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Hedging against Risk in a Heterogeneous Leveraged Market

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Abstract

This paper focuses on the use of interest rates as a tool for hedging against the default risk of heterogeneous hedge funds (HFs) in a leveraged market. We assume that the banks study the HFs survival statistics in order to compute default risk and hence the correct interest rate. The emergent non-trivial (heavy-tailed) statistics observed on the aggregate level, prevents the accurate estimation of risk in a leveraged market with heterogeneous agents. Moreover, we show that heterogeneity leads to the clustering of default events and constitutes thus a source of systemic risk.

Keywords: survival statistics; interest rate; leverage; financial fragility.

JEL Classification Numbers: G23, G24, G32, G33.

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1 Introduction

The recent financial crisis has naturally drawn much attention to the functioning of financial markets both from a policy making and from an economic modeling perspective. This paper focuses on two of the potential sources of instability of the financial markets, namely heterogeneity of the economic agents and leverage, and thus aims to contribute to both of the relevant literatures.

The role of leverage has been studied in a number of papers which develop theoretical models to explain the mechanism through which leverage is affected by volatility and the emergence of a positive relationship between leverage and asset prices. An example is Geanakoplos (2001) who suggest that bad news raises tail volatility and decreases expectations and, as a result, the leverage level. In such an environment there are two effects that tend to decrease prices; lower expectations and lower leverage.[1] Leverage is high during normal and good times and low during bad times, such as during a crisis, which lends empirical support of these findings. This pro-cyclicality of leverage is documented by Adrian and Shin (2008), among others. Lastly, an empirical study by Molina (2005) tries to capture, not only the relationship between leverage and the probability of default, but also the feedback effect which arises between them. This is achieved by relating the default probability and the credit rating and examining the impact of firm's leverage on the credit rating. Here it was found that the positive effect of leverage on the probability of default is three times stronger than it is if the endogeneity of leverage is ignored. This finding highlights the importance of the feedback mechanism in a leveraged economy.

The present paper builds on two recent papers by Thurner, Farmer, and Geanakoplos (2012) and Poledna, Thurner, Farmer, and Geanakoplos (2014) which study the effects of leverage in an economy where the key actors are heterogeneous investors who are called hedge funds (HFs) and a bank. Thurner et al. (2012) show that leverage causes fat tails and clustered volatility. Under benign market conditions HFs become more leveraged as this is then more profitable. High levels of leverage are correlated with increased asset price fluctuations that become heavy tailed. The heavy tails are caused by the fact that when a HF reaches its maximum leverage limit then it has to repay part of its loan by selling some of its assets. Poledna et al. (2014) use a very similar framework to test three regulatory policies: (i) imposing limits on the maximum leverage, (ii) similar to the Basle II regulations, and (iii) a hypothetical perfect hedging scheme in which the banks hedge against the leverage-induced risk using options. They find that the effectiveness of the policies depends on the levels of leverage. They also show that the perfect hedging scheme reduces volatility in comparison to the Basle II scheme but that none of the schemes are able to make the system much safer on a systemic level. These two papers provide a basis for studying both the effects of leverage and heterogeneity on prices within the

[1]See also Brunnermeier and Pedersen (2007), Geanakoplos (2009), and Geanakoplos (1996).

same framework and in this way allow a test of the effectiveness of policies like the one proposed in Basel II. Our model extends this framework in two directions.

Firstly, it provides a microfounded explanation of the behaviour and the source of heterogeneity of the HFs. In both of these papers agents' heterogeneity has to do with their demand for the risky asset. More specifically the HFs are risk neutral and have different demand of the asset given the same information and the same wealth. The characteristic which makes them heterogeneous, is called "aggression" and aims to capture the different responses of the agents to a mispricing signal. Given that the agents are risk neutral it is impossible to provide a rigorous explanation for the difference in aggression. In this paper we establish a model where the agents are risk averse with Constant Relative Risk Aversion (CRRA) and their heterogeneity stems from the difference in the precision of the mispricing signals that they receive from the market.[2] Interestingly, we find that the better informed agents are not necessarily characterised by the lowest default rate.

Second, in our work, the bank does not charge a constant interest rate but rather uses the interest rate as a tool to hedge against the default risk. The rate it charges depends on a historically observed default rate. We show that the failure function (the distribution of waiting times to default) of the heterogeneous HFs is different when observed on the micro and the aggregate level. Within our model we find that the banks systematically underestimate risk. Moreover, we show that there is a heterogeneous effect of hedging on the market and present rigorous results for the connection between heterogeneity and clustering of defaults, constituting systemic risk.

New computational tools have allowed economists to expand the rational representative agent framework and to study how the interactions of heterogeneous agents can give rise to emergent properties of systems that are able to replicate the empirical trends seen in asset prices, asset returns and their distributions (Lux, 1995, 1998; Lux and Marchesi, 1999; Iori, 2002). In Levy (2008), spontaneous crashes are a natural property of a market with heterogeneous investors who are inclined to conform to their peers, under the condition that the strength of the conformity effects is large compared to the degree of heterogeneity of the investors. In other papers, such as Chiarella (1992) and Lux (1995), heterogeneity has to do with the different beliefs and trading rules of the agents (fundamentalists and chartists) which can result to asset price fluctuations and market instability. The seminal work by Brock and Hommes (1997) discusses "rational roots to market instability" in a framework where heterogeneous agents have adaptive behaviour that depends on their investment strategy. Brock, Hommes, and Wagener (2009) discuss the fact that hedging instruments can play a destabilising role in a financial market with agents with heterogeneous expectations. Hedging instruments are represented by Arrow

[2]Other sources of heterogeneity can include the investors' time horizon (LeBaron, 2002), boundedly rational price expectations (Brock and Hommes, 1998) or expectations about future states of the world (Sandroni, 2000)

securities which are added in an asset pricing model holding heterogeneous beliefs. The main finding of this work is that this addition may destabilise the price dynamics and increase volatility.

The remainder of the paper is organized as follows. In Section 2 we present the model. Section 3 presents and discusses the main results. In the last Section we summarize the main points of the paper and draw some conclusions.

2 Model

The model presented here is based on the recent models on leverage with heterogeneous agents proposed by Thurner et al. (2012) and Poledna et al. (2014). These models describe an economy with two assets, one risk-free (cash C) and one risky, three types of agents and a bank. The risky asset exists in a finite quantity equal to N and can be viewed as a commodity or an option. These two assets are held by a representative noise trader and n types of hedge funds (HFs). In addition, a representative fund investor (FI) holds risk-free asset and invests in the risky one indirectly through the HFs. Finally, there is also a bank which lends money to the HFs, using the risky asset owned by the HFs as collateral.

Our model extends the models of Thurner et al. (2012) and Poledna et al. (2014) in two directions. First, it provides a microfounded explanation of the behaviour and the source of heterogeneity of the HFs. In Thurner et al. (2012) and Poledna et al. (2014) the heterogeneity is a result of the HFs having different "aggression" parameters. Here, given the same wealth and mispricing signal, different HFs had a different demand of the risky asset although no explanation was given for the cause of this difference. In our model the HFs have CRRA utility functions and differ in the precision of the mispricing signal that they receive. In our work we assume that the banks rationally estimate the interest rate to charge on the loans that they extend. In Thurner et al. (2012) there was no interest charged while in Poledna et al. (2014) the banks instead hedge by using options without explicitly computing default risk. Our model instead allows the bank to use interest rates as a tool for hedging against the risk of default of the HFs.

The representative noise trader's demand for the risky asset in terms of the cash value d_t^{nt} of the asset at discrete time t depends on a chartist (destabilising) component, a fundamentalist (stabilising) component and a noise component. In this way the price of the asset weakly mean reverts around the fundamental value V which depends on economic fundamentals. More specifically d_t^{nt} is given by

$$\log d_t^{nt} = \rho \log d_{t-1}^{nt} + \sigma^{nt} \chi_t + (1 - \rho) \log(VN), \tag{1}$$

where $\rho \in (0, 1)$ is the relative weight between the fundamentalist and chartist components and $E[\log p_t] = \log V$, where p_t is the price of the risky asset at t. [3] As it is obvious from the above the closer ρ is to 0 the more stable the price behaviour will be. Here σ^{nt} is the variance of the price variations for the noise traders with χ_t is a Gaussian random variable with mean 0 and variance 1.

The representative FI is assumed to be a boundedly rational investor who seeks to invest in the risky asset but does not have any information about the fundamental value of the asset and invests through the HFs. The FI has a chartist behaviour and invests or withdraws money from the HFs based on the latter's historical performance, a benchmark return r_b and a parameter b which controls the amount withdrawn or invested. In this way the investment inflow or outflow at time t, from the FI to HF j is given by the following rule:

$$F_t^j = \max\left[-1, b^j(r_t^{p,j} - r_b)\right] \times \max\left[0, D_{t-1}^j p_t + C_{t-1}^j - L_{t-1}^j i_t\right] \tag{2}$$

where L_{t-1} is the total loan amount at $t-1$, i_t the interest rate charged at t, C_{t-1}^j is the amount of the risk-free asset held by the HF at $t-1$ and $r_t^{p,j}$ is a measure of the HF performance. Eq (2) ensures that FI withdrawals do not exceed the maximum wealth of the HF and that investments are proportional to the fund performance relative to an (industry) benchmark. Here $r_t^{p,j}$ is an exponential moving average of the HF's instantaneous rate of return r_t^j,

$$r_t^{p,j} = (1-\gamma) r_{t-1}^{p,j} + \gamma r_t^j \tag{3}$$

where $\gamma \in (0,1)$ and r_t^j is the rate of return of the HF j, i.e.

$$r_t^j = \frac{D_{t-1}^j (p_t - p_{t-1})}{W_{t-1}^j} \tag{4}$$

The HFs' performance depends on their rate of return over time, i.e. their annual percentage return, a standard measure of fund performance. HFs are represented by risk averse agents whose utility depends on their rate of return r_t^j and their utility is given by:

$$U = 1 - e^{-\alpha r_t^j} \equiv 1 - e^{-\alpha (W_t^j - W_{t-1}^j)/W_{t-1}^j} \tag{5}$$

where $\alpha > 0$, is the Arrow-Pratt-De Finetti measure or relative risk aversion. In this way, the HFs have constant relative risk aversion (CRRA) and their absolute risk aversion is decreasing when wealth increases (DARA). This captures the fact that more wealthy HFs

[3] Similar types of demand have been discussed extensively in relation to exchange rate expectations in a series of papers by of Frankel and Froot (for example see (Frankel and Froot, 1987)); and in relation to Stock Markets in Brock and Hommes (1998) and Boswijk, Hommes, and Manzan (2007). Also see Hommes (2006) for a detailed review.

find it the more difficult to increase their rate of profit and, in order to achieve this, turn to more risky behaviour.

The HFs are spawned with the same endowment W_0 and have information about the fundamental value of the asset, taking advantage of the misprising caused by the noise trader's behaviour. Each HF receives a private noisy signal $\tilde{V} = V + \epsilon_j$ about the fundamental value of the risky asset, where the noise term ϵ is assumed to follow a normal distribution with mean 0 and variance σ_j^2. In this way, the maximization with respect to the expected rate of return yields that the demand for the risky asset of the jth HF with signal precision measured by the variance σ_j^2 is given by

$$D_t^j = \frac{m}{\alpha \sigma_j^2} W_t, \tag{6}$$

where m is the mispricing signal $V - p_t$, given the fundamental value V. Here the source of heterogeneity has to do with the differences in the precision of the signal for the (same) fundamental value. Otherwise the HFs are identical. Without loss of generality in the following we set $\alpha = 1$.

When the demand for the risky asset cannot be met with the cash held by a HF at a given time step, the HF requests a loan from the bank. The bank extends the loan to the HF provided that the HF does not become more leveraged than a limit λ_{max} set externally. Here leverage is defined as the ratio of assets held to the HF's net wealth. In this way a constraint is imposed on the demand of the HF for the risky asset, given its wealth,

$$\frac{D_t p_t}{W_t} \leq \lambda_{max} \tag{7}$$

Consequently, the maximum demand for the risky asset is $D_t = \lambda_{max} W_t / p_t$.

In order to calculate the appropriate interest rate we assume that the banks use a historical measure of the time between defaults τ of a HF, specifically $E[\tau; \lambda^\star]$, conditional that the leverage of the defaulting HF just before default was $\lambda^\star$. Using this table the banks can "look up" an instantaneous probability of default, or hazard rate, for a HF with leverage $\lambda(t)$ as $h = 1/E[\tau; \lambda(t)]$. If the HFs were identical this procedure for computing h would be exact if the bank only received information on the leverage of the HF and not, for example, on the time that it has been trading. However, if the HF failure function is exponential this remains exact in the case of identical HFs, irrespective of whether or not the bank has information on how long the HF has been active. In this fashion, the banks construct a look-up table for $N_\lambda = (\lambda_{max} - 1)/\delta\lambda$ different values of the leverage of borrowers, where $\delta\lambda$ is an arbitrary bin width chosen by the banks.

In order to estimate the the mean time between defaults for an arbitrary value of leverage banks perform linear-log interpolation on their data for $E[\tau; \lambda(t)]$ to obtain a continuous function $\langle\tau(\lambda)\rangle$. [4]

Within this scheme the banks infer that an overnight interest rate that would exactly compensate for default risk would be given by

$$i_t = 1/\langle\tau(\lambda)\rangle \tag{8}$$

The heterogeneity in the HFs, which is unknown to the bank, renders this only an approximation to the necessary rate to hedge default risk for any given HF with a particular signal precision σ_j^2. Nonetheless, it is probably a more precise risk assessment procedure than is feasible for banks, in practice, given the extremely long historical datasets than can be simulated in our computational model to be used by the bank to construct their look-up table for default risk.

The wealth of a HF evolves according to

$$W_t^j = W_{t-1}^j + (p_t - p_{t-1})D_{t-1}^j + F_t^j. \tag{9}$$

When the wealth of a HF falls below $W_{\text{cr}} \ll W_0$, the HF is assumed to be liquidized and goes out of business. After T_r time-steps the bankrupt HF is replaced by an identical one. As a final note, the price of the risky asset is determined by the market clearance condition

$$d_t^{\text{nt}}(p_t) + \sum_{j=1}^{n} D_t^j(p_t) = N. \tag{10}$$

2.1 Choice of Parameters

In all simulations we consider a market with 10 HFs, each one receiving a signal with precision $\beta^j = 1/(\sigma^j)^2$ uniformly distributed in the interval $[5, 50]$. The remaining parameters are chosen as follows: $\rho = 0.99$, $\sigma^{nt} = 0.035$, $V = 1$, $N = 10^3$, $r_b = 5 \times 10^{-3}$, $\gamma = 0.1$, $W_0 = 2$ and $W_{\text{cr}} = 0.2$. Bankrupt HFs are reintroduced after T_r time-steps, randomly chosen according to a uniform distribution in $[10, 200]$.

3 Results and Discussion

Herein, we focus on the survival statistics of the hedge-funds to assess the effects of hedging in a heterogeneous leveraged market, as well as, the ability of the banks to correctly put a price on the default risk. As already mentioned, from the Bank's perspective, HFs are indistinguishable since the heterogeneity is related to the private signals received by the HFs. Consequently, banks, at best, can differentiate between

[4]Banks interpolate using linear-log interpolation because they anticipate that the mean time between defaults for different levels of leverage can vary across many orders of magnitude.

HFs having a different level of leverage. Here we assume that information on collateral encumbrance (leverage) would have to be disclosed to the bank. However, information on the leverage cannot, by itself, be used to distinguish between the heterogeneous HFs since any allowed leverage value can (and, ultimately, will) be accessed by each HF. Nonetheless different HFs will contribute differently to the aggregate default statistics for any leverage value. These are recorded by the bank for the purposes of computing their look-up table for default risk.

To simplify our analysis we study the survival statistics independent of the leverage level of HFs. However, at least qualitatively, our findings hold even when performing independent statistics for different values of leverage as the Bank is assumed to be doing in the model. This simplification, allows us to obtain much finer statistics for each of the HFs, and more importantly, treat the problem analytically.

In Fig. 1 we show the failure density function (FDF) $P(\tau)$, that is the distribution of waiting times τ between defaults, for each of the hedge funds with $\beta = 1/(\sigma)^2 = \{5, 10, 15, \ldots, 50\}$ and $\lambda_{max} = 5$ on a log-linear scale. We observe that the decay of the FDFs can be well described by exponentials, as indicated by the fits shown with black solid lines for all values of β. Therefore, microscopically

$$P(\tau) \approx \mu(\beta) \exp\left[-\mu(\beta)\tau\right]. \tag{11}$$

The exponentially distributed waiting times between defaults indicate that HFs default at approximately a steady rate μ, which depends on β.

However, as mentioned above, statistical analysis on an individual level is not possible by the bank. Consequently, even a bank that is perfectly informed concerning the history of defaults can only infer the distribution of waiting times on the aggregate level. This aggregate distribution will be the weighted sum of the individual exponentials corresponding to HFs with a different β. Hence, the aggregate distribution will be

$$\tilde{P}(\tau) = \sum_{j=1}^{n} w(\beta^j)\mu(\beta^j) \exp\left[-\mu(\beta^j)\tau\right], \tag{12}$$

where $w(\beta^j)$ is the statistical weight corresponding to the j^{th} fund with signal precision β^j.

In Fig. 2 we present the numerically obtained aggregate distribution using logarithmic scale on both axes. The numerical results were obtained by averaging over 2×10^2 simulations, each with a different realization of β^j values, sampling from a uniform distribution in $[5, 50]$. We observe that for sufficiently large waiting times τ the distribution decays according to a power-law (black solid line), $\tau^{-\nu}$ with $\nu \approxeq 3$. The non-exponential (algebraic) asymptotic decay of the aggregate distribution is a result of the mixture of a large number of exponential decays with different default rates μ corresponding to the

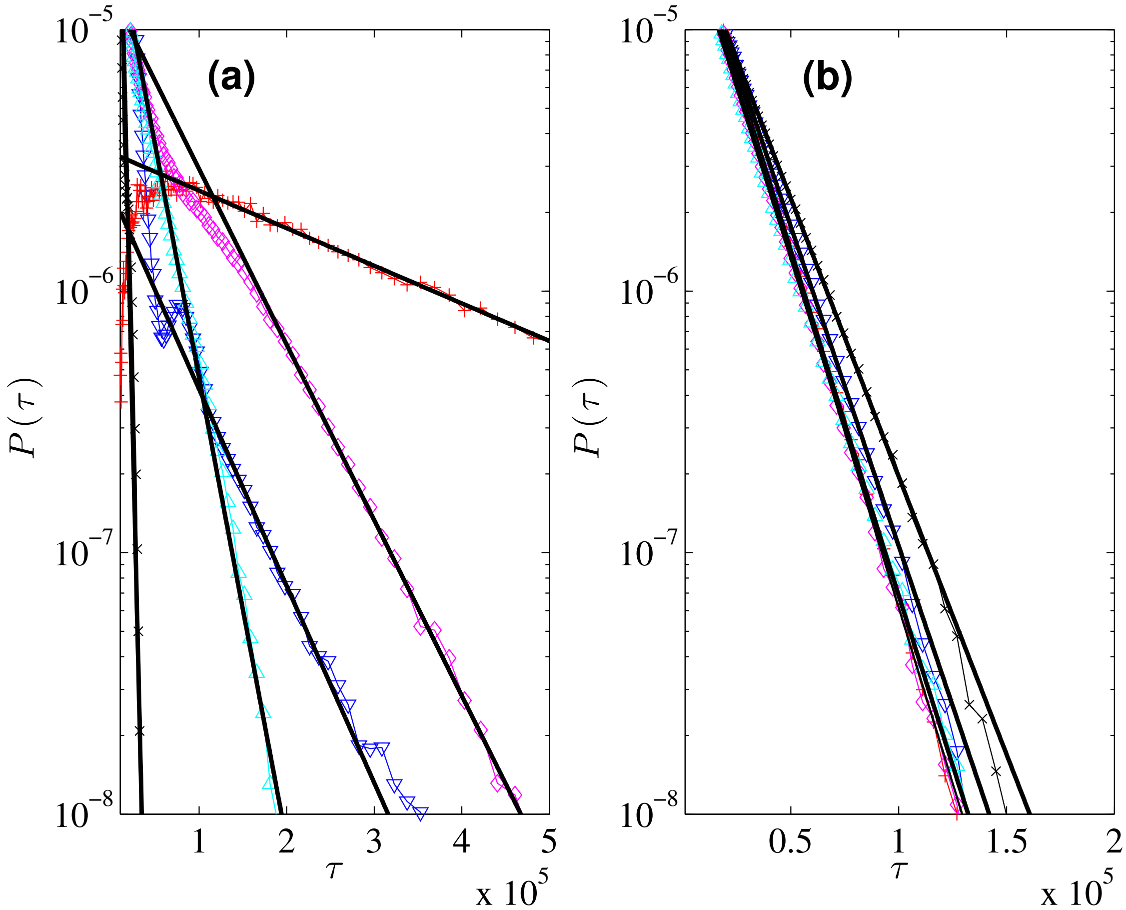

Figure 1: The distribution of waiting times between defaults τ for each HF, having different precision σ: (a) $\beta \equiv 1/\sigma^2 = \{5, 10, 15, 20, 25\}$ (black diagonal crosses, blue downright triangles, red upright crosses, magenta diamonds and cyan upright triangles, respectively) and (b) $\beta = \{30, 35, 40, 45, 50\}$ (with the same color notation). The results were obtained simulating the model for up to 10^8 time-steps and averaging over 2×10^2 different initial conditions, with the maximum allowed leverage set to $\lambda_{max} = 5$. In each case, we perform an exponential fit (black solid lines). Note the log-linear scale.

continuously distributed β. The emergence of the power-law tail on the aggregate level has profound implications on the ability of the bank to correctly assess the default risk, as the slow decay of the FDF shifts the observed mean time between defaults to greater values. In other words, the aggregation performed of the heterogeneous agents results into an underestimation of default rates, as the riskier HFs are "shadowed" by the more stable ones.

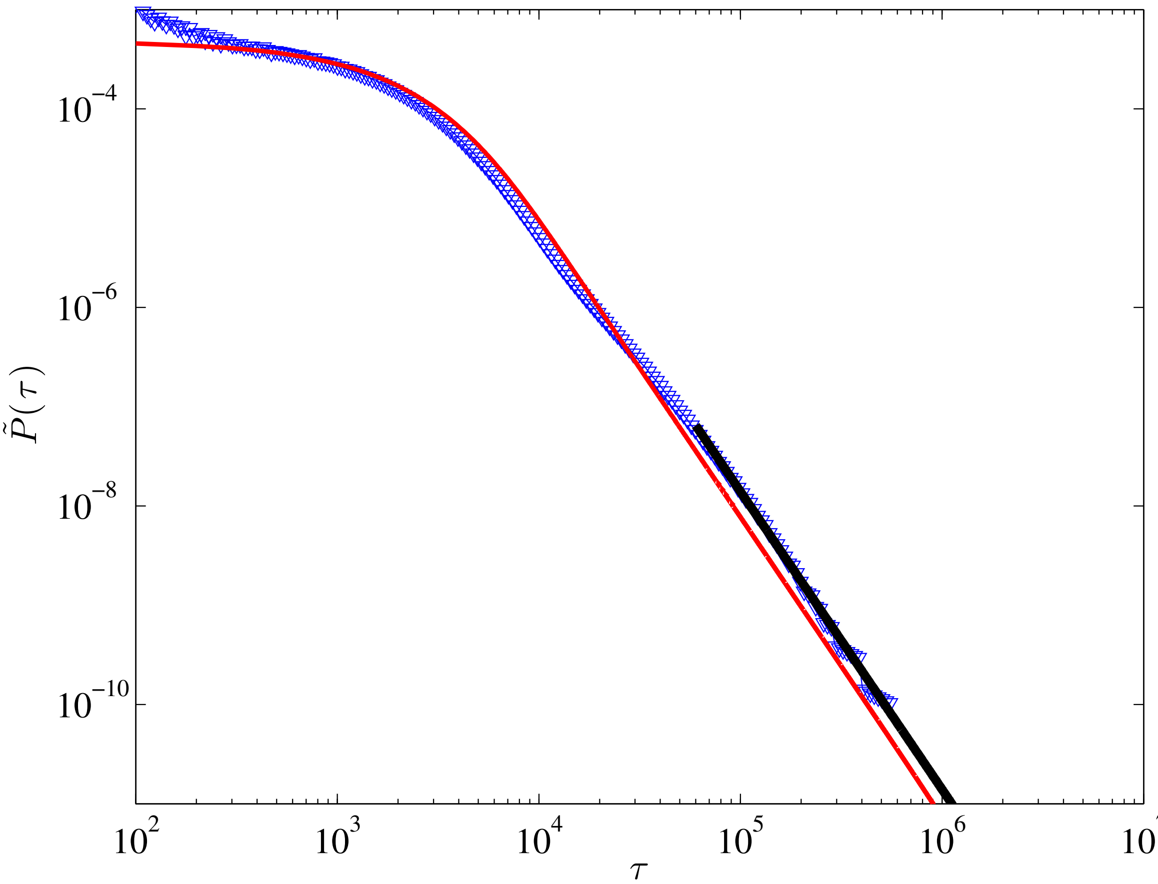

Figure 2: The aggregate distribution of waiting times between defaults (blue downright triangles) obtained on the basis of 2×10^2 simulations of the model with a signal precision β received by each HF randomly chosen according to a uniform distribution in $[5, 50]$, for up to 10^8 time-steps each. For $\tau \gg 1$, the aggregate distribution follows a power-law $\tilde{P}(\tau) \propto \tau^{-\nu}$, with $\nu = -3$ (black thick line). For comparison, the analytical approximation given by Eq. (14) is also shown (red line). The parameters and ϵ were determined numerically $\mu_{\max} = 7.2 \times 10^{-4}$, $\epsilon = 1.2 \times 10^{-3}$.

To be able to theoretically quantify the error (underestimation) of the estimated risk caused by the aggregation of the heterogeneous agents, in the following we derive an analytical model for the numerically obtained distribution $\tilde{P}(\tau)$ presented in Fig. 2. Given that the signal precision is sampled from a continuous distribution, the individual default rates vary also continuously from a minimum value $\mu_{\min}$ to a maximum value $\mu_{\max}$, corresponding to the most stable and unstable HF respectively. Therefore, the sum in Eq. (12) becomes an integral over μ

$$\tilde{P}(\tau) = \int_{\mu_{\min}}^{\mu_{\max}} f(\mu)\mu e^{-\mu\tau} d\mu, \tag{13}$$

where $f(\mu)$ is the distribution of μ given the uniform distribution of β. We can infer the distribution $f(\mu)$ observing that if we allow $\mu_{\min} \to 0$ and $\mu_{\max} \to \infty$, then Eq. (13) coincides with the Laplace transform of $\mu f(\mu)$. Moreover, given that asymptotically $\tilde{P}(\tau) \propto 1/\tau^3$, it follows that $f(\mu)\mu = \mathcal{L}^{-1}\left[\tau^{-3}\right] \propto \mu^2$, where $\mathcal{L}^{-1}$ denotes the inverse Laplace transform. Hence, $f(\mu) \propto \mu$. Introducing further the dimensionless parameter $\epsilon = \mu_{\min}/\mu_{\max}$ in Eq. (13) we find

$$\begin{aligned}\tilde{P}(\tau) =& \frac{2e^{-\tau(\mu_{\max}+\epsilon\mu_{\max})}}{\tau^3\left(\mu_{\max}^2 - \epsilon^2\mu_{\max}^2\right)} \\ &\times \left\{e^{\tau\mu_{\max}}\left[\tau\epsilon\mu_{\max}\left(\tau\epsilon\mu_{\max}+2\right)+2\right]\right. \\ &\left. - \left[\tau\mu_{\max}\left(\tau\mu_{\max}+2\right)+2\right]e^{\tau\epsilon\mu_{\max}}\right\}.\end{aligned} \tag{14}$$

In the limit of an infinitely heterogeneous market, with $\mu_{\max} \gg \mu_{\min}$, ($\epsilon \ll 1$) we can expand Eq. (14) to leading order in ϵ to obtain,

$$\tilde{P}(\tau) = \frac{4}{\tau^3\mu_{\max}^2} - \left[\frac{4e^{-\tau\mu_{\max}}}{\tau^3\mu_{\max}^2} + \frac{4e^{-\tau\mu_{\max}}}{\tau^2\mu_{\max}} + \frac{2e^{-\tau\mu_{\max}}}{\tau}\right] + \mathcal{O}(\epsilon^2). \tag{15}$$

Hence, for $\tau \gg 1$, the slowest decaying term in Eq. (15) dominates and the distribution will converge to

$$\tilde{P}(\tau) \propto \tau^{-3}, \tag{16}$$

as expected. The analytical approximation given by Eq. (14) is presented in Fig. 2 with $\epsilon \equiv \mu_{\min}/\mu_{\max} = 1.2 \times 10^{-3}$ and $\mu_{\max} = 7.2 \times 10^{-4}$ determined numerically (red solid line). As illustrated in Fig. 2, the analytical approximation is in reasonable agreement with the exact (numerical) FDF.

From Eq. (14) we deduce that the interest rate charged by the bank on the basis of the aggregated statistics is approximately

$$i = \frac{1}{\langle\tau\rangle} = \frac{(1+\epsilon)\mu_{\max}}{2} \overset{\epsilon\ll 1}{\approx} \frac{\mu_{\max}}{2}. \tag{17}$$

Therefore, the default risk of the more stable funds will as a result be overestimated, whereas the default risk of the more unstable ones will be underestimated. Consequently, banks will always be overcharging the less likely to default HFs and undercharging the riskier ones. The ϵ dependence of the interest rate reflects the dependence of the magnitude of the error on the extent of the market heterogeneity. In the limit of infinite heterogeneity $\epsilon \ll 1$, the default risk of the most unstable HF in the market will be un-

$\lambda_{\max}$	Bank losses	Interest
5	$(1 \pm 2) \times 10^{-3}$	$(1.2 \pm 0.8) \times 10^{-4}$
10	$(9 \pm 3) \times 10^{-3}$	$(2.0 \pm 1.3) \times 10^{-3}$
15	$(7 \pm 4) \times 10^{-3}$	$(3.2 \pm 0.4) \times 10^{-3}$

Table 1: The mean amount of bank losses and interest payments per time-step. Evindetely, the interest rate charged to the HFs does not cover for the losses due to bankrupties.

derestimated 50%, as suggested by Eq. (17). Another source of error for the estimation of the default risk is the slow convergence of the calculated sample mean of the waiting times to default, used by the bank as a proxy for the default rate, to the true mean, due to the presence of the heavy tail. It is worth mentioning that the overcharging of the more stable HFs does not cover for the loss caused by undercharging the riskier ones as observed in Table 1, where the bank losses and interest payments per time-step are presented.

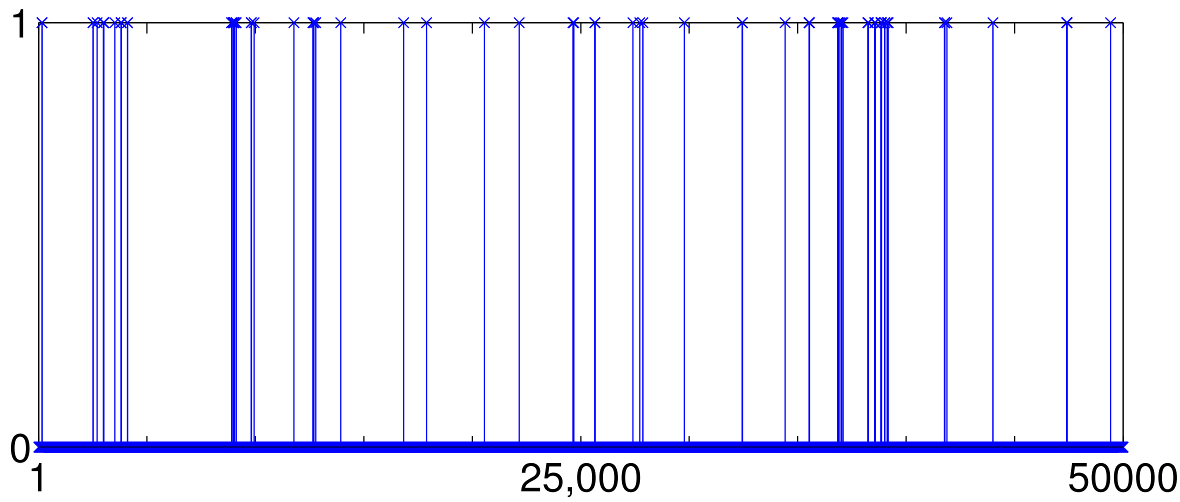

Figure 3: A binary (coarse-grained) representation of the wealth time series S_t as observed on the aggregate level. The binary sequence is contructed by mapping the active phase of each HF to 0 ($W_t > W_{\mathrm{cr}}$) and the default events to 1 ($W_t < W_{\mathrm{cr}}$). We observe the clustering of the default events.

Consequently, due to the presence of heterogeneity in the market and despite the fact that the assumption made by the banks is valid on the micro-level (constant rate of default), the emergent non-trivial survival statistics on the aggregate level hinders hedging against default risk.

Another important effect of the emergent heavy-tail statistics stemming from the heterogeneity of the market, is the absence of a characteristic time-scale for the occurrence of defaults (scale-free asymptotic behaviour). Thus, despite the fact that on the microscopic level there exists a well defined characteristic time-scale for each HF μ, on the aggregate level this quality is lost due to the mixing. The ramifications of the absence of a characteristic time-scale to the clustering of defaults and consequently to the systemic risk run deeply.

To gain insight and facilitate analytical treatment, we employ a coarse graining of the wealth time-series, mapping the phases of activity ($W_t > W_{\rm cr}$) to 0 and bankruptcy events ($W_t \leq W_{\rm cr}$) to 1. In Fig. 3(a) we present a segment of the binary sequence S_t generated by the dynamics after coarse-graining. We observe indeed that the blue vertical lines corresponding to defaults form clusters.

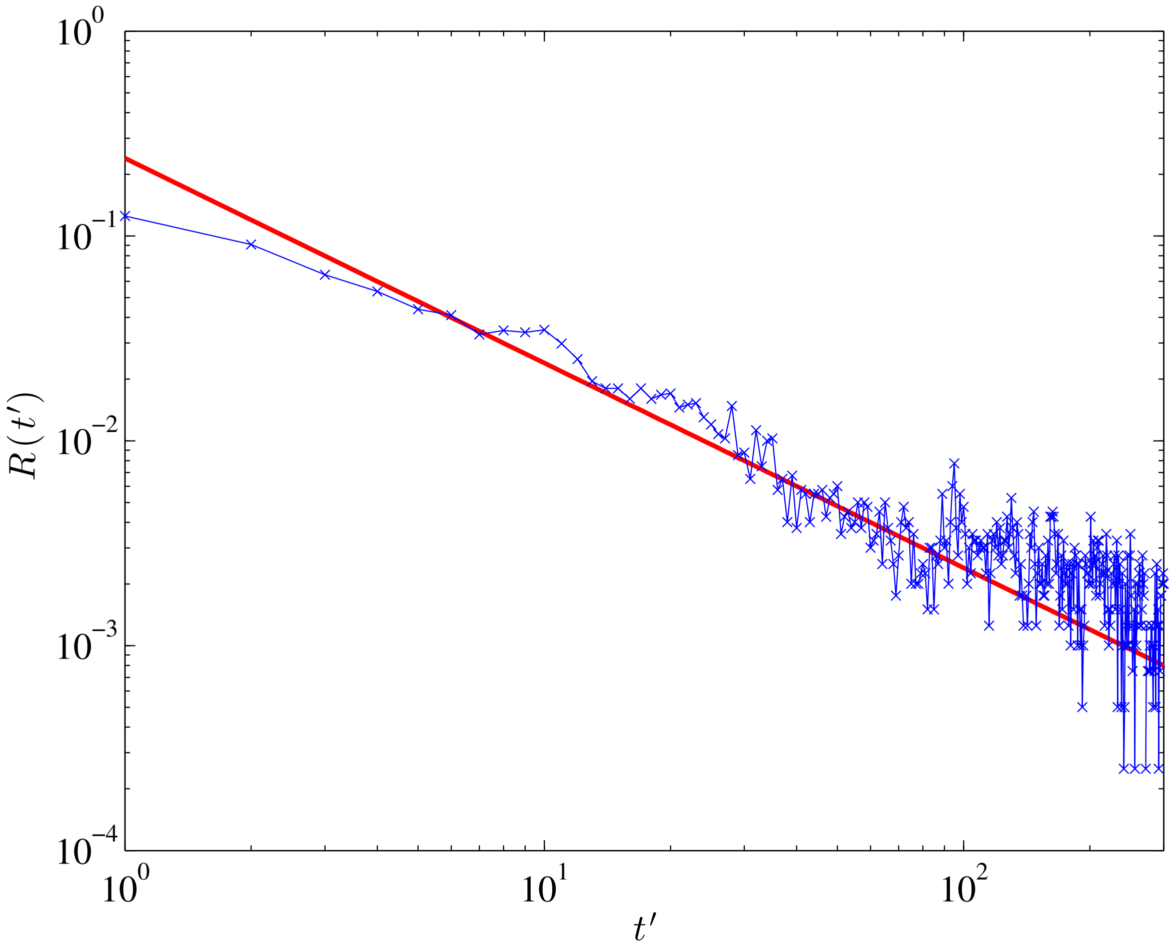

Figure 4: The autocorrelation function of the binary representation of default events $S(t)$) (blue diagonal crosses). The analytical prediction given by Eq. (23) is also shown (red solid line).

The quantification of clustering can be achieved by the autocorrelation function $R(t')$, with t' being the time lag. Following Schuster and Just (2006), the autocorrelation of S_t can be expressed as

$$R(t') = \sum_{\tau=0}^{t'} R(t'-\tau)\tilde{P}(\tau) + \delta_{m,0}, \tag{18}$$

where $\delta_{m,0}$ is the Kronecker delta. We define $\tilde{P}(0) = 0$ and $C(0) = 1$. Since we are interested in the long time limit of the autocorrelation function we can pass on to the continuous time and solve Eq. (18) by Laplace transformation $\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\exp(-st)dt$, utilizing also the convolution theorem. Taking these steps we obtain,

$$R(s) = \frac{1}{1 - \tilde{P}(s)}. \tag{19}$$

The correlation function in the Fourier space $\mathcal{F}\{R(t')$ then is,

$$\mathcal{F}\{R(t')\} = \frac{1}{2}|R(s \to 2\pi i f) - R(s \to -2\pi i f)| \tag{20}$$

Assuming that $\tilde{P}(\tau) \propto \tau^{-3}$ and substituting in Eq. (19) and Eq. (20) we obtain.

$$\mathcal{F}\{R(t')\} = |\log(f)|. \tag{21}$$

Finally, inverting the Fourier transform we obtain the autocorrelation as a function of the time lag t'.

$$R(t') \propto \frac{4\mathrm{Si}(t) - \pi}{t'}, \tag{22}$$

where $\mathrm{Si}(t')$ denotes the sine integral. For $t' \gg 1$ Eq. (22) decays as

$$R(t') \propto \frac{1}{t'}. \tag{23}$$

In Fig. 4 we show the numerically determined autocorrelation function. For comparison, the analytical result given by Eq. (23) is also plotted (red line). We observe that the asymptotic theoretical prediction is in reasonably good agreement with the numerical results.

Finally, to assess the effect of interest on the default risk, we calculate the ratio of default rates of each HF in the presence of interest rate over the corresponding with the interest rate fixed at 0. The results are presented in Fig 5. As can be seen, the introduction of interest rate for high maximum allowed leverage values enhances the probability of default for HFs with signal precision above a critical value $\beta_{\mathrm{cr}} \approx 25$ and, conversely, renders HFs with lower signal precision less likely to default. Thus, the impact of interest in a heterogeneous market is itself heterogeneous.

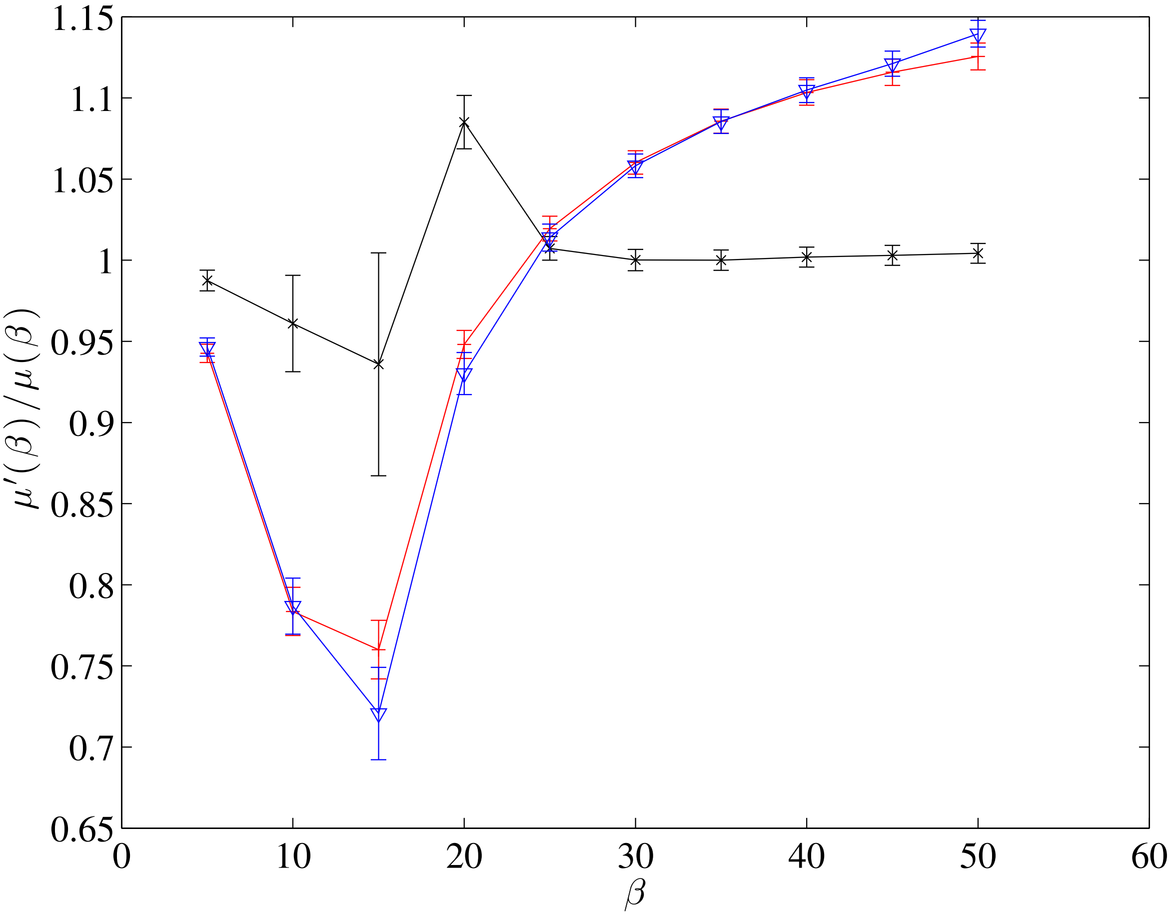

Figure 5: The ratio of the default rate with interest rate being charged μ' over the corresponding one with the interest rate fixed at zero μ, as a function of the precision signal $\beta = 1/\sigma^2$ for three different maximum allowed leverage $\lambda_{\max} = \{5, 10, 15\}$ values (black, red and blue lines, respectively. As observed the presence of interest rate has a heteregoneous effect on the individual default risk, which also depends on $\lambda_{\max}$.

4 Conclusions

We analyse the use of interest rates as a tool of hedging the default risk of heterogeneous hedge funds. We assume that the heterogeneity of the agents stems from the HFs' different quality of the mispricing signals they receive. We study the survival statistics of the HFs, which might be used by a bank in order to charge an interest on the loans that it extends to the HFs. We show that the failure function of the HFs is qualitatively different when observed on the micro and the aggregate level. Specifically, the failure function of all HFs decays exponentially on the micro-level, indicating a constant default rate. On the contrary, the aggregated distribution of waiting times between defaults, in the limit of an infinite level of heterogeneity in the market, tends to a power-law, with an exponent such that the variance becomes infinite (heavy-tail). As a result, we show that the default risk associated with the more unstable HFs will always be underestimated and the converse is true for the more stable ones. Furthermore, we show that there is a heterogeneous effect of hedging on the market, which is enhanced with increasing values of the maximum allowed leverage. Namely, the HFs which are less likely to default become even more creditworthy when an interest rate is charged, while the default rate of the less stable HFs increases when loans are costly. We also show that the scale-free property of the emergent statistics on the aggregate level is intimately connected with the clustering of defaults and, consequently, systemic risk.

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