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ABSTRACT

In our target-initiated theory of takeovers, a target approaches potential acquirers that privately know their standalone values and merger synergies, where higher synergy acquirers tend to have larger standalone values. Despite their information disadvantage, targets can extract all surplus when synergies and standalone values are concavely related by offering payment choices that are combinations of cash and equity. Targets exploit the reluctance of high-valuation acquirers to cede equity claims, inducing them to bid more cash. When synergies and standalone values are not concavely related, sellers can gain by combining cash with securities that are more information-sensitive than equities.

Keywords: Adverse selection; mechanism and security design; bidder-initiated takeovers; decentralized auctions; cash plus securities payments

JEL classification: D44; D82

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Most empirical and theoretical research on takeovers starts with the premise that they are initiated by bidders. However, in recent work, Eckbo, Norli, and Thorburn (2020) establish that "about one half of takeover bids for public targets are initiated by the target, organizing an auction-like process to solicit potential bids." Our paper derives the optimal design of such auctions, including the specifications of the cash-equity composition of bids. We show that this optimal design reconciles a set of empirical regularities that are elusive to explain within a single unified framework: (1) multiple public bidders emerge in few control contests (under 10%) for publicly-traded targets; (2) despite this seeming lack of competition, on average almost all of the combined takeover gains accrue to target shareholders; (3) greater use of cash in bids is associated with higher returns for both acquirer and target; (4) cash acquisitions are more likely with multiple bidders; and (5) acquirers of public targets earn lower returns than acquirers of private targets. Our analysis suggests that empirical patterns in target and acquirer returns will vary according to whether takeovers are target or acquirer initiated; and that empirical work should control for this to avoid confounding identification of the true primitive relationships.

Our model features a target that recognizes the possibility of synergies that would obtain from a merger. The target's problem is that each potential acquirer (bidder) i is privately informed about its standalone value V_i and the synergy $s_i(V_i)$ (net value added) associated with a merger, where potential acquirer types with higher standalone values also tend to generate higher synergies. When acquirers pay with shares of the merged firm's equity, their private information about their standalone values can lead to an adverse selection problem (Myers and Majluf (1984); Che and Kim (2010)). The monetary value of an equity offer is proportional to the value of the combined firm, which is the sum of the target's value under the bidder's control plus the bidder's standalone value. When a bidder type with a lower synergy (NPV) also has a lower standalone value, it may be willing to offer a higher equity share—outbidding bidder types that would provide higher NPVs, for the combined firm—which would drive down the potential revenues that the target can extract. This adverse selection problem is especially severe when synergies rise gradually with standalone values, so that the extent of information asymmetry with respect to standalone values is high relative to that for synergy gains. In such instances, a higher-synergy-type bidder would offer a lower equity share because its standalone value—a share of which it forgoes if it wins—is too high.

Our paper shows how a target can solve this adverse selection problem by having bidders submit bids that involve combinations of securities with *different* levels of information sensitivity. The sensitivity of a security's value to the underlying cash flows is captured by its steepness (DeMarzo, Kremer, and Skrzpacz (2005, DKS)), where call options are steeper than equities, and equities, in turn, are steeper than cash. We show that by combining securities of differing steepness, a seller can obtain high revenues despite adverse selection, as combining different securities allows a target to exploit a reluctance of high-valuation acquirers to pay with security claims whose payments are tied more tightly to their valuations, and thereby overcome its information disadvantage by separating different acquirer types. Moreover, the greater the difference in steepness between the securities, the better it is for the seller.

Indeed, we establish that optimally designed combinations of cash and equity can sometimes allow the target to achieve the first-best outcome, that is, to extract full rents, with the highest-NPV bidder winning and receiving *zero* rent. When this is so, the seller obtains the same revenues as if it knew the private information of all bidders perfectly and simply made a single take-it-or-leave-it offer to the highest-type bidder. The seller exploits the fact that low and high bidder types value cash and equity differently. In particular, high bidder types care more about giving up equity claims to their high standalone values. This allows the seller to screen lower types with high equity offers and screen higher types with high cash offers. The seller constructs its menu of payments so that (1) higher types choose a less steep mix that requires them to pay more cash but give up a smaller equity claim, while (2) lower types choose steeper mixes. When synergies $s_i(V_i)$ increase concavely with standalone valuations, a seller can screen bidders by choosing cash-equity combinations that drive the differential rents between high and low types to zero, extracting all synergy gains, while ensuring global incentive compatibility.

The mechanism is dominant-strategy incentive compatible—if a bidder misreports and wins, its profit is negative, regardless of how the other bidders bid. The mechanism also has the first-price auction property that a winner's payment depends on its own report but not on those of other bidders. The combination of these properties is a novel feature of our mechanism[1] that has important implications. First, the mechanism extracts full rents, even from a single bidder, absent the competition that Crémer and McLean (1985, 1988) exploit when bidders have correlated signals. Second, bidders submit bids that reveal their true types and the seller extracts full rents even when the auction is decentralized so that a seller approaches each bidder privately and bidders do not know how many rivals they face. Later, we discuss how these features reconcile a host of empirical regularities related to takeovers.

When $s_i(V_i)$ is concave, the full-extraction mechanism can be implemented by having bidders bid cash plus an equity component that declines with the cash bid. The mechanism is robust in the sense that a seller can extract almost all of the rents when it only knows the approximate form of $s_i(V_i)$. When $s_i(V_i)$ is convex, full extraction via cash plus equity is not possible. Nonetheless, we identify the optimal cash-plus-equity mechanism in a single-bidder setting, proving that it takes a simple form: a seller offers a single contract that consists of a fixed equity share plus cash.[2] The intuition for how the curvature of $s_i(V_i)$ affects outcomes reflects the facts that (1) the payoff function from a cash-plus-equity contract is linear, (2) when offered multiple contracts, an optimizing bidder will select from the (convex) upper envelope of contract payoffs, and (3) when $s_i(V_i)$ is concave, a bidder's standalone value, which is its opportunity cost of winning, is a convex function of the merged firm's total value. When $s_i(V_i)$ is concave, the upper envelope of contract payoffs can be designed to match the convex curvature of the opportunity cost–total value relationship, separating all

[1] In a standard second-price auction, bidders have dominant strategies but the winner's payment depends on a losing bidder's bid; and in a standard first-price auction, the winner's payment depends on its own bid but bidders do not have dominant strategies.

[2] Payments in the form of a fixed royalty rate plus cash are used in many economic transactions (e.g., off-shore oil leases or timber lease auctions (see Gorbenko and Malenko (2011) and Skrzypacz (2013)).

types and extracting all rents. In contrast, under a convex synergy relationship, the two curvatures go in opposite directions and the best a seller can do is to offer a single contract that pools all participating types.

We next show that under a convex synergy relationship, a seller can combine cash with securities that are steeper than equity to do better. The intuition is that greater separation in steepness makes the combination of cash and a security a more effective screening device, because higher-type bidders are even more reluctant to pay with steeper securities.

Our work provides a framework for understanding a host of empirical regularities associated with takeovers. First, our full-extraction mechanism reconciles the observed distribution of returns between a target and its acquirer. Using different time periods and samples, previous researchers consistently find that the target's abnormal return is high and significant while the average acquirer's return is very low. Indeed, as Dessaint, Eckbo, and Golubov (2019, p1) observe: "an expansive literature benefitting from access to large-scale electronic databases confirms that...target shareholders capture the lion's share of combined takeover gains."

Second, our model provides a framework for understanding the observation that a target often privately approaches multiple potential acquirers but few bidders make public bids (Boone and Mulherin (2007)). Eckbo, Malenko, and Thorburn (2020, EMT) summarize the broad finding that public bids from multiple bidders occur in less than 10% of takeovers. Indeed, only 3.4% of all control contests for publicly traded U.S. targets between 1980 and 2005 had multiple public bidders (Betton, Eckbo and Thorburn (2008, BETa)). Our model explains this rarity. The optimal design has the following properties: (1) each potential acquirer has a dominant strategy, and (2) the winner's payment depends only on its own bid. Taken together, these features imply that neither a potential acquirer's bid nor its payment if it wins depends on what other bidders do. Our model is therefore consistent with a decentralized takeover process in which a target privately approaches each potential bidder, sets the terms of payment contingent on that bidder winning, and then selects the bidder that would increase the target's value by the most, with the target extracting all synergy gains. As such, our work enhances our understanding of auctions versus negotiation. In particular, while Bulow and Klemperer (1996) point to tradeoffs between auctions and negotiation, our work highlights ways in which the two approaches need not conflict.

Third, in our optimal mechanism, the cash-equity composition[3] of an offer is tailored to different bidder types, generating predictions consistent with the asset composition of bids and with the returns of targets and acquirers found in the data. In particular, both target and acquirer returns increase with the cash share of an offer, as does their combined return (Betton, Eckbo, and Thorburn (BETa, 2008, p328), EMT, Betton et al. (2014), and Andrade, Mitchell, and Stafford (2001)). Finally, the target's rent-extracting ability depends on the curvature of the relationship between synergies and standalone values. Our paper can reconcile differences in abnormal returns in acquisitions of public versus private targets when the synergy–valuation relationship varies systematically with target characteristics.

Many researchers have proposed theories of takeovers in which the acquirer has the bargaining power to explain some of these empirical patterns. However, these theories cannot reconcile key features of the data. First, theories in which rational bidders have substantial bargaining power (e.g., making take-it-or-leave-it offers) have difficulty explaining why, unconditionally, acquirer abnormal returns are so low. Second, it is difficult to reconcile the rarity of public bidders (BETa) with existing theories, even with preemptive bidding in which a single-bidder contest results when an initial bidder has a high valuation. The extreme rarity of contests with multiple public bidders means that costly preemption would occur even if an initial bidder has a low valuation (so that the distribution of preempting types is similar to the unconditional distribution). In contrast, a single public bidder and an unbalanced division of surplus between target and acquirer arise naturally in the decentralized target-designed takeover process described by Boone and Mulherin (2007) and Eckbo, Norli, and Thorburn

[3]See BETa, p328, who write that "mixed cash-stock offers are pervasive across the entire (1980 to 2005) sample period," or the survey by EMT (2020)) documenting the wide-spread use of such mixes.

(2020), which our model can capture. In this formulation, a target privately negotiates with potential bidders and extracts all synergy gains even though only one public bid is observed.

A key element of our model is that it is the target that has the bargaining power. To connect our model with the data, we posit that empirically one can approximate "who has bargaining power" with "who initiates the contest." In essence, our premise is that if a target decides to sell itself, then its board runs the process and, consistent with its fiduciary duty, it selects the mechanism that maximizes value. Sections A.2 and B motivate this approximation, detail empirical implications that differentiate our theory from others, and document evidence consistent with our predictions. Together, they demonstrate that target- and acquirer-initiated takeovers reflect different economic forces.[4] In turn, this means that empirical researchers need to account for which party initiates a takeover and the asset composition of an offer in order to extract the true primitive relationships.

A. *Security-bid literature*

DKS provide a comprehensive analysis of security-bid auctions in which payments are made using securities whose values are tied to the cash flows generated by the bidder. The authors show how the steepness of the security used for payment affects a seller's revenue. In particular, they establish that if bidders have private information about the asset's value but their opportunity costs of winning (e.g., standalone values in takeover auctions) are equal and common knowledge, then auctions using steeper securities yield the seller greater expected revenues.[5]

Che and Kim (2010) consider the possibility that opportunity costs are increasing in synergies. They show that if this increase is sufficiently fast then a severe form of adverse selection obtains when bids are in securities: bidders with higher synergies—and thus

[4] Gorbenko and Malenko (2019) develop the first model that seeks to endogenize the initiator of takeovers.

[5] Work on rent extraction and securities auctions/security design includes Hansen (1985, 1987), Fishman (1988), Dasgupta and Tsui (2004), Board (2007), Povel and Singh (2010), Kogan and Morgan (2010), Gorbenko and Malenko (2011), Liu (2012, 2016), Skrzypacz (2013), Burkart and Lee (2015), Sogo, Bernhardt, and Liu (2016), and Lee and Rajan (2020).

higher NPVs—bid less because they care more about retaining claims to their standalone values, so that steeper securities lead to lower seller revenues. The insight of our paper is that sellers can exploit this adverse selection by combining securities of differing steepness. Under severe adverse selection, the differential rents of high over low bidder types change sign between sets of securities with sufficiently differing steepness. Positive mixtures of different sets of securities can balance those differential rents to zero and yield full surplus extraction. Consider the combination of cash plus equity. When bidding strategies are decreasing in pure-equity auctions, low-NPV bidders may extract more rents with equity payments than high-NPV bidders. By contrast, all bidders value cash identically, so that, with cash, a bidder's rent always rises with its NPV. Starting from pure equity, one can reduce the equity payment and increase the cash payment so that the differential rent of a low type over a high type falls, crossing zero at some point.

Combining cash with securities that are steeper than equity can further facilitate the separation of types because higher-valuation types are even less willing to give up securities whose payoffs are tied more tightly to their valuations. The wider spread in steepness better balances bidders' differential rents to zero and helps ensure global incentive compatibility. Taken together, our findings reveal how the insights of DKS on the advantages of steeper securities extend to settings with adverse selection when a seller can combine auction design with security design.

Crémer (1987) shows that when a bidder's opportunity cost is constant and common knowledge, a seller can extract full rents by reimbursing the bidder for its opportunity cost, and demanding all of the equity. Crémer's mechanism has no screening—the seller's one payment choice leaves each bidder type indifferent between truthful reporting and deviating. In contrast, our mechanism relies on the ability to screen lower types with high equity offers and higher types with high cash offers. Conceptually, a further difference exists. Crémer's construction reflects the insight that with constant opportunity costs, equity bids generate more revenues than cash, so a seller should "leverage" by shorting the "inferior" cash component and demanding all of the "superior" equity component. Applying this logic to a setting with severe adverse selection where equity auctions generate less revenues than cash auctions, one might posit that a seller should "leverage" by shorting the "inferior" equity component. In fact, our mechanism uses strictly positive combinations of equity and cash, reflecting the intuition that positive mixtures of cash and equity can balance bidders' differential rents to zero when severe adverse selection obtains.[6]

Ekmekci, Kos, and Vohra (2016) consider the sale of a firm to a single buyer who is privately informed about cash flows and the benefits of control. The seller can offer a menu of cash-equity mixtures, and the buyer must obtain a minimum equity stake to gain control. They identify conditions under which it is optimal to have the buyer acquire either the minimum stake or all shares. In contrast, we show how offering different mixes of equity and cash increases seller revenues, and how and when using steeper-than-equity securities helps further.

Our work also provides a counterpoint to Deb and Mishra (2014). They show that if *utility* is contractible and the type space is finite, then any dominant-strategy implementable social choice rule can be implemented via a combination of cash transfers and equity transfers of *utility*. This result does not hold in our setting in which opportunity costs (standalone values) are not contractible. That is, focusing on cash plus equity is not without loss of generality: we identify settings with finite types where full extraction is impossible using equity plus cash, but the seller can still extract full rents using steeper securities plus cash, and the mechanism is dominant-strategy incentive compatible.

I. The Model

A group of $n \geq 1$ risk-neutral bidders (i.e., acquiring firms) competes to acquire an indivisible asset—a target firm. Each bidder i ($i = 1, ..., n$) privately observes its standalone

[6] We also extend this intuition to settings in which full extraction via cash plus equity is impossible. We derive the optimal mix and provide insights into the advantages of steeper-than-equity securities.

valuation V_i, which is independently distributed according to a continuous and strictly positive probability density f_i with support $\left[\underline{V}_i, \bar{V}_i\right]$, where $0 \leq \underline{V}_i < \bar{V}_i < \infty$. If bidder i acquires the target, this creates a synergy in the combined firm. The synergy is stochastic, with a distribution that can depend on the bidder's identity and the bidder's standalone valuation type V_i. The expected value of the synergy, s_i, is an increasing, continuous, and twice-differentiable function of V_i. Hence, conditional on the winner's type V_i, the expected value of the joint firm is $V_i + V_T + s_i(V_i)$, where V_T is the target's standalone value. The standalone value V_T and the functional forms of f_i and $s_i(\cdot)$ are assumed to be common knowledge.

Our base model assumes that the target is sold via mechanisms in which the winner pays with combinations of equity and cash.[7] Without loss of generality, we consider direct-revelation mechanisms. Let $\mathbf{z} \equiv (z_1, ...z_n)$ be the vector of reported bidder types; let $W_i(\mathbf{z})$ be the probability that bidder i wins, where $\Sigma_i W_i(\mathbf{z}) \leq 1$; and when bidder i wins, let $Q_i(\mathbf{z}) \in [0, 1]$ be the equity share that bidder i retains, and let $M_i(\mathbf{z})$ be its cash payment, which comes out of the revenues of the combined firm.[8]

Let $f(\mathbf{V}) \equiv \Pi_{i=1}^n f_i(V_i)$ denote the joint density of $\mathbf{V} \equiv (V_1, V_2, ..., V_n)$, and define $f_{-i}(\mathbf{V}_{-i}) \equiv \Pi_{k \neq i} f_k(V_k)$, where $\mathbf{V}_{-i} \equiv (V_1, ..., V_{i-1}, V_{i+1}, ..., V_n)$. Let $G_i(z_i)$ be the probability that bidder i wins when it reports having standalone value z_i and all other bidders report truthfully:

$$G_i(z_i) \equiv \int W_i(z_i, \mathbf{V}_{-i}) f_{-i}(\mathbf{V}_{-i}) d\mathbf{v}_{-i}. \tag{1}$$

Similarly, define $q_i(z_i)$ to be the expected equity share that bidder i retains conditional on winning by reporting that it has standalone value z_i when all others report truthfully,

$$q_i(z_i) G_i(z_i) \equiv \int Q_i(z_i, \mathbf{V}_{-i}) W_i(z_i, \mathbf{V}_{-i}) f_{-i}(\mathbf{V}_{-i}) d\mathbf{V}_{-i}, \tag{2}$$

[7] This assumption is without loss of generality when $s_i(\cdot)$ is concave, as we show that this mechanism extracts all surplus.

[8] Cash payments in takeovers typically come out of corporate resources. An earlier version of this paper studied settings in which the cash payment comes out of a winner's personal funds. All general results extend.

and define $\omega_i(z_i)$ to be bidder i's unconditional expected cash payment when it reports z_i,

$$\omega_i(z_i) \equiv \int M_i(z_i, \mathbf{V}_{-i}) W_i(z_i, \mathbf{V}_{-i}) f_{-i}(\mathbf{V}_{-i}) d\mathbf{V}_{-i}. \tag{3}$$

Let $h_i(V_i, z_i)$ be bidder i's expected profit when it has standalone value V_i but reports z_i, and all other bidders report truthfully,

$$h_i(V_i, z_i) \equiv \left[(V_i + s_i(V_i) + V_T)\, q_i(z_i) - V_i \right] G_i(z_i) - \omega_i(z_i) q_i(z_i). \tag{4}$$

Bidder i's equilibrium expected profit is given by $h_i(V_i, V_i)$. Individual rationality requires that

$$h_i(V_i, V_i) \geq 0, \quad \text{for all } i \text{ and } V_i. \tag{5}$$

Incentive compatibility requires that

$$h_i(V_i, V_i) = \max_{z_i} h_i(V_i, z_i), \quad \text{for all } i \text{ and } V_i. \tag{6}$$

A seller's expected profit is the increase in expected social surplus minus the profits of all bidders,

$$\pi_T = \int \Big(\sum_{i=1}^{n} W_i(\mathbf{V})\, s_i(V_i) \Big) f(\mathbf{V})\, d\mathbf{V} - \sum_{i=1}^{n} \int h_i(V_i, V_i)\, f_i(V_i)\, dV_i. \tag{7}$$

The seller seeks to design $W_i(\cdot)$, $Q_i(\cdot)$, and $M_i(\cdot)$ to maximize its expected profit (7) subject to the individual rationality (5) and incentive compatibility (6) requirements for bidders.

Discussion. Although we pose the model in a takeover context, the analysis applies more generally, for example, to oil and gas lease or timber lease auctions that feature cash payments plus equity payments in the form of royalties.[9] One can reformulate our model as an auction in which, rather than there being merger synergies, each bidder has an opportunity cost of winning, V_i, that increases with the bidder's expected total valuation

[9] In such settings, a seller (government) typically has commitment power but no private information.

of the asset, v_i. Because opportunity costs enter bidder payoffs negatively, the curvature restrictions on $V_i(v_i)$ required for full extraction are the opposite of those needed on $s_i(V_i)$, but the analysis is otherwise identical.

We first provide an example to illustrate our mechanism.

EXAMPLE 1: There are $n \geq 1$ ex-ante identical bidders, who have standalone valuations V distributed on $[1, 2]$ and expected synergy $s(V) = 1 + V - 0.1V^2$. The seller's standalone value is zero.

We show that the following mechanism extracts full rents: (1) each bidder reports a type $z \in [1, 2]$, where the highest reported type wins (ties are broken randomly), and (2) the winner pays a fraction $\frac{1-0.2z}{2-0.2z}$ of equity that decreases in z plus a cash payment of $1 + 0.1z^2$ that increases in z. Later, we show how one can exploit the monotonicity of the payment components in z to implement the mechanism using cash payments rather than reports about type.

In this mechanism, the winner retains equity claim $1 - \frac{1-0.2z}{2-0.2z}$ to the firm's residual combined value $V + s(V) - (1 + 0.1z^2)$ after making its cash payment and it forgoes its value V as a standalone entity. To derive the bidding strategy of a type V bidder, let $h(V; z)$ denote its expected profit (in excess of its standalone value) conditional on winning. We then have

$$\begin{aligned} h(V; z) &= \left(1 - \frac{1-0.2z}{2-0.2z}\right) \left(V + s(V) - \left(1 + 0.1z^2\right)\right) - V \\ &= \frac{1}{2-0.2z} \left(1 + 2V - 0.1V^2 - \left(1 + 0.1z^2\right)\right) - V \\ &= \frac{1}{2-0.2z} \left(-0.1V^2 - 0.1z^2 + 0.2zV\right) \\ &= -\frac{0.1}{2-0.2z} (V - z)^2 . \end{aligned} \tag{8}$$

By (8), for all V, the maximized value of $h(V; z)$ is zero, which obtains when a bidder truthfully reports $z = V$; and $h(V; z) < 0$ for all other values of z. Because a bidder's profit is zero when it loses, truthful bidding is optimal regardless of how the other bidders bid. That is, truthful bidding is a weakly dominant strategy. In equilibrium, all bidders receive zero rent, the highest NPV (synergy) bidder wins, and the seller extracts full rents.

A. Analysis

We now show how combining cash and equity helps rent extraction in general. We determine the optimal combination, and identify necessary and sufficient conditions under which full extraction via equity plus cash is possible. We then examine what a seller can do to maximize revenue when these conditions do not hold. We first allow cash payments to be negative. We then identify the conditions needed for full extraction with positive cash payments.

By the Envelope Theorem, bidder i's equilibrium payoff is

$$h_i(V_i, V_i) = h_i(\underline{V}_i, \underline{V}_i) + \int_{\underline{v}_i}^{v_i} (q_i(t) + \frac{ds_i(t)}{dt} q_i(t) - 1) G_i(t) dt,$$

and its derivative is

$$\frac{dh_i(V_i, V_i)}{dV_i} = (q_i(V_i) + \frac{ds_i(V_i)}{dV_i} q_i(V_i) - 1) G_i(V_i). \tag{9}$$

Define

$$S_i \equiv \{V_i | s_i(V_i) > s_j(\underline{V}_j) \text{ for all } j \neq i \text{ and } s_i(V_i) > 0\}$$

to be the set of types V_i such that having bidder i win control of the target can increase expected social surplus relative to having any other bidder win or having the target remain as a standalone entity. That is, if bidder i's type is not in S_i, it is not socially optimal to have i win. If a mechanism using combinations of equity and cash exists that extracts full rents, then

$$h_i(V_i, V_i) = 0, \text{ for all } i \text{ and } V_i, \tag{10}$$

where $G_i(V_i) > 0$ if $V_i \in S_i$, and $G_i(V_i) = 0$ if $V_i \notin S_i$.

For $z_i \in S_i$, divide both sides of (4) by $G_i(z_i)$ to define

$$\hat{h}_i(V_i, z_i) \equiv \frac{h_i(V_i, z_i)}{G_i(z_i)} = (V_i + s_i(V_i) + V_T)q_i(z_i) - V_i - \frac{\omega_i(z_i)q_i(z_i)}{G_i(z_i)}. \tag{11}$$

Here, $\hat{h}_i(V_i, z_i)$ is bidder i's expected profit conditional on winning when it has standalone value V_i but reports z_i and all other bidders report truthfully. Equations (10) and (11) yield

$$\hat{h}_i\left(V_i, V_i\right) = 0, \quad \text{for all } i \text{ and } V_i. \tag{12}$$

Equation (10) and the incentive compatibility condition (6) yield $h_i(V_i, z_i) \leq 0$ for all z_i, and hence $\hat{h}_i(V_i, z_i) \leq 0$, for all $z_i \in S_i$. Thus, for all $V_i \in S_i$, we have

$$\hat{h}_i\left(V_i, V_i\right) = \max_{z_i \in S_i} \hat{h}_i(V_i, z_i) = \max_{z_i \in S_i} \left(\left(V_i + s_i\left(V_i\right) + V_T\right) q_i(z_i) - V_i - \frac{\omega_i(z_i)q_i(z_i)}{G_i(z_i)} \right). \tag{13}$$

Let v_i be the total value of the merged firm under bidder i's control, as a function of V_i:

$$v_i\left(V_i\right) \equiv V_i + s_i(V_i) + V_T. \tag{14}$$

Because v_i strictly increases in V_i, it is invertible. As a result, we can express V_i as a function of v_i. We use this to write (13) as

$$\hat{h}_i(V_i(v_i), V_i(v_i)) = \max_{z_i \in S_i} \hat{h}_i(V_i(v_i), z_i) = \max_{z_i \in S_i} \left(v_i q_i(z_i) - V_i(v_i) - \frac{\omega_i\left(z_i\right) q_i(z_i)}{G_i(z_i)} \right). \tag{15}$$

Adding $V_i(v_i)$ to both sides yields that, for all $V_i(v_i) \in S_i$,

$$\hat{h}_i(V_i(v_i), V_i(v_i)) + V_i(v_i) = \max_{z_i \in S_i} \left(v_i q_i(z_i) - \frac{\omega_i\left(z_i\right) q_i(z_i)}{G_i(z_i)} \right). \tag{16}$$

By (16), $\hat{h}_i(V_i(v_i), V_i(v_i)) + V_i(v_i)$ is the maximum of a family of affine functions of v_i, so it is weakly convex for $V_i(v_i) \in S_i$. Because $\hat{h}_i\left(V_i(v_i), V_i(v_i)\right) = 0$, $V_i(v_i)$ must be weakly convex in v_i, or equivalently v_i must be weakly concave in V_i, for $V_i \in S_i$. In turn, inspection of the definition of v_i in (14) reveals that $s_i(V_i)$ must be weakly concave in V_i.[10] We therefore have the following result.

LEMMA 1 *If $s_i(V_i)$ is not weakly concave over $V_i \in S_i$ for all i, then combinations of equity plus cash cannot extract all surplus, even if the cash component is allowed to be negative.*

We now provide all necessary conditions for the existence of a mechanism using combinations of equity and cash that extracts full rents. These conditions are also sufficient.

THEOREM 1 *A mechanism using combinations of equity and (possibly negative) cash exists that extracts full rents if and only if for all bidders i: (i) if $V_i \in S_i$, then $s_i(V_i)$ is weakly concave in V_i; and (ii) if $V_i \notin S_i$, then $s_i(V_i) \le s_i(z_i) + s_i'(z_i)(V_i - z_i)$ for $z_i \in S_i$.*

PROPOSITION 1 *When conditions (i) and (ii) in Theorem 1 hold, the following dominant-strategy incentive compatible mechanism extracts full rents (i.e., the highest synergy bidder wins and receives zero expected profit):*

- *The winning rule is* $W_i(V_i, \mathbf{V}_{-i}) = \begin{cases} 1 & if \quad V_i \in S_i \text{ and } s_i(V_i) > \max_{j\neq i} s_j(V_j) \\ 0 & otherwise \end{cases}$

  and ties are broken arbitrarily.

- *The winning bidder retains equity share*

$$Q_i(V_i, \mathbf{V}_{-i}) = \frac{1}{1 + \frac{ds_i(V_i)}{dV_i}}, \tag{17}$$

  and makes cash payment

$$M_i(V_i, \mathbf{V}_{-i}) = s_i(V_i) + V_T - V_i \frac{ds_i(V_i)}{dV_i}. \tag{18}$$

[10] If the set S_i is not connected, then the requirement translates to requiring that a weakly concave function of V_i go through $\big(s_i, s_i(V_i)\big)$ for all $V_i \in S_i$.

We leave a formal proof of Theorem 1 and Proposition 1 to the appendix. Here, we provide intuition graphically using Figure 1 for some bidder i.

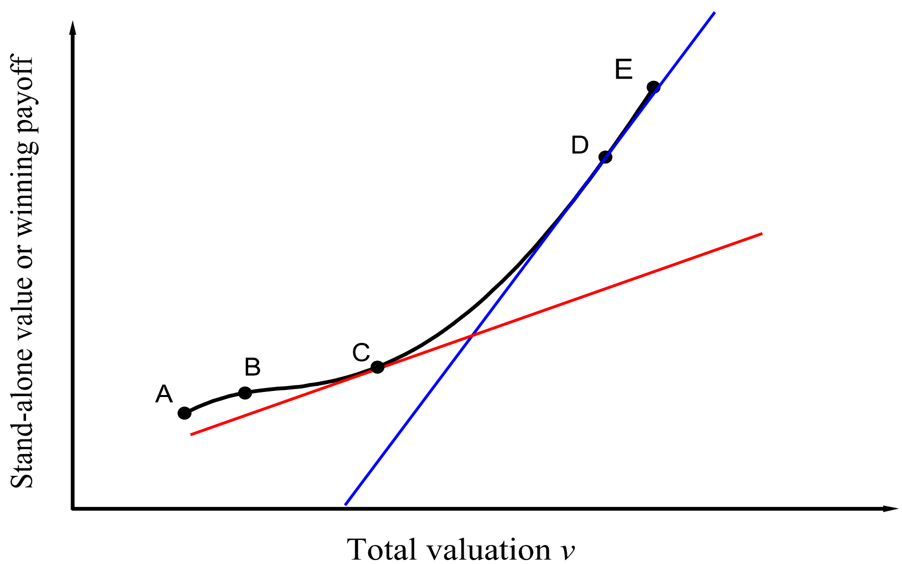

Figure 1: Arc ABCDE plots bidder i's standalone value V_i as a function of its total valuation v_i, where points on CDE$\in S_i$. The tangent line through each point on CDE is the winning payoff from the contract offered to the bidder type associated with that point. The upper envelope of the tangent lines coincides with CDE and lies below ABC.

The horizontal axis is the total valuation v_i in (14). Arc ABCDE depicts bidder i's possible types, plotting V_i as a function of v_i. Points in arc ABC $\notin S_i$ and points in arc CDE $\in S_i$. The winning profit of a type with total valuation v_i from reporting total valuation z_i is

$$h_i^*(v_i, z_i) \equiv \hat{h}_i(V_i(v_i), V_i(z_i)) = \left[v_i q_i(V_i(z_i)) - \frac{\omega_i(V_i(z_i))\, q_i(V_i(z_i))}{G_i(V_i(z_i))}\right] - V_i(v_i). \tag{19}$$

The expression inside the brackets is the contractual payoff to a bidder i with total valuation v_i when it takes the contract meant for type z_i. This payoff is a straight line with slope $q_i(V_i(z_i))$. The last term on the right-hand side is bidder i's standalone value, which represents its opportunity cost of winning. The difference is i's winning profit. Thus, any type point above this straight payoff line would lose profit from this contract, while any type point below the line would realize a positive profit.

Consider the contract offered to some type D on arc CDE. Again the payoff associated with this contract is a straight line. Because type D receives zero profit given full extraction, this line goes through point D. In addition, full extraction at all points in S_i plus the incentive compatibility condition (15) yield $h_i^*(v_i, z_i) \leq 0$ for all $V_i(v_i) \in S_i$ and all z_i. Hence, all other type points (including point A) must be above this line. Because this must hold when point D is replaced by any other point on arc CDE, this arc must be convex (i.e., $s_i(\cdot)$ must be concave, as in part (i) of the theorem), and the line must be a tangent line. The slope of this line is $q_i(V_i(z_i))$ by the payoff condition established earlier, and the slope is $\frac{dV}{dv}$ by the tangency condition. Thus, $q_i(V_i(z_i)) = \frac{dV}{dv}$, which yields (17) after reversing the transformation (14).

Next, consider any point on arc ABC, which is not in S_i. Full extraction requires that the associated type not take any contract offered to types on arc CDE. Thus, all points on arc ABC must be above any tangent line of arc CDE. This delivers condition (ii) of the theorem after reversing the transformation (14). Observe that arc ABC need not be convex—to achieve full extraction, part (i) of the theorem does not need to hold for types outside S_i. The difference in the restrictiveness of the two conditions reflects the fact that for any type *in* S_i, one must ensure that no other type wants to mimic it, but mimicking types not in S_i always loses.

COROLLARY 1 *In the full-extraction mechanism (Proposition 1), higher types choose a flatter mix, paying with less equity and more cash:*

$$\frac{d}{dV_i}(1 - Q_i(V_i, \mathbf{V}_{-i})) = \frac{1}{(1 + \frac{ds_i(V_i)}{dV_i})^2} \frac{ds_i^2(V_i)}{dV_i} \leq 0 \;\; \textit{and} \;\; \frac{dM_i(V_i, \mathbf{V}_{-i})}{dV_i} = -V_i \frac{ds_i^2(V_i)}{dV_i} \geq 0.$$

This corollary reveals the general principle underlying the gains from mixing cash and equity: a seller can tailor its menu of possible cash-equity payments so that (1) higher types that expect to generate higher synergies (and have higher standalone values) choose a less steep mix that requires them to give up a smaller equity claim to those standalone values in return for a higher cash payment,[11] while (2) lower types choose steeper mixes because they care less about giving up greater equity claims to their lower standalone values, and more about the cash payment. The net effect is to reduce the differential rents between high and low types.[12] The mechanism in Proposition 1 is dominant-strategy incentive compatible. A bidder's winning payment depends only on its own report and not on those of other bidders; and if a bidder misreports and wins, its profit is nonpositive. Thus, the mechanism extracts full rents even from a single bidder absent any competition, and bidders would not deviate even in a complete information setting with multiple bidders. Moreover, if $s_i(V_i)$ is strictly concave over $\left[\underline{V}_i, \bar{V}_i\right]$, then the equilibrium enforces itself in a strict sense: bidder i would receive a strictly negative expected profit if it deviated and won.

The cash payments needed in Proposition 1 to extract all rents could be negative. As DKS highlight, mechanisms with negative cash payments can have moral hazard issues. We now provide the additional necessary and sufficient condition for the optimal mechanism to specify that $\omega_i(V_i) > 0$ for all $V_i \in S_i$, that is, for the winner's cash payment to necessarily be positive.

COROLLARY 2 *Suppose that conditions (i) and (ii) in Theorem 1 hold. Then, if, for all i,*

$$\frac{d}{dV_i}\frac{s_i(V_i)+V_T}{V_i} < 0 \quad \textit{for all } V_i \in S_i, \tag{20}$$

then the mechanism specified in Proposition 1 uses strictly positive combinations of equity and cash and extracts all rents, generating strictly higher expected revenues than either pure equity or pure cash. In contrast, if these conditions do not hold, then no mechanism can extract full rents using combinations of equity and strictly positive cash.

Equation (20) is equivalent to the elasticity condition $\frac{d\ln(s_i(V_i)+V_T)}{d\ln V_i} < 1$, that is, the

[11] In contrast, in equity auctions in which standalone values (or opportunity costs or investment costs) are constant, a higher type always gives up a *larger* claim to outbid lower types.

[12] The prediction in Corollary 1 holds for both single- and multiple-bidder contests. This is one way in which our model differs from the single-bidder signaling model of Eckbo, Giammarino and Heinkel (1990). Section B presents additional empirical implications that further differentiate our model from bidder-initiated theories.

bidder's valuation of the asset, $s_i(V_i) + V_T$, increases in the bidder's standalone value V_i less than unit elastically. It is the necessary and sufficient condition for bidding strategies in second-price equity auctions to be decreasing in the total valuation of the combined firm $s_i(V_i) + V_T + V_i$ (Che and Kim (2010)).[13] This condition implies that adverse selection is severe if bidders only pay with equity, but Corollary 2 shows that this condition in fact allows the seller to extract full rents with equity plus strictly positive cash. That is, the seller can exploit the adverse selection to its advantage. When adverse selection is severe, high bidder-types expect to earn less rents than low bidder-types when they pay with equity—so strictly positive mixtures of cash and equity can balance bidders' differential rents to zero. Thus, our mechanism allows the seller to *more than* costlessly overcome the adverse-selection problem: the seller does strictly better than in settings without adverse selection, where positive mixtures of cash and equity cannot extract full rents.

Implementation: The full-extraction mechanism can be implemented via cash plus a *decreasing* equity share. By Corollary 1, the equity share paid falls with V_i and the cash payment rises with V_i. Therefore, the equity share paid can be expressed as a strictly decreasing function of the cash payment. This means that the full-extraction mechanism can be implemented by having each bidder i bid in cash and setting a reserve price of

$$r_i \equiv s_i(V_i^*) + V_T - V_i^* \frac{ds_i(V_i^*)}{dV_i}, \tag{21}$$

which is the cash payment in (18) for type V_i^*, where V_i^* is the zero-synergy type ($V_i^* \equiv \underline{V}_i$ if all types have positive synergies). The bidder with the highest bid exceeding the reserve wins (given ex-ante identical bidders; the approach generalizes) and pays its own bid. However, for each cash bid made, a bidder must pay an additional equity share that

[13]This decreasing property follows because (1) the bidding strategy in second-price equity auctions is $\frac{s_i(V_i)+V_T}{V_i+s_i(V_i)+V_T} = \frac{\frac{s_i(V_i)+V_T}{V_i}}{1+\frac{s_i(V_i)+V_T}{V_i}}$, which decreases in $\frac{s_i(V_i)+V_T}{V_i+s_i(V_i)+V_T}$ if and only if $\frac{s_i(V_i)+V_T}{V_i}$ decreases in $V_i + s_i(V_i) + V_T$; and (2) $V_i + s_i(V_i) + V_T$ increases in V_i.

is uniquely determined by its cash bid via the function determined above. As with the mechanism in Proposition 1, this implementation is dominant-strategy incentive compatible.

To illustrate this implementation, we revisit Example 1. Consider a first-price auction in which a bidder offers cash c plus an associated equity share

$$e(c) = \begin{cases} 1 - \frac{1}{2-0.2\sqrt{10c-10}} & \text{if} \quad c \leq 3.5 \\ 0 & \text{if} \quad c > 3.5 \end{cases}. \tag{22}$$

The cash bid must weakly exceed a reserve price of 1.1, the highest cash bid wins, and the winning bidder pays that bid plus the associated equity.

Let $\pi(c; V)$ be type V's expected profit conditional on winning. Then

$$\begin{aligned} \pi(c; V) &= (1 - e(c))(V + s(V) - c) - V \\ &= (1 - e(c))\left(1 + 2V - 0.1V^2 - c\right) - V. \end{aligned}$$

Direct calculation yields that for all $V \in [1, 2]$, the maximum value of $\pi(c; V)$ is zero, which obtains at

$$c = 1 + 0.1V^2, \tag{23}$$

and that $\pi(c; V) < 0$ for all other values of c.[14]

Because a bidder that loses earns zero profit, bidding according to (23) is optimal regardless of how the other bidders bid. In equilibrium, all bidders receive zero rent. Because (23) strictly increases in V, the highest-valuation bidder wins and the NPV is strictly positive—the seller extracts full rents. This example highlights how, in our first-price auction, optimal combinations of cash with an equity share that declines in the cash bid

[14] When $c \in [1.1, 3.5]$, $\pi(c; V) = \frac{1}{2-0.2\sqrt{10c-10}}\left(1 + 2V - 0.1V^2 - c\right) - V$. Equation (23) is the unique solution to the first-order condition $\frac{\partial \pi(c;V)}{\partial c} = 0$ for $c \in [1.1, 3.5]$, which yields $\pi(c; V) = 0$ for all V. Furthermore, $\frac{\partial \pi(c;V)}{\partial c} > 0$ for $c \in [1.1, 1 + 0.1V^2)$, and $\frac{\partial \pi(c;V)}{\partial c} < 0$ for $c \in (1 + 0.1V^2, 3.5]$. When $c > 3.5$, $\pi(c; V) = 1 + V - 0.1V^2 - c < 0$ for all $V \in [1, 2]$.

yield an equilibrium in which bidders employ dominant strategies.[15]

In fact, to extract full rent, it suffices for a seller to be able to commit to reserve prices—the equity share paid falls with the cash payment, so the cash payment by bidder i can be expressed as a decreasing function $\kappa_i(\cdot)$ of the equity share. The following first-price auction with a decreasing reserve extracts full rents: each bidder i offers a cash-plus-equity combination, and there is an e_i-dependent reserve price for the minimum cash bid: $\max\{\kappa_i(e_i), r_i\}$, where r_i is given in (21). This reserve price falls with the share e_i offered. The bidder with the highest cash bid exceeding the reserve wins (with ex-ante identical bidders, but the approach generalizes to ex-ante heterogeneous bidders). In Example 1, this equity-dependent reserve price is $\max\left\{0.1\left(10-\frac{5}{1-e}\right)^2+1, 1.1\right\}$. The first term inside the max operator is $\kappa(e)$, which is the inverse function of (22).

Robustness: Samuelson (1987) observes that existing mechanisms that extract all (or nearly all) surplus via contingent payments face two robustness concerns. First, in such mechanisms bidders earn zero (or close to zero) rents regardless of whether they report truthfully, rendering them almost indifferent to reporting any values. But then any added factor (even pure white noise) can result in an inefficient bidder type being selected as the winner. Second, such mechanisms require the winning bidder to retain zero equity share, leaving the winner prone to moral hazard, with no incentive to take actions ex post that maximize firm value.

Our full-extraction mechanism addresses both of these concerns. First, if $s_i(V_i)$ is strictly concave, a bidder would receive a strictly negative expected profit from deviating and winning. This makes selection of the "right" bidder robust to small noise. Second, our mechanism almost never requires the winning bidder to give up all equity, mitigating moral hazard concerns. We now establish a third robustness property, deriving a lower bound on seller revenues when full extraction is impossible via combinations of cash and equity. We show that if $s_i(V_i)$ is not quite weakly concave, or if there is slight uncertainty about the value

[15] In contrast, if bidders pay with pure cash, pure equity, or cash plus a *constant* equity share, a first-price auction does not have a dominant-strategy equilibrium.

of s_i conditional on V_i, then combining equity and cash can still extract almost all surplus.

We establish a simple lower bound on a seller's revenue for the case of a single bidder that would generate positive synergies, that is, $s(V) > 0$ for all V. It follows that it is socially optimal to sell to any bidder type, and the maximum social welfare gain is

$$\Pi \equiv \int_{\underline{V}}^{\bar{V}} s(V) f(V) dV.$$

We now define what it means for the synergy function to be close to a (weakly concave) function, and then bound the revenue loss vis-'a-vis basing the mechanism on that function.

Definition: Consider a strictly increasing function $s(\cdot)$ defined over $V \in [\underline{V}, \bar{V}]$, and a strictly increasing function $y(\cdot)$ defined over $V \in [\underline{V} - \frac{\sqrt{2}}{2}\epsilon, \bar{V} + \frac{\sqrt{2}}{2}\epsilon]$, where $\epsilon > 0$. Then $s(\cdot)$ is in the ϵ-neighborhood of $y(\cdot)$ if, for $V_1 \in [\underline{V} - \epsilon, \bar{V} + \epsilon]$ and $V_2 \in [\underline{V}, \bar{V}]$, $V_1 + y(V_1) = V_2 + s(V_2)$ implies that $(V_1 - V_2)^2 + (y(V_1) - s(V_2))^2 \leq \epsilon^2$.

PROPOSITION 2 *If the expected synergy function $s(V)$ is in an ϵ-neighborhood of a weakly concave function $y(V)$, then an equity-plus-cash mechanism exists that generates expected profit of at least*

$$\Pi - \sqrt{2}\epsilon.$$

The bidder reports a type $V \in [\underline{V} - \frac{\sqrt{2}}{2}\epsilon, \bar{V} + \frac{\sqrt{2}}{2}\epsilon]$, wins the asset, retains equity share

$$Q(V) = \frac{1}{1 + \frac{dy(V)}{dV}}, \tag{24}$$

and makes cash payment

$$M(V) = y(V) + V_T - V\frac{dy(V)}{dV} - \frac{\sqrt{2}}{2}\epsilon\left(1 + \frac{dy(V)}{dV}\right). \tag{25}$$

Proof: See the appendix. ∎

Intuitively, for any bidder type with standalone value V and expected synergy $y(V)$, in the prescribed mechanism in Proposition 2 (with $\epsilon = 0$ in (25)), the bidder's expected profit is weakly higher reporting V than reporting anything else. The proof shows that if another type has the same total valuation (i.e., standalone value plus expected synergy equals $V + y(V)$), then that other type is also better off reporting V than reporting anything else. This leaves only to guarantee that individual rationality holds. Reducing the cash payment $M(V)$ by a constant only affects individual rationality constraints and not the incentive compatibility constraints. Thus, by properly choosing this constant (setting it to $\frac{\sqrt{2}}{2}\epsilon$), both the incentive compatibility and individual rationality constraints can be satisfied.

Proposition 2 implies that when $s(V)$ is "almost" weakly concave, a seller can extract almost all surplus. Moreover, this result holds even if s does not evolve deterministically with V. Thus, even if the seller only knows approximately how s is related to V, our mechanism still delivers close to full extraction. With multiple bidders, an additional complication may arise because, with a misspecified model ($y_i(\cdot)$ differs from $s_i(\cdot)$), the highest-synergy bidder may not always be selected. However, as noted earlier, concavity helps our mechanism offset this selection effect, and the mechanism will still extract close to full rents when ϵ is small.

A.1. Optimal Equity-Plus-Cash Mechanism When $s(V)$ is Not Concave

We next characterize how the optimal mechanism is affected when synergies are not a concave function of standalone values. To illustrate, we first consider a single bidder with three possible types (V_i, s_i), $i = 1, 2, 3$, where $0 < V_1 < V_2 < V_3$ and $0 < s_1 < s_2 < s_3$. Let $f_i > 0$ be the probability of a type i bidder, where $f_1 + f_2 + f_3 = 1$. When

$$\tau \equiv \frac{s_3 - s_2}{V_3 - V_2} - \frac{s_2 - s_1}{V_2 - V_1} > 0, \tag{26}$$

concavity is violated, rendering full extraction via equity plus cash impossible. Here $\tau > 0$ measures the convexity of the synergy–value relationship. Note that τ approaches zero as $\frac{s_3 - s_2}{V_3 - V_2} \to \frac{s_2 - s_1}{V_2 - V_1}$, making the relationship almost linear. To solve for the profit-maximizing equity-plus-cash mechanism, without loss of generality, let the seller offer three contract choices $\{c_i, e_i; p_i\}$. A bidder that selects contract i wins with probability $p_i \in [0,1]$, in which case it pays cash c_i and equity share $e_i \in [0,1]$. Denote by $\pi_{i,j}$ a type i bidder's profit when it chooses contract j and wins,

$$\pi_{i,j} = (1 - e_j)(V_i + s_i + V_T - c_j) - V_i. \tag{27}$$

To obtain type i's unconditional expected profit if it picks contract j, multiply by p_j,

$$\Pi_{i,j} = p_j \pi_{i,j}, \qquad \text{for all } i \text{ and } j. \tag{28}$$

Incentive compatibility requires that

$$\Pi_{i,i} \geq \Pi_{i,j}, \qquad \text{for all } i \text{ and } j \neq i. \tag{29}$$

Individual rationality requires that

$$\Pi_{i,i} \geq 0, \qquad \text{for all } i. \tag{30}$$

The seller's expected profit is then

$$\Pi_s = \sum_{i=1}^{3} f_i p_i \left(e_i (V_i + s_i + V_T - c_j) + c_j - V_T \right). \tag{31}$$

The seller's problem is to identify the three contracts $\{c_i, e_i; p_i\}$ for $i = 1, 2, 3$ that maximize its expected profit (31) subject to (29) and (30).

LEMMA 2 *If the synergy–valuation relationship is convex so that $\tau > 0$, then it is optimal to offer a single contract that consists of a fixed equity share $e \in (0,1)$ and fixed cash, with $p = 1$.*

(i) If $\frac{f_2 s_2}{\tau}$ is small so that there are sufficiently few intermediate type 2s, then it is optimal to exclude type 2s and extract all surplus from types 1 and 3.

(ii) If $\frac{f_2 s_2}{\tau}$ is bigger than in (i), then it is optimal to extract full rents from type 2 and either type 1 or type 3, which necessitates giving rents to the other type.

COROLLARY 3 *When $s(V)$ is convex, it is optimal to make a take-it-or-leave-it offer with a fixed equity share plus a fixed cash payment.*

An Internet Appendix provides the details on the construction and properties of the optimal contract. The Internet Appendix is available in the online version of the article at the *Journal of Finance* website. When $\frac{f_2 s_2}{\tau}$ is small, the expected rent gains from type 2 are small relative to the degree of convexity, making it optimal to extract full rents from types 1 and 3, at the cost of excluding type 2, to earn profits of $\Pi_s = f_1 s_1 + f_3 s_3$. The convex synergy–valuation relationship means that any contract that satisfies type 2's individual rationality condition would leave either type 1 or type 3 with strictly positive rents that exceed what the seller could extract from type 2, making it optimal to exclude type 2. Once $\frac{f_2 s_2}{\tau}$ is large enough, it ceases to be optimal to exclude type 2. The optimal design is then either to extract full rents from types 1 and 2, but obtain reduced rents from type 3; or, instead, to extract full rents from types 2 and 3, but obtain reduced rents from type 1. When f_3/f_1 is relatively small, it is optimal to leave rents for type 3; and when f_3/f_1 is larger, it is optimal to leave rents for type 1. In all three cases, the shortfall in seller profits from full extraction approaches zero as $\tau \to 0$.

With three types, the seller can always implement the optimal mechanism with a single contract.[16] We now show that this property extends to a continuum of types as in our main model. With three types, we identified the optimal mechanism in the space of all feasible contracts. In particular, we allow for any p_i between zero and one, and show that in the optimal

[16]In the Internet Appendix we show that the optimal mechanism can also be implemented by multiple contracts. Such contracts have the feature that higher types pay (weakly) higher cash and less equity.

mechanism, $p_i \in \{0;1\}$. That is, with three types, a bidder type is either always excluded or always included in the optimal mechanism. With a continuum of types, to ease analysis we restrict attention to the space of contracts with this feature, that is, with $p_V \in \{0;1\}$ for all V. We show that it remains optimal to offer a single contract when $s(V)$ is convex.

PROPOSITION 3 *Suppose $s(V)$ is strictly convex over $\left[\underline{V}, \bar{V}\right]$ and the contract space is $p_V \in \{0;1\}$ for all V. The optimal mechanism has the following features:*

(i) There exist two bidder types V_1, V_2 with $\underline{V} \leq V_1 \leq V_2 \leq \bar{V}$, such that all types in $[\underline{V}, V_1] \cup \left[V_2, \bar{V}\right]$ participate but intermediate types in (V_1, V_2) are excluded. If $V_1 = V_2$, then all bidder types participate. Full rents are extracted from bidder types V_1 and V_2. All other participating types earn strictly positive rents.

(ii) The optimal mechanism is implemented by a single contract with

$$e = \frac{k}{1+k} \quad \textit{and} \quad c = V_T + s\left(V_1\right) - kV_1, \tag{32}$$

where

$$k \equiv \frac{s\left(V_2\right) - s\left(V_1\right)}{V_2 - V_1} \;\textit{ if } V_1 < V_2; \;\textit{ and } k \equiv \frac{ds(V)}{dV}|_{V=V_1} \;\textit{ if } V_1 = V_2. \tag{33}$$

Proposition 3 characterizes the optimal mechanism when the synergy valuation relationship is convex. It establishes that the key features of the optimal mechanism in the three-type case are general in nature, reinforcing the result that the optimal mechanism has a simple implementation in which the seller makes a take-it-or-leave-it offer with a fixed equity share plus cash.

In sum, if $s(V)$ is concave, then a seller can extract all surplus using the optimal cash-equity menu; while if $s(V)$ is convex, then full extraction using cash and equity is not possible,[17] but the optimal mechanism takes a simple form. To understand why the optimal

[17] In Section II we show that even if $s(V)$ is convex, a seller may still be able to extract full rents by combining cash with steeper-than-equity securities: the convexity of steeper-than-equity security *payments* can offset convexity in the synergy–standalone valuation relationship.

mechanism takes the form of a menu when $s(V)$ is concave but is a single contract when $s(V)$ is convex, consider Figure 2. As in Figure 1, the horizontal axis is the total valuation v_i of the combined firm under the bidder's control, but now the arc ABCDE depicting the bidder's possible types (plotting standalone values V_i as a function of v_i) is concave.

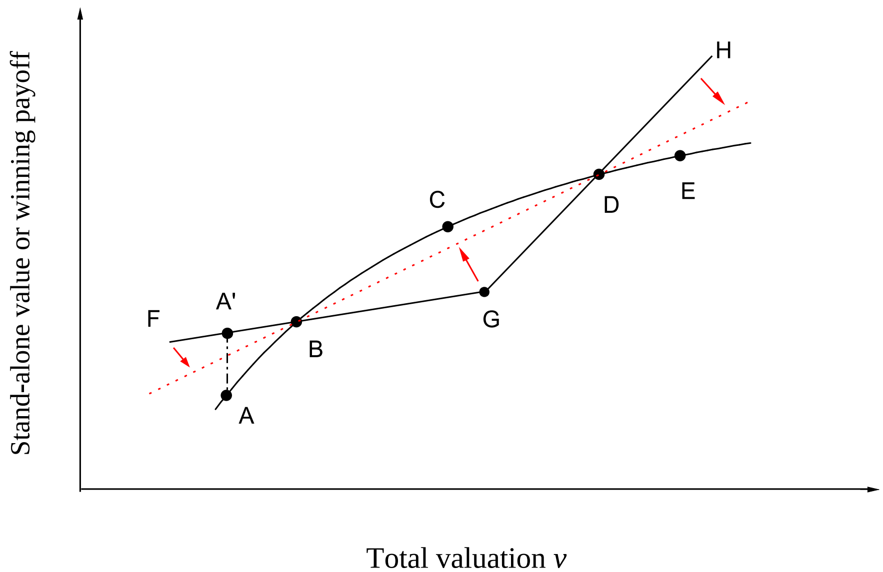

Figure 2: Arc ABCDE plots the bidder's standalone value V as a function of v. Line segments FG and GH are the upper envelope of the winning payoffs from two contracts.

Recall from (19) that the winning contractual payoff to a bidder is a straight line on this V versus v figure. Suppose now that it is optimal for the seller to offer two contracts, illustrated here by lines FG and GH. Because each bidder type selects the contract that maximizes its payoffs, the bidder will select from the upper envelope of these two lines, FGH. In this case, types on curve BCD will not participate, and other types participate and earn positive profit. For example, the profit of type A is the distance between point A and point A′ that is vertically above A on line FB. Now, replace the two contracts by the single contract that extracts full rents from points B and D. The payoff line is the straight line BD. Points on arc BCD still do not participate, but all other points continue to participate and they receive lower rents.

To summarize Figures 1 and 2: when $V(v)$ is convex (i.e., $s(V)$ is concave), one can find a menu of contracts such that the upper envelope of the straight payoff lines associated with the contracts exactly coincides with the $V(v)$ curve, perfectly separating types and leaving each bidder type's winning payoff just equal to its opportunity cost, hence extracting all rents. In contrast, when $V(v)$ is concave (i.e., $s(V)$ is convex), the weakly convex upper envelope of contract payoff lines cannot coincide with $V(v)$. To minimize the bidder's rents—to minimize the vertical distance between the upper envelope and the $V(v)$ curve at places where the former is above the latter—the seller offers a single contract, whose straight payoff line minimizes the convexity of the upper envelope, pooling all participating types.

We now show that if $s(V)$ is approximately affine in V—if $s(V)$ is "close" to being both concave and convex—then a seller can extract almost all surplus by complementing a standard cash auction design with an added fixed equity share payment.

LEMMA 3 *Consider $n > 1$ ex-ante identical bidders with standalone valuations distributed according to $F(V)$ on $[\underline{V}, \bar{V}]$ with associated synergies $s(V) > 0$. The maximum social welfare gain is $\Pi = \int_{\underline{V}}^{\bar{V}} s(V)d\left(F^n(V)\right)$. Suppose $s(\cdot)$ is in an ϵ-neighborhood of an affine function $y(V) = c + bV$, with $b < \frac{ds(V)}{dV}$ at all V. Then in a second-price auction in which bidders bid with fixed equity share α plus cash, expected seller profit increases in $\alpha \in \left[0, \frac{b}{1+b}\right]$ and is no less than $\Pi - \sqrt{2}\epsilon$ at $\alpha = \frac{b}{1+b}$.*

Proof: See the Appendix. ■

Even if a seller can only commit to rejecting offers that do not include a minimal equity share $\alpha \in \left(0, \frac{b}{1+b}\right)$, the mechanism still generates more revenues than pure cash—being able to commit to even a small amount of equity improves the seller's revenue because it reduces the differential rent of a high type over a low type.

A.2. Commitment and Informal Auctions

Our model presumes that a seller has full commitment power so that it is able to reject any offer outside its designed menu. In essence, our premise is that if a target decides to sell itself, then its board runs the process and commits to the mechanism that maximizes value.

A target board's "Revlon duties"[18] mandate that its role is to be "auctioneers charged with getting the best price for the stockholders at a sale of the company." We interpret this rule as requiring the board to select the mechanism that maximizes the ex-ante expected sale price. Consistent with this view, Boone and Mulherin (2007) document that public targets often privately contact potential bidders to solicit bids, and that many public transactions are seller-initiated. The data further suggest that public targets have extensive commitment power, as indicated by the minimal rents obtained by acquiring firms.

Above, we show that when a seller has complete commitment power, the full-extraction mechanism can be implemented by having bidders bid with cash and pay an associated equity share that declines with the cash bid. Moreover, as the alternative solution to Example 1 illustrates, a seller only needs to be able to commit to reserve prices to extract full rents: the seller can use a first-price auction in which a bidder bids cash plus a fixed equity share, where the reserve price on the cash bid decreases with the equity share and the highest cash bid wins. In practice, sellers in auctions do set reserve prices, committing not to sell unless a bid is sufficiently high. Indeed, widespread adoption of strong takeover defenses such as poison pills in the 1980s have endowed targets with considerable commitment power, "halt[ing] the use of hostile bids" (BETa, p298). As Povel and Singh (2006, p1399) observe, "Deal protection devices can be used to enhance a target's commitment to the [optimal selling] procedure."[19]

Still, a private target or a subsidiary of an acquirer may have less commitment power. This leads us to consider the extreme scenario often assumed by bidder-initiated theories in which the target lacks all commitment power and hence is unable even to set a reserve price. That is, we suppose that the acquirer has complete freedom to select the cash–equity mix to offer, and the target cannot reject any offer that leaves it with non-negative expected profit. Outcomes are sharply altered. A target's complete inability to commit results in an informal auction, as in DKS, in which bidders are free to offer any cash–equity combination,

[18] Revlon versus MacAndrews and Forbes Holdings, 506 A.2d 173 (Del. 1986).

[19] Ahern (2012) suggests that a target also has extensive bargaining power in a customer-supply situation when the acquirer relies more on the target as a supplier or customer than the converse.

and the seller chooses the most attractive bid combination ex post. DKS show in their setting that bidders will use the flattest security possible. Lemma IA1 in the Internet Appendix shows that the logic of their argument extends to our setting, leading bidders to offer cash only with no equity component.

In the single-bidder setting often seen in mergers, a complete inability to commit generates the counterfactual implication that the acquiring firm extracts all surplus. The single acquirer now has both an information advantage (it is privately informed) and all bargaining power, so it can ensure winning and extracting full rents by offering a cash bid of V_T, leaving the target with zero profit. With the added freedom to choose any cash–equity combination, it will do no worse; but it can do no better either because in equilibrium the target correctly infers an offer's monetary value and hence will not earn a negative profit.[20] Indeed, if the target cannot set a reserve price, then it cannot do better even if it can commit to a particular equity valuation. That is, suppose a target evaluates an offer $\{c, e\}$ by $c + f(c, e)\, e$, where c is the cash amount and e is the share of equity to be paid, and $f(c, e) \geq 0$ is a function that can potentially depend on c and e, accepting an offer as long as its valuation of the offer $c + f(c, e)\, e \geq V_T$. Once again, because offering a cash bid of V_T ensures full rent extraction, a bidder will do no worse than full extraction.

Nonetheless, as we derived, a target may need only a little commitment to do well. A target that can commit to a fixed equity share plus cash can extract close to full rents when $s(V)$ is approximately affine. Further, when synergies are a convex function of standalone values, a seller cannot extract all surplus, but it can do as well as it possibly can by committing to a fixed equity share (royalty rate) plus cash, a mechanism used in many economic transactions.

[20]The key feature of our model is that one side (the bidders) has private information and the other side (the seller) has full bargaining/commitment power. We show that commitment power alone can completely eliminate the other side's information advantage. Reversing the analysis yields that the acquirer can extract full rents via our mechanism if it has the bargaining power, even if only the target has private information.

A.3. Two-sided Private Information

In the Internet Appendix, we analyze a simple setting in which both the seller and the buyer have private information about their standalone values. We consider a scenario with two possible bidder valuations and two possible seller valuations, where a larger spread in the possible valuations represents an increase in asymmetric information.

With two-sided private information, the menu of contracts that a seller offers may signal information to a buyer about a seller's standalone value. The seller cannot extract all surplus in a separating equilibrium, that is, when different seller types offer distinct menus, revealing their types. Were it possible, then each seller type would earn the associated expected synergy as rent. But then it would be profitable for a low-standalone-valuation type to mimic a high-standalone-valuation type to earn more rents, a contradiction.

However, full extraction is possible in a pooling equilibrium in which both seller types offer the same menu of contracts, *provided that* information asymmetry about a seller's standalone value is not large: the seller uses the mechanism in Theorem 1 and Proposition 1, with the seller's expected standalone value replacing V_T. The seller's expected profit is the expected synergy. However, rents differ conditional on seller type: a low type earns more than full extraction, while a high type earns less. Once information asymmetry about a seller's standalone value is sufficiently high, full extraction ceases to be possible: a high-type seller's rents would be too low, giving it incentives to deviate and leading to strictly positive bidder rents.

Thus, increased information asymmetry about a target, for example, when the target is private rather than public, can lead to increased acquirer rents, consistent with the data.[21] However, in contrast to this prediction of our target-initiated model, when the acquirer has the bargaining power, increased information asymmetry about the target *increases* the target's information advantage, counterfactually *reducing* the acquirer's return.

[21]This fact can also be reconciled by public targets having more bargaining power than private targets.

B. *Empirical Implications and Relationship to the Literature*

In this section, we expand upon how our model reconciles a wide range of empirical regularities associated with takeovers that collectively are difficult for models in which acquiring firms have significant bargaining power to explain. We first list important empirical regularities associated with takeovers. We then show our model can reconcile them. Finally, we contrast our theory with existing theories of takeovers, showing how they can explain only a subset of these regularities.

Empirical Regularity 1: Multiple public bidders for firms are rare (see EMT's summary of the evidence). Indeed, of all contests for publicly traded U.S. targets between 1980 and 2005, only 3.4% had multiple public bidders (BETa).

Empirical Regularity 2: On average, the target grasps almost all gains associated with takeovers of publicly-traded firms. In particular, EMT's review article reports that mean CARs to the target amount to 29% including pre-announcement run-ups due to information leakage, while Dessaint, Eckbo, and Golubov (2019, p3) summarize evidence that "average acquirer returns over the last four decades have remained close to zero and largely flat—both unconditionally and after controlling for the usual observable firm- and deal-specific effects."

Empirical Regularity 3: Abnormal returns to *both* the target and acquirer rise in the cash amount of an offer, as does their combined return (Andrade, Mitchell and Stafford (2001), Table 4). EMT summarize the evidence for bidders, "Bidder abnormal announcement returns are on average highest in all-cash offers, lowest in all-stock offers, and with mixed cash-stock offers in between" (p36); and BETa, p355, summarize the evidence that CARs to targets are greater for cash offers than equity offers. Betton et al. (2014, Table VII) find a strong positive correlation between target and acquirer CARs.

Empirical Regularity 4: Multiple bidders raise the probability of cash use (Betton, Eckbo, and Thorburn (2008b, BETb, Table 1)). Boone and Mulherin (2007, derived from Table IV) find that acquisitions involving equity are twice as likely as pure cash acquisitions to have one bidder.

Empirical Regularity 5: Bidders appear rational, maximizing profits. More specifically, equity-financed takeovers are not associated with poor long-run performance. EMT, citing Eckbo, Makaew and Thorburn (2018), Rhodes-Kropf, Robinson, and Viswanathan (2005), and Li, Taylor, and Wang (2018), summarize the state of knowledge as follows: "Recent empirical evidence indicates that targets do not receive overpriced bidder shares" (p2-3), and "there is little systematic evidence of poor post-acquisition long-run performance of stock-financed takeovers."(p39)

Empirical Regularity 6: Acquirers of public targets earn lower returns than acquirers of private targets. BETa's analysis finds that, on average, public targets extract all synergy gains,[22] whereas in acquisitions of private targets, the acquirer's return are positive.[23] Similarly, Faccio, McConnell, and Stolin (2006, FMS) examine returns to acquisitions of listed and unlisted targets in 17 Western European countries, and find that acquirers of listed targets earn an insignificant average abnormal return of -0.38%, while acquirers of un-listed targets earn a significant average abnormal return of 1.48%.

Our model reconciles all of these observations. In the optimal design each potential acquirer has a dominant strategy, and the winner's payment depends only on its own bid. Thus, our model is consistent with the takeover process identified by Boone and Mulherin (2007), whereby a target privately approaches multiple potential acquirers, sets the terms of payment contingent on that bidder winning, and then selects the bidder that would most increase its value, extracting all surplus despite only one public bid. Thus, the rarity of public bids and the unbalanced division of surplus between target and acquirer arise naturally in our setting.

Our model delivers the patterns regarding the asset composition of bids and returns

[22] BETa calculate combined bidder and target abnormal returns by weighting bidder and target abnormal returns by their associated market capitalizations on day -42. For a publicly-traded target and acquirer, the estimated combined CAR (-41,2) is 1.78%, but the estimated acquirer CAR (-41,2) is an insignificant -0.22%.

[23] Fuller, Netter, and Stegemoller (2002) similarly find that acquirer returns in windows around bids are lower for public targets (insignificantly negative) than for private targets (positive and significant). See also Sundaram (2006), Moeller, Schlingemann, and Stulz (2007), or Bargeron et al. (2007).

because greater cash components reveal (1) higher synergies, from which only the target benefits, and (2) more positive information about the acquirer's standalone value, causing the acquirer to experience greater abnormal returns. Thus, a higher cash component in the offer leads to higher abnormal returns for both the target and the acquirer. The observation that multiple bidders raise the probability that cash is used follows from a simple first-order statistic logic in our model: more bidders increase the probability that the winning bidder has a high synergy, and bidders with higher synergies use more cash.

Finally, the possibility that the nature of the synergy–standalone valuation relationship depends in systematic ways on target characteristics can reconcile the puzzling[24] listing effect: acquirers of publicly-traded targets earn lower returns than acquirers of private-targets. More specifically, a risky start-up or growth firm might succeed hugely with a high quality merger acquirer (associated with higher privately-known standalone values). As a result, the percentage upside synergy gains from acquisitions of younger and smaller target firms might be far higher than those for a mature target, whose value creation might depend less sensitively on its acquirer's quality. This could give rise to a convex synergy-valuation relation for start-ups and growth firms, but a concave one for larger, publicly-traded firms. Our analysis suggests that this can underlie the listing effect: a concave synergy relationship for public targets facilitates full extraction, whereas a convex synergy relationship for private targets leaves rents for the acquirer.[25]

Existing theories of takeovers in which the acquirer has the bargaining power can explain some of these empirical patterns, but they cannot reconcile others. Fishman (1989) develops a bidder-initiated theory of takeovers in which an initial bidder who is privately informed about its valuation can offer either cash or debt. He shows that a high-valuation bidder offers cash to discourage rivals from acquiring information about their valuations

[24] For example, FMS's abstract notes that "The fundamental reason for this listing effect...remains elusive."

[25] In addition, there may also be materially more informational asymmetries about private targets than public targets, and private targets may have less bargaining power. Our analysis of such scenarios shows how they can also contribute to the listing effect.

and then competing for the target. As in our model, his model predicts that the use of more cash is associated with higher acquirer returns. However, it does not predict that higher cash components are associated with higher target returns (since cash use discourages competition for the target even though it provides an initial premium). His model also implies that cash is *less* likely to be used when there are multiple bids, contrary to the predictions of our model and the findings in BETb and Boone and Mulherin (2007).

Eckbo, Giammarino, and Heinkel (1990) study a signaling model of takeovers in which a single bidder and the seller are both privately informed of their valuations.[26] They identify a separating equilibrium in which the cash-equity composition of the bidder's offer reveals the bidder's type. As in our paper, a higher bidder type offers more cash, and is associated with higher CARs for the acquirer. However, in their model a seller accepts an offer as long as it breaks even and the acquirer extracts a large share of the surplus, in contrast to our model and the data. In addition, our model predicts that the target CAR increases in cash, but their model does not. Finally, their single-bidder model is silent on the impact of multiple bidders, whereas ours predicts that multi-bidder settings are more likely to have cash offers.

Researchers have also posited behavioral and agency explanations for both the observed lopsided sharing of takeover synergies and the higher CARs for cash offers than equity offers, including bidder agency costs, managerial hubris and theories based on market misvaluation (leading to bidder opportunism, with bidders paying a mis-informed target with overpriced bidder stocks), but EMT's summary of the evidence (citing Eckbo, Makaew, and Thorburn (2018), Rhodes-Kropf, Robinson, and Viswanathan (2005), and Li, Taylor, and Wang (2018)) is at odds with such theories. In addition, EMT find that cash-rich and cash-poor bidders are equally likely to use equity, suggesting that cash limitations are not a fundamental determinant of these patterns.

[26] See also Gorbenko and Malenko (2018), where cash–equity compositions reflect signaling and financial constraint considerations.

II. Combining Cash with General Securities

We next allow a seller to combine cash with general securities, focusing on a single bidder.[27] We show that combining cash with steeper-than-equity securities can be even more effective at extracting rents than combining cash with equity.

We first illustrate how a dominant-strategy incentive compatible mechanism that employs combinations of call options and cash can extract all surplus when equity plus cash cannot, underscoring how steeper securities can help in the rent extraction.

Example 2. A single bidder has possible standalone values $V \in \{2, 2.6, 3\}$, each with strictly positive probability, with associated expected synergies of $s(2) = 4$, $s(2.6) = 4.4$, and $s(3) = 5$. Expected synergies are thus a convex function of standalone values. The realized synergy of a type with expected synergy s is uniformly distributed on $[s-1, s+1]$. The target's standalone value is zero.

By Theorem 1, the convex synergy relationship means that full extraction is impossible using equity plus cash. We now show that the seller can extract full rents by combining even steeper securities with cash, using call options plus cash. The payment by a bidder who

- reports $V = 2$ consists of a call option with strike 2.25 plus 3.75 in cash;
- reports $V = 2.6$ consists of a call option with strike 2.85 plus 4.15 in cash;
- reports $V = 3$ consists of a call option with strike 3.25 plus 4.75 in cash.

When cash flow is uniformly distributed over $[v-1, v+1]$, where $v > 1$, the expected payment for a call option with strike k is $v-k$ if $k \leq v-1$, $\frac{1}{4}(v+1-k)^2$ if $k \in (v-1, v+1)$, and zero if $k \geq v+1$. Substitution into a bidder's expected profits reveals that truth-telling results in zero payoffs and deviating results in strictly negative payoffs.[28]

[27]The optimal mechanism generalizes to allow the seller to extract full rents from multiple bidders, while maintaining dominant-strategy incentive compatibility.

[28]For instance, consider a type $V = 2.6$ bidder. If it truthfully reports $V = 2.6$, then after paying 4.15 in cash, the total cash flow of the combined firm is uniformly distributed over $[2.85-1, 2.85+1]$. The value of

Thus, the mechanism is incentive compatible and all rents are extracted. In the example, were a bidder cash-constrained and only able to pay with securities, the cash payment could be replaced by a debt payment.[29] The result is a hybrid security of debt plus call option: when cash flow is low it is debt, and when cash flow is high it is a call option. Thus, paying "a call option with strike 2.25 plus 3.75 in cash" can be replaced by a combination of debt with face value 3.75 and a call option with strike 6, $S(y) = g(y) + h(y)$, where $g(y) = min\{y, 3.75\}$ and $h(y) = max\{0, y - 6\}$; paying "a call option with strike 2.85 plus 4.15 in cash" can be replaced by a combination of debt with face value 4.15 and a call option with strike 7; and paying "a call option with strike 3.25 plus 4.75 in cash" can be replaced by a combination of debt with face value 4.75 and a call option with strike 8.[30]

We now turn to our general analysis. With equity payments, only expected cash flows matter. With general securities, the payment's value depends on the details of the cash flow distribution, so more structure is needed. We assume that a type V bidder has expected synergy $s(V)$, but the realized synergy $\widetilde{s}$ is stochastic. Thus, the cash flows generated by the merged firm, $y \equiv \widetilde{s} + V + V_T$, are stochastic with an expected value of $s(V) + V + V_T$.

It eases analysis to use a transformation. The expected cash flow of the target under the bidder's control, $v(V) = E[y|v] = V + s(V) + V_T$, is strictly increasing in V and hence invertible. Thus, V can be expressed as a function of v, which we denote by $V(v)$, and we use v instead of V to denote bidder type. Note that $\frac{ds(V)}{dV} > 0$ implies $\frac{dv}{dV} > 1$, so $\frac{dV(v)}{dv} \in (0, 1)$ for all v, and the concavity of $s(V)$ is equivalent to the *convexity* of $V(v)$. Define $\underline{v} \equiv \underline{V} + s(\underline{V}) + V_T$

the call option is 0.25, so the payoff of 2.85-0.25=2.6 just covers the bidder's standalone valuation. If it reports $V = 2$, then after paying 3.75 in cash, total cash flow is uniformly distributed over $[3.25 - 1, 3.25 + 1]$. The call option has a value of one, so the bidder's payoff of 3.25-1<2.6 fails to cover its standalone valuation. If it reports $V = 3$, then after paying 4.75 in cash, total cash flow is uniformly distributed over $[2.25 - 1, 2.25 + 1]$. The call option has a value of zero, so the payoff of 2.25<2.6 fails to cover the bidder's standalone valuation.

[29] In this example the face value of the debt equals the corresponding cash payment, because the dispersion in the distribution of cash flows is not too great. With more dispersed cash flows, the face value of the debt or the strike value of the call may need to be adjusted to achieve full extraction.

[30] Such securities are piece-wise linear. Payments in the form of piece-wise linear securities are often used in takeovers of private targets. Cain, Denis and Denis (2011) report that "The typical earnout payment is a linear or a stepwise function of the target's performance (subject to a maximum) over the subsequent one to three years. The earnout payments are potentially quite large; on average, if the maximum earnout is paid, it would amount to 33% of the total transaction value."

and $\bar{v} \equiv \bar{V} + s(\bar{V}) + V_T$. We assume that expected synergies are positive for all bidder types.

ASSUMPTION 1 *The cash flow is* $y = \theta(v - \hat{v}) + \hat{v}$*, where* θ *is distributed over* $(0, \infty)$ *with a mean of one and independent of* v*, log(*θ*) has a log-concave density, and* $\hat{v} \in [s(\bar{V}) + V_T, \underline{v})$ *is a constant.*

This structure has the properties that $\mathrm{E}(y) = v$ and that a higher type v is associated with a (stochastically) better distribution of cash flows y. In DKS, $\hat{v} = 0$. Here, $\hat{v} \geq V_T + s(\bar{V})$ ensures that cash flows are high enough that, after the cash payment, the residual cash flows remain nonnegative. The condition $\hat{v} \leq \underline{v}$ ensures that cash flows for all bidder types are positive (such a $\hat{v}$ exists as long as $\underline{V} \geq S(\bar{V}) - S(\underline{V})$).

As in DKS, we consider an *ordered set of securities*, $\left\{S(t, \cdot); t \in (\underline{t}, \bar{t})\right\}$. Each security in such a set is indexed by t: for each t, $S(t, \cdot)$ gives the security's payoff as a function of the realized cash flows. For example, with equity, the index t is the equity fraction and $S(t, y) = ty$.

ASSUMPTION 2 *(i) for all* t *and* $y \in [0, \infty)$*, both* $S(t, y)$ *and* $y - S(t, y)$ *weakly increase in* y *with* $0 \leq S(t, y) \leq y$*.*

(ii) If $t_1 > t_2$*, then* $S(t_1, y) \geq S(t_2, y)$ *for all* $y \in (0, \infty)$*, and* $S(t_1, y) - S(t_2, y)$ *weakly increases in* $y \in (0, \infty)$*.*

Part (ii) says that payments increase in the security index and that the difference in payments widens as cash flows rise. For instance, for call options, a higher index corresponds to a lower strike price.

The bidder pays with combinations of cash and the security. The security payment is nonnegative, so we can assume without loss of generality that cash payments do not exceed $s\left(\bar{V}\right) + V_T$—a bidder that paid more than this would earn negative profits.

When a type v bidder pays cash M plus security $S(t, \cdot)$, let $ES(t, v; M) \equiv E\left[S(t, y - M)|v\right]$ denote its expected security payment, where $y(v)$ is the (before cash payment) stochastic cash flow of the combined firm. Under Assumptions 1 and 2,

$ES(t,v;M)$ is strictly increasing in the security index t, differentiable in t, and twice differentiable in v (DKS, Lemma 1). We denote the first and second partial derivatives with respect to v by $ES_v(t,v;M)$ and $ES_{vv}(t,v;M)$. We assume a full range in the security index. Thus, $\underline{t}$ corresponds to zero payment and $\bar{t}$ corresponds to full payment.[31]

We consider direct-revelation mechanisms. We first leave the sign of cash unrestricted. Given our focus on full-extraction mechanisms and the fact that all bidder types have positive NPVs, we assume without loss of generality that the single bidder wins regardless of the type that it reports. If the bidder reports being type z, let $t(z)$ be the index of the security paid and let $M(z)$ be the cash payment. A type v bidder's expected profit when it reports z is

$$h\left(v,z\right)=v-V(v)-ES\left(t(z),v;M(z)\right)-M(z). \tag{34}$$

Full extraction implies that

$$h(v,v)=0,\ \text{ for all } v. \tag{35}$$

Incentive compatibility yields

$$v\in\arg\max_{z}h(v,z),\quad \text{ for all } v. \tag{36}$$

A mechanism extracts full rents if and only if (35) and (36) hold. It eases analysis to reverse the maximization problem in (36). That is, by (35) and (36), $h(z,z)=0$ and $h(v,z)\leq 0$ for all v and z, so we can rewrite (36) as

$$z\in\arg\max_{v}h(v,z),\quad \text{ for all } z. \tag{37}$$

LEMMA 4 *A mechanism extracts full rents if and only if (35) and (37) hold.*

Lemma 4 replaces the incentive compatibility condition (36) by a reverse-optimization

[31] That is, for any v, any M, and any $\epsilon>0$, we requre (i) a $t^*>\underline{t}$ to exist with $ES\left(t^*,v;M\right)<\epsilon$ and $ES_v\left(t^*,v;M\right)<\epsilon$, and (ii) a $t^{**}<\bar{t}$ to exist with $ES\left(t^{**},v;M\right)>v-\epsilon$ and $ES_v\left(t^{**},v;M\right)>1-\epsilon$.

condition (37). This replacement is valid when a mechanism extracts full rents (or more generally when all bidder types receive the same rent). It simplifies analysis because, although the first-order condition of (37) is the same as the standard envelope condition, its second-order condition is more tractable than that of (36).

Equation (34) and the first-order condition for (37), $\frac{\partial h(v,z)}{\partial v}|_{v=z} = 0$, yield (after substituting v for z)

$$1 - V'(v) - ES_v(t(v), v; M(v)) = 0, \quad \text{for all } v. \tag{38}$$

The second-order condition of (37) gives

$$\frac{\partial^2 h(v,z)}{\partial v^2}|_{v=z} = -\frac{d^2V(z)}{dz^2} - ES_{vv}(t(z), z; M(z)) \leq 0,$$

which, after replacing z with v, yields a necessary condition for full extraction:

$$\frac{d^2V(v)}{dv^2} \geq -ES_{vv}(t(v), v; M(v)), \quad \text{for all } v. \tag{39}$$

When the security is equity, the right-hand side of (39) is zero; (39) just says that $V(v)$ must be weakly convex (i.e., $s(V)$ is weakly concave). By Theorem 1, this is also sufficient for full extraction via equity plus cash. With equities, each security can be expressed as a "fraction" of a base-security. For a general set of ordered securities, such as the family of call options indexed by different strikes, securities cannot be expressed as fractions of each other, complicating analysis. This leads us to identify necessary and sufficient conditions separately and then derive their implications.

Equation (39) yields the following necessary condition.

COROLLARY 4 *Suppose that the payoff function of each security $S(t,\cdot)$ is strictly concave. Then $V(v)$ must be strictly convex ($\frac{dV}{dv} > 0$ for all v) for combinations of (possibly* *negative) cash and security to extract full rent.*

Proof: See the Appendix. ■

From Lemma 5 in DKS, if every security in an ordered set has a concave payoff function, then the set is less steep than equity. Corollary 4 shows that such securities tighten the curvature requirement on $V(v)$ for full extraction vis-à-vis equity.

We now identify sufficient conditions for full extraction. Specifically, we show that even if $V(v)$ is concave, then as long as it is not "too" concave, combinations of steeper-than-equity securities plus cash can extract full rents, even though cash plus equity cannot. We focus on the set $\mathcal{A}$ of steeper-than-equity ordered securities that consists of the set of call options plus sets of securities $S(t,\cdot)$ with strictly convex payoff functions over $(0,\infty)$.

LEMMA 5 *For all $v \in [\underline{v},\bar{v}]$, the system of equations (35) and (38) has at least one solution for $(t;M)$.*

Proof: See the Appendix. ■

To ensure global incentive compatibility of the mechanism, we define a measure of the minimum average second derivative of expected revenues by the security indexed by $t(z)$, where the average is taken over any two distinct points z and $v \in [\underline{v},\bar{v}]$:

$$K \equiv \min_{z,v\in[\underline{v},\bar{v}],z\neq v} \frac{\int_v^z ES_{vv}\left(t(z),\widetilde{v};M(z)\right)d\widetilde{v}}{z-v}, \tag{40}$$

where $(t(z);M\left(z\right))$ solves (35) and (38) for type z.

LEMMA 6 *Consider any ordered set of securities in $\mathcal{A}$. Then $K>0$ for any mechanism that combines cash and these securities and satisfies (35) and (38).*

Proof: See the Appendix. ■

From DKS, if each security in an ordered set has a convex payoff function, then the set is steeper than equity. The steeper is the security, the larger is K. This facilitates full extraction.

THEOREM 2 *Consider any ordered set of securities in $\mathcal{A}$. If $V''(v) \geq -K, \forall v$, then the mechanism that combines cash with these securities and satisfies (35) and (38) is globally incentive compatible. Thus, there exists a mechanism that extracts full rents.*

COROLLARY 5 *Suppose bidding strategies would be strictly decreasing in a pure second-price security-bid auction (without cash payment) for the ordered set of securities in Theorem 2. Then the mechanism in Theorem 2 features strictly positive cash payments and securities, and generates strictly higher revenues than pure cash or pure securities.*

Proof: See appendix. ■

The condition in Theorem 2 that $V''(v) \geq -K$ for all v is sufficient for full extraction via security plus cash. Equity has a linear payoff function, so $K = 0$. Thus, with equity, the condition requires $V''(v) \geq 0$, which is precisely condition (i) in our equity analysis in Theorem 1. When the set of securities is steeper than equities, K is positive by Lemma 6, relaxing the convexity requirement on $V(\cdot)$ needed for full extraction. Intuitively, greater curvature in a security's expected payoff compensates for a lack of convexity in $V(\cdot)$ (i.e., concavity in $s(V)$). That is, the security payment and opportunity cost together comprise a bidder's costs, and convexity in a security's expected payoff can compensate for a lack of convexity in $V(\cdot)$ (i.e., concavity in $s(V)$).[32]

The greater curvature in the securities also relaxes the requirement in Corollary 5 for full extraction with positive cash, which requires bidding strategies to decrease. Steeper securities facilitate this: Che and Kim (2010) show that if equilibrium bidding strategies are decreasing for one class of ordered securities, then they are decreasing for all steeper securities.

Taken together, these findings indicate that a seller does best by combining cash (or debt if the bidder is cash constrained) with the steepest security. Tailored to different types, the mix creates wider variation in steepness, which helps reduce the differential rents

[32]Put differently, the bidder's payoff from steeper-than-equity securities is concave in the total valuation, and the upper envelope of concave functions is no longer restricted to be convex, facilitating rent extraction.

between bidder types, ensuring both the global incentive compatibility of the mechanism and a positive cash payment.

III. Conclusion

Returns to target and acquiring firms in takeovers offer puzzles. Multiple public bidders are rare, but targets receive almost all rents, and mean acquirer returns are very low. The cash-equity mix also matters: acquirer *and* target returns increase with the cash share of an offer. Our theory of target-initiated takeover processes reconciles these and other empirical regularities.

We consider a setting in which potential merger partners are privately informed of their standalone values and merger synergies, and those with higher standalone values also tend to generate higher expected synergies. We show that despite its information disadvantage, a target can design a payment scheme with combinations of cash and equity that extracts all rents if synergies are concavely-related to standalone values, exploiting a reluctance of high-valuation acquirers to give up equity claims that leads them to bid more cash. Our model delivers the result that both acquirer and target returns rise with the cash share of an offer because greater cash components reveal (1) higher synergies from which only the target benefits and (2) more positive information about the acquirer's standalone value, causing the acquirer to experience greater abnormal returns.

Our mechanism applies to decentralized processes in which a target privately approaches each potential acquirer, sets the terms of payment contingent on that bidder winning, and then selects the bidder that would most increase the target's value. As a result, multiple public bidders can be rare. The robust selling mechanism is dominant-strategy incentive compatible—bidders need not know anything about rivals—and a target extracts almost all rents if the synergy–valuation relationship is only approximately concave. When the synergy-valuation relationship is convex, the optimal design of the cash–equity combination is simple: bidders bid cash plus a fixed equity share. Moreover, in such instances, we show that combining cash with even steeper securities like call options can help the seller further.

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V. Appendix

Proof of Theorem 1 and Proposition 1: We first prove that when the conditions in Theorem 1 hold, the mechanism in Proposition 1 extracts full rents. If bidding is truthful, then substituting for $M_i(v_i, \mathbf{v}_{-i})$ and $q_i(v_i)$ in equation (4) reveals that a bidder's equilibrium payoff is zero regardless of his valuation,. That is,

$$[(V_i + s_i(V_i) + V_T - M_i(V_i, \mathbf{V}_{-i}))\, Q_i(V_i, \mathbf{V}_{-i}) - V_i]\, G_i(V_i) = 0.$$

To see that truthful bidding is an equilibrium, suppose that type V_i bids as if it is type z_i. If $z_i \notin S_i$, the bidder would lose, so the deviation is not optimal. If $z_i \in S_i$, then by (17) and (18),

$$q_i(z_i) = \frac{1}{1 + \frac{ds_i(z_i)}{dz_i}} \tag{A1}$$

and

$$\omega_i(z_i) = \left[s_i(z_i) + V_T - z_i \frac{ds_i(z_i)}{dz_i}\right] G_i(z_i). \tag{A2}$$

Then by (4),

$$\begin{aligned}
h_i(V_i, z_i) &= \left[\frac{V_i + s_i\left(V_i\right) + V_T}{1 + \frac{ds_i(z_i)}{dz_i}} - V_i\right] G_i(z_i) - \left[\frac{s_i(z_i) + V_T - z_i \frac{ds_i(z_i)}{dz_i}}{1 + \frac{ds_i(z_i)}{dz_i}}\right] G_i(z_i) \\
&= \left[s_i(V_i) - s_i(z_i) - \frac{ds_i(z_i)}{dz_i}\left(V_i - z_i\right)\right] \frac{G_i(z_i)}{1 + \frac{ds_i(z_i)}{dz_i}} \qquad \text{(A3)} \\
&\leq 0. \qquad \text{(A4)}
\end{aligned}$$

Inequality (A4) holds for all $V_i \in \left[\underline{V}_i, \bar{V}_i\right]$. If $V_i \in S_i$, the inequality follows because the weak concavity of $s_i(\cdot)$ in condition (i) of Theorem 1 implies that $s_i(V_i) - s_i(z_i) - \frac{ds_i(z_i)}{dz_i}\left(V_i - z_i\right) \leq 0$ regardless of whether $V_i \geq z_i$ or $V_i < z_i$. If, instead, $V_i \notin S_i$, the inequality follows from part (ii) of Theorem 1. Thus, deviation is not profitable for all bidder types. Note also that (17) satisfies $Q_i(\mathbf{z}) \in [0, 1]$. Hence, Proposition 1 and the "if" part of Theorem 1 are established.

To prove the "only if" part of Theorem 1, assume that a full-extraction mechanism exists. The necessity of condition (i) was proved in the text. To see that (ii) must hold, we first consider $V_i \in S_i$. Note that full extraction implies (12), and hence $\frac{d\hat{h}_i(V_i,V_i)}{dV_i} = 0$. By (13) and the Envelope Theorem, this yields (17). In turn, (12) yields (18). Thus, (A3) holds for $V_i \in S_i$. Because full extraction also implies that for all types $V_i \notin S_i$, bidding as if the bidder's type is $z \in S_i$ must render a nonpositive profit; by (A3), $G_i(z_i) > 0$ and $\frac{ds_i(z_i)}{dz_i} \geq 0$, and hence we have (ii). ■

Proof of Corollary 2: Observe that when the conditions in Theorem 1 and (20) hold, the mechanism specified in Proposition 1 extracts full rents. Further, by (20), the cash component in (18), $s_i(V_i) + V_T - V_i\frac{ds_i(V_i)}{dV_i} = -V_i^2 \frac{d}{dV_i}\frac{s_i(V_i)+V_T}{V_i}$, is strictly positive. Moreover, a seller must use strictly positive amounts of both cash and equity to extract full rents.

To prove the converse part of Corollary 2, suppose a full-extraction mechanism exists that employs strictly positive cash. For all $V_i \in S_i$, full extraction implies $G_i(v_i) > 0$ and $\frac{dh_i(V_i,V_i)}{dV_i} = 0$, which yields $q_i(V_i) = \frac{1}{1+\frac{ds_i(V_i)}{dV_i}}$ by (9). By $h_i(V_i, V_i) = 0$ and (4),

$$\omega_i(V_i) = \left(1 + \frac{ds_i(V_i)}{dV_i}\right)\left[(V_i + s_i(V_i) + V_T)\frac{1}{1+\frac{ds_i(V_i)}{dV_i}} - V_i\right] G_i(V_i).$$

Since the cash component is positive, $\omega_i(V_i) > 0$, and hence

$$(V_i + s_i(V_i) + V_T)\frac{1}{1+\frac{ds_i(V_i)}{dV_i}} - V_i > 0,$$

which yields (20). ■

Proof of Proposition 2: By (24) and (25), the expected profit of a type V bidder that reports itself to be a type z is

$$\begin{aligned} h(V,z) &= \frac{1}{1+\frac{dy(z)}{dz}}\left(V + s(V) - \left(y(z) - z\frac{dy(z)}{dz} - \frac{\sqrt{2}}{2}\epsilon\left(1+\frac{dy(z)}{dz}\right)\right)\right) - V \quad \text{(A5)} \\ &= h^*(V^*, z) + \delta, \end{aligned}$$

where

$$h^*\left(V^*, z\right) \equiv \frac{\left(V^*+y(V^*)\right)-\left(y(z)+z\right)}{1+\frac{dy(z)}{dz}}+z-V^*,$$

V^* is the solution to $V^*+y\left(V^*\right)=V+s(V)$, and $\delta \equiv V^*-V+\frac{\sqrt{2}}{2}\epsilon$ is independent of z. The same steps as in the proof of inequality (A3) in the proof of Theorem 1 and Proposition 1 yield that for any given V^*, we have that (1) $z=V^*$ maximizes $h^*\left(V^*, z\right)$, and (2) $h^*\left(V^*, z=V^*\right)=0$. Hence, reporting $z=V^*$ maximizes $h(V,z)$. Further, (A5) yields $h\left(V, z=V^*\right)=\delta$, and $\delta \in \left[0, \sqrt{2}\epsilon\right]$ by the premise that $s(V)$ is in an ϵ-neighborhood of $y(V)$. Hence, reporting $z=V^*$ satisfies individual rationality $\left(h\left(V, z=V^*\right) \geq 0\right)$ and leaves each bidder type with rents no greater than $\sqrt{2}\epsilon$. ■

Proof of Proposition 3: Consider an optimal mechanism, $\{e(V), c(V); p(V)\}$. Let κ be the set of V for which $p_V=1$. Let $h(V,z)$ be the expected profit of a bidder with standalone value V that reports $z \in \kappa$:

$$\begin{aligned} h(V,z) &= \left(1-e(z)\right)\left(V+s(V)+V_T-c(z)\right)-V \\ &= s(V)+V_T-\left(e(z)\left(V+s(V)+V_T\right)+\left(1-e(z)\right)c(z)\right). \end{aligned} \tag{A6}$$

Define

$$h^*(V) \equiv \max_{z\in\kappa} h\left(V,z\right) = s(V)+V_T-\min_{z\in\kappa}\left(e(z)\left(V+s(V)+V_T\right)+\left(1-e(z)\right)c(z)\right).$$

A type V bidder type accepts a contract if and only if $h^*\left(V\right)\geq 0$.

To simplify solution, we exploit the fact that the bidder's total (gross of cash payment) valuation of the target, $v(V) \equiv V+s(V)+V_T$, is strictly increasing in V and hence invertible. Thus, we can express V as a function of v, where the strict convexity of $s(V)$ is equivalent to the strict concavity of $V(v)$. Define $\underline{v} \equiv \underline{V}+s\left(\underline{V}\right)+V_T$ and $\bar{v} \equiv \bar{V}+s\left(\bar{V}\right)+V_T$.

Using $s(V) = v - V(v) - V_T$, we express $h^*(V)$ as a function of v:

$$h^{**}(v) \equiv h^*(V(v)) = \eta_a(v) - \eta_b(v),$$

where

$$\eta_a(v) \equiv v - V(v) \quad \text{and} \quad \eta_b(v) \equiv \min_{z \in \kappa} \left(e(z)v + (1 - e(z))\, c(z)\right). \tag{A7}$$

Bidder type v participates if and only if $h^{**}(v) \geq 0$, and its expected profit when it partipates is $h^{**}(v)$. Denote the set of bidder types that participate by κ^*, which is a subset of κ (the two sets may or may not coincide). Then the seller's expected profit is

$$\pi_s = \int_\kappa s(V) f(V) dv - \int_\kappa h^{**}(v(V))\, f(V) dv, \tag{A8}$$

where the first term is the social welfare gain and the second term is the bidder's rents.

Next we show that a single contract can deliver the seller an expected profit of at least (A8). The function $V(v)$ is strictly concave, so $\eta_a(v)$ is strictly convex in v. Further, $\eta_b(v)$ is weakly concave because it is the minimum of a family of affine functions. Thus, $h^{**}(v)$ is strictly convex, so there are at most two solutions to $h^{**}(v) = 0$ over $v \in [\underline{v}, \bar{v}]$. There are three possible cases.

Case 1: $h^{**}(v) = 0$ has two solutions v_1 and v_2 in $[\underline{v}, \bar{v}]$, where $v_1 < v_2$. That is, $\eta_a(v_i) = \eta_b(v_i)$ for $i = 1, 2$. Then strict convexity of $h^{**}(v)$ yields

$$h^{**}(v) < 0 \text{ for } v \in (v_1, v_2)\,; \text{ and } h^{**}(v) > 0 \text{ for } v < v_1 \text{ and } v > v_2. \tag{A9}$$

Consider the single contract that takes the form specified in (32), with $V_i = V(v_i)$ $(i = 1, 2)$ in (33). Denote the expected profit of a type v bidder from this contract by $\hat{h}(v)$. Then

$$\hat{h}(v) = \eta_a(v) - \hat{\eta}_b(v), \tag{A10}$$

where $\hat{\eta}_b(v)$ is given by the right-hand side of (A7) with the contracts in the posited optimal mechanism replaced by the single contract:

$$\hat{\eta}_b(v) = \frac{s(V_2) - s(V_1)}{V_2 - V_1 + s(V_2) - s(V_1)} (v - V_1 - V_T - s(V_1)) + V_T + s(V_1).$$

Recall that $v_i = V_i + s_i(V_i) + V_T$. Substituting yields

$$\hat{\eta}_b(v_i) = \eta_a(v_i) \text{ for } i = 1, 2. \tag{A11}$$

Thus, $\hat{h}(v_i) = 0$ and

$$\hat{\eta}_b(v_i) = \eta_b(v_i) \text{ for } i = 1, 2. \tag{A12}$$

We now use the property that $\hat{\eta}_b(v)$ is affine, and hence both weakly concave and weakly convex. The strict convexity of $\eta_a(v)$ and (A11) yield

$$\hat{\eta}_b(v) > \eta_a(v) \text{ for } v \in (v_1, v_2); \text{ and } \hat{\eta}_b(v) < \eta_a(v) \text{ for } v < v_1 \text{ or } v > v_2. \tag{A13}$$

Comparing (A13) and (A9) reveals that the participation decision of any bidder type under the single contract is the same as that with the posited optimal contracts. Hence, analogous to (A8), the seller's expected profit under the single contract is

$$\hat{\pi}_s = \int_{\kappa} s(V) f(V) dv - \int_{\kappa} (\eta_a(v) - \hat{\eta}_b(v)) f(V) dv, \tag{A14}$$

which, by (A8), yields

$$\hat{\pi}_s - \pi_s = \int_{\kappa} (\hat{\eta}_b(v) - \eta_b(v)) f(V) dv. \tag{A15}$$

The weak concavity of $\eta_b(v)$, the weak convexity of $\hat{\eta}_b(v)$, and (A12) yield

$$\hat{\eta}_b(v) - \eta_b(v) \geq 0 \text{ for } v < v_1 \text{ or } v > v_2,$$

which, upon plugging into (A15) and noticing that κ is the union of $v < v_1$ and $v > v_2$, yields $\hat{\pi}_s \geq \pi_s$. This proves the proposition for Case 1.

Case 2: $h^{**}(v) = 0$ has one solution over $[\underline{v}, \bar{v}]$. Denote the solution by v_1, i.e., $\eta_a(v_1) = \eta_b(v_1)$. Consider three scenarios.

Scenario 1: $\frac{dh^{**}(v)}{dv}|_{v=v_1} = 0$. Then $\frac{d\eta_a(v)}{dv}|_{v=v_1} = \frac{d\eta_b(v)}{dv}|_{v=v_1}$. The convexity of $h^{**}(v)$ yields $h^{**}(v) \geq 0$, or, equivalently, $\eta_a(v_1) \geq \eta_b(v_1)$, for all v. Thus, all bidder types participate. Consider the single contract of the form given in (32) with $e = \frac{d\eta_b(v_1)}{dv}$. Denote the expected profit of a type v bidder from this contract by $\hat{h}(v)$. Then (A10) holds with $\hat{\eta}_b(v)$ given by the right-hand side of (A7) in which the original contract(s) are replaced by the single contract:

$$\hat{\eta}_b(v) = \frac{d\eta_b(v_1)}{dv}(v - V_1 - V_T - s(V_1)) + V_T + s(V_1), \tag{A16}$$

which, by (14), yields $\hat{\eta}_b(v_1) = \eta_a(v_1)$. Hence, $\hat{\eta}_b(v_1) = \eta_b(v_1)$. Further, note that $\frac{d\hat{\eta}_b(v)}{dv} = \frac{d\eta_b(v_1)}{dv}$, and hence $\frac{d\hat{\eta}_b(v)}{dv} = \frac{d\eta_a(v_1)}{dv}$. Using the facts that $\hat{\eta}_b(v)$ is affine, $\eta_a(v)$ is convex, $\hat{\eta}_b(v_1) = \eta_a(v_1)$, and $\frac{d\hat{\eta}_b(v_1)}{dv} = \frac{d\eta_a(v_1)}{dv}$, we have $\eta_a(v) \geq \hat{\eta}_b(v)$ for all v. Thus, all bidder types participate given this single contract, just as with the original contract(s). Further, because $\eta_b(v)$ is concave, $\hat{\eta}_b(v_1) = \eta_b(v_1)$, and $\frac{d\hat{\eta}_b(v_1)}{dv} = \frac{d\eta_b(v_1)}{dv}$, we have $\eta_b(v) \leq \hat{\eta}_b(v)$ for all v. Thus, the seller's expected profit is weakly higher under the single contract. Next, $e = \frac{d\eta_b(v_1)}{dv} = \frac{d\eta_a(v_1)}{dv}$ yields, by (14), $e = 1 - \frac{dV}{dv} = 1 - \frac{1}{\frac{dv}{dV}} = \frac{\frac{ds}{dV}|_{V=V_1}}{1+\frac{ds}{dV}|_{V=V_1}}$, where $V_1 = V(v_1)$. By $e = \frac{k}{1+k}$ ((32)), we have $k = \frac{ds(V_1)}{dV}$, consistent with (33). This proves the proposition for Scenario 1.

Scenario 2: $\frac{dh^{**}(v)}{dv}|_{v=v_1} < 0$. Then $\frac{d\eta_a(v_1)}{dv} < \frac{d\eta_b(v_1)}{dv}$. Because $h^{**}(v) = 0$ has one solution over $[\underline{v}, \bar{v}]$, $h^{**}(v) < 0$ for all $v > v_1$ and $h^{**}(v) > 0$ for all $v < v_1$. Thus, only bidder types $v \leq v_1$ participate. Then $v_1 > \underline{v}$, else no bidder type participates, a contradiction to the premise that the mechanism is optimal. We next consider two sub-cases that show that Scenario 2 cannot arise in optimal mechanism.

Sub-case 1: $v_1 < \bar{v}$. Consider the single contract that takes the form given in (32) with $V_1 = V(v_1)$ and $V_2 = V(v_2)$, where $v_2 \equiv \bar{v}$, in (33). Let $\hat{h}(v)$ be the expected profit of a type v bidder from this contract. Then (A10) holds with $\hat{\eta}_b(v)$ given by (A16). Similar arguments as in Case 1 yield that $\eta_a(v_i) = \hat{\eta}_b(v_i)$ for $i = 1, 2$, and that $\hat{h}(v) < 0$ if and only if $v \in (v_1, \bar{v})$. Thus, bidder types $v \leq v_1$ participate given this contract, just as they did with the original contracts. The only difference is that, with the single contract, bidder type $\bar{v}$ (of measure zero) participates with rents fully extracted, which weakly increases the seller's expected profit. Now consider types $v \leq v_1$. Because $\eta_a(\bar{v}) < \eta_b(\bar{v})$, we have $\hat{\eta}_b(\bar{v}) < \eta_b(\bar{v})$. Further, (i) $\hat{\eta}_b(v_1) = \eta_b(v_1)$, (ii) $\eta_b(v)$ weakly concave, and (iii) $\hat{\eta}_b(v)$ affine, together yield $\frac{d\hat{\eta}_b(v)}{dv} < \frac{d\eta_b(v_1)}{dv}$ and $\hat{\eta}_b(v) > \eta_b(v)$ for all $v < v_1$. Thus, the seller's expected revenue is strictly higher than with the posited optimal contract(s), a contradiction.

Sub-case 2: $v_1 = \bar{v}$. Consider the single contract that takes the form given in (32) with $e = \frac{d\eta_a(v_1)}{dv}$. A similar argument shows that the seller's expected revenue is strictly higher with this contract. Thus, Scenario 2 contradicts the premise that the original contract(s) were optimal.

Scenario 3: $\frac{dh^{**}(v)}{dv}|_{v=v_1} > 0$. We can show that Scenario 3 contradicts the premise of optimality of the mechanism via similar arguments as for Scenario 2, using the single contract taking the form specified in (32), where if $v_1 = \underline{v}$ then $e = \frac{d\eta_a(v_1)}{dv}$, and if $v_1 > \underline{v}$ then $k = \frac{s(V(v_1)) - s(V(\underline{v}))}{V(v_1) - V(\underline{v})}$ in (33). These results prove the proposition for Case 2.

Case 3: $h^{**}(v) = 0$ has no solution over $[\underline{v}, \bar{v}]$. To see that Case 3 cannot arise in the optimal mechanism, observe that it would imply either (1) $h^{**}(v) < 0$ for all $v \in [\underline{v}, \bar{v}]$, or (2) $h^{**}(v) > 0$ for all $v \in [\underline{v}, \bar{v}]$. Neither scenario can be optimal: in (1) no bidder type participates; and in (2) one can add a small positive cash payment to all contracts so that all bidder types still participate but earn strictly less rents. This completes the proof. ■

Proof of Lemma 3: Let $\beta(V)$ denote the cash bid in the second-price auction by type V,

$$\beta(V) = s(V) + V_T - \frac{\alpha}{1-\alpha}V,$$

which strictly increases in V, so the highest bidder type wins. The lowest type $\underline{V}$ earns no rents, and the differential rent of a high type over a low type (by (9)) is positive and strictly decreasing in $\alpha \in \left(0, \frac{b}{1+b}\right)$. Denote the highest and second-highest bidder types by V_1 and V_2. Consider $\alpha = \frac{b}{1+b}$. The winner's profit is

$$\pi = \frac{1}{1+b}\left(s\left(V_1\right) - s\left(V_2\right)\right) - \frac{b}{1+b}\left(V_1 - V_2\right).$$

For $i = 1, 2$ define V_i^* such that $s\left(V_i\right) + V_i = y\left(V_i^*\right) + V_i^*$. Because $\frac{1}{1+b}\left(y\left(V_1^*\right) - y\left(V_2^*\right)\right) - \frac{b}{1+b}\left(V_1^* - V_2^*\right) = 0$, we can rewrite π as

$$\pi = \left(\frac{1}{1+b}\left(s\left(V_1\right) - y\left(V_1^*\right)\right) - \frac{b}{1+b}\left(V_1 - V_1^*\right)\right) + \left(\frac{b}{1+b}\left(V_2 - V_2^*\right)\right) - \frac{1}{1+b}\left(s\left(V_2\right) - y\left(V_2^*\right)\right).$$

For the first term on the right-hand side,

$$\frac{1}{1+b}\left(s\left(V_1\right) - y\left(V_1^*\right)\right) - \frac{b}{1+b}\left(V_1 - V_1^*\right) = \left(s\left(V_1\right) - y\left(V_1^*\right)\right) \leq \frac{\sqrt{2}}{2}\epsilon.$$

Similarly, the second term is also no greater than $\frac{\sqrt{2}}{2}\epsilon$. Therefore, $\pi \leq \sqrt{2}\epsilon$, proving the result. ■

Proof of Corollary 4: For a security with index $t(z)$, denote its payoff function $S\left(t(z), \cdot\right)$ by $\hat{S}\left(\cdot\right)$ for notational ease. Denote the distribution of θ by $f_\theta(\cdot)$. Then

$$ES(t(z), v; M(z)) = \int_0^\infty \hat{S}(\theta\left(v - \hat{v}\right) + \hat{v} - M(z)) f_\theta(\theta) d\theta.$$

Hence,

$$ES_{vv}(t(z), v; M(z)) = \int_0^\infty \theta^2 \hat{S}''(\theta\left(v - \hat{v}\right) + \hat{v} - M(z)) f_\theta(\theta) d\theta.$$

If $\hat{S}(\cdot)$ is strictly concave, then $ES_{vv}(t(z), v; M(z)) < 0$ for all v, z. Then the necessary condition (39) establishes the corollary. ■

Proof of Lemma 5:

Claim 1: At any v, for any $M < v - V(v)$, there is a $t^*(M; v)$ that solves

$$v - V(v) - ES(t, v; M) - M = 0. \tag{A17}$$

Furthermore, $t^*(M; v)$ decreases in M.

Proof of Claim 1: From the assumption that the security index has a sufficient range, the left-hand side of (A17) approaches $v - V(v) - M > 0$ when t is low enough; and the left-hand side approaches $v - V(v) - (v - M) - M = -V(v) < 0$ when t is high enough. Thus, there exists a smallest t that satisfies (A17), which we denote by $t^*(M; v)$ ($ES(t, v; M)$ could be flat at t^*). Because $\frac{\partial ES(t(v),v;M)}{\partial M} \in [-1, 0]$, $\frac{\partial(v-V(v)-ES(t,v;M)-M)}{\partial M} \leq 0$. Because $ES(t, v; M)$ increases in t, $t^*(M; v)$ decreases in M. ■

For any $y \geq 0$, by Claim 1 and Assumption 2, $S(t^*(M; v), y)$ decreases in M. Because $S(t^*(M; v), y)$ is bounded between zero and y, $S(t^*(M; v), y)$ reaches a limit as $M \to v - V(v)$. Define $S^*(y) \equiv \lim_{M \to v-V(v)} S(t^*(M; v), y)$.

Claim 2: $S^*(y) = 0$ at all $y \geq 0$.

Proof of Claim 2: Suppose that $S^*(y^*) = \delta > 0$ at some y^*. Then $S^*(y) \geq \delta$ for all $y \geq y^*$. Then $S(t^*(M; v), y) \geq \delta$ for all M and $y \geq y^*$. We then have

$$\begin{aligned} ES\left(t^*(M), v; M\right) &\geq \delta E\left[\Pr\text{obability}\left(\widetilde{y(v)} \geq y^* + M\right)\right] \\ &\geq \delta E\left[\Pr\text{obability}\left(\widetilde{y(v)} \geq y^* + v - V(v)\right)\right] > 0, \end{aligned}$$

where $\widetilde{y(v)}$ is the stochastic (before cash payment) cash flow of bidder type v. It follows that as M approaches $v - V(v)$, the left-hand side of (A17) becomes strictly negative, a contradiction. ■

Claim 3: $\lim_{M \to v-V(v)} 1 - V'(v) - ES_v\left(t^*(M), v; M\right) > 0$.

Proof: Define $\kappa \equiv 1 - V'(v) > 0$. By the assumption of sufficient range in index, for any $M \in [v - V(v) - 1, v - V(v)]$, there exists a $\hat{t}$ such that $ES_v(\hat{t}, v; M) < \kappa$. By Claim 2, as

M approaches $v - V(v)$, $t^*(M;v)$ must become less than $\hat{t}$. ∎

For any $y \geq 0$, by Claim 1 and Assumption 2, the right-derivative of $S\left(t^*(M;v), y\right)$ with respect to y decreases in M. Because this derivative is bounded between zero and one, it reaches a limit as M approaches $-\infty$. Denote this limiting value by function $d(y)$. Because security payoffs are convex, $d(y)$ is nondecreasing and cannot exceed ine. Hence, when y goes to infinity, $d(y)$ approaches a limit, which we denote by d^*.

Claim 4: d^* =1.

Proof: Suppose, instead, that $d^* < 1$. Then by Assumption 2, Claim 1, and convexity in security payoffs, for all M and y, $S(t^*\left(M;v\right), y)$ is no greater than $d^* y$. Then the left-hand side of (A17) is strictly positive for M sufficiently negative, a contradiction. ∎

By Claim 4, there exists a $y^* > 0$ such that $d\left(y^*\right) > 1 - \frac{V'(v)}{3}$. Thus, there exists M^* such that the right-derivative of $S\left(t^*(M;v), \cdot\right)$, evaluated at y^*, exceeds $1 - \frac{V'(v)}{2}$ for all $M < M^*$. From the convexity of the payoff function, the right-derivative of $S(t^*(M;v), \cdot)$ exceeds $1 - \frac{V'(v)}{2}$ for all $M < M^*$ and all $y > y^*$.

For $M < \min\left\{M^*, -y^*\right\}$, we have

$$ES\left(t^*(M), v; M\right) = E\left[S(t^*(M), \left(\widetilde{y\left(v\right)} - M\right))\right]$$

and

$$ES\left(t^*(M), v + \Delta v; M\right) = E\left[S(t^*(M), \left(y\,\widetilde{\left(v + \Delta v\right)} - M\right))\right],$$

where $\Delta v > 0$. Because the distribution of $y\,\widetilde{\left(v + \Delta v\right)}$ first-order stochastically dominates that of $\widetilde{y(v)}$, it follows that $y\,\widetilde{\left(v + \Delta v\right)}$ can be expressed as the sum of random variable $\widetilde{y(v)}$

plus another random variable $\widetilde{\epsilon}$, where $\widetilde{\epsilon}$ is nonnegative and $\mathrm{E}[\widetilde{\epsilon}] = \Delta v$. Thus,

$$\begin{aligned}
ES\left(t^*(M), v+\Delta v; M\right) &= E\left[S(t^*(M), \left(\widetilde{y(v)} + \widetilde{\epsilon} - M\right))\right] \\
&\geq E\left[S(t^*(M), \left(\widetilde{y(v)} - M\right)) + \left(1 - \frac{V'(v)}{2}\right)\widetilde{\epsilon}\right] \\
&= ES\left(t^*(M), v; M\right) + \left(1 - \frac{V'(v)}{2}\right)\Delta v.
\end{aligned}$$

It follows that $ES_v\left(t^*(M), v; M\right) \geq \left(1 - \frac{V'(v)}{2}\right).$, and hence $1 - V'(v) - ES_v\left(t^*(M), v; M\right) < 0$. Thus, by Claim 3, there exists M such that $1 - V'(v) - ES_v\left(t^*(M), v; M\right) = 0$. This proves Lemma 5. ∎

Proof of Lemma 6: By a similar argument as in the proof of Corollary 4, if $\hat{S}(\cdot)$ is strictly convex, then $ES_{vv}(t(z), v; M(z)) > 0$ for all v, z. It follows that $K > 0$.

Now consider call options. Let $t(z) > 0$ denote the strike price (abusing notation slightly as a higher strike price corresponds to a lower payoff). If $t(z) - \hat{v} + M(z) > 0$, then

$$ES(t(z), v; M(z)) = \int_{\frac{t(z)+M(z)-\hat{v}}{v-\hat{v}}}^{\infty} (\theta\,(v - \hat{v}) + \hat{v} - M(z) - t(z)) f_\theta(\theta) d\theta.$$

Therefore,

$$ES_v(t(z), v; M(z)) = \int_{\frac{t(z)+M(z)-\hat{v}}{v-\hat{v}}}^{\infty} \theta f_\theta(\theta) d\theta$$

and

$$ES_{vv}(t(z), v; M(z)) = \frac{(t(z) + M(z) - \hat{v})^2}{(v - \hat{v})^3} f_\theta\left(\frac{t(z) + M(z) - \hat{v}}{v - \hat{v}}\right) > 0.$$

If $t(z) - \hat{v} + M(z) \leq 0$, we have $ES_v(t(z), v; M(z)) = 1$ and $ES_{vv}(t(z), v; M(z)) = 0$. When $z = v$, we must have $t(z) - \hat{v} + M(z) > 0$, or else the left-hand side of (38) would be $-V'(v) < 0$, a contradiction. Thus, $ES_{vv}\left(t(z), z; M(z)\right) > 0$ for all z. Because $ES_{vv}\left(t(z), v; M(z)\right)$ is continuous in v, $ES_{vv}\left(t(z), v; M(z)\right) > 0$ if $|z - v|$ is small. Because $ES_{vv}\left(t(z), v; M(z)\right)$ is nonnegative for all z and v, its average over z and v must be strictly positive, so (40) yields $K > 0$. ∎

Proof of Theorem 2: By the premise that the mechanism satisfies (35) and (38) (Lemma 5 guarantees the existence of such mechanism), we only need to establish global incentive compatibility. It suffices to show that $\frac{\partial h(v,z)}{\partial v} \geq 0$ for $v < z$ and that $\frac{\partial h(v,z)}{\partial v} \leq 0$ for $v > z$. We have $\frac{\partial h(v,z)}{\partial v} = l(v,z)$, where

$$l(v,z) \equiv 1 - V'(v) - ES_v\left(t(z), v; M(z)\right),$$

and (38) yields $l\left(z,z\right) = 0$. Thus, for $z \neq v$,

$$\begin{aligned} l(v,z) &= l(v,z) - l(z,z) \\ &= -V'(v) - ES_v\left(t(z), v; M(z)\right) + V'(z) + ES_v\left(t(z), z; M(z)\right) \\ &= \int_v^z V''(x)dx + \frac{ES_v\left(t(z), z; M(z)\right) - ES_v\left(t(z), v; M(z)\right)}{z-v}(z-v) \\ &= \int_v^z V''(x)dx + \frac{\int_v^z ES_{vv}(t(z), x; M(z))dx}{z-v}(z-v) \end{aligned}$$

If $z > v$, then by (40) we have

$$l(v,z) \geq \int_v^z V''(x)dx + K\left(z-v\right) = \int_v^z \left(V''(x) + K\right)dx \ \geq 0,$$

and if $z < v$, then

$$l(v,z) \leq \int_v^z V''(v)dv + K\left(z-v\right) = \int_v^z \left(V''(x) + K\right)dx \ \leq 0.$$

Thus, $\frac{\partial h(v,z)}{\partial v} \geq 0$ for $v < z$ and $\frac{\partial h(v,z)}{\partial v} \leq 0$ for $v > z$. We therefore have $h\left(v,z\right) \leq 0$, $\forall z \neq v$. ∎

Proof of Corollary 5:

Claim 1: $ES_v(t, v; M)$ is increasing in t for all v.

Proof: For $t_1 > t_2$, we have

$$ES_v(t_1, v; M) - ES_v(t_2, v; M) = \int [S(t_1, y - M) - S(t_2, y - M)] g_v(y|v)) dy,$$

where $g(\cdot|v)$ is the pdf of cash flows by bidder type v and g_v is its derivative. Because g_v integrates to zero, there exists a y^* such that $g_v(y^*|v) = 0$. The expression $\frac{g_v(y|v)}{g(y|v)}$ strictly increases in y by the strict monotone likelihood ratio property that holds given Assumption 1. Thus, for $y < y^*$, $\frac{g_v(y|v)}{g(y|v)} < 0$ and hence $g_v(y|v) < 0$; and for $y > y^*$, $\frac{g_v(y|v)}{g(y|v)} > 0$ and hence $g_v(y|v) > 0$. Since $[S(t_1, y - M) - S(t_2, y - M)]$ is weakly increasing by Assumption 2,

$$\int_{y<y^*} [S(t_1, y - M) - S(t_2, y - M)] g_v(y|v) dy \geq \int_{y<y^*} [S(t_1, y^* - M) - S(t_2, y^* - M)] g_v(y|v) dy$$

and

$$\int_{y>y^*} [S(t_1, y - M) - S(t_2, y - M)] g_v(y|v) dy \geq \int_{y>y^*} [S(t_1, y^* - M) - S(t_2, y^* - M)] g_v(y|v) dy.$$

Adding yields Claim 1: $ES_v(t_1, v; M) - ES_v(t_2, v; M)$

$$\geq \int_{y<y^*} [S(t_1, y^* - M) - S(t_2, y^* - M)] g_v(y|v) dy + \int_{y>y^*} [S(t_1, y^* - M) - S(t_2, y^* - M)] g_v(y|v) dy = 0.$$

Next, consider any v^*. To show that $M(v^*)$ is strictly positive, suppose that $M(v^*) \leq 0$. Then by (A17), $ES(t(v^*), v^*; M(v^*)) \geq v^* - V(v^*)$. By Assumption 2 and $v^* - V(v^*) > 0$, there exists a $\hat{t} \leq t(v^*)$ such that $ES(\hat{t}, v^*; M(v^*)) = v^* - V(v^*)$. If bidding strategies in a second-price security-bid auction are strictly decreasing, they are still strictly decreasing when the stochastic cash flows for all types v are shifted up by the same amount $|M(v^*)|$. Thus, by the premise of the corollary on the decreasing bidding strategy, $ES(\hat{t}, v^*; M(v^*)) = v^* - V(v^*)$ implies $ES_v(\hat{t}, v^*; M(v^*)) > 1 - V'(v^*)$ (Lemma 1, Che and Kim 2010). By $\hat{t} \leq t(v^*)$ and Claim 1, $ES_v(t(v^*), v^*; M(v^*)) > 1 - V'(v^*)$, contradicting (38). Thus,

$M(v^*) > 0$. Moreover, pure security or pure cash cannot extract full rents because when a seller can combine them, it must use strictly positive amounts of both to extract full rents. ■