WARWICK

ECONOMICS

Positive and Negative Campaigning in Primary and General Elections

Dan Bernhardt & Meenakshi Ghosh

July 2019 No: 1209

This paper has been updated & published in Games and Economic Behavior

Warwick Economics Research Papers

ISSN 2059-4283 (online)

ISSN 0083-7350 (print)

Positive and Negative Campaigning in Primary and General Elections*

Dan Bernhardt[†] Meenakshi Ghosh[‡]

July 2, 2019

Abstract

We analyze primary and general election campaigning. Positive campaigning builds a candidate's reputation; negative campaigning damages a rival's. Each primary candidate hopes to win the general election; but failing that, he wants his primary rival to win. We establish that general elections always feature more negative campaigning than positive, as long as reputations are easier to tear down than build up. In contrast, if the effects of primary campaigns strongly persist, primary elections always feature more positive campaigning than negative. This reflects that a primary winner benefits only from his positive primary campaigning in general elections, and negative campaigning by a rival hurts.

Keywords: Primary, general election, negative and positive campaigning, contest, incumbent, challenger
JEL Code: D72

*We thank Peter Buisseret, Odilon Camara, Abdollah Farhoodi, Mehdi Shadmehr and Nasrin Sarrafi for helpful comments.

[†]Email: danber@illinois.edu; Address: Department of Economics, University of Illinois, Champaign, IL 61820, USA and the Department of Economics, The University of Warwick, Coventry CV47AL, UK.

[‡]Email: mghosh2@illinois.edu; Address: Department of Economics, University of Illinois, Champaign, IL 61820.

1 Introduction

The media has long highlighted how negative campaigning dominates political debate. Indeed, in 2012, 85% of President Obama's advertising was negative, while 91% of Romney ads were negative, and in 2016, over 90% of the ads for both Clinton and Trump were negative.[1] What has received less attention is the fact that primary campaigns are *far* less negative than general election campaigns. For example, in the 2004 Democratic presidential primary, most ads were positive (CMAG); and, in the 2012 Republican primary, while Romney, the strong favorite, ran far more negative ads than positive ads, Gingrich, Paul and Santorum collectively did the opposite.[2] Our paper provides an explanation for why primary campaigns are more positive than general election campaigns.

In our model, two challengers compete in a primary, with the winner advancing to face an incumbent from the opposing party in a general election. Each challenger hopes to win the general election; but failing that, he wants his primary rival to defeat the incumbent. Candidates are described by their initial reputational stocks and resource budgets. A candidate can devote his resources to building his own reputation via positive campaigning and to damaging an opponent's reputation via negative campaigning. Consistent with empirical evidence,[3] we assume that reputations are easier to damage than to build. Electoral outcomes are determined by a contest in which the probability a candidate wins depends on his post-campaigning reputational stock and that of his opponent.

Our first result establishes that the composition of campaigning in general elections reflects their relative effectiveness in influencing who wins the election. It follows directly that general elections feature more negative campaigning than positive; and, indeed, candidates with sufficiently limited resources only campaign negatively. We then establish that primaries campaigns are very different. Most starkly, we prove that if the effects of primary campaigning persist and primary candidates are equally strong, primaries feature *more* positive campaigning than negative—the *opposite* of general elections. This result reflects that a winner's

1 Kantar Media's Campaign Media Analysis Group (CMAG).

2 http://www.washingtonpost.com/politics/study-negative-campaign-ads-much-more-frequent-vicious-than-in-primaries-past/2012/02/14/gIQAR7ifPR_story.html

3 Baumeister et al. (2001) summarize extensive evidence that voters weigh negative information about candidates more than positive information; Soroka and McAdams (2015) also document a 'negativity bias'.

positive primary campaigning also enhances his general election reputation. In contrast, negative primary campaigning only helps a candidate win the primary; and a losing candidate's negative campaigning harms his primary rival in the general election. Thus, our model reconciles the opposing natures of political campaigning found in primaries and general elections.

We next relate our model to the literature on positive and negative campaigning. After deriving the result that primaries are more positive in nature than general elections, an extended discussion develops the consequences of asymmetries between primary challengers. In particular, consistent with Presidential primaries, when one challenger is far stronger than the other, the strong challenger campaigns more negatively, while the weaker challenger campaigns more positively, to avoid harming his strong rival's general election chances.

Literature. Research on positive and negative campaigns dates back to Skaperdas and Groffman (1995) and Harrington and Hess (1996). Skaperdas and Groffman (1995) predict that in two candidate elections, the front-runner engages in more positive and less negative campaigning than his opponent. Harrington and Hess (1996) build a spatial model in which agents can alter initial ideological positions via costly relocation, and an agent's relocation is affected by a rival's actions as well. They predict that a candidate with less attractive attributes is more negative. Chakrabarti (2007) introduces a valence dimension to this model, allowing candidates to influence ideological and valance factors via advertising. He shows that candidates campaign more negatively on the dimension in which they have an advantage.

Polborn and Yi (2006) model negative and positive campaigning when a candidate can reveal a good feature about himself, or a bad feature about a rival, and voters update about information that is not revealed. Brueckner and Lee (2015) explore negative campaigning in a probabilistic voting model, where voting outcomes reflect a random valence shock, showing that a relatively centrist candidate campaigns more negatively than an extreme candidate.

Peterson and Djupe (2005) empirically study the timing and electoral contexts in which primary races are likely to become negative. Using a content analysis of newspaper coverage of contested Senate primaries, they uncover how negativity varies with the status of a Senate seat (e.g., open seat vs. incumbent in primary), and the number and quality of challengers in the primary (based on whether challengers had held office). Da Silveira and De Mello (2011) exploit a natural experiment in Brazil to document the large effects of campaigning expenditures on electoral outcomes. Spenkuch and Toniatti (2018) use a regression discontinuity design to document the effects of political advertising on electoral outcomes.

Our paper relates to political contests in which agents exert positive and negative efforts. Konrad (2000) explores rent-seeking efforts that improve own performance and sabotage efforts that hurt a rival in lobbying contests. Sabotage against one group creates a positive externality for other groups, so more lobbying groups make sabotage less attractive. Gandhi et al. (2016) provide supporting evidence of this. Soubeyran (2009) models two-agent contests with two types of effort—attack and defense. He analyzes how attack (negative campaigning) affects voter turnout.

Analyses of primaries and general elections include Meirowitz (2005). In his model, candidates are uncertain about voter preferences, causing them to adopt ambiguous primary platforms to preserve flexibility, and primaries let voters signal preferences, influencing subsequent platform choices. In Serra (2011), primaries help parties select higher valence candidates at the expense of also selecting candidates whose policies more closely reflect views of primary voters than general election voters (see also Adams and Merrill (2008)). Kartik and McAfee (2007) build a model of primaries and general elections with three possible policies, where high valence candidates are (unobservedly and exogenously) committed to some policy choice—voters value consistency. Hummel (2010) explores outcomes when candidates find it costly to adjust policies between primaries and general elections and face more extreme audiences in primaries.

2 Model

There are three candidates, i, j and I. Candidates i and j belong to the same party, while incumbent I belongs to the opposing party. Challengers i and j first compete in a primary election, with the winner advancing to face I in a general election. Candidates only care about who wins the general election—challenger $k \in \{i, j\}$ receives U from winning, V if his primary opponent wins, and a normalized payoff of 0 if the incumbent wins, where $U > V > 0$. The incumbent receives a positive payoff if re-elected, and nothing if he loses.

Electoral outcomes are determined by a contest, where the probability a candidate wins rises with his reputation, and declines with his opponent's reputation. Specifically, if $\bar{Z}_{i0}$ and $\bar{Z}_{j0}$ are the candidates' respective reputational stocks just prior to the primary election, then candidate i wins with probability $\frac{\bar{Z}_{i0}}{\bar{Z}_{i0}+\bar{Z}_{j0}}$, and candidate j wins with residual probability. An analogous contest determines the winner of the general election. The reduced-form contest formulation lets us isolate the key strategic considerations of primary and general election dynamics in a tractable way.[4] Papers modeling political competition as a contest include Esteban and Ray (1999), Klumpp and Polborn (2006), Epstein and Nitzan (2006), Soubeyran (2009), Herrera et al. (2014), and Bouton et al. (2018); Konrad and Kovenock (2009) explore other multi-contest settings.

Candidate $k \in \{i, j, I\}$ starts out with reputational stock $\bar{X}_k$, and resources $\bar{B}_k$. A candidate can devote resources both to improving his own reputation via positive campaigning and to damaging his opponent's reputation via negative campaigning. If challenger $k \in \{i, j\}$ invests p_{k0} in positive primary campaigning to boost his own reputation, and his rival $\tilde{k}$ spends $n_{\tilde{k}0}$ on negatively campaigning to reduce k's reputation, then candidate k's reputational stock in the primary becomes $\bar{Z}_{k0} = \bar{X}_k \frac{(1+p_{k0})^\alpha}{(1+\rho n_{\tilde{k}0})^\alpha}$. Here $\alpha > 0$ measures the sensitivity of reputations to campaigning and $\rho > 1$ measures the greater effectiveness of negative campaigning than positive campaigning on shaping candidate reputations (Baumeister et al. 1997, Soroka and McAdams 2015). We show this structure reconciles *simultaneously* the preponderance of negative campaigning in general elections, and *positive* campaigning in primaries.

If candidate k wins the primary, he enters the general election with a reputational stock of $\bar{Z}_{k1} = \bar{X}_k \frac{(1+\beta p_{k0})^\alpha}{(1+\beta\rho n_{\tilde{k}0})^\alpha}$. Here $\beta \in [0, 1]$ allows for decay in the effects of primary campaigns on general-election reputations, for example because voters forget primary campaigns, or because the moderate voters who determine a general election winner pay less attention to primary campaigns than party partisans. In a general election, challenger k's final reputational stock is $\bar{Z}_{k2} = \bar{Z}_{k1} \frac{(1+p_{k1})^\alpha}{(1+\rho n_{I1})^\alpha}$, and the incumbent's is $\bar{Z}_{I2} = \bar{X}_I \frac{(1+p_{I1})^\alpha}{(1+\rho n_{k1})^\alpha}$, where p_{k1} and n_{k1} are challenger k's positive and negative campaigning expenditures in the general election.

Challenger k's total electoral resource constraint is $\sum_t (p_{kt} + n_{kt}) \leq \bar{B}_k$, where $p_{kt}, n_{kt} \geq 0$.

[4] Tractability vanishes, for example, in a disclosure game signaling setting (e.g., Polborn and Yi (2006)), where a multitude of equilibria would arise even in settings that are insufficiently rich for our purposes.

Thus, if challenger k wins the primary after spending $p_{k0}+n_{k0}$, he has $\bar{B}_k-(p_{k0}+n_{k0}) \equiv B_{k1}$ left for the general election.[5] The incumbent's resource constraint is $p_{I1}+n_{I1} \leq \bar{B}_I$.

Without loss of generality, we write challenger $k \in \{i,j\}$'s ex-ante expected payoff as

$$\pi_k = (MPr_{k1} - Pr_{\tilde{k}1})Pr_{k0} + Pr_{\tilde{k}1}.$$

$M = U/V$ is the relative payoff from winning the general election versus having a primary rival win, and Pr_{kt} is the probability that k wins the primary ($t=0$) or general ($t=1$) election.[6]

We first characterize equilibrium campaigning in a general election.

Proposition 1. *In a general election, if candidate $k \in \{i,j,I\}$'s resources B_{k1} exceed $\frac{\rho-1}{\rho}$, then k campaigns both positively and negatively, spending more on negative campaigning:*

$$n^*_{k1} = \frac{B_{k1}}{2} + \frac{\rho-1}{2\rho} \quad \text{and} \quad p^*_{k1} = \frac{B_{k1}}{2} - \frac{\rho-1}{2\rho}.$$

If, instead, $B_{k1} \leq \frac{\rho-1}{\rho}$, then candidate k only campaigns negatively, $n^*_{k1} = B_{k1}$ and $p^*_{k1} = 0$.

The proof follows directly. In the general election, challenger k wins with probability

$$Pr_{k1} = \frac{\bar{Z}_{k2}}{\bar{Z}_{k2}+\bar{Z}_{I2}} = \frac{\bar{Z}_{k1}\frac{(1+p_{k1})^\alpha}{(1+\rho n_{I1})^\alpha}}{\bar{Z}_{k1}\frac{(1+p_{k1})^\alpha}{(1+\rho n_{I1})^\alpha} + \bar{X}_I\frac{(1+p_{I1})^\alpha}{(1+\rho n_{k1})^\alpha}},$$

which, written as a function of what k controls in the general election, takes the form

$$Pr_{k1} = \frac{a(1+p_{k1})^\alpha}{a(1+p_{k1})^\alpha + b/(1+\rho n_{k1})^\alpha},$$

where a and b are positive constants obtained by dividing numerator and denominator by $\bar{X}_I\frac{(1+p_{I1})^\alpha}{(1+\rho n_{i1})^\alpha}$. Multiplying numerator and denominator by $(1+\rho n_{k1})^\alpha/b$, and simplifying yields

$$Pr_{k1} = \frac{a[(1+p_{k1})^\alpha(1+\rho n_{k1})^\alpha]/b}{a[(1+p_{k1})^\alpha(1+\rho n_{k1})^\alpha]/b+1},$$

5 This formulation is equivalent to one in which challengers have separate primary and general election budgets, as long as they do not face binding primary budget constraints, and can conserve unspent primary budgets for the general election.

6 Our analysis presumes that M is large enough that even if one challenger is far stronger, his weak rival cares enough about personally winning that he wants to enter the primary, even though it helps the incumbent.

implying that k maximizes the quasi-concave objective $(1+p_{k1})^{\alpha}(1+\rho n_{k1})^{\alpha} \equiv [P_{k1}N_{k1}]^{\alpha}$.

Only relative effectiveness matters for general elections: When $\rho > 1$, it is easier to tear down a reputation than build one up, so candidates spend more on negative campaigns than positive ones, and if their resources are too limited, they only campaign negatively. Thus, very underfunded challengers only campaign "against" an incumbent in a general election, and candidates with more resources devote (weakly) greater shares to positive campaigning.

We now establish that, consistent with the data, candidates campaign relatively more positively in the primary than in the general election.

Proposition 2. *Provided that challenger k devotes any resources to positive campaigning he campaigns relatively more positively in the primary than in the general election: $n^*_{k0} - p^*_{k0} \leq n^*_{k1} - p^*_{k1} = \frac{\rho-1}{\rho}$, where the inequality is strict unless $\beta = 0$.*

The proof in the appendix proceeds by evaluating the relative strengths of the direct and indirect effects of an increase in β on primary campaigning. Challengers campaign relatively more positively in primaries than in general elections for two reasons: (1) the primary winner only benefits from positive primary campaigning in the general election; and (2) when a party rival wins the primary, the adverse effects of a negative campaign against him persist.

We now assume:

Assumption A1 (sufficient resources): Candidates have enough resources that they campaign both positively and negatively (i.e., first-order conditions describe the equilibrium).

Assumption A2 (symmetry): Challengers i and j have identical initial reputations, $\bar{X}_i = \bar{X}_j \equiv \bar{X}_C$ and resources, $\bar{B}_i = \bar{B}_j \equiv \bar{B}_C$.

In a working paper, we establish that under these conditions there exists a $\hat{\beta} \in (0, 1]$ such that for all $\beta < \hat{\beta}$ increases in β cause challengers to reduce negative primary campaigning and increase positive primary campaigning. In the appendix, we prove that an especially striking result obtains when enough of the effects of primary campaigning persist:

Proposition 3. *Under* **(A1)** *and* **(A2)***, for any relative effectiveness of negative campaigning ρ, if the effects of primary campaigning are long-lived (β sufficiently close to one), then $p^*_{k0} > n^*_{k0}$, i.e., challengers campaign more positively than negatively in primaries.*

Thus, when the effects of primary campaigning are long-lived, challengers campaign more positively than negatively in primaries, while the opposite occurs in general elections.

3 Extended Discussion and Conclusion

Negative campaigning dominates US general elections. What has received less attention is that primary campaigns are more positive in nature. Our dynamic theory of campaigning reconciles these conflicting observations. If it is easier to damage an opponent's reputation than to build one's own (Baumeister et al. (2001), Soroka and McAdams (2015)), then general election campaigns are more negative than positive because candidates only care about winning. The composition of primary campaigns reflects that primary winners only gain in the general election from positive primary campaigning; and primary losers harm a rival's general election chances with negative campaigns. We show that if the effects of political campaigns persist, then similarly-qualified challengers campaign more positively than negatively in primaries.

This raises the question as to what happens when challenger differ. In a working paper, we derive how the relative strengths of challengers and the incumbent affect the composition of political campaigning when challengers have similar reputations and resources, and primary campaigning only has small effects on general election reputations (β is small). We prove that (i) challengers reduce both positive and negative primary campaigning in response to facing a stronger incumbent with more resources; and (ii) both challengers increase both positive and negative primary campaigning when one challenger gains slightly more resources, B_i.

Intuitively, facing a relatively stronger incumbent, and consistent with Peterson and Djupe (2005), similarly-situated challengers want to conserve resources for the tougher general election by reducing positive and negative primary campaigning. Strategic complementarities reinforce this: if one challenger spends less on primary campaigning, so does the other. This reflects that if challenger i saves more funds for the general election, i is more likely to defeat the incumbent, so challenger j minds losing the primary by less.

Figures 1 and 2 illustrate how the relative strengths of challengers affect campaigning when they face a strong or a weak incumbent and the effects of campaigning persist. These numerical findings illustrate how with a strong incumbent, if one challenger is far stronger than the other, then making the strong challenger even stronger causes a weak rival to *reduce* primary campaigning, in particular reducing his negative campaigning by more than he increases his positive campaigning. A weak challenger internalizes that his rival has far better chances in the general election that he does not want to harm, and that greater primary campaigning reduces his own low general election chances. In turn, the strong challenger reduces his primary campaigning, as his rival's less aggressive campaign reinforces his advantage.

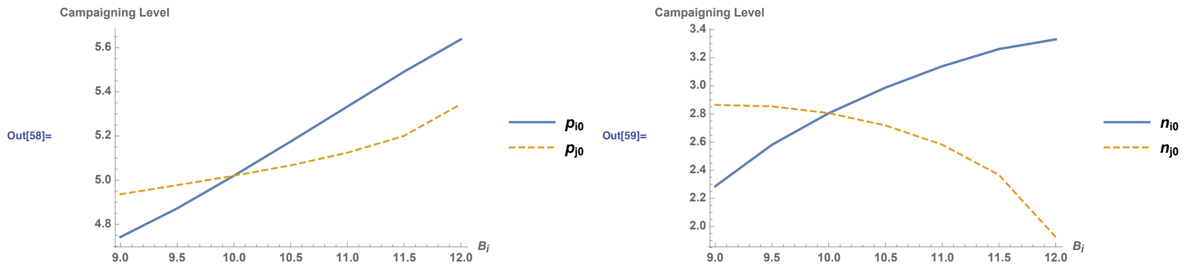

Figure 1: Primary campaigning as a function of challenger i's resources with a strong incumbent when campaigning persists. Parameters: $\bar{X}_j = \bar{X}_i = 6$, $\bar{B}_I = 15$, $\bar{X}_I = 10$, $\bar{B}_j = 10$, $\alpha = 1, \rho = 2, \beta = 1$.

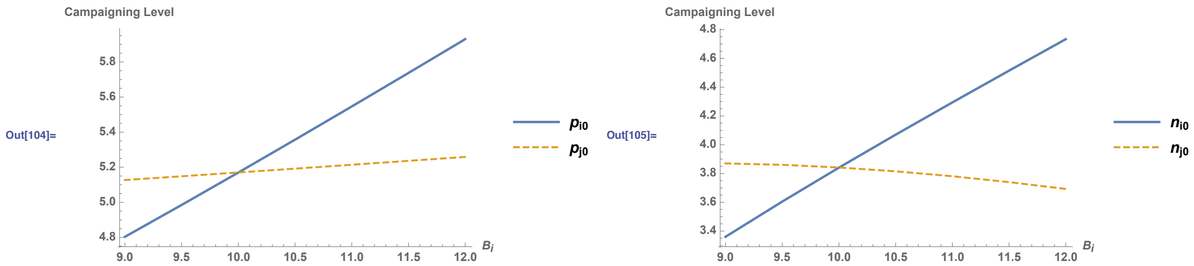

Figure 2: Primary campaigning as a function of challenger i's resources with a weak incumbent when campaigning persists. Parameters: $\bar{X}_j = \bar{X}_i = 6$, $\bar{B}_I = 5$, $\bar{X}_I = 10$, $\bar{B}_j = 10$, $\alpha = 1, \rho = 2, \beta = 1$.

Thus, contrary to general election campaigns, in primaries, a strong challenger campaigns more negatively than a weak one; and a very weak challenger's campaign may be totally positive, consistent with Peterson and Djupe's finding that in *primaries* incumbents face less negative campaigning from weak opponents. Republican presidential primaries illustrate this: in 2012, Romney had far more resources than rivals, and was far more negative; and in 2000, the weak candidate, John McCain, decided against "battling negative with negative...[in response to] the sheer volume of [Bush's] negative assaults."[7]

7 www.nytimes.com/2000/02/16/us/the-2000-campaign-the-arizona-senator-mccain-catches-mud-then-parades-it.html

4 References

Adams, James and Samuel Merrill (2008), "Candidate and Party Strategies in Two-Stage Elections Beginning with a Primary", *American Journal of Political Science*, 52, 344-359.

Baumeister, Roy, Ellen Bratslavsky, Catlin Finkenauer, and Kathleen Vohs (2001), "Bad is stronger than good." *Review of General Psychology*, 5, 323-370.

Bouton, Laurent, Micael Castanheira, and Allan Drazen (2018), "A Theory of Small Campaign Contributions", NBER WP 24413.

Brueckner, Jan and Kangoh Lee (2015), "Negative Campaigning in a Probabilistic Voting Model", *Public Choice*, 164, 379-399 .

Chakrabarti, Subhadip (2007), "A note on negative electoral advertising: Denigrating character vs. portraying extremism", *Scottish Journal of Political Economy*, 54, 136-149.

Da Silveira, Bernado and Joao De Mello (2011), "Campaign Advertising and Election Outcomes: Quasi-natural Experiment Evidence from Gubernatorial Elections in Brazil", *Review of Economic Studies* 78 (2), 590-612.

Djupe, Paul and David Peterson (2002), "The Impact of Negative Campaigning: Evidence from the 1998 Senatorial Primaries", *Political Research Quarterly*, 55(4), 845-860.

Epstein, Gil and Shmuel Nitzan (2006), "The Politics of Randomness", *Social Choice and Welfare* 27(2) 2, 423-433.

Esteban, Joan and Debraj Ray (1999), "Conflict and Distribution", *Journal of Economic Theory*, 87(2), 379-415.

Gandhi, Amit, Daniela Iorio and Carly Urban (2016), "Negative Advertising and Political Competition", *Journal of Law, Economics and Organization*, 32 (3), 433-477.

Harrington, Joseph and Gregory Hess (1996), "A Spatial Theory of Positive and Negative Campaigning", *Games and Economic Behavior*, 17 (2), 209-229.

Herrera, Helios, Massimo Morelli and Tom Palfrey (2014), "Turnout and Power Sharing", *Economic Journal*, 124 (574), 131-162.

Hummel, Patrick (2010), "Flip-flopping from primaries to general elections", *Journal of Public Economics*, 94, 1020-1027.

Kartik, Navin and R. Preston McAfee (2007), "Signaling character in electoral competition", *American Economic Review*, 97, 852-870.

Klumpp, Tilman and Mattias Polborn (2006), "Primaries and the New Hampshire effect", *Journal of Public Economics*, 90 (6-7), 1073-1114.

Konrad, Kai (2000), "Sabotage in Rent-seeking contests", *Journal of Law, Economics, and Organization*, 16(1), 155-165.

Konrad, Kai and Dan Kovenock (2009) "Multi-Battle Contests", *Games and Economic Behavior*, 66 (1), 256-274.

Meirowitz, Adam (2005), "Informational Party Primaries and Strategic Ambiguity", Journal of Theoretical Politics, 17, 107-136.

Peterson, David and Paul Djupe (2005), "When primary campaigns go negative: The determinants of campaign negativity", *Political Research Quarterly*, 58(1), 45-54.

Polborn, Mattias and David Yi (2006), "Informative Positive and Negative Campaigning", *Quarterly Journal of Political Science*, 1(4), 351-371.

Serra, Gilles (2011), "Why primaries? The party's tradeoff between policy and valence" Journal of Theoretical Politics, 23, 21-51.

Skaperdas, Stergios and Bernard Grofman (1995), "Modeling Negative Campaigning", *American Political Science Review*, 89 (1), 49-61.

Soroka, Stuart and Steven McAdams (2015), "News, Politics, and Negativity", *Political Communication*, 32, 1-22.

Soubeyran, Raphael (2009), "Contest with attack and defense: does negative campaigning increase or decrease voter turnout?", *Social Choice and Welfare*, 32, 337-353.

Spenkuch, Jorg and David Toniatti (2018), "Political Advertising and Election Results", *Quarterly Journal of Economics*, 133(4), 1981–2036.

5 Appendix

Proof of Proposition 2. Candidate i wins the primary with probability $Pr_{i0} = \frac{\bar{Z}_{i0}}{\bar{Z}_{i0}+\bar{Z}_{j0}}$, choosing p_{i0} and n_{i0} to maximize

$$\pi^i = (MPr_{i1} - Pr_{j1})Pr_{i0} + Pr_{j1}.$$

Define $P_{it} = 1 + p_{it}$ and $N_{it} = 1 + \rho n_{it}$ for $t \in \{0, 1\}$, and let $Q^*_{i0} = \frac{\bar{X}_j(P^*_{j0}N^*_{j0})^\alpha}{\bar{X}_i(P^*_{i0}N^*_{i0})^\alpha}, Q^*_{i1} = \frac{\bar{X}_I(P^*_I N^*_I(1-\beta+\beta N^*_{j0}))^\alpha}{\bar{X}_i(P^*_{i1}N^*_{i1}(1-\beta+\beta P^*_{i0}))^\alpha}$, with analogous expressions for j. Thus, $Pr_{it} = \frac{1}{1+Q^*_{it}}$. If i has enough resources that $p_{i1} > 0$, then at an interior optimum, (N^*_{i0}, P^*_{i0}) satisfy the first-order conditions:

$$\begin{aligned}
\frac{\partial \pi^i}{\partial N_{i0}} &= (MPr_{i1} - Pr_{j1})\frac{\partial Pr_{i0}}{\partial N_{i0}} + Pr_{i0}\left(M\frac{\partial Pr_{i1}}{\partial N_{i0}} - \frac{\partial Pr_{j1}}{\partial N_{i0}}\right) + \frac{\partial Pr_{j1}}{\partial N_{i0}} \\
&= \left(\frac{M}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}}\right)\frac{\alpha Q^*_{i0}}{N^*_{i0}(1+Q^*_{i0})^2} - \frac{1}{1+Q^*_{i0}}\left(\frac{\alpha M Q^*_{i1}}{N^*_{i1}(1+Q^*_{i1})^2} - \frac{\alpha\beta Q^*_{j1}}{(1-\beta+\beta N^*_{i0})(1+Q^*_{j1})^2}\right) \\
&\qquad - \frac{\alpha\beta Q^*_{j1}}{(1-\beta+\beta N^*_{i0})(1+Q^*_{j1})^2} = 0, \\
\frac{\partial \pi^i}{\partial P_{i0}} &= (MPr_{i1} - Pr_{j1})\frac{\partial Pr_{i0}}{\partial P_{i0}} + MPr_{i0}\frac{\partial Pr_{i1}}{\partial P_{i0}} \\
&= \left(\frac{M}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}}\right)\frac{\alpha Q^*_{i0}}{P^*_{i0}(1+Q^*_{i0})^2} - \frac{\alpha M Q^*_{i1}}{(1+Q^*_{i0})(1+Q^*_{i1})^2}\left(\frac{1}{P^*_{i1}} - \frac{\beta}{1-\beta+\beta P^*_{i0}}\right) = 0,
\end{aligned}$$

where, for example, $\frac{\partial Pr_{i0}}{\partial N_{i0}} = \frac{\partial Pr_{i0}}{\partial Q_{i0}}\frac{\partial Q_{i0}}{\partial N_{i0}} = \frac{-1}{(1+Q_{i0})^2}\frac{-\alpha Q_{i0}}{N_{i0}} = \frac{\alpha Q_{i0}}{(1+Q_{i0})^2}$, and $N^*_{i1} = \rho P^*_{i1} = (1 + \rho + \frac{\rho\bar{B}_i}{2} - \frac{\rho P^*_{i0}+N^*_{i0}}{2})$. Combine the last two terms in $\frac{\partial \pi^i}{\partial N_{i0}}$, divide by $\frac{\alpha}{1+Q^*_{i0}}$ and rearrange:

$$\left(\frac{M}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}}\right)\frac{Q^*_{i0}}{N^*_{i0}(1+Q^*_{i0})} = \frac{MQ^*_{i1}}{N^*_{i1}(1+Q^*_{i1})^2} + \frac{\beta Q^*_{j1}Q^*_{i0}}{(1-\beta+\beta N^*_{i0})(1+Q^*_{j1})^2} \tag{1}$$

Similarly, re-arrange $\frac{\partial \pi^i}{\partial P_{i0}} = 0$ to obtain:

$$\left(\frac{M}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}}\right)\frac{Q^*_{i0}}{P^*_{i0}(1+Q^*_{i0})} = \frac{MQ^*_{i1}}{(1+Q^*_{i1})^2}\left(\frac{1}{P^*_{i1}} - \frac{\beta}{1-\beta+\beta P^*_{i0}}\right). \tag{2}$$

Dividing equation (2) by (1) yields a LHS of $\frac{N^*_{i0}}{P^*_{i0}}$, and then multiplying both sides by the denominator of the RHS yields

$$\frac{N^*_{i0}}{P^*_{i0}}\left(\frac{MQ^*_{i1}}{N^*_{i1}(1+Q^*_{i1})^2} + \frac{\beta Q^*_{j1}Q^*_{i0}}{(1-\beta+\beta N^*_{i0})(1+Q^*_{j1})^2}\right) = \frac{MQ^*_{i1}}{P^*_{i1}(1+Q^*_{i1})^2} - \frac{\beta M Q^*_{i1}}{(1-\beta+\beta P^*_{i0})(1+Q^*_{i1})^2}.$$

Multiplying both sides by P^*_{i0} yields

$$\frac{MQ^*_{i1}}{(1+Q^*_{i1})^2}\left(\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} + \frac{\beta P^*_{i0}}{1-\beta+\beta P^*_{i0}}\right) + \frac{\beta N^*_{i0} Q^*_{j1} Q^*_{i0}}{(1-\beta+\beta N^*_{i0})(1+Q^*_{j1})^2} = 0. \tag{3}$$

At an interior optimum, $n^*_{i0}, p^*_{i0} > 0$, or equivalently $N^*_{i0}, P^*_{i0} > 1$. The second term of equation (3) is positive, so the first grouped term must be negative. Thus,

$$\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} < 0 \iff \frac{P^*_{i0}}{N^*_{i0}} > \frac{P^*_{i1}}{N^*_{i1}} = \frac{1}{\rho},$$

or equivalently,

$$n^*_{i0} - p^*_{i0} < n^*_{i1} - p^*_{i1} = \frac{\rho-1}{\rho}.$$

When $\beta = 0$, equations (1) and (2) reduce to

$$\left(\frac{M}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}}\right)\frac{Q^*_{i0}}{N^*_{i0}(1+Q^*_{i0})} - \frac{MQ^*_{i1}}{N^*_{i1}(1+Q^*_{i1})^2} = 0 \tag{4}$$

and

$$\left(\frac{M}{1+Q^*_{i1}} - \frac{1}{1+Q^*_{j1}}\right)\frac{Q^*_{i0}}{P^*_{i0}(1+Q^*_{i0})} - \frac{MQ^*_{i1}}{P^*_{i1}(1+Q^*_{i1})^2} = 0. \tag{5}$$

Then, as before, it follows that

$$\frac{P^*_{i0}}{P^*_{i1}} - \frac{N^*_{i0}}{N^*_{i1}} = 0 \iff \frac{P^*_{i0}}{N^*_{i0}} = \frac{P^*_{i1}}{N^*_{i1}} = \frac{1}{\rho} \iff n^*_{i0} - p^*_{i0} = n^*_{i1} - p^*_{i1} = \frac{\rho-1}{\rho}. \qquad \blacksquare$$

Proof of Proposition 3. Let **(A1)** and **(A2)** hold. With symmetry, $Q^*_{i0} = 1, Q^*_{i1} = Q^*_{j1}$. Substituting these expressions and $\beta = 1$ into equation (3) yields:

$$\frac{MQ^*_{i1}}{(1+Q^*_{i1})^2}\left(\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} + 1\right) + \frac{Q^*_{i1}}{(1+Q^*_{i1})^2} = 0.$$

Dividing by $\frac{Q^*_{i1}}{(1+Q^*_{i1})^2}$ yields

$$M\left(\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} + 1\right) + 1 = 0.$$

Re-arranging this equality yields

$$\frac{N^*_{i0}}{N^*_{i1}} - \frac{P^*_{i0}}{P^*_{i1}} = -\frac{M+1}{M} = -1 - \frac{1}{M} < -1. \tag{6}$$

Substituting $N_{i1}^* = \rho P_{i1}^*$, the inequality in (6) simplifies to:

$$N_{i0}^*/\rho - P_{i0}^* < -P_{i1}^*. \tag{7}$$

Further, since $P_{i0}^* = 1 + p_{i0}^*$ and $N_{i0}^* = 1 + \rho n_{i0}^*$, and $B_{i1} = \bar{B}_i - (p_{i0}^* + n_{i0}^*) = \bar{B}_i - (P_{i0}^* - 1) - \frac{N_{i0}^* - 1}{\rho}$,

$$P_{i1}^* = \frac{\rho + 1}{2\rho} + \frac{B_{i1}}{2} = \frac{1}{\rho}\left(1 + \rho + \frac{\rho \bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2}\right).$$

Rewrite inequality (7) as

$$\left(\frac{N_{i0}^*}{\rho} - P_{i0}^*\right) < -\frac{1}{\rho}\left(1 + \rho + \frac{\rho \bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2}\right).$$

Multiplying both sides by ρ and rearranging the right-hand side yields:

$$N_{i0}^* - \rho P_{i0}^* < 1 - \rho - \left(2 + \frac{\rho \bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2}\right). \tag{8}$$

From Proposition 1, challengers campaign negatively and positively in the general election if

$$\bar{B}_i - (P_{i0}^* - 1) - \left(\frac{N_{i0}^* - 1}{\rho}\right) > \frac{\rho - 1}{\rho}. \tag{9}$$

Multiplying both sides by ρ and rearranging, we rewrite (9) as

$$1 + \frac{\rho \bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2} > 0. \tag{10}$$

Hence, from (8) and (10),

$$N_{i0}^* - \rho P_{i0}^* < 1 - \rho - \underbrace{\left(2 + \frac{\rho \bar{B}_i}{2} - \frac{\rho P_{i0} + N_{i0}}{2}\right)}_{>0 \text{ from } (10)} < 1 - \rho, \quad \text{i.e.,} \quad N_{i0}^* - \rho P_{i0}^* < 1 - \rho.$$

Substituting $N_{i0}^* = 1 + \rho n_{i0}^*$ and $P_{i0}^* = 1 + p_{i0}^*$ to this inequality yields

$$1 + \rho n_{i0}^* - (\rho(1 + p_{i0}^*)) < 1 - \rho \Leftrightarrow \rho(n_{i0}^* - p_{i0}^*) < 0.$$

Continuity of n_{i0}^*, p_{i0}^* in β implies that the result extends for all β sufficiently large. ■