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Surplus Bounds in Cournot Monopoly and Competition

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Abstract

We characterize equilibria of oligopolistic markets where identical firms with constant marginal cost compete a' la Cournot. For given maximal willingness to pay and maximal total demand, we first identify all combinations of equilibrium consumer and producer surplus that can arise from arbitrary demand functions. Then, as a further restriction, we fix the average willingness to pay above marginal cost (i.e., first best surplus) and identify all possible triples of consumer surplus, producer surplus and deadweight loss.

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1 Introduction

Antoine Augustin Cournot's pioneering mathematical analysis of monopoly and oligopoly, published in his *Recherches sur les Principes Mathematiques de la Theorie des Richesses* (1834), has had an enormous influence in economics.[1] Cournot's model has been a building block for a large number of seminal works in a variety of fields, including international trade (e.g., Brander and Krugman (1983), Atkeson and Burstein (2008)) the study of market power in macroeconomics (Hart (1982)) and industrial organization (Bresnahan and Reiss (1990), Berry (1992)) and antitrust merger policy (Farrell and Shapiro (1990)). After nearly two hundred years, countless papers have explored and extended Cournot's work, which remains a benchmark for theories of price formation in the absence of perfect competition (Vives (1989, 1999)).

In this paper we advance the existing literature, by characterizing all equilibrium outcomes (i.e., triples of consumer surplus, producer surplus and dead-weight loss) that can possibly arise from arbitrary demand functions with given first-best surplus in oligopolistic markets with competition a' la Cournot — including monopoly as a special case. One main assumption is maintained: firms have an identical and constant marginal cost.

Suppose marginal cost of production is zero and there is a unit mass of consumers. Then, the first best surplus of a demand function, let's denoted it with s, is the average willingness to pay of consumers. Let's represent market outcomes as points on the positive quadrant of a Cartesian plane, with profits $n\pi$ (n is the number of competing firms) on the x-axis and consumer surplus, w, on the y-axis. Feasible outcomes satisfy $n\pi + w \leq s$, and deadweight loss is $s - w - n\pi$. As we show in Propositions 2-4, the set of achievable market outcomes is roughly characterized by the triangle described by the points $(n\pi_s, s - n\pi_s), (n\pi^s, 0), (s, 0)$, with $\pi_s \leq \pi^s$.[2] For illustration see Figure 5 at the end of the paper. Remarkably, among all equilibria of all possible demand functions, the one that maximizes consumer surplus is efficient and also minimizes industry profit. Moreover, as $n \to \infty$ then $n\pi_s \to 0$ and $n\pi^s \to s$. Hence, the achievable set converges to the Pareto frontier.

The only-if part of our proofs are constructive, in the sense that for each achievable market outcome (i.e., triple of consumer surplus, producer and loss) we present an (inverse) demand function and a symmetric oligopoly equilibrium quantity that attains it. In particular, our construction relies on a set of demand functions that, in equilibrium, induce a unit-elastic *residual* demand, leaving firms indifferent between playing equilibrium and producing alternative quantities.

Beyond having theoretical interest, obtaining bounds to market outcomes can be useful for a variety of applied purposes. For instance, our results can inform cost-benefit analysis of altering the competitive landscape in cases where good estimates of the demand curve are not available. What's the best case for consumers if a monopoly is introduced in a certain product market? What is the worst case scenario following a merger or a policy that reduces the number of competitors? These are some of the questions whose answers can be informed by our results.

Three papers are most closely related. First, Condorelli and Szentes (2020) identify the highest level of consumer surplus attainable in a monopolistic market, assuming (inverse) demand exhibits a given mean consumer value. The maximum consumer surplus is attained when the demand is unit-elastic and the price is such that all consumers are served. Second, as shown in

[1] Treatment of those subjects remains almost unchanged to this day, to the point that contemporary economics students would hardly notice if excerpts from the *Recherches* were to appear in textbooks.

[2] This description is not precise as for $\pi_s < \pi^s$ the line connecting $(n\pi^s, s - n\pi^s)$ to $(n\pi_s, 0)$ need not be straight.

Neeman (2003) and Kremer and Snyder (2018), it turns out that said unit-elastic demand also generates the minimum monopolist profit. Taken together, these results fully characterize the couples of producer and consumer surplus achievable in a monopoly market with given average consumer value. In our paper, we generalize this characterization to the case of an arbitrary number of firms competing a' la Cournot.

There is a small literature that seek to identify bounds on market outcomes in Cournot oligopoly, based on specific properties of demand functions.[3] Anderson and Renault (2003) derive bounds on the ratios of deadweight loss and consumer surplus to producer surplus based on the degree of curvature of the (inverse) demand function. They show that the "more concave" is the demand, the larger the share of producer surplus to overall surplus and the smaller is the consumer surplus relative to producer surplus. Johari and Tsitsiklis (2005) establish a 2/3 lower bound on the ratio between the sum of consumer and producer surplus and first-best surplus, when the (inverse) demand function is affine and firms are heterogeneous, with their cost function convex. Tsitsiklis and Xu (2014) extend the previous paper by providing smaller lower bounds for general convex (inverse) demand. Moreover, they show that arbitrary high efficiency losses are possible if demand is allowed to be concave. In contrast to these papers, our bounds do not rely on knowledge about the curvature of the demand function. Also, we obtain a complete characterization of all consumer and producer surplus couples for any given first-best surplus. However, we only focus on the case where firms are symmetric and their cost function is linear.

The paper is organized as follows. After introducing the model, we study the case where demand functions are bounded but there is no restriction on first-best surplus. In section 4, which contains the main results of this paper, we impose the additional restriction on the first-best surplus.

2 Model

A market is populated by $n \in \mathbb{N}^+$ firms, all supplying an homogeneous good at common marginal cost $c \in (0, +\infty)$. Firms compete a' la Cournot: each firm i decides the quantity $q_i \in [0, +\infty)$ that it brings to the market and a non-negative price is determined by the market-clearing condition. In particular, let $P : [0, \infty) \to \Re^+$ be a non-negative, left-continuous and non-increasing inverse demand curve faced by the firms. If $Q = \sum_i q_i$ is the total supply, then the market price is $P(Q)$, firm i's profit is $(P(Q) - c)q_i$ and consumer surplus is $\int_0^Q P(x)dx - QP(Q)$.

In this analysis, we maintain that firms are identical and, without loss of generality, we focus on symmetric equilibria, where all firms produce the same quantity.[4] We say that $(q, \ldots, q)$ is a symmetric Cournot equilibrium of P if

$$q = \arg\max_{x \geq 0} \left[P((n-1)q + x)x - cx \right],$$

and in this case we write $q \in \mathcal{E}(P)$. Moreover, we denote with $CS(P, q)$ the consumer surplus and with $\Pi(P, q)$ the profit of each firm in an equilibrium q of P.

[3] A related problem, explored in Carvajal et al. (2013), consists in identifying revealed preference tests for Cournot equilibrium.

[4] We show in Appendix A that for any asymmetric equilibrium there exists a symmetric one where the same total quantity produced is the same. Hence, consumer surplus and total industry profit are the same in the two equilibria.

In addition, we require that $u = \sup\{P(Q) \mid Q \in \Re^+\} \in (c, \infty)$ (i.e., the maximal consumer valuation, denoted u, is above cost and it is finite) and $b = \sup\{Q \mid P(Q) \geq 0\} \in (0, \infty)$ (i.e., the maximal demand, b, is positive and finite). Omitting reference to b and u, let $\mathcal{P}$ be the set of all inverse demand functions that satisfy our restrictions.[5]

Next, let $\overline{\pi} = b(u-c)/n$ and observe that this is the maximum profit that can be made by a single firm in a symmetric equilibrium for a demand in $\mathcal{P}$, since the maximum price is u while the maximum quantity is b. Also, let $\underline{q}(\pi) = \pi/(u-c)$, or for short simply $\underline{q}$ when the profit level is unambiguous, be the individual production that delivers profit π when the price is u. In other words, $\underline{q}(\pi)$ is the minimal quantity that is able to deliver individual profit π for demands in $\mathcal{P}$

3 Bounding Consumer and Producer Surplus

The following family of demand functions in $\mathcal{P}$ plays a key role in the analysis that follows. For each $\pi \in [0, \bar{\pi}]$ and $q \in [\underline{q}(\pi), b/n]$ we define

$$P_{(\pi,q)}(Q) = \begin{cases} u & \text{if } Q \in [0, \frac{\pi}{u-c} + (n-1)q] \\ \frac{\pi}{Q-(n-1)q} + c & \text{if } Q \in (\frac{\pi}{u-c} + (n-1)q, b], \\ 0 & \text{if } Q > b. \end{cases}$$

The next Lemma characterizes equilibria in this family of inverse demand functions.

Lemma 1 *For each $\pi \in (0, \bar{\pi}]$ and $q \in [\underline{q}(\pi), b/n]$, $q \in \mathcal{E}(P_{(\pi,q)})$ and $\Pi(P_{(\pi,q)}, q) = \pi$.*

Proof. Given that $P(nq) = \pi/q + c$, it is easy to verify that $\Pi(P_{(\pi,q)}, q) = \pi$ if q is an equilibrium of $P_{(\pi,q)}$. To see the latter, denote with $P^R_{(\pi,q)}(q_i, q_{-i})$ the residual demand faced by firm i under demand $P_{(\pi,q)}$, with q_{-i} representing the total quantity produced by firms other than i. Observe that for $q_{-i} = (n-1)q$ we have

$$P^R_{(\pi,q)}(q_i, (n-1)q) = \begin{cases} u & \text{if } q_i \in [0, \frac{\pi}{u-c}] \\ \frac{\pi}{q_i} + c & \text{if } q_i \in (\frac{\pi}{u-c}, b-(n-1)q], \\ 0 & \text{if } q_i > b-(n-1)q. \end{cases}$$

It follows firm i is indifferent among any quantity in the interval $[\pi/(u-c), b-(n-1)q]$, as they all provide profit π. To conclude the proof, observe that i's profit is zero for $q_i > b-(n-1)q$ and it is $q_i(u-c) \leq \pi$ for $q_i \leq \frac{\pi}{u-c}$. ■

As explicitly showed in the proof of the Lemma above, the demand function $P_{(\pi,q)}$ exhibits unit-elasticity of the *residual* demand for quantities in $(\pi/(u-c), b-(n-1)q)$, when all other firms supply q. In particular, for each individual firm, producing any quantity in $(\pi/(u-c), b-(n-1)q)$ is a best reply to the other firms producing q and generates profit π.

Figure 1 illustrates the geometry of the demand functions in the class above, for the case where $u = b = 1$ and $c = 0$. The first panel depicts two demand functions, $P_{(\pi,\pi)}$ and $P_{(\pi,1/n)}$,

[5] Following McManus (1964), a symmetric equilibrium exists under the stated assumptions.Equilibria may exists even for unbounded demand. However, no bound can be placed on market outcomes if the demand can be unbounded and no further restriction is imposed, as we shall show toward the end of the next section.

that generate the same profit but induce two different equilibrium quantities, the minimal and the maximal. The dotted curve shows $P_{(\pi,q)}$ as equilibrium q varies in $[\pi, 1/n]$. The second sub-figure depicts two demand functions, $P_{(\pi,1/n)}$ and $P_{(\pi',1/n)}$, that generates different profit levels but induce the same equilibrium quantity. Observe that demands are fully ordered along both dimension π and q. The following useful Lemma is immediate to verify and we state it without proof.

Lemma 2 *$P_{(\pi',q)} \geq P_{(\pi,q)}$ if $\bar{\pi} \geq \pi' \geq \pi \geq 0$ and $P_{(\pi,q')} \geq P_{(\pi,q)}$ if $b/n \geq q' \geq q \geq \underline{q}(\pi)$.*

The next result shows that any demand function that has an equilibrium q and generates individual profit level π is dominated by $F_{(\pi,q)}$ pointwise and that the equilibrium q of $P_{(\pi,q)}$ achieves the largest consumer surplus among symmetric equilibria q of demands in $\mathcal{P}$ if π is equilibrium profit.

Lemma 3 *If $P \in \mathcal{P}$ and $q \in \mathcal{E}(P)$ and $\Pi(P,q) = \pi$ then $P \leq P_{(\pi,q)}$, and $CS(P_{(\pi,q)}, q) \geq CS(P,q)$.*[6]

When $n = 1$ the result is analogous to Lemma 1 in Condorelli and Szentes (2020), which is here generalized beyond the monopoly case.

Proof. First observe that if q is a Cournot equilibrium under P, then for any $x \geq 0$,

$$\pi \geq x\left[P(x + (n-1)q) - c\right].$$

By denoting $Q = x + (n-1)q$ and rearranging, it follows that, for $Q \in [(n-1)q, +\infty)$,

$$P(Q) \leq \frac{\pi}{Q - (n-1)q} + c.$$

Next, we show that $P(Q) \leq P_{(\pi,q)}(Q)$ for $Q \geq 0$. This follows from the inequality above for $Q \geq (n-1)q$. For $Q < (n-1)q \leq (n-1)q + \pi/(u-c)$ the inequality follows from the fact that $P_{(\pi,q)}(Q) = u$ in that range, while $P(Q) \leq u$ by assumption.

We now establish that $P_{(\pi,q)}$ generates (weakly) larger consumer surplus than P. In particular

$$CS(P_{(\pi,q)}, q) - CS(P,q) = \int_0^{nq} (P_{(\pi,q)}(x) - P(x))dx \geq 0,$$

because $P_{(\pi,q)} \geq P$ by the first part of the proposition. ■

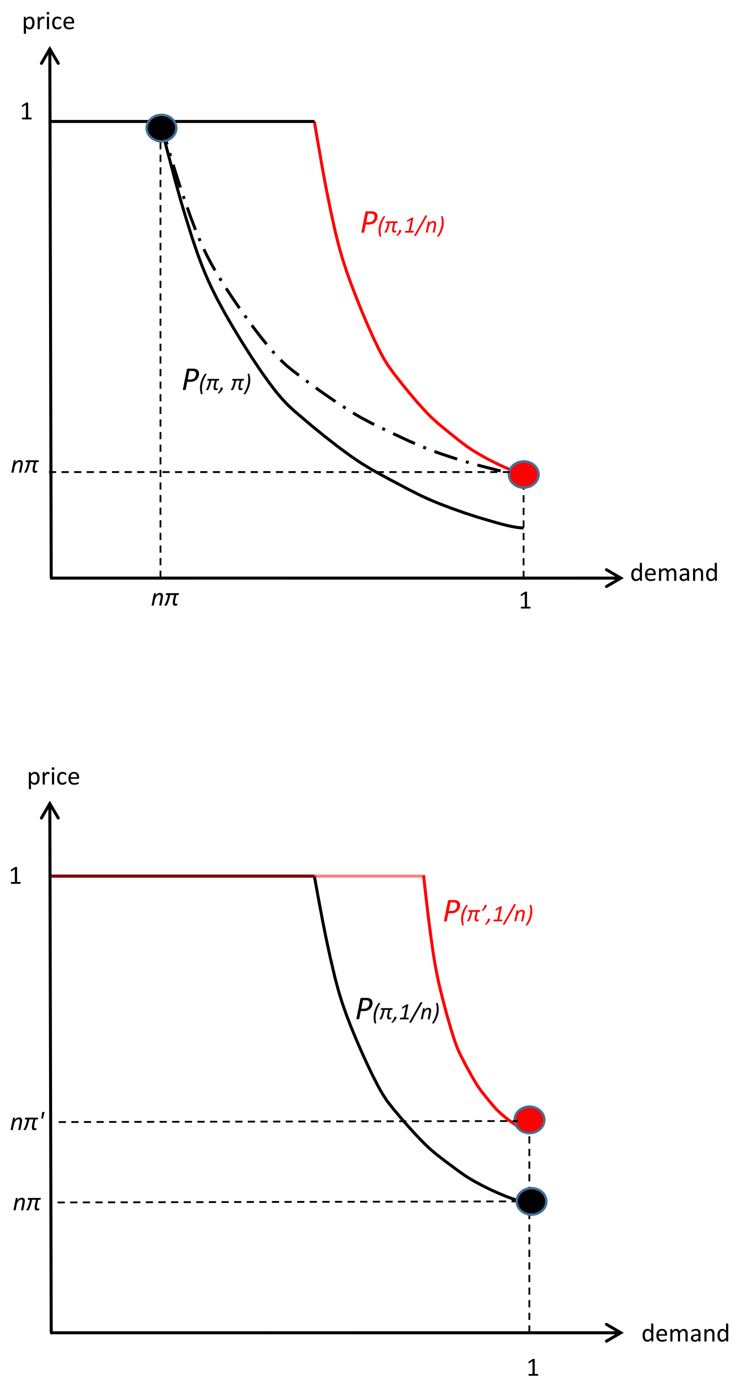
For each $\pi \in (0, \bar{\pi}]$, define the inverse demand function $P_\pi = P_{(\pi,b/n)}$. In the symmetric equilibrium of this demand, all b consumers are served. The following Lemma establishes that there exists no symmetric equilibrium of any demand function generating individual firm profit π that attains a higher consumer surplus than the equilibrium b/n of P_π.

Lemma 4 *For any $P \in \mathcal{P}$ and $q \in \mathcal{E}(P)$ with $\Pi(P,q) = \pi$, we have $CS(P_\pi, b/n) \geq CS(P,q)$.*[7]

[6] Note that, being both decreasing and left-continuous, unless $P = P_{(\pi,q)}$, P and $P_{(\pi,q)}$ will differ on a non-zero measure of the domain $[0, b/n]$.

[7] This result also illustrate that no absolute bound can be placed on the ratio of consumer to producer surplus. In fact, $\lim_{\pi\to 0} CS(P_\pi, b/n)/\pi = \infty$ and $\lim_{\pi\to\bar{\pi}} CS(P_\pi, b/n)/\pi = 0$.

Figure 1: Examples of demands $P_{(\pi,q)}$ for $b = u = 1$ and $c = 0$



Proof. Lemma 1 establishes that b/n is an equilibrium of P_π and q is an equilibrium of $P_{(\pi,q)}$. Lemma 3 establishes that $CS(P_{(\pi,q)}, q) \geq CS(P, q)$. To see that $CS(P_\pi, b/n) \geq CS(P_{(\pi,q)}, q)$ recall the last equation of the Lemma 3 and observe that $b/n \geq q$ and, for each $\pi \in [0, \bar{\pi}]$ and q, q' such that $\pi/(u-c) \leq q' \leq q \leq b/n$, we have $P_{(\pi,q')}(Q) \leq P_{(\pi,q)}(Q)$ for $Q \in [0, \infty)$, by Lemma 2. ■

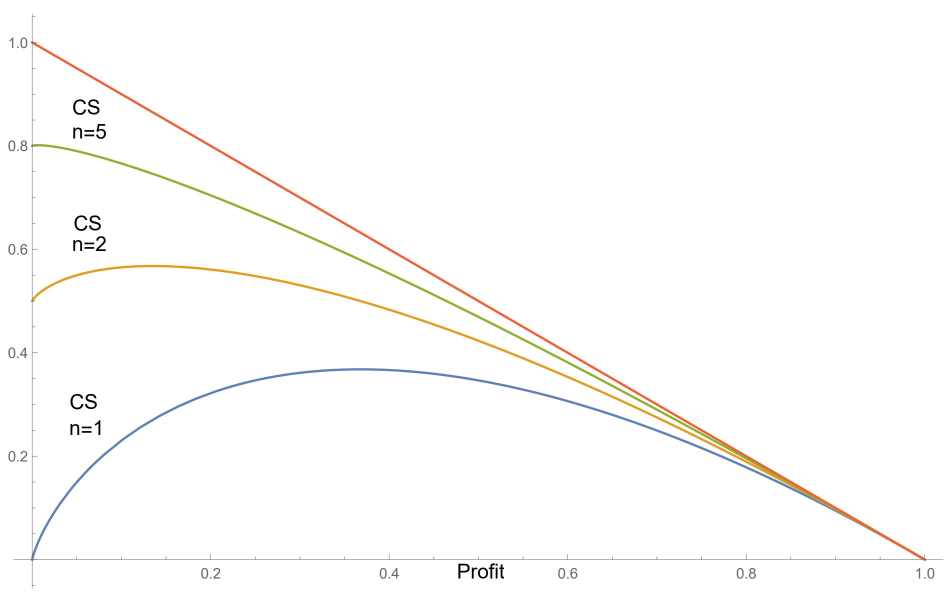
We are now in a position to state the main result of this section, which characterizes all couples of consumer and producer surplus that can arise for some demand function in $\mathcal{P}$.

Proposition 1 *There exists $P \in \mathcal{P}$ and $q \in \mathcal{E}(P)$ such that $\Pi(P, q) = \pi$, $CS(P, q) = w$ if and only if $(\pi, w) \in \{(x, y) : x \in (0, \bar{\pi}], y \in [0, CS(P_x, b/n))\}$.*

Proof. The only-if part is obvious in light of Lemma 4. That is, there can't be any equilibrium payoff couple outside the specified set. To prove the if-part consider $P_{(\pi,q)}$ and observe that, for each $\pi \in (0, b(u-c))$, q is an equilibrium in light of Lemma 1 and $CS(P_{(\pi,q)}, q)$ is continuous and strictly increasing in q, and $CS(P_{(\pi,\underline{q})}, \underline{q}) = 0$. ■

Contour lines for the sets of feasible combinations of consumer surplus, CS, and (total industry) profit are illustrated in the next figure 2 for $n = 1, 2, 5$, assuming $b = u = 1$ and $c = 0$ (i.e., $\lim_{c \downarrow 0}$). As it is apparent from the picture, the set of feasible producer/consumer surplus combinations expand as n grows and the market becomes more competitive. Equilibrium alone imposes very little restrictions on how the surplus is shared in an oligopolistic market populated by a large number of identical firms with constant marginal cost.

Figure 2: Achievable profit and surplus couples in $\mathcal{P}$ $(b = u = 1, c = 0)$



In light of Lemma 4, finding an inverse demand function in $\mathcal{P}$ and an equilibrium that maximizes consumer surplus in a Cournot market with n firms and marginal cost c is equivalent to maximizing $CS(P_\pi, b/n)$ in $\pi \in (0, \bar{\pi})$.

Corollary 1 *Let* $\pi^* = \frac{\bar{\pi}}{e^n}$*, then* $CS(P_{\pi^*}, b/n) = \bar{\pi}(n-1) + \pi^* \geq CS(P, q)$ *for any* $P \in \mathcal{P}$*,* $q \in \mathcal{E}(P)$*.*

Proof.

$$\max_{\pi \in (0,\bar{\pi})} CS(P_\pi, b/n) = \max_{\pi \in [0,\bar{\pi}]} \int_0^b (P_\pi(x) - c)\, dx - n\pi =$$

$$= \max_{\pi \in [0,\bar{\pi}]} \left[\int_0^{\pi/(u-c)+(n-1)b/n} u \; dx + \int_{\pi/(u-c)+(n-1)b/n}^{b} \frac{\pi}{x - b(n-1)/n} + c \; dx - n\pi - bc \right] =$$

$$= \max_{\pi \in [0,\bar{\pi}]} \left[\pi \left(1 - n - \log\left(\frac{\pi}{\bar{\pi}}\right)\right) + \bar{\pi}(n-1) \right].$$

The objective function above is strictly concave and it is routine to verify that setting $\pi = \pi^*$ solves the stated maximization problem. ■

At the consumer-optimal equilibrium, individual firm's profit is π^* (while total industry profit is $n\pi^*$), consumer surplus is $\bar{\pi}(n-1) + \pi^*$, the price is $c + n\pi^*/b$ and the total demand is b. There is no efficiency loss arising from pricing above marginal cost.

Before concluding this section with an example, we observe that $\lim_{\bar{\pi} \to \infty} \pi^* = \infty$. Therefore, as we claimed without proof in footnote 5, if there is no bound on either maximal demand b or maximal valuation u, then no bounds can be placed on consumer and producer surplus either.

Example 1 *The price level for the equilibrium under the consumer optimal demand for* $u = b = 1$*,* $c = 0$ *is* e^{-n}*, while the price level for equilibrium under the standard linear inverse demand curve* $P(Q) = 1 - Q$ *is* $\frac{1}{n+1}$*. The price decreases much faster in the optimal demand curve as opposed to the linear demand, as the number of firms grows. For instance, at* $n = 4$ *the market price is* 0.20 *under the linear demand curve while it is approximately* 0.02 *for* P_{π^*}*. On the other hand, quantity is always* 1 *in the optimal demand while it is* $n/(n+1)$ *in the linear demand.*

4 Bounding Consumer, Producer Surplus and Dead-weight Loss

The aim of this section is to provide a complete characterization of all possible couples of consumer and producer surplus for any given level of the first-best surplus available given demand and production cost.

We define the first-best surplus under demand P, and denote it $FB(P)$, as usual omitting reference to cost, as the consumer surplus that is attained when the quantity supplied is such that exactly all consumers with valuation above the cost are served. More formally, for each demand $P \in \mathcal{P}$, we define

$$FB(P) = \int_0^{\hat{b}(P)} P(x)dx - \hat{b}(P)c,$$

where $\hat{b}(P) = \max\{q : P(q) \geq c\}$ and is well-defined because P is left-continuous and $u > c$. Hence, again omitting reference to c, for $s \in (0, n\bar{\pi}]$ define the set $\mathcal{P}_s = \{P \in \mathcal{P} : FB(P) = s\}$. Note that $FB(P) \leq b(u-c) = n\bar{\pi}$ for $P \in \mathcal{P}$. Hence $0 \leq s \leq n\bar{\pi}$.

Next, observe that, for each $\pi \in [0, \bar{\pi}]$,

$$FB(P_\pi) = CS(P_\pi, b/n) + n\Pi(P_\pi, b/n) = \pi \left[1 - \log\left(\frac{n\pi}{b(u-c)}\right)\right] + (n-1)\bar{\pi},$$

where the first equality follows because $\hat{b}(P_\pi) = b$ given that $P_\pi(b) = c + n\pi/b > c$. Then, observe that $FB(P_\pi)$ is continuous and strictly increasing in π and that $FB(P_0) = (n-1)\bar{\pi}$ while $FB(P_{\bar{\pi}}) = b(u-c) = n\bar{\pi}$.

Now, we define the profit level π_s which, as it will be shown later, is the minimum profit that is attainable when the demand is in $\mathcal{P}_s$.

Definition 1 *For $s \in (0, (n-1)\bar{\pi}]$ let $\pi_s = 0$, for $s \in ((n-1)\bar{\pi}, n\bar{\pi}]$ let π_s solve $s = FB(P_{\pi_s})$.*[8]

In fact, our first result in this section shows that the symmetric equilibrium b/n under $P_{\pi_s} = P_{(\pi_s, b/n)}$ minimizes individual profit among all symmetric Cournot equilibria for inverse demands in $\mathcal{P}_s$. Since for $s < (n-1)\bar{\pi}$, we have $\pi_s = 0$ and $P_0 \notin \mathcal{P}_s$, we abuse notation by defining, only for $s \in [0, (n-1)\bar{\pi}]$

$$P_{\pi_s}(Q) = \begin{cases} \frac{sn}{b(n-1)} + c & \text{if } Q \in [0, b(n-1)/n] \\ c & \text{if } Q \in (b(n-1)/n, b], \\ 0 & \text{if } Q > b. \end{cases}$$

This guarantees that $FB(P_{\pi_s}) = s$ and that b/n is an equilibrium while $\Pi(P_{\pi_s}, b/n) = \pi_s = 0$.

Lemma 5 *$\pi_s = \Pi(P_{\pi_s}, b/n) \leq \Pi(P, q)$ for any $P \in \mathcal{P}_s$ and $q \in \mathcal{E}(P)$*

Proof. The statement is obvious for $s \in (0, (n-1)\bar{\pi}]$ as in equilibrium $\pi_s = 0$. Therefore, we focus on $s \in ((n-1)\bar{\pi}, n\bar{\pi}]$. By way of contradiction, suppose an equilibrium q of some $P \in \mathcal{P}_s$ generates individual profit π' such that $\pi_s > \pi'$. Then, consider that each firm producing b/n is an equilibrium under $P_{\pi'}$. Furthermore, $FB(P_{\pi'}) \geq FB(P)$ given the definition of FB (note $b/n \geq q$ and $P_\pi(b) > c$) and the fact that $P_{\pi'} = P_{(\pi', b/n)} \geq P_{(\pi', q)} \geq P$ by Lemma 2 (first inequality) and Lemma 3 (second inequality). However, since $FB(P_\pi)$ is strictly increasing in π, the assumption that $\pi_s > \pi'$ implies that $s = FB(P_{\pi_s}) > FB(P_{\pi'}) \geq FB(P) = s$, a contradiction. ■

It is a corollary of the previous Lemma that the equilibrium b/n of P_{π_s} not only minimizes producer surplus, but also maximizes consumer surplus among all equilibria for demands in $\mathcal{P}_s$.

Corollary 2 *$s - n\pi_s = CS(P_{\pi_s}, b/n) \geq CS(P, q)$ for any $P \in \mathcal{P}_s$ and $q \in \mathcal{E}(P)$.*

Note that when $n = 1$ and $s > 0$ then $\pi_s > 0$. Hence, as expected, in the monopoly case there is no demand function that would let consumers extract the entire surplus s.

Proof. First, observe that the symmetric equilibrium of P_{π_s} generates consumer surplus equal to $s - n\pi_s$ as the quantity supplied in total is equal to b. Then, by way of contradiction, assume that there exists $P \in \mathcal{P}_s$ and a symmetric Cournot equilibrium of P that generate individual profit π

[8] If $b = u = 1$ and $c = 0$, $\pi_s = \frac{n-1-ns}{nW_{-1}(\frac{n-1-ns}{e})}$ for $s > \frac{n-1}{n}$. Here W_{-1} is the lower branch of the Lambert W function. While it cannot be expressed in terms of elementary functions it is defined by $W_{-1}(xe^x) = x$ for $x \leq -1$.

and larger consumer surplus. Because, $s - n\pi$ is an upper bound of the consumer surplus achievable under P, our contradiction assumption implies that $s - n\pi > s - n\pi_s$, or, equivalently, that $\pi < \pi_s$, which is in contradiction with the result of proposition 5. ■

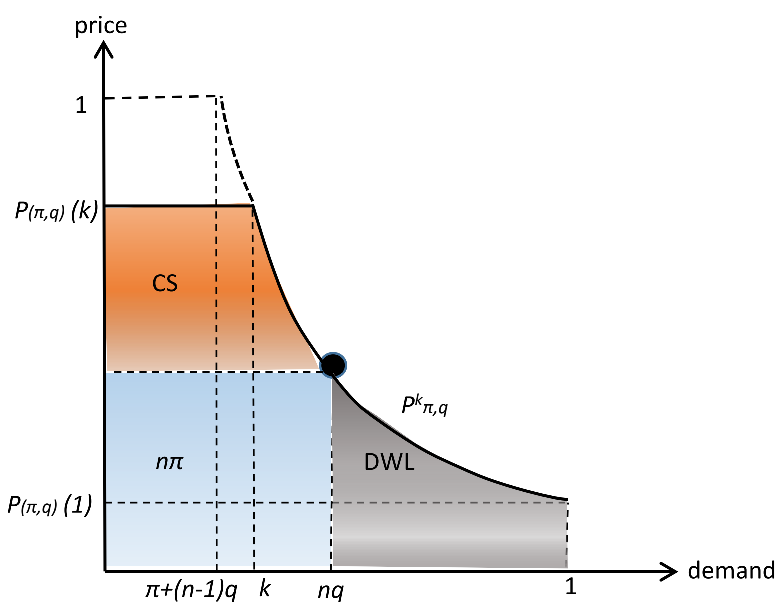
Consider consumer surplus and industry profit, $n\Pi$, in the positive quadrant of a Cartesian plane and identify the total surplus s as a linear constraints that bounds their sum. See the dotted line figure 5 for illustration. Call *achievable* set, the set of consumer surplus, producer surplus and dead-weight loss triples that are achievable in some equilibrium of some demand in $\mathcal{P}_s$. Clearly, any achievable $n\Pi$ is such that $n\pi_s \le n\Pi \le s$, while consumer surplus is bounded above by $s - n\Pi$ and below by zero. We now characterize the upper contour line of the achievable set. More precisely, we now show, that, for any s, any combination of producer and consumer surplus $(n\pi, s - \pi)$ for $\pi \in [\pi_s, s/n]$ can be achieved by some symmetric Cournot equilibrium of some demand in $\mathcal{P}_s$.

Before doing so, we introduce a class of demand functions that will be used to prove further results. For each $\pi \in (0, \bar{\pi}]$, $q \in [\underline{q}(\pi), b/n]$ and $k \in [\underline{q}(\pi) + (n-1)q, nq]$ let,

$$P^k_{\pi,q}(Q) = \begin{cases} P_{(\pi,q)}(k) & \text{if } Q \in [0, k] \\ P_{(\pi,q)}(Q) & \text{if } Q \in (k, b], \\ 0 & \text{if } Q > b. \end{cases}$$

This is a truncated version $P_{(\pi,q)}$. of Because $P^k_{\pi,q}(Q) = P_{(\pi,q)}(Q)$ for $Q \ge k$ and $k \le nq$, light of Lemma 1, it is immediate to see that $P^k_{\pi,q}$ has an equilibrium q that generates individual profit π. The next figure 3 depicts an example of a demand function $P^k_{\pi,q}$ for $b = u = 1$ and $c = 0$ and the division of the first-best surplus between consumer surplus (CS), profits $(n\pi)$ and deadweight loss (DWL) in the q-equilibrium.

Figure 3: Example of a demands $P^k_{\pi,q}$ for $b = u = 1$ and $c = 0$



Proposition 2 *For every $\pi \in [\pi_s, s/n]$, there exists $P \in \mathcal{P}_s$ and $q \in \mathcal{E}(P)$ such that $\Pi(P,q) = \pi$ and $CS(P,q) = s - n\pi$.*

The proof is constructive. For each surplus-profit combination identified in the statement, an inverse demand $P^k_{\pi,b/n} \in \mathcal{P}_s$ is explicitly constructed whose equilibrium b/n achieves them. Note that since the equilibrium is b/n, there is no deadweight loss and $CS(P^k_{\pi,b/n}) + \Pi(P^k_{\pi,b/n}) = s$.

Proof. We focus on showing that for each $s \in (0, n\bar{\pi}]$ and $\pi \in (\pi_s, s/n]$ there exists (unique) $k^0(s,\pi)$ such that $FB(P^{k^0}_{\pi,b/n}) = s$ and therefore $P^{k^0}_{\pi,b/n} \in \mathcal{P}_s$. The proof is concluded by noting that for equilibria b/n of $P^{k^0}_{\pi,b/n}$, consumer surplus is equal to $FB(P^{k^0}_{\pi,b/n}) - n\Pi(P^{k^0}_{\pi,b/n}, b/n) = s - n\pi$.

To show existence of $k^0(s,\pi)$, fix s and observe first that $P^{\pi/(u-c)+(n-1)b/n}_{\pi,b/n} = P_{(\pi,b/n)} = P_\pi$. Then note that because $\pi > \pi_s$ we must have $FB(P_\pi) > FB(P_{\pi_s}) = s$. Finally note that $FB(P^b_{\pi,b/n}) = n\pi \leq s$ because $\pi \leq s/n$. Since $FB(P^k_{\pi,b/n})$ is continuous and strictly decreasing in k in the specified parameter space, we reach our conclusion by the intermediate value theorem. ■

The result shows, roughly speaking, that there need not be an efficiency-equality trade-off when the number of firms is sufficiently large. In particular, as long as each firm is guaranteed π_s, *any* division of the first best surplus between consumers and producers is attained by some demand function without producing dead-weight loss, regardless of the number of firms in the market. Furthermore, note that when the number of firms is sufficiently large, $n > 1/(1-s)$, then $\pi_s = 0$ and so any surplus-profit combination that achieves the first best is attainable.

To complete our characterization, the next results determine, for each feasible (π, s), the *minimum* level of consumer surplus that can be achieved in the equilibrium of some demand in $\mathcal{P}_s$, and also establish that all intermediate levels of consumer surplus between the maximum and the minimum can be achieved. Before proceeding, we need to introduce further notation.

Definition 2 *Let $\pi^s \in [\pi_s, s/n]$ be the solution to $FB(P_{(\pi^s,\underline{q}(\pi^s))}) = s$.*[9]

To see that $\pi^s \geq \pi_s$ exists, note first that $FB(P_{(\pi_s,b/n)})) = s$ and therefore $FB(P_{(\pi_s,\underline{q})}) \leq s$, since by Lemma 2 we have $P_{(\pi_s,\underline{q})} \leq P_{(\pi_s,b/n)}$ and $P_{(\pi_s,b/n)} \geq c$. Second, note that $FB(P_{(s/n,\underline{q})}) \geq s$ as profit under $P_{(s/n,\underline{q})}$ is s/n and $FB(P_{(s/n,\underline{q})}) \geq ns/n = s$. Finally, observe that uniqueness follows by the strict monotonicity of $FB(P_{(\pi_s,\underline{q})})$ in π.[10]

To complete the analysis we now scan over profit levels in $[\pi_s, s/n]$. We consider two cases. First, we look at profit levels in $[\pi^s, s/n]$ (Proposition 3) and, second, we look at profit levels in $[\pi_s, \pi^s]$ (Proposition 4). Before proceeding we find useful to define the deadweight loss generated by an equilibrium q of some demand function P as $DWL(P,q) = FB(P,q) - CS(P,q) - \Pi(P,q)$ and to state some of its properties where the demand is of the type $P^k_{\pi,q}$.

Lemma 6 *$DWL(P^k_{\pi,q}, q)$ is continuous and strictly decreasing in q and independent of k.*

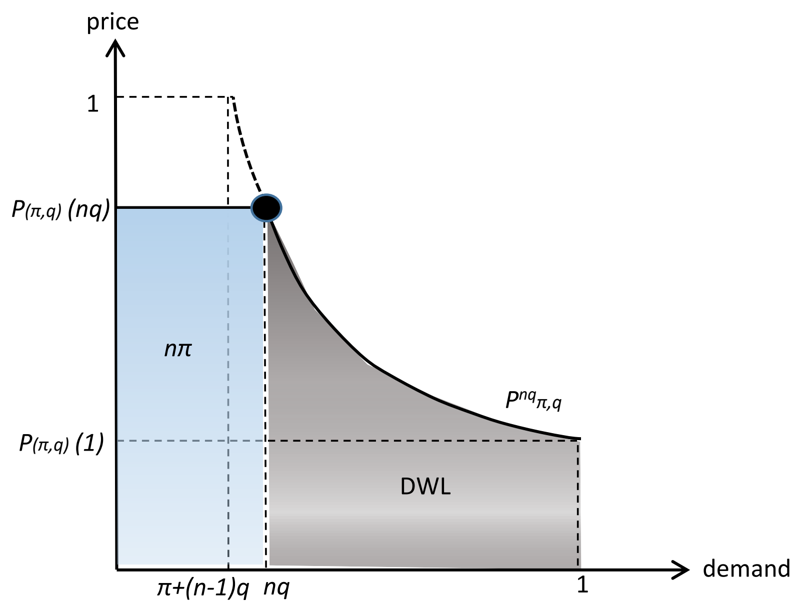
Proof. Since $k < nq$ for $q \in [\underline{q}, b/n]$ and $P^k_{\pi,q}(x) = P_{(\pi,q)}(x)$ for $x \geq nq \geq \pi/(u-c) + (n-1)q$, we have $DWL(P^k_{\pi,q}, q) = \int_{nq}^b (P^k_{\pi,q}(x) - c)dx = \int_{nq}^b (P_{\pi,q}(x) - c)dx = \pi\left[-\log(q) + \log(b - (n-1)q)\right]$. See also figure 3 for a geometric intuition of the last part of the statement. ■

[9] It is worth emphasizing that π^s may but need not be equal to π_s and that $\underline{q}(\pi)$ was defined just before Lemma 1.
[10] For $b = u = 1$ and $c = 0$ we have $\pi^s = \frac{s}{(n-1)s - W_{-1}(-se^{(n-1)s-n})}$ for $s \geq \frac{n-1}{n}$.

Proposition 3 *For every $s \in (0, n\bar{\pi}]$, $\pi \in [\pi^s, s/n]$, and $w \in [0, s - n\pi]$ there exists $P \in \mathcal{P}_s$ and $q \in \mathcal{E}(P)$ such that $\Pi(P, q) = \pi$ and $CS(P, q) = w$.*[11]

The proposition shows that for given s and profit level π above π^s, any feasible combination of consumer surplus and deadweight loss is achievable. The proof is, again, constructive. First, it is showed that zero consumer surplus can be achieved in equilibrium when $\pi \in [\pi^s, s/n]$ using some demand $P^{nq}_{\pi,q}$ for some q. Note that this demand induces zero consumer surplus as illustrated in figure 4, where the equilibrium quantity is indicated with the black dot, the profit is the blue shaded area while the dead-weight loss (DWL) is the gray shaded area. Then it is showed that intermediate levels of consumer surplus are also achievable by demand functions in $\mathcal{P}_s$.

Figure 4: Example of a demands $P^{nq}_{\pi,q}$ for $b = u = 1$ and $c = 0$



Proof. As a first step, for each $s \in (0, (u - c)b]$ and $\pi \in [\pi^s, s/n]$, we determine q such that $FB(P^{nq}_{\pi,q}) = s$. Clearly, if such q exists, then in the equilibrium q of this demand function consumer surplus is zero. In fact, it is immediate to verify that $CS(P^{nq}_{\pi,q}, q) = 0$ because $P^{nq}_{\pi,q}$ is constant between 0 and the equilibrium total quantity nq (see Figure 4).

Hence, we show that for each s and π in the range identified by the statement, there exists $\underline{q}(\pi) \leq q \leq b/n$ and demand $P^{nq}_{\pi,q}$ such that $FB(P^{nq}_{\pi,q}) = s$. To see this, observe that $P^{n\underline{q}}_{\pi,\underline{q}} = P_{(\pi,\underline{q})}$ as $\pi/(u - c) + (n - 1)\underline{q} = n\underline{q}$ since $\underline{q} = \pi/(u - c)$. Then note that $FB(P_{(\pi,\underline{q})}) \geq FB(P_{(\pi^s,\underline{q})}) = s$, where the inequality follows from Lemma 2 observing that $\pi \geq \pi^s$ and $\underline{q}(\pi) \geq \underline{q}(\pi^s)$, while the equality follows from the definition of π^s. Second, consider that $FB(P^b_{\pi,b/n}) = n\pi \leq s$ because $P^b_{\pi,b/n}(Q) = P_{(\pi,b/n)}(b) = n\pi/b + c$ for $Q \in [0, b]$ and $\pi \leq s/n$. The result that such a q exists, call it $q^0(\pi, s)$, follows from the intermediate value theorem by varying q in $FB(P^{nq}_{\pi,q})$ between $\underline{q}$ and b.

[11]This is a complete characterization as consumer surplus can only be in $[0, s - n\pi]$ if profit is π and first-best is s.

To conclude the proof, we now show that, for $\pi \in [\pi^s, s/n]$, all intermediate levels of consumer surplus between 0 and $s - n\pi$ can be achieved by some demand $P^k_{\pi,q} \in \mathcal{P}_s$. As a preliminary step observe two facts: (i) $DWL(P_{\pi,q^0(\pi,s)}, q^0(\pi, s)) = s - n\pi$ because by the first part of this proposition $DWL(P^{nq^0(\pi,s)}_{\pi,q^0(\pi,s)}, q^0(\pi, s)) = s - n$ but deadweight loss of P^k_q does not depend on k (see Lemma 6); (ii) $DWL(P_{\pi,b/n}, b/n) = 0$ by definition. Then, since $DWL(P_{\pi,q}, q)$ is continuous and decreasing in q (see 6), then for any $x \in [0, s - n\pi]$ there exists $\hat{q}$ such that $DWL(P_{\pi,\hat{q}}, \hat{q}) = x$. Then, since $DWL(P^k_{\pi,q}, q)$ does not depend on k (see Lemma 6) and since for any P we have $DWL(P, q) = FB(P) - CS(P, q) - n\Pi(P, q)$, we can establish our result if for all $q \in [q_0(\pi, s), b/n]$ we find k^q such that $FB(P^{k^q}_{\pi,q}, q) = s$. Existence of k^q (where dependence on s and π is omitted to simplify notation) is demonstrated in the remainder of the proof.

To see that for all $q \geq q^0(\pi, s)$ there exists k^q such that $nq \geq k^q \geq nq^0(\pi, s)$ and $FB(P^{k^q}_{\pi,q}) = s$ we can use again the intermediate value theorem after observing the following two things. First, $FB(P^{nq^0(\pi,s)}_{\pi,q^0(\pi,s)}) = s \leq FB(P^{nq^0(\pi,s)}_{\pi,q})$, for $q \geq q^0(\pi, s)$, because, for given k, by Lemma 2 $P^k_{\pi,q} \geq P^k_{\pi,q^0}$ for all $q \geq q^0(\pi, s)$. Second, that $FB(P^{nq}_{\pi,q}) \leq s$ for $q \geq q^0(\pi, s)$ because $FB(P^{nq}_{\pi,q}) = n\pi + DWL(P^{nq}_{\pi,q}) \leq n\pi + DWL(P^{nq^0(\pi,s)}_{\pi,q^0(\pi,s)}, q^0(\pi, s)) = FB(P^{nq^0(\pi,s)}_{\pi,q^0(\pi,s)}, q^0(\pi, s)) = s$, where the inequality follows since we have $DWL(P^{nq}_{\pi,q}) \leq DWL(P^{nq^0(\pi,s)}_{\pi,q^0(\pi,s)}, q^0(\pi, s))$ due to $DWL(P^k_{\pi,q})$ being independent of k and decreasing in q for given π (see Lemma 6). ■

Observe that if $n = 1$, then $\pi_s = \pi^s$ as $P_{(\pi_s,q)} = P_{(\pi_s,q')}$ for any $\pi \leq q \leq b/n$ and so $FB(P_{(\pi,\underline{q})}) = FB(P_{(\pi_s,b/n)}) = FB(P_{\pi_s})$. Hence, the above result completes the characterization of the $n = 1$ monopoly case. In this specific case, for any given s and $\pi \in [\pi_s, s/n]$, all surplus and deadweight loss combinations are achieved as different equilibria of the demand P_π. In particular, the highest consumer surplus $s - n\pi$ is achieved by the equilibrium b/n (see Condorelli and Szentes (2020)) and the lowest, equal to 0, by $\underline{q}(\pi) = \pi/(u - c)$ (see Kremer and Snyder (2018)), while all intermediate levels are achieved by equilibria where quantity ranges from $\underline{q}(\pi)$ to b/n. The achievable set is a triangle characterized by the following three (industry profit, consumer surplus) points: $(n\pi_s, s - n\pi_s)$, $(s, 0)$ and $(n\pi_s, 0)$. See the first column of figure 5 for an illustration.

To achieve the goal of this section, we still need to characterize possible levels of consumer surplus given $\pi \in (\pi_s, \pi^s]$. A last piece of notation is needed for the next and last result.

Definition 3 *For $\pi \in [\pi_s, \pi^s]$, let $\hat{q}(\pi, s)$ solve $FB(P_{(\pi,\hat{q}(\pi,s))}) = s$.*

Note that $FB(P_{(\pi^s,\underline{q})}) = s$. Hence, by Lemma 2, for $\pi \leq \pi^s$ we have $FB(P_{(\pi,\underline{q})}) \leq s$ given that also $\underline{q}(\pi) \leq \underline{q}(\pi^s)$. Then there must exist $\hat{q}(\pi, s)$ such that $FB(P_{(\pi,\hat{q}(\pi,s))}) = s$. This is the case because $FB(P_{(\pi,b/n)}) = FB(P_\pi) \geq FB(P_{\pi_s}) = s$ and $FB(P_{(\pi,q)})$ is continuous and increasing in q.

Proposition 4 *For $s \in (0, n\bar{\pi}]$ and $\pi \in [\pi_s, \pi^s]$, there exists $P \in \mathcal{P}_s$ and $q \in \mathcal{E}(P)$ such that $\Pi(P, q) = \pi$ and $CS(P, q) = w$ if and only if $w \in [CS(P_{(\pi,\hat{q}(\pi,s))}, \hat{q}(\pi, s)), s - n\pi]$.*

Proof. Assume by way of contradiction $P \in \mathcal{P}_s$ and an equilibrium q of P exists such that $\Pi(P, q) = \pi \in [\pi_s, \pi^s]$ and $CS(P, q) < CS(P_{(\pi,\hat{q}(\pi,s))}, \hat{q}(\pi, s))$.

There are three possibilities, either $q < \hat{q}(\pi, s)$ or $q = \hat{q}(\pi, s)$ or $q > \hat{q}(\pi, s)$. First, suppose $q < \hat{q}(\pi, s)$ and note that since we must have $P \leq P_{(\pi,q)} < P_{(\pi,\hat{q}(\pi,s))}$, where the first inequality follows by Lemma 3 and the second from Lemma 2. Recalling the definition of FB at the beginning of this section we must have $FB(P) \leq FB(P_{(\pi,q)}) < FB(P_{(\pi,\hat{q}(\pi,s))}) = s$, which contradicts $P \in \mathcal{P}_s$.

Second, if $q = \hat{q}(\pi, s)$ then $P_{(\pi,q)} = P_{(\pi,\hat{q}(\pi,s))}$. Hence, either $P = P_{(\pi,\hat{q}(\pi,s))}$ and therefore $CS(P, q) = CS(P_{(\pi,\hat{q}(\pi,s))}, \hat{q}(\pi, s))$, a contradiction, or $P < P_{(\pi,\hat{q}(\pi,s))}$ in an interval with positive mass which gives $FB(P) < FB(P_{(\pi,\hat{q}(\pi,s))}) = s$, also a contradiction.

Third, suppose that $q > \hat{q}(\pi, s)$. Observe Lemma 3 implies $P \leq P_{(\pi,q)}$ and therefore

$$DWL(P, q) = \int_{nq}^{\hat{b}(P)} (P(x)-c)dx \leq \int_{nq}^{\hat{b}(P)} (P_{(\pi,q)}(x)-c)dx + \int_{\hat{b}(P)}^{b} (P_{(\pi,q)}(x)-c)dx = DWL(P_{(\pi,q)}, q),$$

where the second inequality follows because $P \leq P_{(\pi,q)}$ and $P_{\pi,q}(Q) \geq c$ for $Q \in [\hat{b}(P), b]$. Then, recall from Lemma 6 that $DWL(P_{(\pi,x)}, x)$ is strictly decreasing in x and conclude that, because $q > \hat{q}(\pi, s)$, we must have

$$DWL(P, q) \leq DWL(P_{(\pi,q)}, q) \leq DWL(P_{(\pi,\hat{q}(\pi,s))}, \hat{q}(\pi, s)).$$

To find a contradiction with the hypothesis that equilibrium q of P generates lower consumer surplus it is then sufficient to observe that

$$CS(P_{(\pi,\hat{q}(\pi,s))}, \hat{q}(\pi, s)) = s - n\pi - DWL(P_{(\pi,\hat{q}(\pi,s))}, \hat{q}(\pi, s)) \leq s - n\pi - DWL(P, q) = CS(P, q).$$

The proof that intermediate levels of consumer surplus can be attained is analogous to the one presented in the previous proposition. In particular, $DWL(P_{(\pi,q)}, q)$ is continuous, strictly decreasing (by Lemma 6) and goes from $s - n\pi - CS(P_{(\pi,\hat{q}(\pi,s))}, \hat{q}(\pi, s))$ to 0 as x goes from $\hat{q}(\pi, s)$ to b/n. Hence, because $DWL(P^k_{\pi,q}) = DWL(P_{(\pi,q)}, q)$ for any k (also by Lemma 6) and $CS(P^k_{\pi,q}, q) = s - n\pi - DWL(P^k_{\pi,q}, q)$ we can conclude the proof if, for all $\pi \in [\pi_s, \pi^s]$, we can find k^q such that $FB(P^k_{\pi,q}) = s$ for all $q \in [\hat{q}(\pi, s), b/n]$. Details are omitted. ■

To summarize, figure 5 fixes the maximum valuation and the maximum demand to one and the cost to zero and illustrates the achievable couples of industry profit and consumer surplus for various levels of first-best surplus and number of firms in the market. Note that, as expected, $\hat{q}(\pi^s, s) = \pi^s/(u - c)$ and therefore $CS(P_{(\pi^s,\hat{q}(\pi^s,s))}, \hat{q}(\pi^s, s)) = 0$. On the other hand $\hat{q}(\pi_s, s) = b/n$ and therefore $CS(P_{(\pi_s,\hat{q}(\pi_s,s))}, \hat{q}(\pi_s, s)) = CS(P_{\pi_s}, b/n) = s - n\pi_s$. That is, as long as $\pi^s > \pi_s$ (see columns 2 and 3 of figure 5), there is a unique consumer surplus level achievable at the minimal profit π_s and the equilibrium is efficient (see columns 2 and 3 of figure 5).

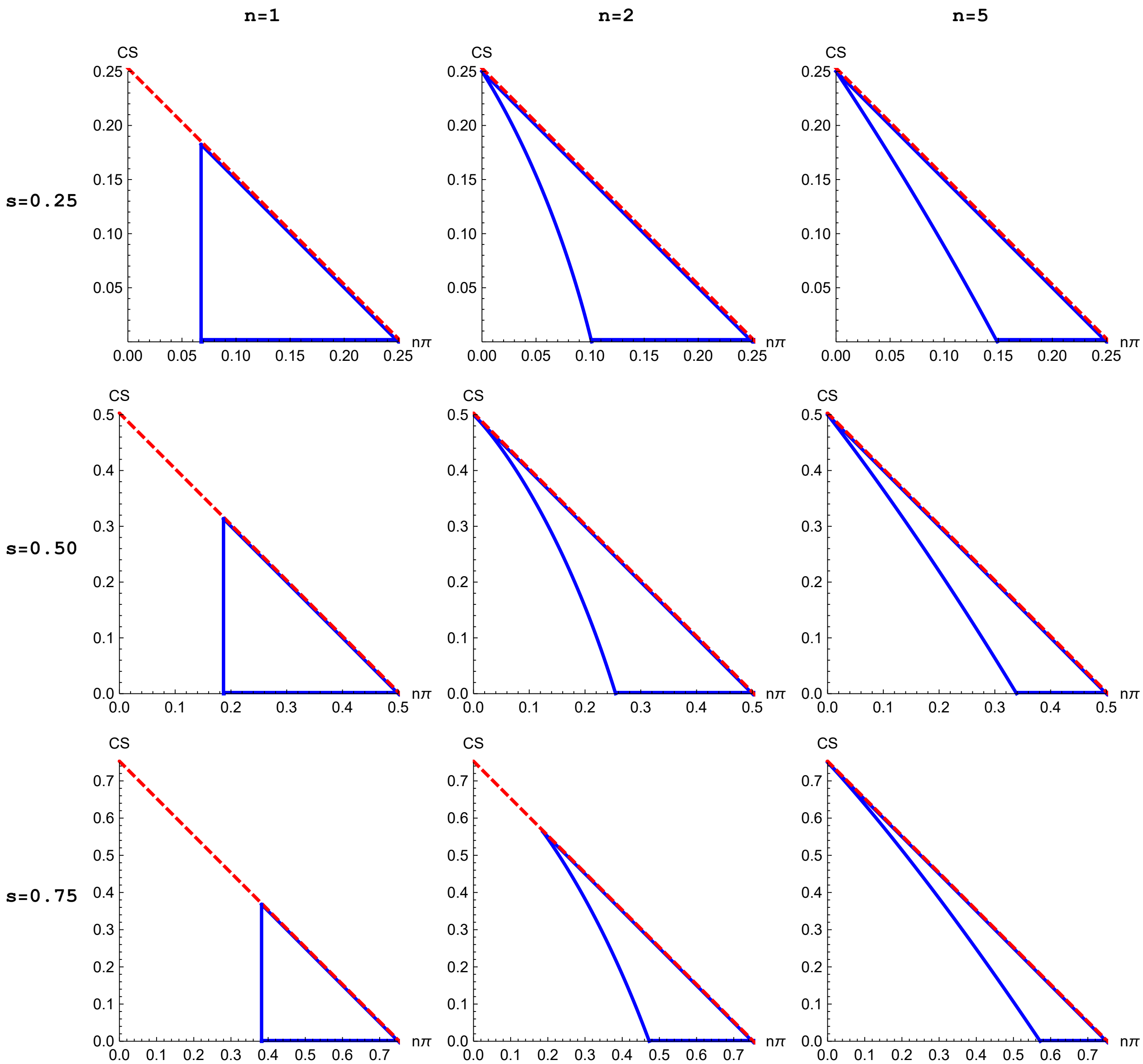
As it can be inferred from figure 5, as the number of firms grows,then $\pi_s \to 0$ and $\pi^s \to s/n$ and, more importantly, the minimum level of achievable consumer surplus given π increases toward $s - n\pi$. This visual insight is confirmed by the following results, which shows that as the number of firms gets large only the Pareto frontier remains in the achievable set.

Corollary 3

$$\lim_{n\to\infty} n\pi_s = 0, \quad \lim_{n\to\infty} n\pi^s = s \textit{ and } \lim_{n\to\infty} CS(F_{(\pi,\hat{q}(\pi,s))}, \hat{q}(\pi, s)) = s - n\pi \textit{ for all } 0 \leq \pi \leq s/n.$$

This confirms conventional wisdom in the Cournot model that inefficiency (but not necessarily profits) disappear as competition increases.

Figure 5: Achievable $(n\Pi, CS)$ couples in $\mathcal{P}_s$ within blue lines, $b = u = 1$ and $c = 0$



Appendix A

We show that if there exists an asymmetric Cournot equilibrium, then there also exists a symmetric equilibrium where the total amount produced, and therefore industry profit, is the same. We illustrate this for the case of two firms, but the argument extends easily to multiple firms.

Suppose there exists equilibrium (q_1, q_2) with $Q = q_1 + q_2$. The following inequalities hold:

$$q_1[P(Q) - c] \geq q'[P(q' + q_2) - c] \quad \forall q'$$
$$q_2[P(Q) - c] \geq q''[P(q'' + q_1) - c] \quad \forall q''.$$

Now substitute $q' = Q/2 - q_2 + \hat{q}$ and $q'' = Q/2 - q_1 + \hat{\hat{q}}$. We can rewrite the above inequalities as

$$q_1[P(Q) - c] \geq (Q/2 - q_2 + \hat{q})[P(Q/2 + \hat{q}) - c] \quad \forall \hat{q}$$
$$q_2[P(Q) - c] \geq (Q/2 - q_1 + \hat{\hat{q}})[P(Q/2 + \hat{\hat{q}}) - c] \quad \forall \hat{\hat{q}}.$$

Summing up the two sets of inequalities we know the following must hold

$$Q[P(Q) - c] \geq (Q/2 - q_2 + \hat{q})[P(Q/2 + \hat{q}) - c] + (Q/2 - q_1 + \hat{\hat{q}})[P(Q/2 + \hat{\hat{q}}) - c] \quad \forall \hat{q}, \hat{\hat{q}}.$$

Since the above must hold for all $\hat{q}, \hat{\hat{q}}$, fix $\hat{q} = \hat{\hat{q}}$. The set of inequalities below must also hold

$$Q[P(Q) - c] \geq (Q/2 - q_2 + \hat{q} + Q/2 - q_1 + \hat{q})[P(Q/2 + \hat{q}) - c] \quad \forall \hat{q}.$$

Finally, noting that $q_1 + q_2 = Q$ and dividing by two we get

$$Q/2[P(Q) - c] \geq \hat{q}[P(Q/2 + \hat{q}) - c] \quad \forall \hat{q}$$

which implies that there exists a symmetric equilibrium where both firms produce quantity $Q/2$.

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