Money Aggregates and Determinacy:
A Reinterpretation of Monetary Policy During the Great Inflation

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March 2018

Warwick Economics Research Papers
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First version: August, 2015
This version: March, 2018

Abstract

Should a policy rule include money? Including money exerts policy inertia and increases inflation aversion. In a New-Keynesian model with trend inflation, these features guarantee price determinacy even when the Taylor principle is not satisfied. Novel Greenbook data confirm money aggregates as U.S. Federal Open Market Committee policy objectives, enabling monetary policy to insulate the U.S. economy from self-fulfilling fluctuations despite positive trend inflation. A high response to inflation and low trend inflation guarantees determinacy post-1982. Cross-country applications highlight the superiority of the rule with money. Raising the inflation target from 2 percent to 4 percent violates the Taylor principle; including money resolves this issue.

Keywords: Determinacy, Great Inflation, Inflation Target, Money Aggregates, Time-Varying Policy

JEL classification: E41, E42, E51, E52, E58, E61, E65

*I am extremely grateful to Thijs van Rens and Marija Vukotić for their excellent guidance. I thank Roberto Pancrazi, Michael McMahon, Olivier Coibion, Krisztina Molnar, Marcus Miller, Leonardo Melosi, Ludger Linnemann, Olivier Loisel, Guido Ascari, Jonathan Benchimol, Peter Sinclair and seminar participants at the University of Warwick, RGS Conference, T2M Conference, GdRE International Symposium, Banque de France, National Bank of Slovakia, and Lahore University of Management Sciences (LUMS) for their useful comments.

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1. **Introduction**

“In our estimates that enforce changes in policy rule, the strongest evidence for monetary policy change is that for shifting emphasis on monetary aggregates in the policy reaction function. This accords with the prominent role monetarism played in policy discussions of the 70’s. If further research succeeds in finding clear evidence of changes in monetary policy behavior in this period, it will most likely be through focusing attention on the changing impact of monetarism on policy behavior.”

(Sims and Zha (2006))

The dramatic rise in the volatility of output and inflation experienced by the U.S. economy during the 1960s and 1970s, followed by a substantial reduction in the 1980s, has been a source of significant debate. Particular attention has been given to examining the role of monetary policy in generating the Great Moderation in macroeconomic activity. Clarida, Gali and Gertler (1998a) suggest that the inability of the U.S. Federal Reserve ("the Fed") to raise nominal interest rates more than one-for-one with inflation, that is, to satisfy the Taylor (1999) principle, induced self-fulfilling expectations-driven fluctuations. This caused price-level indeterminacy, leading to macroeconomic instability. They argue that the leadership of Paul Volcker, who served as Chairman of the Fed from 1979 to 1987, was characterized by a strong response to inflation and a switch from a "passive" to an "active" policy rule, ensuring price equilibrium determinacy and macroeconomic stability.

Yet evidence presented by Orphanides (2002) using the Federal Open Market Committee’s (FOMC) meeting-level data did not detect large changes in the Fed’s response to inflation when comparing the period before and after Volcker’s appointment. Time-varying estimates highlight periods of passive policy even during the 1970s (Boivin (2005)). Coibion and Gorodnichenko (2011) argue that trend inflation may have led to equilibrium indeterminacy. In light of this conflicting evidence, analysis based on money may not only resolve this important debate, as suggested by Sims and Zha (2006), but also provide guidance for the future conduct of monetary policy.\(^1\)

Including money, however, changes both the analytical and the empirical frameworks. The analytical results are highlighted by deriving theoretical conditions that pin down price-level

\(^1\) Although money aggregates were never a formal policy goal, during the FOMC meeting of January 1970, it was decided that ‘[a]n increased stress should be placed on the objective of achieving modest growth in the monetary aggregates’. The policy directive from the second FOMC meeting instructed the manager of the System Open Market Account (SOMA) to ‘seek first and foremost a pattern of growth in a subset of monetary aggregates’, and to maintain ‘money-market conditions consistent with this objective’. In effect, the FOMC acted as if controlling money aggregates were an objective of monetary policy (see, for example, Kane (1974), Meulendyke (1988), Larkin et al. (1988), Friedman (1996), and Mishkin (2007)).
equilibrium for a central bank that has an explicit money growth objective using a New Keynesian model with positive trend inflation. On the empirical side, novel meeting-level Greenbook data are used to identify the role of money in the policy reaction function. The theoretical framework and the policy coefficients are combined to test whether monetary policy contributed to price determinacy in the United States, following the approach of Lubik and Schorfheide (2004), Coibion and Gorodnichenko (2011), and Lubik and Matthes (2014). Finally, policy recommendations are generated based on these results, which may be applicable to contemporary policy issues.

The theoretical conditions suggest that a strong response to money growth guarantees determinacy even when the Taylor principle is not satisfied, and even under positive trend inflation as high as 8 percent. Targeting only money enables the central bank to exert sufficient inertia in interest rate decisions, mitigating the effects of high trend inflation and aiding in ruling out self-fulfilling expectations that cause equilibrium indeterminacy. This extends the findings of Ascari and Sbordone (2014) who find an increase in trend inflation associated with instability and destabilizing inflation expectations under a simple Taylor rule. Including a moderate response to money growth in this rule mitigates a large portion of the indeterminacy region.

Furthermore, I offer a new perspective on the impact of money growth objectives and the money demand relationship on price-level determinacy. When the policy reaction function contains money, price elasticity in the money demand curve contributes to the likelihood of determinacy. Instability in this relationship triggers indeterminacy even with an active policy rule. While responding to money growth enhances the determinacy region, money demand instabilities work to worsen the possibility of determinacy.

To empirically test for the relevance of money in Federal Reserve policy, I compile a novel series on money growth (M1, M2 and M3) from March 1969 through to January 2002 using the briefing forecasts prepared for the Federal Open Market Committee (FOMC) included in the Greenbook. My benchmark results suggest that M1 significantly influenced the policy rate during the pre-Volcker period, entering the reaction function positively, and becomes insignificant during the 1980s. Including money also improves the fit of the benchmark Taylor rule. By contrast, the policy rule without money appears to overestimate the coefficient on inflation. Therefore, the exclusion of money aggregate in the policy rule from previous studies may have produced biased estimates of the reaction function.

Time-varying estimates generally support the fixed-coefficient results: M1 is statistically

\footnote{Qureshi (2016) presents evidence from the transcripts which supports the view that the FOMC switched its focus from a “narrower money aggregate” (M1) to a broader money aggregate” (M3) from early 1983.}
significant from the early 1970s to the late 1970s and from 1982 through to 1984. Surprisingly, money aggregate M2 is only significant from 1980 to 1981, but enters in the policy rule with a negative sign and remains statistically insignificant otherwise. Instead, M3 is found to be significant from 1970 to 1979, from 1981 to 1984, and (briefly) during the 1990s. The coefficient on inflation varies over time, but stays above one, while the coefficient on the output gap declines post-1979, a finding that aligns well with the previous literature (Orphanides (2002), Boivin (2005), and Coibion and Gorodnichenko (2011)).

The policy coefficients based on the real-time data are combined with the analytical framework to test the likelihood of price indeterminacy in the model. My benchmark results suggest a high likelihood of determinacy during the entire time period. Responding positively to money enabled monetary policy to insulate the U.S. economy from self-fulfilling fluctuations, despite positive trend inflation as high as 8 percent during the pre-Volcker era. Money growth objectives assist in preventing self-fulfilling fluctuations, enhancing the coefficient on inflation, injecting interest rate inertia and history dependence. Thus, while my findings support the conclusions of Orphanides (2002), and Sims and Zha (2006) in that the policymakers in the 1970s satisfied the "Taylor principle," the correct policy rule to assess monetary policy during this period includes a central role for money. An even higher probability of determinacy is observed when money aggregate M3 is considered. Furthermore, the rise in response to inflation combined with low trend inflation guarantees determinacy post-1982, complementing the findings of Coibion and Gorodnichenko (2011).

The framework gives rise to several policy-relevant questions. One critical question concerns the conduct of monetary policy under positive trend inflation. Indeed, in response to the Great Recession, proposals to raise the inflation target have been put forward (Blanchard et al. (2010), Krugman (2013), Ball (2014), and Arias et al. (2017)). An important challenge is that raising the inflation target from two to four percent generates price indeterminacy under the classic Taylor rule. I show that including a moderate response to money growth resolves this issue. Another application of this result to more than 200 countries confirms the gains from including a money growth objective to supplement the simple Taylor rule. While conducting policy based on a simple rule may suffice for countries where trend inflation is low, the money growth framework is recommended for countries with even moderately high levels of trend inflation. The results largely confirm that central banks in countries can anchor inflation expectations and avoid self-fulfilling economic fluctuations by including a moderate response to money growth in their policy rule.

To my knowledge, this paper is the first to connect the relationship between money growth objectives and price-level determinacy under positive trend inflation, providing an alternative
framework to analyze monetary policy during the 1970s. The theoretical, empirical and policy results presented in this paper contribute to a broad literature.

First, the novel price-determinacy conditions under money growth targeting extend the theoretical literature – which has focused mainly on the response towards inflation as a prerequisite of active policy (Friedman (2000), Woodford (2001), and Carlstrom and Fuerst (2003)) – and build on the results by Chowdhury and Schabert (2008) and Keating and Smith (2013) by including a role for trend inflation. While it is well known that money demand stability plays an important role when a central bank targets money, this paper is the first to propose its influential role in generating price indeterminacy. This finding may appeal to the literature focusing on price determinacy, and to the recent literature examining monetary policy and the quantity theory (Sargent and Surico (2011), and Teles et al. (2016)).

Second, my empirical results propose a central role for money in FOMC policy deliberations using novel real-time data on money aggregates. This extends a broad literature: on formalizing the role of money as an objective of FOMC policy (DeRosa and Stern (1977), Burns (1979), Sims and Zha (2006), Benati and Mumtaz (2007), Friedman (1996), Chowdhury and Schabert (2008), and Castelnuovo (2012)); on presenting an alternative framework that better describes the monetary reaction function during the Great Inflation (Clarida, Gali and Gertler (1998b), Orphanides (2002), Boivin (2005), Chowdhury and Schabert (2008), Coibion and Gorodnichenko (2011), Hirose et al. (2017)); on the role of money in describing U.S. dynamics (Ireland (2004), Favara and Giordani (2009), Canova and Menz (2011), Benchimol and Fourçans (2017), Qureshi (2016)); on showing that the FOMC targeted M1 and M3 instead of M2, suggesting that one cannot address determinacy issues by ignoring money aggregates, and that the type of money aggregate plays a central role in resolving the conflicting conclusions found in the earlier literature (Castelnuovo (2012)); and on complementing the continued important of money during the post-Great Moderation time period (Belongia and Ireland (2016)).

An additional important empirical finding is that money growth objectives better explain the funds rate as compared to output growth objectives. Policy with money growth better represents FOMC policy, and it is robust to a variety of statistical tests. This contrasts with the findings of Coibion and Gorodnichenko (2011), who attribute part of the macroeconomic stability observed during the Great Moderation to a positive weight on output growth objectives. The similar stabilizing properties of money growth objectives also connect well with the "speed limit" literature (Walsh (2003a)).

My central finding, which suggests that the U.S. economy satisfied price determinacy conditions even under the high positive trend inflation experienced the 1970s, supports the
implications put forward by Orphanides (2002) and Sims and Zha (2006), providing additional support for the well-known view that monetary policymakers satisfied the Taylor principle even before Volcker became Chairman of the Fed. Thus the results confirm the conclusions put forth Orphanides (2002), Sims and Zha (2006), and Chowdhury and Schabert (2008), and contrasting with the findings by Clarida, Gali and Gertler (1998a), Taylor (1999), Coibion and Gorodnichenko (2011), Boivin (2005), Lubik and Matthes (2014) and Hirose et al. (2017). According to my results, money growth objectives play a central role in determining the type of monetary policy in place during the Great Inflation.

Conditional on the estimated policy coefficients, raising the inflation target to 4 percent would be unlikely to lead the U.S. economy to experience indeterminacy. The cross-country results directly follow this result, connecting with the literature emphasizing the usability of money in policy: Christiano and Ljungqvist (1988), Stock and Watson (1989), Krol and Ohanian (1990), Thoma (1994), Ramsey and Lampart (1998), Dotsey, Lantz and Santucci (2000), King and Plosser (1984), Nelson (2002), Dotsey and Hornstein (2003), Coenen, Levin and Wieland (2005), Beck and Wieland (2006), Aksoy and Piskorski (2006), and Hafer, Haslag and Jones (2007). Recently, Blanchard and Summers (2017) have called for a revision of monetary targets for the design of stabilization policies. Going forward, the analysis and the applications offered in this paper suggest the inclusion of money aggregates in this reflective process.

This paper is organized as follows: Section 2 outlines the model. Section 3 presents the determinacy conditions implied by a monetary policy rule with money growth objectives. Section 4 presents empirical evidence of money growth objectives, and combines these results with the analytical framework to study the implications on price determinacy. Section 5 applies this model to study the issues related to raising the inflation target, and presents a cross-country application of this framework. Section 6 concludes and offers suggestions for future research.

2. The Model

To derive the conditions that pin down price-level determinacy under a policy rule that has a money growth objective, I utilize a prototypical New Keynesian DSGE model developed by Walsh (2003b), Galí (2009), and Woodford (2011). Based on Coibion and Gorodnichenko (2011), Ascari and Sbordone (2014), Ascari (2004), Ascari and Ropele (2009), and Ascari, Castelnuovo and Rossi (2011) the model is generalized to allow for positive trend inflation.
2.1. **The Demand Side**

The economy consists of a continuum of households, in which the representative household seeks to maximize the objective function:

\[
E_t \sum_{t=0}^{\infty} \beta^t U \left( C_t, \frac{M_t}{P_t}, N_t \right)
\]  

(1)

The instantaneous utility function, \( U \left( C_t, \frac{M_t}{P_t}, N_t \right) \), is increasing in the consumption of a final good \( (C_t) \) and real money balances \( (M_t/P_t) \), and decreasing in labor \( (N_t) \) according to:

\[
U \left( C_t, \frac{M_t}{P_t}, N_t \right) = \frac{C_t^{1-\sigma} - 1}{1 - \sigma} + \chi_m \frac{(M_t/\eta \pi P_t)^{1-\sigma_m} - 1}{1 - \sigma_m} - \chi_n \frac{N_t^{1+\varphi}}{1 + \varphi}
\]

(2)

where the positive parameters \( \sigma_c \) and \( \sigma_m \) represent the inverse of the intertemporal elasticity of substitution in consumption and real money balances, respectively, while \( \chi_m \) and \( \chi_n \) are positive constants. Based on the implications of the quantity theory of money \( \eta \pi \) is generally normalized to one. However, money holdings may not respond proportionally to prices, at least in the short run. It is on the basis of this argument that Stock and Watson (1993) test if \( \eta \pi \) is indeed equal to one. Maximization of utility curve (2) is subject to a sequence of flow budget constraints given by:

\[
P_t C_t + B_t + M_t \leq P_tw_tN_t + (1 + i_{t-1})B_{t-1} + M_{t-1} + T_t
\]

(3)

for \( t = 0,1,2,... \). \( P_t \) is the price of the consumption good. \( w_t \) denotes the real wage, and \( B_t \) represents the quantity of one period, nominally riskless discount bonds purchased in period \( t \) and maturing in period \( t + 1 \). Each bond pays one unit of money at maturity. \( T_t \) represents lump-sum additions or subtractions to period income. Maximizing (2) subject to the flow budget constraint (3), the necessary first-order conditions at any \( t \) for money demand, consumption Euler, and labor supply, respectively, can be written as:

\[
\chi_m \frac{M_t}{\eta \pi P_t}^{-\sigma_m} C_t^{-\sigma_c} = \frac{i_t}{1 + i_t}
\]

(4)

\[
C_t^{-\sigma_c} = \beta E_t \left\{ C_{t+1}^{-\sigma_c} (1 + i_{t+1}) \left( \frac{P_t}{P_{t+1}} \right) \right\}
\]

(5)

\[
\chi_n C_t^{\sigma_c} N_t^{\varphi} = w_t
\]

(6)
2.2. The Supply Side

In each period \( t \), a final good, \( Y_t \), is produced by perfectly competitive firms that aggregate intermediate goods \( Y_{i,t} \), for a continuum of monopolistically competitive firms, \( i \). Since there is no investment, and government consumption, aggregate consumption must equal the production of final goods:

\[
C_t = Y_t = \left[ \int_0^1 Y(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}}
\]

(7)

where \( \varepsilon \) indicates the elasticity of substitution among intermediate goods. The optimal demand for intermediate inputs is equal to \( Y_{i,t} = (P_{i,t}/P_t)^{-\varepsilon}Y_t \). Profit maximization and the zero profit condition imply that the price index associated with the final good \( Y_t \) is a CES of the intermediate inputs \( P_{i,t} \):

\[
P_t = \left[ \int_0^1 P_{i,t}^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}}
\]

(8)

The production function of intermediate goods producers is:

\[
Y_{i,t} = A_t N_{i,t}^{1-\alpha}
\]

(9)

where \( A_t \) is an exogenous process for the level of technology, which is assumed to be stationary. The labor demand for firm \( i \) is given as follows:

\[
N_{i,t} = \left[ \frac{Y_{i,t}}{A_t} \right]^{\frac{1}{1-\alpha}}
\]

(10)

Marginal costs of firm \( i \) depend upon the quantity produced by the firm, given decreasing returns to scale. Since firms charging different prices would produce different levels of output, they have heterogeneous marginal costs given by the following expression:

\[
MC_{i,t}^e = A_t^{\frac{1}{1-\alpha}} W_t \left[ \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} Y_t \right]^{\frac{\alpha}{1-\alpha}}
\]

(11)

Intermediate firms re-set prices every period a la Calvo (1983): in each period a firm can re-optimize its nominal price, denoted by \( P_{i,t}^* \), with fixed probability \( 1 - \theta \), while with probability \( \theta \) it can index its price to the previous period inflation rate: \( P_{t,t} = \pi_t \varrho P_{t,t-1} \). Based on Christiano, Eichenbaum and Evans (2005), the parameter \( \varrho \in [0,1] \) indicates the
degree of price indexation, and $\pi = P_t / P_{t-1}$ is the price level. In line with Benati (2009), the firm indexes inflation to past inflation only. The problem of the firm $i$, which sets its price at time $t$, is to choose $P^*_i,t$ to maximize expected profits:

$$E_t \sum_{j=0}^{\infty} \theta^j D_{t,t+j} \left[ \frac{P^*_i,t \Pi_{t-1,t+j-1}}{P_{t+j}} Y_{i,t+j} - \frac{W_{t+j}}{P_{t+j}} \left( \frac{Y_{i,t}}{A_t} \right)^{\frac{1}{1-\alpha}} \right]$$ (12)

The firm maximizes (12) subject to the following demand constraint:

$$Y_{i,t+j} = \left( \frac{P^*_i,t \Pi_{t-1,t+j-1}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j}$$ (13)

where $D_{t,t+j} = \beta^j \frac{\lambda_{t+j}}{\lambda_0}$ is the stochastic discount factor, with $\lambda_{t+j}$ denoting the $t+j$ marginal utility of consumption and $\Pi_{t-1,t+j-1}$ indicate cumulative inflation between periods $t$ and $t+j$:

$$\Pi_{t,t+j} = 1 \quad \text{for } j = 0$$

$$\Pi_{t,t+j} = \left( \frac{P_{t+1}}{P_t} \right) \times \left( \frac{P_{t+2}}{P_{t+1}} \right) \times \ldots \times \left( \frac{P_{t+j}}{P_{t+j-1}} \right) \quad \text{for } j = 1, 2, \ldots$$

The first-order condition of this problem can be written as:

$$\left( \frac{P^*_i,t \Pi_{t-1,t+j-1}}{P_{t+j}} \right)^{1+\frac{\varepsilon}{1-\alpha}} = \varepsilon \frac{E_t \sum_{j=0}^{\infty} \theta^j D_{t,t+j} \frac{W_{t+j}}{P_{t+j}} \left( \frac{Y_{i,t}}{A_t} \right)^{\frac{1}{1-\alpha}} \left[ \frac{\Pi_{t-1,t+j-1}}{\Pi_{t,t+j}} \right]^{\frac{\varepsilon}{1-\alpha}} Y_{i,t+j}}{E_t \sum_{j=0}^{\infty} \theta^j D_{t,t+j} \left[ \frac{\Pi_{t-1,t+j-1}}{\Pi_{t,t+j}} \right]^{1-\varepsilon} Y_{i,t+j}}$$ (14)

Note that future expected inflation rates enter both the numerator and the denominator, and thus affect the relative weight on future variables. With positive trend inflation two effects come into play. When intermediate firms are free to adjust, they will set higher prices to try to offset the erosion of relative prices and profits that trend inflation automatically creates. Second, expectation of forward-looking terms are progressively multiplied by larger discount factors. This means that optimal price-setting under trend inflation reflects future economic conditions more than short-run cyclical variations. Price-setting firms become more forward looking. Price indexation mitigates these two effects.

Setting $P_t = \left[ \int_0^1 P_{i,t}^{1-\varepsilon} \, d\varepsilon \right]^{\frac{1}{1-\varepsilon}}$, and $\left( \frac{P^*_i,t}{P_{i,t}} \right) = p^*_i,t$, enables me to write the pricing equation as:

$$1 = \theta \pi_{t-1}^{(1-\varepsilon)} \pi_{t-1}^{(\varepsilon-1)} + (1 - \theta) \left( p^*_i,t \right)^{(1-\varepsilon)}$$ (15)
Notice that equation (14) can be written in a recursive formulation of the optimal price-setting equation:

\[(p^*_t)^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \frac{\varepsilon}{(\varepsilon - 1)(1 - \alpha)} \psi_t \phi_t \tag{16}\]

where, using the definition of the stochastic discount factor \(\lambda_{t+j} = Y_{t+j}^{-\sigma}\), the auxiliary variables can be defined, and written recursively, as:

\[\psi_t = w_t Y_t^{1-\sigma} A_t^{-\frac{\varepsilon}{1-\alpha}} + \theta \beta \pi^{\frac{\varepsilon}{1-\alpha}} E_t \left[ \frac{1}{\pi_{t+1}} \psi_{t+1} \right] \tag{17}\]

\[\phi_t = Y_t^{1-\sigma} + \theta \beta \pi^{\phi_{t-1}-\varepsilon} E_t \left[ \frac{1}{\pi_{t+1}} \phi_{t+1} \right] \tag{18}\]

Using equation (10), and the expression for the intermediate inputs \(Y_{i,t}\) aggregate labor demand is given by:

\[N^d_t = \int_0^1 N^d_{i,t} d\tilde{i} = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{\frac{-\varepsilon}{1-\alpha}} d\tilde{i} \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} = s_t \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \tag{19}\]

Schmitt-Grohé and Uribe (2007) show that the variable \(s_t\) is bounded below at one, and represents the resource costs (or inefficiency loss) due to the relative price dispersion under the Calvo mechanism: the higher is \(s_t\), the more labor is needed to produce a given level of output. Ascari, Castelnuovo and Rossi (2011) show price dispersion to evolve as:

\[s_t = (1 - \theta)(p^*_t)^{\frac{-\varepsilon}{1-\alpha}} + \theta \left( \frac{\pi_t}{\pi_{t-1}} \right)^{\frac{-\varepsilon}{1-\alpha}} s_{t-1} \tag{20}\]

Notice that the variable \(s_t\) directly affects the real wage via the labor supply equation outlined in (6). Finally, the natural rate of output is defined as:

\[y^*_t = \left( \frac{1 - \alpha}{\mu \chi_n} \right) \frac{1}{\varphi + \alpha + \sigma_c (1 - \alpha)} \tag{21}\]

2.3. Log-linearized Model

Equations (4) - (6) are linearized and imposing the market clearing condition \(Y_t = C_t\) yields the money demand curve, the dynamic IS, and the labor supply curve.

\[m_t = \eta_m p_t + \eta_y y_t - \eta_r i_t \tag{22}\]

\(^3\)The output gap is defined as: \(x_t = y_t - y^*_t\).
where $\eta_y = \frac{\sigma_c}{\sigma_m}$ and $\eta_i = \frac{\beta}{\sigma_m(\bar{\pi} - \beta)}$. Similar to Mehra (1991) and Söderström (2005), I take first differences to obtain an expression for the growth rate of the nominal money stock:

$$\Delta m_t = \eta_{\pi}\pi_t + \eta_y\Delta y_t - \eta_i\Delta i_t + \Delta\tau_t$$

(23)

$\Delta m_t$ is the log change in the money stock and $\Delta y_t$ is growth in actual output. $\tau_t$ captures exogenous money demand shocks and $\eta_j$ for $j \in (\pi, i, y)$ represents the (semi-)elasticity of nominal money growth of each of these variables.

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \bar{\pi}_{t+1}) + g_t$$

(24)

Equation (24) is the log-linearized Euler condition, which captures the negative relationship between output and the real interest rate. This is determined by the difference in the nominal interest rate, $i_t$, and expected inflation, $\bar{\pi}_{t+1}$. Due to the inter-temporal substitution effect, higher real returns induce greater savings, depressing aggregate demand. Expectations of positive output expand current output. Aggregate output is subject to a shock $g_t$ that can be interpreted as a shock to government spending, or to the households’ preferences.

The labor equation is obtained directly:

$$\sigma_c y_t + \varphi n_t = w_t$$

(25)

Turning to the log-linearization of (14), and setting $\alpha = \varrho = \varphi = 0$, the term $\Delta_t$ collapses to $\pi_t$, and leads to the following system of difference equations characterizing the generalized NKPC under trend inflation: \footnote{This is only for tractability. The complete model used for analytical purposes allows nonzero values for these parameters.}

$$\pi_t = \lambda(\bar{\pi})y_t + b_1(\bar{\pi})E_t\bar{\pi}_{t+1} + b_2(\bar{\pi})E_t\psi_{t+1} + \kappa(\bar{\pi})a_t$$

(26)

The coefficients are defined as: $\lambda(\bar{\pi}) = \sigma_c\kappa(\bar{\pi}) + b_2(\bar{\pi})$, $\kappa(\bar{\pi}) = \frac{(1 - \theta\bar{\pi}^{-1})(1 - \theta\beta\bar{\pi}^{-1})}{\theta\bar{\pi}^{-1} - 1}$, $b_1(\bar{\pi}) = \beta(1 + \varepsilon(\bar{\pi} - 1)(1 - \theta\bar{\pi}^{-1})$, $b_2(\bar{\pi}) = [\beta(1 - \theta\bar{\pi}^{-1})][1 - \bar{\pi}]$. Notice that the auxiliary variable $\psi_t$ can be written as:

$$\psi_t = [1 - \theta\beta\bar{\pi}^{-1}][w_t - a_t + (1 - \sigma_c)y_t] + \theta\beta\pi^{-\varepsilon}[E_t\psi_{t+1} + \varepsilon E_t\bar{\pi}_{t+1}]$$

(27)
Price dispersion evolves according to:

\[
\begin{align*}
    s_t &= \left[ \frac{\varepsilon \theta \tilde{\pi}^{\varepsilon-1}}{1 - \theta \tilde{\pi}^{\varepsilon-1}} (\bar{\pi} - 1) \right] \pi_t + \theta \tilde{\pi}^{\varepsilon} s_{t-1} \\
\end{align*}
\]

(28)

As suggested by Ascari (2004), Ascari and Ropele (2009) and Ascari, Castelnuovo and Rossi (2011), a positive steady-state inflation rate, \( \bar{\pi} \), affects all coefficients in the Generalized New-Keynesian Phillips Curve (GNKPC), described in equations (26) and (27). It is also influenced by \( s_t \), the process for price dispersion, equation (27). The forward-looking auxiliary process \( \psi_t \) also participates in the determinants of inflation. Since trend inflation leads to a smaller coefficient on current output and a larger coefficient on future expected inflation, the NKPC under positive trend inflation becomes more “forward-looking.” The contemporaneous relationship between inflation and output progressively weakens, and the inflation rate becomes less sensitive to variations in output and more forward looking. Price indexation counterbalances some of the effects of trend inflation. The GNKPC under trend inflation nests the textbook version of the NKPC, which can be derived by setting \( \bar{\pi} = 1 \):

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa y_t
\]

(29)

### 2.4. The Monetary Authority

To examine policy, I consider a rule that considers money growth as an additional policy objective beyond the baseline Taylor rule. The inclusion of a money growth objective draws from a broad literature.

First, empirical evidence based on historical data points towards the role of money in policy, based, in particular, on the money aggregate M1, during the 1970s, and a diminished role during the early 1980s (see, for example, the evidence presented in Burns (1979) and Sims and Zha (2006)). Empirical evidence of the change in objectives has been presented in DeRosa and Stern (1977), wherein M1 is detected to have influenced the setting of interest rates in the early 1970s, compared to its less significant role during the late 1960s. Moreover, legally, the Humphrey-Hawkins Act required the Fed to set one-year target ranges for money supply growth twice a year, and to report the targets to Congress beginning in 1978. A broad literature has pointed to the usefulness of monetary aggregates in monetary policy, emphasizing the use of money as an information variable.\(^5\)

Narrative evidence from the records of the Memorandum of Discussion/Transcripts and the Record of Policy Actions include multiple statements from FOMC members discussing the role of money aggregates in the policy making since the late 1960s and early 1970s. Consider the following excerpts from the Memorandum of Discussion which point to the role of money in policy:

“Mr. Morris ... would urge the Manager to focus on the money stock ... and efforts to resist such growth would be likely to result in undershooting the money supply target”

(Memorandum of Discussion, April 4, 1970, p. 65)

“The Chairman then said it was because he had become seriously concerned about the present stance of monetary policy that he had called a meeting of the Committee for today, one week in advance of the originally scheduled date of January 18. As the members would recall, it had been suggested at the December meeting that it might be necessary for the Committee to assemble before January 18 if the performance of the monetary aggregates did not improve sufficiently. Despite energetic efforts on the part of the Desk, the rate of growth of the money supply – he was thinking chiefly of M1 – that the Committee had set as a major objective was not being attained. ”

(Memorandum of Discussion, January 11, 1972, p. 4-5)

During this period, there is monumental increase in the discussion surrounding money aggregates, including monitoring their behavior and performance, and an increased focus on their projections and performance. Whereas the main money aggregate was M1, its influence over policy diminished in the early 1980s due to its less potent and less reliable role in policy. It was then that the emphasis shifted to broader aggregates within the same money aggregate framework. Beyond the 1970s, the FOMC’s emphasis on money aggregates can be gauged from several quotes. For example, the following excerpt is from the record of policy actions:

“In implementing policy, the Committee agreed that primary emphasis would continue to be placed on the broader aggregates...”

(Record of Policy Actions, July 12-13, 1983, p. 5)

The evidence connects well with the literature that has pointed to the influence of "monetarism" of the 1970s in the FOMC, albeit with it a very specific interpretation: the monetarist

influence on policy could be interpreted as introducing money aggregate as an additional policy objective. Therefore, it may be reasonable to conclude that during the period in question, the operating target was the federal funds rate, and nominal money aggregates possibly played a role in influencing the setting of this policy rate. At the very least, this evidence motivates testing for the inclusion of money growth in the FOMC’s reaction function.

Consequently, a meeting-specific feedback rule which captures the forward-looking behavior of the Fed can be written as follows (with $j$ denoting forecast horizons):

$$i_t = \rho_{1,t}i_{t-1} + \rho_{2,t}i_{t-2} + (1 - \rho_{1,t} - \rho_{2,t})[\psi_{\pi,t}E_t\pi_{t+j} + \psi_{x,t}E_tx_{t+j} + \psi_{m,t}E_t\Delta m_{t+j}] + c_t + \epsilon_t \quad (30)$$

for $j \in (0, 1, ..)$; $\epsilon_t$ is an error term, and the time-varying constant term includes the targets for inflation, output gap and money growth, and the real-interest rate, given by:

$$c_t = (1 - \rho_{1,t} - \rho_{2,t})[(1 - \psi_{\pi})\bar{\pi}_t + r_t - \psi_{m,t}\Delta \bar{m}_t - \psi_{x,t}\bar{x}_t) \quad (31)$$

It is also important to mention that setting $\psi_m = 0$ makes money redundant in this model. Ireland (2004), Benchimol and Fourçans (2012) and Castelnuovo (2012) present a detailed treatment of the various channels through which money may influence macroeconomic variables in a DSGE model.

### 2.5. Bayesian Estimation

The model is first solved using standard Blanchard-Kahn techniques, and then estimated using Bayesian estimation techniques. Because the estimation procedure closely follows the literature, the details are relegated to the appendix. Table 1 reports the prior and posterior densities for the estimated parameters, the mean, and the 10th and 90th percentiles of the posterior distributions.

The estimate of price indexation is conditional on the time period: full-sample estimates suggest a low value, but split-sample estimates point to considerable variation, with a relatively high value in the first sample. Calvo pricing, too, while varying slightly across model estimation and sample, remains close to the benchmark prior value of 0.5, corresponding to an average of a seven-months frequency in price re-setting. This estimate is consistent with that found in Bils and Klenow (2004) and close to the value used by Coibion and Gorodnichenko (2011) in their benchmark analysis. The estimated values of the other structural shocks are also close to what have been proposed. It is comforting to note that they display the usual properties observed in the empirical literature, such as a decline in variance and a
Table 1: Estimated Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Prior (mean, std)</th>
<th>Posterior Mean</th>
<th>[5th pct; 95 pct]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varrho )</td>
<td>P. Indexation Beta</td>
<td>(0.20, 0.15)</td>
<td>0.1449</td>
<td>0.6562; 0.2511</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Calvo Beta</td>
<td>(0.5, 0.1)</td>
<td>0.4947</td>
<td>0.5216; 0.4862</td>
</tr>
<tr>
<td>( \eta_{\pi} )</td>
<td>Inflation Elast. Normal</td>
<td>(1, 0.10)</td>
<td>0.998</td>
<td>0.7113; 0.651</td>
</tr>
<tr>
<td>( \psi_{\pi} )</td>
<td>Resp. Inflation Gamma</td>
<td>(2.00, 0.5)</td>
<td>2.4786</td>
<td>2.421; 4.0846</td>
</tr>
<tr>
<td>( \psi_{y} )</td>
<td>Resp. Output Gamma</td>
<td>(0.125, 0.05)</td>
<td>0.0439</td>
<td>0.0867; 0.0579</td>
</tr>
<tr>
<td>( \psi_{\Delta m} )</td>
<td>Resp. Money Gamma</td>
<td>(0.05, 0.2)</td>
<td>0.001</td>
<td>0.581; 0.0523</td>
</tr>
<tr>
<td>( \rho = \rho_1 + \rho_2 )</td>
<td>Int. Smoothing Beta</td>
<td>(0.9, 0.1)</td>
<td>0.9978</td>
<td>0.8883; 0.9625</td>
</tr>
<tr>
<td>( \rho_a )</td>
<td>Tech. Pers. Beta</td>
<td>(0.5, 0.5)</td>
<td>0.563</td>
<td>0.8784; 0.9581</td>
</tr>
<tr>
<td>( \rho_v )</td>
<td>Policy Pers. Beta</td>
<td>(0.5, 0.5)</td>
<td>0.5226</td>
<td>0.1389; 0.1425</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>IS Pers. Beta</td>
<td>(0.5, 0.5)</td>
<td>0.6968</td>
<td>0.8072; 0.8759</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>Money Pers. Beta</td>
<td>(0.5, 0.5)</td>
<td>0.6729</td>
<td>0.8849; 0.9865</td>
</tr>
<tr>
<td>( \sigma_a )</td>
<td>Tech. Std. Inv. Gamma</td>
<td>(0.005, 2)</td>
<td>0.0156</td>
<td>0.0164; 0.0079</td>
</tr>
<tr>
<td>( \sigma_v )</td>
<td>Policy Std. Inv. Gamma</td>
<td>(0.005, 2)</td>
<td>0.0026</td>
<td>0.0025; 0.0016</td>
</tr>
<tr>
<td>( \sigma_g )</td>
<td>IS Std. Inv. Gamma</td>
<td>(0.005, 2)</td>
<td>0.0076</td>
<td>0.0035; 0.0015</td>
</tr>
<tr>
<td>( \sigma_{\tau} )</td>
<td>Money Std. Inv. Gamma</td>
<td>(0.005, 2)</td>
<td>0.0363</td>
<td>0.0126; 0.0133</td>
</tr>
<tr>
<td>log(( ML ))</td>
<td></td>
<td></td>
<td>-1088.105839</td>
<td>-263.425303; -322.24099</td>
</tr>
</tbody>
</table>

Notes: I use 200,000 draws from the posterior to compute the model. Acceptance rates on average were between 30% to 33%. The log-marginal likelihoods are computed with the harmonic mean estimator presented in Geweke (1999).
rise in persistence over time. As in Smets and Wouters (2007), the estimated persistence of 
the structural shocks is very large in both models.

The estimated policy parameters suggest an aggressive, gradually implemented, long-run 
reaction of the Federal Reserve to fluctuations in inflation, which is close to existing literature 
(Benati and Surico (2009), Boivin and Giannoni (2006), Clarida, Gali and Gertler (1998b), 
and Lubik and Schorfheide (2004)). The response to output, while estimated to be quite low 
for the entire sample, is estimated to be twice as large in the first sample, as compared to 
the second. A high degree of interest rate inertia is observed. Perhaps the largest variation 
in the monetary policy reaction function is observed in the response to money growth and 
inflation. The estimated model confirms evidence of money growth targeting in the first half 
of the sample, which falls to zero in the second sample. The parameter \( \eta_\pi \), suggest a unitary 
value for the entire sample, but which fluctuates across time periods.

3. **PRICE-LEVEL DETERMINACY UNDER MONEY AGGREGATE TARGETING**

This section illustrates the price-level determinacy properties of the monetary policy rule 
when the central bank responds to inflation, output, and to growth in monetary aggregates. 
I study the baseline price-level determinacy condition for the general policy rule, and then 
focus on the different combinations of simple rules under various levels of trend inflation.\(^6\)

3.1. **A GENERAL DETERMINACY CONDITION**

Under the baseline rule in equation (30) - (31) and the log-linearized model, the conditions 
outlined in Woodford (2011) can be used to derive the necessary and sufficient condition 
that guarantees price-level determinacy:

**Proposition 1: Determinacy condition for rules with money growth objectives**

*For any \( \beta \in (0, 1) \), any \( \kappa > 0 \), and \( \bar{\pi} = 0 \), if the monetary authority follows the policy rule*

---

\(^6\)For the benchmark theoretical analysis, I calibrate a subset of the model’s parameters. The discount 
factor \( \beta \) is set to 0.9995, the elasticity of substitution among goods \( \varepsilon \) is set equal to six. Trend inflation 
is set to the average, which is roughly 4 percent for the 1966:I - 2008:II time-period. This is equal to 
1.03974 = 1.0099 to be consistent with how inflation is defined in the model. The value of \( \sigma_m \) is obtained 
by setting \( \eta_i = 4 \), which is the value calculated by Galí (2009), and by and by setting trend inflation equal 
to the average for the United States from 1966:I - 2008:II in equation (24). The degree of price indexation is 
set to zero. The average price duration is assumed to be roughly three quarters, which implies that \( \theta = 0.5 \). 
In the money-demand equation, the parameters \( \eta_\pi \), \( \eta_x \) are normalized to one in the baseline case while \( \eta_i \) is 
set to 4. Finally, the shock parameters are all one unit innovations.
\[ i_t = \rho_1 i_{t-1} + \rho_2 i_{t-2} + (1 - \rho_1 - \rho_2)(\psi_\pi E_t \pi_t + \psi_x E_t x_t + \psi_{\Delta m} E_t \Delta m_t), \]

the following conditions are sufficient for determinacy:

\[
(\beta \sigma)^{-1}(\delta_4 (1 - \beta) + \kappa (\delta_3 - 1) + \delta_5 (1 - \beta) + (\delta_1 + \delta_2) \kappa) > 0 \tag{32}
\]

\[
- (\beta \sigma)^{-1}((\delta_4 - \delta_5)(1 + \beta) + \kappa \delta_3 + (\delta_1 - \delta_2 + 1)(\kappa + 2 \sigma + 2 \beta \sigma)) < 0 \tag{33}
\]

where \( \delta_i \) for \( i \in \{1, 2, 3, 4, 5\} \) are convolutions of the policy parameters and structural parameters underlying the transmission mechanism:

\[
\delta_1 = \frac{\rho_1 + \eta i \psi_{\Delta m}(1 - \rho_1 - \rho_2)}{1 + \eta i \psi_{\Delta m}(1 - \rho_1 - \rho_2)}
\]

\[
\delta_3 = \frac{(\psi_\pi + \eta \pi \psi_{\Delta m})(1 - \rho_1 - \rho_2)}{1 + \eta i \psi_{\Delta m}(1 - \rho_1 - \rho_2)}
\]

\[
\delta_5 = \frac{-\eta x \psi_{\Delta m}(1 - \rho_1 - \rho_2)}{1 + \eta i \psi_{\Delta m}(1 - \rho_1 - \rho_2)}
\]

\[
\delta_2 = \frac{\rho_2}{1 + \eta i \psi_{\Delta m}(1 - \rho_1 - \rho_2)}
\]

\[
\delta_4 = \frac{(\eta x \psi_{\Delta m} + \psi_x)(1 - \rho_1 - \rho_2)}{1 + \eta i \psi_{\Delta m}(1 - \rho_1 - \rho_2)}
\]

**Proof:** See appendix A.3.2

The general determinacy conditions highlight that all policy coefficients feed into guaranteeing insulating the economy from self-fulfilling expectations that lead to equilibrium indeterminacy. As highlighted by the convoluted parameters, \( \delta_i \), introducing money affects the other coefficients on the policy rule, injecting interest rate inertia, an explosive response to inflation and history dependence extending the findings by Svensson (1999), Christiano and Rostagno (2001), Walsh (2003a), Söderström (2005), Keating and Smith (2013), and Qureshi (2016). To better understand the intuition behind the determinacy conditions (32) and (33), I focus on describing price level determinacy under various combinations of the monetary policy rule. For example, setting \( \psi_{\Delta m}, \rho_1 \) and \( \rho_2 \) to zero returns the popular Taylor specification, and the determinacy equations simplifies to the determinacy condition (27) in chapter 3 of Galí (2009) under zero trend inflation.

### 3.2. Responding to Output Gap and Inflation

To connect with the benchmark literature, assume first that the monetary authority only responds to inflation and the output gap. The familiar policy rule can be written as:

\[
i_t = \psi_\pi \pi_t + \psi_x x_t \tag{34}
\]

The necessary and sufficient conditions under this policy rule can be obtained by setting
ψΔm, ρ1 and ρ2 equal to zero in condition (32) - (32).

\[(1 - \beta)\psi_x + \kappa(\psi - 1) > 0\]  \hspace{1cm} (35)

Figure 1 plots the determinacy regions for different values of trend inflation. First, and as discussed in Bullard and Mitra (2002), Galí (2009), and Woodford (2011), this feedback rule satisfies the Taylor principle under zero trend inflation since it implies that in the event of a sustained increase in the inflation rate of \(k\) percent, the nominal interest rate will eventually be raised by more than \(k\) percent, sufficing to determine an equilibrium price level. The top-left plot summarizes this result.

It is immediately clear from the figure, that allowing for positive trend inflation changes the dynamics of the model drastically. As noted by Ascari and Ropele (2009) and Ascari, Castelnuovo and Rossi (2011), indeterminacy is due to the long-run multiplier associated with the response to output gap contains trend inflation, which increases for positive values of trend inflation, thus requiring an even larger response on inflation to achieve determinacy in this model. With the benchmark rule, the minimum response needed by the central bank starts to rise even for trend inflation as low as 1.2 percent. Coibion and Gorodnichenko (2011), Ascari and Ropele (2009), and Ascari, Castelnuovo and Rossi (2011) show that high levels of trend inflation, such as those experienced in the 1970s, required the central bank to raise interest rates by almost 10 times the increase in the inflation rate to sustain a determinate REE. The breakdown of the basic Taylor principle is amplified due to the growing importance of forward-looking behavior in price-setting when trend inflation rises.
3.3. Responding to Inflation and Money Growth

I now turn to describing the main theoretical result of this paper. To analyze price-level determinacy conditions when the monetary authority responds to inflation and money growth, the following policy rule is considered:

\[ i_t = \psi_{\pi} \pi_t + \psi_{\Delta m} \Delta m_t \]  

(36)
Setting \( \rho_1, \rho_2, \) and \( \psi_x \) equal to zero in condition (31) - (32) yield the following novel proposition:

**Proposition 2: Price Determinacy Under Money Growth Objectives** For any \( \beta \in (0, 1) \), and any \( \kappa > 0, \bar{\pi} = 0, \eta_i \geq 0 \) and \( \eta_x \geq 0 \), if the central bank follows the simple rule \( i_t = \psi_\pi \pi_t + \psi_m \Delta m_t \), the following condition is sufficient for determinacy:

\[
\eta_\pi \psi_m + \psi_\pi > 1
\]

Thus, under equation (36), the likelihood of determinacy is affected by the response to money growth and the stability in price elasticity irrespective of the Taylor principle being satisfied. Setting \( \psi_\Delta m = 0 \), for any parameterization of the money demand curve, or setting price elasticity to zero for any parameterization of \( \psi_\Delta m \), condition (37) collapses to the popular Taylor principle (under no trend inflation):

\[
\psi_\pi > 1
\]

Next, I focus on the general price-level determinacy condition, in equation (37). To consider the implications for the nominal rate under the policy rule specified in equation (36), assume a permanent increase in inflation of size \( d\pi \):

\[
di = \psi_\pi d\pi + \psi_m d\Delta m
\]

Simplifying the money demand relationship described in equation (24), the change in the nominal interest rates can be captured by:

\[
di = (\psi_\pi + \eta_\pi \psi_\Delta m) d\pi
\]

Condition (37) is equivalent to the term in brackets in equation (40) being greater than one, implying that the price-level equilibrium will be unique under the interest rate rule in equation (36) whenever \( \psi_\pi \) and \( \psi_\Delta m \) are sufficiently large (or are of the same sign) to guarantee that the real interest rate rises in the face of an increase in inflation of size \( d\pi \). When nominal money growth and inflation are characterized by a unitary relationship (i.e., when \( \eta_\pi = 1 \)), \( \psi_\Delta m > 1 \) also guarantees determinacy.

The determinacy regions under multiple parameterizations of price elasticity are shown in Figure 1. First, when \( \eta_\pi \in (0, 1) \) the determinacy region shrinks and only a larger response to money growth mitigates this channel, conditional on the same response towards inflation. Critically, when \( \eta_\pi \) is restricted to zero, then any response to money growth does
not guarantee determinacy and only a strong response to inflation, of magnitude greater than one (i.e. the Taylor principle), can guarantee price-level determinacy. Thus, price elasticity of money is crucial for determinacy when the central bank targets money.

Figure 2: Determinacy Regions: Varying Price Elasticity ($\eta_\pi$)

Notes: This figure presents determinacy regions based on the feedback rule where the central bank responds to inflation and money growth. The plots consider the effect on the determinacy region when the relationship between nominal money growth and inflation is allowed to vary. The (dark) shaded blue area represents the indeterminacy regions. The y-axis plots the response to inflation, while the x-axis captures the response to money growth.

Moving now to the general case of positive trend inflation, figure (3) summarizes the minimum response to inflation and money growth needed by the central bank to ensure determinacy for different levels of trend inflation rates. The minimum level of inflation response needed for determinacy falls as the response to money growth increases. Determinacy ap-
pears to be guaranteed for moderate levels of positive (4 percent) trend inflation rates when the monetary authority responds by more than one-for-one to both inflation and money growth.

Figure 3: Determinacy Regions: Varying Trend Inflation

Notes: This figure presents determinacy regions based on the feedback rule where the central bank responds to inflation and monetary growth. The plots consider the effect on the determinacy region when trend inflation is allowed to vary. The (dark) shaded blue area represents the indeterminacy regions. The y-axis plots the response to inflation, while the x-axis captures the response to money growth.

Determinacy is guaranteed in a policy rule with money, amplifying the magnitude of the response of real interest rates to expected inflation. In this sense, it supplements the Taylor principle \(\psi_\pi > 1\), which is not sufficient to establish determinacy with positive trend inflation. The convolution of parameters outlined in section 3.1 also suggest that targeting money increases the aggregate also induces history dependence, and guarantees an explosive
response to changes in the macroeconomy. This stabilizing feature of the policy rule with money result has important similarities with output growth objectives, since both features assist with determinacy under trend inflation.

### 3.4. Interest Rate Smoothing

Last, I analyze the case where the monetary authority responds to money growth and partially smooths interest rates. Clarida, Gali and Gertler (1998b) show that incorporating a partial-adjustment mechanism to the original Taylor (1993) rule helps improve the fit of the actual variation in the nominal interest rate observed in the U.S. economy and some large European economies.

\[ i_t = \rho_1 i_{t-1} + \psi \Delta m_{t} \]  \hspace{1cm} (41)

**Proposition 3: Money Growth Objectives and Interest Rate Smoothing** For any \( \beta \in (0,1) \), and any \( \kappa > 0, \bar{\pi} = 0 \), if the central bank follows the simple rule \( i_t = \rho_1 i_t + \psi \Delta m_{t} \), the following condition is sufficient for determinacy:

\[ \psi \Delta m + \rho_1 > 1 \]  \hspace{1cm} (42)

This result can be obtained by setting \( \rho_2, \psi_x \) and \( \psi_\pi \) equal to zero in equation (31) - (32). As before, consider the implications for the nominal rate under the monetary policy rule defined in equation (41) if there was a permanent increase in inflation of size \( d\pi \).

\[ di = \frac{\psi \Delta m}{1 - \rho_1} d\Delta m \]  \hspace{1cm} (43)

If condition (42) is satisfied, then the condition \( \frac{\psi \Delta m}{1 - \rho_1} \) is greater than one, and is enough to guarantee that the real interest rises in the face of an increase in inflation. This tends to counteract the increase in inflation of magnitude \( d\pi \), and acts as a stabilizing force.
Figure 4: Determinacy Regions: Interest Rate Smoothing

Note: This figure presents determinacy conditions based on the feedback rule where the central bank responds only to money growth, under partial interest rate smoothing and varying levels of trend inflation. The (dark) shaded blue area represents the indeterminacy region. The y-axis plots the response to money growth, while the x-axis captures the degree of interest rate inertia to past policy actions.

Figure (4) plots the determinacy regions for simple rule with money growth and interest rate inertia. A number of important results are highlighted. Even under high trend inflation, a strong response to money growth and appropriate interest rate smoothing seem to guarantee determinacy. For example, even with low interest rate smoothing and with $\psi_{\Delta m} > 1$, the Taylor principle is restored for inflation rates as high as 6 percent. This suggests that history dependence is particularly useful in improving the determinacy properties of interest rate rules under trend inflation, aligning with the observations made in Arias et al. (2017). With positive trend inflation, standard inflation targeting may not insulate the
economy from self-fulfilling fluctuations. Targeting money, therefore, may be desirable from a determinacy perspective.

4. **Money Aggregates in Federal Reserve Policy**

In this section, I use meeting-level data to examine Federal Open Market Committee (FOMC) policy. I begin with estimating the benchmark policy rule that includes money aggregates in the Fed’s reaction function. I then turn to split-sample and time-varying analysis of the policy rule. These policy coefficients are then combined with the analytical framework to test the likelihood of price indeterminacy in the model. For the analytical exercise, I utilize the estimated parameters based on the full-sample Bayesian estimation of the model.

4.1. **Benchmark Results**

To test the role of money in formulating policy, the baseline empirical specification for the Fed’s reaction function is a generalized Taylor rule with the following specification:

\[
    i_t = \rho_1 i_{t-1} + \rho_2 i_{t-2} + (1 - \rho_1 - \rho_2) \left[ \psi_\pi \pi_{t+j} + \psi_x x_{t+j} + \psi_m \Delta m_{t+j} \right] + c + \epsilon_t
\]

This framework allows for interest smoothing of order two, as well as a response to inflation, money growth, and the output gap. Indeed, if the coefficient $\psi_m$ tends to zero, it may be reasonable to conclude that the Fed did not target money (Friedman and Kuttner (1992), Friedman et al. (1996)). As before $j$ represents the forecast horizon, with $j = 0$ corresponding to the contemporaneous version of the policy rule. The constant term $c$ consists of the steady-state level of the interest rate ($\bar{r}$), plus the (constant) level of trend inflation ($\bar{\pi}_t$), as well as the target money growth ($\Delta \bar{m}_t$) and output gap ($\bar{x}_t$) and is given by the following functional form:

\[
    c = (1 - \rho_1 - \rho_2) \left[ (1 - \psi_x) \bar{\pi} + \bar{r} - \psi_m \Delta \bar{m} - \psi_x \bar{x} \right]
\]
for the year 1969 does not change the results any of the results presented in this paper.

The remaining data are as follows: the interest rate is the target Federal funds rate set at each meeting; the measure of output gap and inflation is based on Greenbook forecasts, presented in Orphanides (2002). Coibion and Gorodnichenko (2011) and Boivin (2005) have used this dataset to estimate policy using a variant of equation (20). To be consistent with this literature, I also estimate the policy rule where $\psi_m$ is set to 0, and therefore corresponds to the classic Taylor rule estimated in Clarida, Gali and Gertler (1998a). Table (2) summarizes the benchmark estimates of the policy rule against various combinations of the forecast horizon $j$. Column (1) of each version present the estimates of the full policy rule in equation (44) - (45), while column (2) presents estimates of the simple Taylor rule (supressing the coefficient on money to zero).


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Contemporaneous rule</th>
<th>Forward-looking rule</th>
<th>Mixed rule - I</th>
<th>Mixed rule - II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\psi_{\pi,t}$</td>
<td>1.22*</td>
<td>1.19*</td>
<td>1.56*</td>
<td>1.49*</td>
</tr>
<tr>
<td>$\psi_{\pi,t+1}$</td>
<td>1.95***</td>
<td>1.84***</td>
<td>1.55***</td>
<td>1.50***</td>
</tr>
<tr>
<td>$\psi_{x,t}$</td>
<td>0.47*</td>
<td>0.40*</td>
<td>0.49*</td>
<td>0.43*</td>
</tr>
<tr>
<td>$\psi_{x,t+1}$</td>
<td>0.68*</td>
<td>0.60*</td>
<td>0.71***</td>
<td>0.61***</td>
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<tr>
<td>$\psi_{\Delta m,t}$</td>
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<td>0.12</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>1.09***</td>
<td>1.14***</td>
<td>1.04***</td>
<td>1.06***</td>
</tr>
<tr>
<td>$\rho_2$</td>
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<td>-0.19</td>
<td>-0.12</td>
<td>-0.15</td>
</tr>
<tr>
<td>$\rho_1 + \rho_2$</td>
<td>0.93</td>
<td>0.92</td>
<td>0.92</td>
<td>0.91</td>
</tr>
<tr>
<td>AIC</td>
<td>706.19</td>
<td>728.07</td>
<td>690.18</td>
<td>712.2</td>
</tr>
<tr>
<td>BIC</td>
<td>728.45</td>
<td>746.86</td>
<td>712.44</td>
<td>731</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.77</td>
<td>0.75</td>
<td>0.76</td>
<td>0.77</td>
</tr>
<tr>
<td>Observations</td>
<td>302</td>
<td>317</td>
<td>302</td>
<td>317</td>
</tr>
</tbody>
</table>

Determinacy:
---
0% Trend Inflation:
Without Money | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
With Money | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
3% Trend Inflation:
Without Money | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
With Money | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |

Notes: This table presents OLS estimates of the baseline feedback rule. Standard errors are reported in parentheses. $^* p < 0.05$, $^{**} p < 0.01$, $^{***} p < 0.001$ denote significance levels.
Turning to the policy coefficients, while the money aggregate M1 enters with the right sign in the policy rule, it is statistically insignificant, and close in size to that found earlier. The coefficient on inflation varies across the forecast horizon considered; the best-fit policy rule suggests a strong long-run response to inflation and a high weight on the output gap. Combining the estimated policy parameters with the theoretical model introduced in the previous section suggests that the Fed seemed to have followed a rule that yielded price determinacy at the average level of trend inflation of 3 percent. Money is not required to guarantee determinacy at this level of trend inflation.

An observation that aligns with the results presented in Qureshi (2016) is related to the size of the coefficient on inflation once money is included in the policy rule. The model without money appears to overestimate the coefficient on inflation which is more precisely estimated when the money growth objective is introduced. The Taylor rule, without money targeting, appears to suffer from an omitted-variable bias, and, as a result, inaccurately measures the response to inflation.

In terms of fit, the policy rule with money explains the evolution of the funds rate better than the simple Taylor rule. Following Coibion and Gorodnichenko (2011), one way to determine the fit is by estimating AIC and the BIC. These tests also confirm that in terms of timing conventions, a mixed policy rule where the FOMC targeted forward-looking inflation but contemporaneous money and output gap best explains the setting of the funds rate.

The significance of this result is contextualized based on the information available from the transcripts, and the analysis offered earlier: while money aggregate M1 may have been important during the 1970s, its influence seems to have diminished during the 1980s. Thus, the full-sample approach ignores the important policy changes referred to in the literature purely because the fixed-coefficient approach does not highlight these changes. From an empirical perspective, Clarida, Gali and Gertler (1998a), Boivin (2005), and Coibion and Gorodnichenko (2011) provide empirical evidence of important changes in the U.S. conduct of monetary policy over the last 40 years. To address this literature, I undertake split-sample and time-varying analysis of the policy rule.

### 4.2. Split-Sample Estimates

While the complete sample estimation of the reaction function is informative of the type of policy pursued by the Fed, the policy rule may, in fact, have varied over time. Castelnovo (2012) finds money to have influenced policy during the pre-Volcker time period, but not after. It is also fairly uncontroversial that the Fed became more inflation averse under

To incorporate these changes and to study whether money entered the policy rule, a (now standard) single break in the coefficients of the response function around the time of the Volcker disinflation (October 1979) is allowed (Coibion and Gorodnichenko (2011)). The policy rule is estimated for the period 1969 - October 1979, and from 1982 - January 2002.

Table (3) summarizes estimates of the policy rule against various combinations of the forecast horizon $j$ for each period. As before, $j = 0$ corresponds to the contemporaneous version of the policy rule. The table highlight several interesting and novel results. First, with regard to the role of money aggregates in policy, I find significant evidence of their influence on policy during the pre-1979 sample, but not during the post-1982 sample. The best-fit version of the policy rule suggests a large value – close to 0.65 – on money aggregate M1 during this period. Going into the second subsample, the systematic reaction of the Fed to money growth declines from a statistically significant value of 0.65 to a statistically insignificant value of 0.06, signaling lower attention to monetary aggregates as measured by M1. The coefficient on money is remarkably close to the estimates based on historical M2 data in Castelnuovo (2012).
My estimates point to a stronger response by the Fed to inflation in the post-1982 period as compared to the pre-1979 period. Across the two time periods, other response coefficients seem to change in statistically significant ways. First, interest rate decisions become more persistent, in the sense that the sum of the autoregressive components is higher in the latter period than in the early period, and statistically significantly so in two out of three specifications. Second, the Federal Reserve seems to have changed how it responds to the real side of the economy with a lower weight on output gap in the second sample. This behavior of the policy coefficients is consistent with the estimates found in the literature.
Standard AIC and BIC tests also confirm that in terms of timing conventions, a mixed policy rule where the FOMC targeted forward-looking inflation but contemporaneous money and output gap best explains the setting of the funds rate.

An important result concerning the pre-1979 estimates of the policy rule with money suggest that the inclusion of a money growth objective affects the weight on other policy objectives. To highlight these differences, I compare these coefficients with those found in Coibion and Gorodnichenko (2011), who estimate the policy rule for the same time period, but use output growth as a policy objective instead of money growth. Comparing the best-fit versions Coibion and Gorodnichenko (2011) estimate the weight on inflation is 1.04, compared to 0.99 in this paper. They find that the weight on output gap is equal to 0.52, whereas I find this to be equal to 0.79. Critically, while they estimate the weight on output growth to be equal to zero, I find a statistically significant, large emphasis on money growth (0.65).

The direction of bias goes the other way for the post-1982 sample, as the coefficient on inflation is estimated to be higher in the model with money (2.41), as compared to a model without (2.20), and the weight on the output gap is estimated to be lower (0.39), compared to (0.43). Thus, the exclusion of money aggregate in the policy rule seems to distort estimates of the reaction function, which may have important consequences for the role of monetary policy in accounting for the Great Moderation.

Combining the estimated policy parameters with the theoretical model allows me to study price indeterminacy during the Great Inflation. I focus first on the model without trend inflation. I find that the Fed’s response to inflation in the pre-Volcker era satisfied the Taylor principle in forward-looking specifications, but not under the contemporaneous Taylor rule. Because the forward-looking specification is statistically preferred to a contemporaneous response to inflation, the evidence supports the argument of Orphanides (2004) in that the Fed satisfied the Taylor principle in both periods. As my theoretical framework highlights, the inclusion of money aggregates mitigates the uncertainty surrounding estimates of inflation, as the strong response to money enhances the weight on inflation, and injects interest rate inertia into the policy rule. These features enable the monetary authority to rule out self-fulfilling fluctuations, and to guarantee price determinacy.

The table also reports whether determinacy would be possible at 3 percent and 6 percent trend inflation. My empirical results suggest that the Fed was able to rule out self-fulfilling fluctuations during the entire time period (1969 - 2002) even at these levels of trend inflation. Indeed, one would incorrectly conclude that FOMC policy was unable to insulate the U.S.
economy from self-fulfilling fluctuation in a model with trend inflation but without a money growth policy objective. For example, in the fifth column, which reports the best-fit version of the policy rule based on AIC and BIC tests, determinacy is not possible under the simple Taylor rule when trend inflation is 6 percent, but is guaranteed when money aggregates are included in the reaction function.

In this sense, while the estimates of the Fed’s response coefficients without money provide support for the claims of Taylor (1999) and Clarida, Gali and Gertler (1998a) that the failure to satisfy the basic Taylor principle (prior to Volcker) placed the U.S. economy in an indeterminate region, the inclusion of money presents a novel channel that may have enabled the Fed to rule out self-fulfilling fluctuations despite positive trend inflation. These results validate the conclusions forwarded by Sims and Zha (2006), Orphanides (2004), and Chowdhury and Schabert (2008).

4.3. Determinacy Under Time-Varying Monetary Policy

In this section, I relax the assumption of constant coefficients by estimating a similar Taylor rule with time-varying coefficients, following the approach of Boivin (2005). The Taylor rule in equation (20) is generalized to:

\[ i_t = \rho_{1,t}i_{t-1} + \rho_{2,t}i_{t-2} + (1 - \rho_{1,t} - \rho_{2,t})[\psi_{\pi,t}\pi_t + \psi_{x,t}x_t + \psi_{m,t}\Delta m_t] + c_t + \epsilon_t \]  

(46)

where

\[ c_t = (1 - \rho_{1,t} - \rho_{2,t})(1 - \psi_{\pi})\bar{\pi} + r_t - \psi_{m,t}\Delta \bar{m}_t - \psi_{x,t}\bar{x}_t \]  

(47)

I assume that each of the policy parameters are time-varying and follow a driftless random walk:

\[ \Omega_t = \Omega_{t-1} + \omega_t \]  

(48)

\footnote{Other objectives may have influenced the setting of policy during this period. Coibion and Gorodnichenko (2011) find no statistically discernible response to output growth during the pre-Volcker period, but find a much stronger long-run response by the Fed to output growth. I compare my benchmark results with the estimates instead based on the rule with output growth objectives. I focus only on the pre-Volcker period; Qureshi (2016) contends that money growth outperforms output growth objectives in the post-1980 time period. Table 9 in appendix A.3 presents the statistical fit of the policy rule at various forecast horizons, compared against the best-fit rule with money growth. The results confirm that the policy rule with money better represents Fed policy during the pre-1979 period, as compared to the policy rule with output growth. The result is robust for all possible variants of forward-looking and contemporaneous-looking inflation, output gap, and output growth responses, relying on AIC/BIC or log-likelihood measures to select the best specification.}
Similar to Boivin (2005) and Coibion and Gorodnichenko (2011), I allow for breaks to accommodate the difference in volatility of the shocks during the periods 1979 and 1982. The non-linear Kalman filter and smoother estimates the time-varying policy coefficients. To extract a measure of trend inflation, I use the time-varying constant extracted in the policy rule. The methodology to extract trend inflation rests on Coibion and Gorodnichenko (2011), who approximate the real interest rate and the target rate of money growth using the HP filter over each time period. These are then used in equation (47) to extract the measure of trend inflation. Figure 5 presents the estimated parameters, along with one standard deviation confidence intervals.

Figure 5: Time-varying Reaction Function (M1 Aggregate)

Notes: The figure presents the time-varying estimate of the reaction function from 1969 through to 2002. The solid blue line plots estimates of the policy coefficients, while the dotted black line plots standard errors. In the last panel, trend inflation as estimated by Coibion and Gorodnichenko (2011) is also plotted.

Time-varying estimates largely confirm the static estimates of the policy rule outlined earlier. The weight on money growth M1 is estimated to be positive throughout the sample, and statistically significant from early 1970s through to late 1970s, and from 1982 through
to late 1984. In line with the split-sample estimates offered earlier the reaction of the Fed to money growth declines and signals lower attention to monetary aggregates from early 1985, validating the split-sample estimates. The evidence is therefore consistent with the observation that the FOMC paid more attention to money growth during the early 1970s, as suggested in Burns (1979) and Sims and Zha (2006) and during Volcker’s deflation, as suggested in Friedman et al. (1996). Furthermore, time-varying estimates are in line with the narrative record which suggests that the Fed discontinued targeting money aggregate M1 in the 1980s, validating the results presented by Meulendyke (1998).

The coefficient on inflation gradually drifts down from the start of the sample until the early 1980s, with the sharpest fall occurring during the first half of the 1970s, and remaining low during the second half, varying roughly between 0.97 and 2.1. In the second half of 1980s there is a sharp upward drift in the response to inflation, as it rises and remains there for the remainder of the sample. The weight on output gap rises continuously from 1970 through to 1977, falling gradually during Volcker’s deflation, while interest rates become persistent during the post-Volcker sample. Finally, the measure of trend inflation is roughly consistent with popular estimates - the figure plots the measure of trend inflation estimated by Coibion and Gorodnichenko (2011). The target rate of inflation was low, around 3 percent, and rose slightly over the early 1970s. Starting around 1975, we see a substantial increase in the Fed’s target inflation, which peaks at approximately 9 percent in 1978. The latter is reversed during the Volcker disinflation, after which target inflation is progressively reduced to 2 percent by the early 2000s.

To assess the likelihood of determinacy, I combine the estimated policy parameters with the complete model to assess the likelihood of determinacy. These estimates of determinacy rely on numerical solutions, which are based on the complete span of standard errors of the policy parameters. From a distribution of the estimated parameters, 10,000 draws are computed. For each meeting, this algorithm calculates the fraction of draws that yield a determinate rational expectation equilibrium.

The probability of determinacy is calculated under four levels of trend inflation: at 3 percent, 6 percent, time-varying trend inflation, and based on time-varying trend inflation estimated by Coibion and Gorodnichenko (2011). Figure (6) plots the probability of determinacy on each meeting-date.
Figure 6: Benchmark Determinacy Rates

Notes: This figure plots the likelihood of determinacy for each meeting date using four levels of trend inflation, 3 percent, 6 percent, time-varying trend inflation, and time-varying trend inflation estimated by Coibion and Gorodnichenko (2011).

Time-varying results extend and confirm those presented earlier, as monetary policy is found to have ruled out self-fulfilling fluctuations in both pre-Volcker and post-1980s era’s. At the start of the sample, the probability of determinacy is close to one due to the strong response to inflation and the low levels of trend inflation. While the response to inflation declines during the 1970s, the FOMC gradually increased the response to money growth objective during this period which maintained policy as active and insulated the U.S. economy from self-fulfilling fluctuations, confirming the conclusions forwarded by Orphanides (2002), Sims and Zha (2006), and Chowdhury and Schabert (2008).

While the likelihood of indeterminacy rises towards the end of the 1970s, it only falls to below 50% from 1979 - 1982. Therefore, monetary policy during the pre-Volcker period can be categorized as determinate, and does not seem to be responsible for the rise in inflation witnessed during the 1970s. If anything, monetary policy is only indeterminate when trend inflation is declining, and when the Fed (perhaps prematurely) reduced the response to money growth.
aggregate M1. Monetary policy can be characterized as active in the post-1982 period as well; fixed-coefficients confirm that this is due to the strong response to inflation. Note that point estimates confirm policy as active during the pre-1979 and post-1982 period as well.

Analysis based on time-varying estimates confirm that the U.S. economy satisfied price determinacy conditions even under the high positive trend inflation experienced during the 1970s. A major departure from previous studies rests in the specification of the policy rule used to analyze historical policy, which includes an important role for money. These results provide additional support for the well-known view that monetary policymakers satisfied the Taylor principle even before Volcker became Chairman of the Fed.

4.4. Estimates Based on M2 and M3

The policy rule is estimated using money aggregates M2 and M3. As before I use Greenbook forecasts of current and future macroeconomic variables prepared by staff members of the Fed prior to each FOMC meeting. Data on money aggregate M2 are available from 1971 – 2002 while data on M3 are available from 1972 – 2002.

Figure (7) presents the benchmark reaction function estimated using M2 money aggregate instead of M1. The weight on money growth M2, while positive, is statistically insignificant for most of the sample period. Surprisingly, during the late 1970s weight turns to be negative and statistically significant for a brief period, before turning positive, but statistically insignificant towards the mid-1980s till the end of the sample. The other coefficients follow roughly the same pattern as estimated with M1: the coefficient on inflation gradually drifts down from the start of the sample until the early 1980s, it varies roughly between 0.97 and 2.1, rising sharply in the early 1980s. The weight on output gap rises continuously from 1970 through to 1977, and rises falling gradually even after Volcker’s deflation.
Figure 7: Time-varying Reaction Function (M2 Aggregate)

![Figure 7](image-url)

Notes: The figure presents the time-varying estimate of the reaction function from 1971 through to 2002. The solid blue line plots estimates of the policy coefficients, while the dotted black line plots standard errors. In the last panel, trend inflation as estimated by Coibion and Gorodnichenko (2011) is also plotted.

Figure (8) focuses on the policy rule estimated using money aggregate M3. Interestingly, the average response to M3 is much larger as compared to M1 — even during the 1970s, average response stays around 0.70. The estimated coefficient on money growth M3 is significant, and enters with a positive sign during the 1970s, early 1980s and during the early 1990s. Standard errors, first around mid-1980s, and then post 1994, include zero, and the statistical significance of money drops after this period. The continued role for money in estimated Taylor rules over the post-Volcker period is consistent with previous findings by Ireland (2004), and on average match the point estimates proposed in Qureshi (2016).

The coefficient on inflation suggests that the Taylor principle was satisfied throughout the

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8This roughly coincides with the estimated break date in the federal funds rate as found in Qureshi (2016) and is in line with testimony by then-Federal Reserve Bank Chairman Alan Greenspan to the U.S. Congress in July 1993. Conventional literature suggests that his testimony resulted in the de-emphasis of monetary aggregates (see, for example, Friedman and Kuttner (1992)).
sample as standard errors never fall below 1, suggesting that the likelihood of indeterminacy was low during this period. The weight on output gap rises continuously from 1970 through to 1977, falling gradually during Volcker’s deflation, close to the estimates suggested in Bernanke and Mishkin (1992) and Orphanides (2002).

Figure 8: Time-varying Reaction Function (M3 Aggregate)

Notes: the figure presents the time-varying estimate of the reaction function from 1971 through to 2002. The solid blue line plots estimates of the policy coefficients, while the dotted black line plots standard errors. In the last panel, trend inflation as estimated by Coibion and Gorodnichenko (2011) is also plotted.

I calculate the probability of determinacy using the time-varying estimate of trend inflation. Figure (9) plots the probability of determinacy on each meeting date for both M2 and M3 as measures of money aggregate. While the results reinforce the key point that one cannot address determinacy issues by ignoring money aggregates, they also suggest that the type of money aggregate also plays a central role in resolving the conflicting conclusions of the earlier literature. Determinacy is found to depend on the aggregate employed.

At the start of the sample, the probability of determinacy is close to one due to the strong response to inflation and the low levels of trend inflation. While the response to inflation
declines during the first half of the 1970s, the FOMC gradually increased the response to the money growth objective during this period which maintained policy as active and insulated the U.S. economy from self-fulfilling fluctuations. From mid-1970s, determinacy is conditional on the aggregate employed. While estimates based on M1 and M3 suggest a high likelihood of determinacy during this period, estimates based on the M2 aggregate suggest price indeterminacy similar to that found in Coibion and Gorodnichenko (2011). This is because my estimates of the policy rule detect a strong response to M1 and M3 and a small positive response to M2. During the post-1982 period, the U.S. economy has been in a unique equilibrium across all estimates of money aggregates.

Figure 9: Determinacy Rates Based on M2 & M3

![Figure 9: Determinacy Rates Based on M2 & M3](image)

Notes: This figure plots the likelihood of determinacy for each meeting date using money aggregates M2 and M3, based on the estimated time-varying measure of trend inflation.

An important counterfactual analysis present in figure (9) concerns the role of money growth targeting in guaranteeing determinacy. In the figure, estimates based on M2 aggregate suggest qualitatively similar findings to those presented in Coibion and Gorodnichenko (2011), presenting the outcome had the FOMC not included a money growth objective in
its policy rule. Indeed, in that case, monetary policy would have been unable to rule out self-fulfilling fluctuations. Thus, responding to money offers a critical channel that enabled pre-Volcker monetary policy to insulate the U.S. economy from self-fulfilling fluctuations despite positive trend inflation.

The results of this section confirm money as an indicator of policy. Combined with the theoretical results, this suggests that the U.S. economy satisfied price-determinacy conditions even under the high positive trend inflation experienced during the 1970s; this also provides additional support for the well-known view that monetary policymakers satisfied the Taylor principle even before Volcker became Chairman of the Fed. A major departure from previous studies rests in the specification of the policy rule used to analyze historical policy, which includes an important role for money. Thus, the results reinforce the key point that one cannot address determinacy issues by ignoring money aggregates, and that the type of money aggregate plays a central role in resolving the conflicting conclusions found in the earlier literature. This framework enables me to confirm the conclusions forwarded by Orphanides (2002), Sims and Zha (2006), and Chowdhury and Schabert (2008), and yield contrasting conclusions compared with the findings by Clarida, Gali and Gertler (1998a), Taylor (1999), Coibion and Gorodnichenko (2011), Boivin (2005), Lubik and Matthes (2014) and Hirose et al. (2017).

4.5. Robustness

As highlighted by the theoretical conditions, a number of factors related to the structural mechanism could influence this result. Trend inflation, for example, increases the importance of forward-looking behavior in firms’ price-setting decisions. Firms’ profits depend strongly the degree of price stickiness in the model, with greater price stickiness increasing the sensitivity to expectations of future macroeconomic variables, and thereby influencing the probability of indeterminacy. Another important parameter in the New-Keynesian model is the elasticity of substitution, $\varepsilon$, which affects the degree of strategic complementarity in price setting. With higher elasticity of substitution, strategic complementarity in price setting increases so that firms focus more on the pricing behavior of other firms, and, hence, on trend inflation. Therefore, a higher value of $\varepsilon$ tends to increase the effects of positive trend inflation on determinacy. Other important channels, such as the sensitivity of interest rates in the money demand curve, may also influence determinacy, since they affect the degree of feedback from interest rates to money.

Table (4) summarizes both the benchmark results and the counterfactual determinacy
rates based on alternative assumptions on price rigidity, elasticity of substitution, and money demand parameters. The full-sample Bayesian estimates are used to extract the determinacy rates in the first four rows. For example, the first row summarizes the determinacy rates of the complete sample as well as that prevailing during the pre- and post-Volcker periods, as presented in figure (6). The fifth and sixth rows present the probability of determinacy if pre-1979 model estimates are used instead of the full sample estimates. The first column of each time period calculates whether the U.S. economy would be determinate using the point estimates, while the second column calculates the fraction of determinate equilibria given sampling uncertainty.

Table 4: Determinacy Rates: Robustness

<table>
<thead>
<tr>
<th></th>
<th>Full-Sample</th>
<th>Pre-1979 period</th>
<th>Post-1982 period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Point</td>
<td>Fraction of</td>
<td>Point</td>
</tr>
<tr>
<td></td>
<td>Estimates</td>
<td>Determinate</td>
<td>Estimates</td>
</tr>
<tr>
<td>Benchmark</td>
<td></td>
<td>Equilibria</td>
<td>Equilibria</td>
</tr>
<tr>
<td>3% Trend Inflation</td>
<td>Yes</td>
<td>0.9132</td>
<td>Yes</td>
</tr>
<tr>
<td>6% Trend Inflation</td>
<td>Yes</td>
<td>0.8875</td>
<td>Yes</td>
</tr>
<tr>
<td>Nakamura and Steinsson (2008), $\theta = 0.70$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3% Trend Inflation</td>
<td>Yes</td>
<td>0.8252</td>
<td>Yes</td>
</tr>
<tr>
<td>6% Trend Inflation</td>
<td>Yes</td>
<td>0.4456</td>
<td>Yes</td>
</tr>
<tr>
<td>High Elasticity of Substitution, $\epsilon = 10$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3% Trend Inflation</td>
<td>Yes</td>
<td>0.8681</td>
<td>Yes</td>
</tr>
<tr>
<td>6% Trend Inflation</td>
<td>Yes</td>
<td>0.5888</td>
<td>Yes</td>
</tr>
<tr>
<td>Ireland (2009) $\eta_i, \sigma_m = 74.506$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3% Trend Inflation</td>
<td>Yes</td>
<td>0.9161</td>
<td>Yes</td>
</tr>
<tr>
<td>6% Trend Inflation</td>
<td>Yes</td>
<td>0.8864</td>
<td>Yes</td>
</tr>
<tr>
<td>Pre-1979 calibration &amp; Nakamura and Steinsson (2008)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3% Trend Inflation</td>
<td>Yes</td>
<td>0.9127</td>
<td>Yes</td>
</tr>
<tr>
<td>6% Trend Inflation</td>
<td>Yes</td>
<td>0.9068</td>
<td>Yes</td>
</tr>
<tr>
<td>Pre-1979 calibration &amp; $\epsilon = 10$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3% Trend Inflation</td>
<td>Yes</td>
<td>0.9147</td>
<td>Yes</td>
</tr>
<tr>
<td>6% Trend Inflation</td>
<td>Yes</td>
<td>0.9114</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: The table summarises the benchmark results, and robustness results of determinacy using alternative assumptions on price setting, elasticity of substitution, semi-elasticity of interest rates, and pre-1979 estimates of the model. The column "point estimates" uses the estimates of the policy parameters from table 3 to test for a determinate rational expectations equilibrium. The fraction of cases with determinate solutions were computed from 10,000 draws.

The results broadly confirm my findings: the benchmark result in the first row confirm evidence of determinacy during the pre-1979 and the post-1982 periods. Using alternative pricing assumptions based on Nakamura and Steinsson (2008), who propose longer duration of price spells, suggests that determinacy is sustained for the fixed-coefficients. Time-varying
estimates suggest periods of indeterminacy during the pre-1979 period at 6 percent trend inflation, especially during the latter period (1978 - 1979), when the coefficient on M1 is estimated to be lower than the sample average observed during the pre-Volcker period. Using the Bils and Klenow (2004) Calvo pricing estimates, which are also close to those found in this paper, or utilizing the pre-1979 estimates of indexation offsets the effects of longer price duration. The high elasticity of substitution also reduces the probability of determinacy in the model, but determinacy is still guaranteed both using fixed or time-varying coefficients at both 3 percent and 6 percent level of trend inflation using the pre-1979 parameter estimates. Finally, estimates of the semi-elasticity of interest rates by Ireland (2009), have an insignificant impact on the probability of determinacy.

<table>
<thead>
<tr>
<th>Table 5: Determinacy Rates: Counterfactual (Money Aggregate)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
</tr>
<tr>
<td>3% Trend Inflation</td>
</tr>
<tr>
<td>M2 M3</td>
</tr>
<tr>
<td>0.7251 0.9316</td>
</tr>
<tr>
<td>0.4702 0.9385</td>
</tr>
<tr>
<td>0.8721 0.9247</td>
</tr>
<tr>
<td>6% Trend Inflation</td>
</tr>
<tr>
<td>M2 M3</td>
</tr>
<tr>
<td>0.6898 0.9106</td>
</tr>
<tr>
<td>0.4082 0.9158</td>
</tr>
<tr>
<td>0.8566 0.9132</td>
</tr>
<tr>
<td>Nakamura and Steinsson (2008), $\theta = 0.70$</td>
</tr>
<tr>
<td>3% Trend Inflation</td>
</tr>
<tr>
<td>M2 M3</td>
</tr>
<tr>
<td>0.5858 0.9583</td>
</tr>
<tr>
<td>0.2941 0.8766</td>
</tr>
<tr>
<td>0.8071 0.8627</td>
</tr>
<tr>
<td>6% Trend Inflation</td>
</tr>
<tr>
<td>M2 M3</td>
</tr>
<tr>
<td>0.1421 0.5031</td>
</tr>
<tr>
<td>0.0159 0.5208</td>
</tr>
<tr>
<td>0.2415 0.5403</td>
</tr>
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<td>High Elasticity of Substitution, $\varepsilon = 10$</td>
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<tr>
<td>3% Trend Inflation</td>
</tr>
<tr>
<td>M2 M3</td>
</tr>
<tr>
<td>0.6976 0.8914</td>
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<tr>
<td>0.4217 0.9058</td>
</tr>
<tr>
<td>0.8607 0.8917</td>
</tr>
<tr>
<td>6% Trend Inflation</td>
</tr>
<tr>
<td>M2 M3</td>
</tr>
<tr>
<td>0.5013 0.6584</td>
</tr>
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<td>0.1533 0.6549</td>
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<tr>
<td>0.7466 0.7059</td>
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<tr>
<td>Ireland (2009) $\eta, \sigma_m = 74.506$</td>
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<tr>
<td>3% Trend Inflation</td>
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<tr>
<td>M2 M3</td>
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<tr>
<td>0.7249 0.9314</td>
</tr>
<tr>
<td>0.4702 0.9384</td>
</tr>
<tr>
<td>0.8717 0.9246</td>
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<tr>
<td>6% Trend Inflation</td>
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<tr>
<td>M2 M3</td>
</tr>
<tr>
<td>0.6896 0.9105</td>
</tr>
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<td>0.4088 0.9157</td>
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<tr>
<td>0.8561 0.9131</td>
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<tr>
<td>Pre-1979 calibration &amp; Nakamura and Steinsson (2008)</td>
</tr>
<tr>
<td>3% Trend Inflation</td>
</tr>
<tr>
<td>M2 M3</td>
</tr>
<tr>
<td>0.7351 0.9396</td>
</tr>
<tr>
<td>0.4911 0.9433</td>
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<td>0.8756 0.9314</td>
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<td>6% Trend Inflation</td>
</tr>
<tr>
<td>M2 M3</td>
</tr>
<tr>
<td>0.7151 0.9373</td>
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<td>0.4554 0.9391</td>
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<td>0.8672 0.9302</td>
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<td>Pre-1979 calibration &amp; $\varepsilon = 10$</td>
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<td>3% Trend Inflation</td>
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<td>0.7423 0.9418</td>
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<tr>
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<td>6% Trend Inflation</td>
</tr>
<tr>
<td>M2 M3</td>
</tr>
<tr>
<td>0.7342 0.9406</td>
</tr>
<tr>
<td>0.4910 0.9429</td>
</tr>
<tr>
<td>0.8746 0.9332</td>
</tr>
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Notes: The table summarizes the benchmark results, and robustness of results on determinacy using alternative assumptions on price setting, elasticity of substitution, semi-elasticity of interest rates, and pre-1979 estimates of the model. The fraction of cases with determinate solutions were computed from 10,000 draws.

Table (5), while further confirming the robustness of the benchmark result using money aggregate M3, also displays stark conclusions in when money aggregate M2 is used. First,
and as before the results present evidence that monetary policy was able to rule out self-fulfilling fluctuations during the pre- and post-Volcker periods. If anything, estimates based on M3 present an even larger likelihood of determinacy during these periods. This result is robust across alternative degree of price stickiness, degree of complementarity, or upon using alternative assumptions on the money demand parameters. However, using measures of M2 suggests results similar to those of Coibion and Gorodnichenko (2011) whereby only the model’s estimation using pre-and post-classification of the parameters displays some likelihood of indeterminacy.

Analysis based on money aggregates M1, M2, and M3 reveal that only M1 and M3 behave as indicators of policy. When included in the policy rule, these indicators suggest strikingly different conclusions to those based on the existing studies in the literature that have either used M2, or have completely excluded money from the analysis. The results reinforce the key point that one cannot address determinacy issues by ignoring money aggregates, and that the type of money aggregate used plays a central role in resolving the conflicting conclusions of the earlier literature.

5. APPLICATIONS

The benchmark results are extended along two dimensions: to address the issue concerning raising the inflation target, and to address potential indeterminacy issues across countries.

5.1. RAISING TREND INFLATION

An important result that arises from the previous analysis is the effectiveness of the money growth policy objective under positive trend inflation. This result can in turn be applied to better understand the broader "best practices" conduct of monetary policy under positive trend inflation. Indeed, in response to the Great Recession, suggestions to raise the inflation target, and therefore trend inflation, from 2 percent to 4 percent have come to the fore (see, for example Blanchard et al. (2010), Krugman (2013), Ball (2014), and Arias et al. (2017)). The main rationale for a higher inflation target is that it would strengthen the capacity of the monetary authority to reduce interest rates when economic conditions deteriorate.

Raising the inflation target poses questions about its costs and challenges. Arias et al. (2017) study the challenge of achieving price equilibrium determinacy, which is a key indicator of the underlying ability of central banks to anchor inflation expectations and avoid self-fulfilling economic fluctuation. They show that the classic Taylor rule is unable
to guarantee determinacy in a medium-sized monetary model; they instead propose that monetary policy may need to be adjusted.

I re-examine the impact on monetary policy if the inflation target were to be raised from 2 percent to 4 percent or 5 percent. The policy rule is a modified version of the one considered in Taylor (1993). As before, it refers to policymakers’ systematic reaction to the growth rate of nominal money in addition to inflation and output-gap objectives, and can be written as follows:

\[ i_t = \psi_\pi E_t \pi_t + \psi_x E_t x_t + \psi_m E_t \Delta m_t + c_t + \epsilon_t \]  \hspace{1cm} (49)

The model is simulated using three versions of the policy rule: in the first version, the policy coefficients, \( \psi_\pi \), \( \psi_x \), and \( \psi_m \) are set to 1.5, 0.5, and 0, such that the Federal Reserve assigns an equal weight to both nominal and real policy objectives, thereby capturing a version of the classic Taylor (1993) rule. In the second version, \( \psi_m \) is set to 0.5 and added to the Taylor (1993) rule; notice that this type of rule is close to the estimated policy rule of the 1970s considered in section 4, and corresponds to the benchmark rule used in Qureshi (2016). In the third version of the model, the money growth rule is estimated with the price elasticity in the money demand function set to half its value. For the purposes of this section, the structural parameters of the model are fixed to the benchmark calibration used in section 3. Thus, features that aid determinacy such as price indexation are completely closed down.

The table reports an important result: while the simple Taylor rule is sufficient to accommodate a rise in the inflation target from 2 percent to 3 percent, it is unable to accommodate an inflation target of 4 percent or higher because the model yields price indeterminacy. This concern is overturned in the presence of simple money-growth targeting, which is able to accommodate a rise in inflation target of even 5 percent, and which is only problematic under a very weak estimate of price elasticity in the money demand curve.

What drives these results? Targeting money increases the aggregate response of the monetary authority to changes in inflation, induces history dependence, and guarantees an explosive response to changes in the macroeconomy (Svensson (1999), Christiano and Rostagno (2001), Söderström (2005), Keating and Smith (2013), and Qureshi (2016)); thus, targeting money has attributes of good policy (Woodford (2011)). Therefore, while raising the inflation target from 2 percent to 4 percent generates price indeterminacy under the simple Taylor rule, including a moderate response to money growth in the policy rule resolves this issue.
Table 6: Policy Counterfactual

<table>
<thead>
<tr>
<th>Trend Inflation</th>
<th>Policy Parameters</th>
<th>Money Demand</th>
<th>Determinacy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\psi_\pi$</td>
<td>$\psi_y$</td>
<td>$\psi_{\Delta m}$</td>
</tr>
<tr>
<td>2 percent</td>
<td>1.5</td>
<td>0.5</td>
<td>–</td>
</tr>
<tr>
<td>3 percent</td>
<td>1.5</td>
<td>0.5</td>
<td>–</td>
</tr>
<tr>
<td>4 percent</td>
<td>1.5</td>
<td>0.5</td>
<td>–</td>
</tr>
<tr>
<td>5 percent</td>
<td>1.5</td>
<td>0.5</td>
<td>–</td>
</tr>
<tr>
<td>2 percent</td>
<td>1.5</td>
<td>0.5</td>
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<tr>
<td>3 percent</td>
<td>1.5</td>
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<td>4 percent</td>
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<tr>
<td>5 percent</td>
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<td>3 percent</td>
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<td>4 percent</td>
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<td>5 percent</td>
<td>1.5</td>
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</table>

Notes: The table test for a determinate rational expectations equilibrium under alternative calibrations of the policy rule, trend inflation and the money demand curve. "Yes/No" shows whether there is a determinate rational expectations equilibrium when the policy-reaction function is evaluated at the described policy parameters.

5.2. Cross-Country Evidence

Based on the parameterized structure of the economy and the structural shocks, a strong response to money growth guarantees price determinacy even under the presence of moderately high trend inflation. In this context, it may be reasonable to examine the robustness of the relationship between pursuing monetary growth objectives and price determinacy across countries. Moreover, the analysis allows me to reveal the superiority of the policy rule with money compared to a simpler rule without a money growth objective.

Table (7) presents the simulated determinacy outcomes using the model presented in 6.1 for all countries for which data on inflation are available. Annual observations on inflation, as measured by the GDP deflator, are collected for each country for which data are available for the time period 1990 - 2016. Keeping constant the parameter values of the model, trend inflation is matched with each individual country. To check for determinacy, the model is simulated using a simple Taylor rule, and two types of policy rules with money. The three versions of the policy rule are: in the first, the policy coefficients, $\psi_\pi$, $\psi_y$, and $\psi_m$ are calibrated as 1.5, 0.5, and 0 (column 1); in the second, the policy coefficient on money growth is set to 0.5 (column 2); in the third, the coefficients are set to 1.5, 0.5, and 0.75, respectively.
<table>
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<th>Country</th>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>Country</th>
<th>Inflation</th>
<th>(1)</th>
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<th>Inflation</th>
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<th>Inflation</th>
<th>(1)</th>
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<td>8.9772</td>
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<td>4.5038</td>
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</table>

Notes: The table test for a determinate rational expectations equilibrium under alternative calibrations of the policy rule and trend inflation. "Yes/No" shows whether there is a determinate rational expectations equilibrium when the policy reaction function is evaluated at the described policy parameters.
Thus, this exercise demonstrates the full flexibility and applicability of the money growth rule to the conduct of monetary policy across countries.

The results largely confirm that central banks in countries can anchor inflation expectations and avoid self-fulfilling economic fluctuations by including a moderate response to money growth in their policy rule. The table suggests that countries with low trend inflation, such as Canada and Germany would not face any indeterminacy issues if they were to rely on the simple Taylor rule for policy making. However, the indeterminacy problems arise even with trend inflation marginally above 3 percent. A large and wide-ranging group of countries – including, for example, Andorra, India, and the Philippines – may face price indeterminacy issues if they were to rely solely on the simple Taylor rule for the conduct of monetary policy. Indeed, out of the 204 countries in the sample, approximately 65 percent may face indeterminacy issues if they were to rely on only the simple Taylor rule. The number falls to 45 percent if these countries include a moderate response to money growth, and to 34 percent with a slightly stronger response to money growth. Thus, these cross-country applications confirm the superiority of the rule with money.

6. Conclusion

In this paper, I study the role of money growth as an objective of monetary policy by examining its contribution to price-level determinacy. Specifically, I apply this framework to analyze the rise in macroeconomic instability experienced by the U.S. economy during the 1970s. My empirical results suggest that the Federal Reserve Bank was able to rule out self-fulfilling fluctuations even during the high trend inflation of the 1970s. The inclusion of money growth objectives offers a novel channel that enabled the Fed to rule out self-fulfilling fluctuations during this period, validating the conclusions put forward by Sims and Zha (2006), Orphanides (2004), and Chowdhury and Schabert (2008). The contrasting conclusions from the claims of Taylor (1999) and Clarida, Gali and Gertler (1998a) are primarily due to the inclusion of money growth objectives in the policy rule.

I derive novel determinacy conditions that contribute to the theoretical literature, which has hitherto focused only on the response towards inflation as a pre-requisite of active policy. Novel briefing forecasts prepared for the Federal Open Market Committee (FOMC) to estimate a policy rule with money for the 1960 - 2002 time period. Along with inflation and the output gap, money (M1 and M3) were significant policy objectives from the 1970s through to the early 1980s. The exclusion of money aggregate in the policy seems to be an important source of bias compared to previous estimates of the reaction function, and has important
consequences for the role of monetary policy during the Great Inflation. Indeed, the combined weight on inflation and money growth in the policy reaction function supports the fact that Fed policy ruled out self-fulfilling fluctuations even during the pre-Volcker period, a result which is robust for varying levels of trend inflation and across alternative assumptions of the structural parameters.

These results may have important policy implications. An application to more than 200 countries highlights the superiority of the policy rule with money. Moreover, while raising the inflation target from 2 percent to 4 percent generates price indeterminacy under the classic Taylor rule, including a moderate response to money growth in the policy rule resolves this issue. In light of the recent call for a revision of monetary targets for the design of stabilization policies (Blanchard and Summers (2017)), the analysis and the application to policy in this paper suggest the inclusion of money aggregates in this reflective process.
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A. Appendix: For Online Publication

A.1. Proof of determinacy conditions

Proof of proposition 1. In this section I consider the contemporaneous version of the model presented in the section, and find the determinacy condition of the policy rule with money growth. The baseline model and the policy rule are summarized by the following equations:

\begin{align}
x_t &= E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + g_t \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + u_t \\
\Delta m_t &= \eta_\pi \pi_t + \eta_x \Delta x_t - \eta_i \Delta i_t + \eta_y \Delta y_t + \tau_t \\
i_t &= \rho_1 \pi_{t-1} + \rho_2 \pi_{t-2} + (1 - \rho_1 - \rho_2) [\psi_\pi \pi_t + \psi_x x_t + \psi_m \Delta m_t^i] + c + \epsilon_t
\end{align}

(50) (51) (52) (53)

First, by substituting the money demand equation in the monetary policy rule, we can simplify the monetary policy rule and write it in the following form:

\begin{align}i_t = \delta_1 i_{t-1} + \delta_2 i_{t-2} + \delta_3 \pi_t + \delta_4 x_t + \delta_5 x_{t-1}
\end{align}

(54)

where:

\begin{align}\delta_1 &= \frac{\rho_1 + \eta_\pi \psi_\Delta m (1 - \rho_1 - \rho_2)}{1 + \eta_\pi \psi_\Delta m (1 - \rho_1 - \rho_2)} \\
\delta_2 &= \frac{\rho_2}{1 + \eta_\pi \psi_\Delta m (1 - \rho_1 - \rho_2)} \\
\delta_3 &= \frac{(\psi_\pi + \eta_\pi \psi_\Delta m)(1 - \rho_1 - \rho_2)}{1 + \eta_\pi \psi_\Delta m (1 - \rho_1 - \rho_2)} \\
\delta_4 &= \frac{(\eta_x \psi_\Delta m + \psi_x)(1 - \rho_1 - \rho_2)}{1 + \eta_\pi \psi_\Delta m (1 - \rho_1 - \rho_2)} \\
\delta_5 &= -\frac{\eta_x \psi_\Delta m (1 - \rho_1 - \rho_2)}{1 + \eta_\pi \psi_\Delta m (1 - \rho_1 - \rho_2)}
\end{align}

We can write the model in the following compact state space form:

\begin{align}A z_{t+1} = B z_t + C v_t
\end{align}

(55)

and the matrices \(A, B\) are composed of structural parameters, and monetary policy parameters, and \(C\) is a matrix of exogenous variables. The vector \(z_{t+1}\) is composed of the variables \([x_{t+1}, \pi_{t+1}, x_t, i_t, i_{t-1}]\) which contain two non-predetermined variables and three predetermined variables. Thus, if two of the generalised eigenvalues lie outside the unit circle, then the system has a unique solution (proposition 1 in Blanchard and Kahn (1980)).

Alternately, rational expectations equilibrium is determinate if and only if three of the eigenvalues are inside the unit circle. The characteristic polynomial is given by \(p(\lambda) = \)
$a_5 \lambda^5 + a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0$. In this case, define $p(1) = 1 + a_4 + a_3 + a_2 + a_1 + a_0$ and $p(-1) = -1 + a_4 - a_3 + a_2 - a_1 + a_0$. This is given by the following two conditions:

\begin{align*}
p(1) &= (\beta \sigma)^{-1}((\delta_4 + \delta_5)(1 - \beta) + \kappa(\delta_3 - 1) + (\delta_1 + \delta_2)\kappa) \\
p(-1) &= -(\beta \sigma)^{-1}((\delta_4 - \delta_5)(1 + \beta) + \kappa\delta_3 + (\delta_1 - \delta_2 + 1)(\kappa + 2\sigma + 2\beta \sigma))
\end{align*}

(56)

(57)

Conditions (C.13) and (C.14) in Woodford (2011) are necessary for both Cases II and III (and they also rule out Case I). These two conditions are that $p(1) > 0$ and $p(-1) < 0$. Notice that when $\eta_i \geq 0$, the following condition is sufficient for determinacy, since it applies condition 57.

\begin{equation}
| (\beta \sigma)^{-1}((\delta_4 + \delta_5)(1 - \beta) + \kappa(\delta_3 - 1) + (\delta_1 + \delta_2)\kappa) | > 0
\end{equation}

(58)
A.2. IV estimates

The following table presents the least squares and instrumental variable estimates of the benchmark (mixed) Taylor rule with money. Following Coibion and Gorodnichenko (2011), I use past values of inflation, output gap from the Greenbook and lagged money growth as instruments.

Table 8: Estimates of the Policy Rule - OLS and IV estimates

<table>
<thead>
<tr>
<th></th>
<th>Pre-1979</th>
<th>Post-1982</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS (1)</td>
<td>IV (2)</td>
</tr>
<tr>
<td>$\psi_{\pi,t+1}$</td>
<td>0.99</td>
<td><strong>0.97</strong></td>
</tr>
<tr>
<td></td>
<td>(0.56)</td>
<td>(0.61)</td>
</tr>
<tr>
<td>$\psi_{x,t}$</td>
<td>0.79</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.18)</td>
</tr>
<tr>
<td>$\psi_{\Delta m,t}$</td>
<td>0.65</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>1.31</td>
<td>1.31</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>-0.36</td>
<td>-0.37</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$\rho_1 + \rho_2$</td>
<td>0.95</td>
<td>0.94</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>$p-value$</td>
<td>--</td>
<td>0.971</td>
</tr>
</tbody>
</table>

This table presents OLS and IV estimates of the baseline feedback rule. Standard errors are reported in parentheses. The set of instruments are lags of inflation, output gap, the federal funds rate and growth in monetary aggregate M1. The bold letters are the instrumented variables. The bottom panel reports the p-value associated with a test of the models over identifying restrictions (Hausman).
A.3. **Robustness to Output Growth Estimates**

The following table presents the statistical fit of the policy rule at various forecast horizons, compared against the best fit rule with money growth. The results confirm that the policy rule with money better represents Fed policy during the pre-1979 period, as compared to the policy rule with output growth. The result is robust for all possible variants of forward-looking and contemporaneous-looking inflation, output gap, and output growth responses, relying on AIC/BIC or log-likehood measures to select the best specification.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Inflation</th>
<th>Output Gap</th>
<th>Money Growth</th>
<th>Output Growth</th>
<th>AIC</th>
<th>BIC</th>
<th>Log Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>Forecast</td>
<td>Contemp.</td>
<td>Contemp.</td>
<td></td>
<td>102.35</td>
<td>119.36</td>
<td>-45.17</td>
</tr>
<tr>
<td>1</td>
<td>Contemp.</td>
<td>Contemp.</td>
<td>Contemp.</td>
<td>136.2</td>
<td>153.31</td>
<td></td>
<td>-62.1</td>
</tr>
<tr>
<td>2</td>
<td>Forecast</td>
<td>Forecast</td>
<td>Forecast</td>
<td>136.98</td>
<td>154.09</td>
<td></td>
<td>-62.49</td>
</tr>
<tr>
<td>3</td>
<td>Contemp.</td>
<td>Contemp.</td>
<td>Forecast</td>
<td>133.8</td>
<td>150.92</td>
<td></td>
<td>-60.9</td>
</tr>
<tr>
<td>4</td>
<td>Contemp.</td>
<td>Forecast</td>
<td>Contemp.</td>
<td>135.54</td>
<td>152.65</td>
<td></td>
<td>-61.77</td>
</tr>
<tr>
<td>5</td>
<td>Contemp.</td>
<td>Forecast</td>
<td>Forecast</td>
<td>135.44</td>
<td>152.55</td>
<td></td>
<td>-61.72</td>
</tr>
<tr>
<td>6</td>
<td>Forecast</td>
<td>Contemp.</td>
<td>Contemp.</td>
<td>136.2</td>
<td>153.31</td>
<td></td>
<td>-62.1</td>
</tr>
<tr>
<td>7</td>
<td>Forecast</td>
<td>Forecast</td>
<td>Contemp.</td>
<td>137.12</td>
<td>154.23</td>
<td></td>
<td>-62.56</td>
</tr>
<tr>
<td>8</td>
<td>Forecast</td>
<td>Contemp.</td>
<td>Forecast</td>
<td>135.2</td>
<td>152.31</td>
<td></td>
<td>-61.60</td>
</tr>
</tbody>
</table>

Notes: the table reports information criteria and log-likelihood for Taylor rules with the baseline and alternative choices of current versus forecast values of inflation, output growth rate and output gap compared with the best-fit rule with money growth.

A.4. **Bayesian Estimation**

A.4.1. **Priors**

The posterior distribution of the parameters is characterized using the methods outlined in An and Schorfheide (2007). Because the estimation procedure closely follows the literature, the details are relegated to the appendix. Using the priors presented in Ascari, Castelnuovo and Rossi (2011), the model is estimated over the full sample from 1966:I through to 2008:II. The end-of-the-sample dates are chosen to avoid dealing with the Federal Reserve’s unconventional monetary policy that began in September 2008. The model is further estimated for the split-sample, 1966:I - 1979:III and 1984:I - 2008:II, to track the evolution of the model’s paramaters over time.

Before estimating the model, I calibrate a subset of the model’s parameters; $\beta$ is set to 0.9995 to match the average interest rate in my sample, the elasticity of substitution
among goods $\varepsilon$ is set equal to 6, the inverse of the labor elasticity $\varphi = 1$, $\alpha$, the returns to scale parameter in the production function is fixed at $1/3$. Trend inflation is also kept fixed to the average of the sample, which is close to 3%. Since trend inflation is fixed, and subjective interest rate is fixed, the underlying parameters, elasticity of interest rate and output, and therefore the elasticity of consumption are also fixed. Following Ascari, Castelnuovo and Rossi (2011), I pay particular attention to the price indexation parameter, $\varrho$. While there is little evidence to support the presence of price indexation (Coibion and Gorodnichenko (2011) and Christiano, Eichenbaum and Trabandt (2016)), this literature has primarily relied on full-sample estimates or focused on the post-1984 time-period only. However, this parameter may very well not be structural in the sense of Lucas. Thus, the focus here is to estimate its value for the pre-1980 and the post-1980 sample, similar to the approach undertaken in Smets and Wouters (2007).

### A.4.2. Data and Algorithm

An important qualification arises when dealing with models that include money. Since this model includes money, which plays a role in determining the equilibrium values of inflation, output, and interest rates when the policymaker responds to it in its interest rate rule, I employ four observables: the log of the quarterly gross growth rate of the GDP deflator, the log deviation of real GDP with respect to its long run trend, the Federal Funds rate, and the log of the quarterly gross growth rate of money aggregate, M1. This money aggregate is used so as to remain consistent with the empirical analysis offered in section 5, which detects a positive role for money in policy during the pre-Volcker sample.

All data are available from the FRED database. The model is estimated over the full sample period from 1966:I till 2008:II. The corresponding measurement equations of the quarterly model are (the bars denote steady state variables):

\[
\begin{pmatrix}
\frac{dlGDP_t}{dlP_t} \\
FEDFUNDS_t \\
\frac{dlM1_t}{dlM1_t}
\end{pmatrix} =
\begin{pmatrix}
\bar{\gamma} \\
\bar{\pi} - 1 \\
\bar{r} \\
\bar{m}
\end{pmatrix} +
\begin{pmatrix}
y_t - y_{t-1} \\
\pi_t \\
r_t \\
m_t
\end{pmatrix}
\]

Finally, the following algorithm is used to compute the MH procedure:

1. Choose a starting value or prior for our parameters stacked in $\Theta$.

---

9For a detailed review of the various ways money can enter in the model, the reader is referred to Castelnuovo (2012).
2. Draw $\Theta^*$ from $J_t(\Theta^*|\Theta^{t-1})$. The jumping distribution $J_t(\Theta^*|\Theta^{t-1})$ determines where we move to in the next iteration of the Markov chain and contains the support of the posterior.

3. Compute acceptance ratio $r$, according to:

$$r = \frac{p(\Theta^*|y)/J_t(\Theta^*|\Theta^{t-1})}{p(\Theta^{t-1}|y)/J_t(\Theta^{t-1}|\Theta^*)}$$

If our candidate draw has higher probability than our current draw, then our candidate is better so we definitely accept it. Otherwise, our candidate is accepted according to the ratio of the probabilities of the candidate and current draws.

4. Accept $\Theta^*$ as $\Theta^t$ with probability $\min(r, 1)$. If $\Theta^*$ is not accepted, then $\Theta^t = \Theta^{t-1}$.

Candidate draws with higher density than the current draw are always accepted.

A sample of 200000 draws was created, with 2 MH chains, and the first 20% of the sample was rejected. The results remain robust when 200000 draws were created, with 5 MH chains and so forth.