On the Origin of the Family
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On the Origin of the Family*

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Abstract

We present a game theoretic model to explain why people form life long monogamous families. Three components are essential in our framework, paternal investment, fatherhood uncertainty, and, perhaps the most distinctive feature of all, the overlap of children of different ages. When all three conditions are present, monogamy is the most efficient form of sexual organization in the sense that it yields greater survivorship than serial monogamy, group marriage, and polygyny. Monogamy is also the only configuration that fosters altruistic ties among siblings. Finally, our result sheds light to the understanding of why most religions center around the monogamous fidelity family.

Keywords: Overlapping generations; Free riding; Kinship systems; Religion.

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1. Introduction

Why do humans form long lasting monogamous unions? The fact that essentially all vertebrates are known to be non-familial suggests that the emergence of monogamy cannot be taken for granted.\(^1\) We address this question using a game-theoretic overlapping-generations model in which men and women care only for the survivorship and propagation of their own genes into future generations (Hamilton 1964). The main finding of the paper is that monogamy is the most efficient form of sexual organization among humans. The driving force behind this result is the overlap of children of different ages, a feature that sets humans apart from most other species, including anthropoid primates, such as gorillas, bonobos, and chimpanzees.

Besides this overlap, two other features are important in our framework, i.e., paternal investment and fatherhood uncertainty. Fatherhood uncertainty arises from the disproportionate abundance of sperms relative to eggs (Bateman 1948). Women and men trade food for sex within their marital unions consensually. But men also invest in casual sex with women other than their mates. This sex does not involve any food transaction from men to women and is anonymous and non-consensual. There is full intrahousehold labor specialization (Becker 1981/1991): men are the only providers, while women are the only carers.

To capture how the interaction of these three components — overlap of children, paternal investment and uncertain paternity — underpins the emergence of monogamy, we develop an overlapping-generations model in which hunter-gatherers live for five periods, the first two as children, when they rely on parental investment, and the last three as adults. During adulthood, opposite-sex individuals are matched and have children, all striving to maximize the expected number of their own offspring. On the one hand, adult men allocate their resources into three activities, hunting, mate guarding, and casual (or extra-marital) sex. On the other hand, adult women take care of their progenies by feeding them with the food they receive from men. The model fits not only primeval hunter-gatherers’ societies but also all other environments in which women must rely on paternal provisioning to raise their offspring.

We characterize stationary symmetric pure strategy Nash equilibria for a wide range of family conventions, i.e., monogamy, serial monogamy, group marriage (as well as serial group marriage), and polygyny. In the spirit of Young’s (1993) notion, a family convention is a specific matching protocol among men and women that is self-enforcing. As mentioned already, we show that life long monogamy is the most efficient family convention when

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\(^1\)See Lott (1991). Using a looser notion of family — which simply requires offspring to interact, into adulthood, with their parents — Emlen (1995) argues that less than 3 percent of avian and mammal species are known to be familial.
compared to the other family types under analysis. Greater efficiency here means that the monogamous family grows faster than the other family conventions. Why then is monogamy superior?

Consider for simplicity the comparison with serial monogamy. In a monogamous environment a man protects his paternity by guarding and giving food to the same woman during the three adulthood periods. In the serially monogamous convention instead a man does not keep the same partner from one period to the next, and thus has children from more than one woman. The efficiency difference between these two family types arises because mothers redistribute resources among their offspring. If a woman receives resources from two different men, there is a free-rider problem whereby each man realizes that his contribution to his children will be shared with other children who are most likely not his.\(^2\) Thus, every man will shift more of his resources toward extra-marital sex and consequently toward guarding, which are both socially wasteful activities.

Life long monogamy is more efficient than serial monogamy precisely because it is socially less wasteful allowing individuals to channel more resources to child provisioning, thus delivering a higher rate of population growth. Interestingly, this result lines up well with the extensive body of contemporary empirical evidence for advanced societies according to which children who live part of their childhood in a blended (serially monogamous) family have lower educational attainment and experience worse outcomes later in life (e.g., McLanahan and Sandefur 1994; Duncan and Brooks-Gunn 1997; Ginther and Pollak 2004).

The lower efficiency of group marriage is again driven by a free-rider problem, although its mechanism hinges on a classic tragedy-of-the-commons argument. Monogamy is also more efficient than polygyny. In this case, there is no free riding driven by coordination problems among hunters because each woman in a polygynous family will only have one male provider as in the monogamous family. There is however the problem posed by the men who are not matched. They do not face the need to hunt but consume their entire endowment in casual sex in the attempt of stealing paternity from the matched males. Matched men then will respond by increasing their effort in mate guarding. This, together with the fact that only a fraction of males supply food (as the unmatched do not produce), will underpin the greater efficiency of monogamy as compared to polygyny.

In one of the extensions of the model, we also establish that altruistic ties between siblings are uniquely associated with the monogamous family. That is, food transfers among siblings are efficiency enhancing, in the sense that they provide net survivorship gains, but

\(^2\)Private underprovision of public goods is a standard result (Samuelson 1955; Becker 1981/1991; Bergstrom, Blume, and Varian 1986). In Weiss and Willis’s (1985) model of divorce, the free-rider problem is between former spouses who cannot verify each others’ allocative decisions rather than between unrelated men who cannot write down binding verifiable contracts.
in equilibrium this happens only in the context of monogamy and not in the context of the other family conventions. Monogamy is thus the only family configuration in which an adult man values his siblings because they provide him with the assurance that some of his genes will survive into future generations.

The rest of the paper is organized as follows. Section 3 develops a formal model of family formation, discusses the notion of family convention and presents individual strategies and payoffs. As mentioned, the model is best suited to ancestral societies populated by hunter-gatherers, but in fact it can also fit all the environments in which paternal provisioning is necessary to child survival. Section 4 compares monogamy and serial monogamy and shows that the equilibrium that gives rise to the monogamous conventions is more efficient than that arising when individuals are serially monogamous. Section 5 compares monogamy to group marriage (and serial group marriage) and monogamy to polygyny, and demonstrates that monogamy is more efficient than these other conventions. Section 6 extends the model allowing for kin ties among siblings and considering the possibility of maternal food provision. It also discusses the relationship between our notion of family convention and religion, linking our main result to the remarkable fact that many of the major religions center around the monogamous fidelity family. Section 7 concludes. The proofs of all formal propositions are in the Appendix. Before turning to the model, however, Section 2 provides the key underpinnings of our research question and links our contribution to some of the most relevant studies in the literature.

2. Related Literature

Since Becker’s pioneering work (1973, 1974, and 1981/1991), the family has become a prominent area of investigation among economists. Becker (1973, pp. 818–820) emphasizes the importance of own children as the explanation of why men and women live together in an environment in which there is complementarity of male and female time inputs in production. Because own children are important, the notion of uncertain paternity is implicitly called upon in order to justify why unions of several men to one or several women are uncommon. In Becker’s analysis, however, men are not allowed to respond to fatherhood uncertainty (for example with the introduction of implicit contracts or with guarding), and all non-monogamous unions are essentially assumed to be less efficient than monogamous partnerships.

The focus of Becker’s research as well as of other subsequent studies instead has been on

\footnote{See Bergstrom (1996), Weiss (1997), Lundberg and Pollak (2007), and Browning, Chiappori, and Weiss (2011) for important surveys and discussions.}
the gains from trade that a man and a woman can realize by marrying compared to remaining single. The gains to marriage arise from gender specialization in home and market activities, provided that individuals have no direct preferences for spending time in some tasks and not in others, and that the time inputs of household members in the household production of public goods are perfect substitutes (Lundberg and Pollak 2007). In this environment, marriage, which is broadly defined to include both formal unions and cohabitations, corresponds roughly to our notion of the monogamous family. The alternative to marriage, however, is not another form of partnership (such as serial monogamy or polygyny as in the present study) but singlehood. The comparison to singlehood, however, is not compelling in an environment in which individuals care for the survivorship of their genetic endowment into the future.

We follow Becker’s work on the family also in another methodologically important way. That is, we use an individual optimizing approach imposing a minimal set of conditions to characterize how the monogamous family came about and grew in our ancestral past (see Becker [1989] and [1993] for related discussions).4 In particular, as emphasized already, our setup relies on the identification of the three basic conditions (overlap of different cohorts of children, paternal investment, and fatherhood uncertainty) that led humans to depart from other anthropoid primates and enter a runaway evolutionary process in the course of which the most salient aspects of the monogamy family unfolded. This is, in and of itself, an important methodological contribution of the paper. Few scholars in evolutionary biology and primatology have partly followed this design to explain the emergence of language, cultural transmission, maternal behavior, and kinship (McGrew 1992; King 1999; Hrdy 1999; de Waal 2001; Chapais 2008; Gavrilets 2012),5 but they have never used it before to explore the origin of monogamy and, from an epistemological standpoint, it is entirely new in economics.

To see how our three components work in the context of our theory, we consider each of them specifically. The first condition is the presence of overlapping cohorts of dependent children, who rely on large parental investments for a long period of time. Although scholars have long recognized its correlation with the necessity of protracted parental care (e.g., Malinowski 1944; Trivers 1972; Emlen 1995), they have failed to emphasize parenting as a cumulative activity that required parents to support multiple children of different ages at the same time and thus have never fully exploited it to build up a model of the family. We emphasize this as the most original component of the human parental load that sets us apart

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4Biologists and animal scientists, instead, usually identify several pathways that might have triggered the evolution of the family among humans and nonhuman animals alike (e.g., Emlen 1994; Reichardt and Boesch 2003; Chapais 2008; Gavrilets 2012).

5None of these contributions, however, formalizes a model with optimizing agents.
from other species.

A prolonged overlap of dependent children pushes up the demand for food. This underlies the salience of parental investment (Trivers 1972), something humans share with other species, including the complex non-familial colonies of eusocial insects (Wilson 1975) as well as the great apes. But, as argued by Key and Ross (1999), Kaplan et al. (2000), Kaplan and Robson (2002), and Opie and Power (2008), chimpanzee mothers can provide for themselves and their infants, while primeval human mothers cannot. Here is our second key ingredient: women look to men in order to make up their energy deficit. Paternal provisioning is then essential in our model for the emergence of monogamy.

The last basic component is fatherhood uncertainty. Also this is a feature that is not unique to humans, as it is shared, for example, by all males of mammal species in which female ovulation is concealed (Krützen et al. 2004). But most of these other species do not form long lasting families. In the case of humans, instead, fatherhood uncertainty must be linked to the other two previous ingredients. In particular, a man will not have an incentive to invest unless he has a relatively high confidence in his paternity. When fatherhood is uncertain, he must resort to guard his opposite-sex partner(s) if he wishes to reduce the chances that other males steal his paternity and enhance confidence in his biological association with the children he feeds (Hawkes, Rogers, and Charnov 1995). Our theory therefore is that the combination of these three components leads to the formation of the monogamous family.

This combination leads to another important result. When all three ingredients operate together, in fact, we can show that each of the family types analyzed in the paper can be supported as a Nash equilibrium. It is worthwhile stressing that nonhuman primates are known to have relatively rigid mating systems (Reichardt and Boesch 2003; Chapais 2008). For instance, in species that form polygynous units (such as baboons and gorillas), polygyny is the rule across groups of the same species. This, however, is not the case among humans, who exhibit a much greater intraspecific variation in mating arrangements. This multiplicity of equilibria emphasizes the importance of family conventions. A convention is a self-enforcing matching protocol among men and women, and with men and women adopting a specific set of strategies that are customary within a given convention, this does become an equilibrium that everyone expects (Young 1993).

Besides serial monogamy, we also focus on polygyny, which is a family convention that other economists have explored. Earlier studies by Becker (1973 and 1974) and Grossbard

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6Although there is some disagreement on the exact net food production estimates, all these studies reach empirical findings that are qualitatively similar.

(1976) show that polygyny can be explained in a world characterized by male inequality in wealth combined with gender differences in the constraints on reproduction. They do not have an explanation, however, of why polygyny has declined over time in those part of the world where it was once more common. Recent work by Gould, Moav, and Simhon (2008) does provide an explanation based on the increasing relevance of female human capital. Gould and colleagues argue that educated men increasingly value educated women for their ability to raise educated children, and this drives up the value of educated women to the point where educated men prefer one educated partner to multiple unskilled wives. Our model abstracts from male and female heterogeneity but, taking advantage of our three basic components, establishes the greater efficiency associated with monogamy.

Another important strand of economic research explicitly incorporates biological considerations into individual or household behavior (Bergstrom 1995; Robson 2001; Kaplan and Robson 2002; Cox 2007; Bergstrom 2007) as well as economic development and growth (Galor and Michalopoulos 2011; Ashraf and Galor 2013). For instance, Siow (1998) investigates how differential fecundity interacts with market structure to affect gender roles in monogamous societies. Alger and Weibull (2010) examine the strategic interactions between mutually altruistic siblings to assess the extent to which family ties may vary in relation to environmental factors, such as output variability and marginal returns to effort.

Somewhat closer to our approach is another set of studies that emphasize the importance of fatherhood uncertainty. These include Edlund and Korn (2002), Edlund (2006), Saint-Paul (2008), and Bethmann and Kvasnicka (2011). But, unlike ours, such papers rule out casual sex and mate guarding. Rather, they underline the explicit or implicit transfers that take place between a man and a woman engaged in sexual reproduction (as in the case of legal marriages, which are seen as a contractual form of establishing paternity presumption and custodial rights to the man), and examine their consequences in terms of, for example, the matching patterns in marriage markets, the dynamics of human capital accumulation and parental investments in children, and in response to environmental changes that might have altered the demand for marriage (e.g., the introduction of oral contraceptives). By looking at the question of why humans started to form families, we exclude the possibility of binding commitments and enforceable contracts and, rather, concentrate on mate guarding as men’s strategic adaptation to casual sex and uncertain paternity.

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8Earlier work by Becker (1976), which also considered the relationship between sociobiology and economics, argued that economic models can explain biological selection of altruistic behavior toward children and other kin by the advantages of altruism when there are physical and social interactions.

9Interestingly, Alger and Weibull (2010) relate the same environmental factors, including the harshness of the physical environment, to the development of specific religions. In Section 6 we touch upon the successful adaptability of humans to a wide variety of geographic environments, and we provide a brief discussion about religion from an alternative perspective.
The importance of male paternal strategies, which emphasizes the role of fatherhood uncertainty and male provisioning, has been predicated in other models, such as those developed by Hawkes, Rogers, and Charnov (1995), Kaplan et al. (2000), and Kaplan and Robson (2002). These models pursue different objectives from ours. They focus either on the allocation of male effort between caring for infants and competing for paternity or on the development of life-history/human-capital models that help explain the evolution of intelligence and longevity among primates and humans. Nevertheless, none of these models relies on the presence of overlapping cohorts of children and the need of feeding multiple children simultaneously, so they cannot identify monogamy as the most efficient form of sexual organization.

Diamond and Locay (1989) also stress the role played by uncertain paternity in explaining kin ties. They note that if a male is uncertain about paternity, he may wish to invest in his sister’s children with whom he is sure to share some genes. Like in our model, Diamond and Locay’s explanation is that a man values his sister’s children in part because they provide him with the assurance that some of his genes will survive into future generations. As the number of children of his official partner increases, the probability that the male is the father of at least one of them also increases. Thus the value of the assurance provided by sister’s children decreases with either an increase in the paternity probability or an increase in the number of own (presumed) children. As noted, our overlapping-generations model emphasizes an even more fundamental link, that is, the link between older and younger siblings rather than that between an adult man and his sororal nephews and nieces. Lower levels of genetic relatedness (e.g., with nephews and nieces) are expected to be of lower importance (Hamilton 1964). This also emphasizes the genealogical structures linking males, known as agnatic kinship, which figures out prominently in much of the recent evolutionary research (e.g., Chapais 2008). By stressing the individual and societal gains obtained through kinship ties, our result identifies the monogamy family as a key source of exchange among its members and thus a primary engine of primeval economic growth (Ofek 2001; Galor and Michalopoulos 2011).

A final point worth stressing is on interpretation. In our setup, a family convention specifies the matching protocol between men and women. Since extra-marital sex and guarding are socially wasteful, an institution that proscribes them may lead a convention to have an efficiency advantage over another convention that does not. A convention then can be seen as the body of social norms, beliefs, and institutions that may successfully affect survivorship or hamper it. One of such institutions is religion. This reasoning allows us to use our results to understand why most of the major world religions have centered around and supported

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10 See also Alexander (1974) and Kurland (1979) for earlier biological and anthropological research.
the monogamous fidelity family (Wilson 2002; Browning, Green and Witte 2006).\textsuperscript{11} Our paper contributes to the understanding of the broader association between religion and economic outcomes (Freeman 1986; Acemoglu, Johnson and Robinson 2005; Guiso, Sapienza, and Zingales 2006) and, more specifically, to the important debate about whether economic growth is affected by religious beliefs (Iannaccone 1998; McCleary and Barro 2006; Botticini and Eckstein 2007; Becker and Woessmann 2009).

3. The Model

A. Setup

Consider an overlapping-generations model in which identical men and identical women live for five periods. In each period, the sex ratio is equal to one. The first two periods define childhood, during which individuals are infertile, unproductive, and dependent.

The remaining three periods define adulthood. In each of the first two adulthood periods (labeled 1 and 2), opposite-sex individuals from the same cohort are matched and have multiple children. In the last period (labeled 3), individuals are not fertile and use their resources only to support their progeny. Men (and only men) are hunters and provide food for their offspring, while women (and only women) are caregivers and allocate the food between their children.\textsuperscript{12} Allowing mothers to be food providers does not alter any of our main results. We shall come back to this issue in Section 6.

In each period 1 and 2, a man divides his resource (or time) endowment into three activities: (a) hunting, which provides food necessary for child survival; (b) mate guarding, which increases the probability that the children he supports carry his genes; and (c) casual

\textsuperscript{11} Judaism, Christianity, Hinduism, Buddhism, Islam, and Confucianism view the monogamous family as a vital practice that lies at the foundation of society (Browning, Green and Witte 2006). Undoubtedly, there are exceptions. For instance, Confucianism and ancient Judaism permitted powerful men to have concubines. But children of concubines born outside of lawful marriage were usually stigmatized, often severely. Christianity sometimes idealized the sexually abstinent marriage and, with Buddhism, commanded celibacy for its religious leaders. Islam permitted, sometimes encouraged, polygynous marriages, as did Judaism for a time and occasional Christian sects. It should be emphasized that the Qur’an was first revealed to a culture steeped in polygyny. The Islamic approach was to limit the practice substantially (Al-Hibri and El Habti 2006). Interestingly, there are only two Qur’anic verses on polygyny. The first is a conditional permission arising within a very specific context, i.e., the treatment of orphaned girls whose guardians may want to marry them in order to appropriate their wealth. The second verse imposes strong conditions of fairness on men who intend to marry more than one woman, essentially making polygyny highly unlikely for a ‘righteous’ man (Rashid Rida 1975). Interestingly — and in spite of different current practices and different popular perceptions — the permission to marry up to four wives is premised on concerns about the oppression of orphans and appears only in that context (Al-Hibri and El Habti 2006).

\textsuperscript{12} This full gender specialization in home and market tasks echoes Becker (1981/1991). Interestingly, for hunter-gatherer societies, Robson and Kaplan (2006) provide evidence according to which, after taking own consumption into account, women supply 3 percent of the calories to offspring while men provide the remaining 97 percent. For further discussions, see Ember and Ember (1983) and Kaplan et al. (2000).
(or extra-marital) sex, which increases the chance that his genes are represented in the next generation’s gene pool. In period 3, men are only hunters because they (as well as all the women in their cohort) are no longer fertile and thus do not engage in guarding or sex.

Notice there are two distinct types of sexual activity in the model. One is within the marital environment and entails a food-for-sex type of exchange between partners. The other type of sex (what we call casual or extra-marital sex), which occurs outside the marital bond and does not require female consent, is anonymous, pervasive, and does not involve any food transfer from men to women. The asymmetry with which the two types of sex are characterized is driven by the differential reproductive success of men and women (Bateman 1948): a male’s reproductive success is not limited by his ability to produce sex cells but by his ability to fertilize eggs with these cells, while a female’s reproductive success is not limited by her ability to have her eggs fertilized but by her ability to produce eggs. In subsection 4.D, this asymmetry will be reflected in differential payoffs by gender.

We consider four alternative and mutually exclusive family configurations (or conventions): (a) the monogamy family, denoted by $C_M$, where a female is matched with the same male in periods 1 and 2; (b) serial monogamy, denoted by $C_S$, where each female is matched with a male in period 1 and another male in period 2; (c) full promiscuity (or group marriage or cenogamy), $C_G$, in which a group of men are matched to a group of women and all group members share equal parental responsibilities; and (d) polygyny, $C_P$, where one man mates with more than one woman each period.

A family convention therefore specifies the strategy profile for all individuals. In particular, it identifies the matching protocol between men and women, i.e., the woman (or women) a man provides food to and the man (or men) a woman chooses to have sex with. Conventions are not imposed on, but followed by individuals as part of the equilibrium strategy. In any given environment where individuals follow a specific matching practice, men and women are expected to adopt the prevailing sexual practice as long as this is their best response to other individuals’ strategies. Each man and each woman will conform to the protocol and to the equilibrium strategies it implies, expect all the others to conform, and want to conform given everyone else conforms. Thus, a convention emerges as an equilibrium outcome. This means that not every matching protocol is a convention, that is, the requirement that a given convention must be an equilibrium places a restriction on the very notion of convention.

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13 Much evolutionary research on human mating has emphasized the larger and more direct benefits gained by men rather than by women seeking brief sexual encounters outside their marital relationship (e.g., Trivers 1972; Clark and Hatfield 1989; Baker and Bellis, 1995). The literature has also illustrated a number of channels through which casual sex can be either beneficial or harmful to women (Symons 1979; Gregor 1985; Daly and Wilson 1988; Gangestad and Thornhill 1997; Greiling and Buss 2000).

14 As mentioned in the Introduction, this is consistent with the notion of convention proposed by Young (1993).
We focus on stationary symmetric pure strategy Nash (SSPN) equilibria. For clarity of exposition, the framework developed below and in Section 4 considers monogamy and serial monogamy only and shows that there are two SSPN equilibria, one in which individuals choose to be monogamous and another in which they choose to be serially monogamous. Section 5 presents the analysis for group marriage and polygyny.

B. Strategies

An adult man is productive only in periods 1 and 2 and can store food from period to period at no cost. A man’s strategy consists of (i) identifying the female(s) whom he provides food to and guards during his adulthood, and (ii) choosing the allocation of his resources across periods and activities. Because in equilibrium a male does not guard nor transfer food to women who do not select him, without loss of generality, we focus only on the food transfers to and guarding of women who choose him. Specifically, let $x_1$ and $x_2$ define the time equivalent amounts of food a man transfers to the mother of his first-period children in periods 1 and 2, respectively. Similarly, he gives $y_2$ and $y_3$ to the mother of his second-period children in periods 2 and 3. Let $g_t$ denote the amount of time a man devotes to guarding his mate and $k_t$ be the amount of time he spends in casual sex in period $t$, $t = 1, 2$. A male then will face the following lifetime resource constraint

$$1 = \sum_{t=1}^{2} (g_t + k_t + x_t + y_{t+1}),$$

which is defined over the entire adulthood period.\textsuperscript{15} Without loss of generality, each man is endowed with 1 unit of resources.\textsuperscript{16} The exact timing of such decisions is irrelevant, except that guarding and sex (whether marital or not) will not be carried out in the last period, because all adults of the same generation (men and women alike) are no longer fertile.

A woman’s strategy consists of (i) identifying the male(s) with whom she has sex (in exchange of food), and (ii) allocating the food she receives between her offspring. This allocation decision is straightforward in period 1, when she gives all the food she receives, $x_1$, to her first-period children. In period 3, she also cannot affect the amount of food supplied by the male to the second-born for her children’s second stage of childhood, $y_3$. Her decision instead is more complex in period 2, when she has two cohorts of children to nurture. In this case, keeping her partner’s decisions as given, the mother chooses the food

\textsuperscript{15}For simplicity, expression (1) assumes the possibility of free storage and borrowing. However, provided that men are less productive in their last adult period, only saving but not borrowing will occur in equilibrium.

\textsuperscript{16}This normalization gives us a straightforward interpretation of the technological environment described in the next subsection. In the numerical example developed in Section 4, we relax it and allow for a different value of the overall male lifetime resources.
allocation, \( m \in [0, 1] \), that maximizes her payoff, which — contrary to the male’s — includes neither guarding, because motherhood is certain, nor casual sex, because her total fertility is unaffected since her reproductive success is limited by her ability to produce eggs. The amounts of food she allocates to her first-period and second-period children are thus given respectively by \( m(x_2 + y_2) \) and \((1 - m)(x_2 + y_2)\), where \( x_2 + y_2 \) is the total amount of food she receives from her partner(s) in period 2.

C. Technologies

Child survival is stochastic and depends only on the amounts of food children receive during the first two periods of life. These two amounts are generically denoted by \( u \) and \( v \) respectively. From the parents’ viewpoint, the expected number of children of a given cohort who survive into adulthood is given by \( F(u, v) \). The function \( F \) is assumed to satisfy the following conditions.\(^{17}\)

**Assumption 1.** The function \( F : \mathbb{R}^2_+ \to \mathbb{R}_+ \) has the following properties:

(i) \( F \) is increasing, twice differentiable, and strictly 2–concave;\(^{18}\)

(ii) \( F(0, v) = F(u, 0) = 0 \);

(iii) \( \lim_{u \to 0} F_u(u, v) = +\infty, \ \forall v > 0 \); \( \lim_{v \to 0} F_v(u, v) = +\infty, \ \forall u > 0 \);

(iv) \( F_{uv} > 0, \ \forall u > 0 \) and \( \forall v > 0 \);

(v) \( F(u, v) = F(v, u), \ \forall u \geq 0 \) and \( \forall v \geq 0 \).

Assumption 1(i)–(iii) guarantees \( F \) has a number of properties that are similar to those of a standard neoclassical production function. Assumption 1(iv) ensures that \( F \) is supermodular, which in our context is rather natural and equivalent to the dynamic complementarity property discussed by Cuhna and Heckman (2007). Finally, Assumption 1(v), which is invoked only for analytical convenience, imposes a symmetry condition of parental investment timing on survivorship: that is, the probability that a child survives with transfer \( u \) in the first period of childhood and \( v \) in the second is the same as the survival probability obtained

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\(^{17}\)For analytical convenience only, we assume that the actual death of children born in a given period can occur only after the two periods of childhood at the beginning of the adulthood stage. Thus, \( F \) can be interpreted as the number of children who are expected to reach adulthood as *fertile* and *productive* individuals. All children reach the adulthood stage but, without adequate parental investment, they will be unfit to mate and (re-)produce. The model also abstracts from other features which are not essential for our results to hold, such as economies of scale in food production.

\(^{18}\)A function \( f \) is strictly \( \alpha \)-concave if \( f^\alpha \) is strictly concave (Brascamp and Lieb 1976; Kennington 1985). For a formal definition and use of this property in our context, see the Appendix.
when the time order of the transfers is reversed to \( v \) first and \( u \) second. This, in the context of the skill formation model proposed by Cuhna and Heckman (2007), implies that early and late childhood investments are equally critical and equally sensitive.

Consider a man and a woman who select each other as partners. From an individual male’s point of view, let \( \hat{k} \) be the average amount of time spent in extra-marital sex by all other men. The probability that guarding is successful is given by the paternity function \( P(g, \hat{k}) \), while the probability that a man’s guarding is not successful is \( 1 - P \). Suppose the function \( P \) is separable and twice differentiable, and takes the form \( P(g, \hat{k}) = \pi(g)\theta(\hat{k}) \), with \( \pi : [0, 1] \to [0, 1] \) and \( \theta : [0, 1] \to [0, 1] \).

**Assumption 2.** The functions \( \pi \) and \( \theta \) exhibit the following general properties:

1. \( \pi \) is increasing and strictly 2–concave; \( \theta \) is decreasing, and \( 1/\theta \) is strictly concave;
2. \( \pi(0) = 0 \), and \( \pi(g) > 0 \) \( \forall g > 0 \); \( \theta(0) = 1 \), and \( \theta(k) \geq 0 \) \( \forall k \geq 0 \);
3. \( \lim_{g \to 0} \pi'(g) = +\infty \); \( \lim_{k \to 0} \theta'(k) = -\infty \).

Assumption 2 states standard regularity conditions on \( \pi \) and \( \theta \) so that the payoff functions, which will be defined in the next subsection, are strictly concave.\(^{19}\)

We next consider the preference specification conditional on a given family convention. In what follows, the ‘hat’-notation is used to indicate variables that are not a choice under the chooser’s control.

**D. Payoffs**

**Monogamous Man** — It is important to emphasize again that individuals care only for the survivorship of their genes. A monogamous man chooses \( g_1, k_1, x_1, g_2, k_2, x_2, y_2, \) and \( y_3 \) to maximize his payoff (or fitness)

\[
\begin{align*}
\frac{1}{2} \pi(g_1)\theta(\hat{k}_1)F(x_1, \hat{m}(x_2 + y_2)) &+ \frac{1}{2} \pi(g_2)\theta(\hat{k}_2)F((1 - \hat{m})(x_2 + y_2), y_3) \\
&+ \frac{1}{2} \left[1 - \pi(\hat{g}_1)\theta(k_1)\right]F(\hat{x}_1, \hat{m}(\hat{x}_2 + \hat{y}_2)) + \frac{1}{2} \left[1 - \pi(\hat{g}_2)\theta(k_2)\right]F((1 - \hat{m})(\hat{x}_2 + \hat{y}_2), \hat{y}_3)
\end{align*}
\]

subject to (1) and standard nonnegativity constraints \( 0 \leq g_1, k_1, x_1, g_2, k_2, x_2, y_2, y_3 \). The first term in (2) indicates the contribution to the man’s payoff from first period offspring. These

\(^{19}\)Under the conditions imposed by Assumption 2, separability of \( P \) in \( g \) and \( k \) implies that there will be always some guarding even when casual mating tends to disappear. Guarding therefore must be interpreted more broadly than vigilance.
children are genetically related to the adult male with probability $\pi \theta / 2$, while $F(x_1, \hat{m}(x_2 + y_2))$ represents the expected number of such children surviving the first and second periods of childhood. Notice that when casual sex is arbitrarily close to zero and guarding perfectly effective, then $\pi \theta = 1$ and the degree of genetic relatedness with own child will boil down to the standard coefficient of $1/2$. The same considerations apply to the second term which captures the payoff obtained by a man through his second period offspring.

The last two terms in (2) refer to the payoff a male can obtain from casual sex. With probability $1 - \pi \theta$, he is the unofficial father of other children who will not be supported by him. They instead receive food by the official mate of the mother and the cheating male does not have control over such transfers. This is why he takes them as given, and these are denoted with the ‘hat’-notation.

Serially Monogamous Man — Taking into account that the female he guards already has (or will eventually have) children from a different male, a serially monogamous man chooses $g_1, k_1, x_1, g_2, k_2, x_2, y_2,$ and $y_3$, to maximize his payoff

$$
\frac{1}{2} \pi (g_1) \theta (k_1) F(x_1, \hat{m}(x_2 + \hat{y}_2)) + \frac{1}{2} \pi (g_2) \theta (k_2) F((1 - \hat{m})(\hat{x}_2 + y_2), y_3) \\
+ \frac{1}{2} \left[1 - \pi (g_1) \theta (k_1)\right] F(\hat{x}_1, \hat{m}(\hat{x}_2 + \hat{y}_2)) + \frac{1}{2} \left[1 - \pi (g_2) \theta (k_2)\right] F((1 - \hat{m})(\hat{x}_2 + \hat{y}_2), \hat{y}_3)
$$

subject to (1) and the usual nonnegativity constraints. This formulation acknowledges that, from the viewpoint of a given male in period 2, the woman he guarded in the first period (whom he expects to be the mother of his first period children) receives $x_2$ from him and $\hat{y}_2$ from the (different) male who guards her in period 2 and with whom she begets her second period offspring (first term in (3)). The opposite occurs for the mother of his second period children (second term). The last two terms in (3) are identical to the corresponding terms in (2) for the monogamous man.

Adult Woman — A female must decide on how to allocate food to her progeny. The food is given to her by her official partner(s), i.e., the man (men) responsible to guard her. She cannot affect the amount of food given to the first period children in her first adult period, $\hat{x}_1$, nor the amount of food received by the second period children in the third period, $\hat{y}_3$. Thus, irrespective of whether we consider monogamy or serial monogamy, her problem is to choose the food allocation, $m$, in the second period to maximize her own payoff

$$
\frac{1}{2} F(\hat{x}_1, m(\hat{x}_2 + \hat{y}_2)) + \frac{1}{2} F((1 - m)(\hat{x}_2 + \hat{y}_2), \hat{y}_3),
$$

(4)
while taking males’ choices as given. Since she has no uncertainty about her motherhood, the degree of genetic relatedness to each of her children is 1/2.

A brief remark on how casual sex enters into the payoff structure of the model is timely at this point. As mentioned in subsection 3.A, men have a reproductive comparative advantage to access extra-marital sex as opposed to women. Our model reflects this gender asymmetry in the reproductive benefits of extra-marital sex by making casual sex payoff relevant for men and normalizing its net benefits for women to zero.²⁰

4. Monogamy and Serial Monogamy Equilibria

As we have already emphasized in the previous section, a woman chooses a male partner (or a pair of partners, one partner per period) and decides on the allocation of food among her children. A man decides on his female partner (or pair of partners) whom he provides food to and on the time he spends on casual sex and guarding. The “agreement” that women and men enter into, which is governed by the π, θ and F functions, defines the family convention in which men and women choose to live. We focus on stationary symmetric pure strategy Nash (SSPN) equilibria and adopt the following definition.

Definition of Equilibrium. An equilibrium is a 2-tuple \((C_j^*, A_j^*)\), \(j = M, S\), where \(C_j^*\) is the family convention that specifies the matching protocol identifying who matches whom, and \(A_j^*\) is the corresponding allocation profile, with \(A_j^* = (m^*, g_1^*, k_1^*, x_1^*, g_2^*, k_2^*, x_2^*, y_2^*, y_3^*)\), such that \((C_j^*, A_j^*)\) is a stationary symmetric pure strategy Nash equilibrium. That is, given \((C_j^*, A_j^*)\), the best response for any individual is to follow the family convention \(C_j^*\) and choose the proposed allocation \(A_j^*\).

We start by establishing that both family conventions can be supported as a Nash equilibrium. The proof is in the Appendix.

**Proposition 1.** (i) (Monogamy) There exists a unique SSPN equilibrium \((C_M^*, A_M^*)\), in which \(g_1^* = g_2^*, k_1^* = k_2^*, \text{ and } x_1^* = y_3^* = (x_2^* + y_2^*)/2\). (ii) (Serial monogamy) There exists another unique SSPN equilibrium \((C_S^*, A_S^*)\), in which \(g_1^* = g_2^*, k_1^* = k_2^*, \text{ and } x_1^* = y_3^* > (x_2^* + y_2^*)/2\).

Proposition 1 shows that monogamy and serial monogamy share a number of important equilibrium features. In each of the matching protocols, mothers split resources equally

²⁰An interpretation of this gender asymmetry in the payoff structure is that, even if women did not willingly engage in extra-marital sex, men resorted to rape (Thornhill and Palmer 2000). Rape might have been pervasive in ancestral populations. In a study of 186 societies in the Standard Cross-Cultural Sample, Bronde and Greene (1976) show that in one-quarter of societies, rape is accepted or ignored and in another 30 percent is only mildly disapproved or ridiculed.
between the two cohorts of their children (i.e., \( m_M^* = m_S^* = 1/2 \)) and both cohorts receive equal amounts of food over the entire childhood, regardless of birth order. In both periods 1 and 2, all men devote an equal fraction of their resource endowments to guarantee paternity. Similarly, they spend an equal fraction of their time on extra-marital sex in both periods.

There is however a key difference between children in the monogamous family and children in the serially monogamous family: in the latter convention, first-born children receive more food in their first period of life, while second-born children receive more food in their second stage of childhood.\(^{21}\)

The reason for this difference is simple and underlies one of the main insights of the paper. In period 2, each adult individual has two cohorts of children to support. Differently from the monogamous family where a woman receives food for all her children from one man, in the serially monogamous convention, a woman receives food from two different men, and each man provides food for two different women. But when the two men transfer food to one woman, they free ride on each other and supply less food than what they would have done if they were the sole providers. In the second period, food provision is thus less efficient from the serially monogamous male’s viewpoint because he does not have control over the allocation decision of his (current or former) mate, who could use part of his food provision to care not for his offspring but for the offspring of the other man. The free-riding behavior between two competing male providers is the fundamental mechanism here, and will play a key role also in the group marriage configuration (see subsection 5.A).

After having established existence of the two SSPN equilibria, we now compare them and discuss their relative efficiency with the following result.

**Proposition 2.** Survivorship in the monogamous family is always greater than survivorship in the serial monogamy family.

As in Proposition 1, the mechanism underpinning the dominance of monogamy over serial monogamy is the absence of free riding in the monogamous family convention. Compared to the environment when individuals are serially monogamous, food provision in the monogamous family is more efficient. This greater efficiency comes about through two channels. The first is that the absence of free riding has a direct effect on the individual male incentive to hunt and supply food, because all his food transfers are received by his own offspring when he mates monogamously. This means that children of monogamous parents receive more food.

\(^{21}\)This asymmetry result is general and does not depend on the assumption that individuals are fertile for only two periods.
The second channel is a more subtle aspect of the mechanism identified in Proposition 1. Monogamous men and women distribute food to their offspring equally in such a way that the two childhood periods are perfectly symmetric. This guarantees an equal food distribution for both cohorts in both periods. Since the survivorship function $F$ is concave and symmetric, this distribution turns out to be the most efficient intrahousehold allocation.

Now, when greater food provision is efficiency enhancing (irrespective of whether this occurs through the first channel or the second), each man will have an incentive to shift resources away from extra-marital sex into hunting, a shift that in turn will reduce the need for mate guarding. This triggers a multiplier effect. Since cheating and guarding are wasteful from a societal viewpoint, the equilibrium outcome is therefore more efficient in the monogamy family. Conversely, in the serially monogamous family, every man finds it more productive to shift resources away from hunting and into casual sex, which in turn induces greater guarding in equilibrium.

**Example** — We illustrate our results using the following specifications:

$$F = (uv)^\alpha, \quad \pi(g) = g^\beta, \quad \text{and} \quad \theta(k) = 1 - \gamma k^\delta.$$  \hspace{1cm} (5)

Suppose $\alpha = 1/4$, $\beta = 1/10$, $\gamma = 1/4$, and $\delta = 1/3$. With these parameter values, the $F$, $\pi$, and $\theta$ functions satisfy Assumptions 1 and 2. Under such a parameterization and imposing a lifetime male resource endowment of 2, the monogamous convention implies the following approximate equilibrium values: $g^*_t,M = 0.16$ and $k^*_t,M = 0.06$, $t = 1, 2$, and $x^*_1,M + x^*_2,M = 0.78$ (which also equals $y^*_2,M + y^*_3,M$). Such values lead to $\pi(g^*_M) = 0.83$, $\theta(k^*_M) = 0.90$, a survivorship rate of 0.63, and a fitness level derived from each official offspring of 0.47. The same specification when individuals follow the serially monogamous configuration instead implies equilibrium values of guarding and casual sex of approximately 0.19 and 0.09 in each period respectively, and $x^*_1,S + x^*_2,S = 0.48$ (= $y^*_2,S + y^*_3,S$). These in turn imply $\pi(g^*_S) = 0.85$ and $\theta(g^*_S) = 0.89$, which are relatively similar to those found in the monogamous equilibrium, but a survivorship rate of 0.58 and a fitness level of 0.44, which are substantially lower than in the $C_M$ case. This example then clearly demonstrates the results of Proposition 2. In particular, it shows that, as compared to the monogamy convention, free riding in the serial monogamy convention leads to food underprovision as well as to greater extra-marital sex and greater guarding.
5. Other Family Conventions

A. Group Marriage

Group marriage (or cenogamy) is a family structure in which a group of (or all) women share a group of men in the population as partners and, similarly, the same men share the same women as partners in fully promiscuous relationships throughout their entire adult lives (Morgan 1871; Engels 1972; Ingoldsby and Smith 2006; Knight 2008; Allen et al. 2008). Even though cenogamy is not normative in any of the 1170 societies which comprise the Ethnographic Atlas (Murdock 1967a), Lee (1979) reports that it occurs sporadically in several.22 Our analysis provides a direct intuition as to why cenogamy is rare.

In such a convention, men supply food to all the women of the group and, in turn, the women pool the food together communally and use it to feed their children. It is important that this economy is divided up into separate subgroups. To gain a deeper insight, suppose as an example that each family grouping consists of two males and two females. For this matching protocol to be a convention, it must be that each of the two women chooses both men with whom she has sex and children, and each of the two men in turn chooses to guard and provide food to both women. In any given period, therefore, each female of the group will receive half of the resources both men devote to guarding as well as half of the total amount of food supplied by the two males, while the two men of the group share paternity equally.

We can straightforwardly extend this reasoning to any group of arbitrary size and obtain the following result.

Proposition 3. (i) (Existence and uniqueness) There exists a unique SSPN group marriage equilibrium. (ii) (Efficiency) Survivorship in the monogamous family is always greater than survivorship in the cenogamous family. (iii) As the group size increases, survivorship declines.

The existence proof in part (i) of Proposition 3 follows the same steps as those used to establish Proposition 1. The result in part (ii) is again triggered by a free-riding mechanism as illustrated in the two previous propositions. But now the free-rider problem is not driven by the necessity of paternal investment in overlapping cohorts of children, because groups are life

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22Beckerman and Valentine (2002) document that a specific form of group marriage, which they call ‘partible paternity’, is widely distributed across Lowland South America and found among peoples whose traditions diverged millennia ago, including the Mehinaku, the Arawete, the Xocleng, the Barí, the Kuikuru, and the Aché. Among the hunting and gathering Aché, Hill and Hurtado (1996) report that children in group marriages are slightly more likely to reach maturity. This correlation is confirmed by another study on the horticultural Barí (Beckerman et al. 1998).
long unions. Instead it is driven by a standard tragedy-of-the-commons argument (Hirshleifer 1995; Hawkes, Rogers, and Charnov 1995). Promiscuous males in a given marriage group free ride on each other’s food provisions and, compared to monogamous men, spend more time stealing paternity from males in other groups. As a result of this greater threat on their paternity, men’s incentive to guard their mates increases. This happens in all periods, and not just in period 2 as in the serial monogamy family, when competing males fail to internalize the need to coordinate over food provision. In equilibrium, therefore, the group marriage convention will be characterized by severe food underprovision and substantially high levels of male private consumption, i.e., guarding and extra-marital sex. Increasing the size of the group worsens the underprovision problem further. Interestingly, the inefficiency result of Proposition 3 is in line with some of the available anthropological evidence on contemporary cenogamous populations (e.g., Levine and Sangree 1980), but goes against the above-mentioned findings by Hill and Hurtado (1996) and Beckerman et al. (1998).

We also examined an alternative matching practice in which a group of men is matched with a group of women in a given period and with another group in the subsequent period. We call this family type ‘serial group marriage’. It is easy to show the following result.

**Proposition 4.** (i) (Existence and uniqueness) There exists a unique SSPN serial group marriage equilibrium. (ii) (Efficiency) Survivorship in the cenogamy family is always greater than survivorship in the serial group marriage family.

It is again the male free riding behavior in food provision that leads to inefficiency in this environment. This is the joint result of two channels: (a) the coordination problem among hunters in period 2 as we have in the serial monogamy configuration; and (b) the inefficiently large time devoted by males to extra-marital sex and mate guarding as we have in the standard group marriage convention. Such mechanisms place a severe limitation on the population growth that can be achieved by individuals in serial group marriages. This might be one of the reasons why this convention is not observed in modern world societies.

**B. Polygyny**

Polygyny is a family convention in which one man has multiple female partners at the same time. This is a widespread family type among many contemporary societies. For instance, of the 1170 cultures listed in the Murdock’s Atlas, 850 are coded for prevalence of polygyny (Hartung 1982). Tertilt (2005) reports that 28 countries in sub-Saharan Africa

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23Using data from the Standard Cross-Cultural Sample, Marlowe (2000) shows that about 82 percent of the 186 world’s ethnographically described societies exhibit some form of polygyny, although even in these polygynous societies the majority of unions are monogamous (Murdock 1967b).
have polygyny rates greater than 10 percent, and in Cameroon, Burkina Faso, and Senegal more than 40 percent of all men have multiple wives.

The analysis below focuses on the comparison between polygyny and monogamy, since the latter convention has been proved to dominate both serial monogamy and group marriage in terms of greater population growth and individuals payoffs. As we have done so far, we continue to focus on identical women and identical men. In particular, all men (whether matched or unmatched) have the same endowment. Despite this symmetry, the polygynous matching protocol is such that only a subset of the men in the population are chosen by women for a life long match. Each selected man in turn transfers food to all the women who choose him. The other men instead remain unmatched, and, as a result, they do not have the need to hunt but use their entire endowment on casual sex. In equilibrium, therefore, women ignore them and select only food providers.

We shall show that polygyny is less efficient than monogamy. To gain an insight as to why this is the case we stress that, since all individuals care only for the propagation of their genes, unmatched men in a polygynous convention will consume their entire endowment in casual sex in the attempt of stealing paternity from the matched males. Matched men then will respond by increasing their effort in mate guarding. This, together with the fact that only a fraction of males supply food, will drive the result. Notice that in this environment there is no free riding driven by coordination problems among hunters because each woman in a polygynous family will only have one male provider as in the monogamous family.

More formally, assume for simplicity that each matched polygynist has two women in period 1 and the same two women in period 2. It follows that half of the men are matched while the other half are not.\footnote{Both this assumption and the condition that matched polygynist men have two partners can be easily generalized to arbitrary fractions of the population being matched and to arbitrary numbers of female partners, provided that a unity sex ratio for each cohort is preserved.} All women are matched. As in our previous examinations, the objective is to determine the survival rate for individuals who follow the polygynous convention and compare it to the survival rate obtained under monogamy. Our analysis yields the following general result.

**Proposition 5.** (i) (Existence and uniqueness) There exists a unique SSPN polygyny equilibrium. (ii) (Efficiency) Survivorship in the monogamous family is always greater than survivorship in the polygynous family.

Let us elaborate on this result briefly. On the one hand, concavity of $F$ gives men an incentive to divide up their resource among multiple female partners. This could lead the polygynous configuration to enjoy greater population growth than its monogamous counter-
part. On the other hand, however, children of polygynous parents can rely on fewer resources than their monogamous counterparts because each family unit has now a greater number of children. This is aggravated by the presence of unmatched males who are unproductive and whose only activity is having casual sex that jeopardizes the paternity claims of the matched men.\(^{25}\) To have confidence in their fatherhood, therefore, matched men must dissipate part of their resources by increasing guarding significantly compared to the world in which males match monogamously.\(^{26}\) This in turn leads to lower population growth. The latter two forces outweigh the first and the net result is thus a smaller survival rate associated with the polygynous family.

An important point is in order. Previous studies have typically explained the emergence of polygyny through both differential fecundity between men and women and male inequality in wealth, whereby richer (and generally older) men tend to have multiple wives (Orians 1969; Grossbard 1976; Becker 1991).\(^{27}\) Males with more resources acquire multiple female partners on the account of males with fewer resources. In a world with large male endowment inequalities, polygyny is then likely to emerge in equilibrium as a more efficient family convention.\(^{28}\) Proposition 5 instead shows that, provided the differences in male resource endowments are not too large, monogamy dominates polygyny.

6. Extensions and Discussion

A. Kinship

Several studies have shown that individuals care not only for their own progeny but also for other genetically close relatives (Alexander 1974; Kurland 1979; Diamond and Locay 1989; Dunbar, Clark, and Hurst 1995; Knight 2008). They have documented a wide array of kin ties, supporting the notion that the costs and benefits of altruistic acts — in our case, food transfers among kin — are weighted by the closeness of genetic relatedness (Hamilton 1964; 

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\(^{25}\)The association of polygyny with heightened male-male competition has long been studied and documented (e.g., Orians 1969; Trivers 1972; Emlen and Oring 1977). Our contribution here is in linking this observation to the greater efficiency that humans can gain when switching to monogamy.

\(^{26}\)As shown by Hirshleifer (1995) in a different context, the diversion of resources into the socially wasteful activity of guarding (or conflict in Hirshleifer’s case) increases as the number of unmatched males goes up.

\(^{27}\)More recently, Gould, Moav, and Simhon (2008) develop a model in which higher female inequality in the marriage market reduces polygyny.

\(^{28}\)Consider, for example, the case in which half of the men have a unit of resource endowment, while the other half have no resources. In this case, monogamy is an inferior convention while a polygynous convention in which every male with positive resources is matched to two females is more efficient. In addition, it is worthwhile stressing that life long polygyny as described here is more efficient that serial polygyny (in which a male matches with two different groups of women in periods 1 and 2). This is driven by the same free riding mechanism that makes life long monogamy more efficient than serial monogamy as described in Section 4. This result can be easily verified and, thus, its proof is omitted.
Bergstrom 1995; Cox 2007; Alger and Weibull 2010).

An increasing number of scholars suggest that the uniqueness of human society lies in having developed a distinct type of kinship that cannot be found in other primates, which are genetically close to humans, such as chimpanzees, bonobos, and gorillas. This is the agnatic kinship structure, which emphasizes the genealogical structures linking males and the importance of male kin groups (e.g., Langergraber, Mitani, and Vigilant 2007; Chapais 2008).\footnote{The point here is not that other kin forms are unimportant. Much of the evidence presented and discussed in Allen et al. (2008) and Chapais (2008) documents the salience of uterine (matrilineal) kinship associated with early human development. Rather, our emphasis is that agnatic kinship is one of the critical features of the Pan-Homo split, the divergence between the lineage leading to common chimpanzees and bonobos (or Pan) and the lineage leading to humans (Homo).} Patrilineal kinship requires paternity recognition and paternal fitness, which — within our model — are likely to be best accomplished by monogamous families.

This extension therefore formalizes precisely this burgeoning view about agnatic kinship. In particular, we broaden our basic model in a way in which adult men can make food transfers to their presumed younger siblings.\footnote{The opposite transfers (from younger to older siblings) and more complex kinship systems involving individuals with lower levels of genetic relatedness (e.g., cousins, nephews, and nieces) are expected to be of second order importance in our framework and are thus not considered here.} Notice that, differently from the framework used in previous sections, siblings’ birth order now matters, so that older males face a different optimization program from that faced by their younger brothers. This is simply because younger brothers do not have other siblings to support, while older brothers do. The Appendix formalizes such programs.

We turn to our main result, which compares serial monogamy to monogamy. This has been obtained with the same parametric specifications as those given in (5) and used in the example of Section 4.

**Proposition 6.** Assume food transfers between siblings are possible. We characterize two findings:

(a) In the serial monogamy configuration, there exists a unique SSPN equilibrium in which there are no kin transfers.

(b) In the monogamous family convention, there exists a unique SSPN equilibrium in which: (i) there are positive kin transfers; and (ii) compared to the case when kin ties are not allowed, total survivorship is greater.

For the serial monogamy family, part (a) states that kin ties cannot emerge in equilibrium. The intuition is simple. Because each child has the same likelihood of survival and because any given man is genetically closer to his own children than to any other individual in the population, including his own half-siblings, it is optimal for him to provide food only to his
own offspring.

From part (b) we highlight two points. First, positive kin transfers occur in equilibrium because, given the survivorship function $F$ is concave, a man in the monogamous convention finds it advantageous to provide food to both his offspring and his (still dependent) siblings since these are equally genetically close to him. Greater efficiency in food provision translates into greater food transfers, lower levels of extra-marital sex, and lower need for guarding.

Second, in comparison to the case without kinship, firstborn receive strictly more food from their father in both periods of childhood. Later born children instead receive less food from their father, and even after taking into account the positive transfer from their older brothers, their total resources are lower than what they would have received in a context without kin ties. This primogeniture effect emerges because fathers, irrespective of whether they made or received food transfers, internalize the possibility that their firstborn will support their younger siblings. By transferring more food to their first born children, fathers spread their investment across children more equally, which, owing again to the concavity of $F$, increases their payoff. 31

Proposition 6(b) guarantees that a monogamous population in which families are linked through food exchange as opposed to another monogamous population without kin ties will be characterized by greater total food provision, which, in turn, will lead to greater survivorship. Because paternity is uncertain, two males who know they share the same mother could be genetically close to each other as much as they are to their own offspring. In these circumstances, transferring food to younger (still dependent) siblings is efficiency enhancing. Because of this, extra-marital sex goes down and thus the need for guarding recedes too. But lower cheating and lower guarding can only occur within the monogamous convention. The monogamous family therefore allows its members not only to achieve greater survivorship but also to foster “altruistic” behavior among siblings. This result then identifies the monogamy family as a key source of exchange among its members and thus a primary engine of economic growth (Ofek 2001; Chapais 2008; Galor and Michalopoulos 2012).

The notion of investing in siblings and more distant kin is relatively underinvestigated in economic research. Diamond and Locay (1989) offer the first and, to our knowledge, the only model that examines the investment of a risk-averse man in his sister’s children when he is uncertain about his paternity. The overlapping-generation model developed in this paper allows us to focus on the more basic kinship links between older and younger siblings rather

31Recent empirical studies based on contemporary data find evidence that is consistent with our primogeniture effect, with sizeable negative impacts of higher birth order on education, earnings, employment, and teenage fertility. See, among others, Behrman and Taubman (1986) and Black, Devereux, and Salvanes (2005). There is also an extensive literature in psychology and sociology on the effect of birth order on personality and intellectual development (e.g., Zajonc and Markus 1975; Conley 2004).
than those between a man and his sister’s children. This framework then provides us with the natural architecture for extending the analysis to lower levels of genetic relatedness, as in the case of Diamond and Locay, as well as other forms of kin ties, such as matrilineal kinship (Allen et al. 2008).

B. Maternal Food Provisioning

A feature of our model is that the man is the sole investor, in the sense that he is the only food provider (Kaplan et al. 2000; Robson and Kaplan 2006), while the woman has full control over the food distribution among children. Mothers nevertheless can be food providers. There are several examples of modern hunter-gatherer societies in which the female contribution to subsistence, albeit lower than men’s, is positive and not zero (Marlowe 2000; Kaplan and Robson 2002). An interesting question is to see how this possibility would affect our earlier findings. This extension shows that they do not change.

To address this issue we assume that each woman has a given period-specific amount of resources in addition to what she receives from her partner(s). As before, we only consider interior SSPN equilibria. This means that the mother’s food allocation $m \in (0, 1)$ and all paternal food transfers are positive. A sufficient condition for these properties to occur in equilibrium is that the mother’s total resource endowment is not too large. We characterize the following result.

**Proposition 7.** Suppose an adult woman has a given resource endowment, $\mu_t$, $t = 1, 2, 3$, such that interior SSPN equilibria exist. Then Propositions 1–5 hold true.

Proposition 7 ensures that, even in a world where mothers are food providers, monogamy continues to be more efficient than serial monogamy, polygyny, and group marriage, in the sense that it delivers greater population growth.

The intuition behind this result rests on a simple notion of income pooling (Becker 1981) or, more pertinently in our model, food sharing. Men and women take account of the availability of additional resources and make decisions under their new endowment $1 + \sum_t \mu_t = \omega$. The amount of resources received by each child is then the same as if the

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32 There are also few examples of foragers and horticulturalist societies in which women’s mean production is greater than men’s, especially in warm tropical climates, such as in the case of the !Kung and Gwi people of the Kalahari desert in Namibia, Botswana, and Angola (Kaplan and Robson 2002).

33 Because women do not face a trade-off between casual sex and food provision, they will transfer their entire endowment to children.

34 This is a fairly weak condition. When this is not the case, maternal provisioning must be so large relative to its paternal counterpart that mothers will be the sole providers. In that case, then, there exists a resource threshold value above which there is no interior equilibrium. But by focusing on interior equilibria, we rule this case out.
father had a greater total endowment than in the case with no maternal provisioning, i.e., \( \omega > 1 \). This reasoning applies not only to monogamy and polygyny, where this is only one provisioning male, but also to the serial monogamy and group marriage cases where there is free riding between the two or more men linked to the same woman (or women). This is possible because, in equilibrium, the two (or more) men make symmetric food transfers, while the woman (or women) will pool resources with each of the males separately.

Maternal food provisioning therefore neither eliminates nor attenuates the male social dilemma arising from free riding over food supply to offspring (as in the case of serial monogamy and group marriage) and male–male competition over paternity claims (as in the case of polygyny). Monogamy remains the most efficient equilibrium family configuration, provided that the mother’s food contribution is small enough.

The result is different, however, if maternal provisioning becomes more important. As mentioned before, if the female endowment is greater than a given critical threshold, there is no interior SSPN equilibrium. Because of the concavity of the male’s program, men will have an incentive not to provide food at all and free ride on women’s provisioning. In this case, mothers will be the only providers.

It is interesting to notice that among chimpanzees — with whom humans share an exclusive common ancestor (Chen and Li 2001) and an extremely close degree of genetic similarity presently estimated at 99 percent (Wildman et al. 2003; Varki and Altheide 2005) — infants are dependent for food on their mothers (not on their fathers) until they are weaned at about age 5 (Kaplan et al. 2000; Opie and Power 2008). Kaplan and Robson (2002) also document that chimpanzees obtain about 95 percent of their calories from foods that can be gathered by hand. Foragers instead obtain only 8 percent of their calories from easy-to-acquire foods, while the other 92 percent comes from extracted resources and hunted resources. This observation combined with the importance of paternal provisioning may help explain the geographically concentrated distribution of chimpanzee populations as compared to the much wider dispersion of humans who can successfully adapt in all of the world’s major habitats.

C. Religion and Convention

Central to our theory is the observation that, in a world where individuals seek to maximize their reproductive success, pervasive extra-marital sex and high levels of mate guarding are unavoidable, even though these are socially wasteful activities. A recurring point of the paper is that if, in the presence of such activities, a population adopts the monogamous family convention, it will grow more rapidly in equilibrium than another population that adopts a different convention.
Because of the inefficiency they generate, casual sex and guarding might induce the development of social norms, beliefs or institutions that can attenuate their negative effects. One of such institutions is religion. It could be argued that each of the family conventions analyzed in the paper can be seen, in and of itself, as a ‘religion’. The results in Sections 4 and 5, then, allows us to throw light on the remarkable fact that many of the major religions center around the monogamous fidelity family, even though human history has witnessed all kinds of different sexual organizations (Knight 2008).

With a given convention in place, it is hard to affect individual choices further. Yet religions generate moral systems that invariably try to affect equilibrium outcomes at the margin (Csikszentmihalyi 1990; Wilson 2002). Our model provides a simple explanation as to why almost all major world religions have openly condemned a wide range of sexual sins, including adultery and fornication, and stigmatized the product of extra-marital sex, illegitimate children (Browning, Green, and Witte 2006), while allowing or encouraging mate guarding, even if this — like extra-marital sex — is a socially unproductive activity. Reducing extra-marital sex has not only the direct effect of freeing up resources for food provision but also the indirect effect of reducing the necessity of guarding. Both effects increase survivorship. On the contrary, forbidding mate guarding, independently of the difficulty of its implementation, will inevitably give rise to increased cheating.

The notion of a religion or convention that promotes fidelity within the monogamous family is also supported by contemporary empirical evidence. Recent research has documented that, in the last fifty years and across a large cross-section of countries, economic growth has responded positively to religious beliefs (Barro and McCleary 2003) and that greater religious beliefs instill stronger work ethics (McCleary and Barro 2006). Some studies, however, find no evidence of an effect of religion on growth (e.g., Acemoglu, Johnson and Robinson 2001), while others stress the importance of an indirect effect of religion on economic growth through greater human capital accumulation (Botticini and Eckstein 2007; Becker and Woessmann 2009). With its emphasis on moral beliefs about the fidelity monogamous family, our model is likely to bring a new perspective to this important debate.

7. Final Remarks

The answer to the question that opened up the paper is simple: Humans are monogamous because monogamy is the most efficient form of sexual organization. The key component that drives this result is the overlap of children of different ages, which is perhaps the most

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35 This holds true for each of the six major religious traditions mentioned in Section 2, even if there are exceptions, the most notable of which perhaps is Islam. On this, see Section 2.
original component of the parental load faced by humans and not by other species. This overlap in turn may be a successful adaptation for the large size of the human brain, which makes children rely on long-term parental investment before they can become independent (Kaplan and Robson 2002). For early humans then monogamy is the most efficient way to take advantage of their larger brain in terms of the organization of reproductive resources.

Besides the presence of overlapping cohorts of children, paternal investment and fatherhood uncertainty are the other two components of the human phenomenon which underpin the emergence of monogamy. It can be argued that the paper goes full circle back to the seminal work by Becker (1973 and 1974) on marriage. But it does not assume, rather it proves, that family conventions other than monogamy — such as serial monogamy, group marriage, and polygyny — are less efficient. The efficiency gain of monogamy over serial monogamy is triggered by a free-rider problem in which a serially monogamous man realizes that his material contribution to his own children will have to be shared with children of other males. As a result he reduces his food provisioning and shifts more resources to extra-marital sex and thus to guarding, which are both socially wasteful activities. The efficiency gain of monogamy over group marriage hinges on the standard private underprovision of public goods. Finally, the efficiency gain of monogamy over polygyny is driven by the fact that a polygynous man must defend his fatherhood claims against the competition of unmatched men who spend their entire resources in the attempt of stealing his paternity.

The aim of the paper is to give an insight into why monogamy emerged in our ancestral past and, accordingly, the model should be seen in the context of early human societies in which children could not survive without paternal provisioning. However, it is interesting to notice that even in contemporary economies we have many examples of how the monogamous intact family provides children with a comparative advantage with respect other (non-monogamous) family structures (McLanahan and Sandefur 1994; Ermisch and Francesconi 2001; Björklund, Ginther, and Sundström 2007). As the paper documents, monogamy is not the only equilibrium, and it is therefore not surprising to observe other family conventions, both today and in our evolutionary past.36

Despite its simplicity the model delivers also other insights. Here we highlight two. First,

36Indeed, as an example, two-in-five contemporary Americans born between 1950 and 1955 had divorced by age 45, with similar figures found in other countries, such as the United Kingdom and Sweden (Stevenson and Wolfers 2007). Likewise, the proportion of births to unmarried women is sizeable, ranging from about 30% in Germany to approximately 55% in Sweden. For the United States, Martin et al. (2012) estimate that two-thirds of a million babies in 2010 (about 41% of all births) were born to unmarried mothers. As explained below, this contemporary evidence might be driven by factors that are beyond the scope of our analysis (e.g., the welfare state and contraceptive innovations). Clearly our model fits perfectly well a monogamous environment in which men have children from multiple women. This is the obvious result of extra-marital sex.
altruism between siblings can only emerge within a monogamous family convention. This is because every man, who is uncertain about his paternity and yet cares about his reproductive success, values his siblings in part because they provide him with the assurance that some of his genes will survive into future generations. Second, by specifying the matching protocol between men and women, a convention can be seen as a primeval form of religion. Our results then may provide the background to understand why most of the world religions promote fidelity typically within monogamy. We view our paper as a first cut at these exciting issues and expect that much new work will be done in each of such areas.

Our model, of course, captures only a few aspects of the complex problems that primeval women and men must have solved when forming their families. For instance, it depends on the assumption that all men are equally wealthy and attractive and all women are equally desirable and have the same preferences. This is clearly a simplification that may not be innocuous to our results. In fact, as mentioned in Section 5, in an environment in which there is considerable male heterogeneity in resources, polygyny can be more efficient than monogamy. It is not clear, however, how this result could survive when also female heterogeneity is allowed. As shown by Gould, Moav, and Simhon (2008), higher female inequality reduces polygyny. It then becomes crucial to understand how male and female inequalities are intergenerationally transmitted and, thus, how polygyny can be sustained as an equilibrium. This is an interesting issue, but it goes outside the scope of our paper.

Finally, some caution should be taken before hastening to interpret our results in the context of the contemporary family. With the aid of reliable DNA paternity testing, the issue of fatherhood uncertainty might have become less pressing than what it used to be among our ancestral predecessors. Similarly, the widespread availability of effective contraceptive methods has made extra-marital sex substantially less hazardous for women (Guinnane 2011). Another dimension of caution refers to parental investment in children that, albeit still crucial in the contemporary family (e.g., Cunha and Heckman 2010), has been supplemented or replaced by the state or the market (Becker 1991; Lundberg and Pollak, 2007). And yet another dimension is the increased labor force participation of mothers (Goldin 2006). Medical innovations, the market, the state, and the greater economic independence of women might have influenced the nature of exchange within families in ways that our model cannot capture (Laitner, 1997). In spite of all these caveats, the deliberately selective attention of our theory allows us to show how large a problem free riding among males is likely to be and indicates a new original way in rethinking its effects on the formation of the family.

Anderson (2006) and the research cited therein, however, seem to suggest the opposite.
Appendix

Proofs

We begin with the proof of two technical results that will be used to prove the propositions stated in the paper. Throughout the Appendix, the notation $F_i$, $i = 1, 2$, indicates the first derivative of $F$ with respect to its $i$-th argument; similarly, $F_{ij}$, with $i, j = 1, 2$, indicates the derivative of $F_i$ with respect to the $j$-th argument.

Lemma 1. $F_1(x_1, x_2) = F_2(x_2, x_1)$.

Proof. The symmetry condition on $F$, so that $F(x_1, x_2) = F(x_2, x_1)$, implies that

$$F_1(x_1, x_2) = \lim_{h \to 0} \frac{F(x_1 + h, x_2) - F(x_1, x_2)}{h}$$

$$= \lim_{h \to 0} \frac{F(x_2, x_1 + h) - F(x_2, x_1)}{h}$$

$$= F_2(x_2, x_1).$$

Lemma 2. If $x_1 + x_2 > y_1 + y_2$, then either $F_1(x_1, x_2) < F_1(y_1, y_2)$, or $F_2(x_1, x_2) < F_2(y_1, y_2)$, or both inequalities are true.

Proof. Because $x_1 + x_2 > y_1 + y_2$, then $x_1 - y_1 > 0$ and/or $x_2 - y_2 > 0$. Let:

$$\Phi(x_1, x_2) = F_1(x_1, x_2)$$

$$\Psi(x_1, y_2) = F_2(x_1, x_2).$$

An infinitesimal increase in the value of the images of $\Phi$ and $\Psi$, denoted by $\Delta \Phi$ and $\Delta \Psi$, corresponding to an increase $\Delta_1$ in $x_1$ and $\Delta_2$ in $x_2$ can be written as

$$\begin{bmatrix} \Delta \Phi \\ \Delta \Psi \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}.$$ 

By contradiction, assume

$$\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

We now consider the three possible cases.

Case 1. $\Delta_1 > 0, \Delta_2 > 0$.

The system of equations

$$F_{11} \Delta_1 + F_{12} \Delta_2 > 0$$

$$F_{21} \Delta_1 + F_{22} \Delta_2 > 0$$

implies that

$$F_{12} \Delta_2 > -F_{11} \Delta_1$$

$$F_{21} \Delta_1 > -F_{22} \Delta_2.$$

Because all terms are positive, term-wise multiplication of the two equations leads to $(F_{12})^2 > F_{11}F_{22}$, which contradicts the assumption that $F$ is concave.
Case 2. \( \Delta_1 > 0, \Delta_2 < 0 \).

Since Assumption 2(iv) guarantees that \( F_{12} > 0 \), the inequality \( F_{12} \Delta_2 > -F_{11} \Delta_1 \) can never be satisfied because the left-hand term is negative and the right-hand term positive.

Case 3. \( \Delta_1 < 0, \Delta_2 > 0 \).

The inequality \( F_{21} \Delta_1 > -F_{22} \Delta_2 \) can never be satisfied because the left-hand term is negative and the right hand term positive.

Notice the supermodularity condition \( F_{12} > 0 \) is sufficient but not necessary. In addition, the inequalities obtained are for infinitesimal increments \( \Delta_1 \) and \( \Delta_2 \). However, because they hold everywhere, the effect of a change from \( x_1 \) to \( y_1 \) and from \( x_2 \) to \( y_2 \) can be obtained by integrating \( \Delta \Phi \) and \( \Delta \Psi \) over the path from \((x_1, x_2)\) to \((y_1, y_2)\). The corresponding changes \( \Phi(y_1, y_2) - \Phi(x_1, x_2) \) and \( \Psi(y_1, y_2) - \Psi(x_1, x_2) \) keep the same signs as those implied by the infinitesimal increments \( \Delta \Phi \) and \( \Delta \Psi \). Hence, the contradiction holds.

\( \alpha \)-Concavity. The following proofs rely on the \( \alpha \)-concavity of the \( F \) and \( \pi \) functions. As mentioned in Section 3, a function \( f \) is strictly \( \alpha \)-concave if \( f^{\alpha} \) is strictly concave (Brascamp and Lieb 1976; Kennington 1985). We shall make use of the property that if a function \( f \) is \( \alpha \)-concave and a function \( h \) is \( \beta \)-concave than the product function \( fh \) is \( \gamma \)-concave with \( \gamma^{-1} = \alpha^{-1} + \beta^{-1} \). The assumption that the \( F \) and \( \pi \) functions are strictly 2-concave ensures that the objective functions in all our programs are strictly concave.

**Proof of Proposition 1**

**Monogamy** (part (i) — Existence and Uniqueness)

A family convention specifies the strategy profile for all individuals. Consider monogamous matching. This specifies a matching of pairs of individuals of opposite sex, requiring each female to choose one given male as partner for her first two adulthood periods and, similarly, this given male to select that female as the mate he guards and provides food to, again for the first two periods of adulthood. It is easy to verify that, given everyone else adopts the same protocol, abiding by it is the best response for each individual. Therefore, each man’s best response to everyone else’s actions is to guard the woman who selected him to be the father of her children and to provide food only to her. Similarly, given the male choice, it is optimal for each woman to select the man who provides food to her offspring.

We now show that the allocation given by \( g_1^* = g_2^* > 0 \), \( k_1^* = k_2^* > 0 \), \( x_1^* = y_1^* = m^*(x_2^* + y_2^*) > 0 \) and \( m^* = 1/2 \) is the unique stationary symmetric pure strategy Nash (SSPN) equilibrium for the monogamous convention. Let \( z^* = x_2^* + y_2^* \). For notational convenience, here and in what follows, we drop the subscript \( M \) used in the text to denote the monogamy convention. We proceed in five steps.

**Step 1.** We restrict agents to choose symmetric strategies. We shall relax this restriction in Step 4 below. That is, a woman chooses \( m = 1/2 \), while a man chooses \( g_1 = g_2 = g \), \( k_1 = k_2 = k \), and \( x_1 = y_3 = x \). Clearly, given that men adopt symmetric strategies, \( m^* = 1/2 \) is the best response for women.

**Step 2.** Denote the set of symmetric male strategies \( S \subset [0, 1]^4 \) and consider the best response mapping \( \varphi : S \rightarrow S \). That is,

\[
\varphi = \arg \max_{\{g,k,x,z\}} \frac{1}{2} \left\{ \pi(g)\theta(k)F(x, z/2) + \left[ 1 - \pi(g)\theta(k) \right] F(\hat{x}, \hat{z}/2) \right\}
\]
subject to $1 = 2x + z + 2g + 2k$ and $0 \leq g, k, x, z$. Without loss of generality, (A.1) can be rewritten as

$$
\varphi = \arg \max_{\{g, k, x, z\}} \pi(g) \theta(\tilde{k}) F(x, z, x) + [1 - \pi(\tilde{g}) \theta(k)] F(\tilde{x}, 2, \tilde{x})
$$

Because the objective function in (A.2) is strictly concave in $\{k, g, x, z\}$ and the budget set is compact and convex, the best response is a continuous function of $\{g, k, x, z\}$. Therefore, (A.1) can be rewritten as

$$
\phi = \theta(k) \left\{ \pi(g_1) F(x_1, \frac{z}{2}) + \pi(g_2) F(\frac{z}{2}, y_3) \right\} - \left[ \theta(k_1) + \theta(k_2) \right] \pi(g) F(x, \frac{z^*}{2})
$$

Concavity of $-\theta$ implies that at the optimum $k_1 = k_2 = k$. Using this result in (A.3), the male’s program becomes

$$
\max_{\{g_1, x_1, z, g_2, y_3, k\}} \left\{ \pi(g_1) F(x_1, \frac{z}{2}) + \pi(g_2) F(y_3, \frac{z}{2}) \right\} - 2 \theta(k) \pi(g) F(x, \frac{z^*}{2})
$$

subject to $x_1 + 2k + z + y_3 + g_1 + g_2 = 1$. By concavity of $\pi F$ in (A.4) we must have $g_1 = g_2$ and $x_1 = y_3$. Suppose this is not the case. Then, consider the feasible allocations $\tilde{g}_1 = \tilde{g}_2 = \frac{g_1 + g_2}{2}$ (indeed $\tilde{g}$), and $\tilde{x}_1 = \tilde{y}_3 = \frac{x_1 + x_2}{2}$ (indeed $\tilde{x}$). It is easy to verify that the concavity of $\pi F$ guarantees that when $\tilde{g}$ and $\tilde{x}$ are replaced into $\phi$, males obtain a higher payoff and higher survivorship. The concavity of $F$ implies also that at the optimum $x_1 = y_3 = \frac{x}{2}$. It thus follows that $\{g, k, x, z = \frac{z^*}{2}\}$ and $m^* = 1/2$ is an equilibrium. With this shown, we have proved that $\{g^*, k^*, x^*, z^*\}$ and $m^* = 1/2$ is an equilibrium. It is left to show that this is unique.

Step 5. Steps 1–4 guarantee that the first order conditions hold at the fixed point $g^*, k^*, x^* = z^*/2$. Taking account of the previous results and of (A.1)–(A.4), the first order conditions of the problem then become

$$
\frac{\partial \phi}{\partial x} \equiv F_1 \left( \frac{z}{2}, \frac{z}{2} \right) \theta(k) \pi(g) + \lambda = 0
$$
\[
\frac{\partial \phi}{\partial g} = \pi'(g)\theta(k)F\left(\frac{z}{2}, \frac{z}{2}\right) + \lambda = 0
\]
\[
\frac{\partial \phi}{\partial k} = -\pi(g)\theta'(k)F\left(\frac{z}{2}, \frac{z}{2}\right) + \lambda = 0.
\]
These expressions and the budget set can be rewritten as
\[
\frac{\pi'(g)}{\pi(g)} = -\frac{\theta'(k)}{\theta(k)} \quad (A.5)
\]
\[
F_1\left(\hat{z}, \hat{z}\right) = \frac{\pi'(g)}{\pi(g)} \quad (A.6)
\]
\[
1 = 2g + 2k + 2z.
\]
From Steps 1–4, we know that this system of equations has a solution. To see that the solution is unique, note that for any arbitrary given \(e > 0\), (A.6) has a unique solution on the line \(e = g + z\), with \(g > 0\) and \(z > 0\). This must be the case because the left hand side of (A.6) is strictly decreasing in \(z\), while the strict concavity in \(\pi\) implies that the right hand side is strictly decreasing in \(g\). This defines the functions \(g(e)\) and \(z(e)\) which are both strictly increasing in \(e\). Equation (A.5) can then be rewritten as
\[
\frac{\pi'(g(e))}{\pi(g(e))} = -\frac{\theta'(k)}{\theta(k)} \quad (A.7)
\]
Now equation (A.7) has a unique solution on \(1 = 2k + 2e\). Indeed, the left hand side of (A.7) is a decreasing function of \(g\), while the concavity of \(1\theta\) implies that the right hand side is decreasing function of \(k\). Furthermore, \(g(e)\) in an increasing function. This implies the uniqueness of the symmetric fixed point. QED

**Serial Monogamy (part (ii) — Existence and Uniqueness)**

Following the same reasoning used in part (i), if the serial monogamy convention is adopted by all individuals, choosing to be serially monogamous is the best response for each individual. As before, for notational convenience we drop the subscript \(S\) used in the text to denote the serial monogamy convention. We next show that the symmetric allocation \(x_1^* = y_4^* = x^*, x_2^* = y_2^* = y^*, g_1^* = g_2^* = g^*, k_1^* = k_2^* = k^*,\) and \(m^* = 1/2\) is the unique symmetric equilibrium for the serially monogamous convention. Again, we proceed in five steps.

**Step 1.** This is identical to Step 1 in part (i), except that now a man also chooses \(x_2 = y_2\) (= \(y\)). As before, given that men adopt symmetric strategies, \(m^* = 1/2\) is the best response for women.

**Step 2.** Denote the set of symmetric male strategies \(S \subset [0,1]^4\) and consider the best response mapping \(\varphi : S \rightarrow S\). That is,
\[
\varphi = \arg \max_{\{g,k,x,y\}} \frac{1}{2}\left\{ \pi(g)\theta(k)F\left(x, \frac{y + \hat{y}}{2}\right) + \left[1 - \pi(\hat{g})\theta(k)\right]F(\hat{x}, \hat{y}) \right. \]
\[
\left. + \pi(g)\theta(k)F\left(\frac{y + \hat{y}}{2}, x\right) + \left[1 - \pi(\hat{g})\theta(k)\right]F(\hat{x}, \hat{y}) \right\} \quad (A.8)
\]
subject to \(1 = 2g + 2k + 2x + 2y\) and \(0 \leq g,k,x,y\). Without loss of generality, (A.8) can be rewritten as
\[
\varphi = \arg \max_{\{g,k,x,y\}} \pi(g)\theta(k)F\left(x, \frac{y + \hat{y}}{2}\right) + \left[1 - \pi(\hat{g})\theta(k)\right]F(\hat{x}, \hat{y}). \quad (A.9)
\]
Because the objective function in (A.9) is strictly concave in \(\{g, k, x, y\}\) and the budget set is compact and convex, the best response is a continuous function of \(\{k, \hat{g}, \hat{x}, \hat{y}\}\). Then, by Brouwer’s fixed point theorem, a fixed point must exist. This is denoted by \(\{g^*, k^*, x^*, y^*\}\).

**Step 3.** We now prove this fixed point is interior, that is, \(g^* > 0, k^* > 0, x^* > 0, \text{and } y^* > 0\). Let \(\phi\) denote the individual payoff to a man when all other males choose \(\{g^*, k^*, x^*, y^*\}\) and women choose \(m^* = 1/2\). That is, \(\phi = \pi(g)\theta(k^*) F\left(x, \frac{y+y^*}{2}\right) + \left[1 - \pi(g^*)\theta(k)\right] F(x^*, y^*)\). At the fixed point, male \(i\) is also expected to choose the interior strategy \(g^* > 0, k^* > 0, x^* > 0, \text{and } y^* > 0\). By contradiction, instead, assume male \(i\) chooses a different (non-interior) strategy profile. Consider then all the possible corner solutions for the ingredients in \(\phi\). The rest of argument follows exactly the same logic used in Step 3 of part (i) of the proof. As before, we conclude then that, given \(m^* = 1/2\), the fixed point must be an interior solution.

**Step 4.** We now show that imposing male choices \(\{g^*, k^*, x^*, y^*\}\) to be symmetric is not restrictive. For this purpose we rewrite the male objective function as

\[
\frac{1}{2}\left\{\theta(k^*) \left[\pi(g_1) F\left(x_1, \frac{x_2+y^*}{2}\right) + \pi(g_2) F\left(y^* + \frac{y_2}{2}, y_2\right)\right] - \left[\theta(k_1) + \theta(k_2)\right] \pi(g^*) F(x^*, y^*) + 2F(x^*, y^*)\right\}
\]

(A.10)

Concavity of \(-\theta\) implies that at the optimum \(k_1 = k_2\). The same arguments used in Step 4 of part (i) based on the concavity of \(\pi F\) lead us to show that \(g_1 = g_2, x_1 = y_3, \text{and } x_2 = y_2\). We can then verify that \(\{g^*, k^*, x^*, y^*\}\) and \(m^* = 1/2\) is an equilibrium. It is left to show that this is unique.

**Step 5.** Steps 1–4 guarantee that the first order conditions hold at the fixed point \(g^*, k^*, x^*, y^*\). Taking account of the previous results and of (A.8)–(A.10), the first order conditions of the problem then become

\[
\frac{\pi'(g)}{\pi(g)} = -\frac{\theta'(k)}{\theta(k)}
\]

(A.11)

\[
\frac{F_1(x, y)}{F(x, y)} = \frac{\pi'(g)}{\pi(g)}
\]

(A.12)

\[
F_2(x, y)\pi(g) = 2F_1(x, y)\pi(g)
\]

(A.13)

\[1 = 2x + 2y + 2g + 2k.\]

Since \(g^* > 0\), we rewrite (A.13) as \(F_2(x, y) = 2F_1(x, y)\), and notice this equation has a unique solution on \(x + y = q\), for any arbitrary given \(q > 0\). Let the solutions be denoted by \(x(q)\) and \(y(q)\). These are increasing functions in \(q\). Consider next (A.12) which can be rewritten as

\[
\frac{F_1(x(q), y(q))}{F(x(q), y(q))} = \frac{\pi'(g)}{\pi(g)}.
\]

For the same argument used in part (i) of the proof, this equation has a unique solution on \(w = q + g\), for any given arbitrary \(w > 0\). Let \(g(w)\) denote this solution. Finally, consider (A.11) which can be rewritten as

\[
\frac{\pi'(g(w))}{\pi(g(w))} = -\frac{\theta'(k)}{\theta(k)}
\]

which in turn, by the same argument used in part (i) of the proof, has a unique solution on \(1 = w + k\). This establishes the existence of a unique symmetric equilibrium. QED
Useful Property of the Serial Monogamy Equilibrium. Before moving to the proof of Proposition 2, we illustrate a useful property of the serial monogamy equilibrium, which we shall use in what follows. Again, we drop the subscript $S$. From (A.13) and owing to the symmetry of $F$ and Lemma 1, it can be verified that

$$F_1\left(x_1^*, \frac{1}{2}(x_2^* + y_2^*)\right) = \frac{1}{2}F_2\left(x_1^*, \frac{1}{2}(x_2^* + y_2^*)\right) = \frac{1}{2}F_1\left(\frac{1}{2}(x_2^* + y_2^*), x_1^*\right).$$

The fact that $F_1\left(x_1^*, \frac{1}{2}(x_2^* + y_2^*)\right) < F_1\left(\frac{1}{2}(x_2^* + y_2^*), x_1^*\right)$ implies $x_1^* \neq \frac{1}{2}(x_2^* + y_2^*)$. Now, from Assumption 2 we have $F_{11}(u,v) < 0$ and $F_{22}(u,v) > 0$, for all $(u,v)$. This in turn implies $F_1(u,v)$ is decreasing in the first argument and increasing in the second one and thus

$$\frac{1}{2}(x_2^* + y_2^*) < x_1^*. \quad (A.14)$$

A similar reasoning leads us to

$$\frac{1}{2}(x_2^* + y_2^*) < y_3^*. \quad (A.15)$$

**Proof of Proposition 2**

First, we establish a useful result. Consider the expression

$$\frac{\pi'(g)}{\pi(g)} = -\frac{\theta'(k)}{\theta(k)}, \quad (A.16)$$

which coincides with the first order conditions (A.5) and (A.16) for monogamy and serial monogamy respectively. Let $\chi$ be the mapping that links any given $g > 0$ to the $k$ solving (A.11), that is $k = \chi(g)$. We claim that $\chi$ is a strictly increasing mapping. To prove this claim, note that from Assumption 2 the left hand side of this expression is a decreasing function of $g$ since $\pi$ is strictly concave. Similarly, because $1/\theta$ is strictly concave, the right hand side is a decreasing function of $k$. Assume now that $(g_1, k_1)$ solves (A.16). Suppose that $g$ increases from $g_1$ to $g_2$. As the left hand side decreases, the right hand side must also decrease. But as the right hand side is decreasing in $k$, it must be the case that $k_2 > k_1$. Therefore, $g_1 < g_2 \iff k_1 < k_2$ and $\chi$ is strictly increasing.

Second, using this result we proceed with the proof of the proposition. We now introduce the subscripts $M$ and $S$ to indicate monogamy and serial monogamy respectively. From Proposition 1 (part (i)), recall that in the monogamy configuration $x_{1,M}^* = y_{5,M}^* = \frac{1}{2}(x_{2,M}^* + y_{2,M}^*) \equiv x_{1,S}^*$. By contradiction suppose $F(x_{5}^*, y_{5}^*) > F(x_{M}^*, x_{M}^*)$. Proposition 1 demonstrates that the food allocation in monogamy $(x_{1,S}^*, x_{S}^*)$ is symmetric, while the implication from (A.14) and (A.15) is that the allocation in serial monogamy $(x_{5}^*, y_{S}^*)$ is not. This leads to

$$2x_{M}^* < x_{S}^* + y_{S}^*, \quad (A.17)$$

which in turn implies $g_{M}^* + k_{S}^* > g_{S}^* + k_{S}^*$. Then, at least one of the two following inequalities $k_{M}^* > k_{S}^*$ or $g_{M}^* > g_{S}^*$ must hold. Since we know that $g_1 < g_2 \iff k_1 < k_2$, it must be that if $F(x_{5}^*, y_{5}^*) > F(x_{M}^*, x_{M}^*)$, then $g_{M}^* > g_{S}^*$ and $k_{M}^* > k_{S}^*$. From the concavity of $\pi$ and Assumption 1 we have

$$F(x_{5}^*, y_{5}^*)\pi(g_{5}^*) > F(x_{M}^*, x_{M}^*)\pi'(g_{M}^*). \quad (A.18)$$

However, since from (A.12) $F_1(x_{5}^*, y_{5}^*)\pi(g_{5}^*) = F(x_{5}^*, y_{5}^*)\pi'(g_{5}^*)$ and from (A.6) $F_1(x_{M}^*, x_{S}^*)\pi(g_{M}^*) = F(x_{M}^*, x_{M}^*)\pi'(g_{M}^*)$, we obtain $F_1(x_{5}^*, y_{5}^*)\pi(g_{5}^*) > F_1(x_{M}^*, x_{S}^*)\pi(g_{M}^*)$, which implies $F_1(x_{5}^*, y_{5}^*) > F_1(x_{M}^*, x_{S}^*)$. Now from (A.18) we obtain $m^*F_2(x_{S}^*, y_{S}^*)\pi(g^*) > F_2(x_{M}^*, x_{M}^*)\pi(g_{M}^*)$, which in turn
implies \( F_2(x_s^*, y_s^*) > F_2(x_M^*, x_M^*) \). Applying Lemma 2 we then get \( x_s^* + y_s^* < 2x_M^* \), which contradicts (A.17). QED

**Proof of Proposition 3**

*Existence and Uniqueness (part i)*

For notational convenience, the subscript \( G \) is dropped. Suppose, for simplicity, a group consists of two women and two men. The contribution to the male payoff from the children of one of the two women in period 1 is given by

\[
\frac{1}{4}\pi \left( \frac{g_1 + \hat{g}_1}{2} \right) \theta(\hat{k}) F \left( \frac{x_1 + \hat{x}_1}{2}, \frac{\hat{m}(z + \hat{z})}{2} \right),
\]

where, as before, \( z = x_2 + y_2 \). Of course, there are two terms as (A.19) in the man’s payoff function, since a male is matched to two identical women. Similarly the contribution to father’s payoff from the children of one of the two women in period 2 is

\[
\frac{1}{4}\pi \left( \frac{g_2 + \hat{g}_2}{2} \right) \theta(\hat{k}) F \left( \frac{1 - \hat{m}}{2}, \frac{z + \hat{z}}{2} \right),
\]

and again there are two such terms.

We follow the same reasoning used in Propositions 1 and 2. If the group marriage convention is adopted by all individuals, choosing to form a group marriage is the best response for each individual. As before, the proof proceeds in five steps.

**Step 1.** See Step 1 in Proposition 1.

**Step 2.** Let \( z = x_2 + y_2 \). The man’s problem is to choose \( \{g, k, x, z\} \) to maximize

\[
\frac{1}{4}\pi \left( g + \hat{g} \right) \frac{1}{2} \theta(\hat{\kappa}) F \left( \frac{x + \hat{x}}{2}, \frac{1}{2} \frac{z + \hat{z}}{2} \right) + \left[ 1 - \pi(\hat{g}) \theta(\hat{k}) \right] F \left( \frac{\hat{x}}{2}, \frac{1}{2} \frac{z}{2} \right)
\]

subject to \( 1 = 2x + z + 2g + 2k \) and \( 0 \leq x, z, g, k \). Without loss of generality (A.21) can be rewritten as

\[
\frac{1}{2} \left\{ \pi \left( \frac{g + \hat{g}}{2} \right) \theta(\hat{\kappa}) F \left( \frac{x + \hat{x}}{2}, \frac{z + \hat{z}}{4} \right) + \left[ 1 - \pi(\hat{g}) \theta(\hat{k}) \right] F \left( \frac{\hat{x}}{2}, \frac{\hat{z}}{2} \right) \right\}
\]

Since (A.22) is strictly concave in \( \{x, g, k, z\} \) and the budget set is compact and convex, the best response is a continuous function of \( \{\hat{k}, \hat{g}, \hat{x}, \hat{R}\} \). A fixed point, denoted \( \{g^*, k^*, x^*, z^*\} \), therefore, must exist.

**Step 3.** As before, we now prove this fixed point is interior, that is, \( g^* > 0, k^* > 0, x^* > 0 \), and \( z^* > 0 \). The proof uses the same argument as that used in Step 3 of Proposition 1, except that the individual payoff to a man, when all other males choose \( \{g^*, k^*, x^*, z^*\} \), is given by

\[
\phi = \pi \left( \frac{g + x^*}{2} \right) \theta(k^*) F \left( \frac{z + x^*}{2}, \frac{z + z^*}{4} \right) + \left[ 1 - \pi(g^*) \theta(k) \right] F \left( x^*, \frac{z^*}{2} \right).
\]

**Step 4.** We now show that imposing male choices \( \{g^*, k^*, x^*, z^*\} \) to be symmetric is not restrictive. For this purpose we rewrite the male objective function as

\[
\phi = \theta(k^*) \left\{ \pi \left( \frac{g_1 + g^*}{2} \right) F \left( \frac{x_1 + x^*}{2}, \frac{x_2 + z^*/2}{2} \right) + \pi \left( \frac{g_2 + g^*}{2} \right) F \left( \frac{z^*/2 + y_2}{2}, \frac{y_3 + x^*}{2} \right) \right\}
\]

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subject to \( 1 = x_1 + x_2 + y_2 + y_3 + g_1 + g_2 + k_1 + k_2 \) and all variables are nonnegative. By the same concavity arguments as those used in Step 4 of Proposition 1, it follows that \( \{(g^*, k^*, x^*, \frac{z}{2}) = x^* \} \) and \( m^* = 1/2 \) is an equilibrium. It is left to show that this is unique.

**Step 5.** This part of the proof proceed exactly as in the case of Proposition 1. Equation (A.6) as well as the expression for the budget set of the monogamy case apply to the group marriage convention too, while (A.5) for this case becomes

\[
\frac{1}{2} \pi'(g) = \frac{-\theta'(k)}{\theta(k)}. \tag{A.24}
\]

Having proved existence and uniqueness for the case in which a group is composed of two men and two women, it is straightforward to extend the same proof to another environment where the size of the group is an arbitrary integer \( q \) greater than two. This last result can then be summarized as follows:

Let the economy be composed of \( N \) men and \( N \) women. There exists a unique interior SSPN equilibrium with \( N/q \) groups, each of them composed of \( q \) males and \( q \) females, provided \( N/q \) is an integer.

**Efficiency (part (ii))**

Let \( k = \psi_\alpha(g) \) be the solution in terms of \( g \) of the following variant of (A.24)

\[
\pi'(g) = \frac{-\theta'(k)}{\theta(k)}. \tag{A.25}
\]

For any given \( g \), when \( \alpha \) increases the right hand side of (A.25) also increases. Since the right hand side is decreasing in \( k \), the value of \( k \) associated to a given \( g \) must decrease. Therefore \( \frac{d\psi_\alpha(g)}{d\alpha} < 0 \).

We compare the monogamy convention to the group marriage convention, and reintroduce the \( M \) and \( G \) subscripts to identify the two conventions respectively. The proof proceed by contradiction. Assume that survivorship under \( C_G \) is greater than survivorship under \( C_M \). Because the allocations \( A_j, j = M, G \) are both symmetric, we have \( F_1 \left( \frac{z^G_M}{2}, \frac{z^G_M}{2} \right) > F_1 \left( \frac{z^M_M}{2}, \frac{z^M_M}{2} \right) \). This implies

\[
k^*_G + g^*_G < k^*_M + g^*_M. \tag{A.26}
\]

In the case of a group of two men and two women (A.26) leads to \( \psi_{1/2}(g^*_G) + g^*_G < \psi_1(g^*_M) + g^*_M \), which in turn leads to

\[
\psi_1(g^*_G) + g^*_G < \psi_1(g^*_M) + g^*_M \tag{A.27}
\]

because \( \psi_\alpha(g) \) is a decreasing function of \( \alpha \). As \( \psi_1(g) \) is an increasing function of \( g \), (A.27) implies \( g^*_G < g^*_M \). However, because we assumed \( F_1 \left( \frac{z^G_M}{2}, \frac{z^G_M}{2} \right) > F_1 \left( \frac{z^M_M}{2}, \frac{z^M_M}{2} \right) \), at least one of the two following inequalities must hold

\[
F_1 \left( \frac{z^*_G}{2}, \frac{z^*_G}{2} \right) < F_1 \left( \frac{z^*_M}{2}, \frac{z^*_M}{2} \right) \tag{A.28a}
\]

\[
F_2 \left( \frac{z^*_G}{2}, \frac{z^*_G}{2} \right) < F_2 \left( \frac{z^*_M}{2}, \frac{z^*_M}{2} \right). \tag{A.28b}
\]
Note that \((A.28a)\) and \((A.28b)\) are equivalent. We thus only consider the first. This implies
\[
F_1\left(\frac{z'}{2}, \frac{z''}{2}\right) \pi(g_M') < F_1\left(\frac{z'}{2}, \frac{z''}{2}\right) \pi(g_M^*) \text{ because } \pi \text{ is increasing in } g.
\]
On the other hand, we know that
\[
\pi'(g_M^*) F\left(\frac{z'}{2}, \frac{z''}{2}\right) > \pi'(g_M^*) F\left(\frac{z'}{2}, \frac{z''}{2}\right), \text{ because } \pi \text{ is concave.}
\]
Consider then
\[
F_1\left(\frac{z'}{2}, \frac{z''}{2}\right) \pi(g) = F\left(\frac{z'}{2}, \frac{z''}{2}\right) \pi'(g)
\]
which coincides with \((A.6)\) for the monogamy convention and, as explained in part (i) of Proposition 3, must hold also for group marriage. The results obtained above imply that the right hand side of \((A.29)\) is larger in the group marriage convention than in the monogamy convention, while the left hand side is smaller. This is a contradiction. The same conclusion holds for groups of any arbitrary size, as long as the number of men is equal to the number of women. In this case, only the value of the parameter \(\alpha\) has to change.

**Group size (part (iii))**

Let \(1/\alpha\) denote the size of a group. As in part (ii) of this proposition, let \(k = \psi_\alpha(g)\) be the solution in terms of \(g\) of
\[
\frac{\pi'(g)}{\pi(g)} = -\frac{\theta'(k)}{\theta(k)}.
\]

We know that, as the size changes, \(\frac{d\psi_\alpha(g)}{dg} < 0\).

We compare two groups of different size, \(n\) and \(n'\) with \(n' > n\). Let \(\{z^{(n)}, k^{(n)}, g^{(n)}\}\) denote the equilibrium in the group marriage of size \(n\), dropping the ‘star’ superscript for convenience. A similar notation identifies the equilibrium for the group marriage of size \(n'\). Suppose by contradiction that survival in the group of size \(n'\) is greater than survivorship in the group of size \(n\). That is,
\[
F\left(\frac{z^{(n')}}{2}, \frac{z^{(n')}}{2}\right) > F\left(\frac{z^{(n)}}{2}, \frac{z^{(n)}}{2}\right).
\]

Because both allocations are symmetric, this in turn implies
\[
k^{(n')} + g^{(n')} < k^{(n)} + g^{(n)}.
\]

Then, as \(\psi_\alpha(g)\) is a decreasing function of \(\alpha\) and \(1/n' < 1/n\) we have \(\psi_{1/n'}(g^{(n')}) + g^{(n')} < \psi_{1/n}(g^{(n)}) + g^{(n)}\). Now, since \(\psi_\alpha(g)\) is an increasing function of \(g\), this last inequality implies that \(g^{(n')} < g^{(n)}\).

Because of \((A.31)\), however, at least one of the two following inequalities
\[
F_1\left(\frac{z^{(n')}}{2}, \frac{z^{(n')}}{2}\right) < F_1\left(\frac{z^{(n)}}{2}, \frac{z^{(n)}}{2}\right)
\]
\[
F_2\left(\frac{z^{(n')}}{2}, \frac{z^{(n')}}{2}\right) < F_2\left(\frac{z^{(n)}}{2}, \frac{z^{(n)}}{2}\right)
\]
must hold. Since they are equivalent, we only consider the first inequality. This in turn implies \(F_1\left(\frac{z^{(n')}}{2}, \frac{z^{(n')}}{2}\right) \pi(g^{(n')}) < F_1\left(\frac{z^{(n)}}{2}, \frac{z^{(n)}}{2}\pi(g^{(n)})\right)\) because \(\pi\) is increasing in \(g\). But from \((A.30)\) we know
it must be that \( F(\hat{z}^{(n')}, \hat{z}^{(n')}) \pi'(g^{(n')}) > F(\hat{z}^{(n')}, \hat{z}^{(n')}) \pi'(g^{(n)}) \) because \( \pi \) is concave. Following the same reasoning used in part (ii) of the proposition, this leads to a contradiction. QED

**Proof of Proposition 4**

*Existence and Uniqueness* (part (i))

This proof closely follows the proof for part (i) of Proposition 3. Notice expression (A.19) that indicates the contribution to the male payoff from the children of one of the two women in period 1 must be modified and is now given by

\[
\frac{1}{4}\pi \left( \frac{g + \hat{g}}{2} \right) \theta(k) \bar{F} \left( \frac{x_1 + \hat{x}_1}{2}, \bar{m} \left( \frac{x_2 + \hat{y}_2}{2} \right) \right),
\]

(A.32)

and also (A.20) for period 2’s payoff changes accordingly. We again proceed in five steps.

1. **Step 1.** See Step 1 in Proposition 3 restricting to symmetric strategies.

2. **Step 2.** Follow Step 2 of Proposition 3 to prove the existence of best responses. Notice that now the man’s problem is to choose \( \{g, k, x, y\} \) to maximize

\[
\frac{1}{2} \pi \left( \frac{g + \hat{g}}{2} \right) \theta(k) \bar{F} \left( \frac{x + \hat{x}}{2}, \frac{1}{2} \left( \frac{y + \hat{y}}{2} \right) \right) + \frac{1}{2} \left[ 1 - \pi(\hat{g}) \theta(k) \right] \bar{F}(\hat{x}, \hat{y})
\]

subject to \( 1 = 2x + 2y + 2g + 2k \) and \( 0 \leq x, g, k, y \).

3. **Step 3.** Follow Step 3 of Proposition 3 to show that the fixed point is interior. We make use of (A.33) here, but now all the ‘hat’ terms are replaced by ‘star’ terms.

4. **Step 4.** All the arguments used in Step 4 of Proposition 3 hold here to show that the choice of interior symmetric strategies is not restrictive.

5. **Step 5.** This part of the proof again replicates Step 5 of the previous proposition. Notice that the first order conditions and the budget set for this case become

\[
\begin{align*}
\frac{1}{2} \pi'(g) &= -\frac{\theta'(k)}{\theta(k)} \quad \text{(A.34)} \\
\frac{F_1(x, y)}{F(x, y)} &= \frac{\pi'(g)}{\pi(g)} \quad \text{(A.35)} \\
\frac{F_2(x, y)\pi(g)}{F(x, y)\pi(g)} &= 2F_1(x, y)\pi(g) \\
1 &= 2x + 2y + 2g + 2k.
\end{align*}
\]

It should be noticed that (A.35) and (A.36) are identical respectively to (A.12) and (A.13), which were found in the case of serial monogamy. Similarly, the budget set coincides with that characterizing the serial monogamy convention. The difference is in (A.34), where the left hand side is multiplied by \( \frac{1}{2} \). The rest of the proof proceeds as Step 5 of Proposition 2.

Having proved existence and uniqueness for the case in which a group is composed of two men and two women, it is straightforward to extend the same proof to an economy where the size of the group is an arbitrary integer \( q \) greater than two.

*Efficiency* (part (iii))
As in the case of group marriage (part (ii) of Proposition 3), we have \( \frac{dg_\lambda(x)}{dx} < 0 \). Suppose by contradiction that the serial group marriage configuration (denoted with the subscript \( SG \)) yields higher survivorship than the group marriage convention, i.e., \( F(x^*_SG, y^*_SG) > F \left( \frac{z^*_G}{2}, \frac{z^*_G}{2} \right) \), which — because \( (x^*_SG, y^*_SG) \) is asymmetric — implies
\[
k^*_SG + g^*_SG < k^*_G + g^*_G. \tag{A.37}
\]
In the case of a group of two men and two women, (A.37) is equivalent to
\[
\psi_{1/2}(g^*_SG) + g^*_SG < \psi_{1/2}(g^*_G) + g^*_G. \tag{A.38}
\]
Now, as \( \psi_{1/2}(g) \) is an increasing function of \( g \), (A.38) implies \( g^*_SG < g^*_G \). However, because we assumed \( F(x^*_SG, y^*_SG) > F \left( \frac{z^*_G}{2}, \frac{z^*_G}{2} \right) \), at least one of the two following inequalities
\[
F_1(x^*_SG, y^*_SG) < F_1 \left( \frac{z^*_G}{2}, \frac{z^*_G}{2} \right) \quad \text{or} \quad F_2(x^*_SG, y^*_SG) < F_2 \left( \frac{z^*_G}{2}, \frac{z^*_G}{2} \right)
\]
must hold. This implies that at least one of the two following inequalities
\[
F_1(x^*_SG, y^*_SG)\pi(g^*_SG) < F_1 \left( \frac{z^*_G}{2}, \frac{z^*_G}{2} \right) \pi(g^*_G) \quad \text{or} \quad F_2(x^*_SG, y^*_SG)\pi(g^*_SG) < F_2 \left( \frac{z^*_G}{2}, \frac{z^*_G}{2} \right) \pi(g^*_G)
\]
must be true. On the other hand, we know that \( \pi'(g^*_SG)F(x^*_SG, y^*_SG) > \pi'(g^*_G)F \left( \frac{z^*_G}{2}, \frac{z^*_G}{2} \right) \) because \( \pi \) is concave.

Consider then
\[
F_1 \left( \frac{z}{2}, \frac{z}{2} \right) \pi(g) = F \left( \frac{z}{2}, \frac{z}{2} \right) \pi'(g) \tag{A.39}
\]
which coincides to (A.29) and (A.35) and must hold for both group marriage and serial group marriage conventions. The results obtained above imply that the right hand side of (A.39) is larger in the serial group marriage convention than in the group marriage, while the left hand side is smaller. This is a contradiction. The same conclusion holds for groups of any arbitrary size, as long as the number of men is equal to the number of women. In this case, only the value of the parameter \( \alpha \) has to change. QED

**Proof of Proposition 5**

**Existence and Uniqueness** (part (i))

Assume for simplicity that half of the adult male population forms a union so that each man mates with two women in each period, and the other half of the male population remains unmatched. Unmatched males invest all their resources in casual sex. Let \( k^v \) be the amount of casual sex of an unmatched male in period 1. The contribution to the matched (polygynist) male payoff from the children of one of the two women in period 1 is given by
\[
\frac{1}{2} \pi \left( \frac{g_1}{2} \right) \theta \left( \frac{\hat{k}_1}{2} + \frac{k^v}{2} \right) F \left( \frac{x_1}{2}, \frac{x_2}{2} \right), \tag{A.40}
\]
where \( \hat{k}_1 \) is the amount of extra-marital sex by other matched men. There are two terms as (A.40) in the man’s payoff function, since a male is matched to two identical women. Similarly the contribution to the man’s payoff from the children of one of the two women in period 2 is
\[
\frac{1}{2} \pi \left( \frac{g_2}{2} \right) \theta \left( \frac{\hat{k}_2}{2} + \frac{k^v}{2} \right) F \left( \frac{y_2}{2}, \frac{y_3}{2} \right), \tag{A.41}
\]
and again there are two such terms.

We follow the same reasoning used to prove existence and uniqueness in life long monogamy and life long group marriage (Propositions 1 and 3). In particular, our aim is to show that the allocation given by \( g_1^* = g_2^* > 0, \ k_1^* = k_2^* > 0, \ x_1^* = y_2^* = m^*(x_2^* + y_2^*) > 0 \) and \( m^* = 1/2 \) for the polygynist male and the allocation \( k_1^* = k_2^* = 1/2 \) give rise to the unique stationary symmetric pure strategy Nash (SSPN) equilibrium for the polygynous convention. Let \( z^* = x_2^* + y_2^* \). For notational convenience, here and in what follows, we drop the subscript \( P \) used in the text to denote the polygyny convention. As before, the proof proceeds in five steps.

**Step 1.** This is identical to Step 1 of Proposition 1.

**Step 2.** The program for the matched male is to choose \( \{g, k, x, z\} \) to maximize

\[
\frac{1}{2} \left\{ \pi(g) \theta \left( \frac{k}{2} + \frac{k^u}{2} \right) F\left( \frac{x}{2} + \frac{z}{2} \right) + \left[ 1 - \pi\left( \frac{g}{2} \theta \left( \frac{k}{2} + \frac{k^u}{2} \right) \right) \right] F\left( \frac{\hat{x}}{2} + \frac{\hat{z}}{2} \right) \right\}
\]

subject to \( 1 = 2x + z + 2g + 2k \) and \( 0 \leq x, z, g, k \). Without loss of generality (A.42) can be rewritten as

\[
\pi\left( \frac{g}{2} \right) \theta \left( \frac{k}{2} + \frac{k^u}{2} \right) F\left( \frac{x}{2} + \frac{z}{2} \right) + \left[ 1 - \pi\left( \frac{g}{2} \theta \left( \frac{k}{2} + \frac{k^u}{2} \right) \right) \right] F\left( \frac{\hat{x}}{2} + \frac{\hat{z}}{2} \right),
\]

where \( k^u = \frac{1}{2} \), and the same arguments used in Step 2 of Proposition 1 apply.

**Step 3.** Follow Step 1 of Proposition 3 to show that the fixed point is interior. We make use of (A.43), but now all the ‘hat’ terms are replaced by ‘star’ terms.

**Step 4.** The arguments used in Step 4 of Proposition 1 hold here as well to show that the choice of interior symmetric strategies is not restrictive.

**Step 5.** Steps 1–4 guarantee that the first order conditions hold at the fixed point \( \{g^*, k^*, x^* = z^*/2\} \). The rest of the proof again replicates Step 5 of Proposition 1. Notice however that now \( k^* = 0 \) cannot be ruled out invoking \( \lim_{k \to 0} \theta(k) = -\infty \) as stated in Assumption 2(iii). Therefore, when \( k^* > 0 \), the system of equations that pins down the symmetric fixed point \( \{g^*, k^*, x^* = z^*/2\} \) as a unique equilibrium is

\[
\frac{1}{2} \left\{ \pi\left( \frac{g}{2} \right) \theta \left( \frac{k}{2} + \frac{k^u}{2} \right) F\left( \frac{x}{2} + \frac{z}{2} \right) + \left[ 1 - \pi\left( \frac{g}{2} \theta \left( \frac{k}{2} + \frac{k^u}{2} \right) \right) \right] F\left( \frac{\hat{x}}{2} + \frac{\hat{z}}{2} \right) \right\} = 0
\]

This system coincides with that found for the monogamy case in Proposition 1, except for (A.44). When \( k^* = 0 \), instead, (A.45) and the budget constraint are still valid, but (A.44) becomes

\[
\frac{1}{2} \left\{ \pi\left( \frac{g}{2} \right) \theta \left( \frac{k}{2} + \frac{k^u}{2} \right) F\left( \frac{x}{2} + \frac{z}{2} \right) + \left[ 1 - \pi\left( \frac{g}{2} \theta \left( \frac{k}{2} + \frac{k^u}{2} \right) \right) \right] F\left( \frac{\hat{x}}{2} + \frac{\hat{z}}{2} \right) \right\} = \zeta.
\]
Concavity of $F$

Taking the ratio of expressions (A.47) and (A.48) we obtain

$$F_1 \left( \frac{x^*_P}{2}, \frac{x^*_P}{2} \right) \pi \left( \frac{1}{2} - 2x^*_P - k^*_P \right) = F \left( \frac{x^*_P}{2}, \frac{x^*_P}{2} \right) \pi' \left( \frac{1}{2} - 2x^*_P - k^*_P \right).$$  \hspace{1cm} (A.47)

The corresponding expression for monogamy is (A.6) which becomes

$$F_1 (x^*_M, x^*_M) \pi \left( \frac{1}{4} - x^*_M \right) = \pi' \left( \frac{1}{4} - x^*_M \right) F (x^*_M, x^*_M).$$  \hspace{1cm} (A.48)

Taking the ratio of expressions (A.47) and (A.48) we obtain

$$\frac{F_1 (x^*_M, x^*_M) \pi \left( \frac{1}{4} - x^*_M \right)}{F (x^*_M, x^*_M) \pi' \left( \frac{1}{4} - x^*_M \right)} = \frac{F_1 (\frac{x^*_P}{2}, \frac{x^*_P}{2}) \pi \left( \frac{1}{2} - 2x^*_P - k^*_P \right)}{F \left( \frac{x^*_P}{2}, \frac{x^*_P}{2} \right) \pi' \left( \frac{1}{2} - 2x^*_P - k^*_P \right)}.$$  \hspace{1cm} (A.49)

The proof proceed by contradiction. Assume that survivorship under $C_P$ is greater than survivorship under $C_M$. Then $F(\frac{x^*_P}{2}, \frac{x^*_P}{2}) \geq F(x^*_M, x^*_M)$, or equivalently $\frac{x^*_P}{2} \geq x^*$. This implies

$$\frac{1}{4} - x^*_P - \frac{1}{2}k^*_P < \frac{1}{4} - \frac{x^*_P}{2} - \frac{1}{2}k^*_P < \frac{1}{4} - x^*_M.$$  

Now, because $\pi$ is concave, $\frac{x^*_P}{2}$. Thus, (A.49) implies

$$\frac{F_1 (x^*_M, x^*_M)}{F (x^*_M, x^*_M)} = \frac{F_1 (\frac{x^*_P}{2}, \frac{x^*_P}{2})}{F \left( \frac{x^*_P}{2}, \frac{x^*_P}{2} \right)}.$$  \hspace{1cm} (A.50)

Concavity of $F$ and Assumption 1, leading to $F_1 F_{22} > F_{12}^2$ as well as $|F_1| > F_{12}$, guarantee that $\frac{F_1 (x, x)}{F (x, x)}$ is decreasing in $x$. Therefore inequality (A.50) implies $\frac{x^*_P}{2} < x^*_M$. A contradiction. QED

**Proof of Proposition 6**

Our analysis focuses on the specification $F(u, v) = (uv)^\alpha$. As detailed below, with this functional form, we determine the necessary conditions for an interior fixed point analytically. To prove that the fixed point is a global maximum and thus an SSPN equilibrium, we must rely on numerical methods. This analysis was performed using the same specifications used in the example of Section 4, that is, $\pi(g) = g^\gamma$ and $\theta(k) = 1 - \gamma k^\delta$, and for a wide range of parameter values, including those used in the example in the text. We begin with the characterization of the problems in the monogamy convention and then turn to the serial monogamy convention. The comparison is for such two conventions only.

**Monogamy**

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**Adult Woman.** Taking into account that her second period children may receive food from their older brothers, denoted by \( b \), a woman chooses \( m \) to maximize
\[
E_t^1(m(\hat{x}_2 + \hat{y}_2))^\alpha + E \left\{ [1 - m](\hat{x}_2 + \hat{y}_2)]^\alpha (\hat{y}_3 + \hat{b})^\alpha \right\}.
\]  
(A.51)

The expectation operator \( E[\cdot] \) in (A.51) arises because older brothers might not survive to adulthood, and this means \( \hat{b} = 0 \). Letting \( \hat{x}_2 + \hat{y}_2 = \hat{z}_2 \), the woman’s objective is to choose \( m \) to maximize
\[
m^\alpha(\hat{x}_1 \hat{z}_2)^\alpha + (1 - m)^\alpha \hat{z}_2^\alpha E \left[ (\hat{y}_3 + \hat{b})^\alpha \right].
\]  
(A.52)

As food can be supplied only by men, the sex of a child matters. In order to keep a balanced sex ratio and avoid useless complications, every couple is assumed to give birth to an equal number of boys and girls in each period. Siblings’ birth order also matters now, so that the optimization problem faced by older males will differ from that faced by their younger brothers. This is because older children, when adult, can support their younger siblings but not vice versa. Rearranging (A.52), the female’s problem becomes that of choosing \( m \in (0, 1) \) to maximize
\[
m^\alpha(\hat{x}_1 \hat{z}_2)^\alpha + (1 - m)^\alpha \hat{z}_2^\alpha m^\alpha \hat{x}_1^\alpha \hat{z}_2^\alpha (\hat{y}_3 + \hat{b})^\alpha + (1 - m^\alpha \hat{x}_1^\alpha \hat{z}_2^\alpha) \hat{y}_3^\alpha.
\]

The first order necessary condition for an interior solution is then
\[
\left( \frac{m}{1 - m} \right)^{\alpha - 1} \hat{x}_1^\alpha - \hat{y}_3^\alpha - m^{\alpha - 1}(2m - 1) \left\{ \hat{x}_1^\alpha (\hat{x}_2 + \hat{y}_2)^\alpha \left[ (\hat{y}_3 + \hat{b})^\alpha - \hat{y}_3^\alpha \right] \right\} = 0.
\]  
(A.53)

The implicit equation (A.53) gives all equilibrium candidates to interior maxima. As in the case without kin ties, however, there are always two other inefficient non-interior solutions, in which males and females choose \( x_1 = 0 \) and \( m = 0 \) or \( y_3 = 0 \) and \( m = 1 \), which are ignored. Finally, we numerically checked that for the values of the exogenous variables evaluated at the fixed point, i.e., the candidate Nash equilibrium, the solution to (A.53) delivers the global maximum for the mother.

**Elder Male.** Taking account of the possibility that his younger children might receive food transfers from his older children and letting \( x_2 + y_2 = \hat{z}_2 \), the elder male’s program is to choose \( g_1, k_1, g_2, k_2, x_1, z, y_3 \) and \( b \) to maximize
\[
\frac{1}{2} \pi(g_1) \theta(\hat{k}_1)(x_1 \hat{m} z_2)^\alpha + \frac{1}{2} \pi(g_2) \theta(\hat{k}_2)(1 - \hat{m}) z_2)^\alpha \hat{x}_1 \hat{m} z(y_3 + \hat{b})^\alpha
\]
\[
+ \frac{1}{2} \pi(g_2) \theta(\hat{k}_2)(1 - \hat{m}) z_2)^\alpha (1 - \hat{x}_1 \hat{m} a z_2)^\alpha y_3^\alpha + \frac{1}{2} \left[ 1 - \pi(\hat{g}_1) \theta(k_1) \right] (\hat{x}_1 \hat{m} z_2)^\alpha
\]
\[
+ \frac{1}{2} \left[ 1 - \pi(\hat{g}_2) \theta(k_2) \right] (1 - \hat{m}) z_2)^\alpha ((x_1 \hat{m} z_2)^\alpha (\hat{y}_3 + \hat{b})^\alpha
\]
\[
+ \frac{1}{2} \left[ 1 - \pi(\hat{g}_3) \theta(k_3) \right] (1 - \hat{m}) z_2)^\alpha (1 - \hat{x}_1 \hat{m} a z_2)^\alpha \hat{y}_3^\alpha
\]
\[
+ \frac{1}{2} \left[ 1 + \pi(\hat{g}_1) \theta(k_1) \pi(\hat{g}_2) \theta(k_2) \right] (1 - \hat{m}) z_2)^\alpha (\hat{y}_3 + b)^\alpha
\]  
(A.54)

subject to \( 0 = g_1 + x_1 + g_2 + z_2 + y_3 + k_1 + k_2 + b \) and \( 0 \leq x_1, z_2, g_1, g_2, k_1, k_2, y_3, b \) and taking \( \hat{g}_1, \hat{g}_2, \hat{k}_1, \hat{g}_2, \hat{k}_2, \hat{y}_3, \hat{m} \) and \( \hat{b} \) as given. Note that the transfer \( b \) only appears in the budget constraint and in the last term of the objective function (A.54). The problem faced by younger males is similar to that discussed in Sections 3 and 4 of the text and proved in part (i) of Proposition 1.

We begin by ignoring the sign constraints on all the control variables, allowing us to focus on interior solutions, and check afterward that these are positive. Here we focus on the first order
conditions relative to the kin transfer. The issue is to evaluate the sign of the transfer \( b \) the agent would like to make at the fixed point. Using the constraint to eliminate \( g_1 \) from the set of unknowns, the first order condition relative to the transfer \( b \) is

\[
\frac{\alpha}{2} \left[ 1 + \pi(\hat{g}_1)\theta(\hat{k}_1)\pi(\hat{g}_2)\theta(\hat{k}_2) \right] \left[ (1 - \hat{m})\hat{z}_2 \right]^\alpha (\hat{y}_3 + b)^{\alpha - 1} - \pi'(g_1)\theta(\hat{k}_1)(x_1\hat{m}\hat{z}_2)^\alpha = 0. \tag{A.55}
\]

Evaluating the left hand side of (A.55) at \( b = 0 \), we obtain

\[
\frac{\alpha}{2} \left[ 1 + \pi(\hat{g}_1)\theta(\hat{k}_1)\pi(\hat{g}_2)\theta(\hat{k}_2) \right] \left[ (1 - \hat{m})\hat{z}_2 \right]^\alpha \hat{y}_3^{\alpha - 1} - \pi'(g_1)\theta(\hat{k}_1)(x_1\hat{m}\hat{z}_2)^\alpha > 0. \tag{A.56}
\]

Expression (A.56) shows that the derivative of the objective function when evaluated at \( b = 0 \) is strictly positive. Therefore, the unconstrained interior fixed point is characterized by \( b^* > 0 \), and consequently kin transfers must be operative in the original constrained model.

The full characterization of the equilibrium with positive kin transfers is analytically not possible. We performed a numerical analysis using a two-step optimization procedure. Codes of the algorithms are in C++ and are available at <http://privatewww.essex.ac.uk/~cghig/#material>. The first step consists of a constrained optimization algorithm associated with the maximization problem faced by each agent. The elder man solves his problem taking \( \hat{b} \) as given. The solution is a vector \( \{g^*_1, x_1^*, x_2^*, k_1^*, k_2^*, x_3^*, y_3^*, m^*, b^*\} \) in which each component is a function of \( \{\hat{g}_1, \hat{g}_2, \hat{k}_1, \hat{k}_2, \hat{x}_1, \hat{z}_2, \hat{y}_3, \hat{m}, \hat{b}\} \). The mother’s optimal allocation rule, \( m^* \), is instead obtained using the secant algorithm, as a root of the implicit equation (A.53). The second step uses an evolutionary algorithm which iteratively finds the fixed point in actions of the problem by minimizing the difference between the proposed solution to the problem and the other agents’ actions. More precisely, the second step consists of minimizing the Euclidean distance between \( \{\hat{g}_1, \hat{x}_1, \hat{g}_2, \hat{k}_1, \hat{k}_2, \hat{z}_2, \hat{y}_3, \hat{b}, \hat{m}\} \) and \( \{g^*_1, x_1^*, x_2^*, k_1^*, k_2^*, x_3^*, y_3^*, m^*, b^*\} \), by choosing the ‘hat’ variables. The equilibrium is a fixed point, which occurs when \( g_1 = g^*_1, x_1 = x^*_1, g_2 = g^*_2, k_1 = k^*_1, k_2 = k^*_2, z_2 = z^*_2, y_3 = y^*_3, b = b^* \), and \( \hat{m} = m^* \), with the equilibrium allocations depending on the parameter \( \alpha \). Finally, we checked numerically that the solution to the first order conditions is the global maximum of the problem with sign constraints. Therefore, the fixed point \( \{g^*_1, x_1^*, x_2^*, k_1^*, k_2^*, x_3^*, y_3^*, m^*, b^*\} \) is the unique interior Nash equilibrium. This completes the proof of part (i) of result (b) of the proposition.

**Serial Monogamy**

In this configuration second-born children may receive transfers from the older half-brothers on the maternal side, denoted by \( b_M \), as well as from the older half-brothers on the paternal side, \( b_F \). Let \( b = b_M + b_F \).

**Adult Woman.** As a mother can only affect survivorship of her own children, and not that of her nieces and nephews, her choice of \( m \) can only affects the probability of her second period children receiving a transfer from her first period children, \( b_M \). Her objective is then to maximize

\[
m^\alpha(\hat{x}_1\hat{z}_2)^\alpha + m^\alpha\hat{x}_1^\alpha\hat{z}_2^\alpha(1 - m)\hat{z}_2^\alpha \left[ m^\alpha\hat{x}_1^\alpha\hat{z}_2^\alpha(\hat{y}_3 + \hat{b}_M + \hat{b}_F)^\alpha + (1 - m^\alpha\hat{x}_1^\alpha\hat{z}_2^\alpha)(\hat{y}_3 + \hat{b}_F)^\alpha \right]
\]

\[
+ [1 - \hat{m}m^\alpha\hat{x}_1^\alpha\hat{z}_2^\alpha](1 - m)\hat{z}_2^\alpha \left[ m^\alpha\hat{x}_1^\alpha\hat{z}_2^\alpha(\hat{y}_3 + \hat{b}_M)^\alpha + (1 - m^\alpha\hat{x}_1^\alpha\hat{z}_2^\alpha)(\hat{y}_3)^\alpha \right]
\]

It is easy to verify that a solution to this optimization exists and is a function of the parameters exogenous to the mother’s decision.
Elder Male. Taking account of the possibility that his younger children might receive food transfers from their older brothers, his problem is to choose \(g_1, x_1, k_1, g_2, x_2, y_2, k_2, y_3, b_M\) and \(b_F\) to maximize

\[
\frac{1}{2} \pi(g_1) \theta(k_1) x_1^0 (\hat{m}(x_2 + \hat{y}_2)) + \frac{1}{2} \pi(g_2) \theta(k_2) [(1 - \hat{m})(\hat{x}_2 + y_2)]^\alpha \hat{x}_1^0 \hat{m}^\alpha (\hat{x}_2 + y_2)^\alpha (y_3 + \hat{b}_M + \hat{b}_F)^\alpha
\]

\[
+ \frac{1}{2} \pi(g_2)(\hat{k}_2) [(1 - \hat{m})(\hat{x}_2 + \hat{y}_2)]^\alpha [1 - \hat{x}_1^0 \hat{m}^\alpha (\hat{x}_2 + \hat{y}_2)]^\alpha \hat{y}_3^3 + \frac{1}{2} [1 - \pi(\hat{g}_1) \theta(k_1)] \hat{x}_1^0 \hat{m}(\hat{x}_2 + \hat{y}_2)]^\alpha
\]

\[
+ \frac{1}{2} [1 - \pi(\hat{g}_2) \theta(k_2)] [(1 - \hat{m})(\hat{x}_2 + \hat{y}_2)]^\alpha [1 - \hat{x}_1^0 \hat{m}^\alpha (\hat{x}_2 + \hat{y}_2)]^\alpha \hat{y}_3^3
\]

\[
+ \frac{\rho}{4} [(1 - \hat{m})(\hat{x}_2 + \hat{y}_2)]^\alpha (\hat{y}_3 + b_M + \hat{b}_F)^\alpha
\]

\[
+ \frac{\rho}{4} \pi(\hat{g}_1) \theta(\hat{k}_1) \pi(\hat{g}_2) \theta(\hat{k}_2) [(1 - \hat{m})(\hat{x}_2 + \hat{y}_2)]^\alpha (\hat{y}_3 + \hat{b}_M + b_F)^\alpha - 1\frac{\pi'(g_1) \theta(k_1) x_1^0 [\hat{m}(x_2 + \hat{y}_2)]^\alpha = 0}{2}
\]

subject to \(1 = g_1 + x_1 + g_2 + x_2 + y_2 + k_3 + k_4 + b_M + b_F\) and \(0 \leq x_1, x_2, g_1, g_2, k_1, k_2, y_2, y_3, b_M, b_F\), where \(\rho, 0 \leq \rho \leq 1\), is a parameter that can be interpreted as the degree of sibling recognition.

The first order conditions for this problem are formally identical to those without kin ties, except those relative to \(b_M\) and \(b_F\). Eliminating \(g_1\) through the budget constraint, the first order conditions with respect to \(b_M\) and \(b_F\) are respectively given by

\[
\frac{\rho}{4} [(1 - \hat{m})(\hat{x}_2 + \hat{y}_2)]^\alpha (\hat{y}_3 + b_M + \hat{b}_F)^\alpha - 1\frac{\pi'(g_1) \theta(k_1) x_1^0 [\hat{m}(x_2 + \hat{y}_2)]^\alpha = 0}{2}
\]

and

\[
\frac{\rho}{4} \alpha \pi(\hat{g}_1) \theta(\hat{k}_1) \pi(\hat{g}_2) \theta(\hat{k}_2) [(1 - \hat{m})(\hat{x}_2 + \hat{y}_2)]^\alpha (\hat{y}_3 + \hat{b}_M + b_F)^\alpha - 1\frac{\pi'(g_1) \theta(k_1) x_1^0 [\hat{m}(x_2 + \hat{y}_2)]^\alpha = 0}{2}
\]

The other first order conditions lead to a system of equations which is equivalent to (A.11)–(A.13) in part (ii) of Proposition 1. Using the functional specifications given at the start of the proof, it can be verified that, at \(b_M + b_F = 0\) the first order conditions with respect to both \(b_M\) and \(b_F\) are negative. But because transfers cannot be negative, in equilibrium it must be the case that \(b_M = b_F = 0\). We checked numerically that the solution to the first order conditions when \(b_M = b_F = 0\) is a global maximum of the problem with sign constraints. Hence, the unique interior equilibrium is as stated. This completes the proof of part (a) of the proposition.

Part (ii) of result (b) of Proposition 6 is proved numerically. For this, we use the specifications given at the beginning of the proof. As an illustration (and as in the example of Section 4), we also assume \(a = 1/4, \beta = 1/10, \gamma = 1/4, \) and \(\delta = 1/3\), and impose a lifetime male resource endowment of \(2\), the monogamous convention without kin transfers reaches a survivorship value of approximately \(0.627\) for each child. The monogamy convention with positive kin ties (\(b^* = 0.054\)) instead reaches survivorship values of 0.632 and 0.627 for the first- and second-born child respectively. All other values and computations are available at <http://privatewww.essex.ac.uk/~cghig/#material>. QED

**Proof of Proposition 7**

A key feature of the model, which is retained here, is that the mother has full control over the distribution of resources to her children. Assume now that the woman has a given stream of food
endowment in her three adult periods, denoted by $\mu_1, \mu_2$ and $\mu_3$ respectively. The optimization program of the mother is then to choose $z_2$ to maximize

$$
\frac{1}{2} \left[ F(\hat{x}_1 + \mu_1, z_2) + F(W - z_2, \hat{y}_3 + \mu_3) \right],
$$

subject to $0 \leq z_2 \leq W = \hat{R} + \mu_2$, where $(\hat{x}_1, \hat{R}, \hat{y}_3)$ are the amounts of food she receives from her official partner(s) and $z_2$ is the amount of food she distributes to her first period children in the period 2. Because of strict concavity of $F$, the solution is unique. The first order necessary condition associated to an interior solution is

$$
F_2(\hat{x}_1 + \mu_1, z_2) = F_1(W - z_2, \hat{y}_3 + \mu_3).
$$

In this case we denote the solution as $z_2 = m(\hat{R} + \mu_2)$, where $0 < m < 1$. Now, because of the symmetry of $F$, if $\hat{x}_1 + \mu_1 = \hat{y}_3 + \mu_3$ then $m = 1/2$.

Let the food allocations that take account of the mother’s contribution be denoted by $\tilde{x}_1 = x_1 + \mu_1$, $\tilde{\mu} = R + \mu_2$, and $\tilde{y}_3 = y_3 + \mu_3$. Let also $\mu = \sum_{t=1}^3 \mu_t$. The budget constraint of the monogamous man becomes

$$
1 + \mu = g_1 + k_1 + \tilde{x}_1 + \tilde{\mu} + \tilde{y}_3 + g_2 + k_2,
$$

while that of the serial monogamous males is

$$
1 + \mu = g_1 + k_1 + x_1 + x_2 + y_2 + \mu_2 + \tilde{y}_3 + g_2 + k_2
$$

Allowing the mother to be food provider modifies the original problems for $C_M$, $C_S$, $C_G$, and $C_P$ only through additive changes in the variables. Hence, the first order conditions do not change. As an example, consider $C_S$. For this case, we have

$$
m_S^* F_2(\tilde{x}_{1,S}, m_S^*(x_{2,S} + y_{2,S} + \mu_2)) = (1 - m_S^*) F_1(m_S^*(x_{2,S} + y_{2,S} + \mu_2), \tilde{y}_{3,S}^*)
$$

and

$$
F_2(\tilde{x}_{1,S}, m_S^*(x_{2,S} + y_{2,S} + \mu_2)) = F_1(m_S^*(x_{2,S} + y_{2,S} + \mu_2), \tilde{y}_{3,S}^*),
$$

which, as in the original problem, leads to $m_S^* = 1/2$. Similar results hold for the other family conventions. Therefore, the results and properties reported in the text carry over with the total male endowment being replaced by the total resources now available, $1 + \mu$, and provided the solution is interior. Focusing on interior solutions means not only that $0 < m < 1$ but also ensure that all transfers from the father(s) are positive. A sufficient condition for this to hold is that the mother’s own endowments are small relative to $\omega$. Importantly, in $C_S$, as long as $\hat{R} > 0$, there will always be free riding between two competing men and the results will be qualitatively the same as those found earlier. QED
References


Stevenson, Betsey, and Justin Wolfers. 2007. “Marriage and Divorce: Changes and their


