Rational Dialogs

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This paper also appears as *CRETA Discussion Paper No: 19*

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February 22, 2016

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Abstract

Eventual consensus is the only property of a rational dialog.

A *bayesian dialog* [1] is a sequential exchange of beliefs; it is is the prototype of a *rational dialog.* In the simple case of two individuals, at each stage, one of the interlocutors states his beliefs formed after the revision prompted by the beliefs stated by the other at the previous stage. The dialog terminates when nothing is left to be said.

Following Plato [2], the dialog has been a literary genre of choice for the transmission of knowledge; in practice, structured dialogs have been employed extensively to pool the information of experts [3], and more recently, and ambitiously, in the search for consensus, even the resolution of conflict and peacemaking. [4]

Aumann (1976) defined *common knowledge* and proved that consensus is a necessary condition for common knowledge. Geanakoplos and Polemarchakis (1982) proved that bayesian dialogs terminate in consensus and common knowledge [5].

I show that a third party, with access only to the transcript of a dialog, cannot distinguish a bayesian dialog form an arbitrary sequence of alternating utterances: the only property of a rational dialog is eventual consensus. The argument extends to the special case of a didactic dialog, in which an expert is better informed than his interlocutor. The expert never changes his opinion, but the interlocutor follows an arbitrary path to agreement [6].

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[1]Dialogs first appear in the Sumerian literary tablets, composed before the second millennium b.c., much before the earliest Greek and Hebrew literary works, according to Kramer (1963), who adds: *"the disputations and dialogues, eleven in number, ... are the forerunners and prototypes of similarly literary compositions current all over the ancient world as far as India on the east and probably Greece on the west, ... and they provided the literary and stylistic framework for even such profound philosophic works as Plato's dialogues."* Hösle (2012) refers to the discussion between Uddâlaka Âruni and his son Śvetaketu in the seventeenth chapter of the *Chandoygya-Upanishad* and to the *Book of Job.*

[2]Hösle (2012) states: *"[T]he philosophical dialogue was not invented by the Greeks. ... [But,] despite the many dialogues in other cultures, we can maintain that the Greeks succeeded in elevating the genre to new level. ... Important metaphysical speculations were also made in India – but not the comprehensive attempt to ground in reason, and solely in reason, the norms that guide our conduct."*

[3]The Delphi method introduced in the beginning of the cold war, has been documented and assessed in Dalkey (1969).

[4]On *Dialogue*, *Wikipedia* cites Laouris (2015) who reviews the employment of structured dialogs in civil society dialog in Cyprus as well as the formulation of government and social policy.

[5]Bacharach (1979) looked at bayesian dialogs when information is normally distributed.

[6]The dialogs of Plato are, indeed, didactic. Socrates, whose personality dominates and with whom the author, Plato, identifies, knows the truth, to which he guides his interlocutor; indeed, in the course of the dialogue, the knowledgeable expert, Socrates, may lead the interlocutor, temporarily, to error. Unlike Plato, Hume (1779) does not take

If, loosely speaking, one considers common knowledge and agreement as an equilibrium and the dialog that leads to common knowledge as the adjustment path, then it follows that, across fundamentals, equilibrium is a refutable claim, while the rationality of the adjustment path is not. Which bears an analogy with general competitive analysis: as follows from Debreu (1974), the Walrasian tâtonnement that leads to equilibrium, if it does, is arbitrary; nevertheless, equilibrium prices and quantities are not arbitrary, in Brown and Matzkin (1996), and, furthermore, in Chiappori, Ekeland, Kubler, and Polemarchakis (2004), they identify the fundamentals.

Turing (1950) and, in a simpler form, Newman, Turing, Jefferson, and Braithwaite (1952) posed the question whether automatic calculating machines can be said to think: *The idea of the test is that the machine has to pretend to be a man, by answering questions put to it, and it will only pass if the pretense is reasonably convincing . . . . We had better suppose that each jury has to judge quite a number of times, and that sometimes they really are dealing with a man and not a machine. That will prevent them saying 'It must be a machine' every time without proper consideration.*

Quine (1960) put forward the thesis of the indeterminacy of translation: "*manuals for translating one language into another, can be set up in different ways, all compatible with the totality of speech dispositions, yet incompatible with one another.*" According to Kripke (1982), Quine bases his argument from the outset on behavioristic premises and would never emphasize introspective thought experiments, dialogs with one's self, in the way Wittgenstein does. Indeed, in Wittgenstein (1953), the paradox is this: "*no course of action could be determined by a rule, because every course of action can be made out to accord with the rule.*

The argument here is relevant.

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sides in the dialog between the sceptic, Philo, and Cleanthes who defends the argument by design; at least not explicitly.

1   The analytical argument

States of the world are

$$\omega \in \Omega = \begin{pmatrix} Q_{1,1} & \cdots & Q_{1,n} & \cdots & Q_{1,N} \\ & & & & \\ \vdots & & \vdots & & \vdots \\ & & & & \\ Q_{m,1} & \cdots & Q_{m,n} & \cdots & Q_{m,N} \\ & & & & \\ \vdots & & \vdots & & \vdots \\ & & & & \\ Q_{1,M} & \cdots & Q_{M,n} & \cdots & Q_{M,N} \end{pmatrix},$$

a finite set, partitioned into a rectangular array of information cells.

Individuals receive information according to their information partitions. The information sets of individual 1 are the rows of the array of information cells,

$$R_m = (Q_{m,1}, \ldots, Q_{m,n}, \ldots Q_{m,N}),$$

while the information sets of individual 2 are the columns,

$$C_n = (Q_{1,n}, \ldots, Q_{m,n}, \ldots Q_{N,m})^T.$$

At a state of the world, $\omega \in Q_{m,n}$,

$$R(\omega) = R_m, \quad \text{and} \quad C(\omega) = C_n.$$

Events are sets of states of the world.

At ω, individuals 1 *knows* an event, E, if

$$R(\omega) \subset E,$$

and, similarly, for individual 2.

Here,

$$R(\omega) \cap C(\omega') \neq \emptyset, \quad \omega, \omega' \in \Omega,$$

and, as a consequence, the meet (the finest common coarsening) of the information partitions of individuals is $\{\Omega, \emptyset\}$: the only event that is *common knowledge* is Ω.

According to the prior beliefs common to individuals, information cells occur with uniform probability,

$$pr(Q_{m,n}) = \frac{1}{MN};$$

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this circumvents conditioning on 0-probability and, otherwise, it is for simplicity.

The prior conditional probability that the event A occurs is

$$p_{m,n} = pr(A|Q_{m,n}),$$

and the array of prior conditional probabilities is

$$P^0 = \begin{pmatrix} p_{1,1} & \cdots & p_{1,n} & \cdots & p_{1,N} \\ \vdots & & \vdots & & \vdots \\ p_{m,1} & \cdots & p_{m,n} & \cdots & p_{m,N} \\ \vdots & & \vdots & & \vdots \\ p_{1,M} & \cdots & p_{M,n} & \cdots & p_{M,N} \end{pmatrix}.$$

At a state of the world,

$$\omega^* \in Q_{m^*,n^*},$$

individuals 1 and 2 are informed of R_{m^*} and C_{n^*}, and they form posterior beliefs $pr(A|R_{m^*})$ and $pr(A|C_{n^*})$, respectively.

The argument in Aumann (1976), that posteriors that are common knowledge coincide, is, here, evident. Since the only event that is common knowledge is Ω, the posterior beliefs are common knowledge, only if

$$pr(A|R(\omega)) = pr(A|R_{m^*}), \quad \omega \in \Omega,$$

and, as a consequence

$$pr(A|R_{m^*}) = pr(A| \cup_\omega R(\omega)) = pr(A).$$

By a similar argument,

$$pr(A|C_{n^*}) = pr(A| \cup_\omega C(\omega)) = pr(A),$$

and

$$pr(A|R_{m^*}) = pr(A|C_{n^*}).$$

Importantly, for an arbitrary event, A, posteriors need not be common knowledge, and a dialog can commence [7].

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[7] The posteriors, $pr(A|R_{m^*}) = pr(A|C_{n^*})$, may well coincide but not be common knowledge.

The dialog between individual 1 and individual 2 concerns the probability that $\omega^* \in A$: the event A has occurred.

A *dialog* is a finite sequence of utterances,

$$(q^1, q^2, \ldots, q^t, \ldots, q^T), \quad 0 \leq q^t \leq 1,$$

at t odd by individual 1, and at t even by individual 2.

A *bayesian dialog* is defined inductively.

The individual who speaks, at t, bases his beliefs on information available to him at the end of the preceding round of communication, and q^t is the probability that the individual who speaks at t attaches to the occurrence of the event A.

After a permutation of rows and columns [8], $\omega^* = \omega_{1,1}$.

At $t = 1$, individual 1 announces his posterior beliefs,

$$q^1 = \frac{1}{N} \sum_{n=1}^{N} p_{1,n}\,[9].$$

After a further permutation of rows,

$$\frac{1}{N} \sum_{n=1}^{N} p_{m,n} = \frac{1}{N} \sum_{n=1}^{N} p_{1,n} \quad \Leftrightarrow \quad m = 1, \ldots, M^1 \leq M,$$

and the announcement of q^1 by individual 1 reveals that

$$\omega^* \in \cup_{m=1}^{M^1} \cup_{n=1}^{N} Q_{m,n}.$$

After individual 1 has spoken at $t = 1$, is the revised array of conditional probabilities of the event A

$$P^1 = \begin{pmatrix} p_{1,1}^* & \cdots & p_{1,N} \\ \vdots & \vdots & \vdots \\ p_{M^1,1} & \cdots & p_{M^1,N} \end{pmatrix}.$$

Note that $pr(A|R_m^1) = q^1$, for $m = 1, \ldots, M^1$.

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[8] Successive permutations of rows and columns allow for simpler exposition; column R_1 and the row C_1 are not involved in permutations after the initial permutation that places (m^*, n^*) at $(1, 1)$.

[9] $q^1 = pr(A| \cup_{n=1}^{N} Q_{1,n}) = \sum_{n=1}^{N} pr(A|Q_{1,n}) pr(Q_{1,n}| \cup_{n'=1}^{N} Q_{1,n'}) = (1/N) \sum_{n=1}^{N} p_{1,n}.$

At $t = 2$, individual 2 announces his posterior beliefs

$$q^2 = \frac{1}{M^1} \sum_{m=1}^{M^1} p_{m,1}{}^{10}.$$

After a permutation of columns,

$$\frac{1}{M^1} \sum_{m=1}^{M^1} p_{m,n} = \frac{1}{M^1} \sum_{m=1}^{M^1} p_{m,1} \quad \Leftrightarrow \quad n = 1, \ldots, N^2 \leq N,$$

and the announcement of q^2 by individual 2 reveals that

$$\omega^* \in \cup_{m=1}^{M^1} \cup_{n=1}^{N^2} Q_{m,n}.$$

After individual 2 has spoken at $t = 2$, the revised array of prior conditional probabilities of the event A is

$$P^2 = \begin{pmatrix} p_{1,1}^* & \cdots & p_{1,N^2} \\ \vdots & \vdots & \vdots \\ p_{M^1,1} & \cdots & p_{M^1,N^2} \end{pmatrix}.$$

Here, $pr(A|C_n^2) = q^2$, for $n = 1, \ldots, N^2$.

At stage $t = 3, 5, \ldots, (2\tau + 1), \ldots$, individual 1 announces his posterior probability on the event A,

$$q^t = \frac{1}{N^{t-1}} \sum_{n=1}^{N^{t-1}} p_{1,n}{}^{11}.$$

After a permutation of rows,

$$\frac{1}{N^{t-1}} \sum_{n=1}^{N^{t-1}} p_{m,n} = \frac{1}{N^{t-1}} \sum_{n=1}^{N^{t-1}} p_{1,n} \quad \Leftrightarrow \quad m = 1, \ldots, M^t \leq M^{t-1},$$

and the announcement of q^t by individual 1 reveals that

$$\omega^* \in \cup_{m=1}^{M^t} \cup_{n=1}^{N^{t-1}} Q_{m,n}.$$

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[10]$q^2 = pr(A| \cup_{m=1}^{M^1} Q_{m,1}) = (1/M^1) \sum_{m=1}^{M^1} p_{m,1}$.
[11]$q^t = pr(A| \cup_{n=1}^{N^{t-1}} Q_{1,n}) = (1/N^{t-1}) \sum_{n=1}^{N^{t-1}} p_{1,n}$, for $t = 3, 5, \ldots, (2\tau + 1), \ldots$.

The revised array of prior conditional probabilities of the event A is

$$P^t = \begin{pmatrix} p_{1,1}^* & \cdots & p_{1,N^{t-1}} \\ \vdots & \vdots & \vdots \\ p_{M^t,1} & \cdots & p_{M^t,N^{t-1}} \end{pmatrix}, \quad t = 3, 5, \ldots, (2\tau + 1), \ldots.$$

Note that $pr(A|R_m^t) = q^t, \quad m = 1, \ldots, M^t.$

At stage $t = 4, 6, \ldots, 2\tau, \ldots$, individual 2 announces his posterior probability on the event A,

$$q^t = \frac{1}{M^{t-1}} \sum_{m=1}^{M^{t-1}} p_{m,1}{}^{12}.$$

The announcement of q^t by individual 2 reveals that

$$\omega^* \in \cup_{m=1}^{M^{t-1}} \cup_{n=1}^{N^t} Q_{m,n}.$$

The revised array of prior conditional probabilities of the event A is

$$P^t = \begin{pmatrix} p_{1,1}^* & \cdots & p_{1,N^t} \\ \vdots & \vdots & \vdots \\ p_{M^{t-1},1} & \cdots & p_{M^{t-1},N^t} \end{pmatrix}, \quad t = 4, 6, \ldots, 2\tau, \ldots.$$

Here, $pr(A|C_n^t) = q^t, \quad n = 1, \ldots, N^t.$

The argument in Geanakoplos and Polemarchakis (1982), that the exchange and revision of posteriors terminates in consensus is, here, evident. Suppose that at some stage, t, (odd), player 1 does not reveal any information by the announcement of q^t; that is, $M^t = M^{t-1}$, and $P^t = P^{t-1}$. Since $pr(A|R_m^t) = pr(A|R_m^{t-1}) = q^t, \ m = 1, \ldots M^t = M^{t-1}$, while $pr(A|C_n^t) = pr(A|C_n^{t-1}) = q^{t-1}, \ n = 1, \ldots N^t = N^{t-1}$, the posterior beliefs of the two individuals are common knowledge, and they coincide. Evidently, since the dimensions of the array of prior conditional beliefs, P^0, are finite, convergences in finitely many rounds of communication is necessary; and the number of rounds is bounded by M, the cardinality of the collection of information sets of individual 1.

If, in the course of a dialog, $q^t = 0$ or 1, the belief becomes immediately common knowledge and there is consensus. We restrict attention to dialogs with

$$0 < q_t < 1.$$

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[12] $q^t = pr(A| \cup_{m=1}^{M^{t-1}} (Q_{m,1} \cap Q_1)) = (1/M^{t-1}) \sum_n p_{m,1}$, for $t = 4, 6, \ldots, 2\tau, \ldots.$

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Proposition. *Any dialog, $(q^1, q^2, \ldots, q^t, q^{t+1}, \ldots, q^{T-1}, q^T)$, that terminates in consensus: $q^{T-1} = q^T$ is a bayesian dialog.*

Proof: We argue by induction.

Consider an arbitrary dialog of length $T = 3$ that terminates in consensus: (q^1, q^2, q^3), with $q^2 = q^3$.

For

$$P^2 = P^3 = (p_{1,1}),$$

with $p_{1,1} = q^2 = q^3$, the dialogue terminates in consensus and common knowledge [13].

Let

$$P^1 = (p_{1,1}, p_{1,2}, p_{1,3}, \ldots, p_{1,k}, \ldots, p_{1,2+K}),$$

with $2 + K = N^3 = N$ and

$$p_{1,1} = q^2 = q^3,$$

$$p_{1,2} = \varepsilon, \quad 0 < \varepsilon < 1, \quad \text{and}$$

$$p_{1,k} = \tfrac{1}{K}[(K+2)q^1 - q^2 - \varepsilon], \quad k = 3, \ldots, 2 + K.$$

For K large, $0 < p_{1,k} < 1$, while a local variation in ε guarantees that $p_{1,k} \neq p_{1,1}, \ k = 2, \ldots, 2 + K$.

At t = 1, individual 1, informed of R_1, announces

$$q^1 = \frac{1}{N} \sum_{n=1}^{N} p_{1,k} = \frac{1}{N}[q^2 + \varepsilon + (Nq^1 - q^2 - \varepsilon)].$$

AT $t = 2$, individual 2, informed of C_1^1, announces $q^2 = p_{1,1}$. Since, by construction, $p_{1,k} \neq p_{1,1}, \ k = 2, \ldots 2 + K$, the state of information that results is, indeed, $P^2 = (p_{1,1})$; as a consequence, $P^2 = P^3$, and the dialog terminates with consensus: $q^3 = q^2$ that is common knowledge.

The array of prior conditional probabilities $P^0 = P^1$ generates the dialog (q^1, q^2, q^3).

For a dialog of length T that terminates in consensus $(q^1, q^2, \ldots, q^t, \ldots, q^{T-1}, q^T)$, with $q^{T-1} = q^T$, suppose that, for some t that is even, and after

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[13]Here, after the dialog terminates, the information available to individuals coincides with their pooled information; this is not a necessary for a rational dialog.

individual 2 has spoken, the array of conditional probabilities

$$P^t = \begin{pmatrix} p_{1,1} & \cdots & p_{1,N^t} \\ \vdots & \vdots & \vdots \\ p_{M^{t-1},1} & \cdots & p_{M^{t-1},N^t} \end{pmatrix}$$

generates the dialog $(q^t, q^{t+1}, \ldots, q^{T-1}, q^T)$. In particular,

$$q^t = \frac{1}{M^{t-1}} \sum_{m=1}^{M^{t-1}} p_{m,1} = \frac{1}{M^{t-1}} \sum_{m=1}^{M^{t-1}} p_{m,n}, \quad n = 1, \ldots, N^t.$$

Construct the augmented array of conditional probabilities

$$P^{t-1} =$$

$$\begin{pmatrix} p_{1,1} & \cdots & p_{1,N^t} & p_{1,N^t+1} & \cdots & p_{1,N^t+k} & \cdots & p_{1,N^{t-2}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{M^{t-1},1} & \cdots & p_{M^{t-1},N^t} & p_{M^{t-1},N^t+1} & \cdots & p_{M^{t-1},N^t+k} & \cdots & p_{M^{t-1},N^{t-2}} \end{pmatrix}$$

with

$$N^{t-2} = N^t + 1 + M^{t-1}K, \quad K > 0,$$

$$p_{m,N^t+1} = \varepsilon_m, \quad \tfrac{1}{M^{t-1}} \sum_{m=1}^{M^{t-1}} \varepsilon_m \neq q^t, \quad 0 < \varepsilon_m < 1, \quad \text{and}$$

$$p_{m,N^t+k} = \begin{cases} \frac{1}{K}[N^{t-2}q^{t-1} - \varepsilon_m - \sum_{n=1}^{N^t} p_{n,k}], \\ \\ \qquad\qquad N^t + 1 + (m-1)K \leq k \leq N^t + 1 + mK, \\ \\ 0, \qquad\qquad \text{otherwise,} \end{cases} ,$$

for $m = 1, \ldots M^{t-1}$.

For K large, $0 < p_{m,N^t+k} < 1$, while a local variation in ε_m guarantees that $p_{m,N^t+k} \neq M^{t-1}q^t$, $k = 2, \ldots, 2 + K$, $m = 1, \ldots M^{t-1}$.

At $t-1$, individual 1, informed of $R_1^{t-2} = R_1^{t-2}$ announces

$$q^{t-1} = \frac{1}{N^{t-2}} \sum_{n=1}^{N^{t-2}} p_{1,n} = \frac{1}{N}[\sum_{n=1}^{N^t} p_{m,n} + \varepsilon_m + (N^{t-2}q^{t-1} - \varepsilon_m - \sum_{n=1}^{N^t} p_{m,n})].$$

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AT t, individual 2, informed of C_1^{t-1}, announces q^t. Since, by construction, $p_{m,N^t+k} \neq q^t$, $k = 2, \ldots, 2+K$, $m = 1, \ldots M^{t-1}$., the state of information that results is, indeed, P^t. The array of conditional probabilities P^{t-1} generates the dialog $(q^{t-1}, q^t, \ldots, q^{T-1}, q^T)$. Successive repetition of this argument generates an array of conditional probabilities, $P^0 = P^1$, that generates the dialog $(q^1, q^2, \ldots, q^t, \ldots q^{T-1}, q^T)$. $\qquad\square$

The constructive argument above can generate curious dialogs.

Example. We give here the first few steps for the construction of a state space and conditional probabilities that generate a dialog of arbitrary length of the form

$$(\frac{1}{4}, \ldots, \frac{1}{4}, \ldots, \frac{1}{4}, \frac{3}{4}, \frac{3}{4}):$$

$$\left(\begin{array}{c} \frac{3}{4}^* \end{array} \right) \qquad \rightarrow \qquad \left(\begin{array}{cc} \frac{3}{4}, & \frac{3}{4} \end{array} \right),$$

$$\left(\begin{array}{cccc} \frac{3}{4}^* & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{array} \right) \quad \rightarrow \quad \left(\begin{array}{ccc} \frac{1}{4}, & \frac{3}{4}, & \frac{3}{4} \end{array} \right),$$

$$\begin{pmatrix} \frac{3}{4}^* & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & 0 & 0 \\ \frac{1}{8} & \frac{11}{24} & \frac{11}{24} & \frac{11}{24} \\ \frac{1}{8} & \frac{11}{24} & \frac{11}{24} & \frac{11}{24} \end{pmatrix} \quad \rightarrow \quad \left(\begin{array}{cccc} \frac{1}{4}, & \frac{1}{4}, & \frac{3}{4}, & \frac{3}{4} \end{array} \right)$$

Experts At a state of the world, $\omega^* = 1$, an individual, 1, is an expert concerning the event A if

$$pr(A|R_1) = pr(A|Q_{1,n}), \quad n = 1, \ldots N:$$

no information in the join (coarsest refinement of the partitions of the individuals) would cause him to alter his beliefs.

A *dialog with an expert* is a dialog

$$(\bar{q}, q^2, \ldots, \bar{q}, q^{2t}, \ldots, \bar{q}), \quad 0 \leq \bar{q}, q^T \leq 1,$$

at t odd, $q^{2t+1} = \bar{q}$, by individual 1, and at t even, q^{2t}, by individual 2.

Corollary. *Any dialog, with an expert, $(\bar{q}, q^2, \ldots, \bar{q}, q^{2t}, \ldots, \bar{q}, \bar{q})$, that terminates in consensus is a bayesian dialog with an expert.*

Proof. It suffices to set $p_{i,n} = \bar{q}$ for all n, and apply the construction in the proposition starting with $m \geq 2$.

$\square$

Order matters Consider the array of conditional probabilities

$$\begin{pmatrix} 1^* & 0 & 0 \\ & & \\ 0 & 1 & a \end{pmatrix}, \quad a \neq 0, 1.$$

Informed of $R_1 = R(\omega^*)$, individual 1 announces $q^1 = 1/3$.

With $a \neq 0$, this reveals his information set to individual 2, who now knows that $\omega \in Q_{1,1}$ and announces $q^2 = 1$.

In turn, this prompt individual 1 to revise his posterior to $q^3 = 1$.

Alternatively, individual 2 speaks first. Informed of $C_1 = C(\omega^*)$, he announces $q^1 = 1/2$.

With $a \neq 1$, this allows individual 1 to deduce that $C(\omega^*) \in \{C_1, C_2\}$, that is, $C(\omega^*) \neq C_3$, and, as a consequence, announce $q^2 = 1/2$.

The announcement of individual 2 revels no information; the beliefs of individuals are common knowledge, and they coincide.

But, this common posterior belief that assigns probability $1/2$ to the event, reflects information less precise than the information the individuals would have access to at the end of a dialog initiated by individual 1.

Silence The formulation here a dialog is an alternating sequence of utterances; formally, an interlocutor cannot remain silent when it is his turn to speak. One can interpret silence by an interlocutor at t as the repetition of his utterance at $t-2$: that is, $q^t = q^{t-2}$. It is important, nevertheless, that different rounds of responses be distinct, which may not be the case if both interlocutors remain silent.

Public information may obfuscate Consider the arrays of prior and conditional probabilities

$$\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1^* & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Informed of $R_1 = R(\omega^*)$, individual 1 announces $q^1 = 1/2$.

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This reveals his information set to individual 2, who now knows that $\omega \in Q_{1,1}$ and announces $q^2 = 1$.

In turn, this prompt individual 1 to revise his posterior to $q^3 = 1$.

The dialog ends in consensus and common knowledge.

Importantly, the common knowledge information available to individuals coincides with the information pooling would have led them to since $R_1 \cap C_1 = Q_{1,1}$.

Suppose now that, following the realisation of ω^* a public authority, policy maker announces that

$$\omega^* \notin \{Q_{1,3} \cup Q_{2,3}\}.$$

Note that this is something both individuals know; of course, it is not common knowledge, and, as a consequence, it affects the exchange of information between the individuals.

With the truncated array of conditional probabilities

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

the posterior beliefs of the individuals are common knowledge and there is no information to exchange.

The point, as in Dutta and Polemarchakis (2012) is that this common posterior belief that assigns probability 1/2 reflects information that is less precise than the information the individuals would have access to by exchanging beliefs, through a dialog, without the prior public announcement.

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