Warwick Economics Research Paper Series

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Nikos Kokonas and Herakles Polemarchakis

This paper also appears as CRETA Discussion Paper No: 12
It has been published in The Journal of Mathematical Economics, Volume 67 December 2016 pages 120-124)

December, 2015
Series Number: 1102
ISSN 2059-4283 (online)
ISSN 0083-7350 (print)

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Nikos Kokonas [2]   Herakles Polemarchakis [3]

December 22, 2015

[1] We thank Yannis Vailakis for helpful comments.

[2] Department of Economics, University of Bath;
n.kokonas@bath.ac.uk

[3] Department of Economics, University of Warwick;
h.polemarchakis@warwick.ac.uk

Abstract

A reduction in the output of productive assets (trees) in some states of the world can expand the span of payoffs of assets; and, improved risk sharing may compensate for the loss of output and support a Pareto superior allocation. Surprisingly, if short sales of assets are not allowed, improved risk sharing that results from the destruction of output does not suffice to induce a Pareto superior allocation.

Keywords: short sales, destruction, welfare

JEL classification: D52, D61

1 Introduction

In an economy with uncertainty and limited markets for the reallocation of risks, the destruction of output may be Pareto improving. Typically, competitive allocations with an incomplete asset market are not Pareto optimal. A reduction in the output of productive assets (trees) in some states of the world can alter, in particular, expand the span of payoffs of assets; and, improved risk sharing may compensate for the loss of output and support a Pareto superior allocation.

We show here that, surprisingly, if short sales of assets are not allowed, improved risk sharing that results from the destruction of output does not suffice to induce a Pareto superior allocation.

In a renown contribution, Aumann and Peleg (1974) pointed out, through an elementary example, that, in a first-best environment, an individual may benefit from the destruction of some of his endowment: the change in the terms of trade in response to the reduction in resources may be in his favour, and the benefit may compensate for the loss of revenue [1]. Pareto optimality of the initial equilibrium implies that the individual benefits at the expense of others [2]. Our argument, here, is that under uncertainty, an incomplete asset market and a ban on short sales, Pareto improvement is still not possible; an individual may benefit from the destruction of output in some states, but only at the expense of others, even though a Pareto improvement would be possible if assets could be traded with no restrictions.

First, we give an example to show that the destruction of output can indeed be Pareto improving when short-sales are allowed. Subsequently, we show, in a general, non-parametric context, that destruction is never Pareto improving under no short sales. Lastly, we argue that, still, without short sales, an individual may benefit from the destruction of some of the output of assets he holds, even though this reduction in resources is not Pareto-improving.

[1] Effectively, the agent behaves strategically, and, as a monopolist would do, he may restrict supply to benefit from an increase in price.

[2] The argument is not unrelated to the transfer paradox in Leontief (1936) and the long and contentious literature that followed. Donsimoni and Polemarchakis (1994) generalised the result by showing that the distribution of welfare gains and losses resulting from the transfer of resources across individuals is unrestricted; the same holds for the destruction of resources.

2 Pareto improving destruction

Dates are 0 and 1, and two equiprobable states of the world, $s = a, b$, realize at 1; one perishable commodity, c, is exchanged and consumed at every date-even; two individuals, $i = 1, 2$, have identical utility functions

$$U^i = \ln(c_0^i) + \beta \mathrm{E} \ln(c_s^i);$$

and two assets, $j = z, y$, (trees) with identical risk-free dividends, $d_s = d > 0$, each in unit net supply are held each by a different individual. Endowments in commodities are

$$(e_0^1, e_a^1, e_b^1) = (d, 1, 0), \quad (e_0^2, e_a^2, e_b^2) = (d, 0, 1);$$

that the endowment at $t = 0$ coincides with the future output of the tree, d, is only for computational convenience

Since the assets have identical dividend patterns, no risk sharing is possible; the economy is symmetric in individuals and goods, and, at equilibrium, there is no trade: the equilibrium is autarkic, and the allocation coincides with the allocation of endowments augmented with dividends:

$$(c_0^1, c_a^1, c_b^1) = (d, 1 + d, d), \quad (c_0^2, c_a^2, c_b^2) = (d, d, 1 + d).$$

If alternatively, $\epsilon > 0$ units of the output of tree z are destroyed at state b, the optimisation problems, now different across individuals, are

$$\begin{array}{llcll}
\max & \log(c_0^1) + \beta E_{1/2} \log(c_s^1) & & \max & \log(c_0^2) + \beta E_{1/2} \log(c_s^2) \\
\\
\text{s.t} & c_0^1 + q^z z^1 + q^y y^1 = d + q^y & \text{and} & \text{s.t} & c_0^2 + q^z z^2 + q^y y^2 = d + q^z \\
& c_a^1 = 1 + dz^1 + dy^1 & & & c_a^2 = dz^2 + dy^2 \\
& c_b^1 = (d - \epsilon) z^1 + dy^1, & & & c_b^2 = 1 + (d - \epsilon) z^2 + dy^2,
\end{array}$$

with prices of assets q^j and the commodity as numéraire.

Since the asset market is complete, the economy reduces to an economy with trades in contingent commodities,

$$\begin{array}{llcll}
\max & \log(c_0^1) + \beta E_{1/2} \log(c_s^1) & & \max & \log(c_0^2) + \beta E_{1/2} \log(c_s^2) \\
& & \text{and} & & \\
\text{s.t.} & c_0^1 + \sum_s p_s c_s^1 = d + \sum_s p_s e_s^1, & & \text{s.t.} & c_0^2 + \sum_s p_s c_s^2 = d + \sum_s p_s e_s^2,
\end{array}$$

with the commodity at date 0 as numéraire and p_s contingent commodity prices, and endowments

$$(e_0^1, e_a^1, e_b^1) = (d, 1 + d, d), \quad (e_0^2, e_a^2, e_b^2) = (d, d, 1 + d - \epsilon).$$

Demands are

$$c_0^i = \frac{1}{1+\beta}\left(d + \sum_s p_s e_s^i\right), \qquad c_s^i = \frac{1}{2}\frac{\beta}{p_s(1+\beta)}\left(d + \sum_s p_s e_s^i\right),$$

and equilibrium prices are

$$p_a^* = \frac{\beta d}{1+2d}, \quad p_b^* = \frac{\beta d}{1+2d-\epsilon}.$$

Asset prices

$$q^y = p_a^* d + p_b^* d, \quad q^z = p_a^* d + p_b^*(d-\epsilon)$$

and demands

$$z^1 = \frac{c_a^1(p_a^*,p_b^*) - c_b^1(p_a^*,p_b^*) - 1}{\epsilon}, \quad y^1 = \frac{c_b^1(p_a^*,p_b^*) - (d-\epsilon)\frac{c_a^1(p_a^*,p_b^*) - c_b^1(p_a^*,p_b^*) - 1}{\epsilon}}{d},$$

$$z^2 = 1 - z^1, \qquad y^2 = 1 - y^1$$

implement the allocation as a dynamic of equilibrium.

Consider parameters

$$\beta \in (0,1], \quad d \in [0.01, 1], \quad \epsilon \in (0, 0.01);$$

then, $z^1 < 0$, and $y^1 > 1$. It is not difficult to verify that the equilibrium allocation after destruction induces a strict Pareto improvement over the initial equilibrium allocation.

Remark 1. Here, it is the competitive allocation, following the destruction of output, that implements a Pareto improvement; the point of the example is to show that this is possible, even if not it is not necessarily the case. If the competitive allocation is not Pareto optimal, which is the case generically, Pareto superior allocations with destruction of output always exist.

3 Short sales

Dates are 0 and 1, and two states of the world, $s = a, b$, realize in period 1; one perishable commodity, c, is exchanged and consumed at every date-event; a finite number of individuals, i, have utility functions

$$U^i = u^i(c_0^i) + \beta^i E u^i(c_s^i), \; \beta^i > 0,$$

with cardinal index that is strictly concave and satisfies standard regularity and boundary conditions, and endowments in commodities

$$(e^i_0, e^i_a, e^i_b) \gg 0.$$

In addition, assets, $j = y, z$, in unit supply, yield identical and state-independent payoffs in period 1, that, with no loss of generality, we normalise to 1; endowments in assets are

$$1 \geq (\bar{y}^i, \bar{z}^i) \geq 0.$$

The decision problem of an individual is

$$\begin{array}{ll} \max_{c,y,z} & U^i = u^i(c^i_0) + \beta^i E u^i(c^i_s) \\ & \\ \text{s.t.} & c^i_0 + q^y y^i + q^z z^i = e^i_0 + q^y \bar{y}^i + q^z \bar{z}^i \\ & c^i_s = z^i + y^i + e^i_s \\ & z^i, y^i \geq 0; \end{array}$$

the last constraint bans short sales.

Consider the following experiment. Compute an initial equilibrium where short-sales are banned for all assets, $\{c^i, q^y, q^z, z, y\}$. Modify the scenario: destroy a very small part of the dividends of each/some trees in each/some states,– for example, an ϵ^j_s from the dividend of asset j in state s, with dividends reduced to $d^j_s = 1 - \epsilon^j_s$; continue to ban short sales on all assets and compute the new equilibrium, $\{\widetilde{c}^i, \widetilde{q}^y, \widetilde{q}^z, \widetilde{z}, \widetilde{y}\}$. Importantly, the reduction in dividends is such that portfolios span all risks if holdings of shares are unrestricted: the destruction of output introduces insurance opportunities not previously available, the force that drives the Pareto improvement in the preceding example. The question is whether the ban on short sales interferes with the argument; indeed, it does.

Proposition 1. *Absent short sales, a reallocation of commodities and assets following the reduction in dividends cannot be Pareto improving, the augmented insurance opportunities notwithstanding.*

Proof. We argue by contradiction. Suppose $\{\widetilde{c}^i\}$ dominates $\{c^i\}$,

$$u^i(\widetilde{c}^i_0) + \beta^i E u^i(\widetilde{c}^i_s) \geqslant u^i(c^i_0) + \beta^i E u^i(c^i_s),$$

with strict improvement for some.

Strict concavity of the utility function and the previous inequalities imply that

$$\Delta U^i = u^{i'}(c_0^i)(\widetilde{c}_0^i - c_0^i) + E\Big[\beta^i u^{i'}(c_s^i)(\widetilde{c}_s^i - c_s^i)\Big] > 0,$$

and, as a consequence,

$$\tfrac{\Delta U^i}{u^{i'}(c_0^i)} = (\widetilde{c}_0^i - c_0^i) + E\left[\tfrac{\beta^i u^{i'}(c_{1,s}^i)}{u^{i'}(c_0^i)}(\widetilde{c}_s^i - c_s^i)\right] > 0.$$

Substitution from the budget constraints in period 0 yields

$$\tfrac{\Delta U^i}{u^{i'}(c_0^i)} = \bar{y}^i(\widetilde{q}^y - q^y) + \bar{z}^i(\widetilde{q}^z - q^z) - \widetilde{q}^y\widetilde{y}^i - \widetilde{q}^z\widetilde{z}^i + q^y y^i + q^z z^i + \\ E\Big[mrs_s^i\Big(d_s^y\widetilde{y}^i + d_s^z\widetilde{z}^i - (y^i + z^i)\Big)\Big],$$

where

$$\mathrm{mrs}_s^i = \frac{\beta^i u^{i'}(c_{1,s}^i)}{u^{i'}(c_0^i)}.$$

First order conditions for optimization at the initial allocation are

$$E(mrs_s^i) - q^j \leqslant 0, \quad j = y, z,$$

and, after multiplication by z^i and y^i,

$$z^i\Big(E(mrs_s^i) - q^z\Big) = 0,$$

$$y^i\Big(E(mrs_s^i) - q^y\Big) = 0,$$

and

$$\tfrac{\Delta U^i}{u^{i'}(c_0^i)} = \bar{y}^i(\widetilde{q}^y - q^y) + \bar{z}^i(\widetilde{q}^z - q^z) - \widetilde{q}^y\widetilde{y}^i - \widetilde{q}^z\widetilde{z}^i + q^y y^i + q^z z^i +$$

$$E\Big[mrs_s^i\Big(d_s^y\widetilde{y}^i + d_s^z\widetilde{z}^i\Big)\Big] - z^i q^z - y^i q^y;$$

taking the sum over i and, since assets are in unit supply,

$$\textstyle\sum_{i\in I}\tfrac{\Delta U^i}{u^{i'}(c_0^i)} = -q^y - q^z + E\Big[\Big(\sum_{i\in I} mrs_s^i\widetilde{z}^i\Big)d_s^z\Big] + E\Big[\Big(\sum_{i\in I} mrs_s^i\widetilde{y}^i\Big)d_s^y\Big].$$

To complete the proof and derive a contradiction we have to show that the sum is non-positive. The first order conditions at the competitive equilibrium, $E(mrs_s^i) - q^j \leqslant 0$ yield, after multiplication by $\widetilde{z}^i \geq 0$ and $\widetilde{y}^i \geq 0$, summation over i, and since assets are in unit net supply,

$$E\Big[\Big(\sum_{i\in I} mrs_s^i \widetilde{y}^i\Big)\Big] - q^y \leq 0$$

$$E\Big[\Big(\sum_{i\in I} mrs_s^i \widetilde{z}^i\Big)\Big] - q^z \leq 0.$$

Since $d_s^j = 1 - \epsilon_s^j$ and $d_s^j < 1$ for at least one tree and one state, and, importantly, $\widetilde{z}^i, \widetilde{y}^i \geqslant 0$, the expression is non-positive. □

Remark 2. The argument, here, is that the (a) competitive allocation subject to a ban on short sale cannot be improved upon by any allocation that respects the ban; not only the (a) competitive allocation.

Remark 3. Consumption at date 0 does not play a role in the argument and can be dispensed with. We introduce it in order to facilitate comparison with existing literature, Carvajal, Rostek, and Waretka (2012), in particular.

Remark 4. A question that arises is whether Pareto improvement via the destruction of output is possible when short sales are restricted, but not prohibited: asset holdings are simply bounded below. This ins indeed possible, but the gain converges to 0 with the lower bound.

Incentives and intervention

Even though the destruction of output cannot be Pareto improving if short sales are not allowed, an individual may benefit from the destruction of some of the output in his endowment; evidently, the argument and the intuition behind it, is very much as in Aumann and Peleg (1974): one individual effectively exercises market power from the increase in the price of goods that he supplies following the destruction of part of his output.

A variation of the framework parallels Carvajal, Rostek, and Waretka (2012). The economy is populated by two (types of) individuals: two investors, i, who consume in both periods and want to hedge future risks, and an entrepreneur who holds all the assets and has preferences only for period zero consumption. Suppose investors have identical preferences

$$U^i = c_0^i + E_{1/2}\frac{(c_s^i)^{1-\gamma}}{1-\gamma},\ \gamma > 0,$$

and state-contingent endowments

$$(e_a^1, e_b^1) = (1, 0),\ \text{and}\ (e_a^2, e_b^2) = (0, 1).$$

The entrepreneur sells assets that, as above, have identical dividend patterns across states; for simplicity, all 1. The payoff or utility of the entrepreneur is

$$c_0^e = q^y + q^z.$$

Suppose the entrepreneur destroys a very small part of the dividend of tree, y, in state a, ϵ_a^y, and a very small part of dividend of z in the same state, ϵ_a^z, with $\epsilon_a^y \neq \epsilon_a^z$: the return matrix is of full rank. A short-sales constraint for asset z is binding for individual 1, whereas for y, for individual 2. It is not difficult to find values to show that the entrepreneur, nevertheless, has an incentive to destroy in order to increase the amount of period 0 output that he extracts in exchange for the assets.

Remark 5. Finally, the suboptimality of competitive allocations when risk sharing is restricted prompted Geanakoplos and Polemarchakis (1986) to define constrained suboptimality; and to demonstrate that public intervention that employs *instruments that do not augment risk-sharing opportunities* can implement Pareto improvements. The intervention, here, is effective precisely by augmenting insurance possibilities.

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