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Blockholder Disclosure Thresholds and Hedge Fund Activism *

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Abstract

Blockholder disclosure thresholds shape incentives for hedge fund activism, which are jointly determined with real investment and managerial behavior. Uninformed investors value lower thresholds (greater transparency) when the cost of trading against an informed activist outweighs the benefits of the activist’s disciplining of management. Conversely, activists may desire disclosure thresholds if their threat of participation discourages managerial malfeasance, which is their source of profits. Hedge fund activism can be excessive: if market opacity sufficiently harms uninformed investors, the costs of reduced real investment outweigh the social benefits from managerial disciplining, and society benefits from lower thresholds.

Keywords: Hedge fund activism, blockholder disclosure thresholds, informed trading, investor activism.

JEL codes: G34, G14, G18, K22.

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1 Introduction

Hedge fund activism mitigates agency problems that affect governance in publicly-traded companies with dispersed owners (Brav et al. 2008; Bebchuk et al. 2015). However, their business models and the relatively short-term nature of their strategies often generate controversy.\footnote{Brav et al. (2010) find a median duration from disclosure to divestment of 266 days.} Activist hedge funds are, by their nature, informed traders that profit from trading on their information advantages at the expense of uninformed shareholders. As a result, activists may discourage investment, destroying value (Leland 1992; Bernhardt et al. 1995). Blockholder disclosure thresholds can limit the trading profits of activist funds, but this, in turn, affects their incentives to discipline management, and thus managerial behavior. Our paper analyzes how disclosure thresholds determine hedge fund activism, managerial behavior and the initial capital investment by dispersed uninformed investors, i.e., real investment. We derive the optimal policies from the perspectives of investors, activist funds, and society.

Highlighting the issues, in 2011, partners at Wachtell, Lipton, Rosen & Katz (WLRK), a prominent law firm specializing in corporate and securities law, submitted a petition to the SEC advocating that rules governing the disclosure of blocks of stock in publicly-traded companies be tightened. WLRK argued that the current disclosure threshold allows activist investors to secretly accumulate enough stock to create fundamental changes, and they only hold positions for brief periods of time (Brav et al. 2010; Becht et al. 2017). This, WLRK claimed, damages market transparency and investor confidence. Academics responded, arguing that tighter disclosure rules would discourage activists by reducing their ability to profit from stock purchases that do not yet reflect the value of their actions (Bebchuk and Jackson 2012; Bebchuk et al. 2013). This, they argued, would harm small investors, who would then not glean the value-enhancing benefits of hedge fund activism on corporate behavior.

We identify the key forces in the debate about optimal blockholder disclosure thresholds in the presence of activist hedge funds, and study their interplay. Our model reveals how disclosure thresholds affect (i) the incentives of activist funds to engage in costly managerial
disciplining; (ii) the real investment of small uninformed investors; and (iii) the choices by managers of whether to pursue potentially value-destroying activities. Our analysis sheds light on when and how the interests of uninformed investors and activist funds conflict, and where a welfare-maximizing regulator stands.

The driving mechanisms are complex. Uninformed investors benefit from the disciplining effect of hedge fund activism, but incur costs when trading with activists that are privately informed about their own value-enhancing potential. At the other end, activist funds profit from acquiring undervalued stock when their participation has not been revealed; but can be hurt if their trading profits, which represent trading losses for uninformed investors, reduce real investment, or if the possibility of their intervention deters managerial malfeasance.

We show that activists never benefit from a disclosure threshold solely because it boosts real investment, but that they can gain from tighter thresholds that reduce their incentives to intervene, thereby raising the likelihood that managers pursue their own interests at the expense of their firms. We find that, depending on how disclosure thresholds affect managerial actions, either uninformed investors or activist funds may value tighter disclosure thresholds, but that socially-optimal thresholds always lie weakly between their preferred thresholds. This reflects that society does not internalize trading transfers between uninformed investors and activist funds, but it does care about real investment and managerial discipline.

Our base formulation features a continuum of small investors who invest in a project with decreasing returns. The firm’s management can obtain private benefits by taking a value-destroying action. Should management do this, it may be observed by a large activist fund that is external to the firm, which can then engage in a costly intervention that disciplines management and restores project value. The fund’s incentives to incur these costs are provided by the opportunity to secretly acquire stock of the company before its price reflects the value of its intervention. The activist’s sole source of rents is the increase in stock value due to intervention relative to the acquisition price.

The activist endogenously determines how many shares to acquire. In our static deal-
ership model, a competitive market maker posts prices conditional on the net order flow. Then the activist trades along with a random measure of shareholders (initial investors) who receive liquidity shocks that force them to sell their shares. The activist’s order trades off the benefits of a larger block size and the costs of the information revealed. Liquidity trade is drawn from an exponential distribution. This structure allows us to solve for informed trade and its price impacts in closed form (Edmans 2009), and hence how disclosure thresholds affect an activist’s *ex-ante* expected trading profits, prior to an intervention decision.

A disclosure threshold limits the equity position that can be secretly acquired. Crossing the threshold reveals the activist’s position to the market maker, who then updates prices, eliminating any trading rents of the activist. As a result, in equilibrium, the activist’s position does not cross the disclosure threshold and the policy becomes an upper bound on its position and hence trading profits. Lower profits from trading reduce the activist’s incentives to intervene. Importantly, the expected levels of activism, and thus of managerial discipline, determine the profitability of real investment by uninformed investors. In turn, this real investment affects the value of intervention, creating a feedback effect on the incentives of activists to participate. The optimal disclosure threshold policy for each party reflects the tensions each faces with regard to the preferred level of market transparency.

Consider the tradeoffs for uninformed investors. Higher transparency (a lower disclosure threshold) reduces their trading losses, but it also reduces the willingness of hedge fund activists to intervene. In turn, this encourages management to pursue its own interests at the expense of shareholders. Uninformed investors value lower disclosure thresholds when the expected trading losses saved outweigh the benefit of free riding on the activist’s costly managerial disciplining. They gain from the reduced shares that activists acquire when those shares are not needed to induce activism, but are harmed when the share limit discourages activism. Their optimal disclosure threshold trades these considerations off.

Now consider the firm’s management. The manager can take a value-destroying action to obtain private benefits, but incurs a reputation cost if disciplined by the activist. Improve-
ments in the performance and governance achieved by activists often come at the expense of managers and directors who see sharp reductions in compensation and a higher likelihood of replacement (Brav et al. 2010; Fos and Tsoutsoura 2014). This threat of being disciplined by activists improves managerial performance (Gantchev et al. 2019). We capture this mechanism, recognizing the ex-ante disciplining role of activists in discouraging managerial malfeasance. Since higher trading transfers make an activist more willing to act if management misbehaves, they also induce better behavior by management.

This managerial feedback benefits uninformed investors, but, paradoxically, by reducing the likelihood of managerial misbehavior, it reduces an activist’s opportunities to profit from its business of disciplining management. When managers are sensitive to threats of activism, uninformed shareholders want to raise disclosure thresholds, as they only realize trading losses when an activist intervenes. Raising disclosure thresholds both increase activists’ intervention rates (ex-post disciplining) and discourage malfeasance (ex-ante disciplining). The same mechanism represents a tension for the activist, which trades off higher conditional trading profits against a lower probability of profiting. We show that whenever activists value a disclosure threshold, uninformed investors want greater activism, indeed preferring no threshold at all. Shareholders gain from an activist’s willingness to act without having to pay any trading costs. In effect, an activist’s willingness to act discourages excessively—from its perspective, but not shareholders—management’s desire to pursue its own interests at the expense of shareholders.

The final tension reflects that the activist’s trading profits depend on the value of intervention, which reflects the scale of real investment: value-enhancing actions in larger companies have bigger impacts. When the trading losses of initial investors are large relative to the benefits of managerial disciplining, activism reduces real investment and hence the value of the activist’s intervention. The activist does not internalize this investment feedback effect in its trading because initial investment has already been sunk. A disclosure threshold can serve as a commitment device for an activist to limit its trade, thereby raising real
investment. Surprisingly, we establish the activist never benefits from a disclosure threshold just because it boosts real investment: we prove that investment feedback is a second-order effect relative to trading transfers. For the activist, the benefits of increased trading on its information advantage always outweigh the cost of any reductions in real investment.

The negative effect of market opacity on real investment captures the concerns of the Williams Act (1968), which introduced disclosure thresholds. They were designed to “alert investors in securities markets to potential changes in corporate control and to provide them with an opportunity to evaluate the effect of these potential changes.”

2 Uncertainty over managerial behavior and activist trade translate into stock price discounts that benefit activist funds. Trading is a zero-sum game in which expected activist profits represent expected losses to uninformed investors (incurred by initial investors, even if they sold previously to new uninformed traders). When these losses outweigh the benefits of monitoring, hedge fund activism harms the initial investors, causing them to reduce investment. Conversely, activism fosters investment when it benefits uninformed investors. By regulating trading transfers, disclosure thresholds affect real investment. This link between market efficiency and economic efficiency was first made in Bernhardt et al. (1995) in the context of insider trading. Here, we focus on the interplay between corporate governance and real investment. We identify twin real effects of informed trade by hedge fund activists: (i) it encourages activists to create value by intervening in underperforming companies, and (ii) it affects real investment.

We characterize the socially-optimal disclosure threshold and show that it coincides with the preferred policies of uninformed investors and activists only if they all prefer not to have a binding threshold. Society (a regulator) does not internalize the transfer of trading profits from uninformed investors to the activist, caring only about the gross expected value of the firm net of the costs of capital and activism, i.e., caring only about the indirect real effects of trading in financial markets. We show that the socially-optimal disclosure threshold is

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always weakly between the thresholds preferred by shareholders and the activist hedge fund. Practically, our analysis provides guidance to regulators considering revisions to ownership disclosure rules. The multiple considerations entering optimal disclosure thresholds suggest the merits of a tailored policy approach that accounts for market and firm characteristics.

Our base model assumes that an activist only takes positions if it observes management pursuing value-destroying actions, acting whenever expected trading profits outweigh the cost of intervention. Gantchev (2013) finds intervention costs to be sizable. This leads us to consider how outcomes are affected when activists can sometimes engage in stock-picking, profiting from trading on information that management is working to maximize shareholder value, in which case they need not incur costs of intervention. In practice, stock-picking represents a major source of activist profits (Cremers et al. 2020; Feng et al. 2020). We show that such stock-picking has positive spillover effects on disciplining management: stock-picking based on good news raises the likelihood that negative net order flows do not contain activist trade, resulting in lower stock prices that make activism more profitable. This result contributes to the debate on value creation by activist funds, showing that their stock-picking may, in fact, increase profits from disciplining management, improving corporate governance.

Our paper contributes to a long-standing formal literature of governance through voice (Shleifer and Vishny, 1986; Maug, 1998; Kahn and Winton, 1998), and the more recent focus on activist funds (Burkart and Lee, 2018; Brav et al., 2019; Corum and Levit, 2018; Edmans et al., 2019; Burkart and Dasgupta, 2020). Our analysis recognizes the role of financial markets on the incentives of activists to take positions in a target company and intervene. Back et al. (2018) shares this property. They characterize the dynamic trading by an activist investor, showing how the intervention cost function affects outcomes. We simplify the trading process (static) and the cost of intervention (fixed) in order to endogenize firm value in terms of real investment and managerial behavior, and to study the role of market transparency.

The relation between market transparency and real investment is close to seminal papers in the insider trading literature (Leland, 1992; Bernhardt et al., 1995).

Despite the long-term value of hedge fund activism (Brav et al. 2015; Bebchuk et al. 2015), researchers have found that activist funds have short investment horizons (Brav et al. 2008, 2010; Boyson and Mooradian 2011), and that they acquire stock after targeting a firm (Bebchuk et al. 2013). Share prices typically rise sharply when an activist’s presence is revealed because the market anticipates their intervention, and Bebchuk et al. (2015) finds that these post-disclosure spikes in share prices reflect the long-term value of intervention. The main source of rents for activist funds is the price increase caused by their own interventions on the shares acquired prior to revealing themselves (Bebchuk and Jackson 2012; Becht et al. 2017).

Our modelling assumptions are motivated by findings in the empirical literature on hedge fund activism. These findings include: a positive relation between liquidity and hedge fund activism (Collin-Dufresne and Fos, 2015; Gantchev and Jotikasthira, 2017), the value-enhancing nature of interventions by activist funds (Brav et al., 2008; Clifford, 2008; Klein and Zur, 2009; Boyson and Mooradian, 2011; Brav et al., 2015; Bebchuk et al., 2015), and the costs of interventions for activists (Gantchev, 2013) and managers in target companies (Brav et al., 2010; Fos and Tsoutsoura, 2014). La Porta et al. (2006) and Djankov et al. (2008) document evidence of the positive relationship between investor protection and capital formation. Several of our predictions have empirical support, including the extent of the price reaction to the disclosure of an activist fund (Bebchuk et al. 2015), that disclosure thresholds constrain funds’ positions (Bebchuk et al. 2013), and that managers react to the threat of activism (Gantchev et al. 2019; Feng et al. 2020).

Section 2 builds our model of hedge fund activism. Section 3 introduces blockholder disclosure thresholds and derives optimal policies. Section 4 adds stock-picking by funds. Section 5 discusses the applicability of our results, and possible tests of additional predictions. A conclusion follows. An appendix contains all proofs.
2 Hedge Fund Activism

In this section we model hedge fund activism and characterize its inter-linkages with corporate management and real investment. We consider a firm that raises capital for a project whose value depends on the initial investment by uninformed investors and a business plan that may be either good or bad. The manager can deliberately adopt the bad business plan in order to obtain private rents at the expense of shareholders. The bad plan reduces value for shareholders unless an outside activist hedge fund intervenes to discipline management and implement the good plan. All agents are risk neutral. There are four dates, $t = 0, 1, 2, 3$. There is no discounting.

At date $t = 0$, a continuum of dispersed investors choose capital investment $k$ in a project with an expected date $t = 3$ payoff of

$$V = f(k) \left[1 - \delta \cdot 1_{\{m=0\}} \right].$$

(1)

Here, $f$ is a standard production technology with $f'(\cdot) > 0$, $f''(\cdot) < 0$, $f'(0) \to \infty$. The indicator function accounts for the business plan $m \in \{0, 1\}$ implemented by the manager at $t = 1$. The good plan ($m = 1$) yields cash flows $f(k)$ to investors. The bad plan ($m = 0$) yields nothing with probability $\delta \in [0, 1]$. Equivalently, the bad plan destroys a fraction $\delta$ of the project’s value. Investors are uninformed, unable to distinguish between good and bad business plans. The marginal cost of capital is $r > 0$. Initial investors become shareholders who receive claims to terminal project payoffs that they may trade in a market at $t = 2$. We normalize shares outstanding to have measure one.

At $t = 1$, the manager adopts the business plan. If the manager implements the good plan ($m = 1$), she receives a payoff that is normalized to zero at $t = 3$. If, instead, the manager adopts the bad plan ($m = 0$), her payoff depends on whether she is subsequently disciplined by the activist hedge fund. If the activist does not intervene, adopting the bad business plan gives the manager a fixed benefit $\phi$. If the activist disciplines the manager,
she does not receive the private benefit and incurs a privately-observed reputation cost $\rho$. Other market agents share a common prior that $\rho$ is distributed over $[0, R]$ according to a strictly positive density $h$ and associated cumulative function $H$, with $\varphi < R$.

At $t = 2$, initial investors receive liquidity shocks and must sell their stock in a competitive dealership market. These investors place orders to sell a fraction $l$ of stock, where $l$ is exponentially distributed with density

$$y(l) = \begin{cases} 
\mu e^{-\mu l} & \text{if } l \geq 0 \\
0 & \text{if } l < 0.
\end{cases}$$

(2)

Parameter $\mu > 1$ captures market liquidity, with larger values of $\mu$ representing more illiquid markets. On average, liquidity sales represent a proportion $\frac{1}{\mu}$ of the stock.\footnote{Analysis is qualitatively unchanged if initial investors sell fraction $\gamma l$ of shares, where $\gamma \in (0, 1)$ explicitly measures the scale of liquidity shocks relative to firm size.} An activist, who is an outsider to the firm, may also be present in the market. The activist identifies managerial malfeasance (business plan $m = 0$) when it occurs with probability $\lambda < 1$. The activist can discipline management at a cost $c$, forcing the firm to shift from the bad business plan to the good one. The activist privately observes his cost of activism. Other market agents share a common prior that $c$ is distributed on $[0, C]$ according to a strictly positive and weakly decreasing density $g$ and associated cumulative distribution $G$. The activist acquires a fraction $\alpha$ of the company’s shares, which we term his position, trading at the same time as investors receiving liquidity shocks. A competitive market maker observes the net order flow $\omega = \alpha - l$ from the activist and initial investors, but not its components, and sets a price that equals expected project payoffs given $\omega$, i.e., the market maker breaks even in expectation, as in Kyle (1985).

To ease presentation, we assume that the activist cannot trade on private information that the manager maximized shareholder value (choosing $m = 1$). That is, we assume that the activist can only intervene if $m = 0$. Section 4 relaxes this assumption and discusses the
implications of possible stock-picking by the activist fund. For simplicity, we also assume
that if an activist takes a position after management misbehaves (takes action $m = 0$) then
he disciplines management; i.e., he does not “cut-and-run” by selling shares before engaging
with management. Cutting and running becomes unattractive when it impairs the reputation
of activist funds, which Johnson and Swem (2020) find to be important for their profitability.

At $t = 3$, the project delivers cash flows $f(k)$ if the manager implemented the good plan
or if the activist disciplines the manager. Otherwise, the expected cash flows are $(1 - \delta) f(k)$.
The manager receives a payoff of zero from adopting the good business plan, and a payoff
from adopting the bad plan of $\varphi$ or $-\rho$ depending on whether or not the activist intervenes.
Figure 1 summarizes the timing.

$$
\begin{array}{cccc}
 t = 0 & t = 1 & t = 2 & t = 3 \\
 Investors & Manager implements business plan $m \in \{0, 1\}$ & Liquidity traders sell $l \sim \exp(\mu)$. & Cash flows and management payoff realize \\
 invest $k$ & & If the activist observes $m = 0$, he may acquire $\alpha$ and incur cost $c$ to implement $m = 1$. & \\
 & & The market maker observes $\omega = \alpha - l$ and sets price $P = E[V|\omega]$. & \\
\end{array}
$$

Figure 1: Time line

The parameter $\delta$ represents the value destroyed by the bad business plan in expectation; while $\varphi$ represents the private benefits from managerial malfeasance that determine
the manager’s incentives to implement the bad plan. These parameters capture the severity
of the agency problem between management and ownership. If $\delta = 0$, both business plans
yield cash flows $f(k)$, so there are no frictions between investors and the manager, and thus
no room for managerial disciplining; and if $\varphi = 0$, the manager always implements the good
business plan. In contrast, $\delta > 0$ together with $\varphi > 0$ imply that the manager may destroy
shareholder value to obtain private benefits, creating a potential role for hedge fund activism.

Both the manager’s private benefits from malfeasance $\varphi$ and the reputation costs of being
disciplined by an activist $\rho$ allow for multiple interpretations. For instance, managerial ben-
efits from acting against shareholders might be related to increasing executive compensation or empire-building mergers and acquisitions. The costs of being disciplined by an activist include the consequences for career prospects. For example, Fos and Tsoutsoura (2014) report that facing a direct threat of removal is associated with $1.3-$2.9 million in foregone income until retirement for the median incumbent director in their sample; and Keusch (2017) finds that in the year after activists intervene, internal CEO turnover rises 7.4%.

Parameter $\lambda$ captures the ease with which activist funds can identify malfeasance, for example the visibility of such behavior to activists. For instance, Gantchev et al. (2019) find evidence that industry peers of firms targeted by activists have increased perceptions of their exposure to activism, i.e., a higher $\lambda$, reflecting that once a form of malfeasance is uncovered, activists know what to look for in peer firms.

We assume that when the activist correctly identifies the bad business plan, he can discipline management with certainty, and that he buys shares in the target company at a single time where shareholders (investors) face liquidity shocks. In practice, these processes are dynamic (Collin-Dufresne and Fos 2015; Back et al. 2018), with uncertain costs (Gantchev 2013) and outcomes (Becht et al. 2017). We abstract from these mechanics to study the incentives provided by financial markets. What matters for our analysis are the expectations that an activist forms about these costs and outcomes at $t = 2$ when deciding whether to try to discipline management. The decision is based on the balance between expected financial benefits and engagement costs, and the likely dynamic price impacts of trading—and not the particular paths that can be realized given a decision to move forward. In our setting, the cost of activism $c$ is orthogonal to initial investment. This reflects the increasing returns of activism with respect to firm size on a reduced form that keeps our model tractable.

2.1 Market Equilibrium

We solve recursively for the perfect Bayesian Nash equilibrium. At $t = 2$, real investment has been sunk by uninformed investors and is observable to all parties, the manager adopted a
business plan, and the activist observes malfeasance with probability $\lambda$. Uninformed investors receive liquidity shocks and trade simultaneously with the activist in the dealership market with pricing by the risk neutral market maker. At $t = 1$ the manager adopts the plan $m \in \{0, 1\}$ that maximizes her expected private benefits given the expectation that the activist intervenes. At $t = 0$ uninformed investors invest capital, anticipating these subsequent events.

2.1.1 Trading

We begin by deriving the Bayesian Nash equilibrium in the trading subgame at date 2. The activist participation and trade is optimal given the market maker’s pricing function, and the market maker’s pricing function earns it zero expected profits given the activist’s decisions. We use $z$ to denote the endogenous probability of managerial malfeasance, which we characterize in the next section. Proposition 1 summarizes the details:

**Proposition 1** Suppose that at $t = 0$ investors made investment $k$, and that it is common knowledge that the manager adopted the bad business plan ($m = 0$) with probability $z$ at $t = 1$. Then, at $t = 2$, the activist takes a position

$$\alpha^* = \frac{1}{\mu}$$

and disciplines management if he observes managerial malfeasance and the cost of activism is sufficiently small, $c \leq c_t^*$, where

$$c_t^* \equiv [1 - Y(\alpha^*)] \frac{z}{\mu} \left[\frac{1 - \lambda G(c_t^*)}{1 - z\lambda G(c_t^*)Y(\alpha^*)}\right] \delta f(k).$$

Otherwise the activist does not participate.

The market maker, upon observing the net order flow $\omega$, sets prices

$$P(\omega) = \begin{cases} P_l \equiv \frac{1 - z\lambda G(c_t^*)Y(\alpha^*) - z(1 - \lambda G(c_t^*)Y(\alpha^*))\delta}{1 - z\lambda G(c_t^*)Y(\alpha^*)} f(k) & \text{if } \omega \leq 0 \\ P_h \equiv f(k) & \text{if } \omega > 0 \end{cases}.$$
A proof is in the Appendix; here we provide the key intuition. After observing the net order flow, the market maker updates beliefs according to Bayes Rule and sets prices in (5). A net buy order, \( \omega > 0 \), reveals with probability one that the activist took a position, in which case the project is sure to pay \( f(k) \). In contrast, (weakly) net sell orders, \( \omega \leq 0 \), are consistent with both the presence and absence of activism, and allow the activist to extract information rents from uninformed investors. Conditional on the activist acquiring a position \( \alpha \) when participating, the expected value of the project when there is a net sale of stock is

\[
P_l(\alpha) = \left[ \frac{1 - z\lambda G(c_t)Y(\alpha) - z(1 - \lambda G(c_t))\delta}{1 - z\lambda G(c_t)Y(\alpha)} \right] f(k).
\]

(6)

When the activist participates and liquidity shocks outweigh the number of shares that he buys, i.e., when \( l \geq \alpha \), there is a net supply of shares and the activist acquires the stock below its true value at \( P_l < f(k) \). If, instead, \( l < \alpha \), then there is a net demand for stock and the activist pays \( P_h \), making no profit. The probability that the activist camouflages his share purchase with liquidity sales is \( \int_{\alpha}^{\infty} y(l)dl = 1 - Y(\alpha) \). Letting \( a_1 \) denote activism and \( a_0 \) denote the absence of activism, the activist’s expected gross profits from acquiring a position \( \alpha \) are:

\[
E[\Pi_A|a_1] = [1 - Y(\alpha)]\alpha [f(k) - P_l].
\]

(7)

Inspection of (7) reveals that the activist faces a trade-off between the volume of stock acquired \( \alpha \) and the expected cost of information revelation \( 1 - Y(\alpha) \). This captures the adverse price effects by which the expected stock price paid by the activist rises as he buys more shares. The activist’s expected trading profits in (7) are maximized by an equity position \( \alpha^* = 1/\mu \); in equilibrium the market maker sets \( P_l \) in (5) to reflect \( \alpha = \alpha^* \). Greater liquidity, i.e., smaller \( \mu \), makes it easier for the activist to camouflage his trade, encouraging him to acquire a bigger position.

If the activist observes managerial malfeasance, he disciplines management when he expects it to be profitable, i.e., when \( E[\Pi_A|a_1] \geq c \). This relation together with the market
maker’s price policy $P_l(\alpha)$ pin down the activist’s cost participation cutoff in equilibrium:

$$c_t = [1 - Y(\alpha)]\alpha z \left[ \frac{1 - \lambda G(c_t)}{1 - z\lambda G(c_t)Y(\alpha)} \right] \delta f(k),$$  

(8)

which takes the form in (4) when evaluated at the optimal order $\alpha^* = 1/\mu$, i.e., $c_t^* \equiv c_t(\alpha^*)$.

To see that equation (8) only has one solution observe that the right-hand side decreases in $c_t$, implying that the equilibrium cutoff $c_t^*$ is unique. In equilibrium, the activist employs a threshold strategy such that, conditional on observing malfeasance, he takes a position $\alpha^*$ and disciplines management if and only if $c \leq c_t^*$.

The cutoff $c_t^*$ captures two key equilibrium features. First, it represents the activist’s participation threshold, and thus the extent of managerial disciplining. The probability that the activist intervenes to discipline the manager after observing the manager taking an action that reduces shareholder value is $G(c_t^*)$. Thus, a higher $c_t^*$ implies superior governance.

Second, $c_t^*$ captures the activist’s expected conditional trading profits. In equilibrium, the activist’s expected trading profits equal the expected trading losses of uninformed investors because trading is a zero-sum game in which the market maker expects to break even. Thus, $c_t^*$ represents the expected transfer of trading profits from uninformed investors to the activist conditional on the activist intervening.

Conditional trading transfers $c_t^*$ increase with real investment $k$ and with market liquidity $\mu^{-1}$.\footnote{To verify this, use the Implicit Function Theorem in (4) and note that $Y(\alpha^*) = 1 - e^{-1}$.} The positive effect of investment on trading transfers reflects that the greater is the project value, the more valuable is managerial disciplining, and the more profitable it is for the activist to intervene. This follows because the cost of activism does not grow proportionally with the company’s value, so the incentives for disciplining are positively related to stock ownership (Shleifer and Vishny 1986). The positive impact of liquidity on trading profits is a standard feature in settings with informed trading. Higher liquidity reduces information revelation to the market, which allows the activist to increase his position at a reduced risk of discovery (Kahn and Winton 1998; Maug 1998).
2.1.2 Management

At $t = 1$ the manager anticipates that if she adopts the bad business plan, then the activist will intervene to discipline her actions with probability $\lambda G(c_t)$. The manager privately observes her reputation cost of discipline $\rho$ and adopts the bad business plan only when it is expected to be profitable. That is, managerial malfeasance occurs when the expected private benefits from actions that destroy shareholder value (weakly) outweigh the expected cost of being disciplined by an activist fund: $[1 - \lambda G(c_t)]\varphi \geq \lambda G(c_t)\rho$. It follows that the manager employs a threshold strategy, implementing the bad business plan if and only if the reputation cost of discipline is small enough, i.e., if and only if $\rho \leq \rho_t$, where

$$\rho_t \equiv \varphi \left[ \frac{1 - \lambda G(c_t)}{\lambda G(c_t)} \right].$$

Thus, the probability of managerial malfeasance is $H(\rho_t)$. In equilibrium, $\rho^*_t = \rho_t(c^*_t)$, and the characterization of the trading game in Proposition 1 follows directly by setting $z \equiv H(\rho^*_t)$.

The solution for $\rho_t$ reveals that malfeasance declines with the conditional probability of activism $G(c_t)$: the more likely the activist is to participate after observing malfeasance, the less likely is the manager to misbehave. We call the managers’ response to the threat of activism the managerial feedback effect. The effect of activism is negative, reflecting that the threat of activism deters managers from destroying shareholder value. Activism disciplines management via two complementary channels: (i) ex post, the activist intervenes to change the business plan when it is bad; (ii) ex ante, it discourages the adoption of the bad plan.

2.1.3 Investment

At $t = 0$, uninformed investors anticipate trading outcomes and activism levels, and invest capital so as to maximize expected profits. In addition to the investment decision, Proposition 2 below characterizes expected project payoffs and how they are split among market participants in expectation at $t = 0$. This sets the stage for the analysis of the key interacting
forces in the model and the introduction of blockholder disclosure thresholds.

Proposition 2 The expected value at \( t = 0 \) of the project given investment \( k \) is

\[
E[V] = \left[1 - H(\rho^*_t)(1 - \lambda G(c^*_t))\delta\right] f(k) \equiv \pi_V f(k). \tag{10}
\]

The expected gross profits of the activist are:

\[
E[\Pi_A] = H(\rho^*_t)\lambda G(c^*_t) \frac{c^*_t}{f(k)} f(k) \equiv \pi_A f(k). \tag{11}
\]

The expected gross profits of uninformed investors are:

\[
E[\Pi_I] = (\pi_V - \pi_A) f(k) \equiv \pi_I f(k). \tag{12}
\]

The investment \( k \) by uninformed investors solves

\[
\pi_I f'(k) - r = 0. \tag{13}
\]

Total expected cash flows are the product of \( f(k) \) and the probability \( \pi_V \in [0, 1] \) that the project succeeds. Proposition 2 reveals that expected total rents are split between the activist and uninformed investors in proportions \( \pi_A/\pi_V \) and \( \pi_I/\pi_V \) respectively. This follows because the market maker earns zero expected profits, which means that activist trading profits are extracted one-for-one from uninformed investors. The expected gross profits of the activist equal the product of the unconditional probability \( H(\rho^*_t)\lambda G(c^*_t) \) that he participates and the expected trading profits \( c^*_t \) from participating. Uninformed investors obtain, in expectation, the rest of the “pie”, \( (\pi_V - \pi_A) f(k) \). Real investment, characterized by (13), maximizes the ex-ante expected profits of uninformed investors at date 0.

Proposition 2 shows that activism impacts real investment via its effect on the expected profits of uninformed investors. Investors face a tension as to their preferred extent of ac-
tivism. Higher trading transfers $c_t^*$ both incentivize activist participation to discipline management and deter malfeasance, i.e., increase $G(c_t^*)$ and reduce $H(\rho_t^*)$, potentially increasing the expected share $\pi_A$ of cash flows taken by the activist, which reduces the investors’ share $\pi_I$. However, the better governance that follows from increased transfers of trading profits also raises total expected cash flows of the project $\pi_V f(k)$. As a result, greater trading transfers $c_t^*$ to activists need not hurt uninformed investors. In particular, activism fosters real investment when investor gains from managerial disciplining outweigh the associated trading losses, and it discourages real investment otherwise.

This mechanism underscores the investment feedback effect faced by the activist. The value of activism is directly related to the size of the project—the profitability of the activist grows with real investment, i.e., $c_t^*$ grows with $k$. But, expected levels of activism affect investment. Therefore, expected activism affects real investment, which, in turn, affects the extent of activism. Crucially, the activist does not internalize this investment feedback in his trading decision at $t = 2$, because real investment has already been sunk. Thus, when the activist participates, he takes a position $\alpha^*$ that maximizes conditional expected profits (7), i.e., for a given $k$, rather than unconditional expected profits (11).

3 Blockholder Disclosure Thresholds

Blockholder disclosure thresholds require a shareholder to disclose stock holdings once they reach a certain fraction of the overall voting rights in a publicly-traded firm. In recent years, hedge fund activism has led some market participants and commentators to call for an expansion of these rules. However these petitions have often been met with opposition by academics and institutional investors. The model developed in Section 2 captures the key tensions faced by proponents in the two sides of the debate. We use this model to derive the optimal disclosure threshold policies for uninformed investors, activist funds and society.
3.1 The conditional impact

Ownership disclosure rules limit the number of undervalued shares that an activist can acquire, potentially reducing his incentives to participate. If a legal disclosure threshold $\alpha$ is implemented, an activist must publicly announce his position when it crosses the threshold. Then the activist has no incentive to establish a larger position because doing so would reveal his presence causing the stock price to rise to $P_h = f(k)$, which would eliminate his information rents, rendering intervention unprofitable. Corollary 3 follows:

**Corollary 3** A disclosure threshold $\alpha$ is binding if and only if $\alpha < \alpha^*$. In equilibrium, when a disclosure threshold binds the activist sets $\alpha = \alpha$.

The activist’s conditional trading profits $c_t(\alpha)$, characterized by (8), increase with his position for $\alpha < \alpha^*$. Thus, when the activist participates, he acquires position $\alpha = \min\{\alpha, \alpha^*\}$. It follows that a binding threshold necessarily reduces both the profits and extent of hedge fund activism given a firm characterized by $f(k)$ when the activist observed managerial malfeasance. To see this, let $c_t \equiv c_t(\alpha)$ represent the trading profits, and hence participation cutoff, associated with a position determined by a binding threshold $\alpha < \alpha^*$. Because trading profits increase in $\alpha$, activism is now less profitable, i.e., $c_t < c_t^*$, reducing the conditional probability that the activist participates to $G(c_t) < G(c_t^*)$. A direct consequence is that managerial malfeasance is more likely to destroy value. This mechanism is consistent with arguments against expanding ownership disclosure rules. However, our paper shows that they only comprise part of the overall effect.

In particular, the argument is incomplete because it neglects the effects of a disclosure threshold on both managerial behavior and real investment. Changes in expected activism at $t = 2$ alter management incentives at $t = 1$, and both determine real investment at $t = 0$. These, in turn, affect an activist’s opportunities and incentives to participate. A binding disclosure threshold reduces the conditional trading transfers from investors to the activist. However, this may incentivize real investment, creating a positive investment feedback that
increases activist participation, thereby reducing managerial malfeasance. We next study these tensions.

3.2 Optimal Policies

This section analyzes management and investment feedback effects and derives the optimal blockholder disclosure threshold policies. We present our results in terms of the *activism elasticity of management* and the *profit elasticity of activism*, defined respectively as

\[
\varepsilon_m = \frac{\partial H(\rho_t) G(c_t)}{\partial G(c_t) H(\rho_t)} \quad \text{and} \quad \varepsilon_a = \frac{\partial G(c_t)}{\partial c_t} \frac{c_t}{G(c_t)}. \tag{14}
\]

Here, \( \varepsilon_m < 0 \) captures management reaction to the threat of activism: the larger is \( \varepsilon_m \) in absolute value, the larger is the reduction in managerial malfeasance \( H(\rho_t) \) due to a marginal increase in the conditional probability of activism \( G(c_t) \). \( \varepsilon_a > 0 \) captures the responsiveness of activism to informed trading: the larger is \( \varepsilon_a \), the larger is the increment in activism \( G(c_t) \) associated with a marginal increase in expected trading profits \( c_t \). Absent a binding disclosure threshold, when the activist participates he takes a position \( \alpha^* \) and earns expected gross profits \( c^*_t \), the manager adopts the bad business plan if her reputation cost is less than \( \rho^*_t \), and the activism elasticity of management and the profit elasticity of activism are \( \varepsilon_m (c^*_t, \rho^*_t) = \varepsilon^*_m \) and \( \varepsilon_a (c^*_t, \rho^*_t) = \varepsilon^*_a \).

Proposition 4 derives the consequences of disclosure thresholds by characterizing the ordering of the optimal threshold policies for investors, the activist and a welfare-maximizing regulator representing society. We denote these policies \( \alpha_I, \alpha_A, \alpha_R \) respectively, and order the policies as a function of the activism elasticity of management, \( \varepsilon^*_m \). We assume that second-order conditions are well-behaved for investors and the activist; the Appendix shows that they are well-behaved when the activist’s costs of intervention and the management’s reputation costs have uniform distributions.

**Proposition 4** Suppose that the net expected profits of investors and activists are quasicon-
cave in $\alpha$ for $\alpha \leq \alpha^*$. Then there exist cutoffs on the activism elasticity of management for the activist $\varepsilon^{\ast A}_m$, investors $\varepsilon^{\ast I}_m$, and the regulator $\varepsilon^{\ast R}_m$, with $\varepsilon^{\ast A}_m < \varepsilon^{\ast I}_m < \varepsilon^{\ast R}_m$ such that

1. If the activism elasticity of management is sufficiently high, then only the activist benefits from a binding disclosure threshold: $\varepsilon^{\ast I}_m < \varepsilon^{\ast A}_m \Rightarrow 0 < \overline{\alpha}_A < \alpha^* \leq \{\overline{\alpha}_I, \overline{\alpha}_R\}$.

2. If the activism elasticity of management is moderately high, then no one benefits from a binding disclosure threshold: $\varepsilon^{\ast A}_m \leq \varepsilon^{\ast I}_m \leq \varepsilon^{\ast R}_m \Rightarrow 0 < \alpha^* \leq \{\overline{\alpha}_I, \overline{\alpha}_A, \overline{\alpha}_R\}$.

3. If the activism elasticity of management is moderately low, then only investors benefit from a binding disclosure threshold: $\varepsilon^{\ast I}_m < \varepsilon^{\ast}_m \leq \varepsilon^{\ast R}_m \Rightarrow 0 < \overline{\alpha}_I < \alpha^* \leq \{\overline{\alpha}_A, \overline{\alpha}_R\}$.

4. If the activism elasticity of management is low enough, then investors and society gain from a binding disclosure threshold, but activists do not: $\varepsilon^{\ast R}_m < \varepsilon^{\ast}_m \Rightarrow 0 < \overline{\alpha}_I < \overline{\alpha}_R < \alpha^* \leq \overline{\alpha}_A$.

Figure 2 illustrates the results; a full proof with explicit solutions for the three cutoffs is in the Appendix. Optimal disclosure threshold policies are characterized by the first-order conditions (FOCs) of net profit functions with respect to the activist position $\alpha$. Corollary 3 implies that when the optimal position is less than $\alpha^*$, it can be achieved in equilibrium by a binding disclosure threshold. The following subsections derive each of the optimal policies and explain the underlying mechanisms.

\begin{align*}
\varepsilon^*_m & \leq \alpha^* \leq \{\overline{\alpha}_I, \overline{\alpha}_R\} \\
\alpha^* & \leq \{\overline{\alpha}_I, \overline{\alpha}_R, \overline{\alpha}_A\} \\
\overline{\alpha}_I & < \alpha^* \leq \{\overline{\alpha}_R, \overline{\alpha}_A\} \\
\overline{\alpha}_I & < \overline{\alpha}_R < \alpha^* \leq \overline{\alpha}_A
\end{align*}

Figure 2: Optimal disclosure thresholds
3.2.1 Uninformed Investors

Uninformed investors maximize $\pi_I f(k) - rk$. The associated FOC reveals that they benefit from a binding disclosure threshold if and only if

$$H(\rho^*_I) \lambda \left[ g(c^*_I) (\delta f(k) - c^*_I) - G(c^*_I) \right] + \frac{dH(\rho^*_I)}{dc^*_I} \frac{\partial \pi_I}{\partial H(\rho^*_I)} f(k) < 0,$$

which can be rearranged to $\varepsilon^*_m > \varepsilon^*_m^{I}$. That is, when (15) holds, the marginal profits to investors from increasing the activist’s position when $\alpha = \alpha^*$ are negative. Then, investors benefit from a disclosure threshold that limits their trading losses and, in turn, the probability that the activist participates after observing malfeasance. The optimal policy is implicitly characterized by the FOC, i.e., $\alpha_I$ sets the left-hand side of (15) equal to zero. In the Appendix we show that $\alpha_I > 0$, i.e., investors always benefit from some degree of market opacity.

Investors benefit from managerial feedback, which is captured in (15) by the second term on the left-hand side. Here, $\frac{dH(\rho^*_I)}{dc^*_I} = h(\rho^*_I)\frac{\partial \rho^*_I}{dc^*_I} < 0$ captures the manager’s response to a marginal increase in the profitability of activism. A large $h$ implies a high activism elasticity of management $\varepsilon_m$, i.e., a large reduction in malfeasance in response to an increase in the conditional probability that the activist intervenes. Notably, this reduced managerial malfeasance reduces the relative value of activism ex-ante, increasing the share of the pie that uninformed investors obtain in expectation, i.e., $\frac{\partial \pi_I}{\partial H(\rho^*_I)} f(k) < 0$. Thus, increased managerial feedback raises the value of conditional trading transfers for uninformed investors, reducing the desirability of disclosure thresholds. As a result, investors only gain from a binding threshold if the activism elasticity of management is small enough (in absolute terms), i.e., if $\varepsilon^*_m > \varepsilon^*_m^{I}$.

Importantly, a small activism elasticity of management is not sufficient to make a binding disclosure threshold desirable for uninformed investors, i.e., for $\alpha_I < \alpha^*$. To see this, consider $h \to 0$, which eliminates managerial feedback: $\varepsilon_m \to 0$. The FOC in (15) reveals:

**Corollary 5** A necessary condition for uninformed investors to benefit from a binding threshold is $g(c^*_I)(\delta f(k) - c^*_I) < G(c^*_I)$. 

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Holding management behavior constant, \( g(c_t^*) (\delta f(k) - c_t^*) \) represents the marginal benefits to uninformed investors of increasing trading transfers to the activist when \( \alpha = \alpha^* \), i.e., for \( c_t^* \). On the margin, higher transfers increase the probability that the activist participates after observing managerial malfeasance by \( g(c_t^*) \). The net benefit for investors is the difference between the total value enhanced by the activist \( \delta f(k) \) and their trading losses \( c_t^* \). Conversely, \( G(c_t^*) \) captures the conditional loss from marginally higher transfers: with this probability the activist would have participated anyway, even if investors’ expected trading transfers had not been increased. Hence, the conditional marginal profitability of activism must be negative for investors to benefit from a binding disclosure threshold.

Relation \( g(c_t^*) (\delta f(k) - c_t^*) < G(c_t^*) \) can be rearranged to

\[
\varepsilon^*_a < \frac{c_t^*}{\delta f(k) - c_t^*}.
\] (16)

Trading transfers are the cost that investors incur in exchange for managerial discipline. When a marginal increase in trading transfers \( c_t^* \) has a small impact on the conditional probability \( G(c_t^*) \) that the activist intervenes—when the profit elasticity of activism \( \varepsilon^*_a \) is small—then investors gain from a binding disclosure threshold that limits both trading transfers and activist intervention. Corollary 5 shows that this is a necessary condition for investors to find a disclosure threshold desirable.

### 3.2.2 Activist Hedge Fund

The activist maximizes his expected gross revenues \( \pi_A f(k) \) net of the expected cost of intervention, i.e., net of \( H(\rho_t) \lambda G(c_t) E [c | c \leq c_t] \). The Appendix shows that the activist benefits from a disclosure threshold if and only if

\[
H(\rho_t^*) + \frac{dH(\rho_t^*)}{dc_t^*} \left[ c_t^* - E [c | c \leq c_t^*] \right] < 0,
\] (17)

which can be rearranged to \( \varepsilon^*_m < \varepsilon^*_m^A \).
The managerial feedback effect, captured by \( \frac{dH(\rho^*)}{dc^*} [c^*_t - E[c | c \leq c^*_t]] < 0 \), hurts the activist: well-behaving management destroys the raison d’être of activists. A larger position of the activist yields higher trading profits \( c_t \), raising the conditional profitability of activism, and the extent of activism upon managerial malfeasance \( G(c_t) \). However, this, in turn, deters managerial malfeasance, reducing the activist’s opportunity to profit, i.e., \( \frac{dH(\rho_t)}{dc_t} < 0 \). As a result, increasing a binding disclosure threshold, \( \alpha \), which increases trading profits, need not increase the activist’s unconditional expected profits.

Proposition 4 shows that the activist benefits from a disclosure threshold if managerial feedback is strong enough. When management’s behavior is sensitive to the threat of activism, the activist gains from a disclosure threshold that effectively commits it to reducing its intervention rates, thereby encouraging managerial malfeasance. Analysis of (17) reveals that the activist can only benefit from a disclosure threshold due to managerial feedback. *This is not immediate.* The activist also faces an investment feedback effect that he does not internalize. In particular, his position at \( t = 2 \) influences initial investment \( k \), and this determines the trading profits from a given position \( \alpha \). Using (9) in the characterization of trading transfers \( c_t \) and differentiating with respect to \( \alpha \) yields:

\[
\frac{dc_t}{d\alpha} = \frac{\partial c_t}{\partial \alpha}_{\text{mg. net trading transfer}} + \frac{\partial c_t}{\partial k} \frac{\partial k}{\partial \alpha}_{\text{investment feedback}}.
\]

Net trading transfers capture the effect of the activist’s position on transfers at \( t = 2 \) for a given investment \( k \); Proposition 1 shows that these transfers increase with \( \alpha \) for \( \alpha < \alpha^* \). The investment feedback effect captures the impact of the activist’s position on real investment \( \frac{\partial k}{\partial \alpha} \), and hence on trading transfers \( \frac{\partial c_t}{\partial k} \). Real investment always raises trading transfers, and thus the extent of activism, i.e., \( \frac{\partial c_t}{\partial k} > 0 \). However, the activist’s position \( \alpha \) might be large enough to hurt investors, who respond by reducing investment. That is, if \( \alpha > \bar{\alpha}_I \) then \( \frac{\partial k}{\partial \alpha} < 0 \), and the effect of a larger position on trading transfers is determined by the balance of two opposing forces: positive net transfers and a negative investment feedback. Perhaps
surprisingly, this tension is always resolved against the investment feedback effect:

**Corollary 6** \( \frac{dc}{d\alpha} > 0 \) for \( \alpha < \alpha^* \).

This result reflects the subordinated nature of investment feedback with respect to the direct impact of trading transfers. Intuitively, these transfers lead the activist to take a position \( \alpha^* \), which, in turn, affects investment. If the reduction in investment from increasing \( \alpha \) were strong enough to reduce the activist’s trading profits, i.e., if \( \frac{dc}{d\alpha} < 0 \), then it would also increase investor profits because \( g(c_t)[\delta f(k) - c_t] < G(c_t) \) when \( \frac{dk}{d\alpha} < 0 \). But then investors would *increase* investment, not reduce it, benefitting activists.

The characterization of \( \varepsilon^*_A \) in the Appendix reveals that the cutoff increases with \( \varepsilon^*_a \)—the higher is the profit elasticity of activism, the more the activist values a disclosure threshold. When higher trading profits greatly increase the willingness to engage in activism, they may also strongly deter managerial malfeasance. Then, the responsiveness \( \varepsilon^*_a \) of the activist to its potential trading profits harms it. In those circumstances, investors do not want a binding disclosure threshold. This reflects that the activist’s gains from a binding disclosure threshold are due to the increased managerial malfeasance that it causes, malfeasance that destroys project value when the activist does not intervene, and generates trading losses for investors when he does. But then, investors value the extensive discouragement effect of potential activism on managerial malfeasance. In particular, when the marginal value to the activist of tightening the disclosure threshold is positive, it is negative for investors; and vice versa. Formally, \( \varepsilon^*_m < \varepsilon^*_I \), implying that the activist and investors cannot benefit from a disclosure threshold at the same time.

### 3.2.3 Society

Society maximizes total expected value \( \pi_V f(k) \) net of the costs of capital \( rk \) and the expected costs of activism \( H(\rho_t) \lambda G(c_t) E[c|c \leq c_t] \). Society gains from a disclosure threshold
if and only if

\[
\frac{dH(\rho^*_t)}{dc_t^*} \left( \lambda G(c_t^*) (c_t^* - E[c|c \leq c_t^*]) + \frac{\partial \pi_I}{\partial H(\rho^*_t)} f(k) \right) \frac{dc_t^*}{d\alpha} \\
+ H(\rho^*_t) \lambda g(c_t^*) (\delta f(k) - c_t^*) \frac{dc_t^*}{d\alpha} \\
+ \pi_A f'(k) \frac{\partial k}{\partial \alpha} < 0,
\]

which can be rearranged to $\varepsilon_m^* > \varepsilon_m^{*R}$. The condition reveals that society cares about the value-enhancing effects of activism and real investment; but not about transfers between uninformed investors and the activist.

The first line of (19) is positive and captures the social impact of managerial feedback. One can decompose this effect into two components by expanding $\frac{\partial \pi_I}{\partial H(\rho^*_t)}$ and rearranging. One component is the value enhanced by deterring malfeasance, $\delta f(k) [1 - \lambda G(c_t)]$. Here, $\delta f(k)$ is the difference in firm value under good and bad business plans; and $1 - \lambda G(c_t)$ is the probability that the activist stops a bad plan when it is implemented. The other component is the expected cost of disciplining management if the activist intervenes, $\lambda G(c_t) E[c|c \leq c_t]$. Deterring malfeasance means that those costs do not have to be incurred. A regulator wants greater potential activism and hence weaker ownership disclosure when managers respond by more to the threat of discipline, i.e., when the activism elasticity of management is large.

The benefits from managerial feedback reflect the deterrence of malfeasance via the threat of activist intervention (ex ante disciplining). In addition, society, like uninformed investors, benefits from actual interventions that change a bad business plan to a good one (ex post disciplining). The second line of (19) captures this. The activist observes managerial malfeasance with probability $H(\rho_t)\lambda$, while $g(c_t) \delta f(k)$ is the conditional increase in the gross value from intervention, and $g(c_t) c_t$ is the associated increase in the expected cost of activism.

The third line in (19) represents investment feedback that is not internalized by investors. More specifically, real investment solves $\pi_I f'(k) - r = 0$, but the optimal investment for society sets $(\pi_I + \pi_A) f'(k) - r = 0$. Society only benefits from a disclosure threshold if investors
gain, but the converse is not true. For $\varepsilon^*_m > \varepsilon^*_m$ to hold, the investment feedback must be negative, i.e., $\frac{\partial k}{\partial \alpha} < 0$, implying that $\varepsilon^*_m > \varepsilon^*_m$. Intuitively, society only cares about the real economy, and not about secondary markets (trading transfers). The only social cost of increasing managerial disciplining is the potential reduction in investment. If this is sufficiently strong, then (19) holds and the regulator wants to set a binding disclosure threshold. Still, this threshold always exceeds the optimal threshold from the perspective of investors ($\alpha_I < \alpha_R$ when $\alpha_I < \alpha^*$) who also care about trading transfers.

4 Stock-Picking

This section relaxes the assumption in our main setting that the activist can only take a position in a target company after observing managerial malfeasance. Activist funds typically only intervene on a small fraction of their portfolio companies (Feng et al. 2020), and recent empirical evidence suggests the significance of stock-picking, as opposed to activism, for their profitability (Cremers et al. 2020; Feng et al. 2020). This leads us to investigate how speculative trade by activist funds affects their engagement in mismanaged companies and, in turn, managerial behavior, real investment, and optimal blockholder disclosure thresholds.

We now assume that, after the manager implements the good business plan ($m = 1$), the activist can take a position in the company with probability $\theta < 1$. Thus, $\theta = 0$ corresponds to our benchmark model. Parameter $\theta$ captures the informedness of the activist about undervalued stocks of firms when intervention is unnecessary. Here, $\theta \neq \lambda$ allows for the distinct visibility of bad and good business plans or different monitoring technologies of the activist. Taking a position after observing the good business plan is profitable in expectation because
1) the activist has an information advantage, and
2) there is no need for costly intervention. Thus, whenever the manager implements the good plan, the activist trades with probability $\theta$ and does not intervene. Modifying the analysis in Section 2 under our new assumption yields:

**Proposition 7** Managerial disciplining improves with the activist’s informedness about un-
dervalued stocks. Better stock-picking \( \theta \) reduces managerial malfeasance and raises the conditional probability of activism.

Proposition 7 reveals a positive spillover effect of stock-picking by activist funds on managerial disciplining. This effect is, to our knowledge, novel. The economic intuition is as follows: Stock undervaluation occurs because market uncertainty pools together stocks with good and bad plans. More stock-picking (discovery of good plans) leads to more trades when plans are good, sometimes resulting in positive net order flows that reveal the activist’s presence, and reducing the probability conditional on net sell order flow of a stock with a good plan. In turn, this reduces the price of stocks when the net order flow is negative, making stock-picking more profitable. Because activism ‘transforms’ low-value stocks into high-value, the same mechanism applies to activist targets, causing activism to become more profitable. A proof is in the Appendix; here we develop the intuition.

All else equal, the probability that the activist trades on (positive) private information increases with his information about undervalued stocks \( \theta \). Conditional on trading, his optimal position remains \( \alpha^* = 1/\mu \) regardless of whether the activist intends to discipline management or is stock-picking. From the market maker’s perspective, the better the activist is at stock-picking, the more likely it is that a negative order flow \( \omega \leq 0 \) is associated with value destruction by managerial malfeasance, i.e., where \( m = 0 \) and the activist does not intervene. As a result, the associated stock price \( P_l \) decreases with \( \theta \), raising activist profits from trading on its information advantage \( c_l \). In turn, this means that stock-picking makes activism more profitable. In equilibrium, the conditional probability of activist interventions \( G(c^*_l) \) rises with \( \theta \), which, in turn, reduces managerial malfeasance \( H(\rho^*_t) \).

This argument implies that conditional trading transfers \( c^*_l \) increase with stock-picking opportunities \( \theta \), despite both managerial and investment feedback effects. To see this, consider managerial feedback and suppose that, to the contrary, the manager’s response to the threat of activism (reduced malfeasance) was an offsetting force that raised the price set by the market maker. This would reduce trading transfers, and hence the activist’s prof-
its from intervention, and intervention rates themselves. Then the manager would increase malfeasance, not reduce it, a contradiction. A similar argument applies to the response of uninformed investors to an increase in trading transfers, i.e., to the investment feedback effect analyzed in Section 3.2.2 and Corollary 6.

The impact of the activist’s stock-picking for uninformed investors is ambiguous. As in the benchmark setting, investors face a tension between trading losses and managerial disciplining. Greater activist stock-picking improves managerial discipline, but it also raises investor trading losses when the activist takes a position. The dominating effect determines the impact of stock-picking on real investment. With a high enough profit elasticity of activism \( \varepsilon_a^* \) and high managerial feedback (large \( \varepsilon_m^* \) in absolute terms), uninformed investors benefit from increased trading transfers and hence welcome the activist’s stock-picking. Then, the optimal disclosure threshold for uninformed investors \( \alpha_I \) increases with \( \theta \). Instead, if the disciplining effect is offset by higher trading losses, investors prefer lower activist participation rates and their optimal threshold falls with \( \theta \).

Consider the tension faced by the activist. He benefits from increased trading transfers \( c_t^* \) associated with stock-picking, but these transfers raise his own intervention rates to discipline management and thus deter malfeasance. A similar argument is developed in Section 3.2.2; here stock-picking makes it more subtle. In particular, note that deterrence of managerial malfeasance translates to more stock-picking opportunities whenever \( \theta > 0 \). As a result, the activist may benefit from increased transfers in circumstances where, absent stock-picking, managerial feedback would make them unprofitable. It follows that when \( \theta \) is sufficiently large, or the managerial response is sufficiently small, the activist benefits from increased stock-picking, which can only raise the optimal disclosure threshold for the activist \( \alpha_A \). Only when \( \theta \) is sufficiently small and \( \varepsilon_m^* \) is sufficiently large (in absolute terms) can increased stock-picking make a lower disclosure threshold more desirable for the activist.

The social impact of activist stock-picking follows from the intuition developed in Section 3.2.3. Society is keener than investors about stock-picking because it benefits from increased
managerial disciplining and it does not care about the associated higher trading transfers. Thus, the only social cost of activist stock-picking is a potential reduction in real investment caused by hurting uninformed investors. It follows that activist stock-picking benefits society whenever it benefits uninformed investors, but the opposite is not true. Hence, stock-picking contributes to misaligning the interests of uninformed investors and society, increasing the distance between their optimal disclosure thresholds whenever they bind.

5 Discussion

Our model incorporates the main arguments of the debate about the desirability of revising blockholder disclosure thresholds in the presence of activist funds. We now discuss how key mechanisms revealed by our analysis provide a framework for policy evaluation.

Broadly, our model highlights the potential effects of changes in ownership disclosure rules on managerial disciplining and capital formation. For instance, in 2017, then SEC nominee R. Jackson called for an expansion of these rules for activist investors.6 Our analysis stresses the potentially negative impact on managerial disciplining while showing that this might be socially desirable if it fosters real investment sufficiently by increasing investor confidence. Alternatively, the SEC recently proposed to increase disclosure thresholds drastically for asset managers, a measure that would conceal the positions of most activist funds in the US. The comment letter to the SEC Bernhardt and Ordonez-Calafi (2020), based on the analysis here, explained that the potential gains from managerial discipline were likely outweighed by the loss associated with the reduction in market transparency.

Of course, policy evaluation based on our analysis should consider all the moving pieces. Accordingly, recommendations point towards tailored disclosure policies that can account for market and firm characteristics. For example, trading profits are directly related to market liquidity and increased incentives for hedge fund activism. We show that high liquidity can

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lead the costs of adverse selection in financial markets to outweigh the benefits of managerial disciplining, thereby reducing the optimal threshold, while the opposite happens when liquidity is low. Empirical studies that find a positive relation between liquidity and hedge fund activism generally include industry controls (e.g., Edmans et al. 2013; Norli et al. 2015; Gantchev et al. 2019), which, coupled with our results, suggests the potential desirability of industry-specific thresholds based on liquidity measures.

Whether activists’ trading profits are excessive or insufficient from a regulator’s perspective hinges on their costs of engaging management. These costs likely rise with managerial entrenchment and thus with regulation that insulates management from shareholder pressure. Our analysis suggests that a positive relationship between management-friendly regulation (e.g., business judgement rule) and the level of disclosure thresholds is desirable. Notably the extent to which regulation protects or exposes management may depend on the nature of activist demands. For instance, US takeover regulation is relatively permissive with takeover defences (Armour and Skeel 2007), raising the cost of interventions that advocate the sale of a target company. The optimal threshold for takeover-seeking campaigns may therefore exceed those involving interventions on aspects in which management is legally more exposed.

Many countries already require a statement of purpose with the disclosure of ownership (e.g., in 13Ds); our analysis suggests potential benefits of activism objective-specific thresholds. Brav et al. (2010) provide a classification of these objectives.

An important consideration for ownership disclosure rules is target firm size or market capitalization. Some activist funds take positions that do not cross disclosure thresholds; recent studies point to capital costs and financial constraints as key reasons (Becht et al. 2017; Brav et al. 2019). Our setting provides insights into the differential impact of a unique threshold on companies of distinct sizes. Relatively high returns of activism on large companies may make their thresholds too high. With a small profit elasticity of activism, reducing disclosure thresholds can alleviate market adverse selection while maintaining the disciplin-
ing benefits of hedge fund activism. The opposite applies to smaller companies.\textsuperscript{7} Market
capitalization-contingent thresholds, or a combination of dollar and percentage value thresh-
olds, could account for the impacts of firm size heterogeneity—see Edmans and Holderness
(2017) for further arguments on the merits of dollar-ownership measures.

Our main modelling assumptions are motivated by findings in the empirical literature,
and, as our introduction highlights, many of our model predictions have empirical support.
However, regulators and academics may want to investigate the more nuanced mechanisms
in our model—involving the activism elasticity of management and the profit elasticity of
activism—to better understand the effects of policy changes. Gantchev et al. (2019) develop
an empirical framework to study the responsiveness of management to the threat of activism.
Their measures of changes in the probability of companies being targeted by activist funds
may be suitable for an empirical analysis of our setting. Testing the responsiveness of ac-
tivism to increases in trading profits likely requires indirect proxies. One possibility is to
exploit variation in the liquidity of stock in target companies. Our model predicts a positive
relationship between target stock liquidity and activist trading profits and, indeed, Edmans
et al. (2013) find that activism is positively associated with liquidity. Alternatively, one may
be able to exploit changes in disclosure thresholds or the heterogeneity of thresholds across
financial jurisdictions to test the responsiveness of activism to trading profits.\textsuperscript{8}

6 Concluding Remarks

Hedge fund activism has generated debate about the desirability of revising blockholder dis-
closure thresholds. These rules were set to protect small investors from abusive tactics of

\textsuperscript{7}A potential spillover effect of setting low disclosure thresholds is to create incentives for activist funds to
diversify their portfolios. In particular, when trading profits obtained from targeting one company are limited
by the disclosure threshold, an even lower threshold may lead activists to seek new investment opportunities
for the capital that, with a higher threshold, would have been invested in a single company (we thank the ref-
eree for this insight). A pecking order argument suggests that these new projects are typically less profitable.

\textsuperscript{8}Blockholder disclosure thresholds differ across financial systems. For example, investors that intend to
introduce corporate changes in US publicly-listed companies must fill a 13(d) file when their holdings reach
5\% of voting rights. In Canada, disclosure is not required until a 10\% stake is acquired. In the EU, Germany
recently reduced the threshold to the 3\% cutoff used in the UK, while the threshold in France remains at 5\%.
blockholders. We identify the tradeoffs. Disclosure thresholds may discourage activist funds from intervening to protect small investors from corporate managers who take actions that benefit themselves at the expense of firm value; but activist funds are also informed traders who profit from trading on their information advantage about their value-enhancing actions at the expense of uninformed investors. While managerial discipline creates value and incentivizes real investment, the associated trading rents extracted from uninformed investors reduce their profitability and impair investment, destroying value.

We show that the preferences for binding disclosure thresholds of investors, activist funds and society are never aligned. When investors gain from a binding threshold, they benefit more than regulators, and activists are necessarily harmed even though, in this instance, the threshold causes investors to increase investment. Activists can gain from a disclosure threshold because it acts as a commitment device to intervene less frequently. We prove that activists can gain from such commitment when it encourages managerial malfeasance, but not because it fosters real investment. Thus, activists gain only when investors and society are harmed. The threat of activism disciplines managers and raises investment value with no effective cost of intervention, benefiting society. We only find scope for agreement when all market participants gain from non-binding disclosure thresholds. This requires that the willingness of activists to intervene be sufficiently sensitive to the degree of market opacity, but, in turn, that firm management not be too sensitive to the threat of activism in its choices of whether to take actions that benefit itself at the expense of shareholders.

Our analysis provides insights for policy makers. We characterize how optimal disclosure rules that target activist investors (e.g., 13D filings in the US) hinge on multiple factors that differ across firms, suggesting that a tailored approach is desirable. Our model links the desirability of disclosure thresholds to market fundamentals (e.g., liquidity), firm characteristics (e.g., market capitalization and managerial entrenchment) and the regulatory framework (e.g., cost of activism). The mechanisms revealed can help regulators setting thresholds contingent on these characteristics.
7 Appendix: Proofs

7.1 Proof of Proposition 1

Market maker. Let \( \widehat{\alpha} \) be the market maker’s conjecture about the activist’s trade, which is correct in equilibrium. Let \( \widehat{c_t} \equiv c_t(\widehat{\alpha}) \) be the analogous conjecture about his cost participation threshold. The market maker observes \( \omega \). Given \( \omega \), either (i) the activist did not take a position and \( l = -\omega \); or (ii) the activist participates and \( l = -\omega + \widehat{\alpha} \). From our assumptions it follows that the unconditional probability that the activist does not participate is \( [1 - z\lambda G(\widehat{c_t})]y(-\omega) \), and the unconditional probability that he participates is \( z\lambda G(\widehat{c_t})y(-\omega + \widehat{\alpha}) \). Thus, the expected project value is

\[
E[V|\omega] = \left[ \frac{y(-\omega)(1-z) + y(-\omega + \widehat{\alpha})z\lambda G(\widehat{c_t})}{y(-\omega)(1-z) + y(-\omega + \widehat{\alpha})z\lambda G(\widehat{c_t}) + y(-\omega)z[1 - \lambda G(\widehat{c_t})]} \right] f(k) \tag{20}
\]

Suppose the market maker observes \( \omega > 0 \). Then \( y(-\omega) = 0 \), and the activist participates with certainty so \( P(\omega) = P_h \). If, instead, \( \omega \leq 0 \), the market maker does not know whether the activist participates, with \( y(-\omega + \widehat{\alpha}) = \mu e^{\mu(\omega - \widehat{\alpha})} \) and \( y(-\omega) = \mu e^{\mu\omega} \). The term \( \mu e^{\mu\omega} \) cancels out of the numerator and denominator of (20). Using \( Y(\alpha) = 1 - e^{-\mu\alpha} \) yields (6).

Activist. The activist’s position \( \alpha^* = 1/\mu \) is derived in the main text, and the market maker’s conjecture is correct in equilibrium, i.e., \( \widehat{\alpha} = \alpha^* \).

7.2 Proof of Proposition 2

Gross expected profits. Consider an arbitrary position \( \alpha \). The unconditional project value \( E[V] \) in Proposition 2 weighs cash flows \( f(k) \) with the probabilities that (i) the manager implements the good business plan, \( 1 - H(\rho_t) \); (ii) the manager implements the bad plan but is disciplined by the activist, \( H(\rho_t)\lambda G(c_t) \); (iii) the manager implements the bad plan and
is not disciplined by the activist but the project succeeds anyway, \( H(\rho_t)[1 - \lambda G(c_t)](1 - \delta) \).

The activist’s gross profits are obtained by weighting his conditional profits \( E[\Pi_A|\alpha] \) with the probability of participation \( H(\rho_t)\lambda G(c_t) \),

\[
E[\Pi_A] = \pi_A f(k) = H(\rho_t)\lambda G(c_t)[1 - Y(\alpha)]H(\rho_t) \left[ \frac{1 - \lambda G(c_t)}{1 - H(\rho_t)\lambda G(c_t)Y(\alpha)} \right] \delta \]

\[
= H(\rho_t)\lambda G(c_t) \frac{c_t}{f(k)}. \tag{21}
\]

By construction, expected investors’ profits are the residual \( E[\Pi_I] = [\pi_V - \pi_A] f(k) \),

\[
E[\Pi_I] = \pi_I f(k) = [1 - H(\rho_t)(1 - \lambda G(c_t))\delta]
- H(\rho_t)\lambda G(c_t)[1 - Y(\alpha)]H(\rho_t) \left[ \frac{1 - \lambda G(c_t)}{1 - H(\rho_t)\lambda G(c_t)Y(\alpha)} \right] \delta
\]

\[
= [1 - H(\rho_t)(1 - \lambda G(c_t))\delta] - H(\rho_t)\lambda G(c_t) \frac{c_t}{f(k)}. \tag{22}
\]

Proposition 2 provides expressions for expected profits in equilibrium, substituting \( \alpha = \alpha^* = 1/\mu \). Rearranging \( \pi_A \) as a function of \( c_t \) shows that \( \alpha \) affects expected profits only through trading transfers \( c_t \) and capital, i.e., \( E[\Pi_A](c_t(\alpha), k(\alpha)) \) and \( E[\Pi_I](c_t(\alpha), k(\alpha)) \).

**Real Investment.** The first-order condition for investors’ net profits \( \pi_I f(k) - rk \) characterizes real investment. Note that while \( \pi_I \) is a function of both activism and investment, small investors are price takers who do not internalize the effects of their own investment.

### 7.3 Proof of Proposition 4

The proof first studies partial impacts of \( \alpha \) and \( k \) on trading transfers \( c_t \); then it uses the results to derive critical cutoffs \( \{\varepsilon_I^{L}, \varepsilon_I^{A}, \varepsilon_I^{R}\} \). The proof follows with a comparison of the cutoffs. Last, the proof shows that second order conditions hold when the costs of activism
and management reputation costs are uniformly distributed.

### 7.3.1 Partial effects on trading transfers

The characterization of $c_t$ is obtained from using $z \equiv H(\rho_t)$ in (8). Plugging in $\rho_t$ as given in (9) we have that $\frac{dc_t}{d\alpha} = \frac{\partial c_t}{\partial \alpha} + \frac{\partial c_t}{\partial k} \frac{\partial k}{\partial \alpha}$. Here we show $\frac{\partial c_t}{\partial \alpha} > 0$ for $\alpha \leq \alpha^*$ and $\frac{\partial c_t}{\partial k} > 0$.

Use the expression of $c_t$ to define the function $F \triangleq c_t - [1 - Y(\alpha)] \alpha H(\rho_t) \left[ \frac{1 - \lambda G(c_t)}{1 - H(\rho_t) \lambda G(c_t) Y(\alpha)} \right] \delta f(k)$. From the Implicit Function Theorem, $\frac{dc_t}{d\alpha} = -\frac{\partial F}{\partial i} \frac{\partial i}{\partial c_t}$. We obtain:

$$\frac{\partial F}{\partial c_t} = 1 - [1 - Y(\alpha)] \alpha \left( \frac{h(\rho_t) \frac{\partial \mu}{\partial c_t} [1 - \lambda G(c_t)] - H(\rho_t) \lambda g(c_t)}{1 - H(\rho_t) \lambda G(c_t) Y(\alpha)} \right) \delta f(k)$$

$$- [1 - Y(\alpha)] \alpha \left( \frac{H(\rho_t)[1 - \lambda G(c_t)] \left[ h(\rho_t) \frac{\partial \mu}{\partial c_t} \lambda G(c_t) + H(\rho_t) \lambda g(c_t) \right] Y(\alpha)}{[1 - H(\rho_t) \lambda G(c_t) Y(\alpha)]^2} \right) \delta f(k)$$

$$= 1 - [1 - Y(\alpha)] \alpha \left( \frac{h(\rho_t) \frac{\partial \mu}{\partial c_t} [1 - \lambda G(c_t)] - H(\rho_t) \lambda g(c_t) [1 - H(\rho_t) Y(\alpha)]}{[1 - H(\rho_t) \lambda G(c_t) Y(\alpha)]^2} \right) \delta f(k),$$

$$\frac{\partial F}{\partial \alpha} = -H(\rho_t)[1 - \lambda G(c_t)] \delta f(k) \times$$

$$\times \left( \frac{(1 - \mu \alpha)[1 - H(\rho_t) \lambda G(c_t)(1 - e^{-\mu \alpha})] + \mu \alpha e^{-\mu \alpha} H(\rho_t) \lambda G(c_t)}{[1 - H(\rho_t) \lambda G(c_t)(1 - e^{-\mu \alpha})]^2} \right) e^{-\mu \alpha},$$

$$\frac{\partial F}{\partial k} = -[1 - Y(\alpha)] \alpha H(\rho_t) \left[ \frac{1 - \lambda G(c_t)}{1 - H(\rho_t) \lambda G(c_t) Y(\alpha)} \right] \delta f'(k).$$

Notice that $\frac{\partial \mu}{\partial c_t} < 0$ and therefore $\frac{\partial F}{\partial c_t} > 0$. Derivation in (24) expands $Y(\alpha) = 1 - e^{-\mu \alpha}$, and the expression reveals that $\frac{\partial F}{\partial \alpha} < 0$ for $1 - \mu \alpha \geq 0$. Moreover we have that $\frac{\partial F}{\partial k} < 0$.

### 7.3.2 Critical cutoffs

**Investors.** Investor net expected profits are $\pi_I f(k) - rk$, with $\pi_I$ given by (22). To derive their optimal disclosure threshold, we differentiate with respect to $\alpha$. Using $z \equiv H(\rho_t)$ with
\[ \frac{d}{d\alpha} \left\{ \pi_I f(k) - rk \right\} = \left[ \frac{\partial \pi_I}{\partial c_t} \frac{dc_t}{d\alpha} + \frac{\partial \pi_I}{\partial k} \frac{dk}{d\alpha} \right] f(k) + [\pi_I f'(k) - r] \frac{\partial k}{\partial \alpha}. \]  

(26)

Two features simplify the analysis of (26). First, Proposition 2 shows that in equilibrium \( \pi_I f'(k) - r = 0 \), so the last term in (26) vanishes. Second, the activist position that maximizes investor profits also maximizes investment, so any interior maximum of \( \pi_I f(k) - rk \) satisfies \( \frac{\partial k}{\partial \alpha} = 0 \). Using these two features and expansion \( \frac{dc_t}{d\alpha} = \frac{\partial c_t}{\partial \alpha} + \frac{\partial c_t}{\partial k} \frac{dk}{d\alpha} \) reveals that any interior solution \( \overline{\alpha}_I < \alpha^* \) solves \( \frac{\partial \pi_I}{\partial c_t} \frac{dc_t}{d\alpha} f(k) = 0 \). Section 7.3.1 shows that \( \frac{\partial c_t}{\partial \alpha} > 0 \) for \( \alpha \leq \alpha^* \). Hence, if there is an interior maximum \( \alpha^*_I \), it must be characterized by \( \frac{\partial \pi_I}{\partial c_t} = 0 \), where

\[ \frac{\partial \pi_I}{\partial c_t} = H(\rho_t) \frac{\lambda}{f(k)} \left[ g(c_t) (\delta f(k) - c_t) - G(c_t) \right] + \frac{dH(\rho_t)}{dc_t} \frac{\partial \pi_I}{\partial H(\rho_t)}, \]  

(27)

with \( \frac{dH(\rho_t)}{dc_t} = -h(\rho_t) \frac{\varphi g(c_t)}{\lambda G(c_t)^2} \),

\[ \frac{\partial \pi_I}{\partial H(\rho_t)} = - \left[ \delta (1 - \lambda G(c_t)) + \lambda G(c_t) \frac{c_t}{f(k)} \right]. \]

At \( \alpha = 0 \), activist trading profits are zero, so \( c_t = 0 \), and hence (27) > 0 and (26) > 0: investors always value some market opacity, i.e., \( \overline{\alpha}_I > 0 \). To characterize \( \varepsilon_I^m \), rearrange (27) = 0 as

\[ 0 = \frac{H(\rho_t) \lambda}{f(k)} \left[ g(c_t) (\delta f(k) - c_t) - G(c_t) \right] + g(c_t) \frac{\partial H(\rho_t)}{\partial G(c_t)} \frac{\partial \pi_I}{\partial H(\rho_t)}, \]  

(28)

where (28) uses \( \frac{dH(\rho_t)}{dc_t} = g(c_t) \frac{\partial H(\rho_t)}{\partial G(c_t)} \). Substituting \( \varepsilon_a \) and \( \varepsilon_m \), dividing by \( G(c_t) \) and \( H(\rho_t) \),
and rearranging yields

\[
0 = \frac{\lambda}{f(k)} \left[ \varepsilon_a \left( \frac{\delta f(k) - c_t}{c_t} \right) - 1 \right] + \varepsilon_m \frac{g(c_t)}{G(c_t)^2} \partial \pi_I \partial H(\rho_t),
\]

\[
= \frac{\lambda}{f(k)} \left[ \varepsilon_a \left( \frac{\delta f(k) - c_t}{c_t} \right) - 1 \right] + \varepsilon_m \varepsilon_a \frac{1}{G(c_t)c_t} \partial \pi_I \partial H(\rho_t),
\]

\[
\Rightarrow - \left( \lambda G(c_t) \frac{\partial \pi_I}{\partial H(\rho_t)} \right) \left[ \frac{\delta f(k) - c_t}{c_t} - \frac{1}{\varepsilon_a} \right] = \varepsilon_m.
\]

From (21), note that \( \lambda G(c_t) \frac{c_t}{f(k)} = \frac{\partial \pi_A}{\partial H(\rho_t)}. \) The next expression for \( \varepsilon_m^* \) follows directly:

\[
\varepsilon_m^* \equiv - \left( \frac{\partial \pi_A / \partial H(\rho_t^*)}{\partial \pi_I / \partial H(\rho_t)} \right) \left[ \frac{\delta f(k) - c_t^*}{c_t^*} - \frac{1}{\varepsilon_a} \right].
\]

**Activist.** Net expected activist profits are:

\[
\pi_A f(k) - H(\rho_t) \lambda G(c_t) E [c|c \leq c_t] = H(\rho_t) \lambda G(c_t) [c_t - E [c|c \leq c_t]],
\]

where the right-hand side uses the solution for \( \pi_A \) in Proposition 2. Here, \( H(\rho_t) \lambda G(c_t) \) is the unconditional probability that the activist participates. Conditional on intervention, expected profits are the difference between trading profits \( c_t \) and the cost of disciplining management, which is expected to be \( E [c|c \leq c_t] = \left[ \int_{c_t}^c cg(c) \, dc \right] / G(c_t). \) Differentiating yields

\[
\frac{d}{d\alpha} \left\{ H(\rho_t) \lambda G(c_t) [c_t - E [c|c \leq c_t]] \right\}
\]

\[
= \left[ H(\rho_t) \lambda G(c_t) + \frac{dH(\rho_t)}{dc_t} \lambda G(c_t) [c_t - E [c|c \leq c_t]] \right] \frac{dc_t}{d\alpha}.
\]
The result follows because

\[
\frac{dE[c|c \leq c_t]}{d\alpha} = \frac{\partial E[c|c \leq c_t]}{\partial c_t} \frac{dc_t}{d\alpha}
\]

\[
= \frac{g(c_t)}{G(c_t)} \left[ \frac{\partial}{\partial c_t} \left\{ \int_0^{c_t} c g(c) \, dc \right\} - E[c|c \leq c_t] \right] \frac{dc_t}{d\alpha}
\]

\[
= \frac{g(c_t)}{G(c_t)} \left[ c_t - E[c|c \leq c_t] \right] \frac{dc_t}{d\alpha}
\]

where the last line of (33) uses \( \frac{\partial}{\partial c_t} \left\{ \int_0^{c_t} c g(c) \, dc \right\} = g(c_t) \cdot c_t \).

The sign of (32) is determined by the product of the term in big brackets and \( \frac{dc_t}{d\alpha} \); in what follows we prove by contradiction that \( \frac{dc_t}{d\alpha} > 0 \) for \( \alpha < \alpha^* \). Note that \( \frac{dc_t}{d\alpha} = \frac{\partial c_t}{\partial \alpha} + \frac{\partial c_t}{\partial k} \frac{dk}{d\alpha} \) with \( \frac{\partial c_t}{\partial \alpha} > 0 \) for \( \alpha \leq \alpha^* \) and \( \frac{\partial c_t}{\partial k} > 0 \). If \( \frac{dc_t}{d\alpha} < 0 \), the activist gains from a disclosure threshold due to the negative investment feedback, which must satisfy \( \frac{dk}{d\alpha} < -\frac{\partial c_t}{\partial c_t} \frac{\partial \alpha}{\partial k} \). For \( \frac{dk}{d\alpha} < 0 \) to hold, a marginal increase in the activist’s position must hurt investors, implying that \( \frac{\partial \pi}{\partial c_t} < 0 \).

Suppose that the investment feedback satisfies \( \frac{dk}{d\alpha} < -\frac{\partial c_t}{\partial c_t} \frac{\partial \alpha}{\partial k} \) and thus that \( \frac{dc_t}{d\alpha} < 0 \). By assumption increasing, \( \alpha \) reduces \( c_t \), so it must increase investor profits because \( \frac{\partial \pi}{\partial c_t} < 0 \).

But this higher profitability leads investors to increase capital when the activist increases his position \( \frac{dk}{d\alpha} > 0 \), a contradiction. It follows that \( \frac{dc_t}{d\alpha} > 0 \) for \( \alpha < \alpha^* \) and the sign of (32) is determined by the term in big brackets.

We set (32) = 0 to derive \( \varepsilon_a^A \). Using \( \frac{dc_t}{d\alpha} > 0 \) reveals that the condition can be rewritten as

\[
H(\rho_t) + \frac{dH(\rho_t)}{dc_t} [c_t - E[c|c \leq c_t]] = 0.
\]

(34)

Using \( \frac{dH(\rho_t)}{dc_t} = g(c_t) \frac{\partial H(\rho_t)}{\partial G(c_t)} \), plugging in the expressions for \( \varepsilon_a \) and \( \varepsilon_m \), and dividing by \( H(\rho_t) \),
yields the following condition equivalent to (34):

\[
0 = 1 + g(c_t) \frac{\partial H(\rho_t)}{\partial (c_t)} \frac{1}{H(\rho_t)} \left[ c_t - E \left[ c | c \leq c_t \right] \right]
\]

\[
= 1 + \frac{g(c_t)}{G(c_t)} \frac{\partial H(\rho_t)}{\partial G(c_t)} \frac{G(c_t)}{H(\rho_t)} \left[ c_t - E \left[ c | c \leq c_t \right] \right]
\]

\[
= 1 + \varepsilon_m \varepsilon_a \left[ \frac{c_t - E \left[ c | c \leq c_t \right]}{c_t} \right].
\]

The following characterization of \( \varepsilon_m^A \) follows directly:

\[
\varepsilon_m^A \equiv -\frac{1}{\varepsilon_a^*} \left( \frac{c_t^* - E \left[ c | c \leq c_t^* \right]}{c_t^* - E \left[ c | c \leq c_t \right]} \right).
\]

**Regulator.** The regulator maximizes

\[
\pi_V f(k) - rk - H(\rho_t) \lambda G(c_t) E \left[ c | c \leq c_t \right],
\]

where \( \pi_V \) is given in Proposition 2. Differentiating with respect to \( \alpha \) yields:

\[
\frac{d}{d\alpha} \{ \pi_V f(k) - rk - H(\rho_t) \lambda G(c_t) E \left[ c | c \leq c_t \right] \}
\]

\[
= \frac{d\pi_V}{dc_t} \frac{dc_t}{d\alpha} f(k) + \pi_V f'(k) \frac{\partial k}{\partial \alpha} - r \frac{\partial k}{\partial \alpha} - \frac{dH(\rho_t)}{dc_t} \lambda G(c_t) E \left[ c | c \leq c_t \right] \frac{dc_t}{d\alpha}
\]

\[
- H(\rho_t) \lambda g(c_t) E \left[ c | c \leq c_t \right] \frac{dc_t}{d\alpha} - H(\rho_t) \lambda G(c_t) \frac{g(c_t)}{G(c_t)} \left[ c_t - E \left[ c | c \leq c_t \right] \right] \frac{dc_t}{d\alpha},
\]

where the last term incorporates the expression derived in (33). Substitute the equilibrium relationships \( \pi_I f'(k) - r = 0 \) and \( \pi_V = \pi_I + \pi_A \) to rearrange (38). The regulator’s marginal
payoff from increasing $\alpha$ can be represented as:

\[\frac{d}{d\alpha}\{\pi_V f(k) - rk - H(\rho_t)\lambda G(c_t)E[c|c \leq c_t]\}\]  

\[= \frac{d\pi_V}{dc_t} f(k) + \pi_A f'(k) \frac{\partial k}{\partial \alpha} - \frac{dH(\rho_t)}{dc_t} \lambda G(c_t) E[c|c \leq c_t] \frac{dc_t}{d\alpha} - H(\rho_t) \lambda g(c_t) c_t \frac{dc_t}{d\alpha}\]

\[= -\frac{dH(\rho_t)}{dc_t} \left[ \delta f(k) [1 - \lambda G(c_t)] + \lambda G(c_t) E[c|c \leq c_t] \right] \frac{dc_t}{d\alpha}\]

\[+ H(\rho_t) \lambda g(c_t) [\delta f(k) - c_t] - \frac{dH(\rho_t)}{dc_t} \left( \delta f(k)[1 - \lambda G(c_t)] + \lambda G(c_t) E[c|c \leq c_t] \right) \frac{dc_t}{d\alpha}\]

\[+ \pi_A f'(k) \frac{\partial k}{\partial \alpha},\]

where the second equality follows from $\frac{d\pi_V}{dc_t} = H(\rho_t) \delta \lambda g(c_t) - \frac{dH(\rho_t)}{dc_t} \delta [1 - \lambda G(c_t)]$.

The last equality of (39) corresponds to the characterization in (19). Consider this expression and define $\Psi \equiv -\frac{\partial \pi_V}{\partial H(\rho_t)} f(k) + \lambda G(c_t) E[c|c \leq c_t]$ to ease exposition: the second term in the big brackets becomes $\frac{dH(\rho_t)}{dc_t} \Psi$. Moreover, recall that $\frac{dH(\rho_t)}{dc_t} = g(c_t) \frac{\partial H(\rho_t)}{\partial G(c_t)}$. The corresponding first order condition reads

\[0 = \left[ -\frac{\partial H(\rho_t)}{\partial G(c_t)} g(c_t) \Psi + H(\rho_t) \lambda g(c_t) (\delta f(k) - c_t) \right] \frac{dc_t}{d\alpha} + \pi_A \frac{df(k)}{d\alpha}\]

\[= g(c_t) (\delta f(k) - c_t) - \frac{g(c_t) \partial H(\rho_t) G(c_t) \Psi}{H(\rho_t) \lambda} + \frac{\pi_A}{H(\rho_t) \lambda} \frac{df(k)}{dc_t} \frac{dc_t}{d\alpha}\]

\[= \varepsilon_a \left( \frac{\delta f(k) - c_t}{c_t} \right) G(c_t) - \varepsilon_m \varepsilon_a \frac{\Psi}{\lambda c_t} + \frac{\varepsilon_m \varepsilon_a}{\lambda c_t} \frac{df(k)}{dc_t} \frac{dc_t}{d\alpha}\]

\[= \varepsilon_a \left( \frac{\delta f(k) - c_t}{c_t} \right) - \varepsilon_m \varepsilon_a \frac{\Psi}{\lambda G(c_t) c_t} + \frac{\varepsilon_m \varepsilon_a}{\lambda G(c_t) c_t} \frac{df(k)}{dc_t} \frac{dc_t}{d\alpha}\]

\[= -\frac{\varepsilon_m + \lambda G(c_t) c_t}{\Psi} \left[ \frac{\delta f(k) - c_t}{c_t} + \frac{1}{\varepsilon_a} \left( \frac{df(k)}{dc_t} \frac{dc_t}{d\alpha} \right) \right].\]
Note in the last line of (40) that using \( \pi = \pi_I + \pi_A \) we obtain

\[
\frac{\lambda G (c_t) c_t}{\Psi} = \left[ -\frac{\partial \pi_I}{\partial H (\rho_t) f(k)} \frac{\partial \pi_I}{\lambda G (c_t) c_t} - \frac{\partial \pi_A}{\partial H (\rho_t) f(k)} + E \frac{[c|c \leq c_t]}{c_t} \right]^{-1} \tag{41}
\]

\[
= \left[ -\frac{\partial \pi_I / \partial H (\rho_t)}{\partial \pi_A / \partial H (\rho_t)} c_t - E \frac{[c|c \leq c_t]}{c_t} \right]^{-1}.
\]

The next characterization of \( \varepsilon^*_m \) follows directly:

\[
\varepsilon^*_m \equiv \left[ \frac{\delta f(k) - c_t^*}{c_t^*} + \frac{1}{\varepsilon_a} \left( \frac{df(k)/d\alpha}{f(k)} \frac{dc_t^* / d\alpha}{c_t^*} \right) \right] \left[ \frac{\partial \pi_I / \partial H (\rho_t^*)}{\partial \pi_A / \partial H (\rho_t^*)} - \frac{c_t^* - E [c|c \leq c_t^*]}{c_t^*} \right]^{-1} \tag{42}
\]

### 7.3.3 Cutoff relation

To see \( \varepsilon^*_m < \varepsilon^*_m \), note that the relation is equivalent to

\[
\frac{1}{\varepsilon_a} \left( -\frac{\partial \pi_A / \partial H (\rho_t)}{\partial \pi_I / \partial H (\rho_t)} - \frac{c_t}{c_t - E [c|c \leq c_t]} \right) < - \left( \frac{\partial \pi_A / \partial H (\rho_t)}{\partial \pi_I / \partial H (\rho_t)} \right) \left[ \frac{\delta f(k) - c_t}{c_t} \right]. \tag{43}
\]

The left-hand side of (43) is negative because

\[
- \frac{\partial \pi_A / \partial H (\rho_t)}{\partial \pi_I / \partial H (\rho_t)} = \frac{\lambda G (c_t) \frac{c_t}{f(k)}}{- \left[ 1 - \lambda G (c_t) \right] \delta + \lambda G (c_t) \frac{c_t}{f(k)}} \in (0, 1), \tag{44}
\]

whereas \( \frac{c_t}{c_t - E [c|c \leq c_t]} > 1 \). From (44) it also follows that the right-hand side of (43) is positive.

To see \( \varepsilon^*_m < \varepsilon^*_m \), note that a necessary condition for \( \varepsilon^*_m < 0 \) is that \( \frac{\partial k}{\partial \alpha} < 0 \), which implies that investors’ marginal profits decrease and thus \( \varepsilon^*_m < \varepsilon^*_m \). Hence, if \( \varepsilon^*_m = 0 \), then \( \varepsilon^*_m < \varepsilon^*_m \) holds. Thus, for \( \varepsilon^*_m < \varepsilon^*_m \), it is necessary, but not sufficient, that \( \varepsilon^*_m < \varepsilon^*_m \).

### 7.3.4 The uniform-uniform case

We show that when both \( c \) and \( \rho \) are uniformly distributed, second-order conditions hold.

**Investors.** We rewrite the first-order condition (FOC) in (28), first substituting in the uni-
form distribution of the manager’s reputation cost, and then the uniform distribution of the activist’s cost of intervention. Substituting $H(\rho_t) = \frac{\varphi}{R}$ and $h(\rho_t) = \frac{1}{R}$, the condition reads

$$0 = \frac{\varphi}{R} \left[ 1 - \lambda G(c_t) \right] \frac{\lambda}{\lambda G(c_t)} \left[ g(c_t)(\delta f(k) - c_t) - G(c_t) \right]$$

Substituting $G(c_t) = \frac{c_t}{C}$ and $g(c_t) = \frac{1}{C}$ in the second line of (46), the investor’s FOC becomes

$$0 = \delta f(k) \left[ \frac{C + \lambda c_t}{C - \lambda c_t} \right] + c_t \left[ \frac{\lambda c_t}{C - \lambda c_t} - \frac{G(c_t)}{g(c_t)} \right].$$

We prove that there is a unique solution to the investors’ FOC by showing that the right-hand side (RHS) of (47) decreases in $c_t$. Differentiating yields

$$\frac{d}{dc_t} \text{RHS}(47) = \left( \frac{C}{C - \lambda c_t} \right)^2 - \frac{\delta f(k) C}{\lambda c_t^2} - 2 - \left( \frac{1}{1 - \lambda G(c_t)} \right)^2 - \frac{\delta f(k)}{\lambda G(c_t)c_t} - 2,$$
activist, so an upper bound of \( c_t \) is obtained by setting \( Y(\alpha) = 0 \). Moreover (23)-(24) reveal that with \( Y(\alpha) = 0 \), trading transfers are maximized by \( \alpha = \frac{1}{\mu(1 - H(\rho_t)G(c_t))} \). Using the characterization of \( c_t \) in (8) and plugging in \( z \equiv H(\rho_t) \) yields the following equivalent conditions:

\[
c_t \leq \frac{H(\rho_t)}{\mu} \left[ \frac{1 - \lambda G(c_t)}{1 - H(\rho_t)\lambda G(c_t)} \right] \delta f(k),
\]

\[
c_t \leq \frac{\varphi}{\mu R} \left[ \frac{1 - \lambda G(c_t)}{\lambda G(c_t)} \right] \left[ \frac{1 - \lambda G(c_t)}{1 - H(\rho_t)\lambda G(c_t)} \right] \delta f(k)
\]

\[
\frac{\delta f(k)}{\lambda G(c_t)c_t} \geq \frac{\mu R}{\varphi} \left( \frac{1 - H(\rho_t)\lambda G(c_t)}{1 - \lambda G(c_t)} \right)^2.
\]

The second line expands \( H(\rho_t) \) in the numerator using its uniform distribution and the characterization of \( \rho_t \) in (9). Setting \( H(\rho_t) = 0 \) in the third line of (49) yields a weaker sufficient condition:

\[
\frac{\delta f(k)}{\lambda G(c_t)c_t} \geq \frac{\mu R}{\varphi} \left( \frac{1}{1 - \lambda G(c_t)} \right)^2.
\]

Substituting (50) into the second line of (48) reveals that a sufficient condition for \( \frac{d}{dc_t} RHS(47) < 0 \) is \( \mu > \frac{\varphi}{R} \). The condition is satisfied by assumptions \( \mu > 1 \) and \( \varphi < R \).

**Activist.** Substitute \( H(\rho_t) = \frac{\rho_t}{R} \) and \( h(\rho_t) = \frac{1}{R} \) to rewrite the activist’s FOC in (34):

\[
0 = \frac{\varphi}{R} \left[ \frac{1 - \lambda G(c_t)}{\lambda G(c_t)} \right] - \left( \frac{\varphi}{R} \frac{g(c_t)}{\lambda G(c_t)^2} \right) [c_t - E[c|c \leq c_t]].
\]

Multiplying (51) by \( \frac{R}{\varphi} \left[ \frac{\lambda G(c_t)}{1 - \lambda G(c_t)} \right] \) yields a simpler, equivalent condition

\[
0 = 1 - \left[ \frac{g(c_t)}{G(c_t)} \right] \left( \frac{c_t - E[c|c \leq c_t]}{1 - \lambda G(c_t)} \right).
\]

Substitute \( G(c_t) = \frac{C}{C} \) and \( g(c_t) = \frac{1}{C} \) and note that \( c_t - E[c|c \leq c_t] = \frac{\sigma^2}{2} \). It follows that the activist’s FOC satisfies

\[
0 = 1 - \frac{1}{2} \left( \frac{C}{C - \lambda c_t} \right).
\]

The right-hand side of (53) decreases in \( c_t \), implying a unique solution.
7.4 Proof of Proposition 7

First we modify the analysis of Section 2 to account for the new assumption that the activist can trade over positive information \( m = 1 \) with probability \( \theta \). Then we conduct comparative statics with respect to \( \theta \).

7.4.1 Trading

Market maker. Given the market maker’s conjecture about the activist’s trade \( \hat{\alpha} \), which is correct in equilibrium, the unconditional probability that the activist does not participate is 
\[
1 - z\lambda G(\hat{c}_t) - (1 - z)\theta \]
and the unconditional probability that he participates is 
\[
z\lambda G(\hat{c}_t) + (1 - z)\theta.
\]
Thus, the expected project value is

\[
E[V|\omega] = \left[ \frac{y(-\omega)(1 - z)(1 - \theta) + y(-\omega + \hat{\alpha})\left[z\lambda G(\hat{c}_t) + (1 - z)\theta \right]}{y(-\omega)(1 - z)(1 - \theta) + y(-\omega + \hat{\alpha})\left[z\lambda G(\hat{c}_t) + (1 - z)\theta \right] + y(-\omega)z[1 - \lambda G(\hat{c}_t)]} \right] f(k),
\]

(54)

If \( \omega > 0 \), then \( y(-\omega) = 0 \) and the activist participates with certainty so \( P(\omega) = f(k) \equiv P_h \). If \( \omega \leq 0 \), then \( y(-\omega + \hat{\alpha}) = \mu e^{\mu(\omega - \hat{\alpha})} \) and \( y(-\omega) = \mu e^{\mu \omega} \). Algebra and the use of 
\[
Y(\alpha) = 1 - e^{-\mu \alpha}
\]
yields

\[
P_{i(\alpha)} = \left[ \frac{1 - \left[z\lambda G(c_t) + (1 - z)\theta \right] Y(\alpha) - z(1 - \lambda G(c_t))\delta}{1 - \left[z\lambda G(c_t) + (1 - z)\theta \right] Y(\alpha)} \right] f(k).
\]

(55)

In particular, the price (55) is smaller than the price (6) in our benchmark setting.

Activist. The activist’s position remains unchanged with respect to the benchmark setting, \( \alpha^* = 1/\mu \), and the market maker’s conjecture is correct in equilibrium, i.e., \( \hat{\alpha} = \alpha^* \).

Equilibrium. At \( t = 2 \) real investment \( k \) is sunk and observable, and the manager adopted \( m = 0 \) with probability \( z \) and \( m = 1 \) with residual probability. The activist takes a position \( \alpha^* = 1/\mu \) if either (a) he observes the good business plan \( (m = 1) \), or (b) he observes
managerial malfeasance ($m = 0$) and the cost of activism is sufficiently small, $c \leq c^*_t$, where

$$c^*_t \equiv [1 - Y(\alpha^*)] \frac{z}{\mu} \left[ \frac{1 - \lambda G(c^*_t)}{1 - \left[z\lambda G(c^*_t) + (1 - z)\theta \right] Y(\alpha^*)} \right] \delta f(k), \quad (56)$$

and disciplines management in situation $(b)$. Otherwise, the activist does not participate.

The market maker, upon observing the net order flow $\omega$, sets prices

$$P(\omega) = \begin{cases} 
  P_l \equiv \frac{1 - \left[z\lambda G(c^*_t) + (1 - z)\theta \right] Y(\alpha^*) - z(1 - \lambda G(c^*_t))\delta f(k)}{1 - \left[z\lambda G(c^*_t) + (1 - z)\theta \right] Y(\alpha^*)} f(k) \quad \text{if} \quad \omega \leq 0 \\
  P_h \equiv f(k) \quad \text{if} \quad \omega > 0
\end{cases}. \quad (57)$$

### 7.4.2 Management

The analysis of management behavior is equivalent to Section 2.1.2.

### 7.4.3 Investment

The expected value of the project at $t = 0$ given investment $k$, $E[\Pi_V]$, is characterized by (10). The expected gross profits of the activist are

$$E[\Pi_A] = \left[ H(\rho_t^*) \lambda G(c_t^*) + (1 - H(\rho_t^*))\theta \right] \frac{c^*_t}{f(k)} f(k) \equiv \pi_A f(k). \quad (58)$$

The expected gross profits of uninformed investors $E[\Pi_I]$ and investment $k$ are characterized respectively by (12) and the solution to the FOC in (13).

### 7.4.4 Comparative Statics

Using the characterization of $\rho_t$ in (9) we can define \( \frac{dc_t}{d\theta} = \frac{\partial c_t}{\partial \theta} + \frac{\partial c_t}{\partial k} \frac{\partial k}{\partial \theta} \) so that managerial feedback is integrated in the derivations. Here we prove $\frac{\partial c_t}{\partial \theta} > 0$; the main text argues that investment feedback $\frac{\partial c_t}{\partial k}$ cannot determine the sign of $\frac{dc_t}{d\theta}$.

To study $\frac{dc_t}{d\theta}$ consider comparative statics analogous to those in Section 7.3.1. Define function $F^\theta \overset{\Delta}{=} c_t - [1 - Y(\alpha)] \alpha H(\rho_t) \left[ \frac{1 - \lambda G(c_t)}{1 - \left[H(\rho_t) \lambda G(c_t) + (1 - H(\rho_t))\theta \right] Y(\alpha^*)} \right] \delta f(k)$ from the expression of
$c_t$ under the stock-picking assumption. Partial differentiation yields:

\[
\frac{\partial F^\theta}{\partial c_t} = 1 - [1 - Y(\alpha)] \alpha \delta f(k) \times
\]

\[
\times \left( \frac{h(\rho_t) \frac{\partial \rho_t}{\partial c_t} [1 - \lambda G(c_t)] [1 - \theta Y(\alpha)] - H(\rho_t) \lambda g(c_t) [1 - H(\rho_t) Y(\alpha) - (1 - H(\rho_t)) \theta Y(\alpha)]}{[1 - [H(\rho_t) \lambda G(c_t) + (1 - H(\rho_t)) \theta] Y(\alpha)]^2} \right),
\]

\[
\frac{\partial F^\theta}{\partial \theta} = -[1 - Y(\alpha)] \alpha H(\rho_t) \left( \frac{[1 - \lambda G(c_t)] [1 - H(\rho_t)] \theta}{[1 - [H(\rho_t) \lambda G(c_t) + (1 - H(\rho_t)) \theta] Y(\alpha)]^2} \right) \delta f(k).
\]

with $\frac{\partial F^\theta}{\partial c_t} > 0$ and $\frac{\partial F^\theta}{\partial \theta} < 0$. From the Implicit Function Theorem it follows that $\frac{\partial c_t}{\partial \theta} > 0$. 

\(46\)
References


