stocks. Second, the series for labor input is now adjusted for human capital. Notice that the first of these two factors comes into play only to the extent that the hypothetical move of the household from the reference country (the US) to country  $i$  entails losing her initial human capital stock and acquiring the human capital capital stock of country  $i$ . In principle, there is no compelling reason why the thought experiment should be framed in this way, rather than allowing the US household to retain the US human capital stock when moving to

country  $i$ . However, for comparability with the development accounting literature, which assigns a prominent role to cross-country differences in human capital stocks, we opt for assuming that the US household does not retain its human capital with when moving to the comparator country.

The results from repeating the previous exercises bringing human capital into our framework are shown in Figure 9, for our baseline case of optimal spending with distortionary taxes. Comparison with Figure 7 above, which portrays the baseline case without human capital, reveals that the welfare gap with respect to the US is now wider for all countries. However, the magnitude of the change varies across countries. It is especially large for Spain and France, and more modest for the other countries. Further comparison with Figure 7 also shows that the slopes of the various lines are fairly similar across the two figures, so the widening of welfare gaps is roughly the same with or without human capital. At the beginning of the sample period, Spain now shows the largest welfare gap relative to the US, and the UK the smallest one (when ignoring human capital, Japan and France respectively assumed those roles). At the end of the sample period, Spain is behind all countries except Italy, while the relative ranking of the remaining countries is the same as in the case without human capital.

Finally, as we did for the results on within-country welfare growth, we compare our welfare results to those based on traditional measures, namely PPP-adjusted GDP and consumption per capita. The results are in Table 7. Focusing on Panel B, for the final year of our sample, we see that the three measures sometimes give identical results. For example, the US is atop the world rankings by all three measures, although the gap between the US and the second-ranked country is much smaller in percentage terms for welfare (6 percent) than it is for the other two variables (18 or 19 percent). On the other hand, the differences can be striking. For example, Canada, which leads Spain by 20 percent or more in terms of consumption and GDP per capita, is overtaken by Spain and France in our welfare comparison. Indeed, Spain is last within our group of countries in terms of the conventional metrics of consumption and GDP, but ranks fourth in welfare terms, trailing only the US, UK and France. For the other countries, the welfare measure is not so kind. Japan trails the US by only 26 percent in GDP per capita, but double that—52 percent—in terms of welfare. Similarly, Italy has more than 60 percent of the per-capita GDP of the US, but only about one-third the welfare level. On the other hand, France trails the US by 40 percent in consumption per-capita, but by only half that amount in terms of a welfare. Thus, our measure clearly provides new information on welfare differences among countries.

Relative to the traditional indexes of country performance, two new factors we highlight are dynamics (the expectation of future productivity change) and the treatment of leisure. To show their importance for cross-country differences in welfare, we have recalculated our basic index assuming, first, that future levels of productivity in all countries are expected to grow at the US rate, and second by assuming that leisure does not enter the utility function. The results appear in the last two columns of Table 7. When we assume that future productivity in each country is expected to grow at the US rate, the significant changes occur for Italy, Canada, and the UK: the performance of Canada and Italy, whose productivity growth trails that of the US, improves, while

the performance of the UK, with faster productivity growth than the US, worsens. When, instead, differences in leisure are not taken into account in the calculation of the index, Italy, France and Spain, with much lower average hours of work per capita than the US, worsen even more in terms of welfare relative to the US.

#### 5.4 A broader cross-country sample

The empirical exercises so far are limited to a small set of large advanced countries, determined by the availability of the requisite data. However, it may be interesting to assess welfare gaps across countries in a broader sample, including both advanced and developing countries.

Thus, as a final empirical exercise, we extend our cross-country results to a large set of countries for the year 2005. As noted, however, consistent information on hours worked is almost completely unavailable for countries outside the OECD. Hence, as already mentioned, we measure the aggregate labor input using total employment rather than total hours. The immediate consequence is that cross-country differences in work hours per person are ignored in the calculation of cross-country differences in the productivity residual, and thus also in the calculation of differences in welfare.<sup>34</sup>

As with the smaller country sample, we calculated the welfare gaps (always from the perspective of the US household) under different assumptions regarding public spending and taxation, and both including human capital and excluding it. In each case, we estimated country-specific autoregressive models and used them to project the future path of the relevant version of the modified productivity residual. For the sake of space, we only report the results of our baseline specification – optimal government expenditure and distortionary taxes – for the human capital-augmented model. Except for the different measurement of the labor input, the exercise is therefore the same as that reported in Figure 9, but it now includes 63 countries.

Results appear in Table 8. The first column reports the log differences in per capita welfare vis-a-vis the US for the year 2005. Only Luxembourg ranks ahead of the US, the same result found by Jones and Klenow (2010) using a different welfare metric. All the industrial countries in the sample rank above the median, with Italy bringing up the rear in 30th place. Among the six advanced countries in our earlier exercises, relative ranks are the same as those shown in Figure 9, with the only exception being that Spain and Japan trade places, although their respective welfare gaps are numerically very similar (as was the case also in Figure 9).

In turn, most developing countries exhibit fairly large welfare gaps relative to the US. Translating the log-differences shown in the table to percentage terms, we find that a US household moving to South Korea would suffer a welfare loss equivalent to 40 percent of her permanent consumption. Moving instead to Mexico or China would raise the loss above 85 percent of consumption. In addition, inspection of 2005 per capita GDP data (not shown in the table) reveals that, of the 61 sample countries that trail the US in terms of welfare, the vast majority (57 of them) lag fur-

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<sup>34</sup>Recall that, aside from absorption and labor and capital aggregates for the countries involved, numerical comparisons of welfare across countries only require information on factor shares and tax rates for the reference country (which continues to be the US for this exercise).

ther behind in terms of welfare than in per-capita GDP. For these countries, the median difference between the two gaps (in percentage terms) equals 10 percent. In addition, for the six advanced non-US countries in our earlier exercises, the 2005 welfare gaps in Table 8 are in all cases larger than those shown in Figure 9. We conjecture that ignoring cross-country differences in hours of work – as we are effectively doing in the enlarged country sample – may lead, for most countries, to an overestimation of the present value of their labor input relative to the US, and thereby to an overestimation of their respective welfare lag vis-a-vis the US.<sup>35</sup>

As before, we may ask how these welfare-based country comparisons would relate to those obtained on the basis of per capita consumption or GDP. The answer is that the resulting country ranking would show visible differences – for example, Norway had higher PPP GDP per capita than the US in 2005, but, according to the results in Table 8, lower welfare. In contrast, Luxembourg ranks ahead of the US in terms of welfare, but not in terms of consumption per capita (at PPP prices) in 2005. On the whole, however, there is broad agreement among the three measures. Indeed, the correlation coefficients between the log differences in welfare shown in Table 8, and the log differences in per capita PPP GDP or consumption (both vis-a-vis the US) observed in 2005, equal 0.93 and 0.90, respectively. One factor behind this high correlation is probably the lack of data on hours worked for this larger sample, which forces us to ignore the variation in average hours per employee across countries. Perhaps this should not be surprising since, in data sets with a large sample of countries, one cannot account for differences in average hours worked by those employed.<sup>36</sup> It is interesting to note that for the set of countries we consider, the cross-country correlation between the welfare measure (based on flow utility) employed by Jones and Klenow (2010) and (log) PPP GDP per capita is also high (0.95).

Lastly, we examine the extent to which the 2005 cross-country welfare gaps shown in the first column of Table 8 are driven by differences in TFP and by differences in initial capital (both physical and human) per worker. The relevant decomposition is shown in the second and third columns of Table 8. On the whole, TFP accounts for the bulk of the welfare differences. In 56 out of the 61 countries that trail the US in terms of welfare, TFP accounts for over two-thirds of the welfare gap. Across countries, its median contribution equals 79 percent. An extreme case is Norway, whose welfare gap is entirely due to TFP, with part of the gap offset by an initial capital stock above that of the US. At the other extreme, the UK is the only country whose welfare gap relative to the US is fully due to its lower initial capital stock.

## 6 Relationship to the Literature

Measuring welfare change over time and differences across countries using observable national income accounts data has been a long-standing challenge for economists. We note here the similarities

<sup>35</sup>For the countries with data, the correlation between total employment and total hours of work (both in logs) is 0.53, which confirms that the former is a fairly noisy proxy for the latter.

<sup>36</sup>For the sample of seven advanced countries, where we have both hours and employment data, the correlation of GDP per capita with equivalent consumption, correctly computed using total hours, is 0.59. If equivalent consumption is computed, instead, using employment data, the correlation rises to 0.77.

and differences between our approach and ones that have been taken before. We also suggest ways in which our results might be useful in other fields of economics, where the same question arises in different contexts.

Nordhaus and Tobin (1972) originated one approach, which is to take a snapshot of the economy's flow output at a point in time and then go "beyond GDP," by adjusting GDP in various ways to make it a better flow measure of welfare. Nordhaus and Tobin found that the largest gap between flow output and flow welfare comes from the value that consumers put on leisure. Their result motivated us to add leisure to the period utility function in our model, which is standard in business-cycle analysis but not in growth theory. Nordhaus and Tobin's approach has been followed recently by Jones and Klenow (2010) who add other corrections, notably for life expectancy and inequality. However, this point-in-time approach does not take into account the link between today's choices and future consumption or leisure possibilities. For example, high consumption in the measured period might denote either permanently high welfare or low current investment. Low investment would mean that consumption must fall in the future, so its current level would not be a good indicator of long-term welfare. Our approach is to go beyond point-in-time measures of welfare and compute the expected present discounted value of consumers' entire sequence of period utility. Moreover, the intertemporal nature of our approach allows us to frame an appealing thought experiment for cross-country comparisons, where the household of a reference country re-optimizes when faced with the path of exogenous variables of another country.

Our approach echoes the methods used in the literature started by Weitzman (1976) and analyzed in depth by Weitzman (2003), with notable contributions from many other authors.<sup>37</sup> This literature also relates the welfare of an infinitely lived representative agent to observables; for example, Weitzman (1976) linked intertemporal welfare to net domestic product (NDP). Unlike our model which allows for uncertainty about the future, this literature almost always assumes perfect foresight.<sup>38</sup> Allowing for uncertainty is important when forward-looking rules for measurement are applied to actual data. More importantly, the results in these papers are derived using a number of strong restrictions on the nature of technology (typically an aggregate production function with constant returns to scale), product market competition (always assumed to be perfect), and the allowed number of variables that are exogenous functions of time, such as technology or terms of trade (usually none, but sometimes one or two). Most of the analysis in the literature applies to a closed economy where growth is optimal.<sup>39</sup> Taken together, this long list of assumptions greatly limits the domain of applicability of the results.

By contrast, we derive all our results based only on first-order conditions from household optimization, which allows for imperfect competition in product markets of an arbitrary type and

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<sup>37</sup>A far from exhaustive list includes Asheim (1994), Arronson and Löfgren (1995), Mino (2004), Sefton and Weale (2006), Basu, Pascali, Schiantarelli and Serven (2009), and Hulten and Schreyer (2010). Reis (2005) analyses the related problem of computing a dynamic measure of inflation for a long-lived representative consumer.

<sup>38</sup>Arronson and Löfgren (1995) allow for stochastic population growth, and Weitzman (2003, ch. 6) considers shocks coming from stochastic depreciation of capital.

<sup>39</sup>Sefton and Weale (2006) and Hulten and Schreyer (2010) consider an open economy with changes in the terms of trade and Mino (2004) analyses Marshallian spillovers to R&D (all under perfect foresight).

for a vast range of production possibilities, with no assumption that they can be summarized by an aggregate production function or a convex technology set. (This makes it easy to apply our results to modern trade and macro models, for example, since these models often assume imperfect competition with substantial producer heterogeneity.) We do not need to assume that the economy follows an optimal growth path. We are also able to allow for a wide range of shocks, including but not limited to changes in technology, tax rates, terms of trade, government purchases, the size of Marshallian spillovers, monetary policy, tariffs, and markups.<sup>40</sup> Crucially, we do not need to specify the sources of structural shocks to the economy. The key to the generality of our results is that we condition on observed prices and asset stocks without needing to model *why* these quantities take on the values that they do.<sup>41</sup> To our knowledge, in the literature started by Weitzman (1976), this paper is the first to produce empirical measures of intertemporal welfare in a framework that allows productivity to vary over time.

Our results shed light on a variety of issues that bedevil the measurement of productivity and allocative efficiency. For example, Baker and Rosnick (2007), reasoning that the ultimate object of growth is consumption, make the reasonable conjecture that one should deflate nominal productivity gains by a consumption price index to create a measure they call “usable productivity.” We begin from the assumption that consumption (and leisure) at different dates are the only inputs to economic wellbeing, but nevertheless show that output should be calculated in the conventional way, rather than being deflated by consumer prices.

Our work clarifies and unifies several results in other literatures, especially international economics. Kohli (2004) shows in a static setting that terms-of-trade changes can improve welfare in open economies even when technology is constant. Kehoe and Ruhl (2008) prove a related result in a dynamic model with balanced trade: opening to trade may increase welfare, even if it does not change TFP. In these models, which assume competition and constant returns, technology is equivalent to TFP. We generalize and extend these results, and show that in a dynamic environment with unbalanced trade welfare can also change if there are changes in the quantity of net foreign assets or in their rates of return.<sup>42</sup> In general, we show that there is a link between observable aggregate data and welfare in an open economy, which is the objective of Bajona, Gibson, Kehoe and Ruhl (2010). While we agree with the conclusion of these authors that GDP is not a sufficient statistic for uncovering the effect of trade policy on welfare, we show that one can construct such a sufficient statistic by considering a relatively small number of other variables. Our results also

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<sup>40</sup>See also Sandleris and Wright (2011) for an attempt to extend the basic ideas in Basu and Fernald (2002) in order to evaluate the welfare effects of financial crises. These papers try to derive methods to measure the welfare effects of a particular shock, which requires specifying an explicit counterfactual path that the economy would have followed in the absence of the shock.

<sup>41</sup>We do need to forecast the present value of future TFP in order to implement our results in data. It is an open question whether specifying a complete general-equilibrium structure for the model would improve our forecasts substantially. We decided that the possible gain in forecasting accuracy from specifying a general-equilibrium structure would not compensate for the loss of generality of our results.

<sup>42</sup>The result that openness does not change TFP may be fragile in models with increasing returns. If opening to trade changes factor inputs, either on impact or over time, then TFP as measured by Solow’s residual will change as well, which we show has an effect on welfare even holding constant the terms of trade.

shed light on the work of Arkolakis, Costinot and Rodriguez-Clare (2012). These authors show that in a class of modern trade models, which includes models with imperfect competition and micro-level productivity heterogeneity, one can construct measures of the welfare gain from trade without reference to micro data.<sup>43</sup> Our results imply that this conclusion actually holds in a much larger class of models, although the exact functional form of the result in Arkolakis et al. (2012) may not. Finally, since changes in net foreign asset positions and their rates of return are extremely hard to measure, we show that one can measure welfare using data only on TFP and the capital stock, even in an open economy, provided that TFP is calculated using absorption rather than GDP as the output concept.

Our work provides a different view of a large and burgeoning literature that investigates the productivity differences across countries. If we specialize our cross-country result to the lump sum taxes-optimal spending case, we obtain something closely related to the results produced by the “development accounting” literature. We show that in that case, (the present value of) the log differences in TFP levels emphasized by the developing accounting literature need to be supplemented with only one additional variable, namely log level gaps in capital per person, in order to serve as a measure of welfare differences among countries.<sup>44</sup>

A number of recent papers suggest that countries can increase output and TFP substantially by allocating resources more efficiently across firms. Our work implies that the literature is correct to focus on the connection between reallocation and aggregate TFP. An increase in aggregate TFP due to reallocation is as much of a welfare gain for the representative consumer as a change in technology with the same magnitude and persistence. This result implies immediately that estimates of TFP losses due to allocative inefficiency (e.g., Hsieh and Klenow, 2009) can be translated to estimates of welfare losses.

Our results are also related to an earlier literature on “industrial policy.” and to more recent literature on the effect of “reallocation”. Bhagwati, Ramaswami and Srinivasan (1969) and Bulow and Summers (1986) argue welfare would be enhanced by policies to promote growth in industries where there are rents, for example stemming from monopoly power.<sup>45</sup> The TFP term in Proposition 1 or 1’ captures this effect. When firms have market power, their output grows faster than the share-weighted sum of their inputs, even when their technology is constant. Thus, aggregate TFP rises when firms with above-average market power grow faster than average. At first this result sounds counterintuitive, since it implies that welfare is enhanced by directing more capital and labor to the most distorted sectors. However, the logic is exactly the same as the usual result that firms with the greatest monopoly power should also receive the largest unit subsidies to increase their output.

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<sup>43</sup> Atkeson and Burstein (2010) come to similar conclusions in a related model.

<sup>44</sup> As we show, what matters is the present discounted value of TFP differences. Moreover, one needs to compute TFP using different shares than the ones used by the development accounting literature, and switch to a different output concept (based on domestic absorption) in an open economy.

<sup>45</sup> A second-best policy might involve trade restrictions to protect such industries from foreign competition. If lump-sum taxes are available, the optimal policy is always to target the distortion directly through a tax-cum-subsidy scheme.



Finally, our work is closely related to the program of developing sufficient statistics for welfare analysis, surveyed by Chetty (2009). We have proposed such a statistic for a representative consumer in a macroeconomic context. As Chetty notes, such measures can be used to evaluate the effects of policies. Suppose that one wishes to evaluate the effect of a policy change—for example, a change in trade policy, as in Kehoe and Ruhl (2010). The usual method is to relate the policy change to a variety of economic indicators, such as GDP, capital accumulation, or the trade balance, and then try to relate the indicators to welfare informally. Our work suggests that one can dispense with these “intermediate targets,” and just directly relate the welfare outcome to a change in policy, or to some other shock.

## 7 Conclusions

We show that aggregate TFP, appropriately defined, and the capital stock can be used to construct sufficient statistics for the welfare of a representative consumer. To a first order approximation, welfare is measured by the expected present value of aggregate TFP and by the initial capital stock. This result holds regardless of the type of production technology and the degree of product market competition, and applies to closed or open economies with or without distortionary taxation. Crucially, TFP has to be calculated using prices faced by households rather than prices facing firms. In modern economies with high rates of income and indirect taxation, the gap between household and firm TFP can be considerable. Finally, in an open economy, the change in welfare will also reflect present and future changes in the returns on net foreign assets and in the terms of trade. However, these latter terms disappear if absorption rather than GDP is used as the output concept for constructing TFP, and TFP and the initial capital stock are again sufficient statistics for measuring welfare in open economies. Most strikingly, these variables also suffice to measure welfare level differences across countries, with both variables computed as log level deviations from a reference country.

We extend the existing literature on intertemporal welfare measurement by deriving all our results from household first-order conditions alone. The generality of our derivation allows us to propose a new interpretation of TFP that sheds new light on several distinct areas of study. For instance, we show that measures of cross-country TFP differences akin to those produced by the “development accounting” literature are crucial for calculating welfare differences among countries. We also find that readily-available national accounts data can be used to construct welfare measures for open economies, which can be used to evaluate the effects of trade policies and tariff changes. In general, our results imply that all changes in the Solow residual, whatever their source (for example, technology, increasing returns, or reallocation) are equally important for welfare.

We illustrate our results by using national accounts data to measure welfare growth rates and gaps across countries. We find that the assumptions about fiscal policy matter for welfare calculations. For instance, Spain’s welfare growth almost doubles when one allows for distortionary taxation and optimal government spending. Our evidence also suggests that expectations about

future productivity and the presence of leisure in the utility function are important determinants of welfare rankings. Finally, in the vast majority of cases, the bulk of the welfare gap relative to the US, our welfare leader among large countries, is due to the productivity gap, rather than the gap in the initial capital stock.

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## A Appendix: Derivations and numerical simulations

### A.1 Proposition 1

Assume that in the steady state aggregate per capita variables grow at a constant rate  $g$ . Thus, they are proportional to  $X_t = X_0(1+g)^t$ . Rewrite the utility function, the budget constraint and the capital accumulation, equations (1), (2) and (3), in normalized form by dividing by  $X_t$  :

$$v_t = \sum_{s=0}^{\infty} \beta^s U(c_{t+s}; \bar{L} - L_{t+s}) \quad (\text{A.1})$$

$$k_t + b_t = \frac{(1-\delta) + p_t^K}{(1+g)(1+n)} k_{t-1} + \frac{(1+r_t)}{(1+g)(1+n)} b_{t-1} + p_t^L L_t + \pi_t - p_t^C c_t \quad (\text{A.2})$$

$$k_t = \frac{(1-\delta)}{(1+g)(1+n)} k_{t-1} + i_t \quad (\text{A.3})$$

where:  $v_t = \frac{V_t}{X_t^{(1-\sigma)}}$ ,  $c_t = \frac{C_t}{X_t}$ ,  $k_t = \frac{K_t}{X_t}$ ,  $b_t = \frac{B_t}{P_t^I X_t}$ ,  $p_t^K = \frac{P_t^K}{P_t^I}$ ,  $p_t^L = \frac{P_t^L}{P_t^I X_t}$ ,  $p_t^C = \frac{P_t^C}{P_t^I}$ ,  $(1+r_t) = (1+i_t^B) \frac{P_{t-1}^I}{P_t^I}$ ,  $\pi_t = \frac{\Pi_t}{P_t^I X_t}$  and  $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$ .

The first order conditions for this problem are:

$$U_{c_t} - \lambda_t p_t^C = 0 \quad (\text{A.4})$$

$$U_{L_t} + \lambda_t p_t^L = 0 \quad (\text{A.5})$$

$$-\lambda_t + \beta E_t \frac{(1-\delta) + p_{t+1}^K}{(1+g)(1+n)} \lambda_{t+1} = 0 \quad (\text{A.6})$$

$$-\lambda_t + \beta \frac{1}{(1+g)(1+n)} E_t (1+r_{t+1}) \lambda_{t+1} = 0 \quad (\text{A.7})$$

where  $\lambda_t$  is the Lagrange multiplier associated with the budget constraint. Define with  $\hat{x} = \frac{x_t - x}{x}$  the percent deviation from the steady-state of a variable ( $x$  is the steady-state value of  $x_t$ ). Taking a first order approximation of the Lagrangean (which equals the value function along the optimal path), one obtains:

$$\begin{aligned} v_t - v &= E_t \left[ \sum_{s=0}^{\infty} \beta^s (U_c \hat{c}_{t+s} + U_L L \hat{L}_{t+s} \right. \\ &\quad \left. + \lambda p^L L \hat{L}_{t+s} - \lambda p^C \hat{c}_{t+s} - \lambda k \hat{k}_{t+s} - \lambda b \hat{b}_{t+s}) \right. \\ &\quad \left. + \sum_{s=0}^{\infty} \beta^{s+1} \left( \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \hat{k}_{t+s} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \hat{b}_{t+s} \right) \right] \end{aligned}$$

$$\begin{aligned}
& + \sum_{s=0}^{\infty} \beta^s \widehat{\lambda}_{t+s} \lambda \left( -k - b + \frac{(1-\delta) + p^K}{(1+g)(1+n)} k + \frac{(1+r)}{(1+g)(1+n)} b \right. \\
& \left. + p^L L + \pi - p^C c \right) \\
& + \sum_{s=0}^{\infty} \beta^s \lambda \left( p^L L \widehat{p}_{t+s}^L + \frac{p^K k}{(1+g)(1+n)} \widehat{p}_{t+s}^K - p_t^C c \widehat{p}_{t+s}^C + \pi \widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s} \right) \\
& + \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1}
\end{aligned} \tag{A.8}$$

Using the first-order conditions, the first five lines equal zero. Moreover, since we are considering a closed economy case, in equilibrium  $b_t = 0$ . Hence we get:

$$\begin{aligned}
v_t & = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[ p^L L \widehat{p}_{t+s}^L + \frac{p^K k}{(1+g)(1+n)} \widehat{p}_{t+s}^K - p_t^C c \widehat{p}_{t+s}^C + \pi \widehat{\pi}_{t+s} \right] \\
& + \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1}
\end{aligned} \tag{A.9}$$

Linearize the budget constraint and the law of motion for capital:

$$\begin{aligned}
k \widehat{k}_t + b \widehat{b}_t - \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} - \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1} - p^L L \widehat{L}_t - p^L L \widehat{p}_t^L - \frac{p^K k}{(1+g)(1+n)} \widehat{p}_t^K \\
- \frac{rb}{(1+g)(1+n)} \widehat{r}_t - \pi \widehat{\pi}_t + p^C c \widehat{c}_t + p^C c \widehat{p}_t^C = 0
\end{aligned} \tag{A.10}$$

$$k \widehat{k}_t = \frac{(1-\delta)}{(1+g)(1+n)} k \widehat{k}_{t-1} + \widehat{i}_t \tag{A.11}$$

Use (A.10) and (A.11) in (A.9), together with the steady-state version of the FOC for capital. This yields:

$$\begin{aligned}
v_t & = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[ p^C c \widehat{c}_{t+s} + \widehat{u}_{t+s} - \frac{p^K k}{(1+g)(1+n)} \widehat{k}_{t+s-1} - p^L L \widehat{L}_{t+s} \right] \\
& + \lambda \frac{1}{\beta} k \widehat{k}_{t-1}
\end{aligned} \tag{A.12}$$

Take the difference between the expected level of intertemporal utility  $v_t$  as in (A.12) and  $v_{t-1}$  and use the fact that:  $\frac{x_t - x}{x} \simeq \log x_t - \log x$  for a positive  $x_t$ . After adding and subtracting  $E_t x_{t+s}$  for each variable  $x_{t+s}$ , we obtain:



$$\begin{aligned}
\Delta v_t &= E_t \sum_{s=0}^{\infty} \beta^s \lambda [p^C c \Delta \log c_{t+s} + i \Delta \log i_t - p^L L \Delta \log L_{t+s} - \frac{p^K k}{(1+g)(1+n)} \Delta \log k_{t+s-1}] \\
&+ \sum_{s=0}^{\infty} \beta^s \lambda [p^C c (E_t \log c_{t+s-1} - E_{t-1} \log c_{t+s-1}) + i (E_t \log i_{t+s-1} - E_{t-1} \log i_{t+s-1})] \\
&+ \lambda \frac{1}{\beta} k \Delta \log k_{t-1}
\end{aligned} \tag{A.13}$$

Define value added growth (in normalized form) as:

$$\Delta \log y_t = s_C \Delta \log c_t + s_I \Delta \log i_t \tag{A.14}$$

Inserting (A.14) into (A.13) and dividing both terms by  $\lambda p^Y y$  one obtains:

$$\begin{aligned}
\frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} &= E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log k_{t+s-1}] \\
&+ \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s-1} - E_{t-1} \log y_{t+s-1})] \\
&+ \frac{1}{\beta} \frac{k}{p^Y y} \Delta \log k_{t-1}
\end{aligned} \tag{A.15}$$

The definition of equivalent consumption in terms of normalized variables is:

$$v_t = \frac{1}{(1-\sigma)(1-\beta)} c_t^{*1-\sigma} \nu (\bar{L} - L) \tag{A.16}$$

Expanding the right hand side of (A.16) in terms of  $\log c_t^*$  and using the FOC for consumption, it follows that:

$$\frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} = \frac{s_C}{1-\beta} \Delta \log c_t^* \tag{A.17}$$

Using the result above in equation (A.15) one obtains, to a first-order approximation:

$$\begin{aligned}
\frac{s_C}{1-\beta} \Delta \log c_t^* &= E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log k_{t+s-1}] \\
&+ \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s-1} - E_{t-1} \log y_{t+s-1})] \\
&+ \frac{1}{\beta} \frac{k}{p^Y y} \Delta \log k_{t-1}
\end{aligned} \tag{A.18}$$

Using the fact that  $\Delta \log y_t = \Delta \log Y_t - g$ ,  $\Delta \log k_t = \Delta \log K_t - g$ ,  $\Delta \log c_t^* = \Delta \log C_t^* - g$ , equation

(A.18) can be rewritten as:

$$\begin{aligned}
\frac{s_C}{1-\beta} \Delta \log C_t^* &= E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} \\
&+ \sum_{s=0}^{\infty} \beta^s [E_t \log PR_{t+s-1} - E_{t-1} \log PR_{t+s-1}] \\
&+ \frac{1}{\beta} \frac{k}{p^Y y} \Delta \log \frac{K_{t-1}}{N_{t-1}} \\
&+ \frac{s_C}{1-\beta} g - \frac{1}{(1-\beta)} g(1-s_K) - \frac{1}{\beta} \frac{k}{p^Y y} g
\end{aligned} \tag{A.19}$$

where  $\log PR_{t+s}$  is defined in (6). Using equations (A.6) and (A.3) evaluated at the steady-state, one can easily show that  $\frac{s_C}{(1-\beta)} - \frac{1}{(1-\beta)}(1-s_K) - \frac{1}{\beta} \frac{k}{p^Y y} = 0$ , so that the last line in equation (A.19) equals zero. Multiplying both sides of (A.19) by  $\frac{1-\beta}{s_C}$  yields Proposition 1 in the main text.

## A.2 Proposition 2

Consider the maximization problem of a fictitious household, which has the preferences of a household in country  $j$  and faces the prices and per-capita endowments of a household living in country  $i$ . It maximizes (with variables defined in normalized form):

$$\tilde{v}_t^i = E_t \sum_{s=0}^{\infty} \beta^s \frac{\tilde{c}_{t+s}^i 1-\sigma \nu (\bar{L} - \tilde{L}_{t+s}^i)}{1-\sigma} \tag{A.20}$$

subject to:

$$\tilde{k}_{t+s}^i + \tilde{b}_{t+s}^i = \frac{(1-\delta) + p_{t+s}^{iK}}{(1+g)(1+n)} \tilde{k}_{t+s-1}^i + \frac{(1+r_t^i)}{(1+g)(1+n)} \tilde{b}_{t+s-1}^i + p_{t+s}^{iL} \tilde{L}_{t+s}^i + \pi_{t+s}^i - p_{t+s}^{iC} \tilde{c}_{t+s}^i \tag{A.21}$$

where:  $\tilde{v}_t = \frac{\tilde{V}_t}{X_t^i}$ ,  $\tilde{c}_{t+s}^i = \frac{\tilde{C}_{t+s}^i}{X_{t+s}^i}$ ,  $\tilde{k}_t^i = \frac{K_t^i}{X_t^i}$ ,  $\tilde{b}_t^i = \frac{B_t^i}{P_t^{iI} X_t^i}$ ,  $p_t^{iK} = \frac{P_t^{iK}}{P_t^{iI}}$ ,  $p_t^{iL} = \frac{P_t^{iL}}{P_t^{iI} X_t^i}$ ,  $p_t^{iC} = \frac{P_t^{iC}}{P_t^{iI}}$ ,  $(1+r_t^i) = \left(1 + i_t^{B,i}\right) \frac{P_{t-1}^{iI}}{P_t^{iI}}$  and  $\pi_t^i = \frac{\Pi_t^i}{P_t^{iI} X_t^i}$ . A tilde denotes the (unobservable) quantities that the household would choose when facing prices and initial conditions of country  $i$ . To simplify the notation in this proof, all variables without the superscript  $i$  denote utility, preference parameters, quantities and prices in country  $j$ . Linearizing around country  $j$ 's steady state and using the envelope theorem, one obtains:

$$\begin{aligned}
\frac{v}{\lambda p^y y} \frac{\tilde{v}_t^i - v}{v} &= E_t \sum_{s=0}^{\infty} \beta^s [s_L \hat{p}_{t+s}^{iL} + s_K \hat{p}_{t+s}^{iK} + s_\pi \hat{\pi}_{t+s}^i - s_C \hat{p}_{t+s}^{iC} + s_B \hat{r}_{t+s}^i] \\
&+ \frac{1}{\beta} \frac{k}{p^y y} \hat{k}_{t-1}^i + \frac{1}{\beta} \frac{b}{p^y y} \hat{b}_{t-1}^i
\end{aligned} \tag{A.22}$$

Linearize the budget constraint and the law of motion for capital:

$$k \widehat{k}_{t+s}^i + b \widehat{b}_{t+s}^i - \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t+s-1}^i - \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s-1}^i - p^L L \widehat{L}_{t+s}^i - p^L L \widehat{p}_{t+s}^{iL} - \frac{p^K k}{(1+g)(1+n)} \widehat{p}_{t+s}^{iK} - \pi \widehat{\pi}_{t+s}^i + p^C c \widehat{c}_{t+s}^i + p^C c \widehat{p}_{t+s}^{iC} = 0 \quad (\text{A.23})$$

$$k \widehat{k}_{t+s}^i = \frac{(1-\delta)}{(1+g)(1+n)} k \widehat{k}_{t+s-1}^i + \widehat{i}_{t+s}^i \quad (\text{A.24})$$

Using the two equations above in equation (A.22), together with the fact that at  $t-1$ ,  $\widehat{k}_{t-1}^i = \widehat{k}_{t-1}^i$ , we obtain:

$$\frac{v}{\lambda p^y y} \frac{\widehat{v}_t^i - v}{v} = E_t \sum_{s=0}^{\infty} \beta^s \left[ s_C \widehat{c}_{t+s}^i + s_I \widehat{i}_{t+s}^i - s_L \widehat{L}_{t+s}^i - s_K \widehat{k}_{t+s-1}^i \right] + \frac{1}{\beta} \frac{k}{p^y y} \widehat{k}_{t-1}^i \quad (\text{A.25})$$

Now consider the budget constraint of a household in country  $i$  and the law of motion for capital in country  $i$ , linearized around country  $j$ 's steady state.

$$k \widehat{k}_{t+s}^i + b \widehat{b}_{t+s}^i - \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t+s-1}^i - \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s-1}^i - p^L L \widehat{L}_{t+s}^i - p^L L \widehat{p}_{t+s}^{iL} - \frac{p^K k}{(1+g)(1+n)} \widehat{p}_{t+s}^{iK} - \pi \widehat{\pi}_{t+s}^i + p^C c \widehat{c}_{t+s}^i + p^C c \widehat{p}_{t+s}^{iC} = 0 \quad (\text{A.26})$$

$$k \widehat{k}_{t+s}^i = \frac{(1-\delta)}{(1+g)(1+n)} k \widehat{k}_{t+s-1}^i + \widehat{i}_{t+s}^i \quad (\text{A.27})$$

where  $k_t^i = \frac{K_t^i}{X_t^i}$ ,  $b_t^i = \frac{B_t^i}{P_t^i X_t^i}$ ,  $\pi_t^i = \frac{\Pi_t^i}{P_t^i X_t^i}$  and  $p^{iL} = \frac{P_t^{iL}}{P_t^i X_t^i}$ . Using the two budget constraints in equations (A.23) and (A.26), the two laws of motion for capital in equations (A.24) and (A.27) and the fact that bonds are in zero net supply, we obtain:

$$s_C \widehat{c}_{t+s}^i + s_I \widehat{i}_{t+s}^i - s_L \widehat{L}_{t+s}^i - s_K \widehat{k}_{t+s-1}^i = s_C \widehat{c}_{t+s}^i + s_I \widehat{i}_{t+s}^i - s_L \widehat{L}_{t+s}^i - s_K \widehat{k}_{t+s-1}^i \quad (\text{A.28})$$

which implies that equation (A.25) can be re-written as:

$$\frac{v}{\lambda p^y y} \frac{\widehat{v}_t^i - v}{v} = E_t \sum_{s=0}^{\infty} \beta^s \left[ s_C \widehat{c}_{t+s}^i + s_I \widehat{i}_{t+s}^i - s_L \widehat{L}_{t+s}^i - s_K \widehat{k}_{t+s-1}^i \right] + \frac{1}{\beta} \frac{k}{p^y y} \widehat{k}_{t-1}^i \quad (\text{A.29})$$

Similarly, for the household from country  $j$  living in country  $j$ :

$$\frac{v}{\lambda p^y y} \frac{v_t - v}{v} = E_t \sum_{s=0}^{\infty} \beta^s \left[ s_C \widehat{c}_{t+s} + s_I \widehat{i}_{t+s} - s_L \widehat{L}_{t+s} - s_K \widehat{k}_{t+s-1} \right] + \frac{1}{\beta} \frac{k}{p^y y} \widehat{k}_{t-1} \quad (\text{A.30})$$

Using the fact that  $\frac{\widehat{x}_t^i - x}{x} \simeq \log \widehat{x}_t^i - \log x$  for a generic non-negative variable  $x$ , and subtracting equation (A.30) from equation (A.29), we obtain:

$$\begin{aligned} \frac{v}{\lambda p^y y} \frac{\widetilde{v}_t^i - v_t}{v} &= E_t \sum_{s=0}^{\infty} \beta^s \left[ s_C (\log c_{t+s}^i - \log c_{t+s}) + s_I (\log i_{t+s}^i - \log i_{t+s}) \right. \\ &\quad \left. - s_L (\log L_{t+s}^i - \log L_{t+s}) - s_K (\log k_{t+s-1}^i - \log k_{t+s-1}) \right] \\ &\quad + \frac{1}{\beta} \frac{k}{p^y y} (\log k_{t-1}^i - \log k_{t-1}) \end{aligned} \quad (\text{A.31})$$

Using equation (A.17) and the fact that  $\frac{s_C}{(1-\beta)} - \frac{1}{(1-\beta)}(1 - s_K) - \frac{1}{\beta} \frac{k}{p^y y} = 0$ , some algebra yields Proposition 2 in the main text, with the productivity terms defined in (14) and (15).

### A.3 Extensions

#### A.3.1 Government Expenditure

Assume that utility depends upon private consumption and government spending on public consumption, as in equation (20). Assume government expenditure is financed through a lump-sum tax (for expositional simplicity). Re-writing the household maximization problem in normalized variables and proceeding in a similar fashion as in the proof of proposition 1, we obtain:

$$\begin{aligned} v_t &= v + \lambda E_t \sum_{s=0}^{\infty} \beta^s \left[ \frac{U_{c_G} c_G \widehat{c}_{G,t+s}}{\lambda} + p^C \widehat{c}_{t+s} + \widehat{i}_{t+s} - p^L L \widehat{L}_{t+s} - \frac{p^K k}{(1+g)(1+n)} \widehat{k}_{t+s-1} \right] \\ &\quad + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} \end{aligned} \quad (\text{A.32})$$

where  $c_{G,t} = \frac{C_{G,t}}{X_t}$ . The log-change in per-capita GDP, in normalized variables is defined as:

$$\Delta \log y_t = s_C \Delta \log c_{t+s} + s_{c_G} \Delta \log c_{G,t} + s_I \Delta \log i_t \quad (\text{A.33})$$

where  $s_{c_G}$  is the steady state value of  $s_{c_G,t} = \frac{P_t^G C_{G,t}}{P_t^Y Y_t}$  and  $P^G$  is the public consumption deflator.

Using this result and some algebra, equation (A.32) becomes:

$$\begin{aligned}
\frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} &= E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log k_{t+s-1} + (s_{c_G}^* - s_{c_G}) \Delta \log c_{G,t}] \\
&\quad + \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s-1} - E_{t-1} \log y_{t+s-1}) \\
&\quad + \frac{1}{\beta} \frac{k}{p^Y y} \Delta \log k_{t-1}]
\end{aligned} \tag{A.34}$$

where  $s_{c_G}^*$  is the steady state value of  $s_{c_G,t}^* = \frac{U_{c_G,t} c_{G,t}}{\lambda_t}$ . From this point, the algebra is very similar to the benchmark case, and yields (21) and (22) in the main text.

### A.3.2 Open Economy

In an open economy, the household continues to maximize equation (A.1) subject to (A.2). Interpret  $b_t$  as the stock of foreign assets. As  $b_t$  can differ from zero, (A.9) becomes:

$$\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[ p^L L \widehat{p}_{t+s}^L + \frac{p^K k}{(1+g)(1+n)} \widehat{p}_{t+s}^K - p^C \widehat{c}_{t+s}^C + \pi \widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s} \right] \\
&\quad + \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1}
\end{aligned} \tag{A.35}$$

The budget constraint and the law of motion of capital are unchanged with respect to the benchmark case, and therefore equations (A.2) and (A.3) are still valid.

Using these two equations and the steady-state version of the FOC for capital in (A.9) gives us:

$$\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[ p^C \widehat{c}_{t+s}^C + \widehat{i}_{t+s} - \frac{p^K}{(1+g)(1+n)} k \widehat{k}_{t+s-1} - p^L L \widehat{L}_{t+s} \right] + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} \\
&\quad + \lambda \sum_{s=0}^{\infty} \beta^s \left[ b \widehat{b}_{t+s} - \beta \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s} \right]
\end{aligned} \tag{A.36}$$

Using the FOC and the transversality condition for bonds, the last line in the equation above equals zero. As a result, the equations obtained for the closed economy hold also for the open economy, provided the log change of GDP is replaced by the log change in domestic absorption, defined as:

$$\Delta \log A_t = s_C \Delta \log C_t + s_I \Delta \log I_t$$

where  $s_C$  and  $s_I$  are shares out of domestic absorption.

### A.3.3 Summing up

Using the extensions developed in A.3.1-A.3.2 and the others described in the text (distortionary taxes and multiple investment and consumption goods) we can state Proposition 1'. A parallel argument can be used to derive the generalization of Proposition 2, Proposition 2'.

## A.4 Human capital

As in Lucas (1988), assume that non-leisure time can be used either to work or to accumulate human capital. The representative household maximizes intertemporal utility:

$$W_t = \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} U(C_{t+s}; \bar{L} - L_{t+s} - E_{t+s}) \quad (\text{A.37})$$

where  $E_t$  denotes the amount of time devoted to human capital accumulation, under the following budget constraint:

$$P_t^I K_t N_t + B_t N_t = (1-\delta) P_t^I K_{t-1} N_{t-1} + (1+i^B) B_{t-1} N_{t-1} + P_t^L L_t H_{t-1} N_t + P_t^K K_{t-1} N_{t-1} + \Pi_t N_t - P_t^C C_t N_t \quad (\text{A.38})$$

where labor income now depends on the initial level of human capital  $H_{t-1}$ . The human capital accumulation equation is assumed to be as in (33):

$$(H_t - H_{t-1}) + \delta_H H_{t-1} = F(E_t) H_{t-1} \quad (\text{A.39})$$

Linearizing the maximization problem as before, we get:

$$\begin{aligned} v_t - v &= E_t \sum_{s=0}^{\infty} \beta^s \lambda \left( \frac{p^L L h}{(1+g)} \hat{p}_{t+s}^L + \frac{p^K k}{(1+g)(1+n)} \hat{p}_{t+s}^K - p_t^C c \hat{p}_{t+s}^C + \pi \hat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \hat{r}_{t+s} \right) \\ &+ \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} \hat{k}_{t-1} + \left[ U_L \frac{1+g}{F'(E)h} + \frac{\lambda p^L L}{(1+g)} \right] h \hat{h}_{t-1} \end{aligned} \quad (\text{A.40})$$

where  $h_t = \frac{H_t}{X_t}$  and  $p_t^L = \frac{P_t^L}{P_t^I}$  (notice that we do not normalize  $P_t^L$  by  $P_t^I X_t$  as we did in the previous three sections of the appendix). The FOCs for human capital and labor, in normalized terms, are:

$$\begin{aligned} 0 &= -U_{L_t} \frac{1+g}{F'(E)h_{t-1}} \\ &+ \beta E_t \left[ U_{L_{t+1}} \frac{(1+g)h_{t+1}}{F'(E)h_t^2} + \frac{L_{t+1} p_{t+1}^L}{(1+g)} \lambda_{t+1} \right] \end{aligned} \quad (\text{A.41})$$

and

$$0 = -U_{L_t} + \frac{\lambda_t p_t^L h_{t-1}}{(1+g)} \quad (\text{A.42})$$

while the FOCs for consumption, physical capital and bonds are still defined by equations (A.4), (A.6) and (A.7).

Using the FOCs for physical capital and human capital evaluated in the steady state in (A.40) and the log-linearized version of the budget constraint, we obtain:

$$\begin{aligned} v_t - v &= \lambda E_t \sum_{s=0}^{\infty} \beta^s \left[ (p^C c \widehat{c}_{t+s} + i \widehat{i}_{t+s}) - \left( \frac{p^L L h}{(1+g)} (\widehat{L}_{t+s} + \widehat{h}_{t+s-1}) + \frac{p^K}{(1+g)(1+n)} k \widehat{k}_{t+s-1} \right) \right] \\ &\quad + \frac{\lambda}{\beta} k \widehat{k}_{t-1} + \left( \frac{1}{1-\beta} \right) \frac{\lambda p^L L h}{(1+g)} h \widehat{h}_{t-1} \end{aligned} \quad (\text{A.43})$$

The rest of the proof parallels subsection subsection A.1 and yields equations (34) and (35) in the text.

## A.5 Simulations

In order to illustrate the quality of the approximation in Propositions 1, 1', 2 and 2', we consider the standard RBC model augmented to take into account of public expenditure, distortionary and lump-sum taxes and externalities from production.

There is a fixed number of identical household with an infinite time horizon. The representative household chooses consumption, leisure and investments in capital and bonds to maximize the following intertemporal utility function:

$$W_t = E_t \sum_{s=0}^{\infty} \beta^s C_{t+s}^{1-\sigma} (\bar{L} - L_{t+s})^{\frac{1-\gamma}{\gamma}(1-\sigma)}$$

subject to the following budget constraint:

$$I_t + B_t = (1 + i_t^B) B_{t-1} + P_t^L (1 - \tau_t) L_t + P_t^K (1 - \tau_t) K_{t-1} - \Pi_t - C_t$$

In this model public expenditure is pure waste and can be financed through an income tax (with tax-rate  $\tau_t$ ) or a lump-sum tax ( $\Pi_t$ ). Thus, total public expenditure is:

$$C_{G,t} = P_t^L \tau_t L_t + P_t^K \tau_t K_{t-1} + \Pi_t$$

The law of motion for capital is:

$$K_t = (1 - \delta) K_{t-1} + I_t$$

There is a large number of firms which operate under perfect competition and are characterized by the same Cobb-Douglas function:

$$Y_t = E_t A_t L_t^{1-\alpha} K_{t-1}^\alpha$$

where  $A_t$  is the Harrod-neutral technology parameter while  $E_t$  measures an externality that arises from aggregate production:

$$E_t = Y_t^{1-\frac{1}{\epsilon}}$$

In equilibrium, the national income account equation holds:

$$Y_t = C_t + C_{G,t} + I_t$$

There are three potential shocks in this economy to technology  $A_t$ , to the lump-sum tax,  $\Pi_t$ , and to the income tax rate,  $\tau_t$ . The laws of motion of these variables are the following:

$$\log A_t = \rho_1 \log A_t + \varepsilon_{1t}$$

$$\Pi_t = (1 - \rho_2)\Pi^{SS} + \rho_2\Pi_{t-1} + \varepsilon_{2t}$$

$$\tau_t = (1 - \rho_3)\tau^{SS} + \rho_3\tau_{t-1} + \varepsilon_{3t}$$

The following table reports the calibration for the benchmark model:

$\beta$	0.987
$\sigma$	1.1
$\frac{1-\gamma}{\gamma}$	1.78
$\delta$	0.012
$\alpha$	0.4
$e$	2
$\rho_1 = \rho_2 = \rho_3$	0.95
$\Pi^{SS}$	0.16
$\tau^{SS}$	0.29
$sd(\varepsilon_1) = sd(\varepsilon_2) = sd(\varepsilon_3)$	0.0075

In the simulations illustrated in section 4, we have considered four special cases of this model:

1. Ramsey model ( $\gamma = 1; e = 1; \Pi^{SS} = 0; \tau^{SS} = 0$ )
2. RBC model ( $e = 1; \Pi^{SS} = 0; \tau^{SS} = 0$ )
3. RBC model with public expenditure financed by distortionary taxes ( $e = 1; \Pi^{SS} = 0$ )
4. RBC model with public expenditure financed by lump-sum taxes and externalities from production ( $\tau^{SS} = 0$ )



Table 1: Time Series Models for Productivity

<b>Dependent variable: <math>\log PR_t</math></b>							
<i>CASE 1: Wasteful Government. Lump-Sum Taxes</i>							
	Canada	France	Italy	Japan	Spain	UK	USA
log PR(t-1)	0.459 (0.195)	0.967 (0.210)	0.835 (0.153)	1.424 (0.186)	1.174 (0.199)	0.962 (0.200)	0.798 (0.135)
log PR(t-2)		-0.287 (0.204)		-0.582 (0.187)	-0.393 (0.195)	-0.449 (0.179)	
Time	0.0018 (0.0007)	0.0040 (0.0019)	0.0011 (0.0015)	0.0006 (0.0006)	0.0013 (0.0009)	0.0081 (0.0026)	0.0028 (0.0018)
LM1(Prob>chi2)	0.191	0.130	0.444	0.960	0.338	0.450	0.124
<i>CASE 2: Optimal Government. Distortionary Taxes</i>							
	Canada	France	Italy	Japan	Spain	UK	USA
log PR(t-1)	0.689 (0.167)	1.107 (0.202)	0.874 (0.119)	1.468 (0.184)	1.388 (0.172)	1.104 (0.196)	0.801 (0.135)
log PR(t-2)		-0.393 (0.197)		-0.591 (0.186)	-0.623 (0.172)	-0.491 (0.182)	
Time	0.0029 (0.0014)	0.0044 (0.0020)	0.0011 (0.0017)	0.0013 (0.0011)	0.0041 (0.0018)	0.0080 (0.0028)	0.0035 (0.0023)
LM1(Prob>chi2)	0.157	0.274	0.166	0.717	0.309	0.820	0.084

Note: Sample period: 1985-2005 (except Canada: 1985-2004).

Table 2: Annual Average Log Change in Per-Capita Equivalent Consumption

	Wasteful Spending Lump-Sum Taxes	Optimal Spending Lump-Sum Taxes	Wasteful Spending Distortionary Taxes	Optimal Spending Distortionary Taxes
Canada	0.013	0.014	0.021	0.023
France	0.026	0.031	0.026	0.031
Italy	0.018	0.020	0.021	0.023
Japan	0.018	0.025	0.023	0.030
Spain	0.021	0.030	0.030	0.040
UK	0.032	0.036	0.036	0.039
USA	0.025	0.026	0.029	0.030

Note: Sample period: 1985-2005 (except Canada: 1985-2004).

Table 3: Components of the Annual Log-Change in Per-Capita Equivalent Consumption

	Wasteful Spending Lump-Sum Taxes		Wasteful Spending Distortionary Taxes		Optimal Spending Distortionary Taxes	
	Fraction due to:		Fraction due to:		Fraction due to:	
	<i>TFP</i>	<i>Capital</i>	<i>TFP</i>	<i>Capital</i>	<i>TFP</i>	<i>Capital</i>
Canada	0.445	0.555	0.658	0.342	0.690	0.310
France	0.830	0.170	0.827	0.173	0.857	0.143
Italy	0.659	0.341	0.707	0.293	0.724	0.276
Japan	0.429	0.571	0.559	0.441	0.661	0.339
Spain	0.512	0.488	0.663	0.337	0.747	0.253
UK	0.816	0.184	0.833	0.167	0.848	0.152
USA	0.830	0.170	0.852	0.148	0.858	0.142

Note: Sample period: 1985-2005 (except Canada: 1985-2004). TFP includes both expected present value and expectation revision.

Table 4: Annual Average Log-Change in Per-capita Consumption, GDP and Equivalent Consumption

	Consumption	GDP	Equiv. Cons. Opt Gov, Dist Tax	Equiv. Cons. Opt Gov, Dist Tax (US dynamics)	Equiv. Cons. Opt Gov, Dist Tax (No Leisure)
Canada	0.016	0.016	0.023	0.033	0.024
France	0.016	0.016	0.031	0.034	0.031
Italy	0.016	0.017	0.023	0.034	0.023
Japan	0.019	0.018	0.030	0.040	0.029
Spain	0.027	0.027	0.040	0.040	0.041
UK	0.024	0.030	0.039	0.035	0.039
USA	0.020	0.022	0.030	0.030	0.030

Note: Sample period: 1985-2005 (except Canada: 1985-2004).

Table 5: Welfare Gap Relative to USA: 1985, 2005 and Average

	Wasteful Spending Lump-Sum Taxes	Wasteful Spending Distortionary Taxes	Optimal Spending Distortionary Taxes
<i>PANEL A: 1985</i>			
Canada	-0.256	-0.294	-0.295
France	-0.069	-0.176	-0.165
Italy	-0.368	-0.420	-0.437
Japan	-0.511	-0.488	-0.471
Spain	-0.327	-0.396	-0.414
UK	-0.096	-0.182	-0.189
USA	0.000	0.000	0.000
<i>PANEL B: 2005</i>			
Canada	-0.407	-0.455	-0.451
France	-0.078	-0.240	-0.213
Italy	-0.569	-0.641	-0.664
Japan	-0.540	-0.582	-0.526
Spain	-0.396	-0.405	-0.362
UK	0.034	-0.068	-0.059
USA	0.000	0.000	0.000
<i>PANEL C: average 1985-2005</i>			
Canada	-0.328	-0.372	-0.373
France	-0.065	-0.201	-0.181
Italy	-0.445	-0.507	-0.529
Japan	-0.498	-0.505	-0.468
Spain	-0.348	-0.389	-0.375
UK	-0.026	-0.120	-0.116
USA	0.000	0.000	0.000

Note: estimates for Canada refer to 2004 and 1985-2004.

Table 6: Components of Welfare Gap Relative to USA: 1985, 2005 and Average

	Wasteful Spending Lump-Sum Taxes		Wasteful Spending Distortionary Taxes		Optimal Spending Distortionary Taxes	
	Fraction due to:		Fraction due to:		Fraction due to:	
	<i>TFP</i>	<i>Capital</i>	<i>TFP</i>	<i>Capital</i>	<i>TFP</i>	<i>Capital</i>
<i>PANEL A: 1985</i>						
Canada	0.936	0.064	0.945	0.055	0.945	0.055
France	0.968	0.032	0.988	0.012	0.987	0.013
Italy	1.007	-0.007	1.006	-0.006	1.006	-0.006
Japan	1.091	-0.091	1.096	-0.096	1.099	-0.099
Spain	0.855	0.145	0.880	0.120	0.885	0.115
UK	0.379	0.621	0.672	0.328	0.683	0.317
<i>PANEL B: 2005</i>						
Canada	0.900	0.100	0.911	0.089	0.910	0.090
France	0.421	0.579	0.811	0.189	0.788	0.212
Italy	0.972	0.028	0.975	0.025	0.976	0.024
Japan	1.065	-0.065	1.060	-0.060	1.067	-0.067
Spain	0.898	0.102	0.900	0.100	0.888	0.112
UK	-	-	-0.079	1.079	-0.233	1.233
<i>PANEL C: average 1985-2005</i>						
Canada	0.915	0.085	0.925	0.075	0.925	0.075
France	0.666	0.334	0.891	0.109	0.879	0.121
Italy	0.999	0.001	0.999	0.001	0.999	0.001
Japan	1.115	-0.115	1.113	-0.113	1.122	-0.122
Spain	0.887	0.113	0.899	0.101	0.895	0.105
UK	-1.513	2.513	0.448	0.552	0.443	0.557

Note: estimates for Canada refer to 2004 and 1985-2004.

Table 7: Per-Capita GDP, Consumption and Equivalent Consumption relative to USA: 1985, 2005 and Average

	<i>Consumption</i>	<i>GDP</i>	Equiv. Cons. Opt Gov, Dist Tax	Equiv. Cons. Opt Gov, Dist Tax (US dynamics)	Equiv. Cons. Opt Gov, Dist Tax (No Leisure)
<i>PANEL A: 1985</i>					
Canada	-0.179	-0.106	-0.295	-0.188	-0.300
France	-0.285	-0.250	-0.165	-0.190	-0.184
Italy	-0.378	-0.287	-0.437	-0.315	-0.496
Japan	-0.434	-0.211	-0.471	-0.505	-0.449
Spain	-0.624	-0.570	-0.414	-0.522	-0.446
UK	-0.306	-0.304	-0.189	-0.196	-0.195
USA	0.000	0.000	0.000	0.000	0.000
<i>PANEL B: 2005</i>					
Canada	-0.324	-0.177	-0.451	-0.353	-0.455
France	-0.401	-0.317	-0.213	-0.226	-0.240
Italy	-0.501	-0.370	-0.664	-0.553	-0.719
Japan	-0.456	-0.261	-0.526	-0.487	-0.531
Spain	-0.527	-0.419	-0.362	-0.423	-0.376
UK	-0.190	-0.219	-0.059	-0.203	-0.061
USA	0.000	0.000	0.000	0.000	0.000
<i>PANEL C: average 1985-2005</i>					
Canada	-0.248	-0.163	-0.373	-0.259	-0.377
France	-0.338	-0.265	-0.181	-0.166	-0.203
Italy	-0.396	-0.278	-0.529	-0.304	-0.579
Japan	-0.391	-0.176	-0.468	-0.403	-0.460
Spain	-0.537	-0.447	-0.375	-0.405	-0.397
UK	-0.220	-0.251	-0.116	-0.121	-0.120
USA	0.000	0.000	0.000	0.000	0.000

Note: estimates for Canada refer to 2004 and 1985-2004.

Table 8: Welfare Gap Relative to US and its Components in 2005: Extended Sample of Countries

	Welfare Gap	Fraction due to			Welfare Gap	Fraction due to:	
		TFP	Capital			TFP	Capital
Luxembourg	0.507	1.035	-0.035	Iran	-1.835	0.701	0.299
USA	0.000	—	—	Mexico	-1.965	0.777	0.223
Australia	-0.150	0.830	0.170	China	-1.970	0.694	0.306
Singapore	-0.162	0.195	0.805	Jamaica	-2.048	0.823	0.177
Norway	-0.191	1.317	-0.317	Malaysia	-2.185	0.837	0.163
UK	-0.226	-0.013	1.013	Costa Rica	-2.238	0.791	0.209
Sweden	-0.450	0.837	0.163	Thailand	-2.258	0.747	0.253
Belgium	-0.468	0.851	0.149	Panama	-2.470	0.809	0.191
Finland	-0.476	0.735	0.265	Tunisia	-2.484	0.750	0.250
Netherlands	-0.496	0.822	0.178	Argentina	-2.578	0.842	0.158
South Korea	-0.509	0.829	0.171	Guatemala	-2.706	0.680	0.320
Cyprus	-0.518	0.443	0.557	Egypt	-2.728	0.696	0.304
Denmark	-0.529	0.790	0.210	Venezuela	-2.744	0.780	0.220
Austria	-0.538	0.773	0.227	Sri Lanka	-2.746	0.770	0.230
France	-0.542	0.704	0.296	Brazil	-2.920	0.804	0.196
Canada	-0.565	0.855	0.145	India	-2.978	0.673	0.327
Hong Kong	-0.589	0.823	0.177	Indonesia	-3.071	0.748	0.252
Spain	-0.590	0.730	0.270	Peru	-3.152	0.834	0.166
Japan	-0.614	0.964	0.036	El Salvador	-3.335	0.798	0.202
Israel	-0.617	0.835	0.165	Syria	-3.452	0.744	0.256
Switzerland	-0.648	0.926	0.074	Ecuador	-3.480	0.844	0.156
Portugal	-0.701	0.558	0.442	Colombia	-3.683	0.833	0.167
New Zealand	-0.728	0.870	0.130	Honduras	-3.764	0.799	0.201
Ireland	-0.779	0.887	0.113	Philippines	-3.841	0.822	0.178
Romania	-0.936	0.644	0.356	Pakistan	-4.034	0.760	0.240
Italy	-1.068	0.846	0.154	Bolivia	-4.267	0.834	0.166
Trinidad and Tobago	-1.142	0.754	0.246	Lesotho	-4.336	0.795	0.205
Uruguay	-1.516	0.672	0.328	Morocco	-4.417	0.807	0.193
Chile	-1.604	0.759	0.241	Uganda	-4.426	0.727	0.273
Barbados	-1.630	0.890	0.110	Togo	-4.565	0.760	0.240
Turkey	-1.662	0.622	0.378	Paraguay	-5.418	0.874	0.126
Botswana	-1.767	0.791	0.209				

Note: estimates for Canada refer to 2004.

Figure 1: Numerical Experiments: Impulse Responses for log Equivalent Consumption

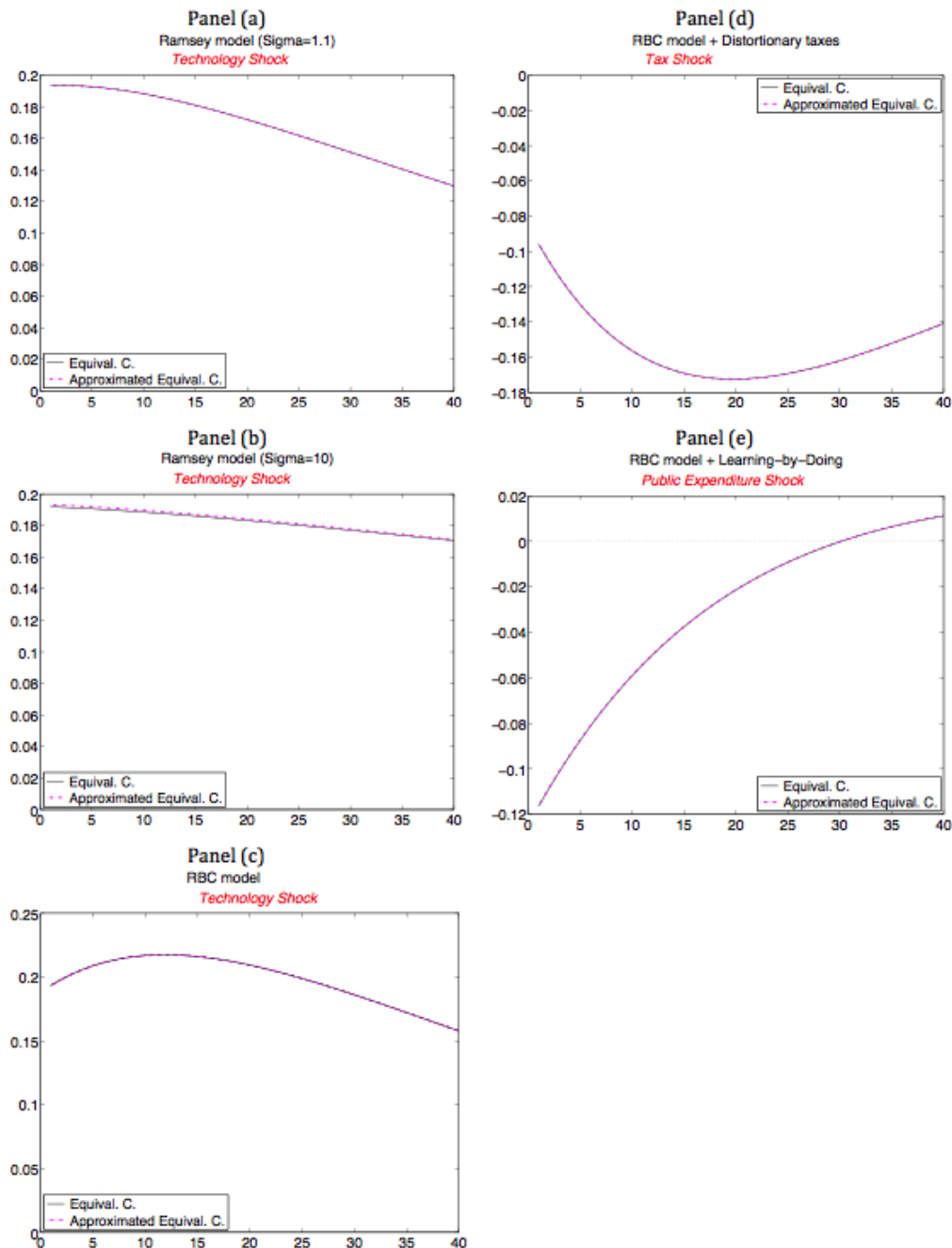




Figure 2: Numerical Experiments: Cross-Country Gaps for log Equivalent Consumption

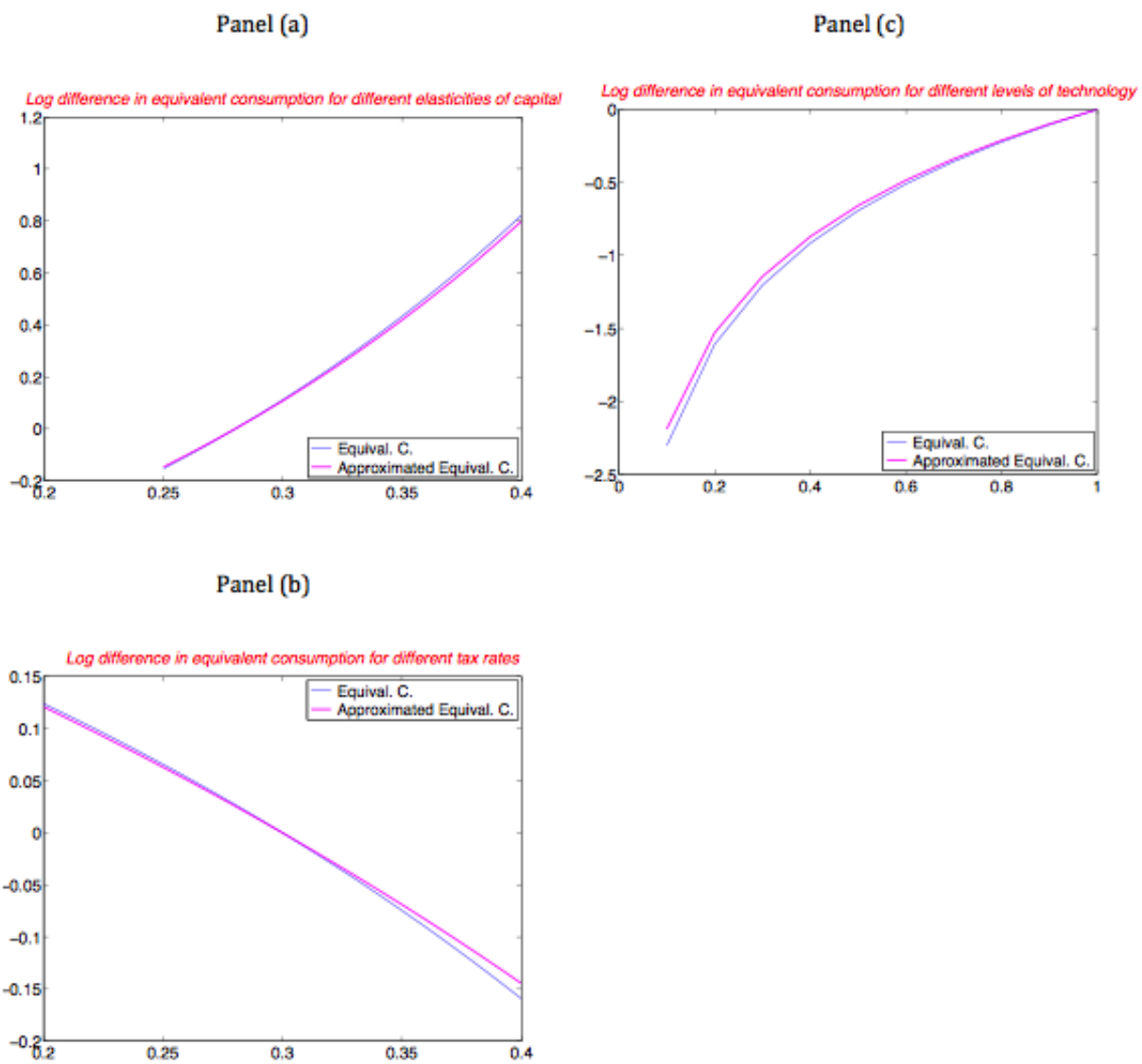


Figure 3: Within Country Welfare Changes (in log Equivalent Consumption): Wasteful Government Spending and Lump Sum Taxes

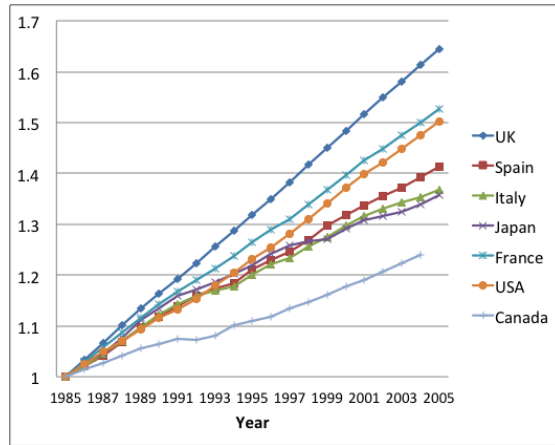


Figure 4: Within Country Welfare Changes (in log Equivalent Consumption): Optimal Government Spending and Distortionary Taxes

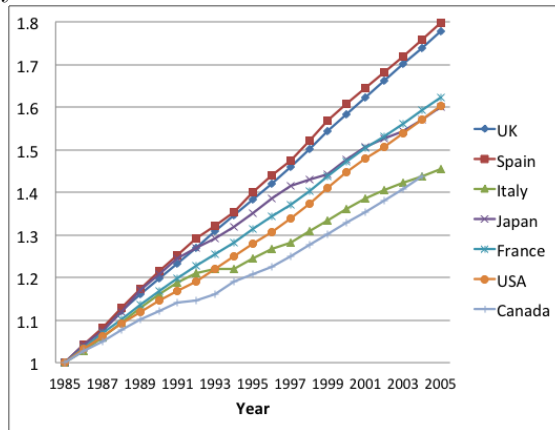


Figure 5: Within Country Welfare Changes with Human Capital (in log Equivalent Consumption): Optimal Government Spending and Distortionary Taxes

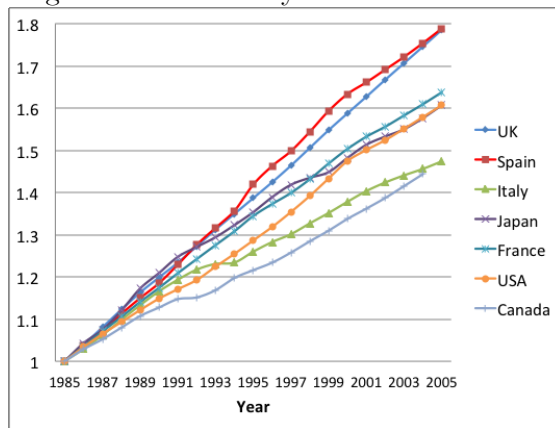


Figure 6: Cross Country Welfare Comparisons with US Preferences (log Equivalent Consumption Gaps vis-à-vis the US): Wasteful Government Spending and Lump Sum Taxes

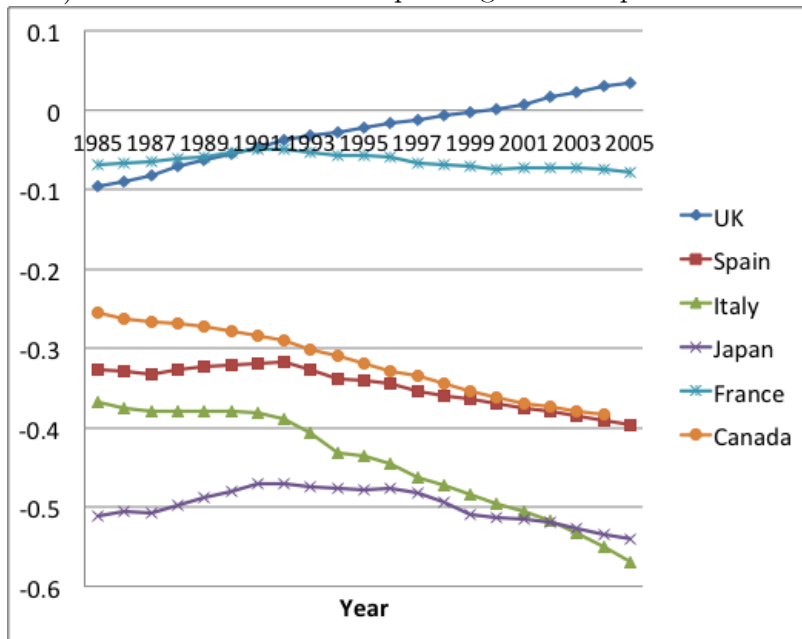


Figure 7: Cross Country Welfare Comparisons with US Preferences (log Equivalent Consumption Gaps vis-à-vis the US): Optimal Government Spending and Distortionary Taxes

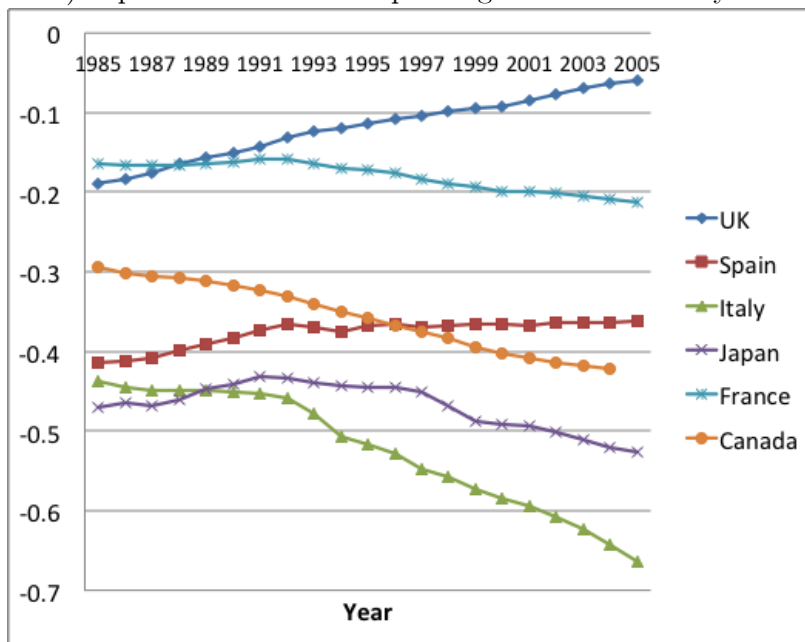


Figure 8: Cross Country Welfare Comparisons with French Preferences(log Equivalent Consumption Gaps vis-à-vis the US): Optimal Government Spending and Distortionary Taxes

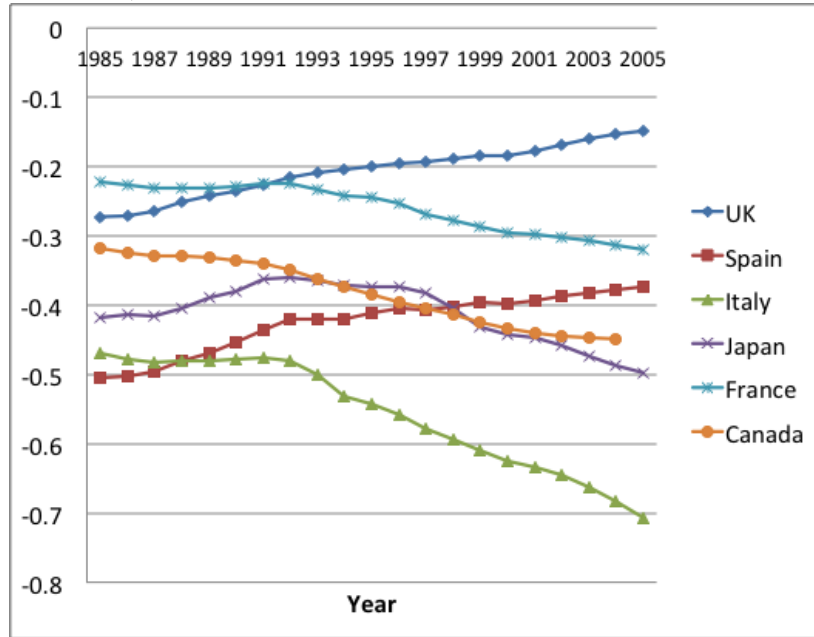


Figure 9: Cross Country Welfare Comparisons with US Preferences and Human Capital (log Equivalent Consumption Gaps vis-à-vis the US): Optimal Government Spending and Distortionary Taxes

