Unemployment Volatility in a Behavioural Search Model

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Abstract

Recent evidence that the opportunity cost of employment is pro-cyclical implies that existing models based around search frictions in the labour market cannot match the large volatilities of unemployment and vacancies observed in the data. In this paper, we incorporate insights from behavioural economics into the search frictions framework. The resultant model can match observed volatilities even if the opportunity cost is strongly pro-cyclical. The key mechanism in the model is that the pro-cyclicality of the opportunity cost has a limited impact on the reference wage of workers; this feeds through into a limited volatility of the wage and so to a large unemployment volatility.

Keywords: behavioural economics, search frictions, unemployment volatility

JEL Classification: E23, E32, J23, J30, J64

1 Introduction

Search models, based around imperfect matching of unemployed workers to open job vacancies, provide a widely used framework for the analysis of the labour market, enabling insights that have guided developments in theory and policy. However, it has proven difficult for the models to explain the large volatilities of unemployment and vacancies that are consistently observed in the data. This "unemployment volatility puzzle" arises because wages are sensitive to the business cycle in the search frictions model with Nash bargaining over wages. As a result, productivity shocks, which drive the business cycle in these models, lead to large changes in the wage; this reduces variability in the number of job vacancies posted by firm, thus suppressing movements in unemployment.

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There are currently two main responses to the unemployment volatility puzzle in the literature. The first changes the wage bargaining protocol from Nash to an alternating offer strategic bargaining set-up that reduces the influence of a worker's outside option on the wage (Hall and Milgrom 2008). The second proposes alternative calibrations of structural parameters within a Nash bargaining framework to increase the elasticities of labour supply and vacancy posting with respect to the wage (Hagedorn and Manovskii, 2008). These responses reduce the importance of output and labour market tightness (the number of vacancies per unemployed worker) in wage determination and increase the importance of the opportunity cost of employment. This results in lower wage volatility and hence greater volatility of unemployment if the opportunity cost of employment is constant, as it is assumed to be in these models.

However, Chodorow-Reich and Karabarbounis (2016) have questioned this assumption. They present evidence that the opportunity cost of employment is in fact strongly procyclical. Since the opportunity cost of employment plays a prominent role in wage-setting in the models of Hall and Milgrom (2008) and Hagedorn and Manovskii (2008), this implies an increased volatility of the wage. Chodorow-Reich and Karabarbounis (2016) argue that this implies that these models cannot generate a large volatility of unemployment and vacancies. As a result, existing explanations of the unemployment volatility puzzle are inadequate. We refer to this as the Chodorow-Reich and Karabarbounis critique. Because wage bargaining is deeply embedded within the literature, the critique applies to other responses to the unemployment volatility puzzle, including Pissarides (2009), Petrosky-Nadeau and Wasmer (2013) and Hall (2017). Thus, the debate on the unemployment volatility puzzle remains active and unresolved. Currently, there is no model in the literature which can address the unemployment volatility puzzle that is not vulnerable to the Chodorow-Reich and Karabarbounis critique (henceforth, the C-R&K critique).

In this paper, we analyse a model that can address the volatility puzzle while being immune to the C-R&K critique. We do this using an alternative version of the search frictions model that incorporates insights from behavioural economics. We argue that simulations of a calibrated version of this model are able to match the observed volatility of unemployment and other moments of the data, even if the opportunity cost of employment is highly procyclical. We are not the first to develop behavioural search models; other examples include Danthine and Donaldson (1990), Danthine and Kurmann (2007, 2010), Eliaz and Speigler (2013) and Kuang and Wang (2017). Our contribution is to argue that this framework enables the derivation of models that are immune to the C-R&K critique.

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1 Other approaches include Pissarides (2009), who introduces fixed vacancy costs and Petrosky-Nadeau and Wasmer (2013), who analyse the effect of search frictions in credit as well as labour markets and Hall (2017), who argues that fluctuations in discount rates are a driving force in the labour market.

2 This close relationship between the wage and the value of non-work concerns commentators such as Hornstein et al (2005), who refer to the "excessive sensitivity" of unemployment to the opportunity cost of labour in some models.
We develop a simple model with undirected search, identical workers and firms and where the only source of volatility is a productivity shock. Output depends on the amount of effort exerted by workers. Wages are posted by firms; if a worker accepts the offered wage, they become employed and determine the amount of effort to expend. As we discuss in detail below, these modelling choices are guided by empirical evidence that wage posting is at least as common as wage bargaining (Hall and Kreuger, 2012, Brenzel et al, 2014), that worker effort is an important input to production (Fehr and Falk, 1999, Kreuger and Mas, 2004, and Gneezy and List, 2006) and that effort is a procyclical function of the wage (Burda et al, 2016).

To derive the optimal supply of effort from workers, we follow Williamson (1985) and Hart and Moore (2008) by assuming that employment contracts are incomplete and that workers have discretion over the amount of effort they exert; see also Gneezy and List (2006) and Eliaz and Spiegler (2013). We assume that workers dislike exerting effort but gain utility from reciprocation; workers respond to what they see as a generous wage offer by increasing effort and "punish" a low wage by reducing effort. This reciprocity effect is supported by evidence discussed in Kahneman et al (1986), Bewley (1999) and Fehr et al (2009). We also follow Fehr and Falk (1999), Brown et al (2004) and Della Vigna and Pope (2018) in assuming that workers assess the wage offer of the firm relative to a reference wage or "fair wage". Using these assumptions, we derive the optimal supply of effort of workers, showing that this is a function of the wage relative to the reference wage.

Firms know the optimal effort function of workers and set the wage to balance the positive impact of higher wages on output against the adverse effect on the wage bill. Optimal wage-setting is characterised by a version of the Solow (1979) condition, modified to allow for search frictions in the labour market. The wage offered to workers is a function of the reference wage and the marginal cost of hiring; this latter affects wage-setting because of search frictions; a higher cost of hiring leads the firm to increase the wage in order to induce increased effort, increasing output along the intensive margin. The relative weight on the reference wage and hiring costs on the wage is a function of the elasticity of effort with respect to the wage. Della Vigna and Pope (2018) estimate this elasticity and find it is small; this implies that the reference wage has a powerful effect on the wage, whereas the impact of marginal hiring costs on the wage is small. This implies that labour market conditions have little direct impact on wage determination. As Hall and Milgrom (2008) point out, this is essential for ensuring the low volatility of the wage across the business cycle that is required if the model is to generate a large volatility of unemployment.

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3Williamson (1985) argues that only minimum job performance can be enforced by an employment contract and that workers have discretion concerning the amount of effort they exert.

4This captures an effect highlighted by Eliaz and Spiegler (2013), who note "the labor contract's inherent incompleteness forces employers to rely to some extent on workers' intrinsic motivation. When workers feel that they have been treated unfairly, their intrinsic motivation is dampened and their output declines".
The Behavioural Economics literature does not provide guidance on how the reference wage varies across the business cycle. Therefore any choice of the reference wage will be to some extent ad hoc. In order to reduce this ad hoc element, we restrict our choice to measures of the reference wage that satisfy two criteria: they must be model-based and must be consistent with empirical evidence in Koenig et al (2017) that the reference wage has low volatility across the business cycle. In doing so, we differ from the existing literature, which mostly follows Danthine and Kurmann (2010), who express the reference wage as a reduced form function of the lagged wage and other variables. Although simple and intuitive, this formulation is ad hoc. As we explain in detail below, we use the annuity value of unemployment as our measure of the reference wage. This captures a workers’ expectation of what their lifetime earnings would be on an alternative career path in which they are currently unemployed. This simple model-based measure has low volatility across the business cycle, so our measure is consistent with the empirical evidence.

The mechanism through which this model generates unemployment volatility is similar to that outlined by Hall (2005): firms respond to a positive productivity shock by posting more vacancies, leading to a fall in unemployment and an increase in labour market tightness. The behavioural aspects of our model give an additional element to this mechanism that amplifies the impact of productivity shocks. This arises because the increase in labour market tightness implies an increase in the job finding rate of unemployed workers. This increases the annuity value of unemployment and so leads to a higher wage. Workers respond to the higher wage by increasing effort, which increases the incentive for firms to post vacancies. This argument relies on pro-cyclical movements in effort. Evidence for this is provided by Burda et al (2016), who show that effort is higher when the unemployment rate is lower.

In addition, the model is able to generate a large volatility of unemployment since the response of wages to productivity shocks is small. Although a productivity shock will increase in the annuity value of unemployment and through this increase the reference wage and thus the wage, this effect is small. This means that firms respond to productivity shocks by making large changes to the level of vacancies they post. By doing so, they generate large movements of unemployment across the business cycle.

The model is able to generate a large volatility of unemployment even if the opportunity cost of employment is strongly procyclical; as a result, the model is immune from the C-R&K critique. This is because cyclical movements in the opportunity cost of employment have only a limited impact on the annuity value of unemployment as transient shocks have only a limited impact on expected lifetime earnings. Since the wage is mainly driven by the annuity value of unemployment, this implies that changes in the opportunity cost of employment have little effect on the wage. The key aspect of this mechanism is supported by evidence in Burda et al (2016), who find only a small impact of unemployment benefits on effort; this is consistent with a small impact of the opportunity cost of employment on the reservation wage.

The remainder of the paper is structured as follows. Section 2) outlines
our behavioural search frictions model. Section 3) presents simulations of the model. Finally, section 4) summarises the paper and raises issues for subsequent research.

2 A Behavioural Search Model

In this section we outline a simple behavioural search model. We use a discrete time model of undirected search with identical workers and firms where the only source of volatility is a productivity shock. After shocks are realised, hiring occurs. Then the firm posts a wage; if the worker accepts this offer, they become employed and determine the amount of effort to expend. Production then occurs followed by exogenous separation. If the worker does not accept the wage offer, they are unemployed for the remainder of period and the firm has an unfilled vacancy.

2.1 Evidence on Wage-Setting and Effort

These modelling choices are guided by empirical evidence on how wages are set in practice, the importance of effort in production and on the factors that underlie the worker’s choice of effort. In this section, we briefly review this evidence. Hall and Kreuger (2012) note that there are two main approaches to wage determination in search frictions models, wage posting, in which firms make take-it-or-leave-it offers to workers, and wage bargaining. They also note that although wage posting is associated with models of directed search, “the assumption of a posted wage in a model with random search would not be unreasonable”.

There are two studies of the incidence of different types of wage setting. Hall and Kreuger (2012) study responses of 1400 workers to survey questions on the circumstances in which they took their most recent job. In total, only 31% of workers reported some type of wage bargaining. This proportion is larger for professional and knowledge-based occupations. It is also larger for workers engaged in on-the-job-search. Brenzel et al (2014) examine the responses of over 9,000 firms to the recurrent German Job Vacancy Survey and find similar results. They find that only 38% of firms report wage bargaining; this proportion falls to 27% in industries covered by a collective wage agreement and to 32% when an unemployed worker is hired. Wage bargaining is much more likely when the job opening requires an experienced and more highly skilled worker. These studies reveal a good deal of diversity in wage-setting. But it is clear that wage bargaining is not common in the type of hiring analysed by the standard search frictions model: hiring to jobs where there is no difference in productivity between workers and where newly-hired workers come from unemployment rather than an alternative employer.

However, as Hall and Kreuger (2012) stress, evidence that wage posting is more common than wage bargaining is not inconsistent with the alternating offer bargaining protocol of Hall and Milgrom (2008) where the first offer made in a
negotiation, assumed to come from the firm, is always accepted. But evidence in Galuščák et al. (2012) argues against alternating offer wage bargaining. Galuščák et al. (2012) examine responses of Chief Executive Officers or Human Resource Managers of around 15,000 firms to a firm-level survey on wage- and price-setting practices in 15 EU countries, conducted under the auspices of the European Central Bank. They find that wages offered to newly-hired workers are more strongly affected by wages offered within the firm than by wages on the wider labour market (the relative weights on internal and external factors are roughly 4:1); similar evidence is found in Levine (1993) and other papers that examine the role of internal and external factors in wage setting. This goes against alternating offer bargaining, where the wage is driven by external factors.

With regard to effort, Williamson (1985) and Hart and Moore (2008) argue that employment contracts are inherently incomplete since only a minimum level of job performance can be legally enforced. As a result, workers have discretion over the amount of effort they devote to the job. There is a large empirical literature documenting the importance of these effects (summarised in Della Vigna, 2009, and Fehr, 2009). This includes laboratory-based studies such as Fehr and Falk (1999), who find wages above the competitive level alongside reciprocal high levels of worker effort, in an experimental labour market in which firms cannot monitor effort. Beyond the laboratory, Kreuger and Mas (2004) document how attempts by major US tyre producers to reduce the wage were perceived as unfair and led to a sharp reduction on the quality of output. Other examples include Gneezy and List (2006), who find that higher paid workers input data into a library information system more rapidly and Cohn et al (2014) who found that workers were productive in distributing free newspapers when they perceived their wages as fair. Further support for the importance of the role of effort in production comes from outside the behavioural literature. A number of studies have analysed the responses of firms to questions about the determinants of wages using data derived from a series of surveys conducted at different dates in a number of different countries. These include Blinder and Choi (1990), Bewley (1999), Campbell and Kamlini (1997) and Levine (1993) for the US, Galuščák et al. (2012) for 15 EU countries, Millard and Tatomin (2015) for the UK and Agell and Lundborg (2003) for Sweden. These studies find that firms consistently cite the adverse impact on the effort of workers as a primary reason for not reducing wages when their economic environment deteriorates.

Finally, considering the determinants of effort, this is often modelled as depending on a worker’s perception of the fairness of the wage offered by the firm, where the fairness of a wage offer is evaluated relative to a worker’s reference wage. The role of the reference wage was stressed by Kahnemann and Tversky (1979), who relate this to the concept of reference dependent preferences (see also Della Vigna, 2009). The relationship between effort and fairness is motivated by reciprocity; this idea was the basis of the "gift exchange" variant of efficiency wage theory proposed by Akerlof (1982) and Akerlof and Yellen (1990); these effects were introduced into the business cycle literature by Danthine and Donaldson (1990). Reciprocity and reference wages were introduced to game
theory by Rabin (1993), whose approach was used by Danthine and Kurmann (2007) to derive a macroeconomic relationship between effort and wages.

2.2 The Model

2.2.1 Workers

There is a continuum of identical workers on the unit interval. In period $t$ a worker is either employed or unemployed. The value function for an employed worker is

$$L_t = w_t - c(e_t) + R(e_t, w_t) + \frac{1}{1 + r} E_t[(1 - \tau)L_{t+1} + \tau U_{t+1}]$$

(1)

The worker earns (and consumes) real wage $w_t$; $c(e_t)$ is the disutility of exerting effort and $R(e_t, w_t)$ is utility gained from reciprocity. The job match dissolves at the end of the period with exogenous probability $\tau$. The value function for an unemployed worker is

$$U_t = z_t + \frac{1}{1 + r} E_t[f_{t+1}L_{t+1} + (1 - f_{t+1})U_{t+1}]$$

(2)

where $z_t$ denotes the opportunity cost of employment, reflecting unemployment benefits and the value of non-work activities. If unemployed, an individual finds a job and is employed in the next period with endogenous probability $f_t$.

2.2.2 Firms

There is a continuum of identical firms on the unit interval. Each firm can hire up to one worker and produces an amount $y_t$, where

$$y_t = \xi s_t e_t$$

(3)

where $s_t$ is a technology shock, $e_t$ is the amount of effort exerted by the worker and $\xi$ is a constant that ensures output equals unity in steady-state. We assume $s_t = e^{\xi t}$ where $e_t^s = \rho e_{t-1}^s + \eta_t^s$ and $\eta_t^s$ is distributed as $N(0, \sigma_s^2)$. The value function of a filled job is

$$J_t = y_t - w_t + \frac{1}{1 + r} E_t[(1 - \tau)J_{t+1} + \tau V_{t+1}]$$

(4)

where $V_t$ is the value function of a vacant job, given by

$$V_t = -\gamma + \frac{1}{1 + r} E_t[q_{t+1}J_{t+1} + (1 - q_{t+1})V_{t+1}]$$

(5)

Firms must pay a real cost of $\gamma$ to post a vacancy. Vacancies are then filled at the start of the next period with probability $q$. We follow the timing convention

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Footnote: The model developed in this section is similar to Martin and Wang (2018). The main difference is the specification of the reference wage, which enables our model to address C-R&K critique. In section 2.2.6, we discuss our previous paper and explain why it is unable to generate a large unemployment volatility if the opportunity cost of employment is pro-cyclical.
of Gertler et al (2015) and assume that new job matches become productive immediately.

We assume free entry of firms, so $V_t = 0$. This implies that the value function for vacancies simplifies to

$$J_t = (1 + r) \frac{\gamma}{q_t}$$

(6)

and so the value function for a filled job becomes

$$(1 + r) \frac{\gamma}{q_t} = y_t - w_t + (1 - \tau) E_t \frac{\gamma}{q_{t+1}}$$

(7)

or

$$y_t = w_t + \lambda_t$$

(8)

where $\lambda_t = (1 + r) \gamma \left[ \frac{1}{q_t} - \frac{1 - \tau}{1 + \tau} E_t \frac{1}{q_{t+1}} \right]$ is the marginal cost of hiring labour.

### 2.2.3 The Labour Market

The labour market is characterised by search frictions. Aggregate hiring is determined by the matching function

$$h_t = mu_t v_t^{1-\alpha}$$

(9)

where $h$ is the number of workers hired, $u$ is unemployment and $v$ are vacancies. $m$ and $\alpha$ are parameters characterising the matching function. unemployment rate. Defining labour market tightness as

$$\theta_t = \frac{v_t}{u_t}$$

(10)

the matching function can also be written as

$$h_t = mu_t \theta_t^{1-\alpha}$$

(11)

The vacancy filling rate, the probability of a firm filling a vacancy, is

$$q_t = \frac{h_t}{v_t} = m \theta_t^{-\alpha}$$

(12)

while the job finding rate, the probability that an unemployed worker finds a job is

$$f_t = \frac{h_t}{u_t} = \theta_t q_t$$

(13)

### 2.2.4 Optimal Effort

A worker chooses effort to maximise the value function in (1). The optimal level of effort is therefore

$$e_c(e_t) = R_c(e_t, w_t)$$

(14)
We follow Rabin (1993) and Danthine and Kurmann (2007) by assuming that
\[ R(e_t, w_t) = g(w_t)d(e_t) \]  
(15)
where \( g(w_t) \) is the worker’s perception of the wage offer of the firm and \( d(e_t) \) is the worker’s reciprocal response. We also assume \( c(e_t) = \frac{\omega_{e_t}^{1+\sigma}}{1+\sigma} \), \( d(e_t) = \frac{\kappa_{e_t}^{1+\gamma}}{1+\gamma} \) and \( g(w_t) = \left(\frac{w_t - w_t^{ref}}{w_t - w_t^{ref}}\right) \), where \( w_t^{ref} \) is the reference wage and \( (w - w_t^{ref}) \) is the differential of the wage over the reference wage in steady-state. The specification of \( g(w_t) \) implies that workers perceive a wage offer that results in a larger wage differential than normal as generous; the specification of \( d(e_t) \) implies that they will respond to this by supplying greater effort. The optimal level of effort is therefore
\[ e_t = \frac{1}{\xi} \left(\frac{w_t - w_t^{ref}}{w - w_t^{ref}}\right)^{\sigma} \]  
(16)
where \( \xi = (\frac{\sigma}{\gamma})^{\gamma} \) and \( \sigma = \frac{1}{\frac{\omega}{\kappa} - 1} \). The parameters of this effort function have been estimated by Della Vigna and Pope (2018); we use these estimates below in calibrating the model. Further supporting evidence for this effort function comes from the study of the amount of time spent on non-work while on the job by Burda et al (2016). Using individual-level longitudinal data from the American Time Use survey, linked to the Current Population Survey, they find that the amount of time spent on non-work is a decreasing function of the wage; as more time spent on non-work implied less effort, this is consistent with effort being an increasing function of the wage.

### 2.2.5 Wage Posting

The firm will choose the wage to maximise the value of a filled job, so
\[ \frac{\partial J_t}{\partial w_t} = 0 \]  
(17)
The optimality condition is
\[ \xi s_t e_{w,t} = 1 \]  
(18)
where \( e_{w,t} \) is the derivative of effort with respect to the wage. In doing so, the firm balances the positive impact of higher wages on output against the adverse effect on the wage bill. Using the production function, this implies the modified Solow Condition (Solow, 1979)
\[ \varepsilon_t = \frac{w_t}{y_t} \]  
(19)
where \( \varepsilon \) is the elasticity of effort with respect to the real wage. Using the optimal effort function, we obtain
\[ w_t = \frac{1}{1-\sigma} w_t^{ref} + \frac{\sigma}{1-\sigma} \lambda_t \]  
(20)
\(^6\)This differs from Danthine and Kurmann (2010), who assume \( c(e_t) = \frac{e_t^2}{2} \), \( d(e_t) = e_t^\kappa \) and \( g(w_t) = \log(w_t) \); our optimal effort function is therefore different.
The marginal cost of hiring enters this wage equation because of search frictions; a higher cost of hiring leads the firm to increase the wage in order to induce increased effort, increasing output along the intensive margin. Cyclicality of the wage arises from cyclicality in the reference wage and in marginal hiring costs. As we discuss below, econometric evidence suggests that the value for $\sigma$ is small. This implies that the reference wage has a powerful effect on the wage, whereas the impact of marginal hiring costs on the wage is small. Thus although marginal hiring costs are highly volatile, this does not feed through into wage volatility. Cyclicality of the wage therefore largely reflects cyclicality in the reference wage. A small value of $\sigma$ increases the importance of main mechanism of the model, which is that, because of the effort effect, the wage is linked to the reference wage. It also reduces the impact of labour market conditions, reflected in the marginal cost of hiring, on wage determination. As Hall and Milgrom (2008) point out, this is essential for generating a mildly cyclical wage. In their alternating offer bargaining model, Hall and Milgrom achieve this by calibrating a very small probability that wage negotiations break down. There is no evidence suggesting this calibration is plausible. In contrast, a small effort effect is supported by recent evidence from the behavioural economics literature\textsuperscript{7}.

### 2.2.6 The Reference Wage

In order to complete the model, we need to specify the reference wage. The Behavioural Economics literature does not provide guidance on how the reference wage varies across the business cycle. Therefore any choice will be to some extent ad hoc. In order to reduce this ad hoc element, we choose to restrict our choice to measures of the reference wages that satisfy two criteria: they must be model-based and must be consistent with available empirical evidence. This evidence comes from Koenig et al (2017), who analyse longitudinal individual-level data on the lowest wage at which workers would take a job, based on data from the British Household Panel Survey, covering 1991-2009, and the German Socio-Economic Panel, covering 1984-2010. We follow Koenig et al (2017) in interpreting this as being informative about the reference wage. Koenig et al (2017) find that the reference wage has a low volatility across the business cycle and is less volatile than the wage\textsuperscript{8}.

Our decision to restrict our choice to model-based measures of the reference wage distinguishes our model from the existing literature, where the reference wage is often expressed as a reduced form function of previous values of the wage and other variables. Danthine and Kurmann (2010) express the reference wage as a function of productivity and the lagged real wage. Koenig et al (2017)\textsuperscript{8}\textsuperscript{,}\textsuperscript{7}

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\textsuperscript{7}A low value of $\sigma$ implies that $\chi$ is small relative to $\varphi$, so the marginal disutility of effort is more responsive to increased effort than is the marginal reciprocity response. In that sense, the main mechanism underlying our model relies on a weak reciprocity effect.

\textsuperscript{8}This evidence is obtained for Germany and the UK. There is no equivalent evidence for the US; the longitudinal data on job search analysed by Krueger and Mueller (2012, 2016) does not span enough time to allow cyclicality of reservation wages to be investigated.
express the reference wage as a function of the lagged wage and the unemployment rate\(^9\). Although simple and intuitive, these formulations are vulnerable to the objection that they are an ad hoc device for adding wage rigidity to the model. Other models, eg Martin and Wang (2018), assume that the reference wage is a function of the "fair wage", where this is modelled by assuming that workers feel entitled to a share of the surplus from their current job match. This gives a wage equation similar to that obtained with Nash bargaining between worker and firm. Shocks to the opportunity cost of employment generate wage volatility which reduces the firm’s incentive to post vacancies and so leads to a smaller response of unemployment. As a result, models based on this approach cannot match the observed volatility of unemployment.

For the reference wage, we use the annuity value of unemployment, given by

\[ w_t^{ref} = \frac{r}{1 + r} U_t \]  

(21)

This measure is simple and intuitive: it is a worker’s expectation of the value of their lifetime earnings on an alternative career path in which they are currently unemployed rather than employed. It satisfies our criteria since it is model-based, reflecting the value function of an unemployed worker and is consistent with the empirical evidence in Koenig et al (2017) as it has a low volatility across the business cycle, being less volatile than the wage. Our preferred measure is equivalent to the reservation wage in steady-state but is less volatile across the business cycle. The reservation wage is not a suitable measure of the reference wage as it is highly volatile across the business cycle\(^10\).11

\(^9\)There are other possibilities. In an early contribution to the literature, Summers (1988) assumes the reference wage reflects the average earnings of employed and unemployed workers, so \( w_t^{ref} = u_t b_t + (1 - u_t) w_t \). As Danthine and Donaldson (1990) argue, this specification implies a highly volatile reference wage. Alternatively, one might adapt an equation for the real wage used by, among others, Krause and Lubik (2007), Shimer (2010) and Gertler et al (2016), who express the reference wage as \( w_t^{ref} = (1 - \omega) w_t^{Nash} + \omega w_{t-1} \) where \( w^{Nash} \) is the real wage implied by Nash wage bargaining. For large values of \( \omega \), this gives a low volatility of the reference wage.

\(^10\)This has been demonstrated using simulations of the theoretical standard search frictions model by, eg, Koenig et al (2017); it can also be shown in the context of the model developed in this paper.

\(^11\)To explore the relationships between the annuity value of unemployment and the reservation wage, we define \( \delta(w_t - w_t^{ref}) = c(w_t - w_t^{ref}) - R(w_t - w_t^{ref}) \) as the cost of exerting effort, net of the reciprocity effect, where we have used (16) to express this as a function of \( (w_t - w_t^{ref}) \). We can then define the reservation wage as the wage that satisfies \( L_t = U_t \), and so, from (1)

\[ U_t = w_t^{res} - \delta(w_t^{res} - w_t^{ref}) + \frac{1}{1 + r} E_t[(1 - \tau)L_{t+1} + \tau U_{t+1}] \]  

(22)

Using this, the reservation wage can be expressed as the implicit equation

\[ w_t^{res} = \frac{r}{1 + r} U_t - \left( \frac{1 - \tau}{1 + r} E_t(L_{t+1} - U_{t+1}) + \frac{1}{1 + r} (E_t U_{t+1} - U_t) \right) + \delta(w_t^{res} - w_t^{ref}) \]  

(23)

The reservation wage is the sum of three terms. The first is our measure of the reference wage, the annuity value of unemployment given by \( \frac{r}{1 + r} U_t \); this has a low volatility across the business cycle. By contrast, the second term is highly volatile across the business cycle and
With this reference wage, the wage offered by firms to workers is

\[ w_t = \frac{r}{(1-\sigma)(1+r)} U_t + \frac{\sigma}{1-\sigma} \lambda_t \]  

(24)

This wage offer is always accepted by workers since it exceeds the reference wage\(^1\). The wage equation has three important properties. First, the low volatility of the reference wage implies that the wage has a low volatility. As a result, as we discuss in the next section, the model is able to generate a large volatility of unemployment. Second, cyclical variation in the opportunity cost of employment has little impact on the reference wage and thus has only a small effect on the wage. The limited impact of the opportunity cost of employment on the reservation wage is supported by evidence in Burda et al. (2016), who find that the amount of time spent on non-work is not affected by unemployment benefits. This is consistent with a small impact of the opportunity cost of employment on the reservation wage, leading to a small impact of the opportunity cost on effort. Third, labour market conditions affect the wage. A rise in unemployment leads to a reduction in the reference wage and thus in the wage. In this way, the model contains the same effects as the existing literature that models the reference wage as a more direct function of unemployment.

### 2.3 Unemployment Volatility

The mechanism through which this model generates unemployment volatility is similar to Hall (2005). Consider a positive productivity shock. Firms respond by posting more vacancies; this leads to a fall in unemployment and an increase in labour market tightness. In response to a negative productivity shock, firms post fewer vacancies, leading to a rise in unemployment and a fall in labour market tightness. Through this mechanism, the model generates variations in unemployment and vacancies across the business cycle.

We note that the behavioural aspects of our model introduce a refinement to this basic mechanism that amplifies cyclical movements in unemployment. The increase in labour market tightness increases the job finding rate, which in turn increases the annuity value of unemployment. From (23), firms respond to this, and to the increase in the marginal cost of hiring, by increasing the wage. Since

\[ w_t - w_t^{eff} = \frac{\sigma r}{(1-\sigma)(1+r)} U_t + \frac{\sigma}{1-\sigma} \lambda_t \]  

(25)

this induces an increase in effort. This increase in effort increases the incentive of firms to post vacancies and so the effort effect amplifies cyclical movements in unemployment. This mechanism is illustrated in section 3.2) below, where we present impulse response functions based on simulations of our model. This accounts for the unsuitability of the reservation wage as a measure of the reference wage. The third term, \( \tau(w_t^{eff} - w_t^{res}) \), also has a low business cycle volatility, since the small value for \( \sigma \) implies that the response of effort to the wage is small\(^1\).

\(^1\) Since it is optimal for firms to offer a wage that exceeds the reference wage, the Diamond Paradox is not applicable in this case.
argument relies on pro-cyclical movements in effort. This is supported by estimates in Burda et al (2016) showing that non-work is higher when the unemployment rate is higher and hence that effort is pro-cyclical. Production function based empirical studies of productivity, eg Basu and Fernand (2001), also suggest that effort is procyclical, although there is no consensus in the literature about this.

The model is able to generate a large volatility of unemployment since the response of the wage to productivity shocks is small. As Shimer (2005) argued, a highly pro-cyclical wage will absorb part of an increase in productivity, reducing the incentive for firms to post vacancies and so dampening cyclical movements in unemployment and vacancies. The wage is not highly pro-cyclical in our model. Following a productivity shock, there is an increase in the annuity value of unemployment. This leads to an increase in the reference wage and thus to an increase in the wage. However, from (24), the increase in the reference wage is only small, since workers understand that transient shocks have only a limited impact on the income they receive throughout their career. As a consequence, firms respond to productivity shocks by making large changes to the level of vacancies they post, but only small changes to the wage. By doing so, they generate large movements of unemployment across the business cycle.

The model is able to generate a large volatility of unemployment even if the opportunity cost of employment is as strongly procyclical as the evidence in Chodorow-Reich and Karabarbounis (2016) suggests. This is because the opportunity cost affects the wage mainly through its impact on the annuity value of unemployment. The cyclicality of the opportunity cost has only a limited impact on the annuity value of unemployment since any fluctuations in the opportunity cost have only a temporary effect on the expected lifetime earnings of a currently unemployed worker. Our model is therefore immune to the C-R&K critique. This is because the opportunity cost affects the wage mainly

13 In our model, the dampening effect of higher wages on vacancy creation is stronger than the stimulus to vacancy creation that comes from procyclical effort. However, the behavioural aspects of the model imply that our model requires less wage rigidity than the standard search frictions models to match the volatility in the data.

14 The model is less able to generate a large volatility of unemployment if wages are set through worker-firm bargaining rather than through wage-posting by the firm. With wage bargaining, the wage is chosen to maximise \[ J_t = (L_t - U_t)^{1-\phi} J_t^{1-\phi} + (1-\phi)(L_t-U_t) J_t^{1-\phi} = 0 \]. Since effort is endogenous, \[ \frac{\partial J_t}{\partial w_t} = \frac{\partial J_t}{\partial w_t} - 1 \]. Compared to the standard search frictions model, the negative impact of higher wages is offset by an increase in output. Since \[ J_t = (1-\phi)(1-\frac{\partial J_t}{\partial w_t}) (L_t - U_t) \], we obtain \[ (L_t - U_t) = \mu_t S_t \], where \[ \mu_t = \frac{\phi}{1-(1-\phi) \frac{\partial J_t}{\partial w_t}} \] is the share of the surplus from a job match accruing to the worker. Since \[ \mu_t > \phi \], workers obtain a larger size of the surplus from the job match when there are effort effects. Since the influence of cyclical labour market conditions on the wage is higher when workers obtain a larger share of the surplus, this implies that the wage will be more volatile under wage bargaining than under wage posting. Therefore a model with wage bargaining will be less able to generate a large volatility of unemployment than a model with wage posting.

15 It would be possible to obtain a similar result using an ad hoc reduced form expression for the reference wage in which the role of the opportunity cost of employment is suppressed.

16 The model would not be immune from the critique if the wage were set through the
through its impact on the annuity value of unemployment. Cyclicality of the opportunity cost has only a limited impact on the annuity value of unemployment since fluctuations in the opportunity cost are only temporary and so have only a limited effect on an unemployed workers expected lifetime earnings. Our model is therefore immune to the C-R&K critique.'

3 Simulation Results

In this section we present simulation evidence to complement the analysis in previous sections. We calibrate the model, as discussed below. The model comprises the production function in (3), the optimality condition in (8), the equations for labour market dynamics in (9)-(13), the effort function in (16), the wage equation in (23) and the exogenous process generating productivity shocks. This model is linearised and solved numerically; it is then simulated and key statistics generated.

3.1 Calibration Strategy

The key parameter of the behavioural search component of our model is the elasticity of the effort function, \( \sigma \). To calibrate this we use estimates of an effort function similar to (16) in Della Vigna and Pope (2018). Della Vigna and Pope (2018) use data on the behaviour of 10,000 participants in an experiment using the Mechanical Turk platform. They estimate a very low elasticity of the effort function. Based on their estimates, we calibrate \( \sigma = 0.02 \). From (16), the implies that \( \varphi \) is much larger than \( \chi \), so the marginal disutility of effort is much larger than the marginal benefit from reciprocation. We follow Danthine and Kurmann (2010) in setting the elasticity of \( d(e_t) \) to \( 1 + \chi = 0.75 \); this implies \( \chi = -0.25 \). These calibrated values for \( \sigma \), and \( \chi \) imply \( \varphi = 49.75 \). We set \( \omega = 0.755 \) and \( \kappa = 0.0093 \) to ensure labour productivity equals unity in steady-state; our results are not sensitive to the specific values chosen\(^{17}\).

We calibrate the remaining structural parameters to ensure that our model matches the first moments and other key features of US data; we do not calibrate parameters in order to ensure that the simulated volatilities implied by our model match the data. In this we differ from Hall and Milgrom (2008) and Hagedorn and Manovskii (2008) who respectively calibrate the probability of strategic bargaining approach of Hall and Milgrom (2008). In that case, the wage can be approximated as the implicit function

\[
\omega_t = \frac{1}{2} [y(e(w_t)) + b_t + \omega + c(e(w_t)) - R(e(w_t), w_t)]
\]

where effort is given by (16) and \( \omega \) is the cost to the firm of a disagreement in bargaining. The model is able to generate a large volatility of unemployment if the opportunity cost of employment is fixed. However it is not able to do so if the opportunity cost is strongly procyclical as the wage is then pro-cyclical, dampening the incentive of firms to post vacancies (Chodorow-Reich and Karabarbounis, 2016).

\(^{17}\)With this calibration, the utility cost of effort and the utility benefits of reciprocation are equal to 1.5% and 1.2% respectively of output in steady-state.
wage negotiations breaking down and the opportunity cost of employment to match observed unemployment volatility. We normalize a time period to be one month. We calibrate $\rho^*$ and $\sigma_s$ to match the persistence and volatility of US productivity growth for 1951-2004 as reported in Hagedorn and Mankovskii (2008); this gives $\rho^* = 0.765$ and $\sigma_s = 0.0084$. We target $u = 0.059$; the average value of the US unemployment rate, 1948-2016. We also target $\theta = 0.72$, in steady-state, the value for average labour market tightness reported by Pissarides (2009). The discount rate is set as $\tau = 0.42\%$, equivalent to an annual discount rate of 5%. The average monthly job separation rate is $f = 0.036^{18}$. Since $fu = \tau(1 - u)$, our values for $\tau$ and $u$ imply the average job-finding rate is $f = 0.576$; this is close to Shimer (2012)'s estimate of $f = 0.594$. We follow Hall and Milgrom (2008) and assume that the average opportunity cost of employment is $z = 0.71$. Chodorow-Reich and Karabarounis (2016) report a range of empirical estimates of the opportunity cost of employment, between 0.47 and 0.96, based on alternative specifications of the flow value of non-work; the mid-point of this range is also $z = 0.71$. For the matching function, we set $\alpha = 0.4$; this is the mid-point of the range of estimates obtained by Petrongolo and Pissarides (2001). We calibrate $\gamma$ so that the model matches $\theta = 0.72$, in steady-state. This gives $\gamma = 0.38$; this value lies in the range of calibrated values of $\gamma$ in the literature$^{19}$. We then calibrate $m$ to satisfy the relationship $f = m\theta^\alpha$; this implies $m = 0.70$. Our calibrated parameter values are summarised in Table 1.

<table>
<thead>
<tr>
<th>Table 1—Calibrated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$\rho^*$</td>
</tr>
<tr>
<td>$\sigma_s$</td>
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<tr>
<td>$\tau$</td>
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<td>$r$</td>
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<td>$z$</td>
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<td>$\gamma$</td>
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</tr>
<tr>
<td>$\kappa$</td>
</tr>
<tr>
<td>$\xi$</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations

$^{18}$In the literature, monthly values of $\tau$ vary between 0.03 (Hall and Milgrom, 2008) and 0.036 (Pissarides, 2009).

As Table 2 shows, we are able to match our calibration targets closely. Being able to do so using standard calibrations of the non-behavioural parameters of the model suggests that adding behavioural effects to an otherwise standard search friction model does not distort the rest of the model. In particular, calibrating $\sigma$ using the small estimated value in Della Vigna and Pope (2018) enables the model to match the data moments in Table 2).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>U.S Data</th>
<th>This Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>Labour Market Tightness</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>$u$</td>
<td>Unemployment Rate</td>
<td>0.059</td>
<td>0.059</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations; the average US unemployment rate is from the BLS; average tightness is from Pissarides (2009)

### 3.2 Simulation Results

We assess our models through their ability to match the volatilities of unemployment, labour market tightness and vacancies and the correlations between unemployment and vacancies and between unemployment and labour market tightness, as reported by Hagedorn and Manovskii (2008) for the US, 1951-2004. These are detailed in the panel a) of Table 3). Panel b) of Table 3) presents our simulation results in the case where the opportunity cost of employment is constant. We obtain large volatilities for unemployment, vacancies and labour market tightness, substantially larger than the volatilities of the underlying shocks. The volatilities of unemployment and labour market tightness are close to the values in panel a); the volatility of vacancies is somewhat larger than the target. The autocorrelations are also large, although for vacancies and labour market tightness they are slightly lower than in the data. The model generates a large negative Beveridge Curve correlation between unemployment and vacancies, although this is not as strong as in the data; the correlations between unemployment and labour market tightness and between vacancies and labour market tightness are also large and close to the values observed in the data.

Panel c) of Table 3) presents results for the case where the opportunity cost of employment is pro-cyclical and $z$ is proportional to $s$, in line with Chodorow-Reich and Karabarbounis (2016). The results are similar to panel b), with large volatilities for unemployment, vacancies, labour market tightness and with similar correlations between the variables.

---

20 The simulated correlations between productivity shocks and unemployment and vacancies are larger than the correlations observed in the data; this is a well-known shortcoming of search frictions models, see Hagedorn and Manovskii (2008), and suggests a wider range of potential shocks might be considered.

21 Chodorow-Reich and Karabarbounis (2016) find that the elasticity of the cyclical component of $z_t$ with respect to the cyclical component of the marginal product of labour exceeds 0.8 and in many specifications is close to unity.
Table 3—Simulation Results


<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$s$</th>
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<tbody>
<tr>
<td>Standard Deviation</td>
<td>0.125</td>
<td>0.139</td>
<td>0.259</td>
<td>0.013</td>
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<tr>
<td>Quarterly Autocorrelation</td>
<td>0.870</td>
<td>0.904</td>
<td>0.896</td>
<td>0.765</td>
</tr>
<tr>
<td>$u$</td>
<td>1</td>
<td>-0.919</td>
<td>-0.977</td>
<td>-0.302</td>
</tr>
<tr>
<td>Correlation Matrix $v$</td>
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<td>0.982</td>
<td>0.460</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td></td>
<td>1</td>
<td>0.393</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td></td>
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</tbody>
</table>

Panel b) Simulation results with constant $z$

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>0.116</td>
<td>0.157</td>
<td>0.253</td>
<td>0.013</td>
</tr>
<tr>
<td>Quarterly Autocorrelation</td>
<td>0.938</td>
<td>0.590</td>
<td>0.765</td>
<td>0.765</td>
</tr>
<tr>
<td>$u$</td>
<td>1</td>
<td>-0.698</td>
<td>-0.895</td>
<td>-0.845</td>
</tr>
<tr>
<td>Correlation Matrix $v$</td>
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<td>0.895</td>
<td>0.944</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Panel c) Simulation results with cyclical $z$

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>0.116</td>
<td>0.157</td>
<td>0.252</td>
<td>0.013</td>
</tr>
<tr>
<td>Quarterly Autocorrelation</td>
<td>0.938</td>
<td>0.590</td>
<td>0.765</td>
<td>0.765</td>
</tr>
<tr>
<td>$u$</td>
<td>1</td>
<td>-0.698</td>
<td>-0.894</td>
<td>-0.844</td>
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<tr>
<td>Correlation Matrix $v$</td>
<td>1</td>
<td>0.895</td>
<td>0.944</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Source: Authors’ calculations; panel (a) is from Hagedorn and Manovskii (2008)

The results in Panel b) show that a model that incorporates behavioural features can match the main features of post-war US labour market data if the opportunity cost of employment is constant. One might view this as merely an addition to the list of models that can do this, albeit one with solid empirical support. However, existing models cannot match the data if the opportunity cost of employment is cyclical, as the estimates in Chodorow-Reich and Karabarbounis (2016) suggest it is. As panel c) of Table 3) shows, the main contribution of our paper is that our model can match the data even when the opportunity cost of employment is highly cyclical. Thus our model is immune from the critique of the existing literature advanced in Chodorow-Reich and Karabarbounis (2016). As things stand, no other model can claim this.

Comparison of Panels b) and c) of Table 3) shows that cyclical fluctuations in the opportunity cost of employment have very little impact on our results. To see why, we note that a temporary productivity shock that increases the opportunity cost leads to only a small increase in the reference wage, as the transient shock has only a small impact on the annuity value of unemployment, defined in (2). As a consequence, cyclicality of the opportunity cost of employment adds little to the response of the wage to a productivity shock, explaining the similarity of
the results in Panels b) and c) of Table 3). However, we note the opportunity cost still plays a key role in wage determination and therefore in determining equilibrium unemployment. Consider a permanent shock which increases the opportunity cost of employment from 0.71 to 0.81. The permanent increase in the opportunity cost leads to a large increase in the reference wage and, through this, to a large increase in the wage. This implies a large increase in the equilibrium rate of unemployment, from 0.055 to 0.083.

Figure 1) illustrates the mechanisms underlying our model by presenting the impulse response functions in the case where the opportunity cost of employment is cyclical. Following a positive productivity shock, firms post more vacancies. So there is an increase in the job finding rate. As discussed above, the increase in the opportunity cost and the job finding rate leads to a small increase in the reference wage, which in turn leads to a small increase in the wage. Although the increase in the wage is small, it is larger than the increase in the reference wage; this is largely due to an increase in marginal hiring costs. The increase in the wage relative to the reference wage leads to an increase in effort. The small increase in the wage, together with the increase in effort, implies a large increase in vacancies. This results in a large fall in unemployment. These latter two effects cause a large increase in labour market tightness which in turn drives the large increases in the job finding rate and in marginal hiring costs.

4 Conclusions

Currently, all models in the literature that are able to match the observed volatilities, correlations and autocorrelations of unemployment, vacancies and labour market tightness in US data assume that the opportunity cost of labour is constant. Evidence in Chodorow-Reich and Karabarbounis (2016) that the opportunity cost of employment is in fact strongly procyclical undermines the ability of these models to match the data. In this paper, we propose a model that can match the data while being immune from the Chodorow-Reich and Karabarbounis critique. We do this by incorporating insights from behavioural economics into the standard search frictions model developed by Diamond (1982), Mortensen and Pissarides (1994) and Pissarides (2000). This results in a model in which output depends on effort exerted by workers and where firms post wages in order to elicit the optimal supply of effort from their workforce.

The model is able to match the observed volatilities of unemployment and vacancies in US data. Intuitively, this is because firms respond to a positive productivity shock by posting more vacancies; this leads to a fall in unemployment. The behavioural elements introduced by our model reinforce this familiar mechanism. The increase in vacancies and the fall in unemployment leads the firm to increase the wage so as to increase effort, increasing output along the intensive margin. This increase in effort increases the incentive of firms to post vacancies and so the effort effect amplifies cyclical movements in unemployment compared to the standard search frictions model. This mechanism operates even if the opportunity cost of employment is strongly pro-cyclical. This is because
the opportunity cost affects the wage through its impact on the annuity value of unemployment. This effect is only small as shocks to the opportunity cost of employment have only a limited impact on the annuity value of unemployment, since workers understand that transient shocks have only a limited impact on the income they receive throughout their career.

We might develop our analysis by investigating whether the model has explanatory power beyond the average characteristics of the US labour market. There are two aspects to this. First, we can analyse the performance of the model in different environments, for example in the European labour market where the vacancy-filling rate is lower than in the US. Doing so has proved a challenge for existing models of search frictions (eg Amaral and Tasci, 2016). Second, we might use our model to address the changes in the cyclicality of the US and other labour markets that have been identified by, among others, Gali and van Rens (2014), in which the volatility of employment and wages has increased. An analysis based around the incentives of workers to supply effort may well be able to contribute to this debate.
References


22


