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Suboptimality with land

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Abstract

In a stochastic economy of overlapping generations subject to uninsurable risks, competitive allocations need not be constrained optimal. This is the case even in the presence of long-lived assets and no short sales.

Key words: long-lived assets; optimality

JEL classification: D52; D61.

Introduction

Long-lived assets, land or "Lucas-trees," restore optimality in economies that extend over an infinite time-horizon. Wilson (1981) developed the argument for economies under certainty, and Santos and Woodford (1997) extended the argument to economies under uncertainty as long as the asset market is complete.

The possible failure of optimality has been studied extensively and is well understood. It derives from the failure of aggregate valuation or, equivalently, an aggregate budget constraint that obtains when the real rate of interest falls short of the rate of growth of output. Samuelson (1958) and Diamond (1965) first argued that aggregate debt or, equivalently, pay-as-you-go social security, may implement optimal allocations, low interest rates notwithstanding. Cass (1972) gave a condition, weaker than aggregate valuation, that is necessary and sufficient for a price path to support a Pareto optimal allocation.

In the presence of uninsurable risks, the optimality properties of competitive allocations are problematic even in finite economies. Not only do competitive allocations typically fail to be optimal, which is not surprising or even relevant, but, typically, they fail to be even constrained optimal: competitive markets fail to make optimal use of the restricted reallocations of risks that fundamentals allow. Geanakoplos and Polemarchakis (1986) showed that, typically, reallocations of the existing assets implement Pareto improvements; Carvajal and Polemarchakis (2011) extended the argument to economies with (purely) idiosyncratic risks. Importantly, constrained suboptimality occurs with multiple commodities or periods of economic activity, and it derives from the variation in relative prices that competitive markets fail to internalize.

Demange (2002) demonstrated an important and surprising result: in an economy of overlapping generations with life-spans of two periods and one, aggregate commodity at each period, long lived assets traded subject to a ban on short sales restore the constrained optimality of competitive allocations in the presence of uninsurable risks: there is no intervention that respects both the prevailing restrictions on risk sharing and the ban on short sales and implements a Pareto improvement. Here, we give a series of examples that demonstrate the dependance of constrained optimality on the restriction to life-spans of two periods and a single commodity. With multiple commodities or multiple periods in the economic life-time of generations, constrained suboptimality may arise.

Lucas and Stokey (1983) and Angeletos (2002) considered optimal fiscal policy under uncertainty, while Diamond and Geanakoplos (2003) and Dutta, Kapur, and Orszag (2000) characterized optimal financial policies for social security. The constrained suboptimality of competitive allocations provides a foundation for intervention. Here, with debt along with long-lived assets, aggregate debt policy implements Pareto improvements.

Land

The economy is stationary, of overlapping generations with life-spans of three periods. Each generation consists of a continuum, of mass 1, of initially identical individuals. There is one perishable commodity at each date. Individuals receive a deterministic endowment, e^y, when young, a stochastic endowment, e^m_s, when middle-aged, and, again, a deterministic endowment, e^o, when old. Personal states , s, occur with probability π_s; there is no aggregate risk. Land, in aggregate supply 1, produces a constant dividend, f. The cardinal utility index, u, satisfies standard curvature, smoothness and boundary assumptions. Consumption is c, and it is numéraire; holdings of land are y and the price of land is q. Time commences at $t = 0$; we omit the subscript, t, that indicates dates, when no ambiguity arises.

The decision problem of an individual is

$$\max\ U = [u(c^y) + E_\pi u(c^m_s) + E_\pi u(c^o_s)],$$

s.t.

$$c^y + qy^y = e^y,$$

$$c^m_s + qy^m_s = e^m_s + (f + q)y^y,$$

$$c^o_s = e^o + (f + q)y^m_s;$$

first order conditions are

$$\tfrac{q}{d+q} u^{'}(c^y) = E_\pi[u^{'}(c^m_s)],$$

$$\tfrac{q}{d+q} = \tfrac{u^{'}(c^o_s)}{u^{'}(c^m_s)}.$$

Equilibrium in the asset markets requires that

$$y^y + E_\pi(y^m_s) = 1;$$

the expectation operator aggregates land holdings of middle-aged individuals. Evidently, middle-aged and old individuals at the initial date, $t = 0$, with appropriate utility functions and asset holdings, implement a stationary equilibrium when activity does not extend to the infinite past.

We shall argue that there are robust situations in which a fiscal authority can implement a Pareto improvement relative to the stationary competitive equilibrium allocation; this, without short sales either before or after the intervention: $y^y, y^m_s > 0$.

The policy instrument is an exogenous specification of investment in land by individuals when young: y^y. Subsequently, individuals trade in competitive markets. In the absence of aggregate risk, and since individuals are identical at the beginning of their lives, the intervention does not go beyond allocations a market could implement.

At a stationary competitive equilibrium

$$\tfrac{q}{d+q}u^{'}(e^y - qy^y) = E_\pi[u^{'}(e^m_s + (f+q)y^y - qy^m_s)],$$

$$\tfrac{q}{f+q} = \tfrac{u^{'}(e^o+(f+q)y^m_s)}{u^{'}(e^m_s+(f+q)y^y-qy^m_s)},$$

$$y^y + E_\pi(y^m_s) = 1.$$

To complete the characterization of a stationary equilibrium we need to specify appropriate initial conditions. There exists an initial old generation with preferences defined, without loss of generality, by $U^o_0(c^o_0) = c^o_0$, and aggregate endowment and holdings of land as the old generation at the stationary equilibrium; and groups, s, of initial middle-aged generations, of size π_s, with life-spans of two periods, preferences $U^m_s(c^m_s, c^o_s) = u(c^m_s) + u(c^o_s)$ and aggregate endowments and initial holdings of land as all future middle-aged generations.

We fix $u(c) = \log(c)$ and endowments $e^m_s = e^m + \epsilon_s > 0$, where $e^m > 0$, $E_\pi(\epsilon_s) = 0$, and $e^o = 0$; the latter guarantees that the middle-aged cohort never short-sell land, while parameter values shall be such that, at the stationary equilibrium, young generations also do not short-sell.

An economy is specified by the parameters $(f, e^y, e^m, \ldots, \pi_s, \ldots \epsilon_s, \ldots)$, and a property is robust if it obtains for an open set of economies.

At all periods, a fiscal authority dictates investment in land, $\widetilde{y}^y_t$, by young individuals; in addition, it redistributes wealth, but, *only* at $t = 0$. Redistribution or transfers at $t = 0$ are τ. Young generations have no discretion on their consumption or savings. On the other hand, middle-aged generations do: they allocate consumption-saving optimally given the land holdings specified by the fiscal authority when young.

We do not restrict attention to stationary interventions, and, as a consequence, we specify t when necessary. Nevertheless, for a Pareto improvement, it suffices for the fiscal authority to set $\{\widetilde{y}_t^y\}_{t=0}^{\infty}$, with $\widetilde{y}_t^y > 0$, and $\widetilde{y}_t^y = \widetilde{y}^y$, for $t \geqslant 1$: the intervention is stationary after the initial date.

From the first order condition of the middle-aged at $t \geqslant 1$,

$$\frac{\widetilde{q}_t}{f + \widetilde{q}_{t+1}} = \frac{e_s^m + (f + \widetilde{q}_t)\widetilde{y}_{t-1}^y - \widetilde{q}_t \widetilde{y}_{t,s}^m}{(f + \widetilde{q}_{t+1})\widetilde{y}_{t,s}^m}$$

or

$$\widetilde{y}_{t,s}^m = \frac{e_s^m + (f + \widetilde{q}_t)\widetilde{y}_{t-1}^y}{2\widetilde{q}_t}.$$

We substitute $\widetilde{y}_{t,s}^m$ into the market clearing for land, $E_\pi(\widetilde{y}_{t,s}^m) = 1 - \widetilde{y}_t^y$, to solve for equilibrium prices,

$$\widetilde{q}_t = \frac{e^m + f\widetilde{y}_{t-1}^y}{2(1 - \widetilde{y}_t^y) - \widetilde{y}_{t-1}^y}.$$

Since we focus on stationary interventions after the initial date, asset prices at $t \geqslant 1$ simplify as

$$\widetilde{q}_1 = \tfrac{e^m + f\widetilde{y}_0^y}{2(1-\widetilde{y}^y) - \widetilde{y}_0^y},$$

$$\widetilde{q} = \tfrac{e^m + f\widetilde{y}^y}{2(1-\widetilde{y}^y) - \widetilde{y}^y}, \; t \geqslant 2$$

For $t = 0$, the first order conditions of initial middle-aged rewrite as

$$\frac{\widetilde{q}_0}{f + \widetilde{q}_1} = \frac{e_s^m + \tau_{0,s}^m + (f + \widetilde{q}_0)y^y - \widetilde{q}_0 \widetilde{y}_{0,s}^m}{(f + \widetilde{q}_1)\widetilde{y}_{0,s}^m}$$

or

$$\widetilde{y}_{0,s}^m = \frac{e_s^m + \tau_{0,s}^m + (f + \widetilde{q}_0)y^y}{2\widetilde{q}_0};$$

y^y are initial asset holdings and $\tau_{0,s}^m$ are transfers they get from the fiscal authority. From period zero asset market clearing,

$$\widetilde{q}_0 = \frac{e^m + E_\pi(\tau_{0,s}^m) + fy^y}{2(1 - \widetilde{y}_0^y) - y^y}.$$

The dynamics of asset prices[1] are $\{\widetilde{q}_t\}_{t=0}^{\infty}$, with $\widetilde{q}_t = \widetilde{q}$, for $t \geqslant 2$: asset prices are stationary after $t = 1$.

We shall demonstrate that a Pareto improvement obtains for $\widetilde{q}_1 > q$ and $\widetilde{q} < q$. Moreover, we restrict the analysis to marginal changes of land holdings and date 0 transfers that translate to marginal changes of prices, allocations and utilities. We outline the construction of a Pareto improving intervention; a complete derivation is in the Appendix. For simplicity, we use the notation $u(c)$, keeping in mind that $u(c) = \log(c)$.

Generations $t \geqslant 2$ are better-off if and only if

$$\frac{dU_t}{u'(c^y)} = -dq\Big(y^y(1 - \frac{q}{f+q}) + (1 - \frac{q}{f+q})E_\pi(\frac{u'(c_s^m)}{u'(c^y)}y_s^m)\Big) > 0; \tag{1}$$

dq is the marginal change of asset prices at $t \geqslant 2$ relative to the competitive equilibrium. The term in parenthesis in (1) is always positive at a stationary equilibrium that is characterised by no short sales of land. Generations $t \geqslant 2$ are better off if and only if $dq < 0$ ($\widetilde{q} < q$): asset prices at $t \geqslant 2$ should be lower than at the stationary competitive equilibrium. The fiscal authority can decrease investment in land by young individuals *after the initial date* to make $dq < 0$ and, as a result, make generations $t \geqslant 2$ better off.

For $t = 0$, and since, then (and only then) the fiscal authority can redistribute revenue, for a Pareto improvement it suffices that

$$dc_0^o + E_\pi(\tfrac{dU_{0,s}^m}{u'(c_s^m)}) + \tfrac{dU_0}{u'(c^y)} > 0$$

or, equivalently,

$$dq_1\Big(\tfrac{q}{f+q} - E_\pi(\tfrac{u'(c_s^m)}{u'(c^y)}y_s^m)\Big) + dq\tfrac{q}{f+q}E_\pi(\tfrac{u'(c_s^m)}{u'(c^y)}y_s^m) > 0. \tag{2}$$

Expression (2) depends only on the marginal change of asset prices at $t = 1$, dq_1, and at $t = 2$, dq; the marginal change of period zero asset price cancels out and transfers (redistribution of revenue) add up to zero. The terms multiplying dq_1, dq are positive at the stationary equilibrium. Since we require $dq < 0$, a necessary condition for (2) to be satisfied is $dq_1 > 0$ ($\widetilde{q}_1 > q$): asset prices at $t = 1$ should be higher than at the competitive equilibrium. The fiscal authority can increase investment in land by young individuals

[1]The assumption of $e^o = 0$ simplifies the dynamics of asset prices. In particular, the asset price at t is not a function of the asset price at $(t + 1)$ but, instead, is pinned down only by the land holdings of young individuals. Our results extend to the case where $e^o > 0$.

at the initial date to satisfy (2) and, with appropriate redistribution, make everyone at $t = 0$ better off.

Lastly, the generation $t = 1$ is better-off if and only if

$$\frac{dU_1}{u'(c^y)} = -dq\Big(- y^y(\frac{q}{f+q} - \frac{dq_1}{dq}) + (1 - \frac{q}{f+q})E_\pi(\frac{u'(c_s^m)}{u'(c^y)}y_s^m)\Big) > 0. \qquad (3)$$

Since we require $dq < 0$, the generation $t = 1$ are better-off if and only if the term in parenthesis in (3) is positive or, equivalently,

$$(1 - \frac{q}{f+q})E_\pi(\frac{u'(c_s^m)}{u'(c^y)}y_s^m) > y^y(\frac{q}{f+q} - \frac{dq_1}{dq}). \qquad (4)$$

To demonstrate that (4) is satisfied and as a consequence generation $t = 1$ is better off, it is convenient to combine (2),(4) as

$$\underbrace{(1 - \frac{q}{f+q})E_\pi(\frac{u'(c_s^m)}{u'(c^y)}y_s^m)}_{=A} > \underbrace{y^y(\frac{q}{f+q} - \frac{dq_1}{dq})}_{=B} >$$

$$\underbrace{y^y\Big(\frac{q}{f+q} + \frac{\frac{q}{f+q}E_\pi(\frac{u'(c_s^m)}{u'(c^y)}y_s^m)}{\frac{q}{f+q} - E_\pi(\frac{u'(c_s^m)}{u'(c^y)}y_s^m)}\Big)}_{=\Gamma}.$$

$A > B$ is identical to (4), while $B > \Gamma$ is equivalent to (2). $A > \Gamma$ is a property of the stationary competitive equilibrium; A, Γ do not depend on marginal changes of asset prices, but, rather, on the characteristics of the stationary competitive equilibrium. There exist interventions at the initial date such that $B > \Gamma$, as argued earlier, and, in addition, B arbitrarily close to Γ. Since $A > \Gamma$, there exist interventions such that $A > B$ and generation $t = 1$ is better off. This completes the argument.

The intuition behind the constrained suboptimality result can be best understood if we focus on the behaviour of generations after date 0. The young members of each generation, taking prices as given, invest too much in land (over-save) in order to insure against the bad realisation of uncertainty. The fiscal authority, by decreasing investment in land by young individuals, induces a non trivial change in asset prices. The latter effect induces a reallocation of wealth among members of each generation that is welfare improving. Effectively, individuals invest too much in land because prices are not "set optimally" at the competitive equilibrium.

Debt

In order to focus on debt, we demonstrate the constrained suboptimality of equilibrium in a two commodity, two period life-span economy of overlapping generations, where individuals hold public debt and invest in land. Multiple commodities serve the same purpose as life-spans of multiple periods. Perturbations of public debt affect the relative price of commodities that, in turn, induces a reallocation of state-contingent wealth that improves ex-ante welfare. As before, there are no short sales of land at equilibrium.

Each generation consists of a continuum, of mass 1, of initially identical individuals. There are two perishable commodities at each date: 1 and 2. Individuals desire both commodities when old and only commodity 1 when young. Commodity 1 is numéraire and its price is normalized to 1, whereas the price of commodity 2 is p. Individuals receive a non-stochastic endowment, e^y_1 when young, a stochastic endowment of commodity 1, $e^o_{1,s}$, and a non-stochastic endowment of commodity 2, e^o_2, when old. Notation and assumptions about land as before. There is a government that issues debt, b, pays interest on debt, i, and levies lump-sum taxes, τ. Time commences at $t = 0$; we omit the subscript, t, that indicates dates, when no ambiguity arises. Notation and assumptions about personal states as before; there is no aggregate risk.

The decision of an individual is

$$\max U = [c^y_1 + E_\pi u(c^o_{1,s}) + E_\pi u(c^o_{2,s})],$$

s.t

$$c^y_1 + b + qy = e^y_1 - \tau,$$

$$c^o_{1,s} + pc^o_{2,s} = e^o_{1,s} + pe^o_2 + (f + q)y + (1 + i)b;$$

consumption of commodities 1 and 2 are c_1 and c_2 respectively. First order conditions are

$$\lambda^y = 1,\ \lambda^o_s = u^{'}(c^o_{1,s}) = \frac{u^{'}(c^o_{2,s})}{p},$$

$$\tfrac{1}{1+i} = \tfrac{q}{f+q} = E_\pi(\lambda^o_s);$$

λ^y, λ^o_s are the Lagrange multipliers on the budget constraints of young and old individuals respectively.

No arbitrage between land and debt implies

$$i = \frac{f}{q}.$$

There exist groups, s, of initial old generations, of size π_s, with preferences $U^o_s(c^o_{1,s}, c^o_{2,s}) = u(c^o_{1,s}) + u(c^o_{2,s})$ and endowments, holdings of land and debt as all future old generations.

The government issues debt and levies taxes in order to finance outstanding debt

$$b_t + \tau_t = (1 + i_{t-1})b_{t-1}.$$

For stationarity,

$$\tau = ib.$$

We compute a robust example to demonstrate the constrained suboptimality of equilibrium; details are presented in the Appendix. An economy is specified by $(u, f, e^y_1, e^o_2, \ldots.., \pi_s, \ldots.., e^o_{1,s}, \ldots..)$ and fiscal policy by (τ, b).

At all periods, the government perturbs debt[2] held by young individuals; in addition, it redistributes wealth, but, *only* at $t = 0$. Subsequently, individuals trade in commodity and land markets. We restrict the analysis to stationary marginal changes of debt.

For $t = 0$, and since, then (and only then) the fiscal authority can redistribute revenue, for a Pareto improvement it suffices that

$$dU_0 + E_\pi(\frac{dU^o_s}{\lambda^o_s}) = \Big(\frac{1}{1+i}dq + db + \frac{b}{1+i}di\Big) + E_\pi(\lambda^o_s(e^o_2 - c^o_{2,s}))dp > 0. \quad (5)$$

Marginal changes of relative prices are

$$dp = \frac{1+i}{e^o_2}\Big(\frac{1}{1+i}dq + db + \frac{b}{1+i}di\Big). \quad (6)$$

To simplify the exposition, define as z the following expression

[2] As a consequence, lump-sum taxes have to be adjusted accordingly in order for the government budget to be satisfied.

$$z = \frac{1}{1+i}dq + db + \frac{b}{1+i}di. \tag{7}$$

The fiscal authority can always change debt appropriately in order to determine the sign of z. To see this, rewrite (7) as

$$z = \Big(\frac{1}{1+i}\frac{\partial q}{\partial b} + 1 + \frac{1}{1+i}\frac{\partial i}{\partial b}\Big)db,$$

and, given the sign of the term in parenthesis, marginal changes of debt determine the sign of z.
Taking into account (6),(7), expression (5) modifies as

$$dU_0 + E_\pi(\frac{dU_s^o}{\lambda_s^o}) = \Big(1 + E_\pi(\lambda_s^o(e_2^o - c_{2,s}^o))\frac{1+i}{e_2^o}\Big)z > 0.$$

The term in big parenthesis is positive; a restriction (inequality) that is satisfied at the stationary competitive equilibrium. Marginal changes of debt such that $z > 0$ imply that the sum of perturbed utilities is positive.

Generations $t \geqslant 1$ are better off if and only if

$$dU_t = \Big(- i + E_\pi(\lambda_s^o(e_2^o - c_{2,s}^o))\frac{1+i}{e_2^o}\Big)z > 0.$$

The term in parenthesis is positive; the second restriction (inequality) that is satisfied at the stationary competitive equilibrium. Since we require $z > 0$, marginal changes of debt induce a Pareto improvement.

Remark 1. It is the pecuniary externality induced by $dp \neq 0$ and trade in the second commodity, $e_2^o \neq c_{2,s}^o$, that drives the constrained suboptimality result. If $dp = 0$ or $e_2^o = c_{2,s}^o$, then the stationary competitive equilibrium is constrained optimal.

Capital

Lastly, we introduce capital and demonstrate that improving interventions are characterized by higher levels of investment in capital relative to the competitive level; the competitive allocation is characterized by under-investment in capital.

The economy is stationary, of overlapping generations with life-spans of two periods. Each generation consists of a continuum, of mass 1, of initially identical individuals. There are two commodities at each date: consumption and labor. Young and old members of each generation are endowed with $\bar{l}$ units of time that they supply inelastically to the market. Idiosyncratic shocks affect the individual efficiency of capital investment. Notation and assumptions about land as before. There is a firm that rents capital, hires labor and produces output using a Cobb-Douglas technology: $f(k,l) = k^a l^{1-a}$; k is capital and l labor employed. Profit maximization requires

$$1 + r = ak^{a-1}l^{1-a},$$

$$w = (1-a)k^a l^{-a};$$

$(1+r)$ is the real interest rate (factor), and w the real wage. Notation and assumptions about personal states as before; there is no aggregate risk.

The decision of an individual is

$$\max U = [c^y + E_\pi u(c_s^o)],$$

s.t

$$c^y + k + qy = w\bar{l},$$

$$c_s^o = (1+r)\varrho_s k + w\bar{l} + (f+q)y;$$

c is for consumption and ϱ_s are shocks to individual capital efficiency; shocks are positive and satisfy the normalization $E_\pi(\varrho_s) = 1$.

First order conditions with respect to capital and land, respectively, are

$$\frac{1}{1+r} = E_\pi\Big(\varrho_s u^{'}(c_s^o)\Big),$$

$$\frac{q}{f+q} = E_\pi\Big(u^{'}(c_s^o)\Big).$$

There is an initial old generation with preferences $U_0^o(c_0^o) = c_0^o$ and

$$c_0^o = f(k_{-1}, l) - wl + w\bar{l} + f + q,$$

where k_{-1} is the initial capital stock and $f(k_{-1}, l) - wl$ are profits of the firm net of capital cost at date 0. Initial old are endowed with one unit of land and $\bar{l}$ units of time.

We demonstrate the constrained suboptimality of equilibrium with a robust example; as before, details are presented in the Appendix. An economy is specified by $(u, f, \alpha, \bar{l}, ..., \pi_s, ..., \varrho_s, ...)$. Moreover, it is important for the arguments that follow to note that $f, \bar{l} > 0,\ 0 < \alpha < 1$, labor market clearing is $l = 2\bar{l}$, and the stationary competitive equilibrium is characterised by $q > 0$ and $k > 0$.

At all periods, a fiscal authority dictates investment in capital, k, by young individuals; in addition, it redistributes wealth, but, *only* at $t = 0$. Subsequently, individuals trade in commodity and land markets. We restrict the analysis to stationary marginal changes of capital.

To proceed it is useful to demonstrate the following result: for linear-concave utilities and Cobb-Douglas technology, the price of land depends negatively on capital investment:

$$\frac{\partial q}{\partial k} < 0.$$

To see this, consider the first order condition with respect to land and substitute in it the budget constraint of old individuals, firm's first order conditions, and market clearing identities. Differentiate q with respect to k,

$$\frac{\partial q}{\partial k} = \frac{1}{\frac{f}{(f+q)^2} - E_\pi u''(c_s^o)} E_\pi \left(u''(c_s^o) \left[\varrho_s (k \frac{\partial r}{\partial k} + 1 + r) + \bar{l} \frac{\partial w}{\partial k} \right] \right);$$

$u'' < 0$, $k(\partial r/\partial k) + 1 + r > 0$, $(\partial w/\partial k) > 0$, from strict concavity, Cobb-Douglas and the first order conditions of the firm, respectively. As a result, $(\partial q/\partial k) < 0$.

For $t = 0$, and since, then (and only then) the fiscal authority can redistribute revenue, for a Pareto improvement it suffices that

$$dc_0^o + dU_0 = \left(\bar{l} \frac{\partial w}{\partial k} \left[\left(\frac{q}{f+q} - \frac{1}{1+r} \right) - \frac{1}{1+r} \right] + \frac{\partial q}{\partial k} \frac{q}{f+q} \right) dk > 0.$$

The term in big parenthesis is positive; a restriction (inequality) that is satisfied at the stationary competitive equilibrium. A policy of higher investment relative to the equilibrium allocation, $dk > 0$, implies that the sum of perturbed utilities is positive.

Generations $t \geqslant 1$ are better off if and only if

$$dU_t = \left(\bar{l} \frac{\partial w}{\partial k} \left[\left(\frac{q}{f+q} - \frac{1}{1+r} \right) + \frac{r}{1+r} \right] - \frac{\partial q}{\partial k} \frac{f}{f+q} \right) dk > 0. \qquad (8)$$

We demonstrate that the term in big parenthesis in (8) is *always* positive at equilibrium. As a result, *only* a policy that increases capital investment relative to the competitive level, $dk > 0$, can induce a Pareto improvement.

According to the previous argument, $(\partial q / \partial k) < 0$ and $(\partial w / \partial k) > 0$. Moreover, $(1 + r - (f+q)/q) > 0$ is the risk premium between the risky investment in capital and the safe investment in land which is positive at equilibrium. A positive risk premium implies $r > 0$. As a result, the term in parenthesis in (8) is positive.

Remark 2. It is the pecuniary externality induced by perturbations of labor income of old members of each generation that drives the constrained suboptimality result.

Remark 3. Carvajal and Polemarchakis (2011) considered a similar economy where idiosyncratic shocks affect only the productivity of labor of old members of each generation. They gave an example where the competitive allocation is characterized by over-investment in capital. Krebs (2003) considered an infinite horizon economy with heterogenous infinite-lived agents that invest in physical and human capital and idiosyncratic shocks affect only the return to human capital. He argued that a reduction in idiosyncratic risk reduces investment in physical, but increases investment in human capital; the equilibrium is characterised by over-investment in physical capital and under-investment in human capital. Geanakoplos and Kubler (2015) considered a two-period economy with heterogenous agents and incomplete markets. They described mechanisms through which, at equilibrium, agents over-borrow.

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Appendix

We complete the argument for each of the sections.

Land

We compute a stationary equilibrium for the economy of section land. Consider two personal states, $s \in \{H, L\}$, $u(c) = \log(c)$, $f > 0$, $e^o = 0$, $e^y > 0$, $e^m_s = e^m + \epsilon_s > 0$, $E_\pi(\epsilon_s) = 0$.

At a stationary equilibrium

$$\tfrac{q}{f+q}\tfrac{1}{e^y - qy^y} = \tfrac{\pi_H}{e^m_H + (f+q)y^y - qy^m_H} + \tfrac{\pi_L}{e^m_L + (f+q)y^y - qy^m_L}, \tag{9}$$

$$\tfrac{q}{f+q} = \tfrac{e^m_s + (f+q)y^y - qy^m_s}{(f+q)y^m_s}, \tag{10}$$

$$y^y + \pi_H y^m_H + \pi_L y^m_L = 1. \tag{11}$$

From (10) we solve for middle-aged asset holdings, y^m_s, as

$$y^m_s = \frac{e^m_s + (f+q)y^y}{2q}. \tag{12}$$

Substitute (12) in (9) to solve for young's asset holdings, y^y

$$\tfrac{q}{f+q}\tfrac{1}{e^y - qy^y} = \tfrac{\pi_H}{e^m_H + (f+q)y^y - q\frac{e^m_H + (f+q)y^y}{2q}} + \tfrac{\pi_L}{e^m_L + (f+q)y^y - q\frac{e^m_L + (f+q)y^y}{2q}}. \tag{13}$$

(13) reduces to the following quadratic expression on y^y

$$\begin{gathered} 3q(f+q)(y^y)^2 + y^y(q(e^m_H + e^m_L + 2\Delta) - 2e^y(f+q)) + \\ \tfrac{q}{f+q} e^m_H e^m_L - 2e^y\Delta = 0, \end{gathered} \tag{14}$$

where $\Delta = \pi_H e^m_L + \pi_L e^m_H$. From (14) we get

$$y^y = \\ \tfrac{-(q(e^m_H + e^m_L + 2\Delta) - 2e^y(f+q)) \pm \sqrt{(q(e^m_H + e^m_L + 2\Delta) - 2e^y(f+q))^2 - 12q(f+q)(\frac{q}{d+q} e^m_H e^m_L - 2e^y\Delta)}}{6q(f+q)}. \tag{15}$$

For restrictions that we specify shortly, the small root in (15) is consistent with negative asset prices. Consider the big root in (15) and substitute it in (12) to solve for y_s^m as a function of q. Substitute the latter together with the big root in (15) in (11) to solve for q. We get

$$\begin{gathered}\frac{-(q(e_H^m+e_L^m+2\Delta)-2e^y(f+q))+\sqrt{(q(e_H^m+e_L^m+2\Delta)-2e^y(f+q))^2-12q(f+q)(\frac{q}{f+q}e_H^m e_L^m-2e^y\Delta)}}{6q(f+q)}+ \\ \frac{e^m-2q}{f+3q}=0.\end{gathered} \tag{16}$$

Equilibrium asset prices are computed from (16). Define the left hand side of (16) as $\Phi(\xi, q)$, where $\xi = (f, e^y, e^m, \pi_H, \pi_L, \epsilon_H, \epsilon_L)$ and $\pi_H\epsilon_H + \pi_L\epsilon_L = 0$. Equilibrium requires $\Phi(\xi, q) = 0$.

Consider the following economy

$$\xi^* = (1.6, 1.05, 4, 0.2, 0.8, 3.2, -0.8).$$

Substituting these parameters in (16) and solving numerically, we get the following solution[3]:

$$q = 2.04859.$$

Asset holdings of agents are

$$y^y = 0.0125451,\ y_H^m = 1.76848,\ y_L^m = 0.792198;$$

individuals do not short-sell land.

A stationary equilibrium exists for economies in a neighbourhood of ξ^*. The argument is as follows. $\Phi(\xi, q) = 0$ defines q as a function of ξ. The derivative of Φ with respect to q is nonzero, $(\partial\Phi/\partial q)(\xi^*, 2.04859) = -2.54416 \neq 0$, and the derivative of Φ with respect to each element of ξ is well-defined. Thus, $(\partial q/\partial\xi)(\xi^*, 2.04859)$ is well-defined.

In the next section we demonstrate the constrained suboptimality of the stationary competitive equilibrium of the ξ^* economy and of the respective stationary competitive equilibrium of an economy in a neighbourhood of ξ^*.

Improving interventions

A fiscal authority dictates investment in land, $\widetilde{y}_t^y$, by young individuals and redistributes wealth only at $t = 0$. Redistribution is $\{\tau_{0,s}^m, \tau_0^y, \tau_0^o\}$, for ini-

[3] Expression (16) implies two extra roots: $q = -9.19947, q = -0.240055$. Negative asset prices are not candidates for equilibrium.

tial middle-aged, initial young and initial old, respectively. For a Pareto-improvement, it suffices to restrict the actions of the fiscal authority to stationary interventions after the initial date: $\widetilde{y}_t^y = \widetilde{y}^y > 0$, $t \geqslant 1$.

As argued earlier, from the first order conditions of middle-aged, we compute the equilibrium prices given the alternative allocation of land and redistribution at date 0. Taking into account the restriction to stationary interventions at $t \geqslant 1$, equilibrium prices are

$$\begin{aligned}
\widetilde{q}_0 &= \tfrac{e^m + f y^y + E_\pi(\tau^m_{0,s})}{2(1-\widetilde{y}^y_0) - y^y}, \\
\widetilde{q}_1 &= \tfrac{e^m + f \widetilde{y}^y_0}{2(1-\widetilde{y}^y) - \widetilde{y}^y_0}, \\
\widetilde{q} &= \tfrac{e^m + f \widetilde{y}^y}{2(1-\widetilde{y}^y) - \widetilde{y}^y},\ t \geqslant 2.
\end{aligned}$$

We compute the marginal change of asset prices, relative to the stationary competitive equilibrium, following a marginal change of policy parameters. Marginal changes in the land holdings of the young at $t = 0$ are denoted by dy_0^y, at $t \geqslant 1$ by dy^y, and marginal changes of transfers at $t = 0$ by $d\tau$.

For $t \geqslant 0$,

$$\begin{aligned}
dq_0 &= \tfrac{2(e^m + f y^y)}{(2(1-y^y) - y^y)^2} dy_0^y + \tfrac{1}{2(1-y^y) - y^y} E_\pi(d\tau^m_{0,s}), \\
dq_1 &= \tfrac{e^m + 2f(1-y^y)}{(2(1-y^y) - y^y)^2} dy_0^y + \tfrac{2(e^m + f y^y)}{(2(1-y^y) - y^y)^2} dy^y, \\
dq &= \tfrac{3e^m + 2f}{(2(1-y^y) - y^y)^2} dy^y,\ t \geqslant 2.
\end{aligned} \tag{17}$$

The derivatives of asset prices with respect to policy parameters, evaluated at the competitive allocation of the ξ^* economy or at the competitive allocation of an economy in a neighbourhood of ξ^*, are positive.

We compute the marginal changes of utilities relative to the stationary competitive equilibrium, taking into account the restriction to stationary interventions after the initial period. For simplicity, we use the notation $u(c)$, keeping in mind $u(c) = \log(c)$.

The marginal change of utility of a typical generation $t \geqslant 2$ is

$$\frac{dU_t}{u'(c^y)} =$$

$$-qdy^y - y^y dq + E_\pi \tfrac{u'(c_s^m)}{u'(c^y)}((f+q)dy^y + y^y dq - qdy_s^m - y_s^m dq)+ \tag{18}$$

$$E_\pi \tfrac{u'(c_s^o)}{u'(c^y)}((f+q)dy_s^m + y_s^m dq).$$

Taking into account the first order conditions for an optimum at the stationary competitive equilibrium, (18) simplifies as

$$\frac{dU_t}{u'(c^y)} = -dq\Big(y^y(1-\frac{q}{f+q}) + (1-\frac{q}{f+q})E_\pi(\frac{u'(c_s^m)}{u'(c^y)}y_s^m)\Big),\ t \geqslant 2.$$

Following a similar argument for $t = 1$,

$$\frac{dU_1}{u'(c^y)} = -dq\Big(- y^y(\frac{q}{f+q} - \frac{dq_1}{dq}) + (1-\frac{q}{f+q})E_\pi(\frac{u'(c_s^m)}{u'(c^y)}y_s^m)\Big).$$

Lastly, for $t = 0$,

$$\frac{dU_0}{u'(c^y)} = d\tau_0^y + y^y\Big(\tfrac{q}{f+q}dq_1 - dq_0\Big) - \Big(dq_1 - \tfrac{q}{f+q}dq\Big)E_\pi\Big[\tfrac{u'(c_s^m)}{u'(c^y)}y_s^m\Big],$$

$$\frac{dU_{0,s}^m}{u'(c_s^m)} = d\tau_{0,s}^m + y^y dq_0 - y_s^m dq_0 + \tfrac{q}{f+q}y_s^m dq_1,$$

$$dc_0^o = (1-y^y)dq_0 + d\tau_0^o,$$

and $(1-y^y)$ are the initial asset holdings of the initial old.

Generations $t \geqslant 2$ are better-off if and only if

$$\frac{dU_t}{u'(c^y)} = -dq\Big(y^y(1-\frac{q}{f+q}) + (1-\frac{q}{f+q})E_\pi(\frac{u'(c_s^m)}{u'(c^y)}y_s^m)\Big) > 0.$$

The term in parenthesis is positive at any well-defined stationary competitive equilibrium that is characterised by no short sales. Generations $t \geqslant 2$ are better-off if and only if $dq < 0$. According to (17), to achieve that target, the fiscal authority must dictate lower investment in land by young individuals after the initial date: $dy^y < 0$.

For $t = 0$, and as we demonstrate at the end of this section, for a Pareto improvement it suffices that

$$dc_0^o + E_\pi(\frac{dU_{0,s}^m}{u^{'}(c_s^m)}) + \frac{dU_0}{u^{'}(c^y)} > 0,$$

or, equivalently,

$$dq_1\Big(\frac{q}{f+q} - E_\pi(\frac{u^{'}(c_s^m)}{u^{'}(c^y)}y_s^m)\Big) + dq\frac{q}{f+q}E_\pi(\frac{u^{'}(c_s^m)}{u^{'}(c^y)}y_s^m) > 0. \tag{19}$$

Marginal changes of asset price at $t = 0$ cancel out when we take sum of perturbed utilities across individuals at $t = 0$. In addition, transfers (redistribution of revenue) add up to zero: $d\tau_0^o + d\tau_0^y + E_\pi(d\tau_{0,s}^m) = 0$. The term multiplying dq_1 is positive at the stationary equilibrium of the ξ^* economy:

$$\frac{q}{f+q} - E_\pi(\frac{u^{'}(c_s^m)}{u^{'}(c^y)}y_s^m) > 0. \tag{20}$$

Since (20) is an inequality, it is satisfied at the corresponding stationary equilibrium of an economy in a neighbourhood of ξ^*. The term multiplying dq is always positive at a stationary equilibrium that is characterised by no short sales. Since we require $dq < 0$, a necessary condition for (19) to be satisfied is $dq_1 > 0$. The next step is to demonstrate that there exists interventions such that (19) is satisfied. To that end, (19) can be equivalently written as

$$dy_0^y > -dy^y\left(\frac{2(e^m + fy^y)}{e^m + 2f(1-y^y)} + \frac{3e^m + 2f}{e^m + 2f(1-y^y)}\frac{\frac{q}{f+q}E_\pi(\frac{u^{'}(c_s^m)}{u^{'}(c^y)}y_s^m)}{\frac{q}{f+q} - E_\pi(\frac{u^{'}(c_s^m)}{u^{'}(c^y)}y_s^m)}\right). \tag{21}$$

In deriving (21), we have substituted for dq_1, dq as a function of dy_0^y, dy^y from (17). The right hand side of (21) is positive since we require $dy^y < 0$ and also the term in big parenthesis is positive at a stationary equilibrium that is characterised by no short sales and satisfies (20). The fiscal authority can increase investment in land by young individuals at $t = 0$ to satisfy (21) and make the sum of perturbed utilities positive. Lastly, if (21) is satisfied, then $dq_1 > 0$.

The generation $t = 1$ is better-off if and only if

$$\frac{dU_1}{u^{'}(c^y)} = -dq\Big(- y^y(\frac{q}{f+q} - \frac{dq_1}{dq}) + (1-\frac{q}{f+q})E_\pi(\frac{u^{'}(c^m_s)}{u^{'}(c^y)}y^m_s)\Big) > 0.$$

Since we require $dq < 0$, generation $t = 1$ is better-off if and only if

$$(1-\frac{q}{f+q})E_\pi(\frac{u^{'}(c^m_s)}{u^{'}(c^y)}y^m_s) > y^y(\frac{q}{f+q} - \frac{dq_1}{dq}) \tag{22}$$

To demonstrate that (22) is satisfied, it is convenient to combine (19), the sum of perturbed utilities at $t = 0$, and (22) as

$$\underbrace{(1-\frac{q}{f+q})E_\pi(\frac{u^{'}(c^m_s)}{u^{'}(c^y)}y^m_s)}_{=A} > \underbrace{y^y(\frac{q}{f+q} - \frac{dq_1}{dq})}_{=B} >$$

$$\underbrace{y^y\Big(\frac{q}{f+q} + \frac{\frac{q}{d+q}E_\pi(\frac{u^{'}(c^m_s)}{u^{'}(c^y)}y^m_s)}{\frac{q}{f+q} - E_\pi(\frac{u^{'}(c^m_s)}{u^{'}(c^y)}y^m_s)}\Big)}_{=\Gamma}.$$

$A > B$ is identical to (22), while $B > \Gamma$ is equivalent to (19). $A > \Gamma$ at the stationary competitive equilibrium of the ξ^* economy. In particular, $A-\Gamma = 0.154927$. Since it is an inequality, it is satisfied at the corresponding stationary equilibrium of an economy in a neighbourhood of ξ^*. The fiscal authority has the discretion to make $B > \Gamma$, as argued before, but also can dictate investment in land such that B can be arbitrarily close to Γ: choose dy^y_0 such that the distance between the left and the right hand side of (21) can be made arbitrarily small. Since $A > \Gamma$, there exist perturbation such that $A > B$. As a result, generation $t = 1$ can be made better off.

To complete the argument we demonstrate that everyone in $t = 0$ can be made better off by appropriate redistribution. We compute transfers such that

$$dc^o_0 = \frac{\epsilon}{3} > 0,\ \frac{dU_0}{u^{'}(c^y)} = \frac{\epsilon}{3} > 0,\ \frac{dU^m_{0,s}}{u^{'}(c^m_s)} = \frac{\epsilon}{3} > 0,$$

and ϵ is defined as

$$\epsilon = dc^o_0 + E_\pi(\tfrac{dU^m_{0,s}}{u^{'}(c^m_s)}) + \tfrac{dU_0}{u^{'}(c^y)} =$$

$$dq_1\Big(\tfrac{q}{f+q} - E_\pi(\tfrac{u^{'}(c^m_s)}{u^{'}(c^y)}y^m_s)\Big) + dq\tfrac{q}{f+q}E_\pi(\tfrac{u^{'}(c^m_s)}{u^{'}(c^y)}y^m_s) > 0.$$

Initial middle-aged individuals are better off, $dU^m_{0,s}/u^{'}(c^m_s) = \epsilon/3 > 0$, if

$$d\tau^m_{0,s} = -y^y dq_0 + y^m_s dq_0 - \frac{q}{f+q} y^m_s dq_1 + \frac{\epsilon}{3}. \tag{23}$$

Substitute for dq_0 as a function of $E_\pi(d\tau^m_{0,s})$ from (17), take into account the asset market clearing identity, multiply each side of (23) with π_s and take the sum of (23) across middle-aged groups in order to solve for $E_\pi(d\tau^m_{0,s})$ as

$$E_\pi(d\tau^m_{0,s}) = \frac{(1-2y^y)\frac{2(e^m+fy^y)}{(2(1-y^y)-y^y)^2} dy^y_0 - \frac{q}{f+q}(1-y^y)dq_1 + \frac{\epsilon}{3}}{\frac{1-y^y}{2(1-y^y)-y^y}}. \tag{24}$$

Substituting (24) into (23) pins down transfers to each group of initial middle-aged.

Initial old are better off, $dc^o_0 = \epsilon/3 > 0$, if

$$d\tau^o_0 = -dq_0(1-y^y) + \frac{\epsilon}{3}$$

Lastly, initial young are better off, $dU_0/u^{'}(c^y) = \epsilon/3 > 0$, if

$$d\tau^y_0 = -y^y\Big(\frac{q}{f+q}dq_1 - dq_0\Big) + \Big(dq_1 - \frac{q}{f+q}dq\Big)E_\pi\Big[\frac{u^{'}(c^m_s)}{u^{'}(c^y)}y^m_s\Big] + \frac{\epsilon}{3}.$$

Transfers add up to zero

$$d\tau^o_0 + d\tau^y_0 + E_\pi(d\tau^m_{0,s}) = \epsilon - \epsilon = 0.$$

Debt

Fix $u(c) = \log(c)$, two personal states, $s \in \{H, L\}$, $e^o_{1,s} = e^o_1 + \epsilon_s > 0$, $E_\pi(\epsilon_s) = 0$, $\epsilon_s \neq 0$ and $f > 0, e^o_2 > 0$.

Combining the first order conditions of consumption and the budget constraints of old individuals we pin down their consumption demands as

$$c^o_{1,s} = \frac{e^o_{1,s} + pe^o_2 + (f+q)y + (1+i)b}{2},$$

$$c^o_{2,s} = \frac{e^o_{1,s} + pe^o_2 + (f+q)y + (1+i)b}{2p}.$$

Combining commodity 2 and land market clearing, $E_\pi(c^o_{2,s}) = e^o_2$ and $y = 1$ respectively, we get

$$p = \frac{1}{e^o_2}\Big(e^o_1 + f + q + (1+i)b\Big). \tag{25}$$

The first order condition on debt modifies as

$$\frac{1}{1+i} = E_\pi\left(\frac{1}{\frac{e^o_{1,s}+pe^o_2+(f+q)y+(1+i)b}{2}}\right). \tag{26}$$

Substituting (25), the no arbitrage relation between debt and land, $i = f/q$, and land market clearing in (26), we get

$$\frac{1}{1+i} = E_\pi\left(\frac{1}{\frac{1}{2}\Big(e^o_{1,s} + e^o_1 + 2f(1+\frac{1}{i}) + 2(1+i)b\Big)}\right). \tag{27}$$

From (27) we compute the equilibrium interest rate as a function of b. To facilitate computations, fix parameters as

$$f = 0.001,\ e^o_1 = 0.3,\ \epsilon_H = 0.25,\ \epsilon_L = -0.25,\ \pi_H = \pi_L = 0.5,\ e^o_2 > 0;$$

e^o_2 cancels out and does not affect the equilibrium interest rate and allocation. Well-defined stationary equilibria, $0 < i < \infty$, that can be improved upon by perturbations of public debt obtain if and only if $b \in (0, 0.65]$; equilibria characterised by $b \in (0, 0.65]$ satisfy all inequalities that were mentioned in the section debt. Following a similar argument as in the section land of the Appendix, there exists stationary equilibria for parameter values in a neighbourhood of the previous parametrization. Since the requirements for a Pareto improvement take the form of inequalities, Pareto improving debt policies exist for parameters in a neighbourhood of the previous parametrization.

Marginal changes of relative prices, interest rate and land price follow from (25), (27) and the no arbitrage relation between land and debt respectively. Marginal changes of utilities are derived using the same methodology as in the section land of the Appendix; we do not repeat the calculations. Lastly, individuals at date 0 can be made better off by appropriate redistribution. The methodology is again similar to the previous section and will not be repeated.

Capital

We demonstrate that there exist economies where the respective equilibrium allocations are constrained suboptimal following a policy of higher investment in capital.

Substituting the budget constraints, land and labor market clearing, and the first order conditions of the firm in the first order conditions of land and capital, we get

$$
\begin{aligned}
\tfrac{q}{f+q} &= E_\pi\Big(u'\Big(\varrho_s \alpha k^\alpha (2\bar{l})^{1-\alpha} + (1-\alpha)k^\alpha(2\bar{l})^{-\alpha}\bar{l} + f + q\Big)\Big), \\
\tfrac{1}{\alpha k^{\alpha-1}(2\bar{l})^{(1-\alpha)}} &= E_\pi\Big(\varrho_s u'\Big(\varrho_s \alpha k^\alpha (2\bar{l})^{1-\alpha} + (1-\alpha)k^\alpha(2\bar{l})^{-\alpha}\bar{l} + f + q\Big)\Big).
\end{aligned}
\tag{28}
$$

From (28), we solve for the equilibrium level of investment, k, and land price, q. Fix an economy: $u(c) = (1/\gamma)c^\gamma$, three equiprobable states, $\{H, M, L\}$, $\varrho_H = 2.05$, $\varrho_M = 0.9$, $\varrho_L = 0.05$, $\alpha = 0.2$, $\bar{l} = 3.6$, $\gamma = -4$, $f = 0.04$. The equilibrium level of investment equals $k = 0.03471$, the price of land $q = 0.01625$ and the risk premium 0.218829. The equilibrium satisfies all required inequalities for a Pareto improvement that were mentioned in the section capital. Lastly, since the requirements for a Pareto improvement take the form of inequalities, Pareto improving policies exist for economies in a neighbourhood of the previous economy.

Marginal changes of utilities are derived using a similar argument as in the section land of the Appendix and will not be repeated. The same applies for redistribution policies at date 0.