Abstract

In this paper we examine the impact of bailout policies in small open economies that are subject to financial frictions. We extend standard endogenous default models in two ways. First, we augment the government’s choice set with a bailout option. In addition to the standard choice of defaulting or repaying the debt, a government can also choose to ask for a third-party bailout, which comes at a cost of an imposed borrowing limit. Second, we introduce financial frictions and a financial intermediation channel, which tie conditions on the private credit market to the conditions on the sovereign credit market. This link has been very strong in European countries during the recent sovereign crisis. We find that the existence of a bailout option reduces sovereign spreads and, through the described link, private credit rates as well. The implementation of a rescue program reduces output losses and increases welfare, measured in consumption equivalent terms. Moreover, bailout benefits emerge even when a government only has the option of asking for a bailout, but does not take advantage of it.

Keywords: Default, Sovereign Spread, Private Spread, Bailouts.

JEL Classification: E44, F32, F34.
1 Introduction

In this paper we develop a quantitative endogenous default model for small open economies with financial frictions and a non-trivial bailout choice to study the effects of stabilization policies on sovereign spreads, private credit rates, and welfare during times of financial distress. Specifically, we start from a baseline endogenous default model, as in Arellano (2008), and extend it in two ways. First, we augment the government's choice set with a bailout option. In particular, in addition to the standard choice of defaulting or repaying the debt, a government can also choose to ask for a bailout from a third party (for instance a monetary authority or an international organization). In that case, however, the third party imposes an upper limit on government's future borrowing, which reflects the conditionality clause of the rescue programs recently proposed in the wake of the European sovereign debt crisis. Second, we introduce financial frictions and financial intermediation, which allow us to link conditions on the private credit market with the conditions on the sovereign credit market. We show that third-party bailouts reduce the exposure of the economy to default, reduce sovereign spreads, and have large effects on welfare.

Accounting for the relationship between sovereign and private lending rates is crucial when evaluating policies aimed at reducing risk in the sovereign bonds market, because of the spillover effects of the stabilization policies onto the private credit markets that may arise through the credit channel. The strong positive relationship between the two rates emerges when examining recent data. In particular, during the recent European crisis, sovereign debt levels and sovereign spreads soared, with private credit conditions dramatically deteriorating at the same time. For example, in the period between April 2007 and January 2012, when the sovereign spreads in Spain increased from 0.05 percent to 5.4 percent, private credit spreads increased from about 0 percent to 3 percent. The literature has started to investigate this comovement: Popov and van Horen (2013) state that a possible source of this link is generated through the large holdings of government debt securities on the balance sheets of European banks. Similarly, Gennaioli et al. (2014b) show that private credit declines because banks’ balance sheets are weakened during the sovereign debt crisis due to their large holdings of sovereign bonds. Allen and Moessner (2012) provide an alternative explanation, according to which large sovereign debt led to a substantial reduction in liquidity on the credit markets during the recent crisis. To the best of our knowledge, the relationship between sovereign spreads and private credit markets has not yet been taken into account in the context of policy evaluation, although the existence of this link is widely recognized in the policy circles. Our paper

\footnote{For instance, Mario Draghi, president of the ECB, addressed this issue (Wall Street Journal, 22 February 2012) by highlighting that “Backtracking on fiscal targets would elicit an immediate reaction by the market. Sovereign spreads and the cost of credit would go up”. This fact serves as one of the underlying motives for the ECB’s 2012 introduction of Outright Monetary Transactions. In addition, Emma Marcegaglia, former president of the General}
fills this gap by evaluating the effects of recently proposed bailout policies aimed at reducing interest rates on sovereign bonds of the troubled economies, taking into account possible effects on private credit conditions as well. Intuitively, sovereign crises pose a great burden on private businesses because of the association between increased sovereign spreads and higher private borrowing costs. This burden is costly because it diverts economic resources that would otherwise be productive pursuits and channels them into the cause of serving higher private lending rates. This inefficiency, in turn, worsens the economic conditions of the country.

Our first contribution is to propose a simple framework for incorporating bailouts into an otherwise standard dynamic stochastic general equilibrium model with endogenous default as in Eaton and Gersovitz (1981) and Arellano (2008). The government of a small open economy chooses which level of assets to hold in order to smooth out consumption in an economy that receives an exogenous endowment in each period. The assets are supplied by risk-neutral international investors. The government cannot commit to honoring its debt and has the option to default. When in the default state, the economy is excluded from the financial markets, but can reenter with an exogenous probability. The price at which the government borrows from the international investors is a function of government’s default probability. In addition to these standard assumptions, we augment the model with a bailout option. When asking for a bailout, the domestic economy receives a transfer from an unmodeled third party. At the same time, the third party imposes a borrowing constraint on the government while the country is in the bailout program. Furthermore, once in the program, the domestic economy cannot request additional policy interventions and it can exit the program with an exogenous probability. Our setting shares some features with the one proposed in Aguiar and Gopinath (2006), which treat bailouts simply as unconditional transfers of resources from an unmodeled third party to creditors when defaults occur. Hence, in their framework bailouts are equivalent to subsidizing default. In contrast, our setup introduces an endogenous decision for entering the bailout program induced by weighting the benefit (additional resources) and the cost (borrowing constraints) with respect to the alternative choices (defaulting or repaying the debt without asking for a bailout).

We incorporate this bailout option into a model that is able to capture the empirical relationship between sovereign spreads and private spreads discussed above. The model is a stochastic general equilibrium model augmented with strategic sovereign default, in which the dynamics are driven by the interaction between the government, firms, international lenders, and financial intermediaries. The government borrows from the international credit markets, paying an interest rate that reflects its endogenous default probability. Firms face a working capital constraint, which requires them

Confederation of Italian Industry, stated that “the spread should decline, otherwise there is a risk of credit freeze to households and businesses” (LaPresse, 11 Nov. 2011).
to hold non-interest-bearing assets to finance a fraction of their wage bill each period. Therefore, faced with this financial friction, firms must borrow funds from domestic financial intermediaries (henceforth, banks) that operate in an intratemporal market, in the spirit of Christiano et al. (2010). The representative bank operates in a competitive environment and its role is to channel resources from lenders to borrowers. Banks collect deposits from firms and foreign agents, and they use these funds to produce credit lines to domestic firms that need to finance their working capital. This setting generates an endogenous relationship between the price of government debt and the rate at which banks lend to the firm, which stems from the pressure on the cost of rising international funding faced by the banks when debt levels are high.

We calibrate the model to match financial variables (sovereign spreads and private rates), output dynamics, and default statistics of Greece, Ireland, Italy, Portugal, and Spain (henceforth, GIIPS). We focus on these countries because arguably they have been affected the most by the recent European sovereign debt crisis. The model is able to replicate the data in several dimensions. As such, the model is suitable for evaluating the effects of bailout interventions.

When we evaluate bailout policies, two important results emerge. First, the presence of bailouts drastically reduces both sovereign and private spreads. In our benchmark specification, the existence of the bailout option cuts sovereign spreads by about 180 basis points. The intuition is simple: international investors internalize that asking for a bailout is now an additional option for the government to avoid default. As a consequence, the lower cost of sovereign bonds allows private banks to produce private loans more efficiently, thus reducing the rates on these loans. In particular, with the benchmark bailout parametrization, private spreads decline by more than 60 basis points. Importantly, even when the government does not ask for a bailout, the mere existence of this option still has a strong positive effect, leading to the reduction of spreads. Second, as a consequence of reduced private rates, the economy becomes more efficient when bailouts are available. To quantify the gain brought by the bailout policy due to reduction of financial frictions, we find that the benchmark bailout policy reduces output losses with respect to GDP by 0.08 percent with respect to an economy without a bailout option. In particular, output losses are reduced from 0.50 percent of GDP in a model with no-bailouts to 0.42 percent of GDP in a model with bailouts. This value is even lower for alternative specifications of the bailout package. Considering the examples of European economies, the gain stemming from eliminating this inefficiency alone would be equivalent to 1.65 billion US$ in Italy and 1.08 billion US$ in Spain.\footnote{To obtain these numbers we calculate the 0.08 percent of the nominal GDP in 2013. For example, in Italy it was $2.071 trillion.}

Finally, we compute the level of welfare in terms of consumption equivalent, associated with having the additional option of asking for a bailout. It is important to stress that our calculation,
when appropriately rescaled, abstracts from the additional resources obtained by the domestic country from the third party when asking for a bailout. Nevertheless, the bailout option carries welfare benefit due to the reduction of sovereign spreads and output losses. On the other hand, the bailout constrains the domestic economy to the borrowing regulations dictated by the third party for the immediate future, thus leading to potential welfare losses. We show that bailout policies are, indeed, highly desirable especially in an economy characterized by financial frictions: the welfare gains range from 2.9 percent to 7.8 percent in terms of consumption equivalents, depending on how strong the bailout conditions are.

**Related Literature**  This paper relates to three distinct strands of the macroeconomic literature. First, it relates to the literature on strategic sovereign default that analyzes the dynamics of sovereign spreads, such as Eaton and Gersovitz (1981), Arellano (2008) and Aguiar and Gopinath (2006). These papers, however, do not examine the interaction of sovereign spreads with private-sector interest rates because they do not model private debt explicitly. Mendoza and Yue (2012) introduce financial frictions in a sovereign-default model, with the objective of reconciling default and business-cycle stylized facts in emerging economies. While our work here relates to Mendoza and Yue (2012), we extend the baseline default model in a different direction. We focus on the link between private and sovereign spreads and its impact on the bailout decision by the government. With this objective in mind, we consider a simpler production side of the economy. A variety of other papers has focused on default, but as a result of a self-fulfilling crisis, such as Lorenzoni and Werning (2013) and Conesa and Kehoe (2012).

Our paper also relates to the growing literature that studies the interaction between financial intermediation and sovereign risk, such as Bianchi (2012), Acharya et al. (2011) and Bocola (2013). Regarding bailouts, Aguiar and Gopinath (2006) model a bailout as an unconditional and automatic transfer of resources from a non-modeled third party to creditors in case of default. Hence, as they acknowledge, it is not surprising that default rates increase because a bailout in that framework is viewed as a subsidy for default. As a result, their model is not suitable for studying bailout policies because they do not model the option to choose a bailout or the conditionality that bailout imposes. In our setting, instead, choosing a bailout constrains the domestic economy as it comes with imposed borrowing regulations. Also, sovereign default and banking crisis have raised the attention of many recent papers in the international macroeconomics literature. For instance, Reinhart and Rogoff (2011) and Gennaioli et al. (2014a) conduct empirical studies to uncover the relationship between sovereign debt and banking crisis, with the former using an aggregate data and the later using a cross-country panel data on banks. In addition, Padilla (2013) develops a model where banks are exposed to the risk of sovereign default as they lend both to the government and the corporate
sector. Mallucci (2013) uses a similar model with wholesale funding to study the implications of a relaxation of collateral eligibility requirements by a central bank.

Finally, our paper also relates to the literature on financial frictions, with a focus on credit conditions in the private sector and their interaction with interest rates, as in Neumeyer and Perri (2005), Garcia-Cicco et al. (2010), Uribe and Yue (2006), and Corsetti et al. (2013), among others.

The remainder of the paper is as follows. Section 2 illustrates how we introduce a bailout choice, by using a simple endogenous default model. Section 3 describes the full model in detail. Section 4 presents empirical evidence regarding the relationship between private and sovereign spreads in GIIPS countries. Section 5 illustrates the calibration and performance of the model. Section 6 introduces bailout programs in the full model and describes their implications on sovereign risk, private credit, output losses, and welfare. Section 7 concludes.

2 Simple Default Model with Bailout Option

Our final goal is to study the effects of a bailout option in a relatively rich model. Before we explain the details of that model, we first illustrate the trade-off that this additional bailout option generates in a much simpler framework as in Arellano (2008). The government borrows from or lends to international investors in order to smooth out consumption in an economy that receives an exogenous endowment in each period. In each period the government chooses the level of one-period bond holdings, \( b_0 \), where \( b_0 \) denotes the amount to be repaid in the next period. When country issues bonds, \( b_0 < 0 \), and when it purchases bonds, \( b_0 > 0 \). The price of the bond is denoted by \( q \).

The government cannot commit to honoring its debt and has the option to default. When in the default state, the economy is excluded from the financial markets and it reenters with an exogenous probability, \( \theta \).

In addition to these standard assumptions, we augment the model with a bailout option. When the government asks for a bailout, the domestic economy receives a transfer, \( g(b) \), from an unmodeled third party. At the same time, the third party imposes a borrowing constraint, \( \bar{b} \), on the government, where \( b_0 \geq \bar{b} \) and \( \bar{b} < 0 \). This constraint is in effect the whole time the country is in the bailout program. Furthermore, once in the program, the domestic economy cannot request additional interventions and it can exit the program with an exogenous probability, \( (1 - \mu) \). We denote the participation in the bailout program with the variable \( s = \{0, 1\} \); with \( s = 0 \) denoting that the domestic economy is not in the bailout program and \( s = 1 \) denoting that the economy is in the bailout program.

Let us express with \( c = c(b, y, q, s) \) the level of consumption as a function of the assets level,
exogenous income, price of the assets, and the bailout state indicator. Here, the optimal value of the government that is not yet in a bailout program is given by:

\[ v^o(b, y, 0) = \max \left\{ v^c(b, y, 0), v^p(b, y, 1), v^d(y) \right\}, \]

where \( v^c(b, y, 0) \) is the value of repaying the debt without asking for a bailout, \( v^p(b, y, 1) \) is the value of repaying by asking for a bailout, and thus immediately entering the bailout program, and \( v^d(y) \) is the value of default. When the economy is already in the bailout program, \( s = 1 \), the government cannot ask for another transfer from the third party, and the corresponding optimal value is given by:

\[ v^o(b, y, 1) = \max \left\{ v^c(b, y, 1), v^d(y) \right\}. \]

The value function of repaying the debt while not asking for a bailout is:

\[ v^c(b, y, 0) = \max_{b^0} u(c(b, y, q, 0)) + \beta \mathbb{E}[v^o(b', y', 0)], \]

and the value function of repaying the debt while asking for a bailout is:

\[ v^p(b, y, 1) = \max_{b \geq b^0} u(c(b, y, q, 1) + g(b)) + \beta \mathbb{E}[\mu v^o(b', y', 1) + (1 - \mu)v^o(b', y', 0)]. \]

In addition, the value of being in a bailout is as follows:

\[ v^e(b, y, 1) = \max_{b \geq b^0} u(c(b, y, q, 1)) + \beta \mathbb{E}[\mu v^o(b', y', 1) + (1 - \mu)v^o(b', y', 0)]. \]

Notice that no new transfers of resources occur, but the economy cannot borrow above the borrowing limit imposed by the third party for the entire duration of the bailout program. Finally, the default value is standard and is given by:

\[ v^d(y) = u(y^{def}) + \beta \mathbb{E}[\theta v^o(0, y', 0) + (1 - \theta)v^d(y')], \]

where \( \theta \) is the probability of leaving financial autarky.

This setup highlights the main mechanism behind the bailout option, which will be also present in the richer framework described in the next section. When deciding whether to enter a bailout program, the government faces a clear trade-off: the government weights the benefit of receiving additional resources in time of high leverage against the disutility stemming from the limit on future borrowing. In the next section, we incorporate the bailout option in a more general framework.

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4For instance, this function in the Arellano (2008) model would be \( c = c(b, y, q) = y + b - q(b', y)b' \). In our simple variant of this model with a bailout option it would be \( c = c(b, y, q, 0) = y + b - q(b', y, 0)b' \), with 0 indicating that the government has not entered the bailout program. On the other side, when it does enter the bailout program, consumption (net of the transfer \( g(b) \)) would be \( c = c(b, y, q, 1) = y + b - q(b', y, 1)b' \). Finally, during defaults, consumption is simply given by \( c = y^{def} \), where \( y^{def} \) denotes a penalized output level.
3 The Full Model

This section describes the full endogenous sovereign default model, which we later use for policy evaluations. In particular, the model is augmented with financial frictions and a bailout option. Our framework consists of a small open economy populated by households, firms, banks, and a government. Households choose consumption and receive transfers from the government. The government borrows at the international markets in order to smooth households’ consumption, but it cannot commit to honoring its debt. Firms make production decisions subject to a working capital constraint, which requires them to finance a wage bill before production takes place. Financial intermediaries borrow from a variety of sources and lend to firms. The rest of the world is populated by risk-neutral international investors. A detailed description of the model follows.

3.1 Households

Households are identical, risk averse, and maximize the present value of their expected utility given by:
\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \]
where \( c_t \) represents households’ consumption in period \( t \), \( \beta \in (0, 1) \) is the discount factor, and \( u(\cdot) \) represents a utility function, which is strictly increasing and strictly concave. Households are endowed with one unit of time. They supply labor, \( h_t \), inelastically and receive an hourly wage of \( w_t \). They also own and rent a given and constant amount of capital \( k \) at the rate \( u_t \).

Additionally, we assume that households receive government transfers, \( \tau_t \), and also own firms, which distribute profits, \( \pi_t \). Given these assumptions, the budget constraint of the households at any period \( t \) reads:
\[ c_t = h_t w_t + k u_t + \tau_t + \pi_t. \]

3.2 Firms

Firms produce a single good and rent capital and labor services taking all prices as given. They are subject to financial frictions in the form of a working capital constraint, i.e. firms have to...
advance a share of the wage bill before production takes place, as in Neumeyer and Perri (2005) and Mendoza and Yue (2012). Firms production function is given by:

\[ y_t = \varepsilon_t F(k, h_t), \]

where \( \varepsilon_t \) represents an exogenous productivity shock, \( y_t \) denotes output and \( F(\cdot) \) satisfies the assumptions of the neoclassical production function. We assume that the logarithm of the total factor productivity follows an \( AR(1) \) process, with Gaussian innovations of mean 0 and variance \( \sigma^2 \).

We denote the net interest rate that firms face when financing the working capital constraint by \( r^d_t \). Formally, the working capital constraint is given by:

\[ \kappa_t \geq \eta w_t h_t, \]

where \( \eta \geq 0 \) and \( \kappa_t \) denotes the amount of working capital held by a representative firm in period \( t \). Thus, firm’s profits are:

\[ \pi_t = y_t - w_t h_t - u_t k - r^d_t \eta w_t h_t. \]

Firms choose labor according to its first order condition:

\[ F_h(k, h_t) = w_t \left[ 1 + \eta r^d_t \right]. \]

Hence, the return to labor is affected by the degree of financial friction in the private sector, \( \eta \), and by the private lending rate, \( r^d_t \).

### 3.3 International Lenders

The modeling choice of international investors is quite standard, as in Yue (2010) and Arellano (2008) for example. Specifically, international investors are risk-neutral and, in addition to one-period government bonds, they have access to a risk-free asset with net return \( r > 0 \). When pricing sovereign bonds, they take into account that government can default on its debt with probability \( \delta_t \). Given the risk-neutrality assumption, they price sovereign bonds such that they break even in expected terms. Investors demand small open economy bonds \( b_{t+1} \) in order to maximize profits, \( \phi \), given by:

\[ \phi = \rho b_{t+1} - \frac{1 - \delta_t}{1 + r} b_{t+1}. \]

Hence, equilibrium bond price will be:

\[ q_t = \frac{1 - \delta_t}{1 + r}. \]

The default probability is endogenous and depends on the government’s incentives to repay its debt obligation. In particular, when foreign asset holdings are negative, foreign investors account for a
positive probability of default, while when asset holdings are positive, default incentives are zero and the price of bonds equals the inverse of the risk-free rate.

3.4 Financial Intermediaries

To finance the working capital constraint firms need credit lines from banks that operate in an intratemporal market. The representative bank operates in a competitive environment and its role is to channel resources from lenders to borrowers. Banks collect deposits from firms and foreign agents, and they use these funds to produce credit lines to domestic firms that need to finance their working capital. If extra funds are produced, the banks accumulate reserves, \( b_t \), which pay the risk-free rate. The resulting bank’s balance sheet, at the end of period \( t \), is presented in the following table:

Table 1 – Bank’s balance sheet

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working capital loans ( \kappa_t )</td>
<td>International investors deposits ( d_t )</td>
</tr>
<tr>
<td>Reserves ( b_t )</td>
<td>Firms deposits ( d_f^t = \kappa_t )</td>
</tr>
</tbody>
</table>

At the beginning of period \( t \), immediately after the government issues debt, the bank collects deposits from firms and foreign investors. Firms make a deposit equal to the working capital they need to finance, \( \kappa_t \). The firms do not earn interest on these deposits. If the economy is not in default, the bank combines firms’ deposits with foreign investors’ deposits to produce credit lines using a “credit production technology” given by \( L(\kappa_t, d_t) \). These credit lines are offered to firms that finance working capital and, if there is any excess credit, the bank stores it as reserves using a technology that pays the risk-free rate.

An arbitrage condition implies that foreign investors must be paid at least the return equal to the one that they would get by lending to the government. For this reason, we assume that the bank purchases a unit of deposit from foreign investors at a price \( q_t \) and promises to repay 1 at the end of period \( t \). The end of period \( t \) is arbitrarily close to the beginning of period \( t + 1 \), which is the time when the government decides whether to honor the foreign debt or not. This timing assumptions is similar to the one in Neumeyer and Perri (2005). If the government defaults,

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6When modeling the financial sector, we borrow some ingredients from Christiano et al. (2010).
7This assumption is only a normalization and none of the results depend on it. For instance, Christiano et al. (2010) use the same setup but assume that firms receive a return on their deposits and then have to pay a larger interest rate on their credit lines.
the bank cannot make repayments to foreign investors and those resources are immediately lost. Hence, foreign investors are subject to the same risk when depositing in a domestic bank and when lending to the government. This implies that the expected return on the bank deposits of foreign investors must be $1/q_t$.

The problem of the bank can be written as:

$$\max \pi_t^b = (1 + r_t^d)\kappa_t - \kappa_t + q_t d_t - d_t + (1 + r)b_t^r - b_t^r,$$

subject to

$$b_t^r + \kappa_t = L(\kappa_t, d_t) = A \left( \lambda \kappa_t^\gamma + (1 - \lambda)(q_t d_t)^\gamma \right)^{\frac{1}{\gamma}} - \psi \kappa_t,$$

with $b_t^r \geq 0$. The bank combines firms’ deposits, $\kappa_t$, and foreign investors’ deposits, $q_t d_t$, using a credit production technology with efficiency level $A$. The credit production technology is a standard CES Armington aggregator, with these two types of deposits as inputs. The Armington elasticity of substitution between firms’ and foreign deposits is defined as $\frac{1}{\gamma-1}$ and $\lambda$ is the weight on firms’ deposits. However, in order to access this technology, the bank must pay a linear cost, $\psi \kappa_t$. As shown in Appendix A, optimality conditions imply the following relationship between the private lending rate and the price of sovereign bonds:

$$r_t^d = r(1 + \psi) - Ar\lambda \left[ \frac{\lambda(1 - \lambda)q_t^\gamma}{(\frac{1-q_t}{q_t(1-\lambda)})^{\frac{1}{\gamma-1}} - (1 - \lambda)q_t^\gamma} + \lambda \right]^{\frac{1-s}{s}},$$

stating that the smaller is the price of government bonds, the larger is the pressure on private lending rates. This relationship stems from the fact that high sovereign debt levels put an upward pressure on the borrowing costs of the banks and make it harder for the banks to raise funds on the international markets.

Foreign investors cannot deposit funds in the banks in case of default, since country is in autarky. In this case, the problem of the bank is to maximize profits,

$$\max \pi_t^b = (1 + r_t^d)\kappa_t - \kappa_t + (1 + r)b_t^r - b_t^r,$$

subject to:

$$b_t^r + \kappa_t = L(\kappa_t) = A\lambda^\frac{1}{2} \kappa_t^\frac{1}{2} - \frac{\psi}{2} \kappa_t^2,$$

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8This assumption keeps the sovereign default decision as close as possible to the baseline default model. Another option would be to assume that the government confiscates and distributes these funds to households or that banks are able to use them for liquidity production purposes. However, these assumptions would affect government default decisions by increasing default incentives.

9The advantage of this functional form, as opposed to a Cobb-Douglas, is that in this case liquidity production still takes place even if one input is not available. In particular, the liquidity production function holds during normal times, but also during default.
and $b_t^r \geq 0$, where we assume that the cost of accessing the liquidity technology is quadratic.\textsuperscript{10} This assumption depicts the fact that during default times it is more costly to provide credit. At the same time, it also depicts the fact that the more credit firms require, the larger is the rate they face. Then, optimality conditions in default times imply that:

$$r_t^d = r \left[ 1 + \psi \kappa_t - A \lambda_t^2 \right].$$

Notice that banks operate both in normal and default times. Moreover, given the quadratic costs faced by banks during default, the private interest rate is defined even during periods of default, when it continues to affect economic activity. Therefore, even during default when the country is excluded from the international markets, the economy will suffer an endogenous output loss due to the presence of financial frictions.\textsuperscript{11}

### 3.5 Government

The government issues debt and has three options. In particular, it can choose to: 1) default on its debt obligations; 2) repay the debt and issue new debt in international markets; or 3) repay its debt obligation while entering in a bailout program. In the last case, a third party (a monetary authority or an international organization) transfers funds to the domestic economy and imposes a limit that constrains borrowing during the length of the program. The borrowing limit imposed on a participating country depicts the fiscal conditionality clause of bailouts. We use $s_t = \{0, 1\}$, where $s = 1$ denotes that government is in the bailout program, and $s = 0$ denotes that government is not in the bailout program. If the government chooses to repay its debt, it remains in the contract and chooses the new level of assets, $b_{t+1}$. As the government is benevolent, its objective is to maximize households’ lifetime utility using foreign debt, the default option and the transfers, subject to the following resource constraint of the economy:

$$c_t = y_t - r_t^d \eta w_t h_t + b_t - q_t \left( b_{t+1}, \varepsilon_t, s_t \right) b_{t+1}.$$

The second term on the right hand side of the resource constraint represents a resource cost to the economy resulting from the existence of working capital constraint. Once we introduce the additional bailout option, the price of government bond depends on $s_t$ as well.

When the government defaults, the country is excluded from financial markets for a random number of periods. In this case, the country experiences productivity losses that capture the

\textsuperscript{10}This assumption guarantees an interior solution.

\textsuperscript{11}For some parameterization, $r_t^d$ in default periods could be lower than the risk-free rate, $r_t$. To avoid this issue, we impose that in default $r_t^d = \max \left( r, r \left[ 1 + \psi \kappa_t - A \lambda_t^2 \right] \right)$.
disrupting effects of default in the domestic economy. In particular,
\[ c_t = \tilde{y}_t - r_t^d \eta w_t h_t, \]
where \( \tilde{y}_t = \varepsilon^{def} F(e, h_t) \) and \( \varepsilon^{def} = \gamma(\varepsilon) \). We assume that \( \gamma(\cdot) \) is a penalty function as in Arellano (2008). With an exogenous probability \( \theta \), government reenters the international credit markets where all past debt is forgiven.

When the government enters the bailout program, it receives a transfer of resources from a third party. While in the program, a country cannot ask for additional bailout funds or exit the program voluntarily; nevertheless, the government can choose to default on its debt. For simplicity we assume that exiting from the bailout program occurs with an exogenous probability. Implicitly, we assume that the domestic economy does not incur any pecuniary cost from receiving a bailout. However, once in the program, the government is financially constrained on its asset position in that it cannot borrow more than \( \bar{b} \) with \( \bar{b} < 0 \), a limit imposed by the third party. When the government enters the bailout program, consumption is given by:
\[ c_t = y_t - r_t^d \eta w_t h_t + b_{t+1} - q(b_{t+1}, \varepsilon_t, s_t) b_{t+1} + g(b_t), \]
where \( g(b_t) \geq 0 \) is the size of the bailout injected. We assume that the bailout size is zero if level of assets held by the domestic economy is positive and that the function \( g(\cdot) \) is a non-increasing function of \( b_t \). This means that \( g(b) = 0 \) if \( b \geq 0 \) and \( \frac{\partial g}{\partial b} \leq 0 \). These assumptions capture the notion that bailouts are present only when the domestic country has debt and that they are proportional to the degree of the fiscal imbalances of the country. For simplicity, in this paper we assume that \( g(b_t) \) is linear in \( b_t \), that is: \( g(b_t) = G b_t \), with \( G < 0 \).\(^{12}\)

## 4 Private Lending Rates and Sovereign Spreads

In the previous section, we showed that our model generates a positive link between the interest rate on sovereign bonds and the interest rate on private lending, as captured by equation (1). In order to confront our model with the data, in this section we investigate the empirical relationship between these two rates using the data of the GIIPS economies.\(^{13}\) In particular, we compute sovereign spreads as the difference between the return on 10-year sovereign bonds of the five GIIPS economies and the return on 10-year German sovereign bonds.\(^{14}\) As a measure of the private

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\(^{12}\)Since resources are transferred only when the country runs debt, i.e. \( b < 0 \), this specification guarantees that the transfer is positive.

\(^{13}\)This link recently has attracted the attention of applied macroeconomists and policymakers. See, for example, Albertazzi et al. (2012), Bofondi et al. (2013), and Zoli (2013).

\(^{14}\)We use Reuters monthly data from January 2003 to December 2011. In our computation of sovereign spreads, it is implicit that the expected inflation for Germany and the rest of the countries is the same.
sector cost of borrowing, we consider the annualized agreed rate for 10-year loans to non-financial corporations from the ECB. Given that this measure is largely affected by the prime interest rate set by the ECB and that our model does not depict any monetary ingredient, we isolate the movements in the cost of borrowing from this rate. Consequently, we compute deviations of the agreed rate from the EU marginal lending facility rate and we add the average ECB rate to capture the correct level of the private lending rate. We refer to this variable as the “private lending rate”. To compute private spreads, we subtract the German interest rate from the private lending rate.

![Graph showing sovereign spreads and private lending rates for Greece, Ireland, Italy, Portugal, and Spain from 2003 to 2012. The top panel displays monthly annualized spreads for these countries, with spreads starting to increase after 2008, a trend that remains a striking characteristic of the ongoing European sovereign debt crisis. The bottom panel shows private lending rates for the same economies, with rates starting to increase after 2008, after a period of stability or even a slight decline. These figures provide evidence that the recent sovereign crisis influenced the conditions on the private credit markets as well, although the magnitude of a change in private rates is smaller than that of government debt.]

**Figure 1 – Sovereign Spreads and Private Lending Rates**

Note: The top panel displays monthly annualized spreads for Greece, Ireland, Italy, Portugal, and Spain. The bottom panel displays private lending rates for the same economies. The vertical line depicts the first month of 2008. The construction of sovereign and private spreads is explained in the main text.

The top panel of Figure 1 displays the sovereign spreads in the GIIPS countries. It is evident that spreads started to increase after 2008, a trend that remains a striking characteristic of the ongoing European sovereign debt crisis. The bottom panel shows private lending rates in the five GIIPS countries. The increase in risk premium affected all five countries, although unequally, with Greece and Portugal being affected the most. Interestingly, private lending rates also started to increase after 2008, after a period of stability or even a slight decline. These figures provide evidence that the recent sovereign crisis influenced the conditions on the private credit markets as well, although the magnitude of a change in private rates is smaller than that of government debt.
spreads.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Greece</td>
<td>0.23</td>
<td>1.6</td>
</tr>
<tr>
<td>Spain</td>
<td>0.03</td>
<td>-0.4</td>
</tr>
<tr>
<td>Italy</td>
<td>0.21</td>
<td>0.5</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.11</td>
<td>0.5</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.01</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Note: This table reports the mean of sovereign and private spreads and their correlation. Variables are in annualized percentages. Quarterly variables are computed by averaging monthly rates. “Pre-Crisis” denotes the sub-sample from January 2003 until December 2007 and “Crisis” denotes the sub-sample from January 2008 until December 2011. The construction of sovereign and private spreads is explained in the main text.

Table 2 reports mean sovereign spreads and private lending rates, as well as their correlation, in the pre-crisis period (2003-2007, left panel) and crisis period (2008-2011, right panel). There are several things worth noticing here. First, not surprisingly, the average sovereign spread increased by a very large margin in all five countries during the crisis period. Second, private spreads also increased (except for Ireland) by as much as 100 basis points. Third, the correlation between these two rates changed significantly after the crisis outburst. In particular, the correlation is small and negative during the pre-crisis period, while it is large and positive during the crisis.

As Popov and van Horen (2013) point out, a possible source of the link between private and sovereign spreads exists through the large holdings of government debt securities on the balance sheets of European banks. Similar explanation is offered by other authors as well. For example, Gennaioli et al. (2014b) show that private credit declines because bank’ balance sheet is weakened during the crisis due to large holdings of sovereign bonds. Another potential explanation is that sovereign debt contributes to the liquidity dry up in credit markets, which substantially worsened during the crisis, as suggested by Allen and Moessner (2012). Our modeling assumption relates to this mechanism, since external investments to domestic banks facilitate liquidity provision, which becomes more costly in times of sovereign crisis.
5 Empirical Strategy

This section presents the calibration and dynamics of the full model presented in Section 3 when we impose that $s_t = 0 \; \forall t$, which reflects the case when the government does not have the option of asking for a bailout. We decide to abstract from the bailout option at this stage simply because we base our calibration on the historical data which includes periods in which bailout option was arguably unexpected. We document that the model with financial frictions and without bailouts can successfully replicate the data. Then, when we evaluate the effects of bailout option on the dynamics of our economy - debt level, default decision, sovereign and private rates, and welfare - we augment the model with a bailout choice. In what follows, we first describe the calibration of the full model without a bailout option, and then the choice of the bailout option parameters.

5.1 Calibration of the Financial Intermediaries

The first step in the calibration is to use the empirical link between the private and sovereign rates uncovered in Section 4 to pin down the parameters that characterize the liquidity technology of the financial intermediaries. Notice that modeling choice of financial intermediation does not restrict the sign of the relationship between sovereign net rates, $\frac{1}{\psi_t} - 1$, and private net rates, $r^d_t$, as displayed in equation (1). The parameters that determine this relationship are $A$, $\psi$, $\lambda$ and $\zeta$.

![Figure 2 – Sovereign and private spreads](image)

Note: Blue dots denote coordinate pairs of sovereign and private spread observed in the data and red points the ones implied by the model. The construction of sovereign and private spreads is explained in the main text.

We take pairs of observations of these two variables in the GIIPS countries over the period 2003-2012, restricting our attention to the data points in which the sovereign rates are larger than...
2 percent in annualized terms to avoid negative sovereign spreads that are not seen in the model. Given these data points, we calibrate our four parameters such that the relationship implied by equation (1) closely mimics the one implied by the data. Figure 2 presents the scatter plot of the data (blue) and the one implied by the model (red). The parameter values that generate red dots on the scatter plot are $A = 6.98$, $\psi = 6.15$, $\lambda = 0.88$ and $\zeta = 0.15$.

5.2 Calibration of the Rest of the Model

Here we describe in detail choice of functional forms and the remaining parameters of the model, when abstracting from the bailout option. Regarding the functional forms of the utility and production functions, we assume CRRA utility function with coefficient of risk aversion denoted by $\sigma$, and a Cobb-Douglas production function with capital share denoted by $\alpha$. This choice is much in line with the previous literature.

The exogenous technology process is assumed to follow an $AR(1)$ process. The persistence parameter, $\rho_\varepsilon$, and the standard error of its innovations, $\sigma_\varepsilon$, are calibrated to match quarterly real output per capita of the GIIPS countries over the 1960-2008 period. We follow Garcia-Cicco et al. (2010), and remove a cubic trend from output and compute the first order autocorrelations and standard deviations.\footnote{We obtain similar moments if we remove a quadratic trend or a linear trend. Moreover, moments are also similar if we extend the sample until 2014, although slightly larger volatility is observed.} As displayed in Table 3, in all five economies output is highly persistent with the volatility levels of similar magnitudes.

<table>
<thead>
<tr>
<th></th>
<th>Greece</th>
<th>Ireland</th>
<th>Italy</th>
<th>Portugal</th>
<th>Spain</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(y)$</td>
<td>0.85</td>
<td>0.97</td>
<td>0.93</td>
<td>0.98</td>
<td>0.97</td>
<td>0.94</td>
</tr>
<tr>
<td>$\sigma(y)$</td>
<td>0.05</td>
<td>0.06</td>
<td>0.02</td>
<td>0.06</td>
<td>0.04</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Note: $\rho(y)$ denotes the first order autocorrelation and $\sigma(y)$ denotes the standard deviation of log cubic detrended output.

We use quarterly real output per capita for the period 1960-2008.

It is helpful to describe now some key features of default episodes in Europe during the last 200 years that we will use to calibrate some parameters of the model as described below.\footnote{We use the evidence provided in Reinhart and Rogoff (2009). Here, a default episode is defined as follows: “A sovereign default is defined as the failure of a government to meet a principal or interest payment on the due date (or within the specified grace period). These episodes include instances in which rescheduled debt is ultimately extinguished in terms less favorable than the original obligation.”} Table 4 presents the number of default episodes, the quarterly default probability of a default episode, the share of periods that the economy spent in default relative to the total quarters in our sample, and

\begin{itemize}
  \item A sovereign default is defined as the failure of a government to meet a principal or interest payment on the due date (or within the specified grace period). These episodes include instances in which rescheduled debt is ultimately extinguished in terms less favorable than the original obligation.
\end{itemize}
the average length of a default episode. Based on this data we calculate the average probability
of default episode per quarter to be 0.64 percent, average share of periods in default to be 17.6
percent, and finally average length of default to be 30 quarters.

Table 4 – Default statistics

<table>
<thead>
<tr>
<th></th>
<th>Greece</th>
<th>Ireland</th>
<th>Italy</th>
<th>Portugal</th>
<th>Spain</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default episodes</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>Frequency default per quarter (%)</td>
<td>0.70</td>
<td>0</td>
<td>0.17</td>
<td>0.71</td>
<td>1.64</td>
<td>0.64</td>
</tr>
<tr>
<td>Share of periods in default (%)</td>
<td>50.6</td>
<td>0</td>
<td>3.4</td>
<td>10.6</td>
<td>23.7</td>
<td>17.6</td>
</tr>
<tr>
<td>Average length of default</td>
<td>72</td>
<td>N.A.</td>
<td>20</td>
<td>14</td>
<td>14.36</td>
<td>30</td>
</tr>
</tbody>
</table>

Note: Greece (1832-2009), Ireland (1919-2009), Italy (1861-2009), Portugal (1800-2009) and Spain (1812-2009). Statistics
are in quarters. Default statistics are taken from Reinhart and Rogo (2009).

The discount factor, $\beta$, is calibrated to match the observed default frequency of 0.64 percent
across the GIIPS countries, as reported in Table 4. The probability of re-entering the asset market,
$\theta$, is calibrated to match the average length of default in a GIIPS countries (30 quarters).

A subset of parameters is fixed following the existing literature. Specifically, the relative risk
aversion parameter, $\sigma$, is set to 2 and capital share is set to 0.3. We fix hours worked to 0.3 and the
level of capital to 16.6 in order to normalize the steady state level of output to 1.\(^{17}\) The risk-free
rate, $r$, is set to match the real rate on 10-year German bond over the sample period. The default
penalty, captured by the parameter $\gamma$, is set to 0.948, which is in line with existing literature.

Finally, the parameter $\eta$ determines the size of firm loans. We calibrate this parameter such
that the model generates an average level of loans in line with the ratio of business loans to GDP
observed in the GIIPS countries, which is, on average, 55 percent during the period 2007-2011.\(^{18}\)
The resulting value of $\eta$ is 0.78, which is also in line with existing literature. For instance, Mendoza
and Yue (2012) set their share of import goods that has to be paid using working capital to 0.7,
which is still lower than standard values used in the literature, such as in Neumeyer and Perri
(2005) for example. Table 5 summarizes the baseline calibration.

We discretize the exogenous process for total factor productivity using 41 nodes, following
the procedure proposed by Tauchen (1986). In the same spirit as Arellano (2008), we discretize
the asset space using 300 nodes and, to compute the model moments, we simulate the economy
for 100000 periods. Given the simulated data, we identify the default episodes and compute the
average duration of default, the default frequency and the share of quarters in default. Finally, we

\(^{17}\)This normalization does not affect the dynamic properties of the model.
\(^{18}\)We use the data on total business loans from the Statistical Data Warehouse at the ECB.
Table 5 – Baseline calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_t$</td>
<td>Persistence of TFP shock</td>
<td>0.96</td>
</tr>
<tr>
<td>$\sigma_t$</td>
<td>Std. deviation of TFP shock</td>
<td>0.02</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>0.3</td>
</tr>
<tr>
<td>$k$</td>
<td>Fixed level of capital</td>
<td>16.6</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Intertemporal discount factor</td>
<td>0.895</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Relative risk aversion coefficient</td>
<td>2</td>
</tr>
<tr>
<td>$r$</td>
<td>Risk-free rate</td>
<td>1.0069</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Default penalty</td>
<td>0.948</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Prob. re-entering asset markets</td>
<td>0.033</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Working capital constraint coef.</td>
<td>0.78</td>
</tr>
</tbody>
</table>

use all the subsamples of at least 275 quarters that do not contain default episodes to compute all the relevant moments.

Table 6 reports empirical and theoretical moments. Our model is able to match the targeted moments as well as to reproduce several aspects of the economy that are not used as targets in our calibration. Specifically, the default episodes generated by our model replicate fairly well the experienced default episodes of GIIPS countries (on average). Also, the model can successfully replicate the relevant asset prices in the model, specifically the average sovereign bond spread and private loan spread both in the normal times and crisis-times. Importantly, the model captures the increasing correlation of the two spreads during a crisis time, from a mild negative value, $-0.15$, to a large positive value, 0.77, in line with the data. This is a very important feature of our model since most of the existing models either do not even model private rate or assume one-to-one relationship with the sovereign rate.

We compute the average output loss as a percentage of average GDP implied by the model during periods of crisis and during normal times. Recall that output loss is generated in our model through the interaction between working capital constraint of the firms and the technology that banks employ to produce private loans. Therefore, implied average output loss is informative about the importance of financial frictions in the model. As seen in the table, output losses are quantitatively important in the model: during normal times they account for 0.45 percent of GDP, whereas they increase to 0.66 percent of GDP during a sovereign crisis. This is a feature that the standard default model without financial frictions cannot capture. Hence, our model is able to incorporate the tightening of the link between sovereign rates and private rates during a sovereign crisis.
crisis, which, in turn, endogenously generates higher output losses through the presence of financial inefficiencies. Another novel contributions of this paper is to explore how international policies (such as bailout programs) are affected by this important, and relatively unexplored, consequence of the crisis. We describe the parametrization of the bailout program next.

<table>
<thead>
<tr>
<th>Table 6 – Data and Model Moments</th>
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<tbody>
<tr>
<td><strong>Moments</strong></td>
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<tr>
<td><strong>Output moments</strong></td>
</tr>
<tr>
<td>$\rho(y)$</td>
</tr>
<tr>
<td>$\sigma(y)$</td>
</tr>
<tr>
<td><strong>Default</strong></td>
</tr>
<tr>
<td>Average duration of default</td>
</tr>
<tr>
<td>Default frequency (%)</td>
</tr>
<tr>
<td>Share of quarters in default (%)</td>
</tr>
<tr>
<td><strong>Asset Prices</strong></td>
</tr>
<tr>
<td>Pre-crisis average sovereign spread (%)</td>
</tr>
<tr>
<td>Crisis average sovereign spread (%)</td>
</tr>
<tr>
<td>Pre-crisis average private spread (%)</td>
</tr>
<tr>
<td>Crisis average private spread (%)</td>
</tr>
<tr>
<td>Pre-crisis private-sovereign spread correlation</td>
</tr>
<tr>
<td>Crisis private-sovereign spread correlation</td>
</tr>
<tr>
<td><strong>Loans and Output Loss</strong></td>
</tr>
<tr>
<td>Private loan-to-GDP Ratio</td>
</tr>
<tr>
<td>Pre-crisis average output loss (% of output)</td>
</tr>
<tr>
<td>Crisis average output loss (% of output)</td>
</tr>
</tbody>
</table>

### 5.3 Calibration of the Bailout Program

Having a model that is able to capture the empirical evidence regarding default, sovereign rates, and private rates reasonably well, we now introduce an additional option of asking for a bailout. There are three parameters that characterize the bailout policy intervention: the size of the bailout, $G$, the probability of exiting the bailout program, $(1 - \mu)$, and the upper limit on borrowing while in the bailout program, $\bar{b}$. 
Since there is no realistic counterpart of a similar bailout intervention, we first assume a rather conservative parametrization of the bailout program. We then analyze the robustness of our findings under various alternative parametrizations of the policy, as described below. Specifically, our benchmark bailout parametrization consists of a bailout size \((G)\) of 0.15, which implies that when a government of a troubled economy asks for a bailout, the third party provides resources that amount to 15 percent of the troubled economy’s national debt.\(^{19}\)

Setting the borrowing limit imposed by the third party when a country enters the bailout program, \(\bar{b}\), and the probability of exiting the program, \((1 - \mu)\), is less straightforward than the case of the bailout size as there is even less information in the data. To address this issue, we consider a different set of values for both parameters and compare their implications. We first find the median value of debt in the economy without bailouts, and use that value as the borrowing limit imposed on a bailout economy when it enters the bailout program. Notice that, were the no-bailout-option economy to be constrained, this value of borrowing limit would imply it to be 50 percent of the time. Finally, we assume that the bailout regime is not an absorbing state, in that it is possible to exit this regime and reenter the unconstrained regime with probability \((1 - \mu)\), equal to 0.3. After presenting the results for our benchmark parametrization, we show how our findings change when we assume a larger or smaller size of the bailout (i.e. \(G=0.05\) and 0.25), a smaller or larger borrowing limit (i.e. \(\bar{b}\) such that the country without bailout option would be constrained 40 percent or 60 percent of the time), and a shorter or longer bailout duration (i.e. \((1 - \mu)\) equal to 0.35 or 0.25).

6 The Effects of the Bailout Program

We first show that the introduction of the bailout option affects default sets. Intuitively, the presence of this additional option reduces the default set because there is a region of the state space for which requesting a bailout intervention is preferred to defaulting. Figure 3 illustrates how optimal decisions of the government change when an additional option is introduced. In particular, Figure 3 illustrates how optimal decisions of the government change when the bailout option is introduced by plotting the state-space (asset holdings, x-axis, and income, y-axis) for which it is optimal to default (black area) or ask for a bailout (grey area). For comparison, we

\(^{19}\)This is a rather conservative value for the third-party transfers. For instance, during the last three years third-party transfers to troubled economies are around the following level or even higher: In May 2010, Greece obtained a bailout of $110 billion (US) (36 percent of GDP); in November 2010, Ireland obtained a bailout of $113 billion (51 percent of GDP); in May 2011, Portugal obtained a bailout of $116 billion (48 percent of GDP); in June 2012, the Spanish banking sector obtained a bailout of $125 billion (9 percent of Spanish GDP).
also report default set implied by the the no-bailout-option model with the same parametrization (lighter shaded areas). The top panel shows the optimal decision sets conditional on not being in

![Figure 3 – Repayment, default and intervention sets](image)

Note: The top figure represents the default and intervention sets when the economy is in the state where it has repaid the debt without asking for bailouts, and the bottom panel represents these sets conditional on being already in a bailout program. The lighter shaded area, in both panels, displays the default set in a model with the same calibration but in which bailouts are not available, the grey area displays the region where it is optimal to ask for a bailout and the black area displays the region where default is optimal. The vertical line represents the borrowing limit under intervention.

the bailout program \((s_t = 0)\): the default set is significantly smaller than in the no-bailout-option economy, and there is a significant portion of the asset space where asking for a bailout is optimal. The bottom panel of the figure shows the same sets conditional on already being in the bailout program \((s_t = 1)\). Since additional bailouts are not feasible when a country is already in the bailout program, the intervention set is empty by construction. For our benchmark parameterization, the default region while in the bailout program is empty, since only asset levels to the right of the vertical dashed line, which represent the borrowing limit \(\bar{b}\), are feasible. Notice, however, that this is not a general result, since for different bailout calibration there might be a non-empty default region even when the country is in the bailout program.

The shrinking of default sets impacts the conditions on the sovereign credit market. In fact, international investors understand that countries have smaller default regions as a result of the
Figure 4 – Price bond schedule

Note: Each figure, from top to bottom, represents the price bond schedules conditional on low-, average- and high-output levels. The thick blue lines represent the price functions in model where bailouts are available and the dashed red lines represent those of a model without bailout option.

bailout option and, as a consequence, they request a smaller premium when lending to the government. This observation is made clear in Figure 4, where we report the bond price schedule in an economy with a bailout option, conditional on repaying the debt and not asking for a bailout, \( q(b', \varepsilon, 0) \), (solid blue lines) and a bond price schedule in a no-bailout-option economy (red dashed lines). Each panel represents the price of bonds in the two economies as a function of the asset holdings (x-axis) given a level of output: low-level (recession), average (normal times) and high-level (boom), from top to bottom respectively. The only difference between the two economies is that one has a bailout option and the other does not. This difference turns out to be important as it drives the wedge between the bond prices offered by the international investors in the two economies.

The effects of bailouts on bond prices can be better visualised when plotting sovereign spreads during recessions. Figure 5 plots sovereign spreads for the economy with and without bailout option (solid blue lines and dashed red lines, respectively) for a mild recession where output is 1 percent lower than average (upper panel) and for a strong recession where output is 3.5 percent lower than average (bottom panel). The shaded area represents the region where repayment with
bailout is optimal. In the case of a mild recession, it is not optimal for the economy to ask for a bailout for any level of debt (as suggested by the absence of the darker shaded area). Nevertheless, the existence of the bailout option still affects sovereign spreads. In fact, in the model without a bailout option, the sovereign spread drastically rises even for lower levels of debt, whereas in the model with a bailout option, the spread is very low even for much larger levels of debt. Specifically, when bailout option is not available, spreads increase up to 25 percent. When a bailout option is available, for the same level of debt and output, the spreads are lower than 5 percent. This result is very important because it shows that the existence of the bailout program effectively reduces spreads even for levels of debt and output for which bailout is not optimally requested. In addition, notice that in a model without bailout the government defaults for a lower level of debt than in a model with a bailout option, as indicated by the asset value for which the spread lines disappear.\footnote{Spreads are not defined when a country defaults since, by assumption, the country is excluded from international markets. Hence, we can infer default regions in Figure 5 by the level of debt where the spread function disappears.}
When the recession is stronger, there is a region of the state space where the government finds it optimal to ask for a bailout. In this case, sovereign spreads in presence of the bailout option are small, since the international investors understand that asking for a bailout allows the country to obtain resources to repay the debt.

Table 7 displays some quantitative implications of the benchmark bailout programs, and the robustness to alternative specifications. First, the presence of bailouts drastically reduces both sovereign and private spreads. In our benchmark specification, the existence of the bailout cuts sovereign spreads by about 180 basis points, ranging from 70 basis points for the more constrained bailout size to more than 200 basis points for the less constrained bailout size. The intuition is simple: international investors are aware that asking for a bailout is now an additional option for the government to avoid default. As a consequence, the lower cost of sovereign bonds allows private banks to run more efficient liquidity technology to produce private loans. In particular, with the benchmark bailout parametrization, private spreads decline by more than 60 basis points.

Second, as a consequence of reduced private rates, the economy becomes more efficient when bailouts are available. To quantify the gain brought by the bailout policy due to reduction of financial frictions, we find that the benchmark bailout policy reduces output losses with respect to GDP by 0.08 percent with respect to an economy without a bailout option. In particular, output losses are reduced from 0.50 percent of GDP in a model with no-bailouts to 0.42 percent of GDP in a model with bailouts. This value is even lower for alternative specifications of the bailout package. Hence, when taking into account the spillover effect that sovereign risk transmits to the private sector, bailout policies are even more desirable. Considering the examples of European economies, the gain stemming from eliminating this inefficiency alone would be equivalent to 1.65 billion US$ in Italy and 1.08 billion US$ in Spain.²¹

²¹To obtain these numbers we calculate the 0.08 percent of the nominal GDP in 2013. For example, in Italy it was $2.071 trillion.
Table 7 – Quantitative implications of Bailouts

<table>
<thead>
<tr>
<th>No Bailout</th>
<th>Bailout</th>
<th>Bailout Parameters</th>
<th></th>
<th></th>
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<tbody>
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<td></td>
<td></td>
<td>Benchmark</td>
<td>Size(+)</td>
<td>Size(-)</td>
<td>Constraint(-)</td>
<td>Constraint(+)</td>
<td>Length(-)</td>
<td>Length(+)</td>
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<tr>
<td>Bailout size, $G$</td>
<td></td>
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<td>0.25</td>
<td>0.05</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
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<tr>
<td>Constraint, ( %)</td>
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<td>50</td>
<td>50</td>
<td>40</td>
<td>60</td>
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</tr>
<tr>
<td>Exit probability, $(1 - \mu)$</td>
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<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
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<td>0.35</td>
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Moments

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<th>Mean sovereign spread</th>
<th>Mean private spread</th>
<th>Output loss</th>
<th>Default frequency</th>
<th>Intervention frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Bailout</td>
<td>2.63</td>
<td>1.66</td>
<td>0.50</td>
<td>0.63</td>
<td>-</td>
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<tr>
<td>Bailout</td>
<td>0.79</td>
<td>1.04</td>
<td>0.42</td>
<td>0.82</td>
<td>16.2</td>
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Welfare

<p>| | | |</p>
<table>
<thead>
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</thead>
<tbody>
<tr>
<td>Welfare</td>
<td>-</td>
<td>4.9</td>
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</tbody>
</table>

Note: The constraint parameter of the bailout is associated to $\bar{b}$ and it refers to the percentage of periods in which debt is greater than $\bar{b}$ in the non-bailout model. All moments are conditional on not being in default. The spreads are annualized and given in percentages. Default and intervention frequencies are also given in percentages; output loss is in percentage of GDP. The welfare statistics should be interpreted as the percentage of consumption that the representative agent in the domestic economy without bailout is willing to give up to live in an economy with bailout, while keeping the average amount of resources in the economy constant. To do that, we rescale the output endowment in the economy without bailout to have the same mean as the total amount of resources (output level plus resources obtained from the bailout) as in the economy with bailout. Hence, this adjustment isolates the welfare effects of bailouts stemming from the reduction of the cost of debt obligation and of output losses.
Finally, we are able to compute a welfare gain attributable to the presence of the bailout policy. In particular, we compute the percentage of consumption level that an agent who lives in an economy where bailouts are not available is willing to give up to live in an economy where bailouts are available, i.e. we compute the level of welfare in consumption equivalent terms. Importantly, we want to compute a welfare gain that abstracts from the extra resources obtained by the small economy from the third party when asking for a bailout. To achieve this goal, we inject to the no-bailout-option economy the same amount of resources that the bailout economy receives as transfers.

Specifically, we first compute the total amount of extra resources obtained by the country in the entire simulation. Denote this value with \( G_T = \sum_{j=0}^{J} g(b_j) \), where \( J \) is the number of bailout episodes in the entire simulation of length \( T \).\(^{22}\) Hence, the country obtains on average \( \bar{G} = \frac{G_T}{T} \) of extra resources in each period. To abstract from these resources received by the bailout economy, when computing the lifetime utility of the no-bailout-option economy, we increase the amount of output produced in the economy by \( \bar{G} \) in each period. Our measure of welfare is, then, a percentage of consumption, \( \lambda \), that has to be given in each state and time to an agent living in an economy without bailout but with an extra amount of income \( \bar{G} \) in each period such that she has the same expected lifetime utility of an agent living in an economy with bailout. This measure is given by:

\[
E_0 \sum_{t=0}^{T} \beta^t u(c_t) = E_0 \sum_{t=0}^{T} \beta^t u(\tilde{c}_{t}^{\text{NoBailout}}(1 + \lambda)),
\]

where \( \tilde{c}_{t}^{\text{NoBailout}} \) is the optimal consumption in the non-bailout model when increasing the amount of output produced in the economy by the level \( \bar{G} \) in each period.

In our benchmark specification, the welfare gain from bailouts is rather large and equal to almost 5 percent in consumption equivalent terms. We interpret this result as a consequence of the link between sovereign and private spreads stemming from our private banking setting. In fact, the existence of the bailout option not only reduces the cost of borrowing, but also reduces the inefficiency of the private sector by allowing private banks to supply cheaper loans to domestic firms. The link between the magnitude of output losses and the welfare gain for different specifications of the bailout package supports this intuition. The magnitude of the welfare gain from bailout is sizable for the different specifications of the bailout design.

\(^{22}\)Recall that in our model the domestic economy receives a bailout transfer only in the period it enters in the bailout program.
7 Conclusion

In this paper, we propose a parsimonious framework of modeling bailouts in order to shed some light on the effectiveness of the recently proposed bailout policies for troubled European economies. In fact, during the recent European crisis, debt levels and sovereign spreads soared, which lead policymakers to implement bailout policies aimed at reducing sovereign spreads. However, as the link between sovereign spreads and private credit conditions during crisis times strengthens - dismal conditions on the sovereign markets immediately translate onto private credit markets - it should not be ignored when evaluating bailout policies. Intuitively, sovereign crises pose a great burden on private businesses because of the association between increased sovereign spreads and higher private borrowing costs. This burden is costly because it diverts economic resources that would otherwise be productive pursuits and channels them into the cause of serving higher private lending rates. This inefficiency, in turn, worsens the economic conditions of the country. Therefore, this paper evaluates recently proposed bailout policies that are aimed at reducing interest rates on sovereign bonds faced by troubled economies, taking into account possible effects on private credit.

We examine the impact of bailout policies in small open economies subject to financial frictions. In addition to standard choices of defaulting or repaying the debt, a government can choose to ask for a third-party bailout which comes at a cost of imposed borrowing limit. By modeling financial frictions and financial intermediaries, we are able to generate negative spillover effects from the sovereign credit markets onto the private credit markets, a characteristic that is present in the ongoing European sovereign debt crisis.

We find that the existence of a bailout option decreases default incentives and therefore reduces rates on sovereign bonds, as international investors understand that default incentives are now smaller. In addition, because of the generated link between private credit markets and sovereign credit markets, when sovereign rates decline, private rates decline as well. Importantly, these effects are present even when government does not choose a bailout option; the mere presence of a bailout option has a positive beneficial effect on sovereign spreads, as investors assign lower probability to the default scenario.

Ultimately, we are interested in computing the benefits brought by the implementation of stabilization policies. Our paper is a step in this direction. In particular, the implementation of our rescue program, which we model as a third-party bailout that comes at a cost of imposed borrowing limit, reduces output losses by 0.08 percent of GDP and increases welfare by 5 percent measured in consumption equivalent terms.
References


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A Appendix: Banks Problem

The problem of the bank is:

$$\max \pi^b_t = (1 + \tau^d_t)\kappa_t - \kappa_t + q_t d_t - d_t + (1 + r)b^r_t - b^r_t,$$

subject to:

$$b^r_t + \kappa_t = A(\lambda\kappa^c_t + (1 - \lambda)(qd_t)^c)^{1\over\xi} - \psi\kappa_t,$$

with $$b^r_t \geq 0.$$  

Optimality conditions, dropping the time subscript for convenience, imply:

\[ \frac{\partial L}{\partial \kappa} : \quad r^d + \Omega \left[ 1 + \psi - A(1 - \xi)\lambda (\lambda\kappa^c_t + (1 - \lambda)(qd_t)^c)^{1-\xi-\xi\over\xi} \kappa^{\xi-1} \right] = 0, \]

\[ \frac{\partial L}{\partial d} : \quad q - 1 - \Omega \left[ A(1 - \xi)(1 - \lambda) (\lambda\kappa^c_t + (1 - \lambda)(qd_t)^c)^{1-\xi-\xi\over\xi} q^c d^{\xi - 1} \right] = 0, \]

\[ \frac{\partial L}{\partial b^r} : \quad r + \Omega = 0, \]

where \( \Omega \) is the Lagrangian multiplier associated with the constrain (2).

Substituting, we obtain:

$$r^d = r(1 + \psi) - Ar\lambda \left[ \frac{\lambda(1 - \lambda)q^c_t}{\left( \frac{1-q}{Aq_t r(1-\lambda)} \right)^{1\over\xi} - (1 - \lambda)q^c_t} + \lambda \right]^{1-\xi\over\xi}. $$

Also, optimality conditions imply the following equilibrium quantity of \( d \) and \( b^r \):

$$d = \left[ \frac{\lambda\kappa^c_t}{\left( \frac{1-q}{Aq_t r(1-\lambda)} \right)^{1\over\xi} - (1 - \lambda)q^c_t} \right]^{1\over\xi}, $$

$$b^r = (h^b)^c A(\lambda\kappa^c_t + (1 - \lambda)(qd_t)^c)^{1-\xi\over\xi} - \kappa - \psi\kappa. $$